## NEAR EAST UNIVERSITY

## Faculty of Engineering

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## ROBOT FOLLOWING A PLANE IN SPACE (MATLAB PROGRAM)

Graduation Project<br>EE - 400

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#### Abstract

A 3-DOF Stewart Platform has upper and lower platforms with equilateral triangular shape. In the present project this platform is augmented by locating an extendible limb at the mass center of the upper platform along the direction of the unit normal vector. Previously obtained results for the kinematics of 3-DOF Stewart Platform is extended to study the kinematics of this augmented mechanism.

In particular the problem that is studied is to determine the motion of all the limbs when the tip of the extendible limb is constrained to move in space from one point to another through a set of parallel adjacent lines, which create a plane in space as a result of the motion, with a suitable velocity and acceleration profiles. If the positions of the four points in space, and also the $z$ component of position vector of the mass center for the upper platform are given then the rest of the information about the inverse kinematics of the mechanism is supplied by a written Matlab program. Through this program one can easily calculate the lengths, velocity and acceleration of all the limps.


## 1. OVERVIEW TO ROBOTICS

### 1.1 Introduction

Robotics is a very broad topic. No two sources will give the same definition of the term. In Japan, for example, any mechanical device that operates in a factory and performs a single, simple task, time and time again, is considered to be a robot, whereas in America, such devices are considered "automation systems".

The field of robotics is currently undergoing a change. While in the past, robots where predominately used in factories for purposes such as manufacturing and transportation, a new generation of "service robots" has recently begun to emerge. Service robots cooperate with people, and assist them in their everyday tasks. A landmark service robot is Helpmate Robotics's Helpmate robot, which has already been deployed at numerous hospitals worldwide(King \& Weiman1990). Helpmate, however, does not interact with people other than by avoiding them. In the near future, similar robots are expected to appear in various branches of entertainment, recreation, healthcare, nursing, and others, and we expect them to interact directly with people.


Figure 1.1

The wasubot- a humanoied miusican at Tsukuba Expo' 85 it can sight read music recognize a limited set of voice commands.

### 1.2 History of Robotics and Robots

A brief review of robot development is important because it puts the current machines and interest in them into a historical perspective. The following list of dates highlights the growth of automated machines, which led to the development of the industrial robots currently available.

## 1801

Joseph Jacquard invents a textile machine, which is operated by punch cards. The machine is called a programmable loom and goes into mass production.

1830
American Christopher Spencer designs a cam-operated lathe.
1892
In the United States, Seward Babbitt designs a motorized crane with gripper to remove ingots from a furnace.

1921
The first reference to the word robot appears in a play opening in London. The play, written by Czechoslovakian Karel Capek, introduces the word robot from the Czech robota, which means a serf or one in subservient labor. From this beginning the concept of a robot takes hold.

1938
Americans Willard Pollard and Harold Roselund design a programmable paintspraying mechanism for the DeVilbiss Company.

## 1946

George Devol patents a general-purpose playback device for controlling machines. The device uses a magnetic process recorder. In the same year the computer emerges for the first time. American scientists J. Presper Eckert and John Mauchly build the first large electronic computer called the Eniac at the University of Pennsylvania. A second computer, the first general-purpose digital computer, dubbed Whirlwind, solves its first problem at M.I.T.
1948
Norbert Wiener, a professor at M.I.T., publishes Cybernetics, a book that describes the concept of communications and control in electronic, mechanical, and biological systems.

## Chapter One

1951
A teleoperator-equipped articulated arm is designed by Raymond Goertz for the Atomic Energy Commission.

1954
The first programmable robot is designed by George Devol, who coins the term Universal Automation. He later shortens this to Unimation, which becomes the name of the first robot company.
1959
Planet Corporation markets the first commercially available robot.
1960
Condec Corporation purchases Unimation and development of Unimate Robot Systems begins. American Machine and Foundry, later known as AMF Corporation, markets a robot, called the Versatran, designed by Harry Johnson and Veljko Milenkovic.

## 1962

General Motors installs the first industrial robot on a production line. The robot selected is a Unimate.

1964
Artificial intelligence research laboratories are opened at M.IT., Stanford Research Institute (SRI), Stanford University, and the University of Edinburgh.

## 1968

RS Mosher at general electric research built a quadruped walking machine (shone in figure 1.2), this walking truck was over three meter long, weighted 1400 kg and was powered by 68 kW petrol motor.

1970
At Stanford University a robot arm is developed which becomes a standard for research projects. The arm is electrically powered and becomes known as the Stanford Arm.

1973
The first commercially available minicomputer-controlled industrial robot is developed by Richard Hohn for Cincinnati Milacron Corporation. The robot is called
the T3, The Tomorrow Tool.

## 1974

Professor Scheinman, the developer of the Stanford Arm, forms Vicarm Inc. to market a version of the arm for industrial applications. The new arm is controlled by a minicomputer.

## 1976

Robot arms are used on Viking 1 and 2 space probes. Vicarm Inc. incorporates a microcomputer into the Vicarm design.
1977
ASEA, a European robot company, offers two sizes of electric powered industrial robots. Both robots use a microcomputer controller for programming and operation. In the same year Unimation purchases Vicarm Inc.
1978
The Puma (Programmable Universal Machine for Assembly) robot is developed by Unimation from Vicarm techniques and with support from General Motors. 1980

The robot industry starts its rapid growth, with a new robot or company entering the market every month.
When, in 1954 George C. Devol filed a U.S. patent for a programmable method for transferring articles between different parts of a factory, he wrote: "The present invention makes available for the first time a more or less general purpose machine that has universal application to a vast diversity of applications where cyclic control is desired".


Figure 1.2 Mosher's walking truck: a research prototype of a four-legged quadruped built at general electric research and development center in 1968.

## Robots Today

Today's mobile robots are beginning to achieve the dreams of early researchers. While they're not yet performing on the level of college students in human tasks like speaking and manipulating objects, robots are useful. We have learned that robots' roles can be more important in assisting and extending the reach and vision of humans, rather than trying to duplicate or replace them.

We have learned that robots can outperform people on highly specialized, repetitive tasks that bore human beings. Robots can lift, pry or inspect objects and places that are difficult, distant or hazardous for humans. Teams of robots can gather data from far a field and combine it for analysis. We and our computers can use the results to monitor the environment, investigate unknown places, and gain more perspectives on phenomena that were far beyond our abilities to understand before. Together, robots and human beings are venturing into places and knowledge spaces where no human can travel alone. Truly, we are beginning to turn science fiction into fact.

### 2.3 Parts of a Robot

A robot is basically made up of

- Base
- Brain
- Sensors
- Actuators.

The base of the robot can be stationary, (fixed) or mobile. Robots used in manufacture are examples of fixed robots. They can not move their base away from the area they are working. Mobile bases are typically platforms with wheels or tracks attached. Instead of wheels or tracks, some robots employ legs in order to move about.

The brain of a robot is a computer. However, computer is by design very sensitive to movement, vibration and dust. Also computers have a finite minimum size, which limit their uses. Fixed robots are not generally limited in the size of the computer they can use, as the "brain" can be placed in an unused corner and then linked to the robot by long cables. On the other hand, mobile robots are limited in the size of the computer they can use, as the "brains" are transported on the platform (there are a few exceptions). The constraints placed on mobile "brains" are size and weight, the larger in size, the larger in weight. However small size also generally means little processing power, and large computers are as powerful as "dumb worms" as it is, so the processing power of most mobile robots is severely limited.

Sensors used by robots vary between robots depending on their needs and uses. Each robot needs certain information in order to work properly. The actual sensors take many shapes and forms.

## Generally the sensors used by robots are:

- Visual sensors
- Inertial, Acceleration and Heading sensors
- Range finding devices
- Force/torque, accelerometers, tactile sensors
- Sonar sensors
- Pan/tilt mechanisms
- Measuring linear motion
- Interfacing sensors

Actuators used in robotics are almost always combinations of different electromechanical devices. Sometimes robots use hydraulics, particularly in the car building industry. The electro-mechanical devices range from 'muscle-wires' to inexpensive RC-servo and motors. There are several types of motors available including:

- Synchronous
- Stepper
- AC servo
- Brushless DC servo
- Brushed DC servo

These are then connected to cable, gears, axles, pulleys and alike to give the robot movement, and the ability to interact with it's environment.

### 1.4 Applications of Robots

Robotics are used in many diverse applications, from turtle robots in school classrooms, to welding robots in car manufacturing factories, to the teleoperated arm on the space shuttle. Each application has its own set of problems, and consequently, its own set of robotic requirements. Many new industries have arisen as a result, and we are likely to see more in the future, as new concepts are developed in research laboratories. While many of the consumer-oriented robots are of more novelty value than practical use, the introduction of robots into factories has already had a considerable impact on manufacturing processes.

American manufacturing sector has decreased, while the number of people employed by small, high-technology companies has increased by a greater amount.
Robotics technology can contribute to employment in small factories, because of the increased flexibility it gives to small-volume, batch-oriented manufacturing. This flexibility allows the company to manufacture a wider range of products with less
equipment.

### 1.4.1 Industry

Robots are used in a wide range of industrial applications. The earliest applications were in materials handling, spot welding, and spray painting (see figure 1.3). Robots were initially applied to jobs that were hot, heavy, and hazardous such as die-casting, forging, and spot welding. One problem in these industries is finding people who are willing to work with the poor equipment and under the poor conditions, which exist in some factories. For example, in die casting and forging a lot of existing plant is so old that it will have to be replaced before robots can be used.


Figure 1.3 spray painting whit robots

Robots are used in many other applications, for example: grinding, tending machine tools, molding in the plastics industry, applying sealants to motor car windscreens, and picking items up off conveyors and packing them on to fork lift pallets. These applications, and the problems involved, are reported innumerous conference proceedings. Innovative new applications include laser-beam welding and water-jet cutting. In the following sections, three industrial applications that are in various stages of research are examined.

### 1.4.1.1 Arc welding

Arc welding, which is potentially a large application for robots, places high demands on the technology. Unlike spot welding, where the weld has to be placed at a fixed spot, an arc weld has to be placed along a joint between two pieces of metal. Commercial arc welding systems (see Figure 1.4) rely on people to accurately fix the parts to be welded, and then the robot goes through a programmed welding sequence


Figure 1.4 Arc welding whit robots

The only advantage this has over manual welding is the consistent quality of the weld. The human operator is now left with the tedious job of fixing. Having rotating fixture tables speeds up productivity, so that the operator can be fixing one set of parts while the robot is welding another

For all types of joints, a minimum requirement for arc weld sensors is that they are capable of indicating the proper tracking position. Further requirements are that the weld is placed accurately, end's of the required size and shape. To achieve these conditions, the robot must hold the electrode at the correct orientation to the seam, at the correct distance from the seam, and move at a constant velocity so that a constant amount of material flows into the joint. These problems are more complex on three-
dimensional objects than on flat plates, and often require geometric modeling to plan the robot motion.

### 1.4.1.2 Assembly

One long-term goal of manufacturing technology is the totally automated factory, where a design is conceived at a computer graphics terminal and no further human intervention is required to manufacture the article. Manufacturing in a totally automated environment would include the following steps:

- Product conception
- High level specification
- Product design
(All done interactively by human designers), materials ordering, generation of machine tool commands, generation of parts flow strategies through the factory, control of parts transfer and machine tools, automatic assembly and inspection (all done automatically using robotics technology).
Today, there are two examples of highly automated factories where very few people are employed. One is the processing of film by photographic companies, and the other is the machine tool center operated by Fujitsu Fanuc. Roth is examples of successful hard automation with very few robots. The automation of photographic processing is possible because it is a high-volume, single-task process. The machine center consists of numerical controlled machines, combined with robots and conveyor belts for tool changing and parts transfer.

A major potential application for robots is the automation of assembly, this is currently a very labor intensive process and much more difficult than it appears at first sight.

### 1.4.1.3 Machining metals

Mass production was made possible by the ability to repeatedly machine parts to the designer's specification. There are eight basic ways of machining metals (see figure 1.5) as listed bellow:

## - Drilling

- Milling


Figure1．5 Basic ways of machining metals（a）Drilling（b）Grinding（c）Boring
（d）Planning（e）Milling（f）Turning（g）Shaping（h）Slotting

## 1．4．2 Laboratories

Robots are finding an increasing number of applications in laboratories．They are good at carrying out repetitive tasks，such as placing test tubes into measuring instruments，relieving the laboratory technician of much tedious work．At this stage of their development，robots are used to perform manual procedures automatically．A typical sample－preparation system（Figure 1．21）consists of a robot and laboratory stations such as balancers，dispensers，centrifuges，and test tube racks．Samples are moved from laboratory station to laboratory station by the robot under the control of user－programmed procedures．
Manufacturers of these systems claim they have three advantages over manual operation：increased productivity，improved quality control，and reduced exposure of humans to harmful chemicals

### 1.4.3 Kinestatic manipulators

Robotics technology found its first application in the nuclear industry with the development of teleoperators to handle radioactive material. More recently robots have been used for remote welding and pipe inspection in high-radiation areas. The accident at the Three Mile Island nuclear power plant in Pennsylvania in 1979 has spurred the development and application of robots to the nuclear industry.

Rover (or RRV), a Remote Reconnaissance Vehicle developed at Carnegie-Mellon University (Figure 1.6), was used to inspect the basement of the reactor containment buildings, to obtain concrete core samples from the walls, and to remove the top layer of concrete from floors in parts of the reactor building using a pneumatically powered scabbing machine.


Figure 1.6 Remote Reconnaissance Vehicles ready to enter the basement of Three
Mile Island reactor two equipped will core boring-drills by reds.

### 1.4.4 Agriculture

One of the most successful projects so far has been the development of a sheepshearing robot in Australia (see Figure 1.7). The trajectory of the cutting shears over the body of the sheep is planned using a geometric model of a sheep. To compensate for variations in the size of a sheep from the model, and for its changing shape as it breathes, data from sensors mounted on the cutter is used to modify the trajectory in real-time, as the wool is removed. Over 200 sheep have been shorn (though not completely) with fewer injuries to the sheep than occur with human shearers.
Other experimental applications of robots in agriculture include transplanting of seedlings, pruning grapevines in France, and picking apples. All these systems are in experimental stages, but they have each demonstrated their potential.


Figure 1.7 Australian sheep shearing robot.

### 1.4.5 Space

Space exploration poses special problems for robots; they cannot yet be used to replace people. Teleoperators, which combine human intelligence with mechanical manipulation, require a person in the loop. Future applications of robots in space include planetary rovers with manipulator arms, free-flying general-purpose robots
within space stations, satellite maintenance robots, manipulator arms for space manufacturing, and construction robots for the construction of space stations and space ships.

In November 1970, Lunokhod I, the Russian unmanned lunar rover, landed on the Moon. In July 1976, Viking I landed on Mars. It carried a robot arm, which was used to dig a small trench in the Martian soil and to scoop up soil samples for analysis. After a soil sample was screened, a miniature conveyor carried it to a highly sophisticated biological laboratory. In March 1982 the teleoperator arm (RMS: Remote Manipulatory System) on board the space shuttle Columbia was first used to move scientific payloads out of the hold into space.


Figure 1.8 Marsokhod used in space applications by NASA

### 1.4.6 Submersible vehicles

Two events during the summer (in the northern hemisphere) of 1985 increased the public's awareness of undersea applications of robotics. In the first - the crash of an Air India jumbo jet into the Atlantic Ocean off the coast of Ireland - a remotely guided submersible robot, normally used for cable laying, was used to find and recover the
black boxes from the jetliner. The second was the discovery of the Titanic at the bottom of a canyon, where it had settled after hitting an iceberg in 1912, four kilometers below the surface. A remotely controlled submersible vehicle was used to find, explore, and film the wreck.

### 1.4.7 Education

Robots are appearing in the classroom in three distinct forms. First, educational programs using simulation of robot control as a teaching medium. The programming language Karel the Robot, a subset of Pascal, is used as an introductory programming language. Karel has the control structures and syntax of Pascal, but a robot has replaced variables, objects for the robot to manipulate, and a grid-based environment.

The second, and currently most common, use of robots in education is the use of turtle robots (Figure 1.9) in conjunction with the LOGO language to teach computer awareness. LOGO was intended to create an environment where learning mathematics would be natural and fun. The turtle is an object to think with and to be used to draw geometric patterns. While LOGO is used for this purpose, the language has been so well human engineered, from an educational point of view, that it provides a natural environment for a child's first excursions into programming.

The third use is in the robotics classroom. A range of low cost manipulators, mobile robots, and complete systems has been developed for use in robotics educational laboratories. Owing to their low cost, many of these systems suffer from poor mechanical reliability, low accuracy, non-existent sensors, and inadequate software.


Figure1.9 Tasman turtle robot use in school.

### 1.4.8 Assisting the handicapped

Potential robotic aids for the disabled range from automatic wheelchairs, which carry the occupant around a hospital in response to voice commands to robots, which feed severely handicapped people. The overriding goal of this research is to make machines, which restore some of the autonomy the user lost when he or she lost the use of his bodily functions.

### 1.5 Future directions

The writings of some researchers look more like science fiction than the reality that is to be found in research laboratories. Most of the futuristic predictions of androids are unlikely to be achieved in the near future. The complexity of the human brain is such that monitoring many of its activities is beyond current technology. Researchers investigate small sections of the brain in detail to try and pinpoint which areas perform. which functions. At present, we don't understand the sensory process of hearing and interpreting speech, which involves the analysis of a single analogue signal. Our knowledge of the brain is similar to that of a person who, when looking at a printed circuit board, can draw a map of the connections between the integrated circuits, but has little understanding of what operations are performed inside the integrated circuits and even
less idea of how the circuits interact to perform their overall function.

At a more practical level, one of the significant problems in robotics is communication between computers, sensors, and robots. Lack of common communications standards (both hardware networks and software protocols) creates real problems when integrating robots and sensors into a work cell. To overcome these problems, some manufacturers have adopted the Manufacturing Automation Protocol (MAP) standard.

### 1.6 Robot classification

The power of the software in the controller determines the utility and flexibility of the robot within the constraints of mechanical design and sensor availability. Robots have been classified according to:

- Their generation
- Their intelligence level
- Their level of control
- Their programming language level.

These classifications overlap, but they all reflect the power of the software in the controller, in particular, the sophistication of sensor interaction.

### 1.6.1 Robot classification based on their generation

The generation of a robot is determined by the historical order of developments in robotics. Five generations are normally assigned to industrial robots. The third generation is used in industry, the fourth is being developed in research laboratories, and the fifth generation is largely a dream. The generations are:

1. Playback robots, which play back a sequence of recorded instructions, such as in spray painting and spot welding. These robots often have open loop control - for example, pick-and-place robots which use mechanical stops to limit travel.
2. Sensor-controlled robots, which have closed-loop control of manipulator motions, and decision making based upon sensor inputs.
3. Vision-controlled robots where the robot can manipulate an object using information from a vision system.
4. Adaptively controlled robots where the robot can automatically reprogrammed its actions on the basis of sensor inputs.
5. Artificially intelligent robots where the robot uses the techniques of artificial intelligence to make its own decisions and solve problems.

### 1.6.2 Robot classification based on their intelligence level

The Japanese robot association (ITRA) has classified robots into six classes on the basis of their level of intelligence

1. Manual handling devices controlled by a person.
2. Fixed sequence robots. 3. Variable sequence robots where an operator can modify the sequence easily.
3. Playback robots where the human operator leads the robot through the task.
S. Numerically controlled robots where the operator supplies a movement program, rather than teaching it the task manually.
4. Intelligent robots, which can understand and interact with changes in the environment.

### 1.6.3 Robot classification based on their level of control

The programs in a robot controller can be grouped according to the level of control they perform:
I. Artificial intelligence level, where the program will accept a command such as, Pick up the bearing, and decompose it into a sequence of lower level commands based on a strategic model of the task.
2. Control mode level where the motions of the system are modeled, including the dynamic interactions between the different mechanisms. Trajectories planned, and grasp points selected. From this model a control strategy is formulated, and control commands issued to the next lower level.
3. Servo system level where actuators control the mechanism parameters using feedback of internal sensory data, and paths are modified on the basis of external sensory data. Also failure detection and correction mechanisms are implemented at this level.

### 1.6.3 Robot classification based on their programming language level.

The final classification we will consider is programming language level. The key to the effective application of robots to a wide variety of tasks is the development of highlevel robot languages. Many robot-programming systems exist, although the most advanced are currently available only in research laboratories. Existing robot programming systems fall into three broad categories.

1. Guiding systems, in which the user leads the robot through the motions to be performed.
2. Robot-level programming systems, in which the user writes a computer program to specify motion and sensing.
3. Task-level programming systems, in which the user specifies operations by their actions on the objects the robot is to manipulate.

## 2. STEWART PLATFORM

### 2.1 Introduction

To achieve higher accuracy during the machining process, high rigidity and stability of the machine's structure are necessary. The majority of today's industrial muti-axis machines consist of serially connected axes. A serial mechanism, one whose links and joints alternate with one another in a long chain, is inherently non-rigid due to its cantilevered structure. Therefore, alternative machine-tool designs have been investigated based on parallel structures where the actuators form closed kinematic chains, enhancing rigidity, accuracy, and force/torque capacity. The Stewart Platform is an example of a parallel architecture offering excellent stability and high stiffness. The structure, although originally developed by Gough for testing tires, is widely regarded as the Stewart Platform since he popularized it through its use as a flight simulator.

### 2.2 Construction

Stewart Platform has upper and lower platforms with equilateral triangular shape as shown in figure 2.1. There are many different type of Stewart Platforms studied in the literature where shape of the base and upper platform are either assumed to be hexagonal or triangular. Also some of the platforms have either six legs or three legs. The connections of the legs are provided by either through ball joints or universal (cardan) joints.


Figure 2.1: An example of 6-DOF Stewart platform

### 2.3 Application

Stewart Platform has attracted the attentions of many researchers and gained an increased popularity within the industrial society nowadays. This attention brought such applications as machine tool technology, crane technology, underwater research, air-tosea rescue, flight simulation and satellite dish positioning ... etc.

To clarify this some examples of Stewart Platform applications are listed bellow:

The Stewart Platform was first designed and built to test tires at the Performance and Stressing Department as shown in figure 2.2


Figure2.2 Testing tires machine

Over a decade before Stewart published his article in 1965, Stewart has proposed the use of a platform for flight simulators, which can be designed as shown in figure 2.3


Figure 2.3 Pictorial view of Flight Simulator

The Inverted Stewart Platform can be suspended from a single floating vessel or it can be suspended from three strategically located ships to place an underwater drilling platform (as shown in figure 2.4), lay pipelines, or it can be used for underwater salvage.


Figure 2.4 Impression of possible design of underwater oil drilling machine

The Inverted Stewart Platform can be used to replace conventional crane technology The Inverted Stewart Platform crane provides the crane operator with greater control of the crane hoist mechanism. The national Institute of Standards and Technology has developed a crane, known as ROBOCRANE (see figure 2.5), utilizing the Stewart Platform technology.


Figure 2.5 Robocrane used in machining metal

An Inverted Stewart Platform can replace the conventional single cable hoisting technology currently being used on helicopters for use as an air crane or in air-to-sea rescue. And also the Stewart Platform manipulator could be used for positioning satellite dishes on land or on a pitching and rolling ship at sea.

Stewart Platform is used in complicated heart surgery to assist the surgeon and ensure accuracy in dealing with critical position in human body.


Figure 2.6 Stewart Platform used as surgeon assistant.

An Inverted Stewart Platform could be modified for use as a Lunar Rover. Equipped with the necessary wheels it could be used to lift heavy objects or it could serve as drilling platform.

The Stewart Platform concept is currently being applied in machine tool technology in a machine manufactured by Ingersoll known as a Hexapod. The Hexapod is a unique milling machine, which can be manipulated through six degrees of freedom. Conventional milling machines can only be operated in 3 to 4 axes ( $x, y, z$, and rotational).


Figure 2.7 Impression of possible design of universal mill

### 2.4 Forward and inverse kinematics

Forward and inverse kinematics are terms to describe mapping from the space of inputs to the space of outputs of a non-dynamic mechanical system as shown in figure 2.8 Forward kinematics involves solving the forward transformation equation to find the location of the hand in terms of the angles and displacements between the links, while inverse kinematics involves solving the inverse transformation equation to find the relationships between the links of the manipulator from the location of the hand in space.

Figure 2.9 shows a typical application of the forward and inverse kinematics algorithm. A trajectory for a device such a milling machine or welding system is to be followed. The controller, using the inverse kinematics algorithm (and calculating desired changes in position), computes the control signal given to the actuator. At the output of the platform we measure the lengths of the links controlled by the actuators. Then the forward kinematics algorithm transforms those lengths into (platform) position. The position signal is compared to the assigned position and added to the controller's input.

Because of the Stewart platform closed kinematics chain, the rigidity of the end effectors, and the accuracy of the links length servos, a reasonably accurate feedforward assignment is possible. Due to the complexity of older forward kinematics algorithms, most Stewart platforms are controlled without using feedback. The proposed algorithm will allow the calculation of the forward kinematics in real time. This open the opportunity to increase the accuracy of the control algorithms and broaden the current uses of the platform.

figure 2.8 Forward and Inverse Kinematics


Figure 2.9 An example of using transformation of forward and inverse kinematics for control purpose

### 2.5 Advantages of Stewart Platform

Some of the advantages of Stewart platform may be listed as follows:
Stewart platform is a parallel link mechanism, which has the major mechanical differences from the typical serial link robot. The basic idea in this parallel link mechanism is to connect two platform with a number of links, and to distribute the loads between the legs, which results in improved load carrying capabilities, and to improve both accuracy and repeatability, as errors will tend to average out rather than accumulating as in the case of serial link mechanisms.

In addition to the significant improvements over conventional technology in terms of performance and cost, the Stewart Platform offers the possibility for further improvements towards ultra-fine-tolerance machining. An important means for achieving this goal is the development of error compensation capabilities based on the precise understanding of inaccuracy factors which cause position and orientation errors during the tool's motion.

The great advantage of the Stewart platform is that no bending forces are applied to its six struts: they are in pure tension or pure compression

If an object is transported on a mobile platform, all accelerations of the mobile platform affect the object. This is of course undesirable, since accelerations can move or even damage the object. Stewart-platforms are mostly used for simulation, where the platform generates accelerations that increase the simulation's quality. Vice versa, it's possible to use a Stewart-platform mounted on a mobile platform to compensate the unwanted accelerations by generating a tilt. The necessary movement of the platform is calculated by a washout-filter. Applications of this combination include the transport of liquids in open boxes and medical transports, where the patients must not be affected by any acceleration.

## 3. 4-DOF STEWART PLATFORM

### 3.1 Introduction

The 4-DOF Stewart Platform manipulator shown in Fig. 3.1 is used as the basic part of the manipulator considered in this topic. It consists of equilateral triangular upper and lower platforms. This platform here is augmented by locating an extendible limb at the mass center of the upper platform along the direction of the unit normal vector. All the previously obtained results for 3-DOF Stewart Platform are used and extended for this 4-DOF manipulator.

In particular the problem that is studied in this topic is to determine the motion of all the limbs when the tip of the extendible limb is constrained to move in space from one point to another along a set of parallel adjacent lines passing through these points with a suitable velocity and acceleration profiles.

If the positions of the four points in space, and also the $z$ component of position vector of the mass center for the upper platform are given then the rest of the information about the inverse kinematic of the mechanism is supplied by a written Matlab program. Through this program one can easily calculate the lengths, velocity and acceleration of all the limps.


Figure 3.1 4-DOF Stewart Platform

### 3.2 Applications

This design can be applied in variety fields.
For instance:
1.The important use of the 4-DOF Stewart Platform in the areas of painting it representing useful applications. Through that application, things that have to be taken under considerations:

- The distance between the plan and the tip of the extendable limb should be constant.
- The flow rate of the paint should be constant.
2.In the industrial areas it represents useful applications in machining of metals, for example it can be used as a shaper machine to keyway in rods like crankshaft of the cars. In this case there is necessary thing have to be considered are :
- The orientation of the tip should be constant.
- The depth of the keyway should be equal to the length of the cutting tool.
- The width of the cutting tool should be equal to the separation between the adjacent lines.


### 3.3 Fundamental For Stewart Platform Calculations

A 3-DOF Stewart Platform has upper and lower platforms with equilateral triangular shape as shown in Figure 3.1.

With respect to the selected coordinate frame $S(0 X Y Z)$ for the position vectors of the corners of the lower platform we have

$$
\mathbf{q}_{1}=\left[\begin{array}{l}
R  \tag{3.1}\\
0 \\
0
\end{array}\right], \quad \mathbf{q}_{2}=\left[\begin{array}{c}
-R / 2 \\
(\sqrt{3} / 2) R \\
0
\end{array}\right], \quad \mathbf{q}_{3}=\left[\begin{array}{c}
-R / 2 \\
-(\sqrt{3} / 2) R \\
0
\end{array}\right]
$$

Similarly for the coordinate frame $\Sigma(0 x y z)$ for the position vectors of the corners of the upper platforms we have

$$
\mathbf{p}_{1}=\left[\begin{array}{l}
r  \tag{3.2}\\
0 \\
0
\end{array}\right], \quad \mathbf{p}_{2}=\left[\begin{array}{c}
-r / 2 \\
(\sqrt{3} / 2) r \\
0
\end{array}\right], \quad \mathbf{p}_{3}=\left[\begin{array}{c}
-r / 2 \\
-(\sqrt{3} / 2) r \\
0
\end{array}\right]
$$

Initially it is assumed that $\Sigma$ and $S$ are coincident and sharing the same origin 0 . After the displacement of the upper platform with center $G$ (center of the equilateral triangle), its corners $P_{i}(i=1,2,3)$ lie on the respective vertical planes $\Pi_{1}, \Pi_{2}$, and $\Pi_{3}$ passing through the vertical $0 Z$ axis. Let the position vector of $G$ be

$$
\begin{equation*}
\xi=\left[x_{G} \mathbf{y}_{\mathrm{G}} \mathbf{z}_{\mathrm{G}}\right]^{\mathrm{T}} \tag{3.3}
\end{equation*}
$$



Fig3.2 Upper and lower platforms

Note that each limb has two degree of freedom:
A limp can rotate about the revolute joint at $\mathrm{Q}_{\mathrm{i}}(\mathrm{i}=1,2,3)$ and also its length $l_{i}$ may be changed; however the limb remains within the fixed vertical plane $\Pi_{i}$. If

$$
\mathbf{T}=\left[\begin{array}{lll}
\mathbf{t}_{1} & \mathbf{t}_{2} & \mathbf{t}_{3}
\end{array}\right]
$$

represents the transformation matrix which transforms the initial position of the upper platform to its displaced position, then for the position vectors of the joints $P_{i}$ one can write

$$
\left.\begin{array}{l}
\mathbf{a}=\mathbf{T} \mathbf{p}_{1}+\xi \\
\mathbf{b}=\mathbf{T} \mathbf{p}_{\mathbf{2}}+\xi  \tag{3.5}\\
\mathbf{c}=\mathbf{T} \mathbf{p}_{3}+\xi
\end{array}\right\}
$$

According to Rodrigues' formula we have:

$$
\mathbf{T}(\mathbf{n}, \theta)=\left[\begin{array}{lll}
\mathbf{t}_{1} & \mathbf{t}_{2} & \mathbf{t}_{3} \tag{3.6}
\end{array}\right]=\cos \theta \mathbf{I}+(1-\cos \theta) \mathbf{m} \mathbf{n}^{\mathrm{T}}+\sin \theta \mathbf{N}
$$

$\mathrm{T}=\cos \theta\left[\begin{array}{lll}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right]+(1-\cos \theta)\left[\begin{array}{ccc}n_{1}^{2} & n_{1} n_{2} & n_{1} n_{3} \\ n_{2} n_{1} & n_{2}^{2} & n_{2} n_{3} \\ n_{3} n_{1} & n_{3} n_{2} & n_{3}^{2}\end{array}\right]+\sin \theta\left[\begin{array}{ccc}0 & -n_{3} & n_{2} \\ n_{3} & 0 & -n_{1} \\ n_{2} & n_{1} & 0\end{array}\right]$
Where $\mathbf{n}=\left[\begin{array}{lll}n_{1} & n_{2} & n_{3}\end{array}\right]^{T}$ is the unit vector lying on the rotation axis and $\mathbf{N}$ is its skewsymmetric matrix representation while $\theta$ is the rotation angle about the rotation axis.

Constraint equations of 3-DOF Stewart Platform derived in [1,2] are reproduced in (3.8). Where C and S stand for $\cos (\theta)$ and $\sin (\theta)$, respectively. Third equation in (3.8) states that either $\sin (\theta)=0$ (which means that $\theta=\mathrm{k} \pi$ with $\mathrm{k}=0, \pm 1, \pm 2 \ldots$ ) or $n_{3}=0$. Where the occurrence of the second case is most probable.

Therefore assuming that $n_{3}=0$ and hence $n_{1}^{2}+n_{2}^{2}=1$, the transformation matrix $\mathbf{T}$ in (3.6) takes the form given in (3.9).

$$
\left.\begin{array}{c}
x_{G}=-\frac{r}{2}(1-C)\left(n_{2}^{2}-n_{1}^{2}\right) \\
y_{G}=-r(1-C) n_{1} n_{2} \\
0=S n_{3}
\end{array}\right\} \quad \begin{array}{ccc}
C(\mathbf{n}, \theta)=\left[\begin{array}{ccc}
C+(1-C) n_{1}^{2} & (1-C) n_{1} n_{2} & S n_{2} \\
(1-C) n_{1} n_{2} & C+(1-C) n_{2}^{2} & -S n_{1} \\
-S n_{2} & S n_{1} & C
\end{array}\right]=\left[\begin{array}{lll}
\mathbf{t}_{1} & \mathbf{t}_{2} & \mathbf{t}_{3}
\end{array}\right]
\end{array}
$$

From these expressions one can deduce the following important property: The rotation axis and hence $\boldsymbol{n}$ lies in the 0 XY plane. The unit vector $\mathrm{v}=\boldsymbol{t}_{3}$ is perpendicular to both the upper platform giving its orientation and also to the rotation axis as shown in Figure3. 3

### 3.4 Inverse Kinematic Of Augmented Platform

In this article 3-DOF platform is augmented by locating an extendible limb at the mass center of the upper platform along the direction of the unit normal vector as shown in figure3.1. We assume that the positions of the points $A S, A L, A S n, A L n$ in space and $z_{G}$ component in (3.3) are known. The problem here is to determine the motion of all the limbs when the tip $K$ of the extendible limb is constrained to move in space following a rectangular plane. That is to move along a set of parallel adjacent lines. We are going to determine the motion of all the limb when the tip K constrain to move in space from the point AS1 to the point AL1 along the straight line with a suitable velocity and acceleration profiles, and by repeating this procedure for the parallel adjacent lines we can get the length, velocity and acceleration profiles.

For position changes of the end point of the extendible limb, a function $\lambda(t)$ indicated in Figure 3.3 must be selected so that its velocity and acceleration become zero at the beginning and at the end of the motion. To satisfy these conditions one way of selecting $\lambda(t)$ is:

$$
\begin{equation*}
\lambda(t)=\frac{1}{2 \pi}(t-\sin t) \quad 0 \leq t \leq 2 \pi \quad(0 \leq \lambda \leq 1) \tag{3.10}
\end{equation*}
$$



Figure 3.3 Vector representation for the path


Figure 3.4 Interpretation of the transformation

From Figure 3.3 one can write

$$
\begin{equation*}
\mathbf{a a}=\mathbf{a s}+\lambda(\mathrm{t})(\mathbf{a l - a s}) \tag{3.11}
\end{equation*}
$$

To determine the limb lengths $I_{1}, l_{2}, l_{3}$ a method similar to that used in [1] may be repeated: Since $z_{G}$ is given, point $G$ lies on the horizontal plane II shown in Fig. 5. Consider the projection $\mathrm{K}_{1}$ of the tip K on $\Pi$ at a definite time. If the undermined length of the limb $K G$ is indicated by $L$, then $G$ is further constrained to lie on a circle $\left(C_{1}\right)$ in $\Pi$ of radius

$$
\begin{equation*}
r_{1}=\left(a a_{z}-z_{G}\right) \tan (\theta)=L \sin (\theta) \tag{3.12}
\end{equation*}
$$

with center at $\mathrm{K}_{1}$. In this expression $\theta$, due to the property mentioned at the end of the previous section, that is, the rotation axis and $\mathbf{n}$ lies in the $0 X Y$ plane. The unit vector $\mathbf{v}=\mathbf{t}_{\mathbf{3}}$ is perpendicular to both the upper platform giving its orientation and also to the rotation axis as shown in Figure 3.3 - is the same as the rotation angle. $\left(\mathrm{C}_{1}\right)$ is actually is the intersection of the plane $\Pi$ with the sphere of radius L with the center at K . On the other hand $G$ may also be considered to lie on another circle $\left(\mathrm{C}_{2}\right)$ with an undetermined radius $r_{0}$. In fact $\left(C_{2}\right)$ is the intersection of the plane $\Pi$ with the sphere of radius $|s|$ centered at 0 .

For the distance $R_{0}=\mid \overline{0_{1} K_{1}}$ of the point $K_{1}$ from the $O Z$ axis and for the radius $r_{0}$ we have the expressions

$$
\left.\begin{array}{c}
R_{0}=\sqrt{a a_{x}^{2}+a a_{y}^{2}}  \tag{3.13}\\
r_{0}=\sqrt{x_{g}^{2}+y_{g}^{2}}
\end{array}\right\}
$$

Since both the circles $\left(\mathrm{C}_{1}\right)$ and $\left(\mathrm{C}_{2}\right)$ are lying in $\Pi$, their intersection points determine the location of G . In order to obtain a unique solution we shall consider the case where the circles are tangent to each other. However to keep G as close as possible to the OZ axis we consider the case indicated in Figure 3.6.

Depending upon whether $R_{0}-r_{1}$ is positive or negative we have

$$
\begin{equation*}
\overline{O_{1} G}=k \overline{0_{1} K_{1}} \tag{3.14}
\end{equation*}
$$



Figure 3.5: Vector representation on a vertical plane and rotation angle


Figure 3.6: Geometry for the location of $G$ in $\Pi$ plane

And from (3.12) and (3.13) we write:

$$
k=\frac{R_{0}-r_{1}}{R_{0}}=1-\frac{\left(a a_{z}-z_{G}\right) \tan (\theta)}{\sqrt{a a_{x}^{2}+a a_{y}^{2}}}
$$

Therefore

$$
\begin{equation*}
x_{G}=k \mathrm{aa}_{\mathrm{x}}, \quad \mathrm{x}_{G}=k \mathrm{aa}_{\mathrm{y}} \tag{3.16}
\end{equation*}
$$

In these expressions $\theta$ is the only unknown parameter to be determined. On the other hand from the constraint equations in (3.8), after squaring and adding the first two equations and considering the fact that $n_{1}^{2}+n_{2}^{2}=1$, we have

$$
\begin{equation*}
r_{0}=\frac{1}{2} r(1-C) \tag{3.17}
\end{equation*}
$$

Then the relation $r_{0}=R_{0}-r_{1}$ becomes

$$
\begin{equation*}
\sqrt{a a_{x}^{2}+a a_{y}^{2}}=\frac{1}{2} r(1-C)+\left(a a_{z}-z_{G}\right) \tan (\theta) \tag{3.18}
\end{equation*}
$$

And its solution for $\theta$ determines $\mathrm{x}_{\mathrm{G}}$ and $\mathrm{y}_{\mathrm{G}}$ in (3.16). From Figure 3.5 and from the expression of the transformation matrix $\mathbf{T}$ in (3.9) we have:

$$
\mathbf{a} \mathbf{a}^{=}\left[\begin{array}{l}
a a_{x}  \tag{3.19}\\
a a_{y} \\
a a_{z}
\end{array}\right]=\left[\begin{array}{l}
x_{G} \\
y_{G} \\
z_{G}
\end{array}\right]+L\left[\begin{array}{c}
S n_{2} \\
-S n_{1} \\
C
\end{array}\right]
$$

And the unknown components of the unit vector $\mathbf{n}$ on the rotation axis can be obtained as

$$
\begin{equation*}
n_{2}=\frac{a a_{x}-x_{G}}{L S}, \quad n_{1}=\frac{y_{G}-a a_{y}}{L S} \tag{3.20}
\end{equation*}
$$

Note that the unit normal vector $v$ is identical to $t_{3}$ in (3.9):

$$
\mathbf{v}=\left[\begin{array}{lll}
S n_{2} & -S n_{1} & C \tag{3.21}
\end{array}\right]^{T}
$$

With these information T is calculated from (3.9) or (3.6) and $\mathrm{a}, \mathrm{b}, \mathrm{c}$ from (3.5), all in turn give the limb lengths as

$$
\begin{gather*}
l_{1}=\left|\mathbf{a}-\mathbf{q}_{1}\right|, \quad l_{2}=\left|\mathbf{b}-\mathbf{q}_{\mathbf{2}}\right|, \\
l_{3}=\left|\mathbf{c}-\mathbf{q}_{\mathbf{3}}\right| \tag{3.22}
\end{gather*}
$$

Velocity of the limbs can be calculated by remembering that

$$
\begin{equation*}
l_{1}^{\prime}=\frac{l_{1}(t+\Delta t)-l_{1}(t)}{\Delta t} \tag{3.23}
\end{equation*}
$$

Similarly acceleration of the limb lengths can also be obtained.

A written Matlab program given in appendix A calculates all these expressions. A close study yields that the in the result of the example considered below the speed and acceleration of the legs are not quite zero at the beginning and end of the motion. This situation may be improved if the expression of $\lambda$ in (3.10) is used twice:

$$
\begin{equation*}
\lambda(l t)=\frac{1}{2 \pi}\left(\left[\frac{2 \pi}{T t} l t-\sin \left(\frac{2 \pi}{T t} l t\right)\right]-\sin \left[\frac{2 \pi}{T t} l t-\sin \left(\frac{2 \pi}{T t} l t\right)\right]\right) \tag{3.24}
\end{equation*}
$$

Where Tt is the duration of time ( lt ).

## Example 3.1:

Let $\mathrm{r}=7$
$\mathrm{R}=10$
$Z_{G}=20$
$\mathbf{A S 1}=\left[\begin{array}{lll}4 & 10 & 30\end{array}\right]$
$\mathrm{AL} 1=\left[\begin{array}{lll}10 & 12 & 40\end{array}\right]$
$\mathbf{A S n}=\left[\begin{array}{lll}4 & 14 & 30\end{array}\right]$
$\mathbf{A L n}=\left[\begin{array}{lll}10 & 1640\end{array}\right]$
And $\mathrm{T}=3$

### 3.5 The results of the Matlab program

The following figures demonstrate the path taken by the extendable limb and the length, velocity, and acceleration of the four limbs.

The path of the limb explained in figure 3.7, which illustrate that the tip starts to move in the next line form the end of the present line.

As the result of the motion we obtained expected and logical results for the length of the limbs as shown in figures ( $3.8,3.11,3.12,3.13$ )

The velocity and acceleration of the limbs start in each line from zero and decreased at the end to come to zero as illustrated in figures (3.8,3.9,3.14-3.19).


Figure 3.7 The tip motion plan


Figure 3.8 The length of the extendable limb.


Figure 3.9 The velocity of the extendable limb.


Figure 3.10 The acceleration of the extendable limb.

gure 3.11 The length of the first limb


Figure 3.12 The length of the second limb.


Figure 3.13 The length of the third limb.


Figure 3.14 The velocity of the first limb


Figure 3.15 The velocity of the second limb


Figure 3.16 The velocity of the third limb.


Figure 3.17 The acceleration of the first limb


Figure 3.17 The acceleration of the first limb


Figure3.18 The acceleration of the second limb


Figure3.19 The acceleration of the third limb

## CONCLUSION

In this project we made a complementation from straight line motion of a special 4DOF manipulator to a motion in plane, where the tip of one of the limbs is constrained to move through a set of parallel adjacent lines, with a restriction of having zero velocity and acceleration at the beginning and at the end of each line.

The oscillations appear on the velocity and acceleration curves, indicated in the figures on the result of example 3.1, caused by the restriction of having zero velocity and acceleration at the initial point and at the end of each line.

In this procedure we assumed that the robot moves in uniform rectangular plane. With a minor modification this plane can be non-uniform like trapezoid

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## APPENDEX A

## Main MATLAB program (neucee.m)

clear
$\mathrm{ks}(3)=20$;
$r=7$;
$\mathbf{R}=10$;
$\mathrm{q} 1=[\mathrm{R} ; 0 ; 0]$;
$\mathrm{q} 2=\left[-\mathrm{R} / 2 ;\left(3^{\wedge} 0.5 / 2\right)^{*} \mathrm{R} ; 0\right] ;$
$q 3=\left[-R / 2 ;-\left(3^{\wedge} 0.5 / 2\right)^{*} R ; 0\right] ;$
$\mathrm{pl}=[\mathrm{r} ; 0 ; 0]$;
$\mathrm{p} 2=\left[-\mathrm{r} / 2 ;\left(3^{\wedge} 0.5 / 2\right)^{*} \mathrm{r} ; 0\right]$;
$\mathrm{p} 3=\left[-\mathrm{r} / 2 ;-\left(3^{\wedge} 0.5 / 2\right)^{*} \mathrm{r} ; 0\right]$;
as $1=\left[\begin{array}{lll}4 & 10 & 30\end{array}\right]$;
all $=\left[\begin{array}{llll}1 & 1 & 12 & 40\end{array}\right]$;
$a s n=\left[\begin{array}{lll}4 & 14 & 30\end{array}\right]$;
aln $=\left[\begin{array}{lll}10 & 16 & 40\end{array}\right]$;
incr $=0.2$;
$p p=0$;
$\mathrm{j}=0$;
$\mathrm{sa}=0$;
$q \mathrm{a}=0$;
$\mathrm{ss}=0$;
$\mathrm{qq}=0$;
secas=asl(2)-incr;
slt=0;
$\mathrm{fla}=1$;
for secn=all(2):incr:aln(2)
secas=secas+incr;
$a s=[\operatorname{asl} 1(1)$ secas asl (3) $]$;


```
al=[al1(1) secn all(3)];
if rem(fla,2)==0
tem=as;
as=al;
al=tem;
end
fla=fla+1;
Tt=3;
C=2*pi/Tt;
dt=0.05;
for lt=slt:dt:slt+Tt,
j=j+1;
z(j)=0;
ltt(j)=lt;
zg(j)=ks(3);
A=sin(C*
B=cos(C*}(\textrm{lt-slt)})
D=C*(lt-slt)-A;
landa=(1/(2*pi))*(D-sin(D));
aa=as+landa*(al-as);
hh=((al-as)*(al-as)}\mp@subsup{)}{}{\wedge}^0.5
hiz(j)=(1/Tt)* (1-B)* (1-\operatorname{cos}(\textrm{D}))*hh(1);
ivme(j)=(2*pi)/((Tt)^2)*}(\textrm{A}-\mp@subsup{\textrm{A}}{}{*}\operatorname{cos}(\textrm{D})+(1-\textrm{B}\mp@subsup{)}{}{\wedge}\mp@subsup{2}{}{*}\operatorname{sin}(\textrm{D})\mp@subsup{)}{}{*}hh(1)
ww(j)=aa(1)/aa(2);
aax(j)=aa(1);
aay(j)=aa(2);
aaz(j)=aa(3);
fl
%%%%%%%%%
```

```
ll(j)=(aa(3)-zg(j))/ct;
if j>1;
sa=sa+1;
ltth(sa)=lt;
llh(sa)=(ll(j)-11(j-1))/dt;
if sa>1;
qa =qa+1;
ltti(qa)=1t;
lli(qa)=(llh(sa)-1lh(sa-1))/dt;
end;end;
k=(1/(2*aa(2)))*((r/2)*(1-ct))}\mp@subsup{)}{}{\wedge}2+(aa(1)^2+aa(2)^2)/(2*aa(2))-(aa(3)-
ks(3))^2/(2*aa(2))*(st/ct)^2;
kk(j)=k;
ca=(aa(1)^2+aa(2)^2)/(aa(2)^2);
cb=(aa(1)/aa(2))*k;
x(j)=cb/ca;
y(j)=-(aa(1)/aa(2))*x(j)+k;
yy(j)=-ww(j)*x(j)+kk(j);
zzz=(11(j)*st);
n2(j)=-(x(j)-aa(1))/zzz;
nl(j)=(y(j)-aa(2))/zzz;
n=[n1(j) n2(j) 0];
v=[st*n2(j) -st*n1(j) ct];
vx(j)=v(1);
vy(j)=v(2);
vz(j)=v(3);
ks=[x(j); y(j); ks(3)];
T=[ct+(l-ct)*nl(j)^2(1-ct)*nl(j)*n2(j)v(1);(l-ct)*nl(j)*n2(j)ct+(l-ct)*n2(j)^2v(2);-v(l)
-v(2) ct];
a=T*
b}=\textrm{T}*\textrm{p}2+\textrm{ks}
```

$\mathrm{c}=\mathrm{T}^{*} \mathrm{p} 3+\mathrm{ks} ;$
$111=-q 1+a ;$
$122=-q 2+b ;$
$133=-q 3+c ;$
$11 \mathrm{~s}=\left(111^{\prime *} 111\right)$;
$11(j)=11 s(1)^{\wedge} 0.5 ;$
$12 \mathrm{~s}=\left(122^{*} 122\right) ;$
$12(\mathrm{j})=12 \mathrm{~s}(1)^{\wedge} 0.5$;
$13 \mathrm{~S}=\left(133^{\prime} * 133\right)$;
$13(\mathrm{j})=13 \mathrm{~s}(1)^{\wedge} 0.5$;
if $\mathrm{j}>1$,
$\mathrm{SS}=\mathrm{SS}+1$;
$1 \mathrm{th}(\mathrm{ss})=1 \mathrm{t}$;
$11 \mathrm{~h}(\mathrm{ss})=(11(\mathrm{j})-11(\mathrm{j}-1)) / \mathrm{dt}$;
$12 \mathrm{~h}(\mathrm{ss})=(12(\mathrm{j})-12(\mathrm{j}-1)) / \mathrm{dt} ;$
$13 \mathrm{~h}(\mathrm{ss})=(13(\mathrm{j})-13(\mathrm{j}-1)) / \mathrm{dt}$;
if $s s>1$;
$q q=q q+1 ;$
lti(qq) $=1 \mathrm{l}$;
$11 \mathrm{i}(\mathrm{qq})=(11 \mathrm{~h}(\mathrm{ss})-1 \mathrm{lh}(\mathrm{ss}-1)) / \mathrm{dt} ;$
$12 \mathrm{i}(\mathrm{qq})=(12 \mathrm{~h}(\mathrm{ss})-12 \mathrm{~h}(\mathrm{ss}-1)) / \mathrm{dt} ;$
$13 \mathrm{i}(\mathrm{qq})=(13 \mathrm{~h}(\mathrm{ss})-13 \mathrm{~h}(\mathrm{ss}-1)) / \mathrm{dt} ;$
end;
end;
end;
$\mathrm{slt}=\mathrm{slt}+2^{*} \mathrm{Tt}$;
end;

## Subprogram f11.m

## format long

$\mathrm{tl}=1$;
12 $2=89$;
f22
while $\mathrm{abs}(\mathrm{ub})>0.0000001$
$\mathrm{t} 1=\mathrm{teb}$;
t2 $=$ tea;
f22
end;
$\mathrm{ct}=\cos (\mathrm{teb} * \mathrm{pi} / 180)$;
$\mathrm{st}=(1-\mathrm{ct} \wedge 2)^{\wedge} 0.5$;

Subprogram $\mathbf{2 2}$
$i=0$;
$\mathrm{ti}=(\mathrm{t} 2-\mathrm{t}) / 10 ;$
for $t e=-1$ : $\mathrm{ti}: \mathrm{t}_{2}$,
$i=i+1$;
$\mathrm{t}(\mathrm{i})=\mathrm{te}$;
teta=te*pi/180;
Ro $=\left(\mathrm{aa}(1)^{\wedge} 2+\mathrm{aa}(2)^{\wedge} 2\right)^{\wedge} 0.5$;
$u(i)=\operatorname{Ro}-\left(r / 2^{*}(1-\cos (\right.$ teta $))+(\operatorname{aa}(3)-z g(j))^{*} \tan ($ teta $\left.)\right) ;$
if $i>1$,
if $\operatorname{sign}(u(i)) \sim=\operatorname{sign}(u(i-1))$,
tea $=\mathrm{t}$ (i);
teb=t(i-1);
$u a=u(i)$;
$u b=u(i-1)$;
end;
end;
end;

