# NEAR EAST UNIVERSITY 

## ENGINEERING FACULTY

## ELECTRICAL \& ELECTRONICAL

## DEPARTMENT

## SYMMETRICAL COMPONENTS IN

## Power Analysis

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## Chapter 1

## Phasors

## THE USE OF VECTORS

## Introduction:-



Most engeneers are familiar with the purpose of vectors, but perhaps not quite so familiar with their use.

Those who use them know how solvable they are for expressing voltage and current relations in a way that can easily be understood, and how natural it becomes to visualize even complex conditions in term of vectors, it should also be understood that, in addition to their having an implicit value, vectors are the basics of many fault calculation.

## BASIC CONCEPTS OF VECTORS-TECHNIQUE.

Sinusoidal quantati es such as alternating current and voltage can be represented by a vector.

In vector-technique as applied to electrical engineering, the main concern is with a pictorial representation of relations between two or more alternating quantities, such as current and voltage, ie. their relative magnitudes and phase-displacements, and not with their values at any instant of time. For example $e 1=E 1 \sin \omega t$ and $\quad e 2=E 2 \sin (\omega \mathrm{t}+\Phi)$,
two voltages represented by two vectors having lengths proportional to E1 and E2, and with angular displament $\Phi$ between them, as shown in Fig. 1-1


Fig. 1-1

Since they have the same angular velocity, their magnitude and phase-displacement are not affected by their relations but are the same at all instants.

When drawing vector diagrams, it is usual to shoose an arbitary line (referencevector) and give one of the vectors zero phase-displacement by drawing it along this line, as E1 in the Fig. 1-1, we will consider right hand side as positive position. So any vector will be drawnswill be related to zero postion vector. (E1 in this figure).

In the electric point of view the vector length is usually made proportional to the R.M.S. value of the quantity represented, rather than its maximum value.

By way of example, consider the vectors relating to a three-phase system. In Fig. 1-2(a), Ean, Ebn and Ecn are the voltage of the phase. Terminals $a, b$ and $c$ realtive to the netural n .

In Fig. 1-2(b), Ena, Enb and Enc are the voltages of tye neutral-terminal $n$ relative to the phase-terminals $\mathrm{a}, \mathrm{b}$ and c .


1-2(a)


Fig. 1-2(b)

In Fig. 1-3(a), $\mathrm{Ia}, \mathrm{Ib}$ and Ic are the currents in the phase-wires, shown lagging behine their driving voltages Ean, Ebn and Ecn by angular $\Phi 1, \Phi 2$ and $\Phi 3$. It is often said that the netural-current in is the vector-sum of the line cuurent, but since we consider the currents acting away from the source $\mathrm{In}=-(\mathrm{Ia}+\mathrm{Ib}+\mathrm{Ic})$. As shown in Fig. 1-2(a)



Fig. 1-3(a)


Fig. 1-3(b)

The following figures shows the lagging, leading interphase vectors:


Lagging


Ebc Leading

* phase-sequence:

The term "phase-sequence" is use to describe the order in which vectors are palced in relation to one another for counter-clockwise relation. The phase order a-b-c is called positive-phase-sequence as shown in the figure above.

The phase order $a-b-c$ is called negative-phase-sequence as shown in the figure. If all the vectors are in phase, they are said to have zero-phase-sequence.

## VOLTAGE DROPS:

The voltage between any two points in a system of impedences is the vector-sum of the cource-voltage and the voltage-drop in the impedence between the source and the points under consideration as shown in Fig. 1-4,


Fig. 1-4

## VECTOR ALGEBRA

An impedence may be expressed as a vector-quantity comprising resistance and reactance component at right angles, and may therefor be written algebrically as $\mathrm{Z}=\mathrm{R}+\mathrm{jX}$ (inductive) and $\mathrm{Z}=\mathrm{R}-\mathrm{j} \mathrm{X}$ (capacitive). The numerical value of the impedance is the modulus $|Z|=\sqrt{R^{2}+X^{2}}$.

The angle between the vector and the reference-line is the argument, $A=\tan ^{-1} X / R$.

## CHAPTER 2

## 2- SYMMETRICAL COMPONENTS:-

The system impeedence in each phase are identical and the three phase voltage and currents throughout the system are completly balanced, in other words they have equal magnitudes in each phase and are displaced in 3 phase by 120 degree.

In a balnced system, analysis can proceed on a single-phase basis. The knowledge of voltage and current in one phase is sufficint to completly determite (V\&C) in the other two phases. Real \& reactive powers are simply three times the coressponding per phase values.

Unbalanced system operation can result in an otherwise balanced system due to unsymmetrical fault e.g live to ground fault or line to line fault. System operation may also become unbalanced when laods are unbalaned as in the presence of large single phase loads.

A more convenient method of analyzing unbalanced operation is through symmetrical component when the three phase ( $\mathrm{V} \& \mathrm{C}$ ) which may be unbalanced are transformed in to three sets of balanced ( $\mathrm{V} \& \mathrm{C}$ ) called symmetrcal components. In such a transformation, the impedances presented by various power system elements to
symmetrical components are decoupled from each other resulting in independent system networks for each component as balanced set.

## 2-2 SYMMETRICAL COMPONENT TRANSFORMATION

A set of three balanced voltages (phasers) $\mathrm{Va}, \mathrm{Vb}, \mathrm{Vc}$ is characterized by equal magnitudes and interphase differences of 120 degree. The set is said to have a phase sequence abc (positive sequence) if Vb lags Va by 120 degree and Vc lags Vb by 120 degree. The three phasers can then be expressed in terms of the refernce phasor Va as

$$
\mathrm{Va}=\mathrm{Va}, \mathrm{Vb}=\alpha^{2} \mathrm{Va}, \mathrm{Vc}=\alpha \mathrm{Va}
$$

where the complex number operator $\alpha$ is defined as

$$
\alpha=e^{j 120^{\circ}}
$$

It has the following properties

$$
\left.\begin{array}{l}
\alpha=e^{j 240^{\circ}}=e^{-j 120^{\circ}}=\alpha^{*}  \tag{2-1}\\
\left(\alpha^{2}\right)^{*}=\alpha \\
\alpha^{3}=1 \\
1+\alpha+\alpha^{2}=0
\end{array}\right]
$$

If the phase seqyence is acb (negative sequence), then

$$
\mathrm{Va}=\mathrm{Va}, \mathrm{Vb}=\alpha \mathrm{Vd}, \mathrm{Vc}=\alpha^{2} \mathrm{Va}
$$

Thus a set of balanced phasors is fully characterized by its refernce phasors (say Va) and its phase sequence (positve or negative).

Suffix 1 is commonly used to indicate positive sequence. A set of (balanced) positve sequence phasors is written as

$$
\begin{equation*}
\mathrm{Val}, \mathrm{Vb} 1=\alpha^{2} \mathrm{Va} 1, \mathrm{Vc} 1=\alpha \mathrm{Va} 1 \tag{2-2}
\end{equation*}
$$

Similarly, suffix 2 is used to indicate negative sequence. A set of (balanced) negative sequence phasors is written as

$$
\begin{equation*}
V a_{2}, \quad \mathrm{Vb}_{2}=\alpha \mathrm{Va}_{2}, \quad V c_{2}=\alpha^{2} \mathrm{Va}_{2} \tag{2-3}
\end{equation*}
$$

A set of three voltages (phasors) equal in magnitude and having the same phase is said to have zero sequence. Yhus a set of zero sequence phasors is written as

$$
\begin{equation*}
\mathrm{Vao}, \quad \mathrm{Vbo}=\mathrm{Vao}, \quad \mathrm{Vco}=\mathrm{Vao} \tag{2-4}
\end{equation*}
$$

Consider now a set of three voltages (phasors) $\mathrm{Va}, \mathrm{Vb}, \mathrm{Vc}$ which in general may be unbalanced. According to Fortesque's therom the three phasors can be the sum of positve, negative and zero sequence phasors defined above. Thus

$$
\begin{align*}
& V a=V a 1+V a 2+V a o  \tag{2-5}\\
& V b=V b 1+V b 2+V b o  \tag{2-6}\\
& V c=V c 1+V c 2+V c o \tag{2-7}
\end{align*}
$$

The three phasors sequence (positive, negative and zero) are called the symmetrical components of the original pkasors set $\mathrm{Va}, \mathrm{Vb}, \mathrm{Vc}$. The addition of symmetical components as per Eqs. (2-5) to (2-7) to generate $\mathrm{Va}, \mathrm{Vb}, \mathrm{Vc}$ indicated by the phasor digram of Fig1-1.


Let us now express Eqs. (1-5) to (1-7) in terms of refernce phasors Va1, Va2,
Vao.
Thus

2

These equations can be expressed in the matrix form

$$
\left|\begin{array}{c}
\mathrm{Va}  \tag{2-11}\\
\mathrm{Vb} \\
\mathrm{Vc}
\end{array}\right|=\left|\begin{array}{ccc}
1 & 1 & 1 \\
\alpha^{2} & \alpha & 1 \\
\alpha & \alpha^{2} & 1
\end{array}\right|\left|\begin{array}{c}
\mathrm{Val} \\
\mathrm{Va2} \\
\mathrm{Vao}
\end{array}\right|
$$

or

$$
\begin{equation*}
\mathrm{Vp}=\mathrm{AVs} \tag{2-12}
\end{equation*}
$$

where

$$
\mathrm{Vp}=\left|\begin{array}{c}
\mathrm{Va} \\
\mathrm{Vb} \\
\mathrm{Vc}
\end{array}\right|=\text { vector of original phasor }
$$

$$
\mathrm{Vs}=\left|\begin{array}{c}
\mathrm{Va} 1 \\
\mathrm{Va} 2 \\
\mathrm{Vao}
\end{array}\right|=\text { vector of symmetrical component }
$$

$$
A=\left|\begin{array}{lll}
1 & 1 & 1  \tag{2-13}\\
\alpha^{2} & \alpha & 1 \\
\alpha & \alpha^{2} & 1
\end{array}\right|
$$

we can write Eq. (2-12) as

$$
\begin{equation*}
\mathrm{Vs}=\mathrm{A}^{-1} \mathrm{Vp} \tag{2-14}
\end{equation*}
$$

Computing $\mathrm{A}^{-1}$ and utilizing relations (2-1), we get

$$
\mathrm{A}^{-1}=\frac{1}{3}\left|\begin{array}{ccc}
1 & \alpha & \alpha^{2}  \tag{2-15}\\
1 & \alpha^{2} & \alpha \\
1 & 1 & 1
\end{array}\right|
$$

In expanded form we can write Eq. (2-14) as

$$
\begin{align*}
& \mathrm{Va} 1=1 / 3\left(\mathrm{Va}+\alpha \mathrm{Vb}+\alpha^{2} \mathrm{Vc}\right)  \tag{2-16}\\
& \mathrm{Va} 2=1 / 3\left(\mathrm{Va}+\alpha^{2} \mathrm{Vb}+\alpha \mathrm{Vc}\right)  \tag{2-17}\\
& \mathrm{Vao}=1 / 3(\mathrm{Va}+\mathrm{Vb}+\mathrm{Vc}) \tag{2-18}
\end{align*}
$$

Equations (2-16) to (2-18) give the necessary relationships for obtaining symmetrical components of the original phsors, while Eqs. (2-5) to (2-7) give the relationships for obtaining original phasors from the symmetrical components.

The symmetrical component transformations through given abpve in terms of voltages hold for any set of phasors and therefore automatically apply for a set of currents. Thus

$$
\begin{equation*}
\mathrm{Ip}=\mathrm{A} \mathrm{I} \tag{2-19}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{Is}=\mathrm{A}^{-1} \mathrm{Ip} \tag{2-20}
\end{equation*}
$$

where

$$
\mathrm{Ip}=\left|\begin{array}{l}
\mathrm{IA} \\
\mathrm{Ib} \\
\mathrm{Ic}
\end{array}\right| ; \quad \text { and } \mathrm{Is}=\left|\begin{array}{c}
\mathrm{Ia} 1 \\
\mathrm{Ia} 2 \\
\mathrm{Iao}
\end{array}\right|
$$

Of course A and $\mathrm{A}^{-1}$ are the same as given earlier.

In expanded form the relations (2-19) and (2-20) can be expressed as follows:
(I) Construction of current phasors from their symmetrical components:

$$
\begin{align*}
& \mathrm{Ia} 1=\mathrm{I} \mathrm{I} 1+\mathrm{I} \mathrm{I} 2+\mathrm{Ia} 0  \tag{2-21}\\
& \mathrm{Ib}=\alpha^{2} \mathrm{I} \mathrm{I} 1+\alpha \mathrm{I} 22+\mathrm{Iao}  \tag{2-22}\\
& \mathrm{Ic}=\alpha \mathrm{Ia} 1+\alpha^{2} \mathrm{I} \mathrm{I} 2+\mathrm{Iao} \tag{2-23}
\end{align*}
$$

(ii) Obtaining symmetrical components of current phasors:

$$
\begin{align*}
& \mathrm{I} \mathrm{I}=1 / 3\left(\mathrm{Ia}+\alpha \mathrm{Ib}+\alpha^{2} \mathrm{Ic}\right)  \tag{2-24}\\
& \mathrm{Ia} 2=1 / 3\left(\mathrm{Ia}+\alpha^{2} \mathrm{Ib}+\alpha \mathrm{Ic}\right)  \tag{2-25}\\
& \mathrm{Iao}=1 / 3(\mathrm{Ia}+\mathrm{Ib}+\mathrm{Ic}) \tag{2-26}
\end{align*}
$$

Certain observation can now be made regarding a three-phase system with neutral return as shown in Fig. 1-2.


The sum of the three line voltages will always be zero. Therefore, the zero sequence component of line voltages is always zero, i.e.

$$
\begin{equation*}
V a b o=1 / 3(V a b+V b c+V c a)=0 \tag{2-27}
\end{equation*}
$$

On the other hand, the sum of phase voltages (line to neutral) may not be zero so that their zero sequence component Vao may exist.

Since the sum of the three line current equals the current in the neutral wire, we have

$$
\begin{equation*}
\mathrm{Iao}=1 / 3(\mathrm{Ia}+\mathrm{Ib}+\mathrm{Ic})=1 / 3 \mathrm{In} \tag{2-28}
\end{equation*}
$$

i.e. the current in the neutral is three times the zero sequence line current. If the neutral connection is servered,

$$
\begin{equation*}
\mathrm{Iao}=1 / 3 \mathrm{In}=0 \tag{2-29}
\end{equation*}
$$

i.e. in absence of a neutral connection the zero sequence line current is always zero. Power inveriant means that the sum of powers of the three symmetrical component equals the three-phase power.

Total complex power in a three-phase circut is given by

$$
\begin{equation*}
S=V p^{2} I p^{*}=V a I a^{*}+V b I b^{*}+V c I c^{*} \tag{2-30}
\end{equation*}
$$

or

$$
\begin{align*}
\mathrm{S} & =[\mathrm{A} \mathrm{Vs}]^{T}[\mathrm{Ais}]^{*} \\
& =\mathrm{Vs}^{T} \mathrm{~A}^{T} \mathrm{~A}^{*} \mathrm{Is} * \tag{2-31}
\end{align*}
$$

Now

$$
\begin{align*}
& \mathrm{A}^{T} \mathrm{~A}^{*}=\left|\begin{array}{lll}
1 & \alpha^{2} & \alpha \\
1 & \alpha & \alpha^{2} \\
1 & 1 & 1
\end{array}\right|\left|\begin{array}{ccc}
1 & 1 & 1 \\
\alpha & \alpha^{2} & 1 \\
\alpha^{2} & \alpha & 1
\end{array}\right|=3\left|\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{array}\right|=3 \mathrm{U}  \tag{2-32}\\
& \therefore \mathrm{~S}=3 \mathrm{Vs}^{T} \mathrm{UIs}^{*}=3 \mathrm{Vs}^{T} \mathrm{Is}^{*} \\
& =3 \mathrm{Va} 1 \mathrm{Ia} 1^{*}+3 \mathrm{Va} 2 \mathrm{Ia} 2^{*}+3 \text { Vao Iao* }  \tag{2-33}\\
& =\text { sum of summetrical component powers }
\end{align*}
$$

Example 2-1 A delta connected balanced resistive load is connected across an unbalanced three-phase supply as shown in Fig. 2-3. With currents in lines A and B specified, find the symmetrical components of line currents. Also find the symmetrical components of delta currents. Do you notice any relationship between symmetrical components of line and delta currents. Comment.


Fig. 1-3

$$
\text { Solution } \quad \mathrm{IA}+\mathrm{IB}+\mathrm{IC}=0
$$

or

$$
\begin{aligned}
& 10 \angle 30^{\circ}+15 \angle-60^{\circ}+\mathrm{Ic}=0 \\
& \therefore \mathrm{Ic}=-16.2+\mathrm{j} 8.0=18 \angle 154^{\circ} \mathrm{A}
\end{aligned}
$$

From Eqs. (2-24) to (2-26)

$$
\begin{align*}
\text { IA2 } & =1 / 3\left(10 \angle 30^{\circ}+15 \angle\left(-60^{\circ}+120^{\circ}\right)+18 \angle\left(154^{\circ}+240^{\circ}\right)\right) \\
& =10.35+\mathrm{j} 9.3=14 \angle 42^{\circ} \mathrm{A}  \tag{i}\\
\text { IA2 } & =1 / 3\left(10 \angle 30^{\circ}+15 \angle\left(-60^{\circ}+240^{\circ}\right)+18 \angle\left(154^{\circ}+120^{\circ}\right)\right) \\
& =-1.7-\text { j4.3 }=4.65 \angle 248^{\circ} \mathrm{A}  \tag{ii}\\
\text { Iao } & =1 / 3(\text { IA }+ \text { IB }+ \text { Ic })=0 \tag{iii}
\end{align*}
$$

From Eq. (2-2)

$$
\begin{array}{ll}
\text { IB1 }=14 \angle 282^{\circ} \mathrm{A} & \text { Ic1 }=14 \angle 162^{\circ} \mathrm{A} \\
\text { IB2 }=4.65 \angle 8^{\circ} & \text { Ic2 }=4.65 \angle 128^{\circ} \mathrm{A} \\
\text { Ibo }=0 \mathrm{~A} & \text { Ico }=0 \mathrm{~A}
\end{array}
$$

Check:

$$
\mathrm{IA}=\mathrm{IA} 1+\mathrm{IA} 2+\mathrm{IAo}=8.65+\mathrm{j} 5=10 \angle 30^{\circ}
$$

Converting delta load into equivalant star, we can redraw Fig. 2-3 as in Fig. 2-4.


Fig. 2-4

Delta currents are obtained as follows

$$
V_{A B}=1 / 3\left(\mathrm{R}\left(\mathrm{I}_{\mathrm{A}}-\mathrm{I}_{\mathrm{B}}\right)\right.
$$

Now

$$
\mathrm{IAB}=\mathrm{VAB} / \mathrm{R}=1 / 3(\mathrm{IA}-\mathrm{IB})
$$

similarly,

$$
\begin{aligned}
& \mathrm{IBC}=1 / 3(\mathrm{IB}-\mathrm{IC}) \\
& \mathrm{ICA}=1 / 3(\mathrm{IC}-\mathrm{IA})
\end{aligned}
$$

Substituting the values of IA, IB and IC, we have

$$
\begin{aligned}
& \mathrm{IAB}=1 / 3\left(10 \angle 30^{\circ}-15 \angle-60^{\circ}\right)=6 \angle 86^{\circ} \mathrm{A} \\
& \mathrm{IBC}=1 / 3\left(15 \angle-60^{\circ}-18 \angle 154^{\circ}\right)=10.5 \angle-41.5^{\circ} \mathrm{A} \\
& \mathrm{ICA}=1 / 3\left(18 \angle 154^{\circ}-10 \angle 30^{\circ}\right)=8.3 \angle 173^{\circ} \mathrm{A}
\end{aligned}
$$

The symmetrical components of delta currents are

$$
\begin{aligned}
\mathrm{IAB1} & =1 / 3\left(6 \angle 86^{\circ}+10.5 \angle\left(-41.5^{\circ}+120^{\circ}\right)+8.3 \angle\left(173^{\circ}+240^{\circ}\right)\right)(\text { iv }) \\
& =8 \angle 72^{\circ} \mathrm{A}
\end{aligned}
$$

$$
\begin{align*}
\mathrm{IAB} 2 & =1 / 3\left(6 \angle 86^{\circ}+10.5 \angle\left(-41.5^{\circ}+240^{\circ}\right)+8.3 \angle\left(173^{\circ}+120^{\circ}\right)\right)(\mathrm{v}) \\
& =2.7 \angle 218^{\circ} \mathrm{A} \\
\mathrm{IAB} & =0 \tag{vi}
\end{align*}
$$

IBC1, IBC2, IBC0, ICA1, ICA2 and ICA0 can be found by using Eq. (2-2).

Comparing Eqs. (i)and (iv), and (ii) and (v), the following relationship between symmetrical components of line and delta currents are immediately observed:

$$
\begin{align*}
& \mathrm{IAB} 1=\frac{I A 1}{\sqrt{3}} \angle 30^{\circ}  \tag{vii}\\
& \mathrm{IAB} 2=\frac{I A 2}{\sqrt{3}} \angle-30^{\circ} \tag{viii}
\end{align*}
$$

We can verify these by calculating IAB1 and IAB2 from Eqs. (vii) and comparing the results with Eqs.(iv)and (v).

We need to discuss the standard polarity marking of a single-phase transformer as shown in Fig.1-5. The transformer ends marked with a dot have the same polarity.


Therefore, voltage VHH is in phase with voltage VLL`. Assuming that the small amount of magnetising current can be neglected, the primary current Il entering the dotted end cancels the demagnetising ampere-turns of the secondary current I2 so that I1 and I 2 with directions of flow as indicated in the diagram are in phase. If the directions of I2 is reversed, I1 and I2 will be in phase opposition.

Consider now a star-delta transformer with primary side star connected and secondary side delta connected as shown in Fig. 2-6. Windings shown parallel to each other are magnetically coupled. The polarity markings are indicated on each phase. With phases marked ABC on the star side, there are a number of ways of labelling the phases abc on the delta side. The labelling indicated on the diagram corresponds to $+90^{\circ}$ connection in which the positive sequence phase a to neutral voltage (delta side) leads phase A to neutral voltage (star side) by $90^{\circ}$ and so also the line currents in a and A .

This labelling is computationally convenient. If we reliable delta as $(b \rightarrow a, c \rightarrow b$, and $a \rightarrow c)$. we get the standard $\mathrm{Yd} 1,-30^{\circ}$ connection. If the polarities on delta side are also reversed, we get the standard $\mathrm{Yd} 11,30^{\circ}$ connection [30].

Double suffixes will be used for line-to-line voltages and delta currents; while single suffixes will be used for line currents and line to neutral (phase) voltages. Line-toline transformation ratio will be taken as unity.


Figure 2-7 shows positive and negative sequence voltages on primary (star) and secondary (delta) sides of the transformer. Fig. 2-8 shows the positive and negative sequence currents on the two sides of the transformer. The following observations can easily be made from these figures:

$$
\begin{array}{lr}
V a 1=j \text { VA } 1, & I a 1=j I A 1  \tag{2-34}\\
V a 2=-j V A 2, & I a 2=-j I A 2
\end{array}
$$

If the power flow reverses, that is, if it flows from delta to star, the voltage phasors do not change, while all the current phasors reverse. The phasor relationships between star and delta voltages and current therefore remain unchanged.


Positive sequence voltages


Star side

Delta side

Negative sequence voitoges

By examination of Fig. 2-7, the following relationship can be written down between line to neutral sequence voltages and line-to-line sequence voltages.

$$
\begin{array}{ll}
\mathrm{VAB}=\sqrt{3} \text { Val } \angle 30^{\circ} ; & \mathrm{Vab} 1=\sqrt{3} \mathrm{Val} \angle 30^{\circ}(2-35) \\
\mathrm{VAB} 2=\sqrt{3} \text { VA2 } \angle-30^{\circ} ; & \mathrm{Vab} 2=\sqrt{3} \mathrm{Va} 2 \angle-30^{\circ}
\end{array}
$$

Also by examination of Fig. 2-8, we can write the following relationship between sequence line current and sequence delta currents.

$$
\mathrm{Iabl}=\frac{I a 1}{\sqrt{3}} \angle-150^{\circ}
$$

$$
\begin{equation*}
\mathrm{I} a b 2=\frac{I a 2}{\sqrt{3}} \angle 150^{\circ} \tag{2-36}
\end{equation*}
$$

## 2-4 SEQUENCE IMPEDANCE OF PASSIVE ELEMENTS

Fig. 2-10 shows three impedance's $\mathrm{Za}, \mathrm{Zb}, \mathrm{Zc}$ carrying currents Ia , Ib , Ic which return via the neutral (ground) impedance Zn . The phase voltages at the two ends of the impedance's are $\mathrm{VA}, \mathrm{VB}, \mathrm{VC}$ and $\mathrm{Va}, \mathrm{Vb}, \mathrm{Vc}$, respectively. This circuit model can represent a three-phase transmission line with active sources (synchronous machines) at each end and with a ground return circuit. If $\mathrm{Va}, \mathrm{Vb}, \mathrm{Vc}$ are regarded as zero, the circuit represents three star connected impedance's with neutral return through Zn . Now
$\mathrm{VA}-\mathrm{Va}=\mathrm{Vaa}$, the voltage drop in Za and Zn
$\mathrm{VB}-\mathrm{Vb}=\mathrm{Vbb}$, the voltage drop in Zb and Zn
$\mathrm{VC}-\mathrm{Vc}=\mathrm{Vcc}$, the voltage drop in Zc and Zn


We can therefore write

$$
\mathrm{VAa}=\mathrm{Za} \mathrm{Ia}+\mathrm{Zn}(\mathrm{Ia}+\mathrm{Ib}+\mathrm{Ic})
$$

$$
\begin{aligned}
& \mathrm{VBb}=\mathrm{Zb} \mathrm{Ib}+\mathrm{Zn}(\mathrm{Ia}+\mathrm{Ib}+\mathrm{Ic}) \\
& \mathrm{VCc}=\mathrm{Zc} \mathrm{Ic}+\mathrm{Zn}(\mathrm{Ia}+\mathrm{Ib}+\mathrm{Ic})
\end{aligned}
$$

or, in vector matrix form
\(\left.\left|$$
\begin{array}{c}\mathrm{VAa} \\
\mathrm{VBb} \\
\mathrm{VCc}\end{array}
$$\right|=\left|\begin{array}{ccc}\mathrm{Za}+\mathrm{Zn} \& \mathrm{Zn} \& \mathrm{Zn} <br>
\mathrm{Zn} \& \mathrm{Zb}+\mathrm{Zn} \& \mathrm{Zn} <br>

\mathrm{Zn} \& \mathrm{Zn} \& \mathrm{Zc}+\mathrm{Zn}\end{array}\right|\)| I |
| :---: |
| l l |
| lc | \right\rvert\,

or

$$
\begin{equation*}
\mathrm{Vp}=\mathrm{Z} \mathrm{Ip} \tag{2-38}
\end{equation*}
$$

Applying symmetrical component transformation, we get

$$
\text { Avs }=\mathrm{Z} \text { Ip }
$$

or

$$
\begin{equation*}
\mathrm{Vs}=\mathrm{A}^{-1} \mathrm{ZA} \mathrm{Is}=\mathrm{Zs} \text { Is } \tag{2-39}
\end{equation*}
$$

where
$\mathrm{Zs}=\mathrm{A}^{-1} \mathrm{ZA}=$ symmetrical component impedance matrix.

$$
\begin{aligned}
& Z_{\mathrm{s}}=\frac{1}{5}\left[\begin{array}{ccc}
1 & \alpha & \alpha^{2} \\
1 & \alpha^{2} & \alpha \\
1 & 1 & 1
\end{array}\right]\left[\begin{array}{ccc}
Z_{\mathrm{a}}+Z_{\mathrm{n}} & Z_{\mathrm{n}} & Z_{\mathrm{n}} \\
Z_{\mathrm{n}} & Z_{\mathrm{b}}+Z_{\mathrm{n}} & Z_{\mathrm{n}} \\
Z_{\mathrm{n}} & Z_{\mathrm{n}} & Z_{\mathrm{c}}+Z_{\mathrm{n}}
\end{array}\right]\left[\begin{array}{ccc}
1 & 1 & 1 \\
\alpha^{2} & \alpha & 1 \\
\alpha & \alpha^{2} & 1
\end{array}\right] \\
& Z_{\mathrm{s}}=\left[\begin{array}{lll}
\frac{1}{3}\left(Z_{\mathrm{a}}+Z_{\mathrm{b}}+Z_{\mathrm{c}}\right) & \frac{1}{3}\left(Z_{\mathrm{a}}+\alpha^{2} Z_{\mathrm{b}}+\alpha Z_{\mathrm{c}}\right) & \frac{1}{3}\left(Z_{\mathrm{a}}+\alpha Z_{\mathrm{b}}+\alpha^{2} Z_{\mathrm{c}}\right) \\
\frac{1}{5}\left(Z_{\mathrm{a}}+\alpha Z_{\mathrm{b}}+\alpha^{2} Z_{\mathrm{c}}\right) & \frac{1}{2}\left(Z_{\mathrm{a}}+Z_{\mathrm{b}}+Z_{\mathrm{c}}\right) & \frac{1}{( }\left(Z_{\mathrm{a}}+\alpha^{2} Z_{\mathrm{b}}+\alpha Z_{\mathrm{c}}\right) \\
\frac{1}{3}\left(Z_{\mathrm{a}}+\mathrm{c}^{2} Z_{\mathrm{b}}+\alpha Z_{\mathrm{c}}\right) & \frac{1}{\left(Z_{\mathrm{a}}+\alpha Z_{\mathrm{b}}+\alpha^{2} Z_{\mathrm{c}}\right)} \frac{\frac{1}{3}\left(Z_{\mathrm{a}}+Z_{\mathrm{b}}+Z_{\mathrm{c}}\right)+3 Z_{\mathrm{n}}}{}
\end{array}\right]=
\end{aligned}
$$

or

Zs is computed below:

$$
\left.\mathrm{Zs}=1 / 3\left|\begin{array}{ccc}
1 & \alpha & \alpha^{2}  \tag{10-40}\\
1 & \alpha^{2} & \alpha \\
1 & 1 & 1
\end{array}\right|\left|\begin{array}{ccc}
\mathrm{Za}+\mathrm{Zn} & \mathrm{Zn} & \mathrm{Zn} \\
\mathrm{Zn} & \mathrm{Zb}+\mathrm{Zn} & \mathrm{Zn} \\
\mathrm{Zn} & \mathrm{Zn} & \mathrm{Zc}+\mathrm{Zn}
\end{array}\right| \begin{array}{ccc}
1 & 1 & 1 \\
\alpha^{2} & \alpha & 1 \\
\alpha & \alpha^{2} & 1
\end{array} \right\rvert\,
$$

Substituting Zs from Eq. (2-40) in Eq. (2-39), we can write

where the symmetrical component self and mutural impedances are defined as

$$
\begin{aligned}
& \mathrm{Z} 11=\mathrm{Z} 22=1 / 3(\mathrm{Za}+\mathrm{Zb}+\mathrm{Zc}) \\
& \mathrm{Z} 00=1 / 3(\mathrm{Za}+\mathrm{Zb}+\mathrm{Zc})+3 \mathrm{Zn} \\
& \mathrm{Z} 12=\mathrm{Z} 20=\mathrm{Z} 01=1 / 3\left(\mathrm{Za}+\alpha^{2} \mathrm{Zb}+\alpha \mathrm{Zc}\right) \\
& \mathrm{Z} 10=\mathrm{Z} 21=\mathrm{Z} 02=1 / 3\left(\mathrm{Za}+\alpha \mathrm{Zb}+\alpha^{2} \mathrm{Zc}\right)
\end{aligned}
$$

It is immediately seen from above that the voltage drop of any particular sequence will be produced by currents of all sequence (positive, negative and zero). Therefore, the method of symmetrical component is no simpler that three-phase circuit analysis. However, there is a redeeming feature due to the fact that the components of a power
system present balanced impedences and a fault imbalannces an otherwise balanced system. For a balanced system

$$
\mathrm{Za}=\mathrm{Zb}=\mathrm{Zc}=\mathrm{Zt}
$$

The symmetrical component impedence matrix now simplifies to

$$
\mathrm{Zs}=\left|\begin{array}{ccc}
\mathrm{Zt} & 0 & 0  \tag{2-41}\\
0 & \mathrm{Zt} & 0 \\
0 & 0 & \mathrm{Zt}+3 \mathrm{Zn}
\end{array}\right|=\text { a diagonal matrix }
$$

We can now write Eq. (2-39) in expanded form as

$$
\begin{align*}
& \text { VAa } 1=\mathrm{Zt} \text { Ia } 1=\mathrm{Z} 1 \text { Ia } 1 \\
& \text { VAa } 2=\mathrm{Zt} \text { Ia } 2=\mathrm{Z} 2 \text { Ia } 2  \tag{2-42}\\
& \text { VAa } 0=(\mathrm{Zt}+3 \mathrm{Zn}) \text { Ia } 0=\text { Z } 0 \text { Iao }
\end{align*}
$$

2-6 SEQUENCE IMPEDENCES OF SYNCHRONOUS MACHINE
$\mathrm{Zl}=\mathrm{Z1}=$ positive sequence impedence
$\mathrm{Z} 2=\mathrm{Z} 1=$ negative sequence impedence
$\mathrm{Z} 0=\mathrm{Z} 1+3 \mathrm{Zn}=$ zero sequence impedence

## a) Positive sequence impedence and network

Since a synchronous machine is designed with symmetrical winding, it has induced emfs of positive sequence only, i.e. no negative or zero sequence voltages are induced in it. When the machine carries positive sequence currents only. The armature reaction field caused by positive sequence currents rotate at synchronous speed in the same direction as the rotor, i.e. it is stationary with respect to field excitation. The machine equivalently offers a direct axis reactance whose value reduces from subtransient reactance (Xd") to trasient reactance ( $\mathrm{Xd}^{\prime}$ ) and finally to state (synchronous)reactance (Xd) as the short circuit transient progresses in time. If armature resistance is assumed negligible, the positive sequence of the machine is

$$
\begin{align*}
\mathrm{Z1} & =j \mathrm{Xd} \mathrm{"}^{\prime} \text { (if } 1 \text { cycle trasient is of interest) }  \tag{2-46}\\
& =j \mathrm{Xd} \mathrm{~d}^{\prime} \text { (if 3-4 cycle transient is of interest) }  \tag{2-47}\\
& =j \mathrm{Xd} \text { (if steady state value is of interest) } \tag{2-48}
\end{align*}
$$

If the machine short circuit takes place from unloaded conditions, the terminal voltage constitutes the positive sequence volatge; on the other hand, if the short circuit from loaded conditions, the voltage behind appropriate reactance (subtransient, trasient or synchronous) constitutes the positive sequnce voltage.

Fig. 2-12a shows the three-phase positive sequence network model of a synchronous machine. Zn does not appear in the model as $\mathrm{In}=0$ for positive sequence currents. Since it is a balanced network it can be reprsented by the single-phase network model of Fig. 2-12b for purposes of analysis. The reference bus for a positive sequence network is at neutral potential. Further, since no current flows from ground to neutral, the neutral is at ground potential.


With refernce to Fig. 2-12b, the positive sequence voltage of terminal a with respect to the reference bus is given by

$$
\begin{equation*}
\mathrm{Va} 1=\mathrm{Ea}-\mathrm{Z} 1 \mathrm{Ia} 1 \tag{2-49}
\end{equation*}
$$

## b) Negative Sequence I mpedence and Network

It has already been said that a synchronous machine has zero negative sequnce induced volyage. With the flow of negative sequence currents in the stator a rotating field
is created which rotates in the opposite direction to that of the positive sequence field and, therefore, at double synchronous speed with respect to rotor. Currents at double the stator frequency are therefore induced in rotor field and damper winding. In sweeping over the rotor surface, the nagative sequence mmf is alternately presented with reluctances of direct and quadrature axes. The nagative sequence impedance presented by the machine with consideration given to the damper windings, is often defined as

$$
\begin{equation*}
\mathrm{Z} 2=\mathrm{j}\left(\mathrm{Xq}{ }^{\prime \prime}+\mathrm{Xd} \mathrm{~A}^{\prime \prime}\right) / 2 \tag{2-50}
\end{equation*}
$$

Negative sequence network model of a synchronous machine, on a three-phase and single-phase basis are shown in Fig. 2-13a and b, respectively. The reference bus is of course at neutral potentioal which is the same as ground potential.

From Fig. 2-13b the negative sequence votalge of terminal a with respect to refernce bus is

$$
\begin{equation*}
\mathrm{Va} 2=-\mathrm{Z} 2 \mathrm{Ia} 2 \tag{2-51}
\end{equation*}
$$


c) Zero Sequence Impedence and Network

We state once again that no zero sequence voltages are induced in a synchronous machine. The flow of zero sequence current creates three mmfs which are in time phase but are distributed in space phase by $120^{\circ}$. The resultant air gap field casued by zero sequence is therefore zero. Hence, the rotor winding present leakage reactance only to the flow of zero sequence currents.

Zero seqeunce network models on a three- and single-phase basis are shown in Fig. 2-14a and $b$. In Fig. 2-14a the current flowing in the impedance Zn between nautral and ground is $\operatorname{In}=3 \mathrm{Ia} 0$. The zero sequence voltage of terminal a with respect to ground, the refernce bus is therefore

$$
\begin{equation*}
\mathrm{Va} 0=-3 \mathrm{ZnIa} 0-\mathrm{Z} 0 \mathrm{~g} \mathrm{Ia} 0 \tag{2-52}
\end{equation*}
$$

where Z 0 g is the zero sequence impedance per phase of the machine. Sine the singlephase zero sequence network of Fig. 2-14b carries only per phase zero sequence current, its total zero sequence impedance must be

$$
\begin{equation*}
\mathrm{Z} 0=3 \mathrm{Zn}+\mathrm{Z} 0 \mathrm{~g} \tag{2-53}
\end{equation*}
$$

in order for it to have the same voltage from a to refernce bus. The refernce bus here is, of course, at ground potential.

From Fig. 2-14b zero sequence voltage of point a with respect to the refernce bus is

$$
\begin{equation*}
\mathrm{Va} 0=-\mathrm{Z} 0 \mathrm{I} a 0 \tag{2-54}
\end{equation*}
$$




Order of Values of Sequence Impedence of a Synchronous Generator

Typiacl values of seqeunce impedence of a turbo-generator rated $5 \mathrm{MVA}, 6.6 \mathrm{KV}$,
$3,000 \mathrm{rmp}$ are:

$$
\begin{aligned}
& \mathrm{Z} 1=12 \% \text { (subtransient) } \\
& \mathrm{Z} 1=20 \% \text { (transient) } \\
& \mathrm{Z} 1=110 \% \text { (synchronous) } \\
& \mathrm{Z} 2=12 \% \\
& \mathrm{Z} 0=5 \%
\end{aligned}
$$

For typiacl values of positive, negative and zero sequence reactancees of a synchronous machine refer to Table 9.1.

A fully transposed three-phase line is completely symmetrical and therefore the per phase impedance offered by it is independent of the phase sequence of a balanced set of currents. In other words, the impedances offered it to positive sequence current are identical.

When only zero sequence flow in a trasmission line, the currents in each phase are identical in both magnitude and phase angle. Part of these currents return via the ground, while the set rest return through the overhead groung wires. The ground wires being grounded at several towers, the return currents in the ground wires are not necessarily uniform along the entire length. The flow of zero sequence currents through the transmission lines, ground wires and ground creates a magnetic field pattern which is very different from that caused by the flow of positive or negative sequence currents where the currents have a phase different of $120^{\circ}$ and the return currents is zero. The zero sequence impedence of a transmission line also accounts for the ground impedence ( $\mathrm{Z} 0=\mathrm{Z} 10+$ 3 Zg 0 ). Since the ground impedence heavily depends on soil conditions, it is essential to make some simplifying assumptions to obtain analytical results. The zero sequence impedence of transmission line usually ranges from 2 to 3.5 times the positive sequence impedence. This ratio is on the higher side for double circuit lines without ground wires.

## 2-8 SEQUENCE IMPEDENCE AND NETWORK OF TRANSFORMERS

It is well known that all present day installations have three-phase transformers since they entail lower initial cost, have smaller space requirements and higher efficiency

The positive sequence series impedence of a transformers equals its leakage impedence. Since a transformers is a static device, the leakage impedence does not change with alteration of phase sequence of balanced applied voltage. The transformer negative sequence impedence is also therefore equal to its leakage reactance. Thus, for a transformer

$$
\begin{equation*}
\mathrm{Z} 1=\mathrm{Z} 2 \text { = Zleakage } \tag{2-55}
\end{equation*}
$$

Assuming such transformer connection that zero sequence currents can flow on both sides, a transformer offers a zero sequence impedence which may differ slightly from the corresponding positive and negative sequence values. It is, however, normal practice to assume that the series impedence of all sequence are equal regardeless of the type of transformer.

The zero sequence magnetizing current is somewhat higher in a core type than in a shell type transformer. This difference does not matter as the magnetizing current of a trasformer is always neglected in short circuit analysis.

Above a certain rating $(1,000 \mathrm{kVA})$ the reactance and impedence of a transformer are almost equal and are therefore not distinguished.

## Zero Sequence Networks of Transformers

Before considering the zero sequence network of various types of transformer connections, three important observations are made
(i) When magnetizing current is neglected, transformer primary would carry current only if there is current flow on secondary side.
(ii) Zero seqeunce currents can flow in the legs of star connection only if the star point is grounded which provides the necessary return path for zero sequence currnts. This fact is illustrated by Fig. 2-15a and b.

(a) Ungrounded star

(b) Grounded star

(iii) No zero sequence cûrrents can flow in the lines connected to a delta connection as no return path is available for currents. Zero sequence currents can, however, flow in the legs of a delta-such currents are caused by the p resence of zero sequence volatgews in the delta connection. This fact is illustrated by Fig. 2-16


Let us now consider various types of transformer connection.

## Case1: Y-Y transformer ba nk with any one neutral grounded

If any one of the two neutrals of a Y-Y tranformer is ungrounded, zero sequence currents cannot flow in the ungrounded star and consequently, these cannot flow in the grounded star. Hence, an open circuit exists in the zero sequence network between H and L, i.e. between the two parts of the system connected by the transforemer as shown in Fig. 2-17.


Case 2: Y-Y transformer bank both neutrals grounded

When both the neutral of a Y-Y transformer are grounded, a path through the transformer exists for zero sequence currents in both windings via the two grounded
neutrals. Hence, in the zewro sequence network $H$ and $L$ are connected by the zero sequence impedance of the transformer as shown in Fig. 2-18.


## Case 3: Y- $\Delta$ transformer bank with grounded $Y$ neutral

If the neutral of star side is grounded, zero sequence currents can flow in star becasue a path is available to ground and the balancing zero sequence currents can flow in delta. Of cours no zero sequence currents can flow in the line on the delta side. The zero sequence network must therefore have a path from the line H on the star side through the zero sequence impedance of the transformer to the reference bus, while an open circuit must exist on the line L side of delta (see Fig. 2-19). If the star neutral is grounded through Zn , an impedednce 3 Zn apperas in series with Z 0 in the sequence network.


## Case 4: Y- $\Delta$ transformer bank with ungrounded star

This is the special case of case 3 where the neutral is grounded through $\mathrm{Zn}=\propto$. Therefore no zero sequence current can flow in the transformer windings. The zero sequence network then modifies to that shown in Fig. 2-20.


## Case 5: $\Delta-\Delta$ transformer bank

Since a delta circuit provides no return path, the zero sequence currents cannot flow in or out of $\Delta-\Delta$ transformer; however, it can circulate in the delta windings. Therefore, there is an open circuit between H and L and Z 0 is connected to the refernce
bus on both ends to account for any circulating zero sequence current in the two deltas (see Fig. 2-21).


## 2-9 CONSTRUCTION OF SEQENCE NETWORK OF A POWER SYSTEM

The positive sequence network is constructed by examination of one-line diagram of the system. It is to be noted that positive sequence voltages are present in synchronous machines (generator and motors) only. The transition from positive sequence network to negative network is straight forword. Since the positive and negative sequence impedence are identical for static elements (lines and transformers), the only change necessary in positive sequence network to obtain negative sequence network is in respect of synchronous machines. Each machine is represented by its negative sequence impedence, the negative sequence voltage being zero.


The refernce bus for positive and negative sequence networks is the system neutral. Any impedence connected between a neutral and ground is not included in these sequence networks as neither of these sequence cuurent can flow in such an impedence.

Zero sequence subnetworks for various parts of a system can be easily combine to form complete zero sequence network. No voltage sources are present in the zero sequence network. Any impedance included in generator or transformer neutral becomes three times its value in a zero sequence network. The procedure for drawing sequence network is illustrsted through the following examplpes.

Example 2-3 A $25 \mathrm{MVA}, 11 \mathrm{kV}$, three-phase generator has a subtrasient reactance of $20 \%$. The generator supplies two motors over a transmission line with transformers at both ends as shown in the one-line digram of Fig. 2-22. The motors have rated inputs of 15 and 7.5 MVA, both 12 kV with $25 \%$ subtransient reactance. The three-phase transformers are both rated $30 \mathrm{MVA}, 10.8 / 121 \mathrm{kV}$, connection $\Delta-\mathrm{Y}$ with leakage reactance of $10 \%$ each. The series reactance of the line is 100 ohms. Drdw the positive and negative sequence networks of the system with reactance marked in per unit.

Assume that the negative sequence reactance of each machine is equal to its subtransient reactance. Omit resistances. Select generator rating as base in the generator circuit.

Solution A base of $25 \mathrm{MVA}, 11 \mathrm{kV}$ in the generator circuit requiers a 25 MVA base in all other circuit and following voltage bases.

Transmission line voltage base $=11 \times 121 / 10.8=123.2 \mathrm{kV}$
Motor voltage base $=123.2 \times 10.8 / 121=11 \mathrm{kV}$
The reactance of transformers, line and motors are converted to pu values on appropriate bases as follows:

$$
\begin{aligned}
& \text { Transformers reactance }=0.1 \times 25 / 30 \times(10.8 / 11)^{2}=0.0805 \mathrm{pu} \\
& \text { Line reactance }=100 \times 25 /(123.2)^{2}=0.164 \mathrm{pu} \\
& \text { Reactance of motor } 1=0.25 \times 25 / 15 \times(10 / 11)^{2}=0.345 \mathrm{pu} \\
& \text { Reactance of motor } 2=0.25 \times 25 / 7.5 \times(10 / 11)^{2}=0.69 \mathrm{pu}
\end{aligned}
$$

The required positive sequence network is presented in Fig. 2-23.


Since all the negative sequence reactance of the system are equal to the positive seqeunce reactances, the negative sequence network is identical to the positive sequence network but for the omission of voltage sources. The negative sequence network is drown in Fig. 2-24.


Example 2-4 For the power system whose one-line diagram is shown in Fig. 2-25 sketch the zero sequence network.


Example 2-5 Draw the zero sequence network for the system discribed in example 2-3. Assume zero sequence reactances for the generator and motor of 0.06 per unit. Current limiting reactance of 2.5 ohms each are connected in neutral of the generaot and motor No. 2. The zero sequence reactance of the transmission lines is 300 ohms .

Solution The zero sequence reactance of the transformers is equal to its positive sequence reactance. Hence

Transformers zero sequence reactance $=0.0805 \mathrm{pu}$
Generatorr zero sequence reactance $\mathrm{s}=0.06 \mathrm{pu}$
Zero sequence reactance of motor $1=0.06 \times 25 / 15 \times(10 / 11)^{2}$

$$
=0.082 \mathrm{pu}
$$

Zero sequence reactance of motor $2=0.06 \times 25 / 7.5 \times(10 / 11)^{2}$

$$
=0.0164 \mathrm{pu}
$$

Reactance of currents limiting reactors $=2.5 \times 25 /(11)^{2}=0.516 \mathrm{pu}$
Reactance of current limiting reactor included in zero sequence network

$$
=3 \times 0.516=1.548 \mathrm{pu}
$$

Zero sequence reactance of transmission line $=300 \times 25 /(123.2)^{2}$

$$
=0.494 \mathrm{pu}
$$



The zero sequence network is shown in Fig.2-27.


## CHAPTER 3

## SYMMETRICAL FAULT ANALYSIS

## 3-1 INTRODUCTION

The symmetrical short circuits faults arecaused in the system accidentally through insulation failure of equipment or flashover of lines initiated by lightning stroke or through accidental faulty operation. The system must be protected against flow of heavy short sircuits currentsby disconnecting the faulty part of the system by means of circuits breaker operated by protective relaying.

The majority of the system faults are not three-phases faults but faults involving one line to ground or occasionally two lines to ground. These are unsymmetricall faults. Through the symmetrical faults are rare, the symmetrical fault analysis must be carried out, as this type of fault generally leads to most severe fault current flow against which the system must be protected. Symmetrical fault analysis is, of course, simpler to carry out.

A power network comprises synchronous generators, transformers, lines and loads. Thogh the operating conditions at the time of fault are important, the lods can be neglected during fault, as voltage dip very low so that currents drawn by loads can be neglected in comparison to fault current.

The syndronous generator during short circuit has a characteristic time-varying behaviour is .In the event of a short circuits, the flux per pole undergoes dynamic change with associated transient in damper and field windings. The reactance of the circuits model of the machine changes in the first few cycles from a low subtransient
reactance to a higer transient value, finally settling at a still higher synchronous (steady state) value. Depending up on the arc interruption time of circuit breakers, a suitable reactance value is used for the circuit model of synchronous generators for short circuit analysis.

For selecting a circuit breaker we must, therefore, determint the initial current in the transient that flows at the time of circuit interruption.

## 3-2 TRANSIENT ON A TRANSMISSION LINE

i) The line is fed from constant voltage source.
ii) Short circuit takes place when the line is unloaded.
iii) Line capacitance is nigligible and the line can be represented by a jumped $R L$ series circuit. So our circuit will be such as in figure 3.1


The parameter $\alpha$ controls the instant on the voltage wave when short circuit occurs after short circuit here

$$
\begin{aligned}
& i=i_{s}+i_{t} \\
& i_{s}= \\
& 2=\frac{\sqrt{2 V}}{|Z|} \sin (w t+\alpha-\theta) \\
& =\left(R^{2}+w^{2} L^{2}\right)^{1 / 2}<\left\langle\theta=\tan ^{-1} \frac{w L}{R}\right\rangle \\
& i_{t}=\text { Transient current } \\
& =i_{s}(0)_{e}-(R / L / 2 t \\
& =\frac{\sqrt{2 V}}{|Z|} \sin (\theta-\alpha) e^{-(R / T)} \\
& i=\frac{\sqrt{2 \bar{V}}}{|Z|} \sin (w t+\alpha-\theta)+\frac{\sqrt{2 V}}{|Z|} \sin (\theta-\alpha) e^{-(R / L) t} \\
& \text { Symmetrical short DC off - set curnt } \\
& \text { circuit current }
\end{aligned}
$$

In power system terminology, the sinusoidal steady state current is called the symmetrical short circuit current unindirectional transient component is called the DC off set current, unsymmmetrical till the transient decays.

The maximum momentary short circuit current $\boldsymbol{i}_{m m}$ corresponds to the first peak. If the decay of transient current in this short time is neglected

$$
\begin{equation*}
i_{m m}=\frac{\sqrt{2 V}}{|Z|} \sin (\theta-\alpha)+\frac{\sqrt{2 V}}{|Z|} \tag{3.2}
\end{equation*}
$$

Since transmission line resistance is small, $\theta \approx 90^{\circ}$.

$$
\begin{equation*}
i_{m m}=\frac{\sqrt{2 V}}{|Z|} \cos \alpha+\frac{\sqrt{2 V}}{|Z|} \tag{3.3}
\end{equation*}
$$

This has the maximum possible value for $\alpha=0$, i.e.
short circuit occurring when the voltage wave is going through zero.
Thus

$$
\begin{align*}
i_{m m(\max \text { possible) })}= & 2 \frac{\sqrt{2 V}}{|Z|}  \tag{3.4}\\
= & \text { twice the maximum of symmetrical short circuit } \\
& \quad \text { current (doubling effect) }
\end{align*}
$$

For a selection of circuit breakers momentary short circuit current is taken corresponding to its maximum posiible value (a safe choice).

The next question is 'waht is the current to be interrupted?. As has been pointed out earlier, the modern day circuit breakers are designed to interrupt the current in the first few cycles (faive cycles or less). With reference to figure 3.2 it means that when the current is interrupted, the DC off-set (it) has not yet died out and so contributes the current to be interrupted.


FIGURE 3.2 Waveform of short circuit current on a transmission line

## 3-3 SHORT CIRCUIT OF A SYNCRONOUS MACHINE (ON NO LOAD)

Under steady state short circuit conditions, the armature reaction of a syncronous generator produces a demagnetizing flux. In terms of a circuit this effect is modelled as a reactance $\mathrm{X}_{\mathrm{a}}$ in series with the induced emf. This reactance when combined with the leakage reactance Xl of the machine is called synchronous reactance Xd . Armature resistance being small can be geglected.

(a) Steady state short circuit model of a synchronous machine


Consider now the sudden short circuit of a synchronous generator initially operating under open circuit conditions. The machine undergoes a transient in all the three phases finally ending up in steady state conditions. The circuit breaker must, of course, interrupt the current much before steady conditions are reached. Immediately up on short circuit, the DC off-set currents appear in all the three phases, each with a different magnitude since the point on the voltage wave at which short circuit occurs is different for each phase. These DC off-set currents are accounted for separately on an impirical basis and, therefore, for short circuit studies we need to concentrate our attention on symmetrical short circuits current only.Immediately in the event of short circuit, the symmetrical short circuit current is limited only by the leakage reactance of the machine. Since the air gap flux cannot change intantaneously to counter the demagnetization of the armature short circuit current, currents appear in the field winding as well as in the damper winding in a direction to help the main flux. These currents decay in accordance with the winding time constants. The time constant of the damper winding which has low leakage inductance is much less than of the field winding, which has high leakage inductance. Thus during the initial part of the short circuit, the damper and field windings have
transformer currents induced in them so that in the circuit model their reactances Xr of feild winding and Xdw of damper winding appear in parallel square with Xa in figure 3.3b. As the damper winding currents are first to die out, Xdw effectively becomes open circuited and at alater stage Xt becomes open circuited. The machine reactance thus changes from the parallel combination of $\mathrm{Xa}, \mathrm{Xf}$ and Xdw during the initial period of the short circuit to Xa and Xf in parallel in the middle period of the short circuit, and finally to Xa in steady state. The reactance presented by the machine in the initial period of the short circuit.

$$
\begin{equation*}
X_{l}+\frac{1}{\left(1 / X_{a}+1 / X_{f}+1 / X_{d w}\right)}=X_{d}^{*} \tag{3.5}
\end{equation*}
$$

Subtransient reactance of the machine; while the reactance effective after the damper winding currents have died out,

$$
\begin{equation*}
X_{d}^{\prime}=X_{l}+\left(X_{a} \| X_{f}\right) \tag{3.6}
\end{equation*}
$$

Transient reactance. Of course the reactance under steady conditions is the syncronous reactance of the machine. Obviously $\mathrm{Xd}^{\prime \prime}<\mathrm{Xd}^{\prime}<\mathrm{Xd}$. The machine thus offers a time-varying reactance which changes from $\mathrm{Xd}^{\prime \prime}$ to $\mathrm{Xd}^{\prime}$ and finally to Xd .

(a) Symmetrical short circuit armature current in synchronous machine

(b) Envelope of synchronous machine symmetrical short circuit current

Fig. 3.4

The short circuit current can be devided into three periods-Initial subtransient period when the current is large as the machine offers subtransient reactance, the middle transient period where the machine offers transient reactance, and finally the steady state period when the machine offers synchronous reactance.

In term of the oscillogram, the currents and reactances discussed aboe we can write

$$
\begin{align*}
& |I|=\frac{O a}{\sqrt{2}}=\frac{\left|E_{g}\right|}{X_{d}}  \tag{3.7a}\\
& \left|I^{\prime}\right|=\frac{O b}{\sqrt{2}}=\frac{\left|E_{g}\right|}{X_{d}^{\prime}}  \tag{3.7b}\\
& \left|I^{\prime \prime}\right|=\frac{O c}{\sqrt{2}}=\frac{\left|E_{g}\right|}{X_{d}^{\prime}} \tag{3.7c}
\end{align*}
$$

where
$|\mathrm{I}|=$ Steady state current (rms)
$\left|I^{\prime}\right|=$ Transient current (rms) excluding DC component
$\left|\mathrm{I}^{\prime \prime}\right|=$ Subtransient currnt (rms) excluding DC component
$\mathrm{X}_{\mathrm{d}} \quad=$ Direct exis synchronous reactance
$\mathrm{X}^{\prime}{ }_{\mathrm{d}} \quad=$ Direct axis transient reactance
$\mathrm{X"d}=$ Direct axis subtransient reactance
$\left|E_{g}\right|=$ Per phase no load voltage

To find the transient $\mathrm{Oa}, \mathrm{Ob}, \mathrm{Oc}$ reactance we can calculate the intercept Ob .

$$
\begin{aligned}
& \Delta i^{\prime \prime}=\Delta i_{0}^{\prime \prime} \exp \left(-t / \tau_{d w}\right) \\
& \Delta i^{\prime}=\Delta i_{0}^{\prime} \exp \left(-t / \tau_{f}\right)
\end{aligned}
$$

Where $\tau_{\mathrm{dw}}$ and $\tau_{\mathrm{f}}$ are respectively damper, and field winding time constants with $\tau_{\mathrm{dw}} \ll \tau_{f .}$ are time $t \gg \tau_{d w}, \Delta i^{\prime \prime}$ practically dies out and we can write

$$
\log \left(\Delta i^{\prime \prime}+\Delta i^{\prime}\right)_{t>d \mathrm{dw}} \approx \log \Delta i^{\prime}=-\Delta i_{0}^{\prime} t / \tau_{f}
$$


$\log \left(\Delta i^{\prime \prime}+\left.\Delta i^{\prime}\right|_{t>d d w} \approx \log \Delta i^{\prime}=-\Delta i_{0}^{\prime} / \tau f\right.$

The plot of $\log \left(\Delta i^{\prime \prime \prime}+\Delta i^{\prime}\right)$ versus time for $t>\tau d w$ therefore, becomes a stright line with a slope of $\left(-\Delta i_{0}^{\prime}\right.$ As the stright line portion of the plot is extrapolated, te intrcept corresponding to $t=0$ is
$\left.\Delta i^{\prime}\right|_{t=0}=\left.\Delta i_{0}^{\prime} \exp \left(-t / \tau_{f}\right)\right|_{t=0}=\Delta i_{0}^{\prime}=a b$


Table 3.1 Typical/values of synchronous machine reactances

| Type of <br> Machine | Turbo-Alternator <br> (Turbine Generator) | Salient Pole <br> (Hydroelectric) | Syncronous <br> Comensator <br> (Condensor/ | Syncronous <br> Motors* |
| :--- | :--- | :--- | :--- | :--- |
| Capacitor) |  |  |  |  |

Through the machine reactance are dependent upon magnetic station, the values of reactances normally lie within certain predictable limits for different types of machines. Table 3.1 gives typical values of machine reactances which can be used in fault calculations and in stability studies.

Normally both generator and motor subtransient reactances are used to determine the momentary current following on occurance of a short circuit. To decide the interrupting capacity of circuit breakers, except those which open instantaneously, subtransient reactance is used for generators and transient reactance for synchronous motors.

Since the system in on no prior to occurance of the fault, the voltage of the two generators are indentical and are equal to 1 pu .

$$
\begin{aligned}
\text { Total impedance }= & (j 1.5 \| j 1.25)+(j 1.0)+(0.744+j 0.99) \\
& +(j 1.6)+(0.93+j 0.55) \\
= & 1.674+j 4.82=5.1<70.8^{\circ} \quad p u
\end{aligned}
$$

$$
\begin{aligned}
& I s c=\frac{1<0}{5.1<70.8^{\circ}}=0.196<-70.8^{\circ} \quad p u \\
& i_{\text {base }}=\frac{100 * 10^{3}}{\sqrt{3} * 6.6}=8,750 \quad \mathrm{~A}
\end{aligned}
$$

$$
\text { Isc }=0.196 * 8,750=1,715 \quad \mathrm{~A}
$$

Total Impedance between F and Il kV bus

$$
\begin{aligned}
& =(0.93+j 0.55)+(j 1.6)+(0.744+j 0.99)+(j 1.0) \\
& =1.674+j 4.14=4.43<76.8^{\circ} \quad \mathrm{pu}
\end{aligned}
$$

Voltage at ll kV bus $=4.43<67.8^{\circ} * 0.196<-70.8^{\circ}$

$$
=0.88<-3^{\circ} p u=0.88 * l l=9.68 \mathrm{kV}
$$

## Example 3-1

A 25 mva, 11 kV generator with $\mathrm{Xd"}$ connected through a transformer, line and a transformer to a bus that supplies three identical motors as shown in Fig 3.8. Each motor has $\mathrm{Xd}^{\prime \prime}=25 \%$ and $\mathrm{Xd}^{\prime}=30 \%$ on a base of $5 \mathrm{MVA}, 6.6 \mathrm{kV}$. The three-phase rating of the step-up transformer is $25 \mathrm{MVA}, 11 / 66 \mathrm{kV}$ with a leakage reactance of $10 \%$ and that of the step-down transformer is $25 \mathrm{MVA}, 66 / 6.6 \mathrm{kV}$ when a three-phase fault occurs at the point F. For the specified fault, calculate
a) The subtransient current in the fault,
b) The subtransient cutrrent in the breaker,
c) The momentary cutrrent in breaker B, and
d) The current to be interrupted by breaker B in five cycles.

Given: Reactance of the transmission line $=15 \%$ on a base of $25 \mathrm{MVA}, 66 \mathrm{kV}$.
Assume that the system is operating on no load when the fault occurs.


Fig 3.8

## Solution

Choose a system base of 25 MVA.
For Generator voltage base of 11 kV , line voltage base is 66 kV and motor voltage base is 6.6 kV .
a) For each motor

$$
X^{\prime \prime} d m=j 0.25 * \frac{25}{5}=j 1.25 . p u
$$

Line, transformers and generator reactance are already given on paper base values.

The circuit model of the system for fault calculations is given in Fig 3.9a. The system being initially on no load, the generator and motor induced emfs are indentical. The circuit can be therefore be reduced to that of fig. 3.9 b and then to fig. 3.9 c . Now

$$
\begin{aligned}
& I s c=3 * \frac{1}{j 1.25}+\frac{1}{j 0.55}=-4.22 \mathrm{pu} \\
& \text { Base current in } 6.6 \mathrm{kV} \text { circuit }=\frac{25 * 1,000}{\sqrt{3} * 6.6}=2,187 \mathrm{~A}
\end{aligned}
$$

$$
I s c=4.22 * 2,187=9,229 \mathrm{~A}
$$

b) From Fig 3.9c, current through circuit breaker B is

$$
\begin{aligned}
\operatorname{Isc}(B) & =2 * \frac{1}{j 1.25}+\frac{1}{j 0.55}=-j 3.42 \\
& =3.42 * 2,187=7,479.5 \mathrm{~A}
\end{aligned}
$$



Fig. 3.9
c) For fin ding mon mentary current through the breaker we must add the DC off-setcurrent to the symmetrical subtransient current obtained in part (b). Rather than calculating the DC off-set current allowance is made for it on an empirical basis.

$$
\text { Momentary current through breaker } \begin{aligned}
\mathrm{B} & =1.6 \times 7,473.5 \\
& =11,967 \mathrm{~A}
\end{aligned}
$$

d) To compute the current to be interrupted by the breaker, motor subtransient reactance ( $\mathrm{Xd} \mathrm{C}^{\prime \prime}=\mathrm{j} 0.25$ ) is replaced by transient reactance $\left(\mathrm{Xd}^{\prime}=\mathrm{j} 0.30\right)$.
$\mathrm{X}^{\prime} \mathrm{d}$ (motor) $=\mathrm{j} 0.3 \times 25 / 5=\mathrm{j} 1.5 \mathrm{pu}$

The reactances of the circuit of fig 3.9 c now modify to that of fig. 3.9 d . Cutrrent (symmetrical) to be interrupted by the breaker (as shown by arrow)

$$
=2 * \frac{1}{j 1.5}+\frac{1}{j 0.55}=3.1515 \mathrm{pu}
$$

Allowance is made for the DC off-set value by multiplying with a factor of 1.1 ( sec .3 .5 ). Therefore, the current to be interrupted is

$$
1.1 * 3.1515 * 2,187=7,581
$$

### 3.4 SHORT CIRCUIT OF A LOADED SYNCHRONOUS MACHINE

The analysis of short circuit on a loaded synchronousmachine is complicated and is beyond the scope of this project. WE shall however, present here the methods of computing short circuitcurrent when short circuit occurs under loaded condition.

The circuit model of a synchronous generator operating under steady conditions supplying a load current $I^{\circ}$ to the bus at the terminal voltage $V^{\circ}$. Eg is the induced emf under loaded condition and Xd is the direct axis synchronous reactance of the machine. When short circuit occurs at the terminals of this machine, the circuit model to be used for computing short circuit current is given for subtransient current, and for transient current. The induced emfs to be used in these models are given by


Fig. 3.10 Circuit model of a loaded machine

$$
\begin{align*}
& E_{g}^{*}=V^{o}+j I^{o} X_{d}^{\prime \prime}  \tag{3.8}\\
& E_{g}^{\prime}=V^{o}+j I^{o} X_{g}^{\prime} \tag{3.9}
\end{align*}
$$

The voltage $\mathrm{E}_{8}$ is known as the voltage behind the subtransient reactance and the voltage $\mathrm{E}_{\mathrm{g}}^{\prime}$ is known as the voltage behind the transient reactance. In fact, if $I^{\circ}$ is zero (no load case), $\mathrm{E}_{\mathrm{g}}^{\prime}=E_{g}^{\prime}=E_{g}$, the no loads volatage, in which case the circuit model reduces discussed before.
(a) Circuit model for computing

(b) Circuit modal for compuling
transient current

Fig. 3.11

The synchronous motors have internal emfs and reactances similar to that of a generator except that the current direction is reserved. During short circuit conditions voltage can be replaced by similar circuit models except that the voltage behind subtransient/transient reactance is given by

$$
\begin{align*}
& E_{m}^{*}=V^{o}+j I^{o} X_{d}^{\prime}  \tag{3.10}\\
& E_{m}^{\prime}=V^{o}+j I^{o} X_{d}^{\prime} \tag{3.11}
\end{align*}
$$

Whenever we are dealing with short circuit the synchronous machines (generator and motors) are replaced by their voltage behind subtransient (transient) reactance is series with sulransient (transient) reactance. The rest of the network behind passive remains unchanged.

## EXAMPLE 3-3

A synchronous generator and a synchronous motor each rated $25 \mathrm{MVA}, 11 \mathrm{kV}$ having $15 \%$ subtransient reactancare connected though transformers and line as shown in fig 3.12. The transformers are rated 25 MVA, $11 / 66 \mathrm{kV}$ and $66 / 11 \mathrm{kV}$ with leakage reactance of $10 \%$ eachTHe line has a reactance of $10 \%$ on a base of a $25 \mathrm{MVA}, 66 \mathrm{kV}$. The motor is drawing 15 MW at 0.8 power factor leading and a terminal voltage of 10.6 kV when symmetrşcal three-phase fault occurs at the motor terminals. Find the subtransient current in the generator, motor and fault


Fig 3.12

## Solution

All reactances are given on a base of 25 MVA and appropriate voltages.

$$
\begin{aligned}
& \text { Prefault voltage } V^{\circ} \frac{10.6}{11}=0.9636\left\langle O^{\circ} \mathrm{pu}\right. \\
& \begin{aligned}
\text { Loaded } & =15 \mathrm{MW}, 0.8 \mathrm{pf} \text { leading } \\
& =\frac{15}{25}=0.6 \mathrm{pu}, 0.8 \mathrm{pf} \text { leading }
\end{aligned} \\
& \text { Prefault current } I^{\circ}=\frac{0.6}{0.9636^{*} 0.8}\left\langle 36.9^{\circ}=0.7783\left\langle 36.9^{\circ} \mathrm{pu}\right.\right.
\end{aligned}
$$

Volatage behind transient reactance (Generator)

$$
\begin{aligned}
E_{g}^{*} & =0.9636<0^{\circ}+j 0.45 * 0.7783<36.9^{\circ} \\
& =0.7536+j 0.28 \quad p u
\end{aligned}
$$

Voltage behind subtransient (Motor)

$$
E_{m}^{*}=0.9636<0^{\circ}-j 0.15^{*} 0.7783<36.9^{\circ}
$$

$$
=0.0336-j 0.0933 \quad p u
$$

The prefault equivelant circuit is shown in Fig. 9.12b. Under faulted condition (Fig. 9.12c)
$I_{m}^{*}=\frac{0.7536+j 0.2800}{j 0.45}=0.6226-j 1.6746 \mathrm{pu}$
$I_{m}^{*}=\frac{0.0336-j 0.0933}{j 0.15}=-0.6226-j 6.8906 \quad \mathrm{pu}$

$$
=0.0336-j 0.0933 \quad p u
$$

current in fault
$I_{f}=I_{g}^{*}+I_{m}^{*}=-j 8.5653 \quad p u$
Base current $($ gen $/$ motor $)=\frac{25 * 10^{3}}{\sqrt{3} * 11}=1,3122.2 \quad p u$

Now
$I_{g}^{*}=1,312.2(0.6226-j 1.6746)=(816.4-j 2,197.4) \quad p u$
$I_{m}^{*}=1,312.2(-0.6226-j 6.8906)=(-816.2-j 9,041.8) \quad A$
$I=-j 11,239 \quad A$

## SHORT CIRCUIT (SC) CURRENT COMPUTATION THROUGH THE THEVENIN THEOREM

An alternate method of computing short circuits is through the application of the Thevenin theorem. This method is faster and easily adopted to systematic computation for large networks.

Consider synchronous generator feeding a synchronous motor over a line. Figure 313 b shown the circuit model of the system under conditions of steady load. Fault computations are to be made for a fault at F , at the motor terminals. As a first step the circuit model is replaced by the one shown in fig 3.13 b .

As seen from FG the Thevenin equivalent circuit of fig. 3.13 b is drawn in fig. 3.13c. It comprises prefault voltage V digree in series with the passive Thevenin impedance network. It is noticed that the prefault current I digree does not appear in the passive Thevenin impedance network. It is therefore to be remembered that this current must be accounted for by superposition after the SC solution is obtained through use of the Thevenin equivalent.

Consider now a fault at F through an impedance Z . Figure 3.13 shows the Thevenin eqivalent of the system feeding the fault impedance. We can immediately write
$I^{f}=\frac{V^{\circ}}{j X_{\text {Th }}+Z^{f}}$

Current caused by fault in generator circuit
$\Delta I_{\mathrm{g}}=\frac{X_{d m}^{\prime}}{\left(X_{d g}^{\prime}+X+X_{d m}^{\prime}\right.} I^{f}$


FIGURE 3.13 Computation of SC current by the thevenin equivalent

Current Caused by fault in motor circuit
$\Delta I_{m}=\frac{X_{d g}^{\prime}+X}{\left(X_{d m}^{\prime}+X+X_{d g}^{\prime}\right)} I^{\delta}$

Postfault currents and voltage are obtained as follows by superposition:
$I_{g}^{f}=I^{\circ}+\Delta I_{g}$
$I_{m}^{f}=-I^{\circ}+\Delta I_{m}$ (in the direction of $\Delta I_{m}$
Postfault voltage

$$
\begin{equation*}
V^{f}=V^{\circ}+\left(-j X_{T h} I^{f}\right)=V^{\circ}+\Delta V \tag{13.16}
\end{equation*}
$$

Where $\Delta V=-j X_{t h} I^{\wedge} f$ is the voltage of the fault point $F^{\prime}$ on the Thevenin passive network (with respect to the reference bus $G$ ) caused by the flow of fault current $I^{\wedge} f$.

An obsevation can bae made here. Since the prefault current flowing out of fault point $F$ is always zero, the postfault current out of $F$ is independent of load for a given prefault voltage at F .

The above approach to SC computation is summarized in the following four steps:

## STEP 1:

Obtain steady state solution of loaded system

## STEP 2:

Replace reactances of synchronous machines by subtransient/transient values. Short circuit all emf sources. The result is the passive Thevenin network.

## STEP 3:

Excite the passive network of step 2 at the fault point by negative of prefault voltage in series with the impedance. Compute voltages and currents at points of interest.

## STEP 4:

Postfault currents and voltages are obtained by adding results of steps 1 and 3 .

The following assumptions can be saely made in SC computations leading to considerable computational simplification:

Assumption 1: All prefault voltage magnitudes are 1 pu .
Assumption 2: All prefault currents are zero.

The first assumtion is quite close to actual conditions as under normal operation all voltages (pu) are nearly unity.

The changes in current caused by short circuit are quite large, of the order of 10 20 pu and are purely reactive; whereas the prefault load currents are almost purely real. Hence the total postfault current which is the result of the two currents can be taken in magnitude equal to the large component (caused by the fault).

This justifies assumption 2.

Let us illustrate the above method by recalculating the results of example 3.3

The circuit model for the system of example 3.3 for computation of postfault condition is shown in fig 3.14


Figure 3.14 F is the fault point on the passive Thevenin network
$I^{f}=\frac{V^{\circ}}{(j 0.15 \| j 0.45}=\frac{0.9636^{*} j 0.60}{j 0.15 * j 0.45}=-j 8.565 \quad \mathrm{pu}$
Change in generator current due to fault,
$\Delta I_{g}=-j 8.565 * \frac{j 0.15}{j 0.60}=-j 2.141 \quad p u$
Change in motor current due to fault,
$\Delta I_{m}=-j 8.565 * \frac{j 0.45}{j 0.60}=-j 6.424 \quad p u$
To these changes we add the prefault current to obtain the subtransient current in machines.
Thus
$I_{g}^{\prime \prime}=I^{\circ}+\Delta I_{g}=(0.623-j 1.674) \quad p u$
$I_{m}^{*}=-I^{\circ}+\Delta I_{m}=(-0.623-j 6.891) p u$

Which are the same as calculated already.

We have thus solved Example 9.3 alternatively through the Thevenin theorem and superposition. This, needed, is a powerful method for large networks.

### 3.5 SELECTION OF CIRCUIT BREAKERS

Two fo the circuit breaker rating require the computation of SC current are: rated momentary current and rated interrupting current. Symmetrical momentary SC current is obtained by using subtransient reactances for synchronous machines. Momentary current (rms) is then calculated by multiplying the symmetrical momentary current by a factor of 1.6 to account for the presence of DC off-set current.

Symmetrical current to be interrupted is computed by using subtransient reactances for synchronous generator and transient reactances for synchronous motors-induction motors are neglected. The DC off-set value to be added to obtain the current to be interrupted is accounted for by multiplying the symmetrical SC current by a factor as tabulated below:

| Circuit breaker Speed | Multiplying Factor |
| :--- | :---: |
| 8 cycles or slower | 1.0 |
| 5 cycles | 1.1 |
| 3 cycles | 1.2 |
| 2 cycles | 1.4 |

If SC MVA (explained below) is more than 500, the above multiplying factors are increased by 0.1 each. The multiplying factor for air breakers rated 600 V or lower is 1.25 .

The current that a circuit breaker can interrupt is inversely proportional to the operating voltage over a certain rage, i.e.

Amperes at operating voltage
$=$ amperes at rated voltage * rated voltage operating voltage

Of course, operating voltage cannot exceed the maximum design value. Also, no matter how low the voltage is, the rated interrupting current cannot exceed the rated maximum interrupting current. Over this range of voltage, the product of operating voltage and interrupting current is constant. It is therfore logical as well as convenient to express the circuit breaker rating in terms of SC MVA that can be interrupted, defined as

Rated interrupting MVA (three-phase) capacity

Obviously, rated MVA interrupt capacity of circuit breaker is to be more than (or equal to) the SC MVA required to be interrupted.

For the selection of a circuit breaker for a particular relation, we must find the maximum possible SC MVA to be interrupted with respect to type and location of fault and generating capacity (also synchronous motor load connected to th system. A threephase fault though rare is generaly the one which gives the highest SC MVA and circut breaker must be capable of interrupting it. An exception is an LG (line-to-ground) fault close to a synchronous generator. In a simple system the fault location which gives the highest SC MVA may be abvious but in a large system verious possible location must be tried out to obtain the highest SC MVA requiring repeated SC computations.

### 3.6 ALGORITHM FOR SHORT CIRCUIT STUDIES

So far we have carried out short circuit calculations for simple systems whose passive networks can be easily reduced. In this article we extend our study to large systems. In order to apply the four steps of short circuit computation developed earlier to large systems, it is necessary to evolve a systematic general algorithm so that digital computer can be used. The first step towards short circuit computation is to obtain prefault voltages at all buses and currents in all lines through a load flow study. Let us indicate the prefault bus voltage vector as
$V_{B U S}^{0}=\left[\begin{array}{c}V_{1}^{0} \\ V_{2}^{0} \\ \cdots \\ \cdots \\ V_{m}^{0}\end{array}\right]$

Let us assume that the rth bus is faulted through a fault impedance $\mathrm{Z}^{\wedge} \mathrm{f}$. The postfault bus voltage vector will be given by
$V_{B U S}^{f}=V_{B U S}^{0}+\Delta V$
where $\Delta \mathrm{V}$ is the vector in changes in bus voltage caused by the fault.

As step 2, we draw the passive Thevenin network of the system with generators replaced by transient/subtransient reactances with their emfs shorted (Fig. 3.21)


Fig 3.21


As per step 3 we now excite the passive Thevenin network with $-\mathrm{V}^{\circ}{ }_{\mathrm{r}}$ in series with $\mathrm{Z}^{\wedge} \mathrm{f}$ as in figure 3.21. The vector $\Delta \mathrm{V}$ comprises the bus voltages of this network.

Now

$$
\begin{equation*}
\Delta V=Z_{B U S} J^{F} \tag{3.20}
\end{equation*}
$$

WHERE

$$
Z_{B U S}=\left[\begin{array}{rrr}
Z_{11} & \ldots \ldots \ldots . . Z_{1 n} \\
\ldots & \ldots \\
Z_{n 1} & \ldots \ldots \ldots \ldots Z_{n n}
\end{array}\right]=\text { Bus impedance matrix of the passive Thevenin network(3.21) }
$$

$J^{f}=$ bus injection vector
Since the network is injected with current $-I^{f}$ only at the rth bus,

In the above relationships, $\mathrm{V}^{\circ} 1 \mathrm{~s}$, the prefault bus voltages are obtained from a load flow study. YBUS matrix is assembled for the network of Fig. 3.20 (this includes the generator reactances) using the rules given The ZBUS matrix is then obtained by inversion of the YBUS matrix.

Postfault currents can now be obtained in all branches of the network. The postfault current $\mathrm{I}_{\mathrm{ij}}$ in the (ij)th line (with positive current direction from i to j ) is given by
$I_{i j}^{f}=y_{i j}\left(V_{i}^{f}-V_{j}^{f}\right)$
Where $\mathrm{Y}_{\mathrm{ij}}$ is the admittance of the (ij)th line e .

