NEAR EAST UNIVERSITY

GRADUATE SCHOOL OF APPLIED AND SOCIAL SCIENCES

FUZZY PORTFOLIO SELECTION USING GENETIC ALGORITHM

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Ph.D. Thesis

Department of Computer Information Systems

Nicosia-2006

Mustafa Menekay : Fuzzy Portfolio Selection Using Genetic Algorithm

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ABSTRACT

The business environment is fully characterized with uncertainties. The decision maker, when selecting a portfolio, frequently deals with insufficient data. In this conditions, to minimize risk and maximize future returns proper portfolio model must be designed. The use of deterministic and stochastic models leads to unrealistic results. Using fuzzy models allows the removal of this drawback and also permits the incorporation of the expert knowledge. In this thesis the application of fuzzy theory to portfolio selection is presented.

The state of application problem of fuzzy theory to portfolio selection has been given. It was found that the existing portfolio selection models are mainly oriented to partial fuzzification of deterministic linear programming models (mainly to model uncertainty in the return) without the incorporation of fuzzy risk. These models do not always allow effective management of the conflict between expected return rate and risk and suffer from high computational complexity resulted from using the classical fuzzy linear programming approach. In this thesis a fuzzy portfolio selection model based on fuzzy linear programming solved by genetic algorithm is proposed. Fuzzy logic is utilized in the estimation of expected return and risk. Using fuzzy logic, managers can extract useful information and estimate expected return by using not only statistical data, but also economical and financial behaviours of the companies and business strategies.

In the formulated fuzzy portfolio model, fuzzy set theory gives chance of possibility trade-off between risk and return: This is obtained by assigning satisfaction degree between criteria and constraints and defining tolerance for the constraints in order to obtain goal value in objective risk function. Using the formulated fuzzy portfolio model, a Genetic Algorithm (GA) is applied to find optimal values of risky securities with a computational complexity than by existing methods.

The modelling of portfolio model have been performed by usingstatistical data taken from Istanbul Stock Exchange. The simulation of fuzzy portfolio model has been done by using Matlab Package. The comparative results of fuzzy model with deterministic portfolio model satisfy the efficiency of the proposed method.

ACKNOWLEDGEMENTS

First and foremost, my sincere thanks go to Dr. Suat İ. Günsel for providing me with the opportunity to start this thesis and for "putting on the pressure" which enabled me to complete it.

Also, I would like to greatest thank my supervisor, Assoc. Prof. Dr. Rahib Abiyev for great help and whom made it all possible. Once again thank you.

I would like to thank my supervisor Prof. Dr. Rafik Aliev for his kind help and advise.

I also would like express my great thanks Prof Dr. Şenol Bektaş and Prof. Dr. Fahreddin M. Sadıkoğlu for their provided extensive support during my Ph.D. thesis.

I would like to thank Prof. Dr. Berna Kocaman for her kind help.

I would like to thank Prof. Dr. Umit Hassan for his support and trust.

I would like to thank Prof. Dr. Hasan Ali Bıçak for his kind help and advise to completed my thesis.

I would like to thankful my father Mehmet and my mother Eşmene for their support, help , trust, and provide me everything. Without them and god helping I can not reach this moment.

Also I will not forget my friends Şerife, Ahmet, Tülen and Mehmet . Thank you

By the way thanks for my cousins Saed and Mert.

And last but not least, my deepest thanks and appreciation to my family, especially my dearest wife Hayriye, my son Uzay and my daughter Eşmene who shared my burden of completing this thesis during the long, sometimes stressful, but meaningful and productive process.

TABLE OF CONTENTS

List of Tables **Page**

List of Figure **Page**

List of Figures in Appendix **Page**

INTRODUCTION

The financial and economic performance of companies are determined by analyzing company financial statements, that includes such important information as profit margins, return to stockholders, growth in sales, return on equity and minimization of risks. This information is useful for investors, creditors and other external users to plan their work. In this thesis we take a closer look at how information can be combined, analyzed and modelled to find many important financial decisions in investment. This is achieved by the development of portfolio model. Portfolio selection model determines optimal strategy for finding optimal values of portfolio proportions for each of the securities.

Business managers make financial investments to improve their income and to meet the target (desired) return for the assets. To implement this idea they use different optimization models. These models are basically probabilistic models. The presence of uncertainty in investment and insufficiency of information about problem domain forces managers to be cautious in expressing their views. The uncertainty in finance is traditionally dealt with by probabilistic approaches. The probability theory is one of the main tools used for analyzing uncertainty. But it cannot describe uncertainty completely since there are such uncertainty factors that differ from the random ones and a number of empirical studies show limitation of using probabilistic approaches in characterizing the uncertainty in finance. The frequently changeable economical and political situations influence company activities and financial markets. For this reason in most cases information content of investors allows to express their views "fuzzy" or vague in terms to structure portfolios, so that the target return, which is assumed to be higher than risk-free rate, is met. This condition requires managers to hold portfolio of risky assets. Such portfolios may be structured on the base of imprecise views.

Fuzzy set is a powerful tool used to describe an uncertain environment with vagueness, ambiguity or some other type of fuzziness, which are always involved in not only the finance but also the behaviour of the financial managers' decisions. In a financial optimization model using fuzzy approaches, quantitative analysis and qualitative analysis, the experts' knowledge and the managers' subjective opinions can be better integrated.

During the construction of a portfolio model it is important to have statistical data taken from companies financial operations that determine average return and standard or absolute deviation defining risk of investment. Additionally an investor can take into account and complete his knowledge with other information, such as economical and financial behaviours of the companies, government policies, business strategies, etc. Also subjective factors, such as manager's accuracy when focusing on certain portfolio of assets may be useful. This information depends on many factors that can be estimated quantitatively by expert perceptions. All qualitative and quantitative information are dominant to define expected return of portfolio. In order to deal with all mentioned information it is proposed to use fuzzy set theory proposed by Zadeh [1].

There are number researches works about construction of fuzzy portfolio selection. In these fuzzy portfolio optimization models securities expected returns are accepted as arithmetic means of historical returns. That is, the expected returns are determined with the crisp values. However, this would not lead to the best result when there are uncertainties in finance and/or available data are insufficient. In these conditions, the investor usually formulates his view about the expected return by using fuzzy terms. In this paper the fuzzy expected return is introduced and used in construction of portfolio selection model.

The fuzzy portfolio selection is performed in several complex situations. These are its difficult uncertain nature, selection of assets by their expected profitability. This is complex optimization problem. The existing fuzzy portfolio selection models are mainly oriented to partial fuzzification of deterministic linear programming models. The solutions of these models are suffer from high computational complexity resulted from the conversion of fuzzy linear programming models into standard crisp one. In this situation a better solution can be obtained by applying directed random search method such as Genetic Algorithms (GA). The use of GA in portfolio optimization allows us to find global optimal solution with a computational complexity less than by existing method.

The aim of this thesis is the construction of fuzzy portfolio selection model in investment based on GA that will help to minimize portfolio risk for a given level of return (or maximize portfolio return for a given level of risk). To construct such a portfolio model we have to take into account the character and source of uncertainty, which are appropriate to financial operations of most companies. In this thesis the fuzzy set theory is applied to deal with such types of uncertainties.

The constructed portfolio will take into consideration expected return, accessible risk preference of investors and nature of assets. Taking into account the above mentioned, the expected values of returns and accessible portfolio risk are determined by expert perception in fuzzy form. While determining these values manager can use qualitative (statistical) and quantitative information. Using the fuzzy values of these parameters the construction of fuzzy optimization model is considered.

The thesis consists of introduction, four chapters, a conclusion, a list of references and appendices.

In chapter I a review on portfolio selection problem is given, the developed portfolio selection model is described, The state of art understanding of fuzzy portfolio selection is given, and the research problem is described.

In chapter II, the state of application problem of fuzzy portfolio selection for security investment is given. The concepts of fuzzy expected return and fuzzy risks are explained. The fuzzy constraints and fuzzy objective function are formulated. Fuzzy portfolio selection model is presented.

In chapter III the application of Genetic Algorithm (GA) for solving deterministic and fuzzy portfolio selection problem are considered. GA operators are used for finding optimal values of securities. The main operations of genetic algorithms (GA) that are often used for optimization problem have been described. The implementation of algorithm for deterministic and fuzzy portfolio selection is given.

In chapter IV the computer modelling of fuzzy portfolio selection is given. Software development for fuzzy portfolio selection is described. As a numerical example the application of fuzzy portfolio selection to Istanbul Stock Exchange is considered. Analysis of the efficiency of obtained results is presented. Comparative results of fuzzy portfolio selection with deterministic one are described.

Conclusion includes the important results obtained from the thesis.

CHAPTER I

REVIEW ON USAGE OF FUZZY TECHNOLOGY IN PORTFOLIO CONSTRUCTION

1.1 Overview

Construction of portfolio starts with defining future performances of available securities and construction of the model that performs the choice of portfolio. In this chapter state of art of portfolio selection for security investment problem will be considered. The application problem of fuzzy technology to portfolio selection problem will be analyzed. Research problem statement will be presented.

1.2 State of application problem of fuzzy systems for portfolio selection

The main goal of portfolio construction in investment is to determine, as accurately as possible, the combination of assets that will produce the most satisfactory outcome for the investor, over a defined investment period. Each individual investor from the point of view of investment management tries to increase income by making optimal decision in his investment policy. Investors have different financial goals, different levels of risk tolerance and different personal preferences. These characteristics are often defined as objectives and constraints. Mainly in objectives and constraints return and risk that is handled can take parts. It's a balancing act between risk and return with each investor having unique requirements, as well as a unique financial outlook – essentially a constrained utility maximization objective.

The finding relation between these parameters is the major problem of portfolio management in finance. The ideal goal in portfolio management is to create an optimal portfolio derived from the best risk-return opportunities available given a particular set of risk or return constraints. To be able to make a decision, it must be possible to quantify the degree of risk in particular opportunities. In a investment portfolio the more shares or assets held, the greater the risk reduction. Factors that may influence risk in any given investment vehicle include uncertainty income, interest rate, inflation, exchange rates, tax rates, the state economy . In addition, an investor will assess the risk of given investment (portfolio) with the context of other types of investments. One way to control portfolio risk is via diversification, whereby investments are made in a wide variety of assets so that the exposure to the risk of any particular security is limited. If an investor owns shares in only one company, that investment will fluctuate depending on the factors influencing that company. If that company goes bankrupt, the investors may lose all of the investment. If, however, the investor owns shares in several companies in different sector, then the likelihood of all of those companies going bankrupt simultaneously is greatly diminished. Thus, diversification reduces risk. However, it is impossible to eliminate all risk completely even with an extensive diversification. Although bankruptcy risk has been considered here, the same principle applies to other forms of risk.

The goal is to hold a group of investments or securities within a portfolio within a portfolio potentially to reduce the risk level suffered without reducing the level of return. To measure the success of a potentially diversified portfolio, covariance and correlations re considered. Covariance measures to what degree the returns of two risky assets move in tandem. A positive covariance means that the returns of the two assets move together whilst a negative covariance means that they move in opposite direction. Covariance for two investments x and y is defined as :

$$\text{Cov}(x,y) = \sum p(\overline{x}-x)(\overline{y}-y).$$

Where p is the probability.

Trying to minimize risk it is not enough to invest in many securities. It is necessary to avoid investing in securities with high covariance among themselves. The investors would diversify across industry because firms in difference industries especially industries in difference economic characteristics have lower covariances than firms within an industry [1].

As mentioned above, in portfolio selection an investor must make choice about optimal distribution of assets among different selection. Designing the correct portfolio model of assets requires modern, powerful and reliable mathematical tools and programs. The finding optimal values of expected return and risk measured by the variance are main problem of portfolio selection. It is the aim of the portfolio manager to find a portfolio that maximizes expected return under given risk level or a portfolio that minimizes risk under given return level. Unfortunately asset having high returns usually have high risk.

There are number of research works about construction of portfolio models. The first contribution in portfolio analysis was given by Prof. Harry Markowitz [2]. According to his model, investors facing portfolios with the same risk will choose the one with bigger profitability, and facing portfolios with the same profitability will choose the one with the minimum risk. This model is called mean-variance model of Markowitz. The core of the Markowitz mean-variance model is to take the expected return of a portfolio as the investment return and the variance of the expected returns of a portfolio as the investment risk. According to Markowitz for a given return rate, one can derive the minimum investment risk by minimizing the variance of a portfolio; or for a given risk level which the investor can tolerate, one can derive the maximum returns by maximizing the expected returns of a portfolio. In this model for given $x_i \in S$ securities, where S is the set of investment alternatives (securities), and a level ρ of expected return the model is formulated as follows:

$$z = \frac{1}{T}\sum_{i \in S} \sum_{j \in S} \sigma_{ij} x_i x_j \rightarrow \text{minimize} \tag{1.1}$$

Subject to

$$\sum_{j-1}^{n} R_j x_j \geq \rho D,$$

$$\sum_{j-1}^{n} x_j = D$$

$$x_1, x_2, \ldots, x_n \geq 0 \tag{1.2}$$

where xi represents the percentage of money invested in security i , Rj =E(Rj) is random variable representing the return of security i, σij is the covariance between returns of security i and of security j, D is total portfolio expenditure, ρ is minimal rate of return required by investor. The most commonly adopted assumption for this model is multivariate normally distributed rates of return. This model is known as quadratic programming model.

The main input data for the Markowitz mean-variance model are expected returns and variance of expected returns of these securities. Simplifying the number and types of the input data has been one of the main research topics in this field. Although some breakthroughs, such as the Index Model, have been implemented, all of these methods have some drawbacks due to some known reasons [3].

Some problems arise when applying mean-variance model. These are following:

Increasing the number of securities of portfolios affects the increasing the size of the covariance matrix that will be difficult to estimate.

quadratic programming models are usually far more difficult to solve than correspondingly sized linear ones.

in mean variance model the rates of return of assets follow a multivariate normal distribution.

To overcome these disadvantages, some authors have suggested to transform the nonlinear portfolio problem into a linear one [4, 5], which avoids to compute second order moments, and then to solve it using linear programming algorithms. Following Sharpe's work on linear approximation to the mean-variance model, many attempts have been made to introduce risk measures which (for discrete random variables) result in solving Linear Programming (LP) problems. While the simplest LP computable risk measures of the mean absolute deviation or the Gini's mean absolute difference may be viewed as some approximations to the variance, shortfall or quantile risk measures are recently gaining more popularity in various financial applications.

Konno and Yamazaki proposed to use absolute deviation risk function as objective function and formulated mean absolute deviation (MAD) portfolio optimization [4]. The MAD model, a special case of the piecewise linear risk model, has been shown to be equivalent to the Markowitz model under the assumption that returns are multivariate normally distributed. That is, under this assumption, the minimization of the sum of absolute deviations of portfolio returns about the mean is equivalent to the minimization of the variance. They approximate the expected value of random variable by the average of the realization of the random variable over the T periods of the time horizon. This model support the following three arguments:

a) In the formulation of the MAD model, there is no requirement for the covariance matrix of asset returns,

b) the relative ease with which a linear program can be solved compared to a quadratic one- thus large scale problems can be solved faster and more efficiently,

c) mean absolute deviation portfolios have fewer assets- this fact implies lower transaction costs in portfolio revisions.

This model is formulated as

$$z = \frac{1}{T}\sum_{t-1}^{T}\left|\sum_{j=1}^{n}(r_{jt} - R_j)x_j\right| \rightarrow \text{minimize} \tag{1.3}$$

Subject to

$$\sum_{j-1}^{n} R_j x_j \geq \rho D,$$

$$\sum_{j-1}^{n} x_j = D$$

$$x_1, x_2, \ldots, x_n \geq 0 \tag{1.4}$$

The random variable Rj still represents the rate of return, while xj is the amount of money invested in security j. According to Konno and Yamazaki, rjt is the realization of the random variable Rj during the period t and is obtainable through historical data. Alternative models in which different scenarios for the rates of returns are taken into account are described in [5]. In particular, they assume that the mean of Rj can be estimated as

$$R_j = E[R_j] = \frac{\sum_{t-1}^{T} r_{jt}}{T} \tag{1.5}$$

where T is the length of the time horizon. By using auxiliary variables the model can be written as

$$z = \frac{1}{T}\sum_{t-1}^{T} y_t \rightarrow \text{minimize} \tag{1.6}$$

Subject to

$$y_t + \sum_{j=1}^{n}(r_{jt} - r_j)x_j \geq 0,\ \mathrm{t} = 1,...,\mathrm{T}$$

$$y_t - \sum_{j=1}^{n}(r_{jt} - r_j)x_j \geq 0,\ \mathrm{t} = 1,...,\mathrm{T}$$

$$\sum_{j-1}^{n} R_j x_j \geq \rho D,$$

$$\sum_{j-1}^{n} x_j = D$$

$$x_1, x_2, ..., x_n \geq 0 \tag{1.7}$$

The largest part of the portfolio selection models which have been proposed in the literature are based on the assumption of a perfect fraction ability of the investments in such a way that the portfolio fraction for each security could be represented by a real variable. In the real world, securities are negotiated as multiples of a minimum transaction lot (the so called rounds). As a consequence of considering rounds, solving a portfolio selection problem requires finding the solution of a mixed integer programming model. When applied to real problems, the tractability of the integer model is subject to the availability of algorithms able to find a good, even if not optimal, general mixed integer model including real characteristics of the problem has been presented in [5].

Based on mean absolute deviation model, Speranza and Mansini used deviation of the portfolio return below the average as the risk and formulated a semi absolute deviation portfolio selection model [6]. In this model investor penalizes the negative semi-absolute deviation instead of the absolute deviation. In [7] the Linear Programming solvable portfolio optimization models based on extensions of the Conditional Value at Risk (CVaR) measure is described. The models use multiple CVaR measures thus allowing for more detailed risk aversion modelling. The theoretical description of the models and their performances on the real-life data taken from the Milan Stock Exchange are given. The mathematical formulation of semiabsolute deviation portfolio selection model can be formulated as follows

$$z = \frac{1}{T}\sum_{t-1}^{T}\left|\min\{0, \sum_{j-1}^{n}(r_{jt} - R_j)x_j\}\right| \rightarrow \text{minimize} \tag{1.8}$$

Subject to

$$\sum_{j-1}^{n} R_j x_j \geq \rho D,$$

$$\sum_{j-1}^{n} x_j = D$$

$$x_1, x_2, ..., x_n \geq 0 \qquad (1.9)$$

Here Ri is average return in security j over the entire period, xj is portfolio allocation in security j over the entire period T. ρ is minimum rate of return, D is total budget invested in portfolio. rit is return of security j over period t.

By using auxiliary variables the model can be written as

$$z = \frac{1}{T}\sum_{t-1}^{T} y_t \rightarrow \text{minimize} \qquad (1.10)$$

Subject to

$$y_t + \sum_{j=1}^{n}(r_{jt} - r_j)x_j \geq 0, \ \mathrm{t} = 1,...,\mathrm{T}$$

$$\sum_{j-1}^{n} R_j x_j \geq \rho D,$$

$$\sum_{j-1}^{n} x_j = D$$

$$x_1, x_2, ..., x_n \geq 0 \qquad (1.11)$$

In [6] Speranza and Mansini deal with the portfolio problem with minimum transaction lots. They show that the problem of finding a feasible solution is, independently of the risk function, NP-complete. Moreover, given the mixed integer linear model, new heuristics are proposed which starting from the solution of the relaxed problem allow to find a solution close to the optimal one. The algorithms are based on the construction of mixed integer subproblems (using only a part of the securities available) formulated using the information obtained from the solution of the relaxed problem. The heuristics have been tested with respect to two disjoint time periods, using real data taken from the Milan Stock Exchange.

In an effort to final a solution for portfolio selection models the elements of sofcomputing have been effectively used. In [8] a new model for portfolio selection in which the expected returns of securities are considered as variables rather than as the arithmetic means of securities is purposed. A genetic algorithm is designed to solve the optimization problem which is difficult to solve with the existing traditional algorithms due to its nonconcavity and special structure. The obtained results of model that use a numerical example is compared with the results derived from the traditional model of Markowitz.

Different approaches are designed for solving portfolio selection problems and constructing efficient frontier of the portfolio [10-20].

The mentioned deterministic models are effectively used in cases where a sufficient amount of data can be collected. However, when there is uncertainty in finance, available data is insufficient then knowledge about problem area is incomplete and the constructed model based on probability theory does not give proper results.

The presence of uncertainty in finance allows managers to be cautious in expressing their views about portfolio assignment. The uncertainty in finance is traditionally dealt with probabilistic approaches. A number of research work show limitation of using probabilistic approaches in characterizing the uncertainty in finance reference. The probability theory cannot describe uncertainty completely since there are such uncertainty factors that differ from the random ones. Fuzzy set is a powerful tool used to describe an uncertain environment with vagueness, ambiguity or some other type of fuzziness, which are always involved in, not only the finance but also the behaviour of the financial managers' decisions.

When economical states of the companies that are involved in investment are characterized with uncertainties, then existing portfolio models that are based on probability theory do not give desired results. In such situations, during the construction of portfolio model, an investor besides taking statistical data from companies' financial operations can additionally take into account other information, such as economical and financial behaviours of the companies, government policies, business strategies, etc. in order to complete his or her knowledge. Moreover, there may be some subjective factors, such as manager's accuracy when focusing on certain portfolio on determined assets. This information depends on many factors that can be estimated quantitatively by expert perceptions. All qualitative and quantitative information are dominant to define expected return of portfolio. In order to deal with all mentioned it is proposed to use fuzzy set theory, proposed by Zadeh. In a financial optimization model using fuzzy approaches, quantitative analysis, qualitative analysis, the experts' knowledge and the managers' subjective opinions can be better integrated.

The reason to use fuzzy technology in portfolio selection is based on one's perception about environment, uncertain character of information. Fuzzy portfolio selection can be considered as general multiple objective multiple constraint fuzzy optimization. Fuzzy optimization is the collection of techniques that formulate optimization problems with flexible, approximate or uncertain constraints and goals. Some fuzzy optimization methods have been proposed in the literature in order to deal with different aspects of soft constraints. Zimmerman in [20] has considered the fuzzy optimization as a symmetric problem. In this formulation, fuzzy sets represent both the problem goals and the flexible (soft) constraints. In this formulation the fuzziness arises because of definition of fuzzy maximization and the approximate inequality. These are defined by fuzzy goal and fuzzy constraints. In this formulation the fuzzy goals and the constraints are aggregated to a single function that is maximized. This framework can handle crisp constraints as well as fuzzy constraints.

In [21] the fuzzy set is applied to solve optimization problem with soft constraints. In this work fuzzy optimization is divided into two categories: 1) To represent uncertainty in the constraints and the goals (objective functions), 2) To represent flexibility in the constraints and the goals. Here the satisfaction of the constraints and goals is used, where preference for different constraints and goals can be specified by the decision-maker. The difference in the preference for the constraints is represented by a set of associated weight factors, which influence the nature of trade-off between improving the optimization objectives and satisfying various constraints. The weighted satisfaction of the problem constraints and goals are demonstrated by using a simple fuzzy linear programming problem.

A number of research works is devoted for introducing and developing fuzzy portfolio optimization using linear programming. In [22] using vague goals for the expected return and risk the fuzzy portfolio selection problem is considered.

The possibility theory is applied to handle uncertainty and solve portfolio optimization problem [23, 24]. Two kinds of portfolio selection models are proposed based on fuzzy probabilities and possibility distributions, respectively, rather than conventional probability distributions in Markowitz's model. In proposed model possibility distribution is used to characterize experts' knowledge. Possibility distribution is identified using returns of securities associated with possibility grades offered by portfolio experts. A numerical example of a portfolio selection problem is given. However it is not always easy for an investor to specify possibility grades.

In [25] a fuzzy approach is proposed to repair infeasibility in portfolio optimization problem. The fuzzy optimization scheme for managing a portfolio in the framework of risk–return trade-of is considered. The performance on a numerical example is illustrated.

In [26, 27] α level procedure with Lagarange Multiplier method is used to solve fuzzy portfolio optimization problem. The securities values of the model are determined by using gradient method. The use of α level procedure with gradient method allows extension of the fuzziness of obtained results which decrease accuracy.

In [28] using linear interval programming the solution of portfolio selection is considered. Describing the uncertain returns of assets by intervals the portfolio modeling based on semiabsolute deviation measure of risk, which can be transformed to a linear interval programming model is presented.

In [29] an approach to portfolio selection using fuzzy decision theory is presented. The approach is such that a given target rate of return is achieved for an assumed market scenario. If the assumed market scenario turns out to be incorrect, the portfolio is guaranteed to secure a given minimum rate of return. The methodology is useful in the management of assets against given liabilities or in forming structured portfolios that guarantee a minimum rate of return.

In [30] the constraint on the level of return is fuzzified and the technique of fuzzy evolutionary programming is employed to select an optimal portfolio of securities with low risk and with highly acceptable level of total return. Experimental results show the method is highly effective. The problem of selecting a portfolio with low risk and with high probability of expected return is resolved in the same manner. In this fuzzy portfolio optimization models securities expected returns are accepted as arithmetic means of historical returns. That is, the expected returns are determined with the crisp values. But this is not the best result when there are uncertainties in finance and/or available data is insufficient. Under these circumstances, the investor usually formulates his view about expected return by using fuzzy terms.

In [31] the application of Fuzzy Theory to term based portfolio selection is considered. In order to maximize future returns, proper structuring of portfolio is highly critical. This thesis documents that fuzzy decision theory can be successfully applied to selection of securities to form short-term liquidity portfolio with varying maturity period. The results indicate that Fuzzy Logic aids in the extraction of useful information from sample dataset where a portfolio manager may have low confidence in his prediction. As an example a portfolio of treasuries such as Bonds, Notes and Bills, is used to emphasize how Fuzzy Logic can be utilized in deciding the distribution of securities across maturities of varying value, to maximize returns.

In [32] the modified S-curve membership function methodology and fuzzy linear programming are used in a real life industrial problem of mix product selection. This problem occurs in production planning management where a decision maker plays an important role in making decision in a fuzzy environment. A formulated approach it is used to find a good solution for the decision maker to make a final decision. An industrial application of fuzzy linear programming (FLP) through the S-curve membership function has been investigated using a set of real life data collected from a Chocolate Manufacturing Company. The problem of fuzzy product mix selection has been defined. The objective of this thesis is to find an optimal units of products with higher level of satisfaction with vagueness as a key factor. This problem has been considered because all the coefficient such as technical and resource variables are uncertain. Using 29 constraints and 8 variables a sufficiently big problem is considered. Since there are several decisions that were to be taken, a table for optimal units of products with respect to vagueness and degree of satisfaction has been defined to identify the solution with higher level of units of products and with a higher degree of satisfaction. Optimal units of products and satisfactory level have been computed using Fuzzy Linear programming (FLP) approach. The fuzzy outcome shows that higher units of products need not lead to higher degree of satisfaction. The result of this work indicates that the optimal decision depends on vagueness factor in the fuzzy system of mix product selection problem. Further more, the high level of units of products is obtained when the vagueness in the system is low.

Main progress in portfolio selection using fuzzy set theory is represented in [34]. The differences of fuzzy portfolio model from probabilistic portfolio model are given. The important of application of fuzzy set theory to the development of practical portfolio models is shown. It is emphasized that the uncertainty of the complex financial markets is not only influenced by random events. To describe the uncertainty of financial markets the integrated methodologies that combine fuzzy sets, probability theory and chaotic methods should be future investigated [34].

Analysis of existing research works demonstrate that, in practice due to large number of securities, the constraint on expected total return is tolerable. When the investment risk is high, it needs to be lowered. In this thesis the satisfaction degree is used between criteria and constraints in order to decrease risk to some value and to find a required optimal solution. The use of such an approach allows to construct more flexible model than existing ones and to make trade-off between risk and return.

In this thesis, by taking into consideration the above mentioned, and by using fuzzy expected return and flexible constraints the construction of fuzzy portfolio selection model is considered. The constructed portfolio model will take into consideration expected return and accessible risk preference of an investor.

The fuzzy portfolio selection is performed in several complex situations. These are its difficult uncertain nature, transcendence selection of assets by their expected profitability. This is a complex optimization problem. In this situation the traditional methods used for solving fuzzy portfolio optimization problems do not give fully satisfactory results. A better solution can be obtained by applying Genetic Algorithms (GA). The use of GA in portfolio optimization allows us to find global optimal solution. In this thesis the GA is used to solve formulated fuzzy portfolio optimization.

1. 3 Statement of research problem

When there is uncertainty in finance and/or available data is insufficient then the solving of portfolio selection problem becomes very difficult. The fuzzy set theory is applied within this thesis to manage risk–return trade-off and make a good decision. The constructed fuzzy portfolio model will allow investors to make optimal decision about securities. In this thesis to solve fuzzy portfolio selection problem the following steps are carried out

Analysis of different optimization models and formulating portfolio selection model for security investment problem

Defining fuzzy risk, fuzzy expected return and formulating fuzzy objective function and constraints in optimization model.

Developing fuzzy optimization model for portfolio selection.

Description GA operations, and developing efficient algorithm for deterministic portfolio optimization problem.

Using GA developing efficient algorithm for fuzzy portfolio optimization problem.

Developing computer model (software) for fuzzy portfolio selection.

Applying of fuzzy portfolio selection to Istanbul Stock Exchange and proving the efficiency of obtained results.

The development of fuzzy portfolio selection model will considerably increase the efficiency of portfolio management system and allow investors (managers) effectively manage their stocks that they want to make investment to companies.

1. 4 Summary

When economical states of companies are characterized with uncertainties, then existing deterministic portfolio models based on probability theory do not give desired results. In order to deal with uncertainties, fuzzy set theory is applied to construct portfolio model. The state of application problem of fuzzy set theory for construction of investment portfolio has been given. In this thesis to formulate fuzzy portfolio model an investor will take into account qualitative and quantities information characterizing companies behaviours. These are statistical data taken from companies' financial operations, economical and financial behaviours of the companies, government policies, business strategies. Using the formulated fuzzy portfolio model, a Genetic Algorithm (GA) is applied to find optimal values of risky securities. The steps of implementation of fuzzy portfolio selection model are presented.

CHAPTER II

FUZZY PORTFOLIO SELECTION FOR SECURITIES INVESTMENT

2.1 Overview

Designing correct portfolio of assets requires modern, powerful and reliable mathematical tools that take into account uncertainty of problem domain, insufficiency of information. In these conditions the probability model does not effectively describe the investigated problem and the accurate estimation of the input parameters. To construct a model, the use of human perception and intuition are needed. In this chapter the analysis and estimation of expected return and risk are considered. The fuzzy risk and fuzzy expected return are introduced. The satisfaction degree is introduced to provide the flexibility of constraints. The formulation of fuzzy optimization problem and construction of fuzzy portfolio selection model are presented.

2.2 Fuzzy Risk

The aim of portfolio management is to reduce or control risk. There are several different factors that cause risk and lead to variability of return on an individual investment. Factors that may influence risk in any given investment include uncertainty of income, interest rates, inflation, exchange rates, tax rates, the state of economy, default risk and liquidity risk. In addition, investors will assess the risk of given investment (portfolio) with the context of other types of investments. One way to control portfolio risk is via diversification, whereby investments are made in a wide variety of assets so that the exposure to the risk of any particular security is limited [36].

The traditional approach is used to model risky choices to describe choices involving risk in terms of their underlying probability distributions and associated utilities. In financial literature, risk is defined as a product of severity of loss and probability of loss.

Intuitively, risk exists when loss is possible and its financial impact is significant. In general, risk is evaluated qualitatively rather than quantitatively. In fact, in the real world, the possibility and financial significance of loss cannot be defined with precision.

In general, depending on the perception of risk, measures of risk can conditionally be divided into two groups [37-39]. The first group is classified as symmetric measure that quantifies risk in terms of probability-weighted dispersion of results around a specific reference point, usually the expected value. Measures in this category use negative as well as positive deviations from a pre-specified target. In this group the most well-known and widely applied risk measures are variance or standard deviation and the expected or mean absolute deviation (MAD). The second group is classified as asymmetric measures of risk. It comprises measures which quantify risk according to results and probabilities below reference points. Such risk measures include the Expected Value of Loss, the Semi-Variance proposed by Markowitz, Safety Risk, Value at Risk - VaR and its extension Conditional VaR (CVaR) and α-t criterion. A financial institution faces a second dilemma of deciding which of the two main risk metric categories - symmetric or asymmetric measures of risk – represent its attitude towards risk and, therefore, should be utilized. The simplest approach is that of comparing the performance relative to the portfolio's past history. This is achieved by computing the risk measure as a function of the portfolio composition and the random returns of the assets. Typically, the standard deviation would then reflect the deviation of the asset returns from the expected portfolio return. On the other hand, the portfolio performance can be measured relative to a benchmark index or an alternative investment opportunity. In this case, the risk measure is also a function of a target level of return. The standard deviation in this case would then reflect the deviation of the asset returns from the expected target return.

The main difference of the symmetric measures of risk, when compared with the asymmetric, is that returns above the pre-specified target are also included. In that case, the returns used to calculate the risk measures can take values between [-∝,+∝].

The two symmetric risk metrics are the Variance and MAD. The classical representation of variance desalts with measuring the spread of the expected returns relative to the average expected portfolio return.

$$\sigma^2 = \frac{1}{T}\sum_{t=1}^{T}(\bar{r} - r_t)^2 = E\{(\bar{r} - r)^2\} \quad \text{or} \quad \sigma^2 = E\{(R - r)^2\} \tag{2.1}$$

Here $\bar{r}$ is average value of returns, R is target value of returns.

Mean Absolute Deviation measures of risk can be represented as:

$$MAD = E\{|\,\bar{r} - r\,|\} \quad \text{or} \quad MAD = E\{|\,R - r\,|\} \tag{2.2}$$

Asymmetric measure can be defined by α-t model.

$$F_{\alpha}(R) = E\{(\max[0, R - r])^{\alpha}\},\ \alpha > 0 \tag{2.3}$$

where α characterize the moment of return distribution or may be taken as indicating different attitudes towards risk.

Safety First is a special case of the □-t risk when □→̃.

$$SR = E\{(\max[0, R - r])^{\alpha \to 0}\} \tag{2.4}$$

Expected Downside Risk can be obtained from α-t model. when □=1.

$$\overline{D} = E\{(\max[0, R - r])\} \tag{2.5}$$

If the target is set equal to the expected portfolio return then the measure can be viewed as a special case of the MAD risk measure where only the negative deviations from the target are considered. This measure called the Semi-MAD measure.

$$Semi - MAD = E\{(\max[0, \bar{r} - r])\} \tag{2.6}$$

The semi-variance is a special case of the □-t model, for □=2.

$$\sigma^{-2} = E\{(\max[0, R - r])^{2}\} \tag{2.7}$$

For □→∝ the □-t model defines the worst-case scenario

$$WCS = E\{(\max[0, R - r])^{\alpha \to +\infty}\} \tag{2.8}$$

The Value-at-Risk (VaR) of a portfolio at the β probability level is the left quantile of the losses of the portfolio, i.e, the lowest possible value such that the probability of losses less than VaR exceeds β ×100%. The VaR is defined by 1- β.

Traditionally, the major challenge in risk analysis is considered to estimate some probability distribution. It is true, if and only if the risks in a system are statistical risks. However, in many risk systems, randomness is just one of risk natures. A probability distribution is just a relation between events and probabilities of occurrence where an event and a probability value can be regarded as a state and an input, respectively. For many systems, it is impossible to precisely calculate the relation, and we face the problem of imprecise probability.

In many cases, it is difficult to obtain the equations for risk and all data are unnecessary to study the risk. Probabilistic methods simplify the procedure. However, it isn't reasonable to describe risk by probability theory. But sometimes there are sample problem, where the data is too scanty to make a decision in any classical approach. It means that it is difficult to obtain a precise relation between events and probabilities of occurrence.

In the decision theory risk may have a three-dimensional concept involving the following natures:

- Adverse outcome for individuals;
- Uncertainty over the occurrence, timing, site, or magnitude of adverse outcome;
- Complexity to show precisely by a state equation or a probability distribution.

When the available data is insufficient are, there is uncertainty in the environment then the probabilistic approach does not adequately describe the problem because there are such uncertainty factors that differ from the random ones. Fuzzy set is a powerful tool used to describe an uncertain environment with vagueness.

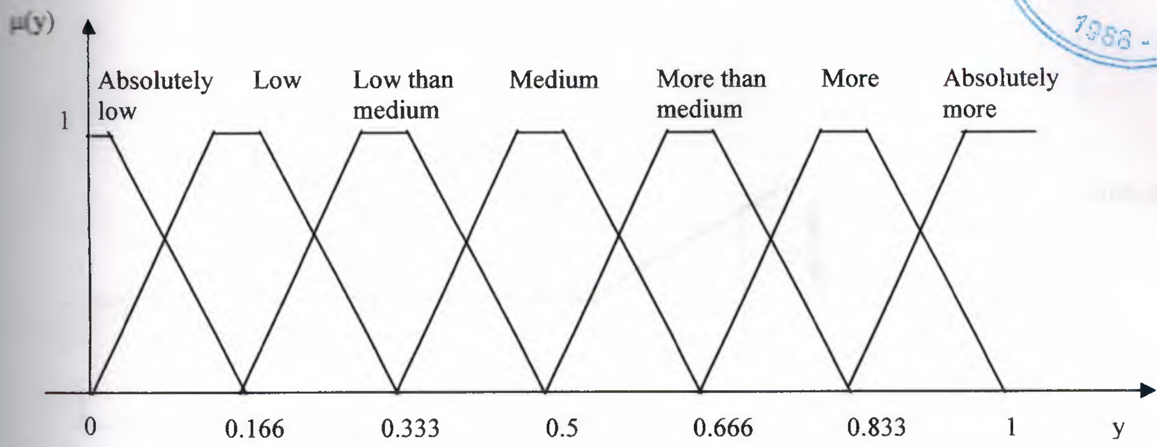
Risk is either precise or fuzzy. Risk is expressed in terms of the probability-risk only when a risk phenomenon can be studied by a probability method.

Fuzzy risk can be defined as an approximate representation of the risk describing a fuzzy relation between loss events and concerning factors. In this work the mathematical measure of fuzzy risk is taken as the fuzzy semi-variance model, defining fuzzy distance that determines deviation of the portfolio return below the expected return.

$$fuzzy\ risk = E\{(\max[0, \widetilde{R} - r])\} \qquad (2.9)$$

Here $\widetilde{R}$ is fuzzy value of target return

Figure 1. Linguistic terms assigned for fuzzy risk

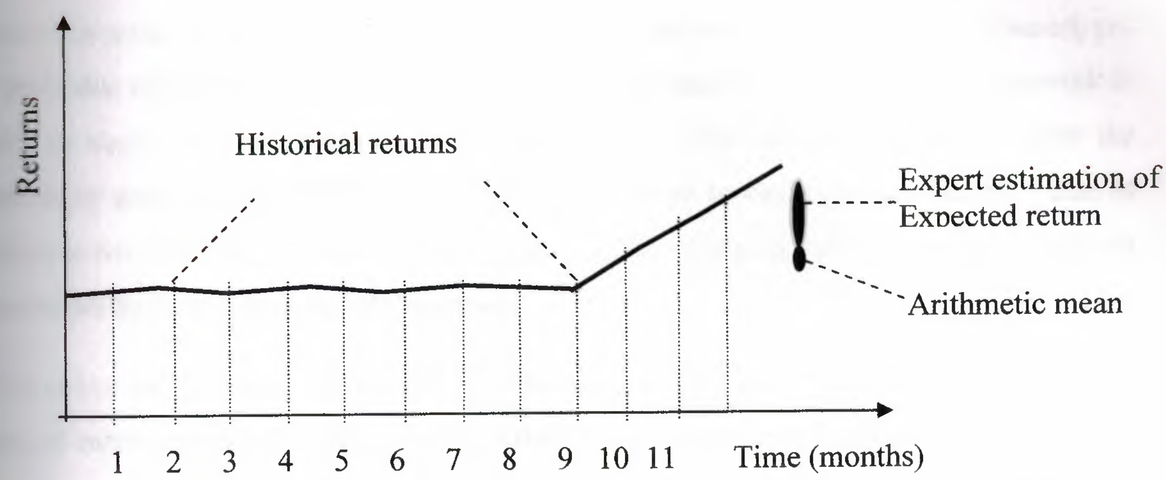


In portfolio analysis, decision makers usually use three types of gradations for estimation of risk. These gradations are risk averse, neutral and risk seeker. Using fuzzy logic we have possibility to specify risk more accurately, based on linguistic terms, such as absolutely risky, more risky, medium risky, less risky etc. (figure 1). In existing previous works the risk estimation for portfolio construction problem is implemented based on of crisp measure. The fuzzy measure is used for estimation of portfolio risk.

Another important parameter in portfolio modeling is return rate. Portfolio models utilize the concepts of risk and return in a combined paradigm. In many portfolio models expected returns of securities are accepted as arithmetic means of historical returns. But this is not correct when there are uncertainties in finance. If long run historical returns are stable, returns for the last few months are increasing and the economic conditions are improving then expected return will be larger than calculated arithmetic mean. In this case arithmetic mean may be lower limit of expected return. Also by analyzing financial statements of companies, the upper limit can be assigned based on expert knowledge . In inverse case arithmetic mean will be upper limit of expected return.

In Figure 2 trajectory of historical returns of security (solid line), arithmetic mean of historical return and expert estimation of expected return are shown. As shown, arithmetic mean doesn't characterize the expected return for the next month. Expert estimation is fuzzy and arithmetic mean will be lower limit of expected return.

Figure 2. Expert estimation of expected return



In this work the trapezoidal fuzzy numbers are used to represent uncertain values of variables. For example, expected return for j-th asset in given time interval can be accepted as:

$\widetilde{R}_j = (0.03;\ 0.036; 0.040; 0.045)$

This means that return of j-th asset will be at least 3%, desirable to be between 3.6% and 4% and it will not be higher than 4.5%. The use of such type fuzzy numbers is very adaptable to the structure of human mind.

2.3 Flexible fuzzy constraints for portfolio optimization.

In practice during construction of portfolio it becomes difficult to obtain investor's goal value in objective risk function, under given constraints defined for the return. This is existed due to presence of the conflict between risk and return. The use of fuzzy set theory gives chance of possibility trade-of between risk and return.

The portfolio selection models are optimization models. Optimization is an important activity in many fields of science and engineering. The classical framework for the optimization is the minimization (or maximization) of the objectives, given the constraints for the problem to be solved. Many design problems are characterized by multiple objectives, where a trade-off amongst various objectives must be made. Moreover, some flexibility may be present for specifying the constraints of the problem. Furthermore, some of the objectives in decision making may be known only approximately.

Often, the objectives can be expressed approximately in linguistic terms, but a precise mathematical formula is not available in many cases. Also, the decision constraints may be relaxed in some situations, as long as the decision objectives can be improved. These types of problems require an extension of the classical optimization and constraint framework in order to deal with the flexibility of the constraints. This situation if occurs, when the number of securities is high. In such cases it is difficult to find optimal minimal value of objective risk function of portfolio, which satisfies investor goal value under given difficult constraints defined bye the portfolio return.

This exists due to presence of conflict between return and risk. Fuzzy set theory provides ways of representing and dealing with flexible or soft constraints, in which the flexibility at the constraints can be exploited to obtain additional trade-off between improving the objectives and satisfying the constraints, that is trade-of between risk and return. Various fuzzy optimization methods have been proposed in the literature in order to deal with different aspects of soft constraints. In formulation of fuzzy optimization due to Zimmermann [20], concepts from Bellman and Zadeh model of fuzzy decision making [39] are used for formulating the fuzzy optimization problem. In this formulation, fuzzy sets represent both the (aspired) problem goals and the flexible (soft) constraints.

The optimal trade-off amongst the problem goals and the constraints is determined by the maximizing fuzzy decision, where the optimal decision is found by maximizing the simultaneous satisfaction of the optimization objectives and the constraints.

The asymmetry between the problem goals and the problem constraints disappears in this formulation, and the fuzzy goals and the constraints are aggregated to a single function that is maximized. It should be noted that this framework is general enough to handle crisp constraints as well as fuzzy constraints.

In the fuzzy optimization model of Zimmermann [20], simultaneous satisfaction of the decision goals and the constraints is sought. No further distinction is made amongst the constraints and the goals. When there is a possibility to make a trade-off between improving the objective and satisfying the constraints, however, the user of the optimization algorithm (i.e. the designer, the decision maker, the controller, etc.) can choose to trade a particular constraint or goal preferentially with respect to the other ones.

For example, satisfaction of a particular constraint may be more important than the satisfaction of another one. Within the classical framework, constraints of different importance are distinguished by ordering them hierarchically according to their importance and to admit them into the optimization problem one by one, often by first starting with the most constraining set and then gradually removing the constraints one at a time. In [22] fuzzy optimization admits another model by introducing weight factors that represent the importance of the constraints for the optimization problem. Since there is no distinction between the fuzzy goals and the fuzzy constraints in Zimmermann's formulation of fuzzy optimization, the weight factors can also be applied to the optimization objectives. In [21] the trade-off amongst the objectives and various constraints can be influenced by changing the associated weight factors. Recently proposed weighted extensions of fuzzy t-norm operators are used for the aggregation.

The proposed framework is rather general, and it can be applied to various fuzzy non-linear programming problems with multiple objectives and constraints. It is assumed that a general optimization algorithm is available and has been implemented for performing the final (crisp) optimization in order to obtain the optimal solution to the fuzzy optimization problem. Various well-known algorithms with different complexity can be used for this purpose.

In this thesis the suggested fuzzy portfolio optimization model gives degree of satisfaction between criteria and constraints and defines tolerance for the constraints in order to obtain goal value in objective function. The use of satisfaction degree allows to represent flexibility in the constraints and the goals. The used approach allows to make risk-return trade-off. The provided model helps portfolio managers effectively manage risk than the existing approaches that are based on probability theory.

The general formulation of portfolio optimization in the presence of flexible goals and constraints is given by

$$\textit{fuzzy} \underset{x \in X}{\text{maximize}} [f_1(x), f_2(x), \ldots, f_n(x)]$$

$$\text{subject to } g_i(x) \mathrel{\widetilde{\leq}} 0, \quad i = 1,2,\ldots,m. \qquad (2.10)$$

The sign $\tilde{\leq}$ denotes that gi(x)≤0 can be satisfied to some satisfaction degree, which can be smaller than or equal 1.This fuzzy maximization corresponds to achieving the highest possible aspiration level for the goals fi(x), given the fuzzy constraints to the problem. This problem can be solved by using Zadeh fuzzy decision making [39].

In portfolio modeling the investor opinion is used to choose satisfaction degree on the base of risk-return trade-of analysis. If the objective function attains its value more than or equal investor goal value then there is no need to define satisfaction degree to constraint and investor is fully satisfied. If the objective function attains its value less than investor goal value then there is need to define satisfaction degree for the constraints.

The following membership function can be used as membership degree for the constraints $(Ax)_i \tilde{\leq} b_i$.

$$\mu_{B_i}(y_i) = \begin{cases} 1 & \text{if } y \leq b_i \\ g_i(y_i) & \text{if } b_i \leq y < b_i + q_i \\ 0 & \text{otherwise} \end{cases} \qquad (2.11)$$

Here $\mu_{B_i}(y)$ is membership degree of the constraint. qi is maximum violation (tolerance) designated for the constraint.

Similarly we can define degree of satisfaction of inequality constraint $(Ax)_i \tilde{\geq} b_j$

$$\mu_{B_j}(y_j) = \begin{cases} 1 & \text{if } y \geq b_j \\ g_j(y_j) & \text{if } b_j - q_j \leq y < b_j \\ 0 & \text{otherwise} \end{cases} \qquad (2.12)$$

The functions gi(yi) and gj(yj) defined for the constraints can be nonlinear or linear.

3. Formulation of fuzzy portfolio optimization

et's consider fuzzy optimization problem for portfolio modeling.

efinition 1: Let x and f be vectors and defined on nonempty space $X \in R+$ and $F \in R$, orrespondingly. $\widetilde{c}$, $\widetilde{b}$ are vectors and $\widetilde{a}$ is matrix that are defined in $P \in R$. Fuzzy ptimization problem is defined as

$$\text{Fuzzy} \underset{x \in \Sigma}{\text{minimize}} \; f(\widetilde{c}, x)$$

2.13)

$$g(\widetilde{a}, x)\, \Omega \widetilde{b}$$

2.14)

ere Σ is nonempty subset of X. Σ is called to as design search space or feasible region. Ω fuzzy extension of the relations >, =, <.

n order to obtain feasibility the constraints must be verified at a certain degree. The fuzzy easibility is defined as follows.

efinition 2: Let g be the function defined in R and P is given set of parameters. $\mu_{a_i}(x)$ nd $\mu_{b_i}(x)$ are membership function of fuzzy parameters $\widetilde{a}$ and $\widetilde{b}$, respectively. Let Ωi, $\in$M be the fuzzy relation with corresponding membership function μ_{Ω_i}. The fuzzy set X f feasible solutions given by membership function μX is defined as

$$X(x)= \quad \min\{\mu_{\Omega_1}(g1(x;a1),b1), \ldots, \mu_{\Omega_m}(gm(x;am),bm)\}$$

his is called feasible solution of the fuzzy optimization problem of (2.13) and (2.14).

Jsing (2.10) ÷ (2.14) lets define fuzzy portfolio construction model that would take into ccount investor preferences about risk-return trade-of and define tolerance for the onstraints in order to obtain goal value in objective function.

t first, consider portfolio optimization problem given by (2.13) and (2.14). Assume that onstraints in (2.14) are in linear order.

$\text{Minimize}_{x \in \Sigma} \; f(\tilde{c}, x)$ (2.15)

$$\tilde{A}_1 x \geq \tilde{B}_1$$
$$\tilde{A}_2 x \leq \tilde{B}_2$$
$$x \geq 0$$

(2.16)

Here $\tilde{A}_1, \tilde{A}_2$ are matrices and $\tilde{B}_1, \tilde{B}_2$ are vectors that are defined in R.

In particular case investor may also provide set of bounds for securities, such as

$$L \leq x \leq U, \qquad \sum_{i=1}^{n} x_i = 1$$

Here L and U lower and upper bound defined for the securities. This assumption does not imply any loss of generality of model.

Depending on importance of objective function or constraints, we can define tolerance for them, in order to find the optimal solution. Let's consider the case when constraints in (2.16) are tolerable, that is we may relax the constraints in order to find an optimal compromised solution.

The satisfaction of the constraints will be acceptable to some certain membership degree. This degree is called satisfaction degree that allows organizing flexibility of the constraints. In the portfolio modeling the investor opinion is used to choose satisfaction degree on the base of risk-return trade-of analysis. If the objective function attains its value more than or equal investor goal value then there is no need to define satisfaction degree to constraint and investor is fully satisfied. If the objective function attains its value less than investor goal value then there is need to define satisfaction degree for the constraints.

The membership function can be nonlinear or linear. In the thesis we use following function for membership degree. The degree of satisfaction of inequality constraint $\tilde{A}x \geq \tilde{B}_1$ in (2.16) is defined as follows.

$$\mu_{B_i}(y) = \begin{cases} 1 & \text{if } y \geq b_i \\ 1 + \dfrac{y - b_i}{q_i} & \text{if } b_i - q_i \leq y < b_i \\ 0 & \text{otherwise} \end{cases} \qquad (2.17)$$

Here $\mu_{B_i}(y)$ is membership degree of the constraint. qi is maximum violation (tolerance) designated for the constraint. The value of qi should be provided by investor that makes decision.

Similarly we can define degree of satisfaction of inequality constraint $\widetilde{A}x \leq \widetilde{B}_2$ (figure 3)

$$\mu_{B_j}(y) = \begin{cases} 1 & \text{if } y \leq b_j \\ 1 - \dfrac{y - b_j}{q_j} & \text{if } b_j \leq y < b_j + q_j \\ 0 & \text{otherwise} \end{cases} \qquad (2.18)$$

After defining satisfaction degree we can formulate fuzzy optimization problem as

$$\underset{x \in \Sigma}{\text{Minimize}} \; f(\widetilde{c}, x) \qquad (2.19)$$

$$\begin{aligned} &\widetilde{A}_1 x \geq \widetilde{B}_1 - q(1 - \mu_B(y)) \\ &\widetilde{A}_2 x \leq \widetilde{B}_2 + q(1 - \mu_B(y)) \\ &x \geq 0 \end{aligned} \qquad (2.20)$$

Here $tol = q(1 - \mu_B(y))$ is tolerance designated for the constraints. For $\leq$ and $\geq$ constraints the different tolerances and satisfaction degrees can be assigned.

Taking into account above mentioned the fuzzy portfolio optimization model will be formulated as follows.

$$z = \frac{1}{T}\sum_{t-1}^{T}\left|\min\{0, \sum_{j-1}^{n}(r_{jt} - \widetilde{R}_j)x_j\}\right| \rightarrow \textit{fuzzy}\ \text{minimize} \qquad (2.21)$$

$$\widetilde{R}_j x_j \geq \rho\widetilde{D} - q(1-\mu_B(y)),$$

$$x_j = \widetilde{D}$$

$$x_2,\ldots,x_n \geq 0 \qquad (2.22)$$

re $\widetilde{R}_j$ is fuzzy value of expected return. xj are fuzzy values of portfolio allocation in urity j over the entire period T and $\widetilde{D}$ is total budget invested in portfolio. rjt is return of urity j over period t. The problem is to determine such values of xj under fuzzy qualities and equality conditions (2.22), by using them in objective function (2.21) the zy value of objective function would be minimum.

mmary

e concepts of fuzzy risk and fuzzy expected return are introduced. The values of these ameters are estimated on the base expert perception. The soft constraints are introduced. obtain optimal solution, for each constraint the satisfaction degree is assigned. signing satisfaction degree can provide flexibility of the constraints. Fuzzy flexible istraints are introduced. Using semiabsolute deviation model, the portfolio selection del is formulated for security investment to provide flexibility of the constraints. The olied model has soft constraint allowing to estimate the investor's preference about risk-urn trade-off. In this model for the each constraint the satisfaction degree is assigned. e fuzzy portfolio selection model is formulated for security investment that takes into count fuzzy value of expected return and fuzzy risk

CHAPTER III

MATHEMATICAL METHODS FOR PORTFOLIO SELECTION PROBLEM SOLUTION

3.1 Overview

1. In this chapter the application of Genetic algorithms (GAs) for solving of deterministic and fuzzy portfolio selection problem is considered. GAs are global optimization algorithms based on the mechanics of natural selection and natural genetics. GAs have a number of specific peculiarities by which they differ from the other methods of optimization. These are the following:

2. Genetic algorithms employ only the objective function, not the derivative one or some other function. It is very convenient in case that the function is neither differentiable nor discrete.

3. Genetic Algorithms employ a parallel multi-point search strategy by maintaining a population of potential solutions, which provides wide information of the function behaviour and exclude the possibility of arising the local extreme of the function, while the traditional search methods, such as gradient, etc, can not cope with this problem.

4. Genetic Algorithms use probability-transitive rules instead deterministic ones.

5. Besides, Genetic Algorithms are very simple for computer solution.

6. To solve portfolio optimization problem the main operators of GA - selection, crossover and mutation are described. The algorithms for solving the deterministic and fuzzy portfolio optimization problem using GA's are presented.

3.2. Genetic based optimization of securities of investment portfolio based on crisp model

1. The mathematical approach used for designing the correct portfolio of assets requires powerful methods of computation to find global optimal solution for portfolio optimization problem. One of the approaches to solve portfolio optimization problem is the use of gradient method. The gradient method is iteration method and time consuming. Sometimes for complicated processes gradient method has local minima problem and could not find global optimal solution of optimization problem. Taking into consideration above mentioned, in this thesis genetic algorithm (GA) is used to solve portfolio selection problem to find optimal values of securities.

2. GAs are adaptive heuristic search algorithm based on the evolutionary ideas of natural selection and genetics. As such they represent an intelligent exploitation of a random search used to solve optimization problems. GAs are not all random, they also exploit historical information to direct the search into the region of better performance within the search space. The basic techniques of the GAs are designed to simulate processes in natural systems necessary for evolution. GAs seek to solve optimization problems using the methods of evolution, specifically survival of the fitness. In a typical optimization problem, there are a number of variables which control the process, and a formula or algorithm which combines the variables to fully model the process.

3. The problem is then to find the values of the variables which optimize the model in some way. If the model is a formula, then the purpose of the algorithm is usually to catch the maximum or minimum value of the formula. There are many mathematical methods which can optimize of this nature (very quickly) for fairly "well-behaved" problems. These traditional methods tend to break down when the problem is not so "well-behaved". It should be inceled that evaluation (in nature or anywhere else) is not a purposive or directed process. That is, there is no evidence to support the assertion that the goal of evolution is to produce Mankind. Indeed, the processes of nature seem to boil down to different individuals competing for resources in the environment. Some are better than

others. Those that are better are more likely to survive and propagate their genetic material. In nature, the encoding for genetic information is done in a way that admits asexual.

4. An effective GA representation and meaningful fitness evaluation are the keys for success in GA applications. The appeal of GAs comes from their simplicity and elegance as robust search algorithms as well as from their power to discover good solutions rapidly for difficult high-dimensional problems. GAs are useful and efficient when: The search space is large, complex or poorly understood.

5. Domain knowledge is scarce or expert knowledge is difficult to encode to narrow the search space.

6. No mathematical analysis is available.

7. Traditional search methods fail.

8. The advantage of the GA approach is the ease with which it can handle arbitrary kinds of constraints and objectives; all such things can be handled as weighted components of the fitness function, making it easy to adapt the GA scheduler to the particular requirements of a very wide range of possible overall objectives.

9. GAs are applied to many scientific, engineering problems, in business and entertainment, including: Optimization, Automatic Programming, Machine and robot learning, Economic models, Immune system models, Ecological models, Population genetics models, Models of social systems.

10. Most traditional optimization methods used in science and engineering applications can be divided into two broad classes: direct search methods requiring only the objective function values and gradient search methods requiring gradient information either exactly or numerically.

11. These methods work on point-by-point basis. They start with an initial guess and a new solution is found iteratively.

12. Most of them are not guaranteed to find the global optimal solutions. The termination criterion is the value of gradient of objective function becomes close to zero.

13. They work with coding of the parameter set, not the parameters themselves.

14. Advantage of working with a coding of variable space is that the coding discretizes the search spaces even though the function may be continuous.

15. Since function values at various discrete solutions are required, a discrete or discontinuous function may be tackled using GAs.

16. They search from a population of points, not single point so it is very likely that the expected GA solution maybe a global solution

17. They use objective function values and not derivatives.

18. Probabilistic transition rules are used, not deterministic. The search can proceed in any direction.

19. GAs were introduced as a computational analogy of adaptive systems. They are modelled loosely on the principles of the evolution via natural selection, employing a population of individuals that undergo selection in the presence of variation-inducing operators such as mutation and recombination (crossover). A fitness function is used to evaluate individuals, and reproductive success varies with fitness.

20. The steps of algorithm are described below.

21. Randomly generate an initial population M(0)

22. Compute and save the fitness u(m) for each individual m in the current population M(t)

23. Test fitness function. If it has acceptable small value, stop operation. In other case go to step 4.

24. Define selection probabilities p(m) for each individual m in M(t) so that p(m) is proportional to u(m).

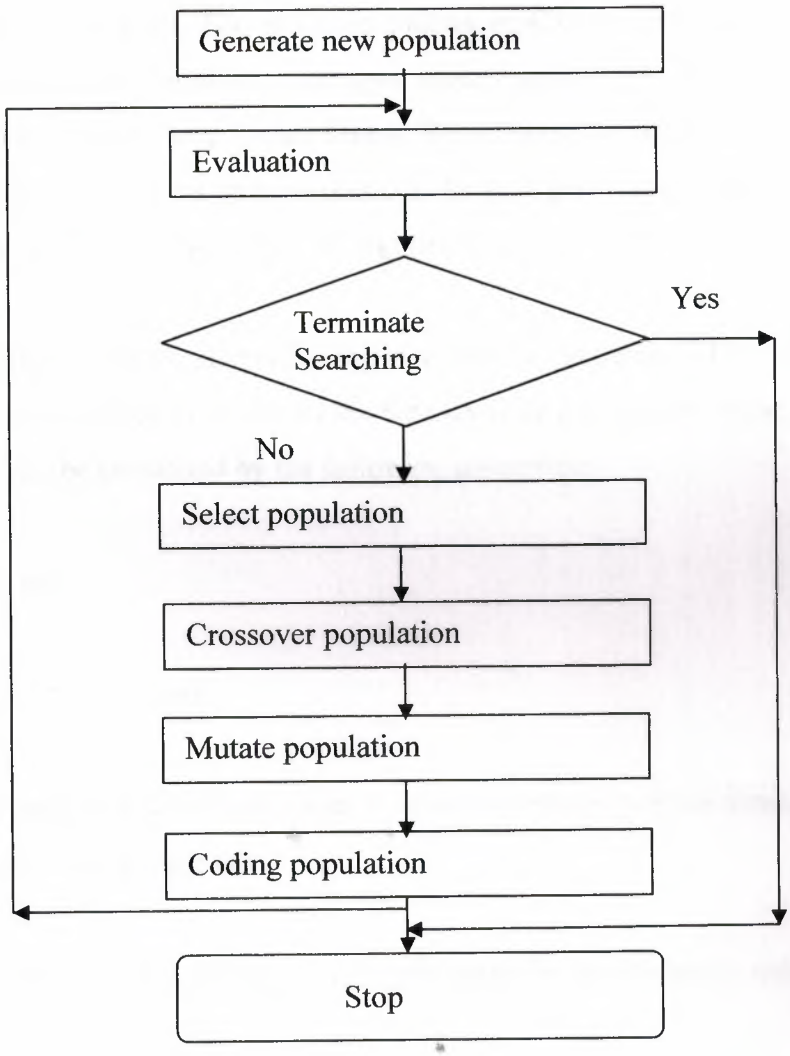
25. Generate M(t+1) by probabilistically selecting individuals from M(t) to produce offspring via genetic operators selection, crossover and mutation.

26. Repeat step 2 until satisfying solution is obtained.

27. The paradigm of GAs described above is usually the one applied to solving most of the problems presented to GAs. Though it might not find the best solution. more often than not, it would come up with a partially optimal solution.

28. The flow chart that demonstrates basic operations of Genetic algorithm is given in figure 3.1.

Figure 3.1.Basic operations of GA



The GA is applied to solve portfolio selection problem. In this problem the searching parameters are values of assets. Set of parameters are represented by chromosomes that is the genes represent the assets and form a chromosome. During portfolio optimization procedure the number of chromosomes are generated randomly. Using GA learning asset values in chromosomes are adjusted in order to find their optimal values. GA learning is carried out by GA operators. The main operations in GA are selection, crossover and mutation. The aim of the selection is to give more reproductive chances to population members (or solutions) who have higher fitness. Parent selection, in a simple term, is to let the current population members mate to produce the next generation. There are number of selections. In the thesis "roulette selection" algorithm is used.

At first iteration ps random numbers having ri values are generated. Then the conditions qi< ri <qi+1 are tested. Then qi+1 is selected for following population. Here qi cumulative probability, that can be calculated by the following procedure.

qi= p1 +p2 +...+ pps,

where pi =yi/(y1+y2+...+yps)

pi probability of selection of i-th individual to new population. yi is the fitness value of the i-th population. ps is population size.

Probability of selection of i-th individual pi is calculated by the following expression.

$$p_i = \frac{eval(v_i)}{F}$$

$$F = \sum_{i=1}^{ps} eval(v_i)$$

Here F is the total fitness of the population, eval(vi) is the fitness value for each chromosome vi (i=1, ..., ps).

The selection process is implemented ps times. Obviously, some chromosomes would be selected more than once; the best chromosomes get more copies; the average stay even, and the worst die off.

Then recombination operators- crossover and mutation are applied to the individuals in the new population. Crossover and mutation are two main components in the reproduction process in which selected pairs mate to produce the next generation.

The purpose of crossover and mutation is to give the next generation of solutions chances to differ from their parental solutions. Both parent components intend to give children chances to differ from their parents, and hope that some of the children can be closer to the optimal destination than their parents. For implementing crossover operation the probability of crossover pc parameters of a genetic system is defined. This probability gives us the expected number ps of chromosomes which undergo the crossover operation. This is done in the following way:

For each chromosome in the (new) population:

- Generate a random (float) number r from the range [0,1];

- If r < pc , select given chromosome for crossover;

The selected chromosomes mate randomly. There are some forms of crossover: one-point, two-point, multipoint and uniform. In one-point crossover operation the whole chromosome is divided into two parts. The dividing position pos can be generated randomly or by programmer. This position indicates the crossing point. Two chromosomes

(b1 b2 … bpos bpos+1 … bm)

(c1 c2 … cpos cpos+1 … cm)

are replaced by a pair of their offspring:

(b1 b2 … bpos cpos+1 … cm)

(c1 c2 … cpos bpos+1 … bm)

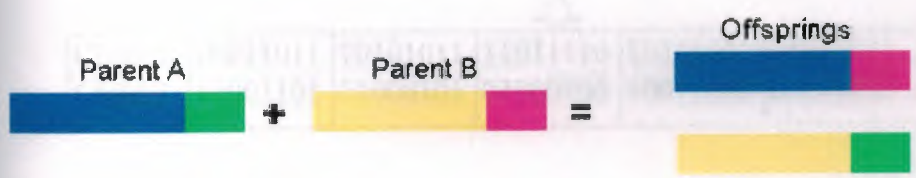
If we use binary numbers then one-point crossover operation will have following form

01100111 ⇨ 01101101

10101101 10100111

The graphic representation of one-point crossover operation is shown in figure 3.2

Figure 3.2. One-point crossover operation



Two-point crossover operation is shown below

10100101 | 10111111 ⇨ 10101001 | 10111011

10011001 | 00111011 10010101 | 00111111

The graphic representation of two-point crossover operation is shown in figure 3.3.

Figure 3.3. Two-point crossover operation

In case, when chromosomes are very long the use of one or two point crossover operation doesn't give desired results and learning of parameters values takes more computational time. In such cases the increasing number of crossover points gives desirable result and allow us to decrease the learning time. In the work using multipoint crossover operation the learning of the values of unknown parameters are performed.

As an example in figure 3.4 the multipoint crossover operation is shown. Here genes of parent 1 are described with grey colour, genes of parent 2 with white colour.

Figure 3.4 Multipoint crossover operation

Parent 1	10011101	10100101	11010100	10111111	01011100...
Parent 2	11001011	01001011	01001110	10011001	00111011...

Child 1	10011011	10101011	11011110	10111001	01011011...
Child 2	11001101	01000101	01000100	10011111	00111100...

Like crossover, mutation is another way to cause chromosomes created during a reproduction to differ from those of their parents. There is a mutation rate associated with the operator. The lower rate gives less chance to the chromosomes of the children differ from their parents.

The operator is applied to a bit string which represents a chromosome, it sweeps down the list of bits, replacing each bit by a randomly selected bit if a probability test is passed. In other words, for each bit in the bit string, the operator generates a random number between zero and one. If the random number is smaller than the mutation rate (r<pm), then the operator replaces the bit by a randomly generated bit (either zero or one). In binary strings, 1s are changed to 0s and 0s to 1s. Mutation operation on eight genes shown below. In binary string 0 is changed to 1.

Mutate

10011101 $\Longrightarrow$ 10111101

Above described selection, multipoint crossover and mutation operators are applied for learning the values of securities. In figure 3.5 the structure of chromosome that contain n assets is shown

Figure 3.5. Structure of chromosome that contain n assets are shown

asset 1	asset 2	asset 3	…	asset n

The deterministic portfolio optimization model for n asset can be given as follows

$$z = \frac{1}{T}\sum_{t=1}^{T}\left|\min\{0, \sum_{j=1}^{n}(r_{jt} - R_j)x_j\}\right| \rightarrow \text{minimize} \tag{3.1}$$

$$\sum_{j=1}^{n} R_j x_j \geq \rho D,$$
$$\sum_{j=1}^{n} x_j = D$$
$$x_1, x_2, \ldots, x_n \geq 0 \tag{3.2}$$

The learning algorithm for finding optimal values of securities for portfolio selection includes the following steps.

Step1. Define objective risk function and constraints.

Step 2. Determine population size. For each risky security (unknown parameter) randomly generate set of chromosomes equal to population size.

Step 3. Apply decoding to transform the binary representation of the securities to the variable representation and determine decimal representation of the values of securities.

Step 4. Test the satisfaction of the constraints for each solution by using formula (3.2). The solutions that satisfy constraints are selected for the next operation. The other solutions that do not satisfy constraints are not take part in the next step 5.

ep 5. Using selected solutions evaluate fitness functions. Choose fitness function as 1/(1+z). Here z is objective function which is calculated by formula (3.1).

ep 6. Select the solution that corresponds to the maximum value of fitness function. For e selected new solution compare the value of objective function with the value of jective function corresponding to the selected solution that is determined in previous ep. The solution that corresponds to the maximum value of objective function is selected d saved in the file.

ep 7. Number of learning iteration is tested. If current number is less than or equal user efined number then learning is continued and control is given to step 8, in other case ogram is stopped and control is given to step 9.

tep 8. Apply GA operators – selection, crossover and mutation to change binary values of ecurities. After correction of the values of securities go to step 3.

tep 9. Analyze and save the obtained results. If for the obtained solution the values of risk nd return of portfolio are preferable by investor then stop operation. In other case go to tep 2.

Jsing described algorithm the learning unknown parameters of the optimization model has een performed. The flowchart of genetic portfolio optimization algorithm is given in ppendix A

3.3 GA based method for fuzzy portfolio selection

The input data for fuzzy portfolio selection model are historical returns and fuzzy values of expected returns in future. The expected values of returns are determined by expert perception in fuzzy form, the fuzzy value of risk is accepted as fuzzy distance that is determined by deviation of return rates below the expected return.

In fuzzy portfolio selection main problem is finding the fuzzy vales of securities. One of the approaches to solve fuzzy portfolio optimization problem is the use of gradient method with α- cut in fuzzy operations . This is time consuming and decreases the accuracy of obtained results. Also for complicated processes the local minima problem is another disadvantage fuzzy optimization problem. To avoid above mentioned the GA is used to solve fuzzy portfolio selection problem and find optimal values of securities. In figure 3.6 the structure of chromosomes that contain n fuzzy values of securities is shown. As shown in figure the fuzzy values of securities are described by tree numbers- left, middle and right.

Figure 3.6. Structure of chromosome that contains twelve fuzzy assets are shown

asset1	asset2	asset3		asset n
$x_l(1), x_m(1), x_r(1)$	$x_l(2), x_m(2), x_r(2)$	$x_l(3), x_m(3), x_r(3)$	. . .	$x_l(n), x_m(n), x_r(n)$

To describe fuzzy values of the parameters in the objective function and constraints the triangular forms are used. Each triangular fuzzy number is characterized by three parameters- left, middle, and right. Fuzzy value of each asset is characterized by left, middle and right sides. Chromosome contains all fuzzy vales of assets and consists of number of genes that are represented by binary numbers 0 and 1.

The selection, multipoint crossover and mutation operators that are described in previous chapter are applied for learning the fuzzy values of securities.

The learning algorithm for finding optimal values of securities for portfolio selection includes the following steps.

Step1. Define objective risk function and constraints. Take value of satisfaction degree μ=1.

Step 2. Determine population size. For each risky security (unknown parameter) randomly generate set of chromosomes equal to population size.

Step 3. Apply decoding to transform the binary representation of the fuzzy values of securities to the variable representation and determine decimal representation of the values of securities.

Step 4. Test the satisfaction of the constraints for each solution by using formula (2.22). The solutions that satisfy constraints are selected for the next operation. The other solutions that do not satisfy constraints are not take part in the next step 5.

Step 5. Using selected solutions evaluate fitness functions. Choose fitness function as J=1/(1+z). Here z is calculated by (2.21).

Step 6. Select the solution that corresponds to the maximum value of fitness function. For the selected new solution compare the value of objective function with the value of objective function corresponding to the selected solution that is determined in previous step. The solution that corresponds to the maximum value of objective function is selected and saved in the file.

Step 7. Number of learning iteration is tested. If current number is less than or equal user defined number then learning is continued and control is given to step 8, in other case program is stopped and control is given to step 9.

Step 8. Apply GA operators – selection, crossover and mutation to change binary values of securities. After correction of the values of securities go to step 3.

Step 9. Analyze the obtained results. If for the obtained solution the values of risk and return of portfolio are preferable by investor then stop operation. In other case ask investor preference to change satisfaction degree to minimize risk. After changing satisfaction degree go to step 2.

Applying described steps the finding unknown parameters of the fuzzy portfolio optimization model is carried out.

The flowchart of the program for fuzzy genetic portfolio optimization algorithm is given in figure 3.7.

Block 1. Start of the program.

Block 2. Entering expected return, historical data describing company return rates, portfolio return ρ, satisfaction degree μ. Entering the parameter values for population size ps, crossover rate pc, mutation rate pm, chromosome's length for left, middle, right sides of fuzzy parameters, number of parameters n, number of learning iterations endloop, crossover point pos.

Block 3. Generating random integer numbers 0 and 1 for ps chromosomes.

Block 4. Initialization of loop.

Block 5. If current loop number is not less than equal end of loop number then stop operation and go to the block 6.

Block 6. End of the program.

Block 7. Initializing initial values of fuzzy fitness functions describing the fuzzy portfolio optimization model and total fitness for left middle and right sides to zero.

Block 8. Control the loop of chromosomes and selecting first chromosome.

Block 9. Converting the chromosome that describe the fuzzy value of securities consisting of 0 and 1 to decimal values.

Block 10. Calculating values of fitness function for left, middle and right sides describing portfolio optimization model.

Block 11. Defuzzification of the fuzzy value of fitness function.

Block 12. Compare the calculated value of fitness function with previous one.

Block 13. Save calculated value of fitness function.

Block 14. Calculate total fitness

Block 15. Control the loop of chromosomes and selecting first chromosome.

Block 16. Calculating cumulative probability.

Block 17. Generating ps random numbers r(i).

Block 18. Control the loop of random numbers and selecting first random number.

Block 19. Control the loop of chromosomes and selecting first chromosome.

Block 20. Check condition r(i)>q(i1). If it is true check the following condition. In other case select next random number.

Block 21. Check the condition r(i)<q(i1+1). If it is not true select next random number.

Block 22. Save i-th chromosomes in v(i,j).

Block 23. Generating ps random numbers r(i).

Block 24. Initialization index k to 1.

Block 25. Control the loop of random numbers and selecting first random number.

Block 26. Check the condition r(i)<pc. If it is not true select next random number.

Block 27. Select i-th chromosome for crossover operation, ind(k)=i;

Block 28. Increment index k.

Block 29. Control the loop of selected chromosomes for crossover operation. Select first chromosome.

Block 30. Control the loop of genes in the chromosome until the crossover point.

lock 31. Making crossover operation and swapping the place of genes.

lock 32. Generating random numbers r(i,j).

lock 33. Control the loop of random numbers and selecting first random number.

lock 34. Control the loop of genes in the chromosome and selecting first gene.

lock 35. Check the condition r(i,j)<pm. If condition is not true select next random number and gene.

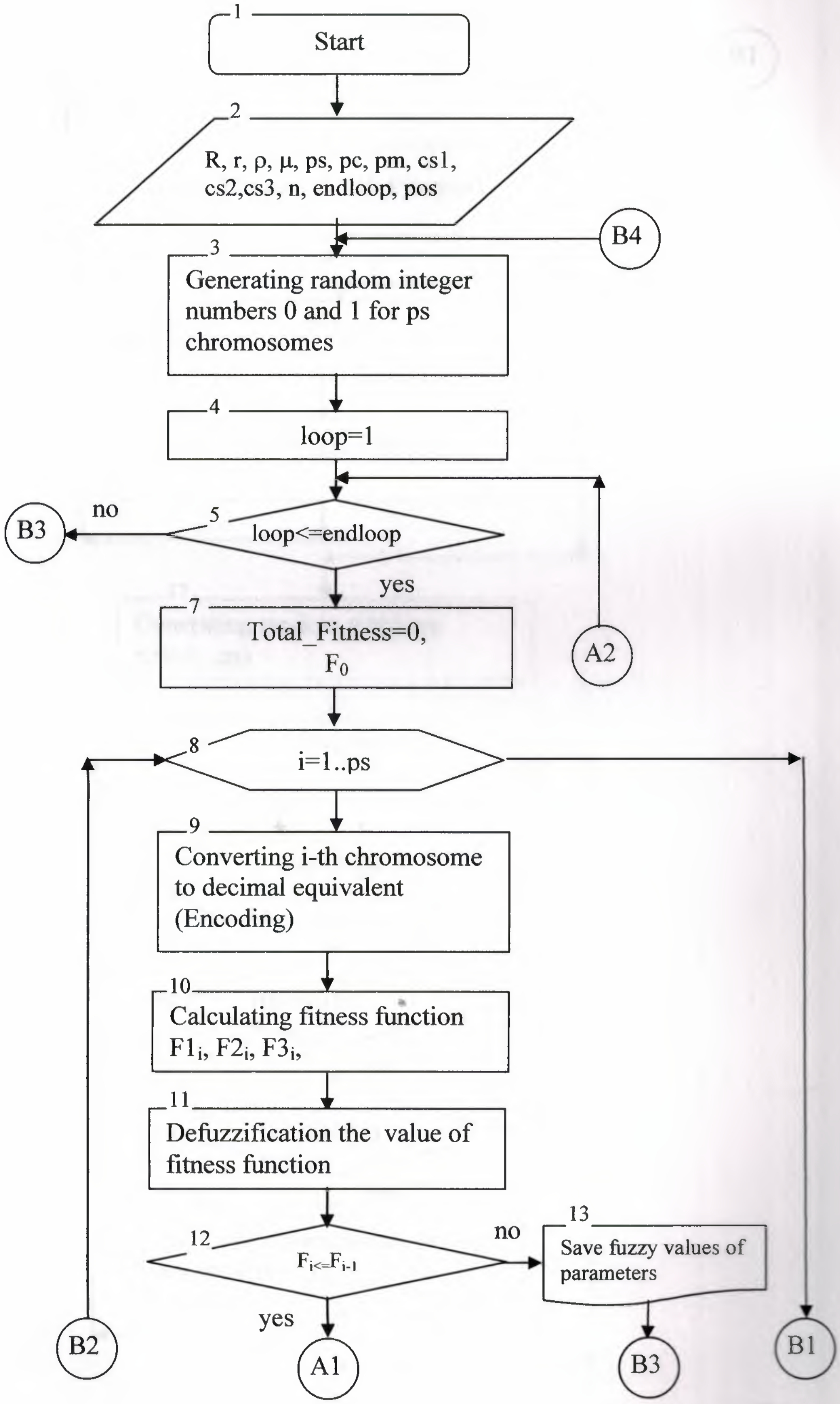
lock 36. Check gene's value (condition v(i,j)=0).

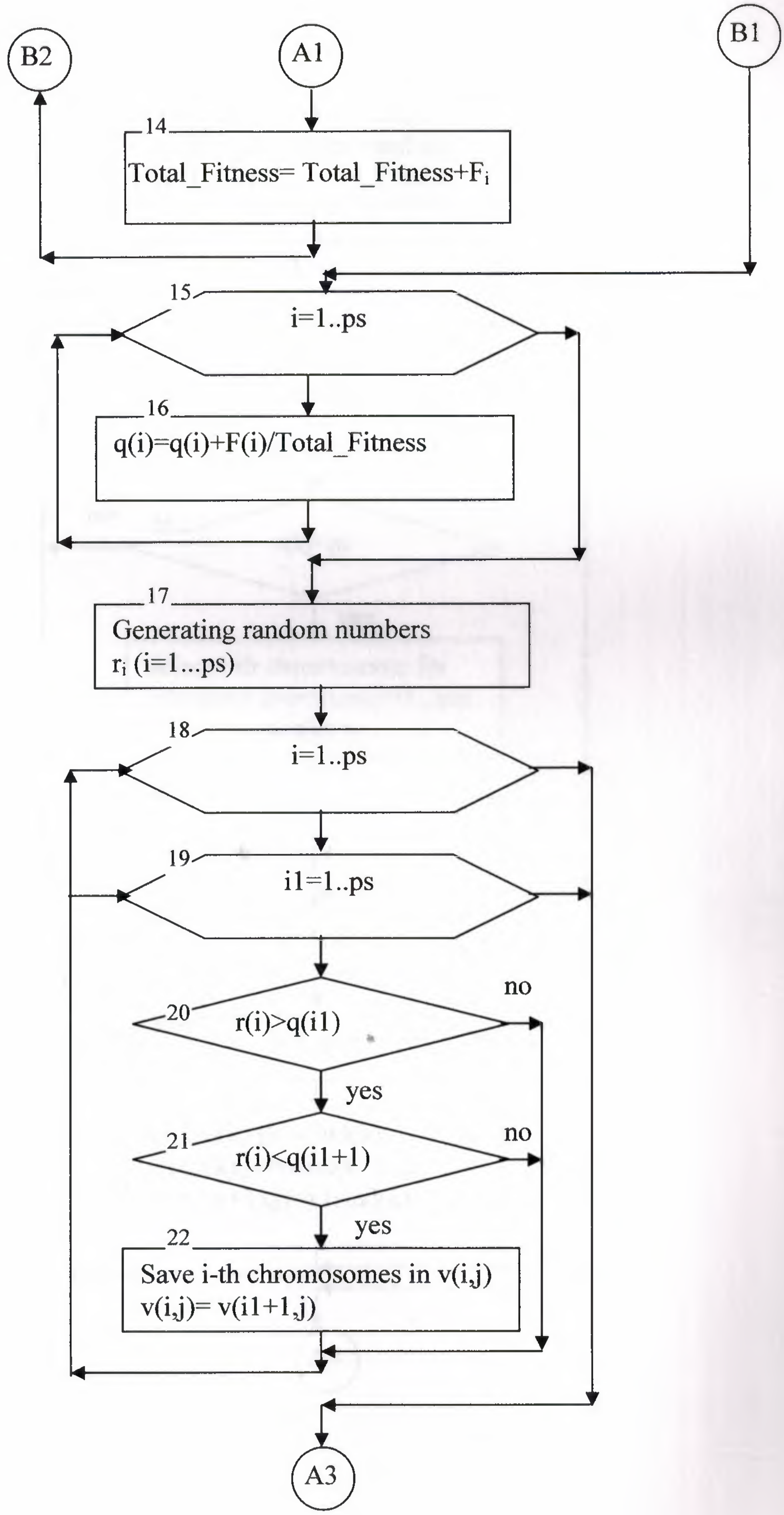
lock 37. Change value of gene to 1.

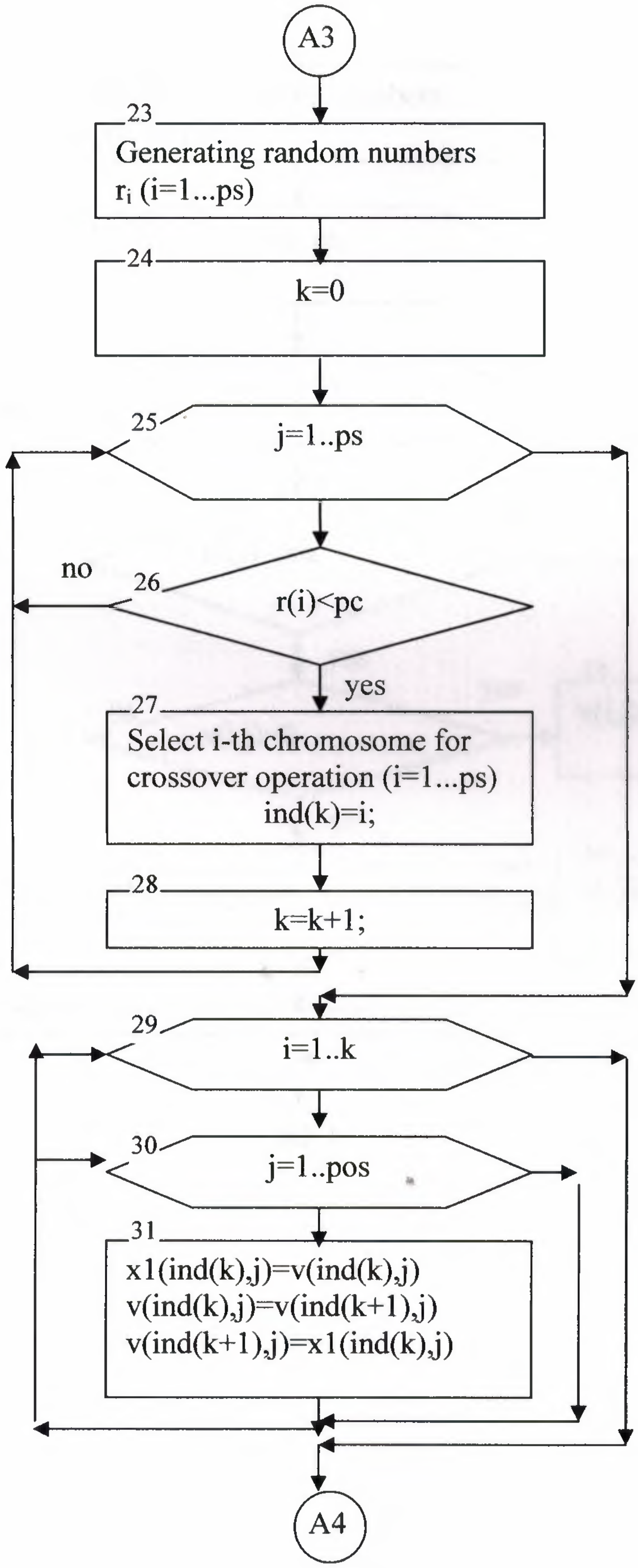
lock 38. Check gene's value (condition v(i,j)=1).

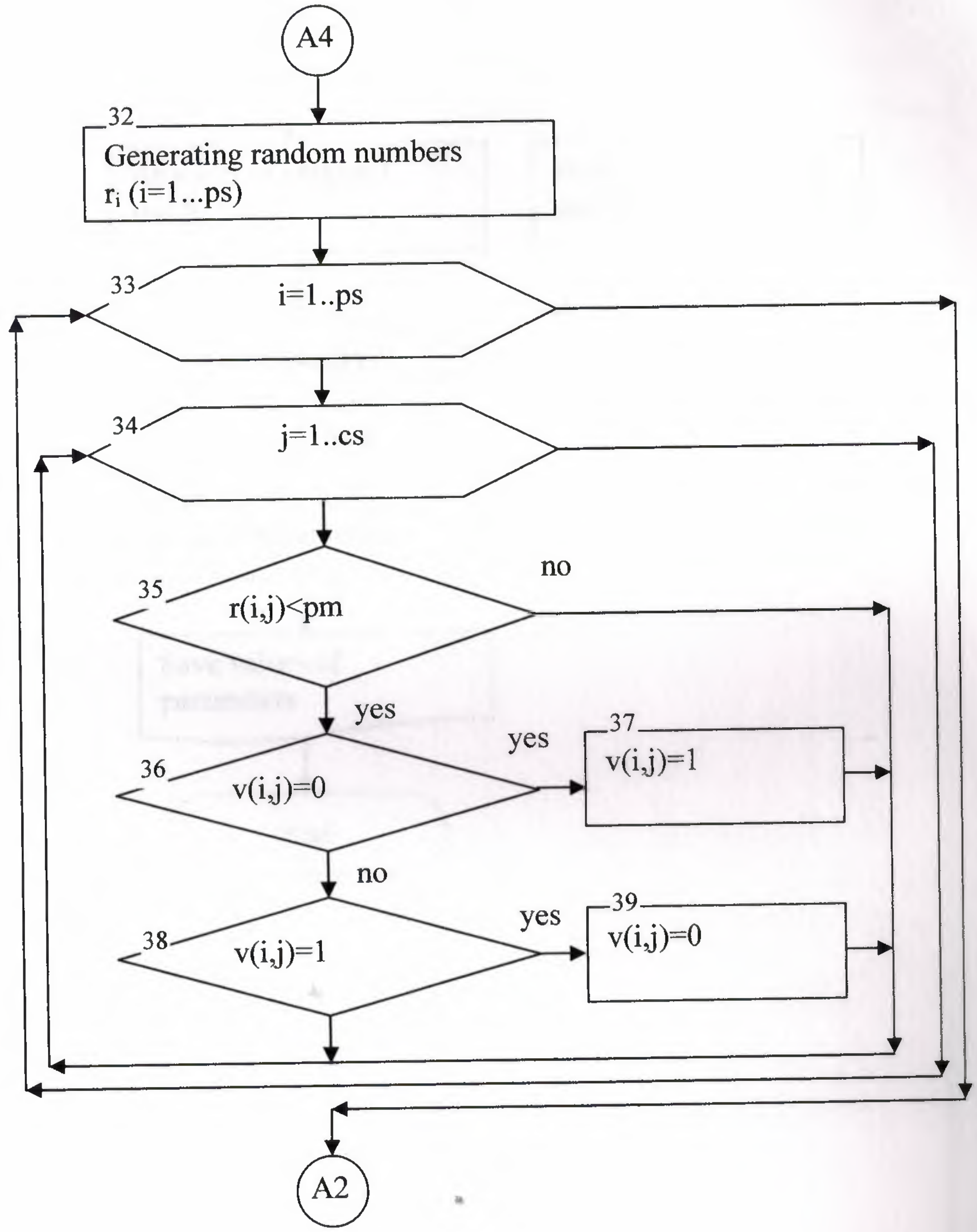
lock 39. Change value of gene to 0.

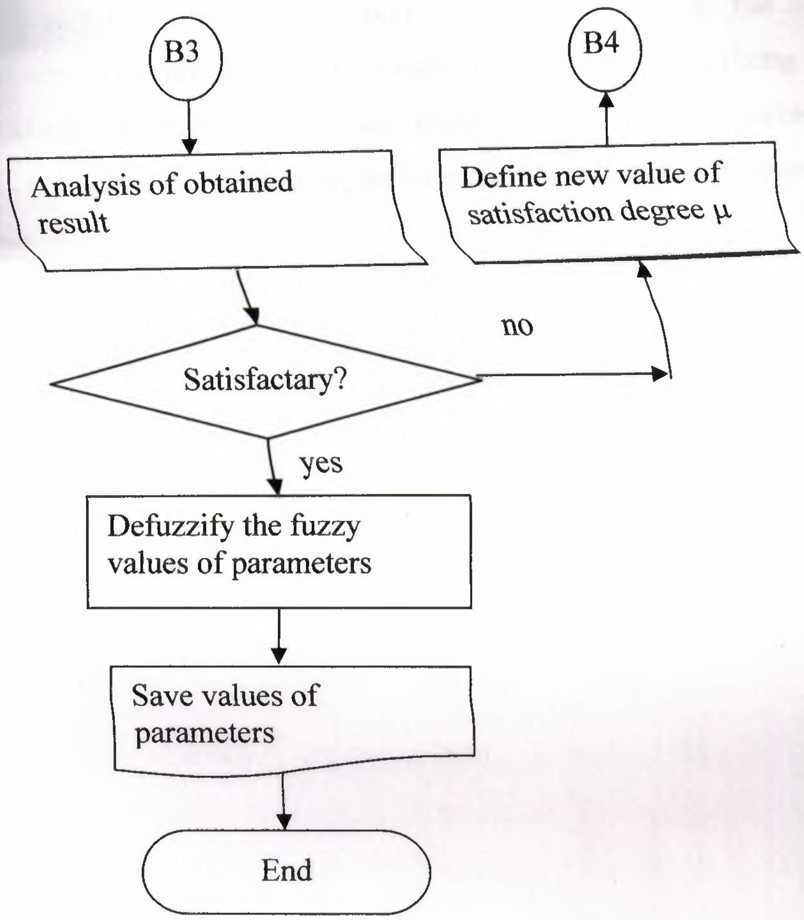
Figure 3.7. Flowchart of fuzzy genetic portfolio optimization algorithm




B2
A1
B1
14
Total_Fitness= Total_Fitness+F$_i$
15
i=1..ps
16
q(i)=q(i)+F(i)/Total_Fitness
17
Generating random numbers
r$_i$ (i=1...ps)
18
i=1..ps
19
i1=1..ps
20
r(i)>q(i1)
no
yes
21
r(i)<q(i1+1)
no
yes
22
Save i-th chromosomes in v(i,j)
v(i,j)= v(i1+1,j)
A3


A3
23
Generating random numbers
r_i (i=1...ps)
24
k=0
25
j=1..ps
no
26
r(i)<pc
yes
27
Select i-th chromosome for
crossover operation (i=1...ps)
ind(k)=i;
28
k=k+1;
29
i=1..k
30
j=1..pos
31
x1(ind(k),j)=v(ind(k),j)
v(ind(k),j)=v(ind(k+1),j)
v(ind(k+1),j)=x1(ind(k),j)
A4


A4
32
Generating random numbers
r_i (i=1...ps)
33
i=1..ps
34
j=1..cs
35
r(i,j)<pm
no
yes
36
v(i,j)=0
yes
37
v(i,j)=1
no
38
v(i,j)=1
yes
39
v(i,j)=0
A2


B3
B4
Analysis of obtained result
Define new value of satisfaction degree μ
no
Satisfactary?
yes
Defuzzify the fuzzy values of parameters
Save values of parameters
End

Summary

The description of genetic algorithm used for portfolio optimization is given. The main operators of GAs - selection, crossover and mutation operators are described. Using GA operators the mathematical solution of deterministic and fuzzy portfolio selection problems are described. The block schemes of described algorithms allow computer realization of represented algorithms.

HAPTER IV

OMPUTER MODELING OF FUZZY PORTFOLIO SELECTION

.1 Overview

esigning the correct portfolio model of assets requires modern, powerful and reliable athematical tools and programs. In this chapter using GA the software modeling of eterministic and fuzzy portfolio selection is considered. The computer modeling has been erformed using statistical data taken from Istanbul Stock Exchange. For the formulated odel the fuzzy values of expected returns for assets have been determined. Using istorical return rates of companies the portfolio selection problem is run. In the result of odeling the optimal values of securities for each company have been determined. The omparative result of simulation of fuzzy portfolio selection model with deterministic one s given.

.2. Development of software for deterministic portfolio selection

ssume that a portfolio manager wants to allocate his assets among risky securities based n recent historical data or the corporation's financial report. To find the amount of assets or each security the semi absolute deviation portfolio optimization model is used. The GA escribed in chapter 3.2 is applied to find optimal values of assets. At first step these assets re represented by chromosomes. Each chromosome consists of number of genes that are epresented by binary numbers 0 and 1. In figure 4.1 structure of chromosome that contain ix assets are shown.

he sum of assets must be equal the value of total assets invested to securities. The roblem is to find such optimal values of invested securities by using they in equation (3.1) he value of risk that is given as objective function will be minimized. The GA operators re applied to find optimal values of parameters.

Figure 4.1. Structure of chromosome that contain six assets are shown

asset 1	asset 2	asset 3	asset 4	asset 5	asset 6

At the beginning the set of solutions for risky securities are generated randomly. For these values the constraints are tested and the values of objective functions are calculated. Among these solution there is the solution that satisfies the given criteria and is minimum then it is selected as optimal solution and is saved in file. After encoding is applied to all solution and they are transformed to the binary forms consisting of 0 and 1. The GA operators- selection, crossover and mutation are applied to adjust values of risky securities.

Then decoding is used to translate the GA representation of the problem to the variable representation. These values are again tested in equations (1). If among these solution there is solution that satisfies (1) and is optimal than previous solution which was accepted as optimal then last solution is taken as optimal and iteration is continued.

The learning iterations are continued until the satisfactory optimal solution is found.

During simulation the historical values of return rates for six stocks from Istanbul Stock Exchange Market for twelve month are taken. The values of returns of six stocks for twelve months are given in table 4.1.

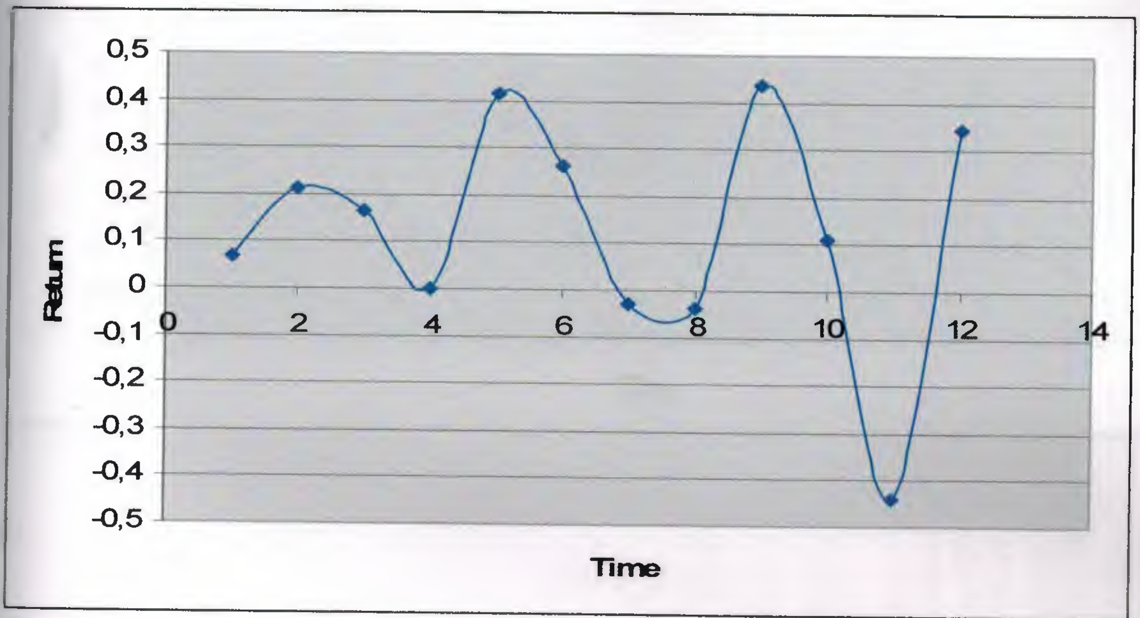
Table 4.1. Return of securities over 12 month

	Stocks					
	X1	X2	X3	X4	X5	X6
	0.0667	0.3200	0.1100	0.1560	0.1600	0.0500
		0.1300	0.1030	0.4500	0.3300	0.1700
	0.2121	0.1530	0.0860	0.4000	0.2700	0.3800
Times		-0.1900	-0.0790	-0.2200	-0.1580	-0.3600
	0.1647	0.1560		0.0000	0.0330	-0.1970
	0.0000	0.1300	0.0000	-0.0960	-0.0980	-0.1400
		0.0100	0.0350	0.0000	0.0100	0.0760
	0.4167	-0.0780	0.3100	-0.1560	-0.1140	0.0560
		0.6200	-0.0540	0.2070	0.4070	0.0870
	0.2632	-0.0300	0.5900	0.0820	-0.0690	0.3530
	-	-0.2100	0.1200	-0.2500	-0.0417	-0.0730
	0.0306	0.0240	-0.2700	0.0520	0.0345	0.1000
	-		0.0010			
	0.0392					
	0.4366					
	0.1094					
	-					
	0.4386					
	0.3412					

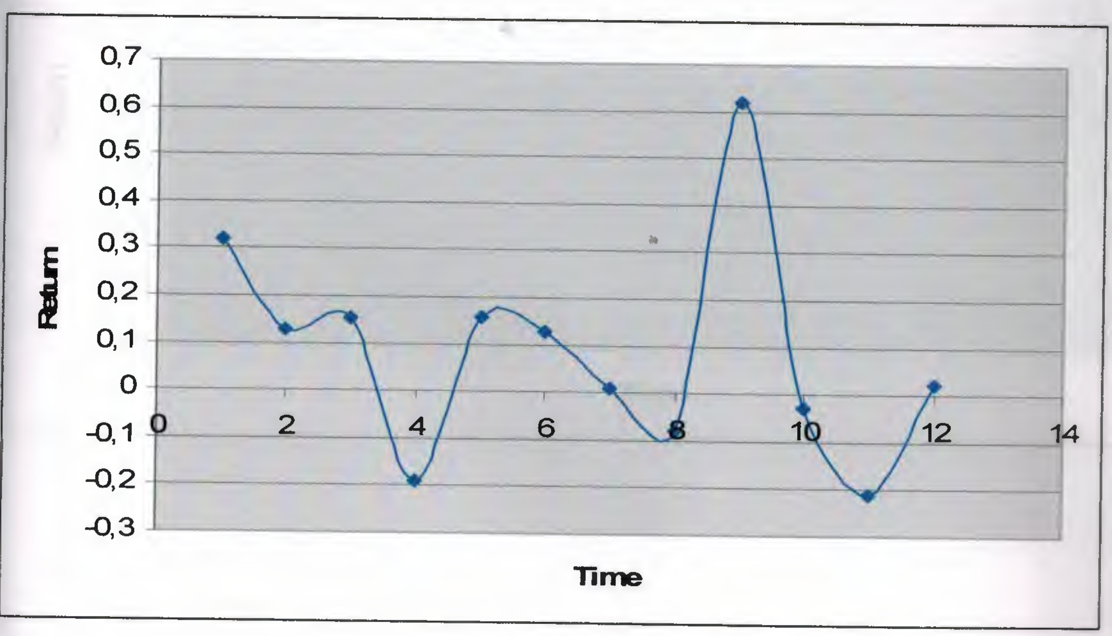
The plots of return rates for securities are given in figure 4.2. As shown from figure most of the securities are high risky. The determining investment for these securities and construction portfolio are very important.

Figure 4.2. Plot of returns for securities over 12 month: a) X1, b) X2, c)X3, d)X4, e)X5, f)X6

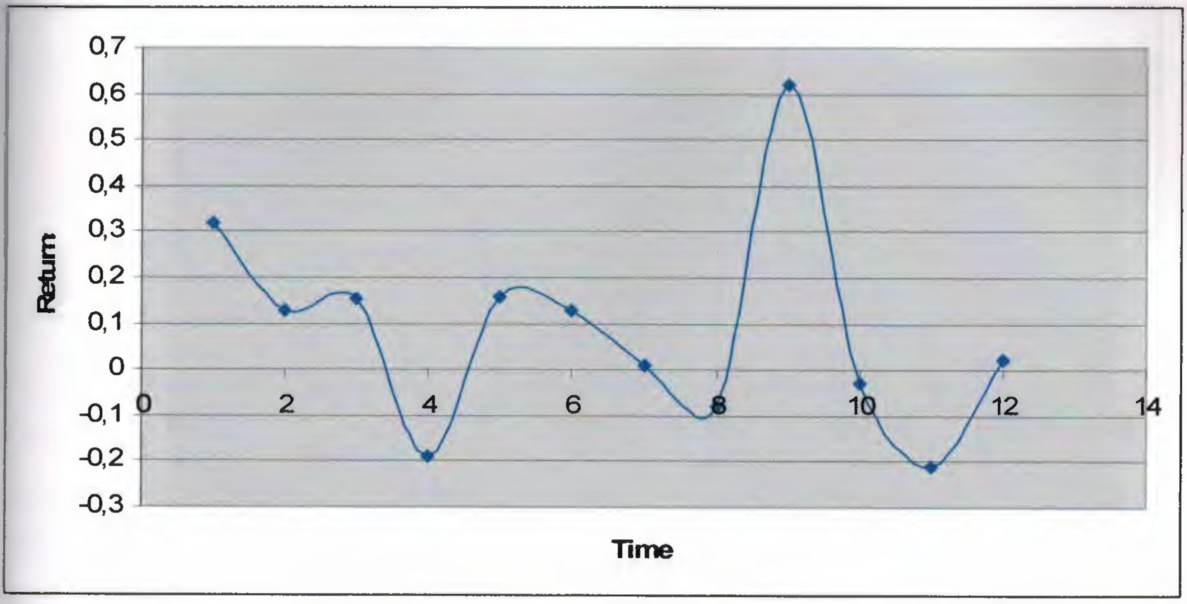
a)



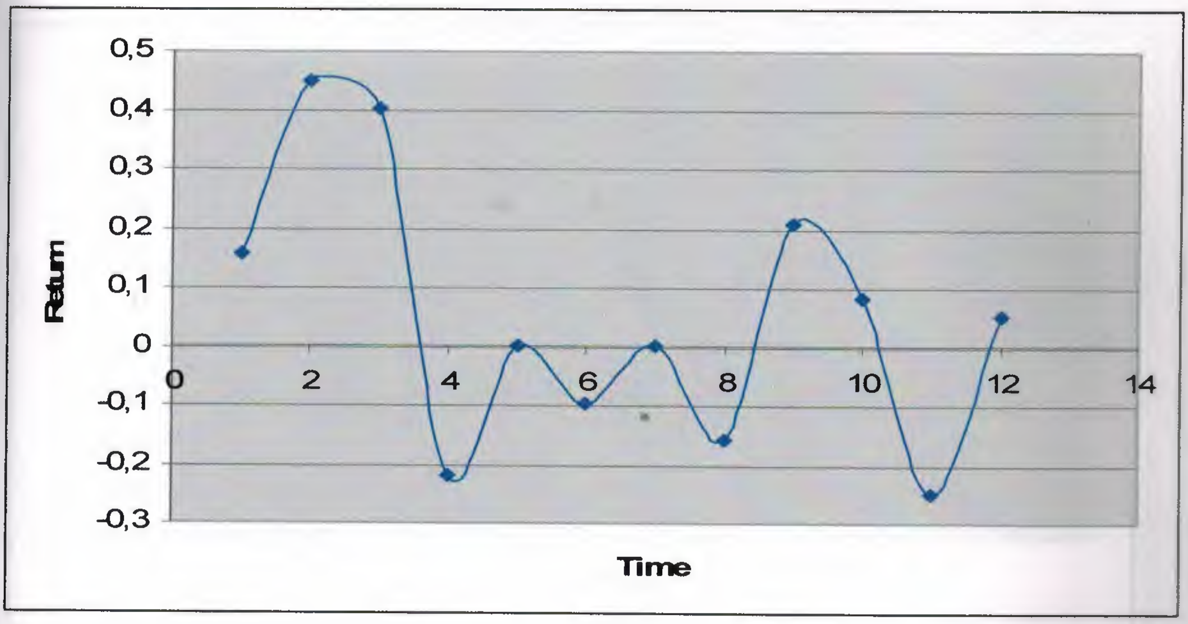
b)



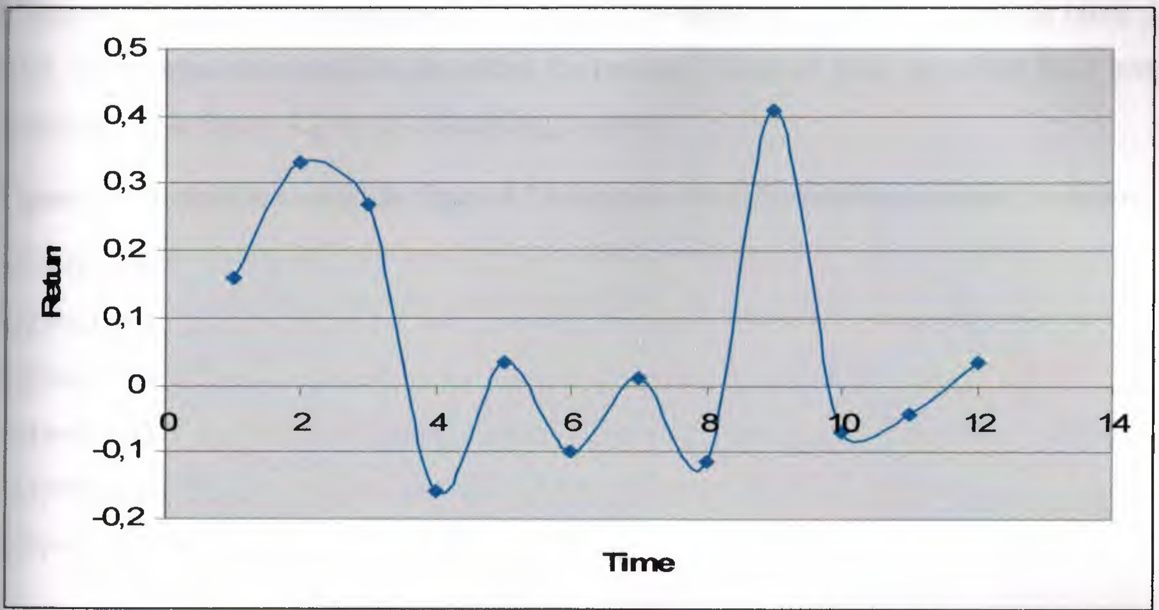
c)


0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
Return
0
2
4
6
8
10
12
14
Time

d)

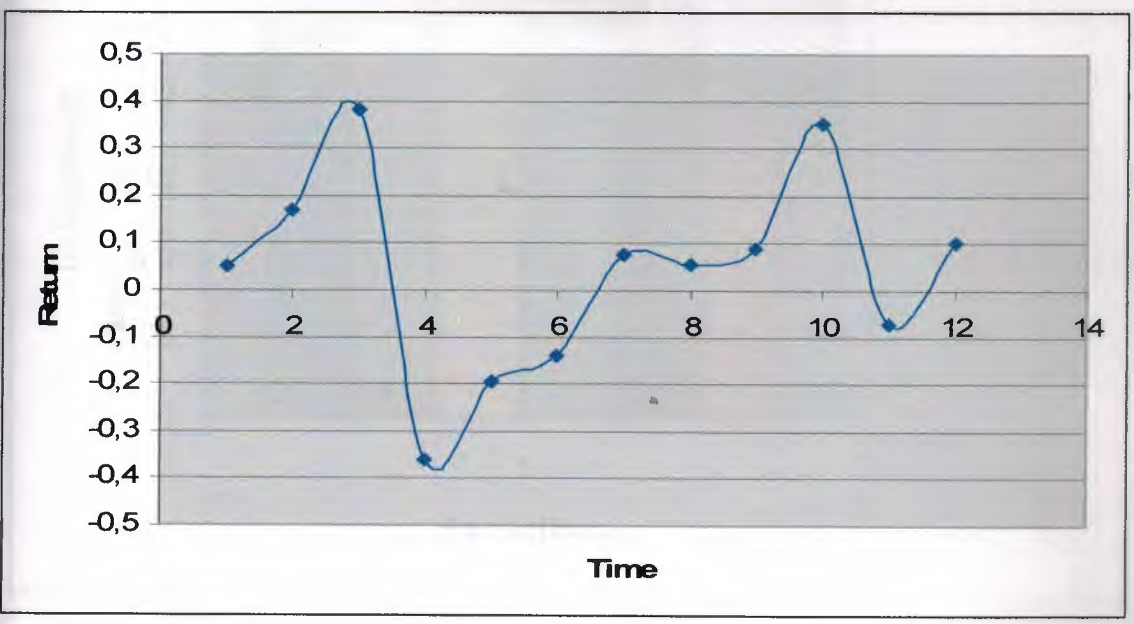

0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
Return
0
2
4
6
8
10
12
14
Time

e)



Return
Time
0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
0
2
4
6
8
10
12
14

f)



Return
Time
0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
-0,4
-0,5
0
2
4
6
8
10
12
14

sing expert opinion the values of expected returns for six stocks are estimated as (0.1252, .0862, 0.0793, 0.0521, 0.0637, 0.042), correspondingly. Using these input data the bjective function and constrained are formed. The value of portfolio return B is taken as .03. Using input data and GA algorithm the optimal values of risky securities have been ound in (1). In figure 4.2 result of learning is given.

igure 4.2. Simulation result ,In figure 4.3 histogram for six portfolio securities is shown

(1)=0.2586

(2)=0.0244

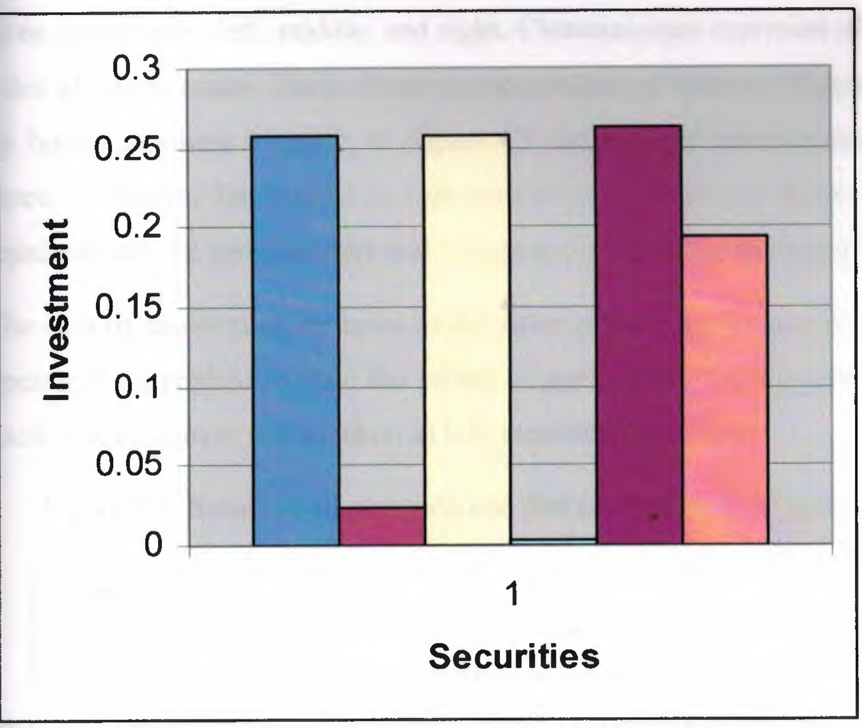
(3)=0.2575

(4)=0.0033

(5)=0.2630

(6)=0.1931

Figure 4.3 Histogram for six portfolio securities



As shown from figure if investor wants make investment to first, third, fifth and sixth ecurities the portfolio return will be maximized. Second and fourth securities will not give igh benefit.

4.3 Development of software for fuzzy portfolio selection

Consider fuzzy modeling of portfolio selection problem. Assume that a portfolio manager wants to allocate his assets among n risky securities based on recent historical data or the corporation's financial report and he want to minimize risk under some given level of portfolio return. Let xj are proportion of the total investment devoted to the risky security j, j=1,2,...,n. Assume that the data is obtained for the risky security j at period T. Obtained data are rate of return of risky security j at period t, where t=1,2,...,T. To find the amount of assets for each securities the above-described (2.21) and (2.22) fuzzy optimization model is applied.

The problem is to find such optimal values of invested securities by using them in equation (2.21) the value of risk that is given as objective function will be minimized. GA is applied to solve optimization problem and find optimal fuzzy values of securities. During modeling to describe fuzzy values of the parameters in the objective function and constraints the triangular forms are used. Each triangular fuzzy number is characterized by three parameters- left, middle, and right. Chromosomes represent all left, middle and right sides of fuzzy values. Each chromosome consists of number of genes that are represented by binary numbers 0 and 1. In figure 4.1 structure of chromosome that contains twelve assets is shown. Each asset is represented with 36 genes, where 12 genes are used to represent left, 12 genes middle and 12 genes right sides of the fuzzy values of assets.

The sum of assets must be equal to the value of total assets invested to securities. The GA operators are applied to train the values of parameters. During learning 50 populations are used. The crossover rate is taken as 0.8, mutation rate – 0.08.

Figure 4.1. Structure of chromosome that contain twelve fuzzy assets are shown

asset1	asset2	asset3		asset12
$x_l(1), x_m(1), x_r(1)$	$x_l(2), x_m(2), x_r(2)$	$x_l(3), x_m(3), x_r(3)$	...	$x_l(12), x_m(12), x_r(12)$

At the beginning the set of solutions, equal to population size, for each risky securities are generated randomly. For these values the constraints are tested and the objective function is evaluated. Then GA learning is applied to the solutions in order to find optimal one.

4.4 Numerical example. Application of fuzzy portfolio selection to Istanbul Stock Exchange.

During simulation the historical values of return rates for twelve stocks from Istanbul Stock Exchange for twelve month are taken as an input data. The values of returns of stocks for twelve months are given in table 1.

Table 1. Return of securities over 12 month

	Times					
	1	2	3	4	5	6
r1t	0.1613	-0.0159	0.2990	-0.0202	-0.0481	0.0833
r2t	0.0787	0.0595	0.1667	-0.1724	0.0482	0.3387
r3t	0.2405	0.0	0.2553	0.1325	-0.1170	-0.2656
r4t	0.2603	0.1587	0.1667	-0.1000	0.0	-0.0769
r5t	0.2857	0.1667	0.2923	-0.3229	-0.0400	0.0204
r6t	0.1091	0.0377	0.3119	-0.1920	0.1111	0.0227
r7t	0.1463	0.9680	0.0714	-0.4355	0.1481	-0.1290
r8t	-0.0149	0.0984	0.3708	-0.2054	0.3176	-0.2917
r9t	0.0750	0.0811	0.3704	-0.2603	-0.0135	0.0571
r10t	0.4590	-0.0469	0.5422	-0.3712	-0.0959	-0.1310
r11t	0.5152	0.4143	0.5054	-0.2377	-0.0758	-0.0294
r12t	0.1169	0.6383	0.9583	-0.2208	-0.0833	-0.2186

	Times					
	7	8	9	10	11	12
r1t	-0.0303	-0.1281	0.8095	-0.2588	0.1184	0.1692
r2t	-0.0159	-0.0597	0.2523	0.4658	-0.4786	-0.2391
r3t	0.3333	-0.0204	0.4627	-0.0290	-0.3000	-0.0877
r4t	0.0947	-0.0686	0.4571	-0.0139	-0.2577	0.1412
r5t	0.1136	-0.0784	0.3247	0.1324	-0.2609	0.0
r6t	0.0233	-0.1224	0.6000	0.0533	-0.1758	0.1098
r7t	0.6316	1.6389	0.0	0.2414	-0.5167	-0.1864
r8t	0.4286	0.0769	0.2188	-0.0303	-0.0294	0.0462
r9t	0.0294	-0.1707	0.7083	0.0435	-0.0980	-0.0556
r10t	0.0633	0.4630	0.4400	0.3636	-0.4149	0.1750
r11t	0.0099	-0.1140	0.4805	0.0132	-0.1648	0.0581
r12t	0.0238	0.0095	0.1798	-0.0220	-0.3259	-0.0690

The plots of the return rates of twelve securities are given in appendix A.

The values of expected returns are determined through arithmetic means of return rates or distribution function. This formula in case of uncertainty, for all stocks does not give accurate values of expected returns. Istanbul Stock Exchange is characterized with big oscillations of stocks indices. The use of arithmetic means of assets as expected return in portfolio model might not give desired result. For this reason the expert opinion is used to evaluate more appropriate values of expected returns. Based on analysis of pervious historical data and financial reports of companies the membership function is defined by the expert for each expected return. In the thesis the fuzzy values of expected returns are taken in triangular form. The values of expected returns for assets are evaluated as

R1=[0.0749 0.032 0.0454 0.0585 0.0478 0.0641 0.1948 0.0771 0.0549 0.1155 0.1096 0.0622];

R2=[0.0849 0.037 0.0504 0.0635 0.0528 0.0691 0.2048 0.0821 0.0589 0.1205 0.1146 0.0722];

R3=[0.0949 0.042 0.0554 0.0685 0.0578 0.0741 0.2148 0.0871 0.0639 0.1255 0.1196 0.0822];

Here R1, R2, R3 are left, middle and right parts of fuzzy values of expected return.

Using input data the objective function and constraints are formed. The accessible fuzzy value of portfolio return in the constraint is taken as [0.025 0.03 0.035].

Using GA learning algorithm the optimal values of risky securities have been found. Here left, middle and right sides of fuzzy values of risky securities are given in table 2.

Table 2. Simulation result

x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12
0.1843	0.1744	0.0113	0.0334	0.0517	0.0720	0.0152	0.1565	0.0426	0.0114	0.0286	0.0207
0.1998	0.2180	0.0146	0.0426	0.0668	0.0998	0.0701	0.1757	0.0469	0.0157	0.0318	0.0257
0.2154	0.2358	0.0167	0.0457	0.0780	0.1059	0.1076	0.1854	0.0711	0.0456	0.0441	0.0467

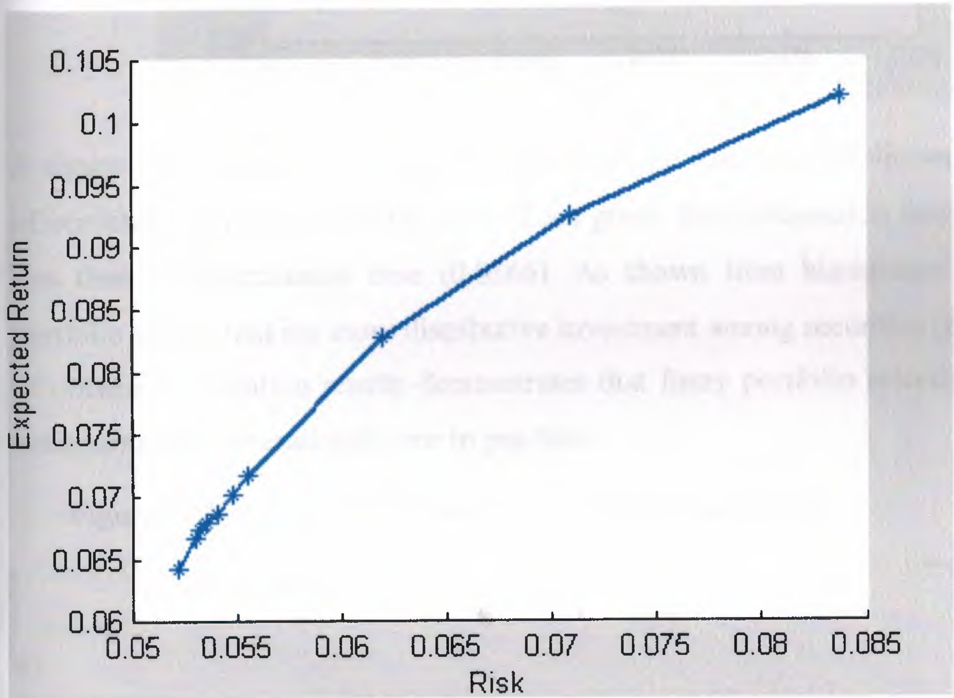
Each security is characterized by three numbers. These numbers demonstrate the acceptable fuzzy values of portion of investment for each security. For example x1=(0.1643, 0.1808, 0.1954) demonstrate that the portion of investment for first security must be at least 16.5%, maximum 19.5%. To find acceptable average value of the portion of investment for the security the Center of Average defuzification method is applied. In the result of defuzification it is found that acceptable average value for first security will be 18.06%.

4.5 Analysis of the efficiency of obtained results

For obtained values of securities (table 2) the calculated value objective risk function was 0.0548 and the value of portfolio return was 0.0701. Taking different satisfaction degrees 0.55 and 0.1 and maximum tolerance q=0.02 the model again is run and new values of securities have been determined. When satisfaction degree is taken as 0.55, for the obtained values of securities the value of risk function is determined as 0.0520 and the value of return- 0.0670. When satisfaction degree is taken as 0.1 the value of risk function is determined as 0.0501 and the value of return- 0.0652. The use of satisfaction degree allows decreasing of the value of risk function to some certain values. At the same time, the value of return is also decreased.

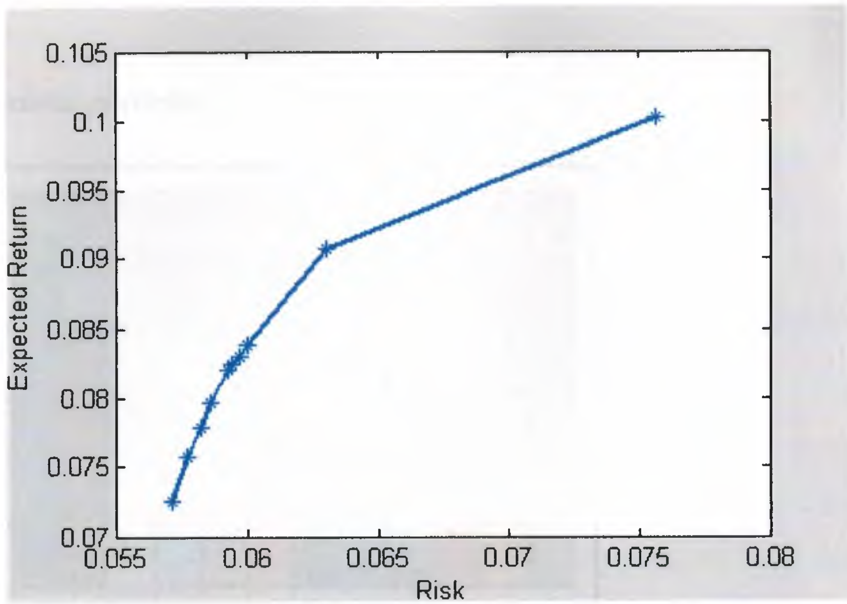
Also using the described model and statistical data the efficient frontier for portfolio model is constructed. During simulation fifty chromosomes are generated for each security. In figure 4.4 using GA searching algorithm the portfolio efficient frontier for twelve securities have been constructed. The efficient frontier describes the defuzification results of fuzzy expected returns and risks.

Figure 4.4. Fuzzy portfolio efficient frontier



In the same initial condition the modeling of deterministic portfolio selection has been carried out. Using arithmetic means of historical return rates the expected returns are determined for each security. Then applying deterministic semiabsolute model and genetic algorithm the optimal values of securities have been determined. In figure 4.5 the efficient frontier constructed by deterministic model is given. As shown from the figures the values of objective risk function in the fuzzy portfolio are less than in deterministic one.

Figure 4.5. Efficient frontier obtained from deterministic model

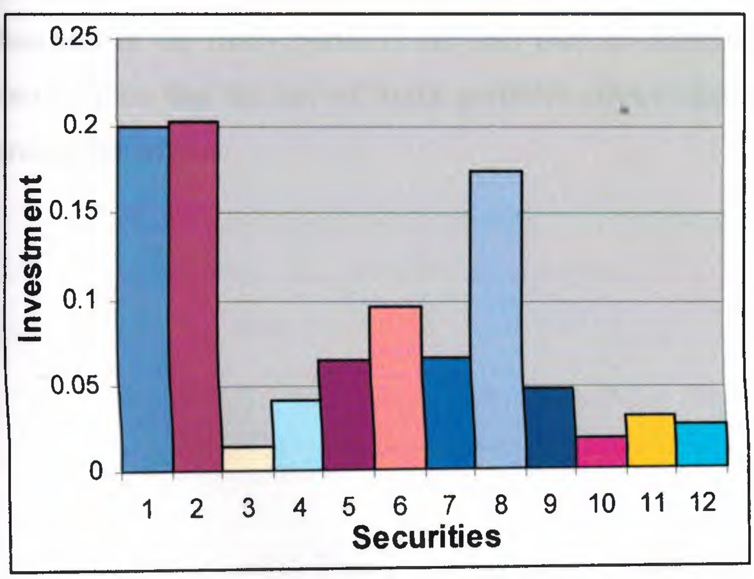


In figure 5 the histograms for fuzzy (a) and deterministic (b) portfolio securities in the case of minimum portfolio return rate ρ=0.03 are given. Risk obtained in fuzzy case (0.0548) is less than in deterministic case (0.0566). As shown from histograms the use of fuzzy portfolio allows making more distributive investment among securities (figure 5). Analysis of obtained simulation results demonstrates that fuzzy portfolio selection model is more reasonable than deterministic one in practice.

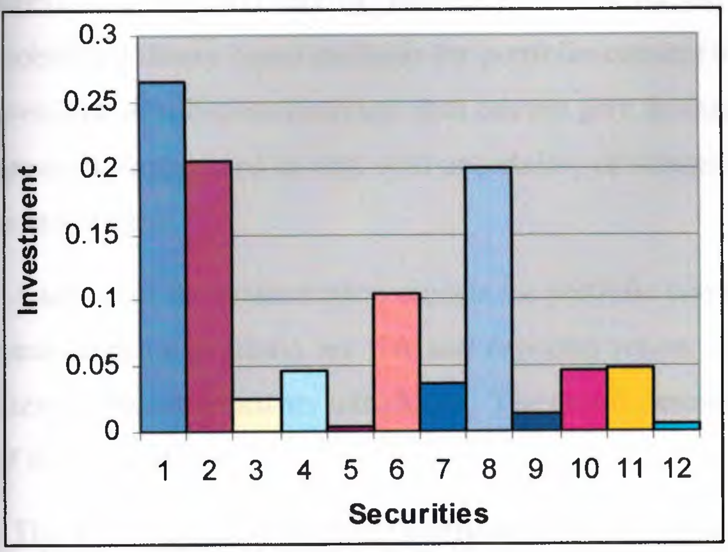
Figure 5. Histograms of the portions of portfolio securities:

Fuzzy portfolio

a)



b) Deterministic portfolio



4.6. Summary

The simulation of deterministic and fuzzy portfolio selection models have been performed using statistical data taken from Istanbul Stock Exchange. Using GAs the software is developed for finding optimal values of securities in investment. The realization of developed models is carried out using Matlab programming language. The obtained results from the modeling satisfy the efficiency of the presented fuzzy approach in portfolio selection.

Comparative results of developed models demonstrate that the values of objective risk function in the fuzzy portfolio are less than in deterministic one. Obtained histograms demonstrate that the use of fuzzy portfolio allows making more distributive investment among securities.

Conclusion

- Analysis of financial and economical statements of companies demonstrates that they are functioning in condition of uncertainties, insufficiency of information. The existing probability theory based methods for portfolio construction in case lack of information or even lack of sufficient historical data can not give desirable result. In this case fuzzy logic approach is purposed to deal with uncertainty of research problem and construct portfolio in investment.

- Analysis of the mathematical models for portfolio construction problem shows that basic parameters for portfolio are risk and expected return. The conceptions of fuzzy risk and fuzzy expected return are introduced. The expert perception is used to estimate the values of these parameters.

- The soft constraints for fuzzy portfolio model are introduced. To obtain optimal solution the satisfaction degree for each constraint are assigned to dealt with flexibility of the constraints. Fuzzy flexible constraints are introduced for this problem

- Using semi absolute deviation model, the portfolio selection model is formulated for security investment to provide flexibility for the constraints. The applied model has soft constraints allowing to estimate the investor's preference about risk-return trade-off.

- The fuzzy portfolio selection model is formulated for security investment. This model takes into account fuzzy value of expected return and fuzzy risk. Satisfaction degree is assigned for each constraint to provide flexibility of the constraints.

- Using GAs the algorithm is developed to solve deterministic portfolio selection problem. This algorithm is used to develop program for finding optimal values of securities in investment.

- Using fuzzy GAs the algorithm is developed to solve fuzzy portfolio selection. Based on developed algorithm the program is developed for finding optimal fuzzy values of securities in investment.

- The software realization of developed portfolio models is carried out using Matlab programming language. The developed models are tested using statistical data taken from Istanbul Stock Exchange. The obtained results from the modeling satisfy the efficiency of the presented fuzzy approach in portfolio selection.

- Comparative results of developed models demonstrate that the values of objective risk function in the fuzzy portfolio are less than in deterministic one. Obtained histograms demonstrate that the use of fuzzy portfolio allows making more distributive investment among securities.

APPENDIX

APPENDIX A

The flowchart of genetic portfolio optimization algorithm

Block 1. Start of the program.

Block 2. Entering the values of population size ps, crossover rate pc, mutation rate pm, chromosome's length cs, number of learning loops endloop, crossover point pos.

Block 3. Generating random integer numbers 0 and 1 for ps chromosomes.

Block 4. Initialization of loop.

Block 5. If current loop number is not less than equal end of loop number then stop operation and go to the block 6.

Block 6. End of the program.

Block 7. Initializing initial value of fitness function describing portfolio optimization model and total fitness to zero.

Block 8. Control the loop of chromosomes and selecting first chromosome.

Block 9. Converting the chromosome that consists of 0 and 1 to decimal values.

Block 10. Calculating values of fitness functions that describe portfolio optimization model 3.1 and 3.2.

Block 11. Compare the calculated value of fitness function with previous one.

Block 12. Save calculated value of fitness function.

Block 13. Calculate total fitness

Block 14. Control the loop of chromosomes and selecting first chromosome.

Block 15. Calculating cumulative probability.

Block 16. Generating ps random numbers.

Block 17. Control the loop of random numbers and selecting first random number.

Block 18. Control the loop of chromosomes and selecting first chromosome.

Block 19. Check condition r(i)>q(i1). If it is true check the following condition. In other case select next random number.

Block 20. Check the condition r(i)<q(i1+1). If it is not true select next random number.

Block 21. Save i-th chromosomes in v(i,j).

Block 22. Generating ps random numbers.

Block 23. Initialization index k to 1.

Block 24. Control the loop of random numbers and selecting first random number.

Block 25. Check the condition r(i)<pc. If it is not true select next random number.

Block 26. Select i-th chromosome for crossover operation, ind(k)=i;

Block 27. Increment index k.

Block 28. Control the loop of selected chromosomes for crossover operation. Select first chromosome.

Block 29. Control the loop of genes in the chromosome until the crossover point.

Block 30. Making crossover operation and swapping the place of genes.

Block 31. Generating random numbers.

Block 32. Control the loop of random numbers and selecting first random number.

Block 33. Control the loop of genes in the chromosome and selecting first gene.

Block 34. Check the condition r(i,j)<pm. If condition is not true select next random number and gene.

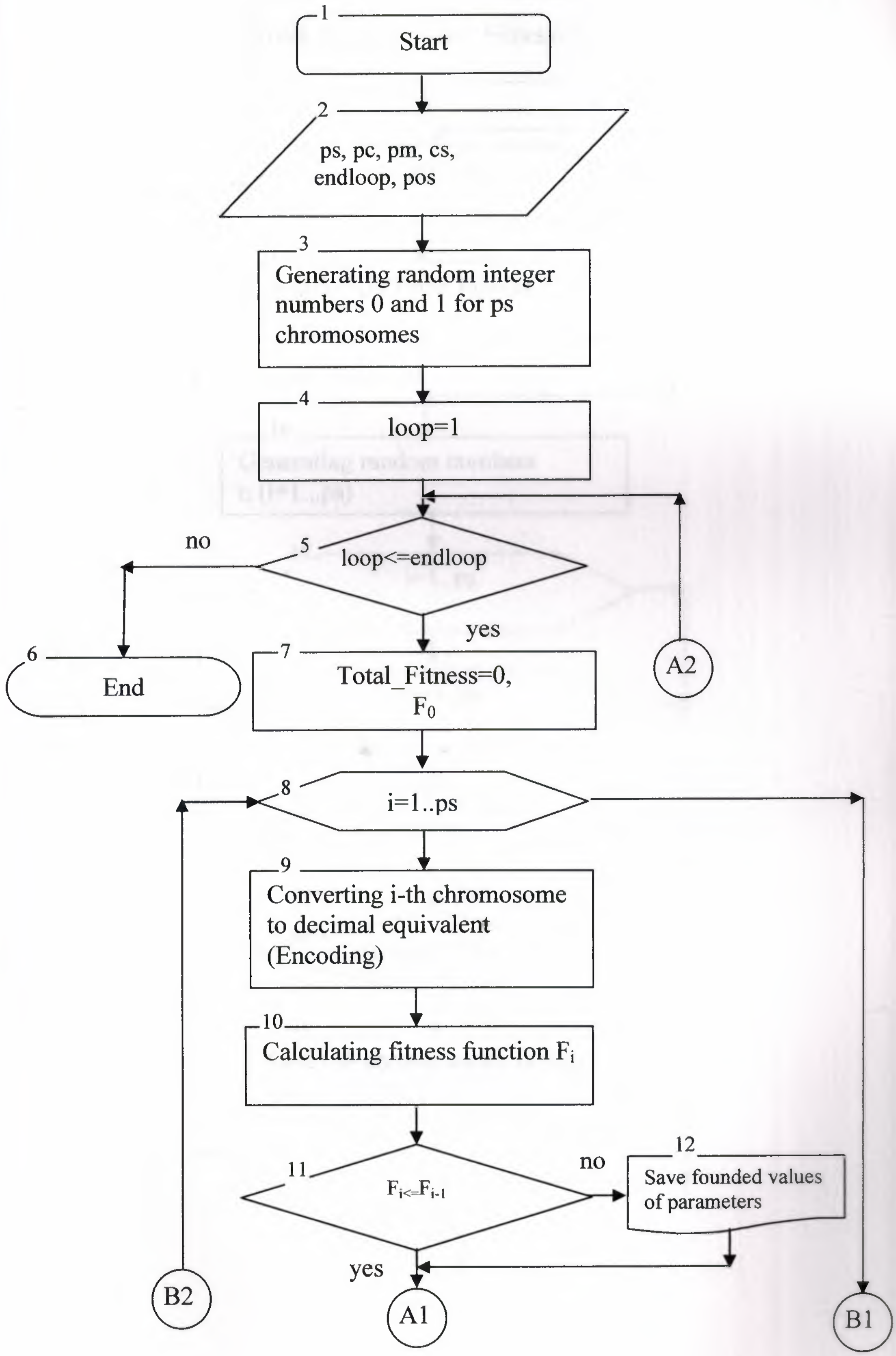
Block 35. Check gene's value (condition v(i,j)=0).

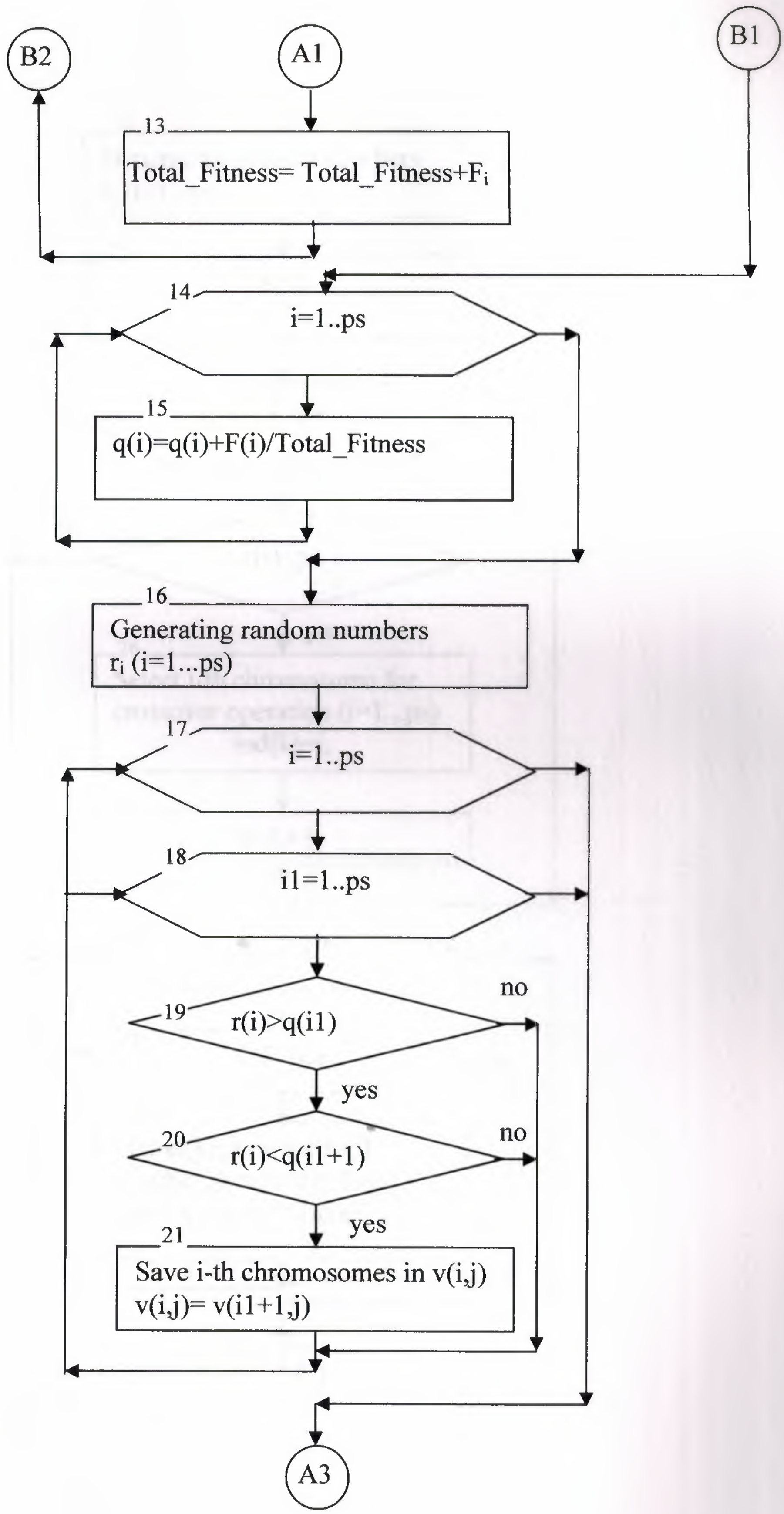
Block 36. Change value of gene to 1.

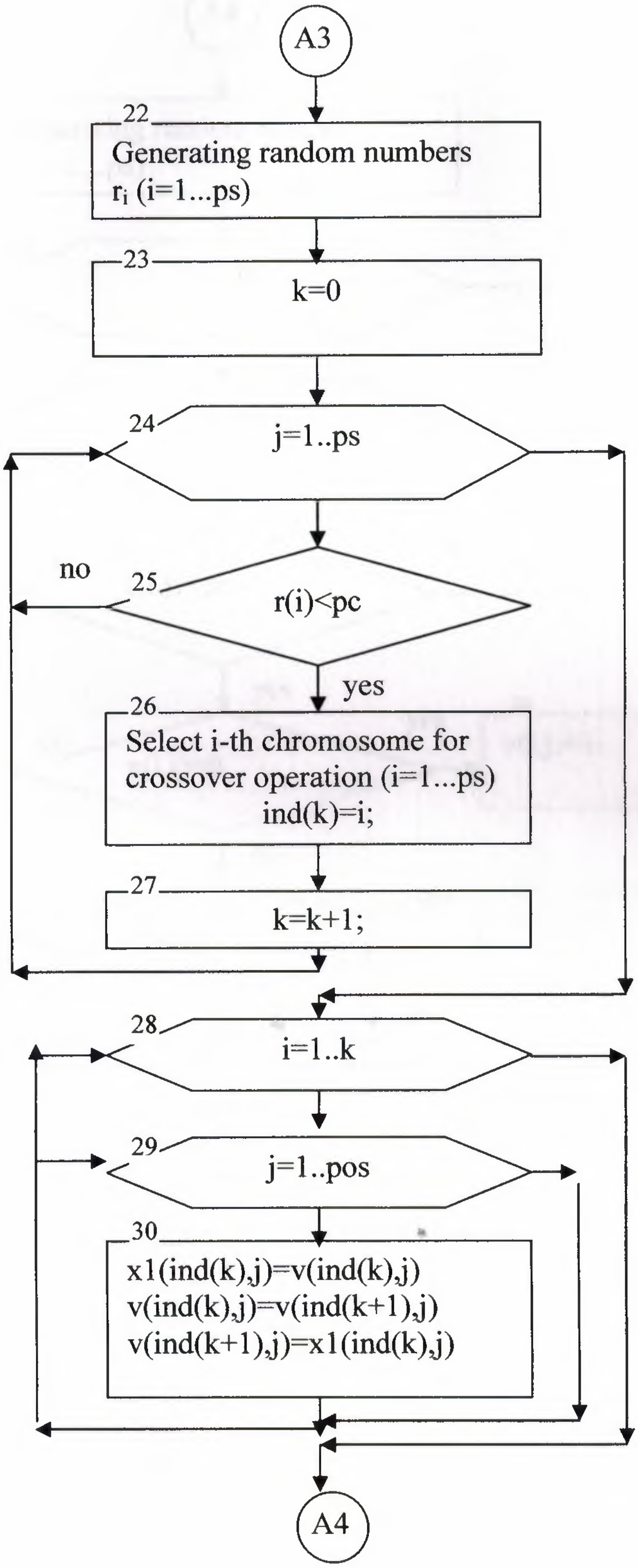
Block 37. Check gene's value (condition v(i,j)=1).

Block 38. Change value of gene to 0.

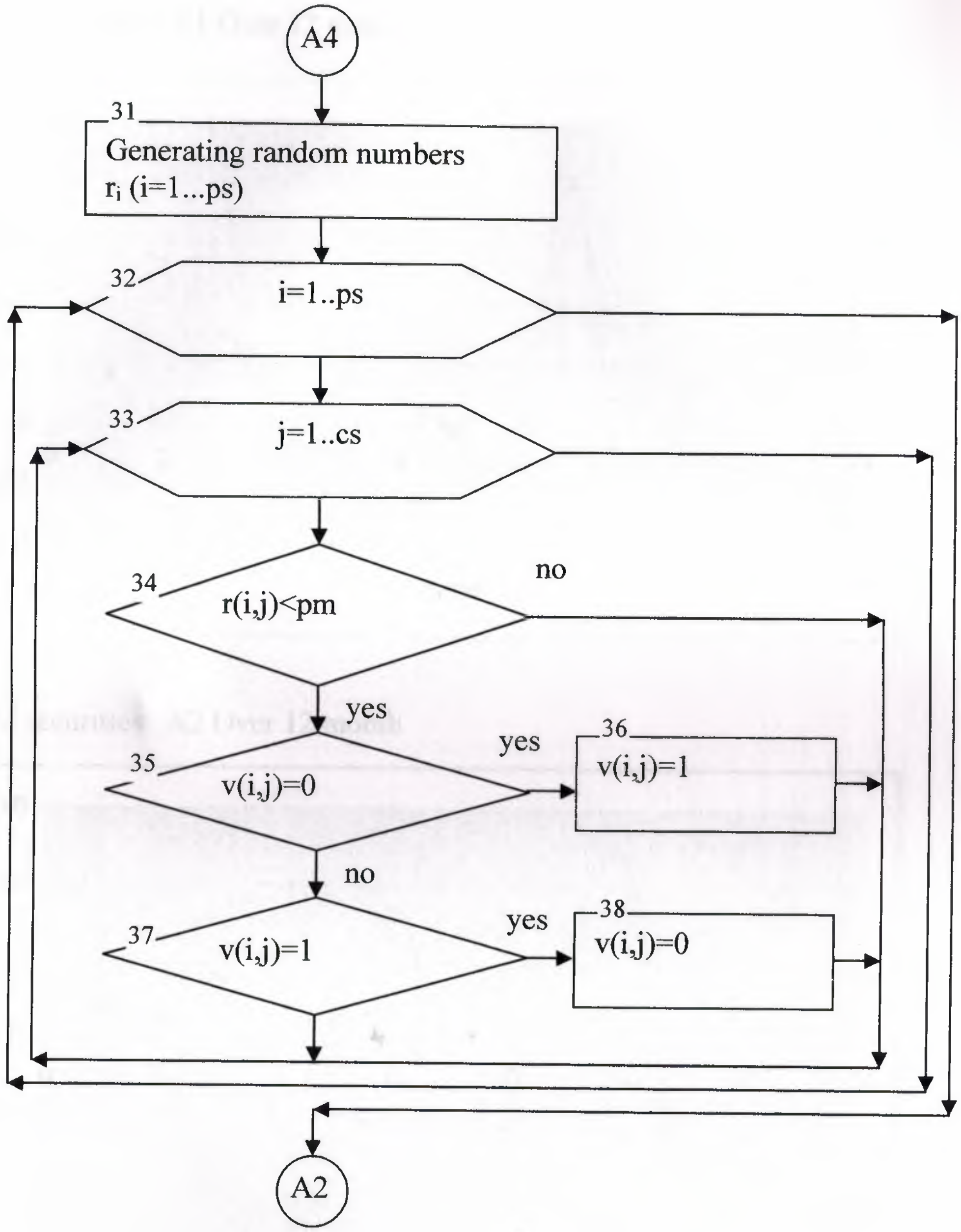
Flowchart of genetic portfolio optimization algorithm




B2
A1
B1
13
Total_Fitness= Total_Fitness+F_i
14
i=1..ps
15
q(i)=q(i)+F(i)/Total_Fitness
16
Generating random numbers
r_i (i=1...ps)
17
i=1..ps
18
i1=1..ps
19
r(i)>q(i1)
no
yes
20
r(i)<q(i1+1)
no
yes
21
Save i-th chromosomes in v(i,j)
v(i,j)= v(i1+1,j)
A3

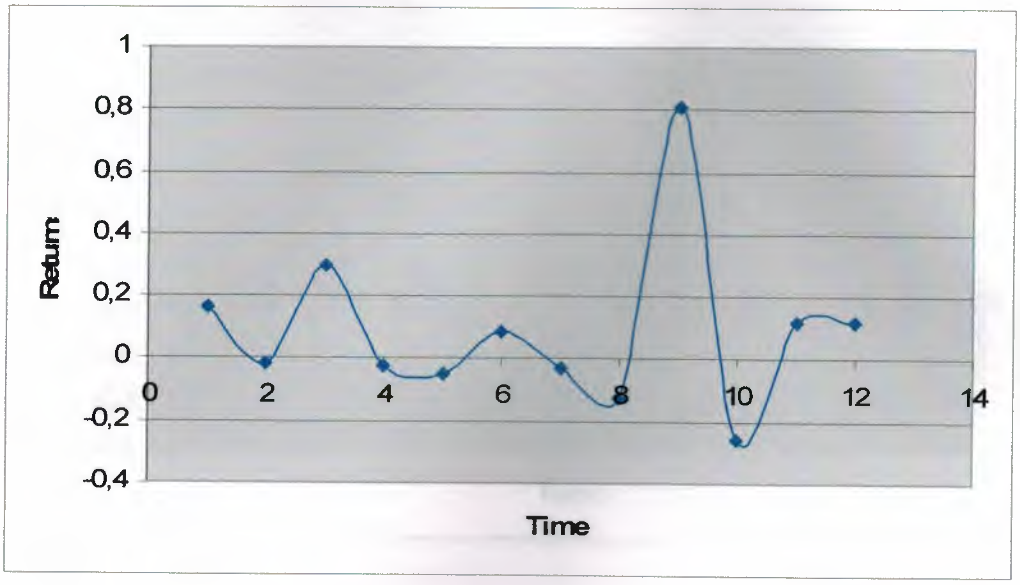


A3
22
Generating random numbers
r_i (i=1...ps)
23
k=0
24
j=1..ps
no
25
r(i)<pc
yes
26
Select i-th chromosome for
crossover operation (i=1...ps)
ind(k)=i;
27
k=k+1;
28
i=1..k
29
j=1..pos
30
x1(ind(k),j)=v(ind(k),j)
v(ind(k),j)=v(ind(k+1),j)
v(ind(k+1),j)=x1(ind(k),j)
A4

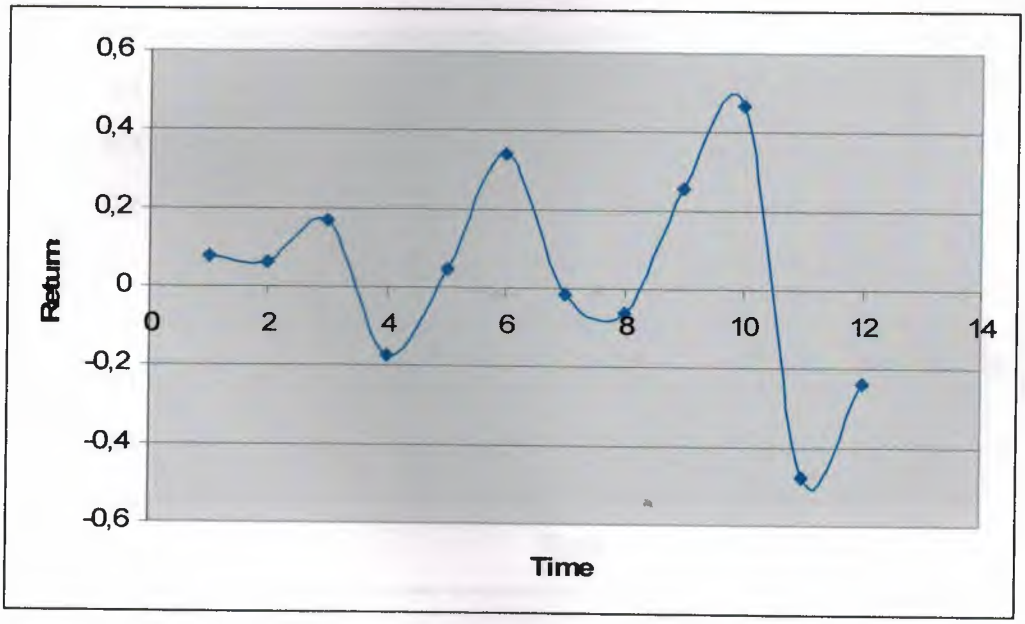

A4
31
Generating random numbers
r_i (i=1...ps)
32
i=1..ps
33
j=1..cs
34
r(i,j)<pm
no
yes
35
v(i,j)=0
yes
36
v(i,j)=1
no
37
v(i,j)=1
yes
38
v(i,j)=0
A2

APPENDIX B

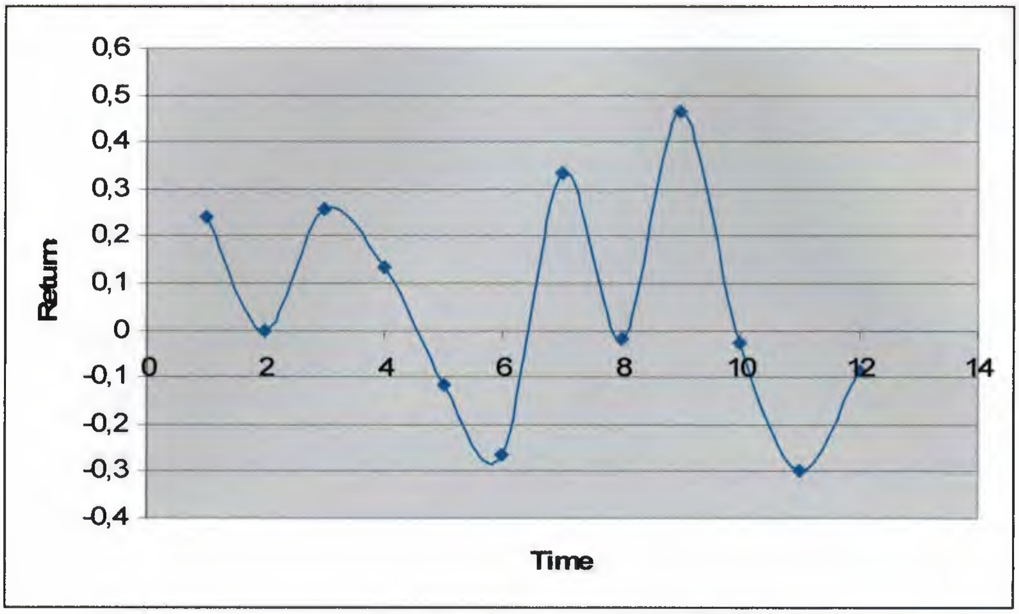
Return of securities X1 Over 12 month



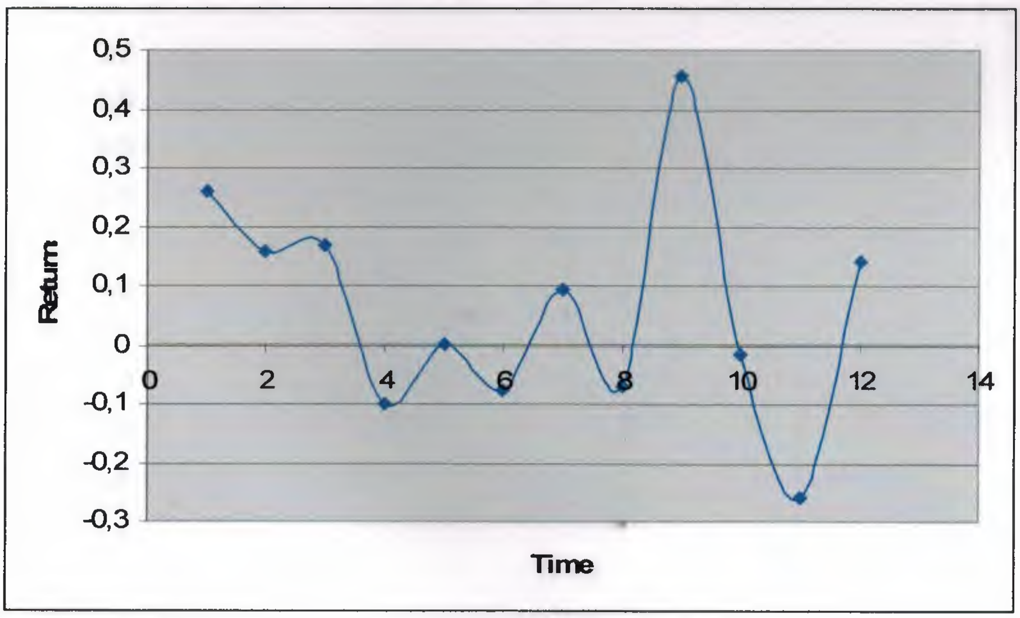
Return of securities X2 Over 12 month



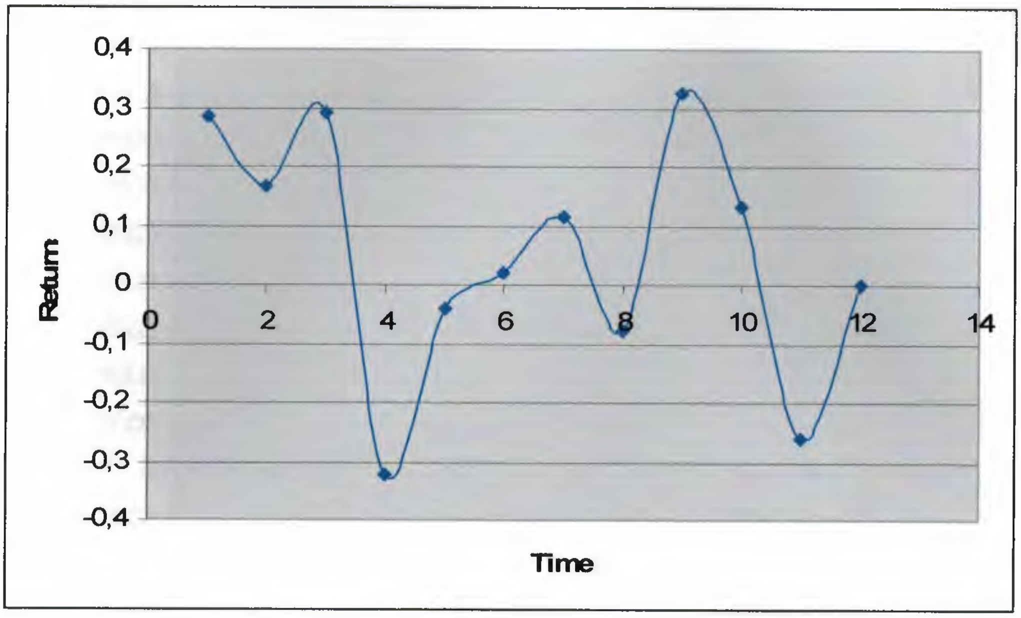
Return of securities X3 Over 12 month



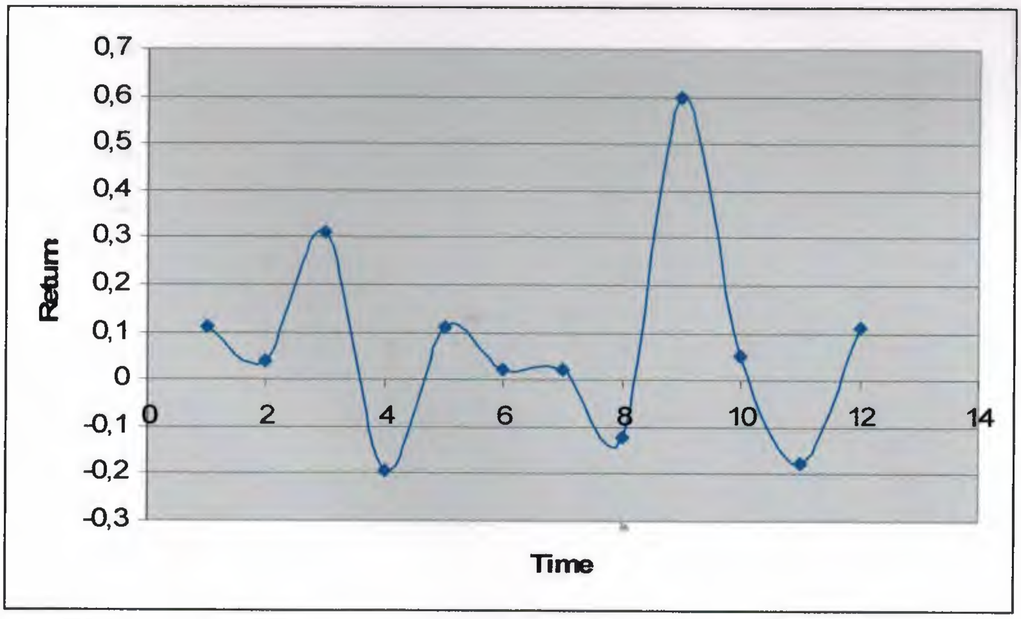
Return of securities X4 Over 12 month



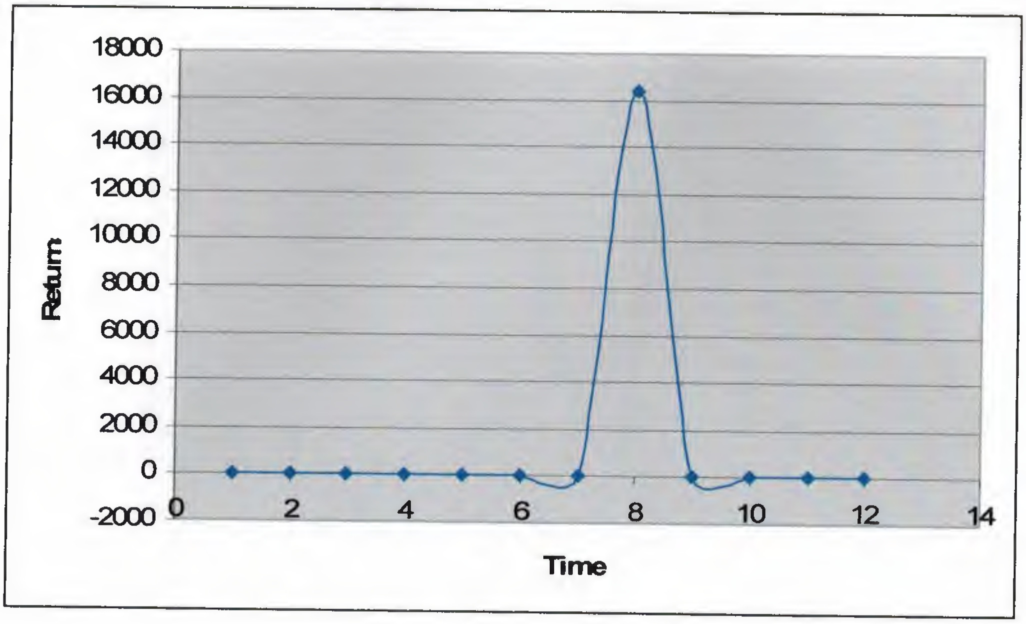
Return of securities X5 Over 12 month



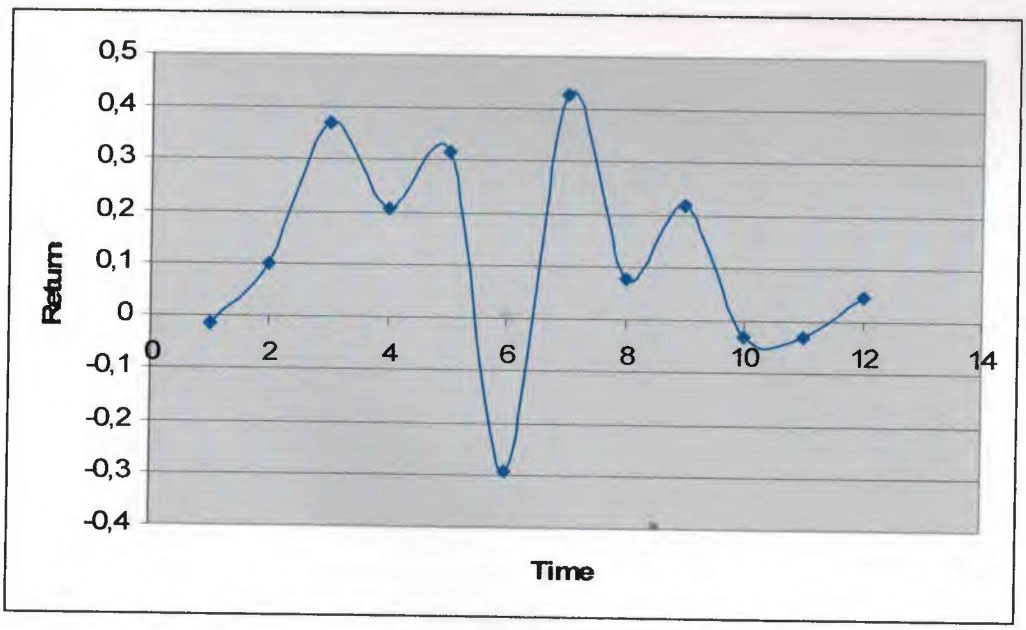
Return of securities X6 Over 12 month



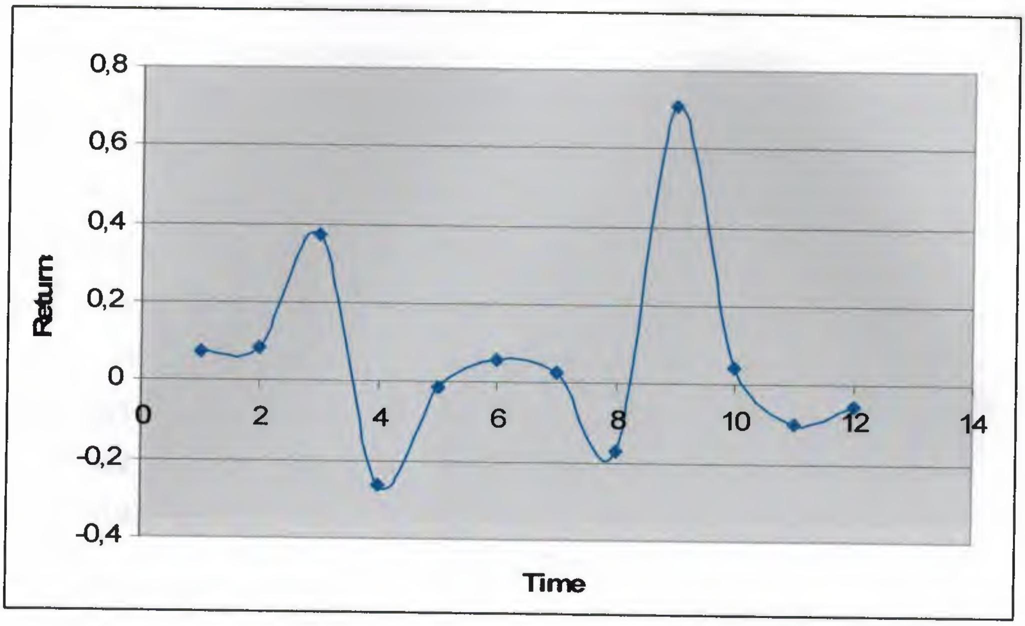
Return of securities X7 Over 12 month



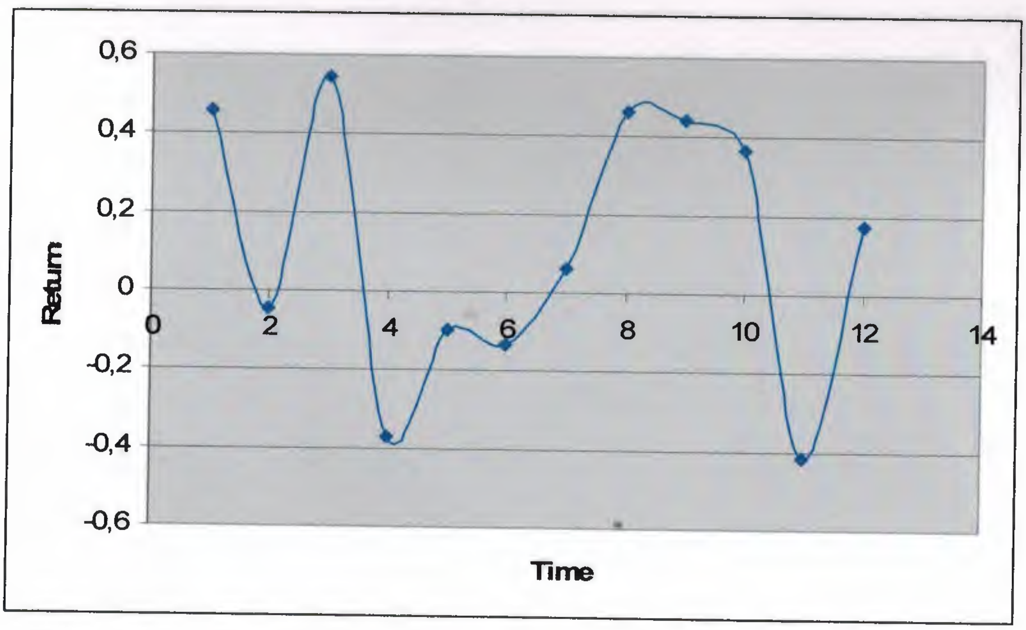
Return of securities X8 Over 12 month



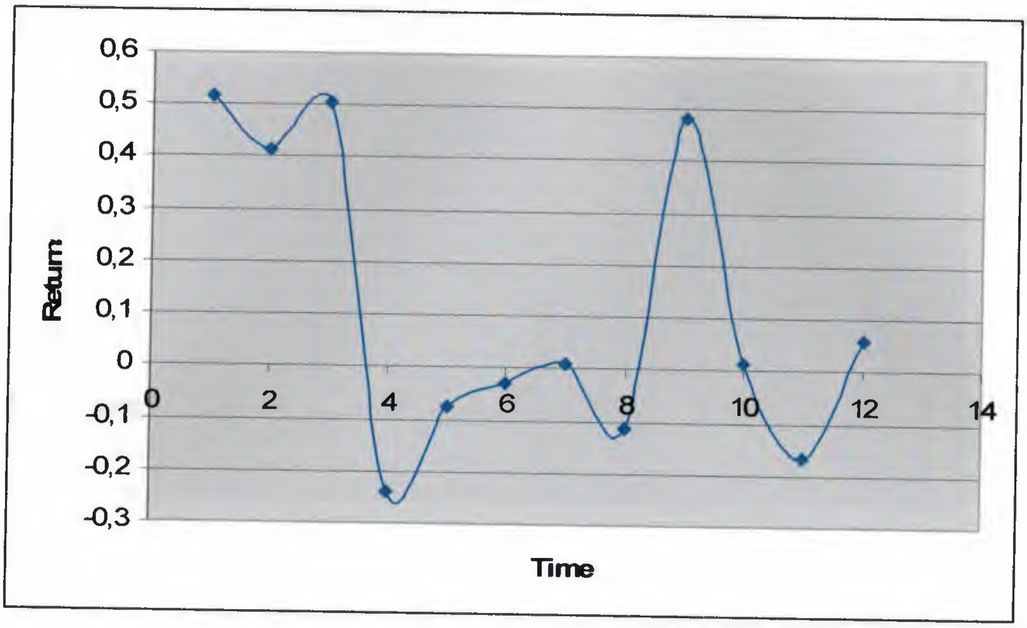
Return of securities X9 Over 12 month



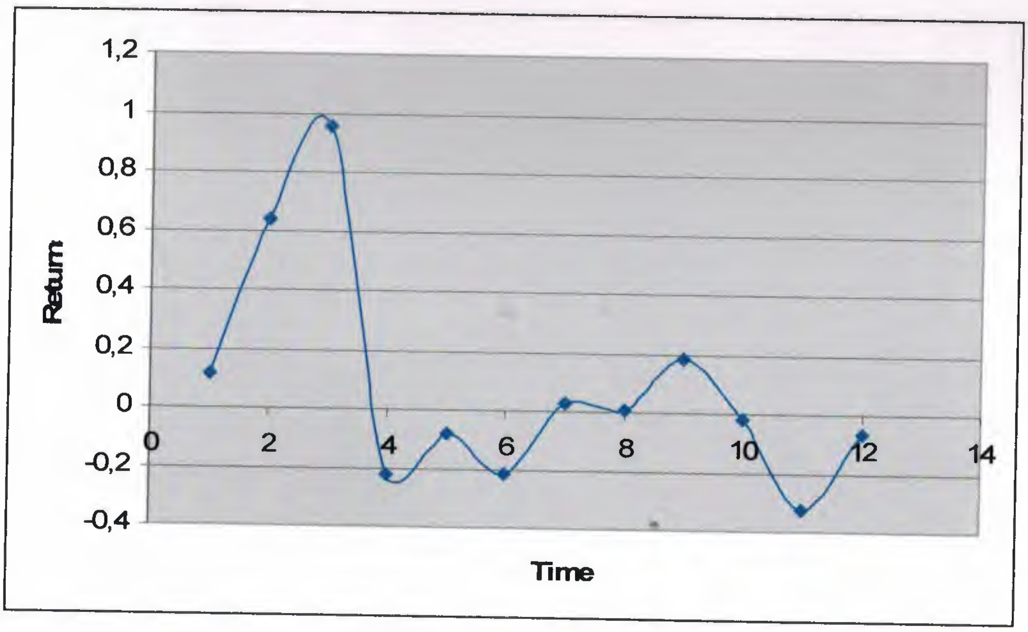
Return of securities X10 Over 12 month



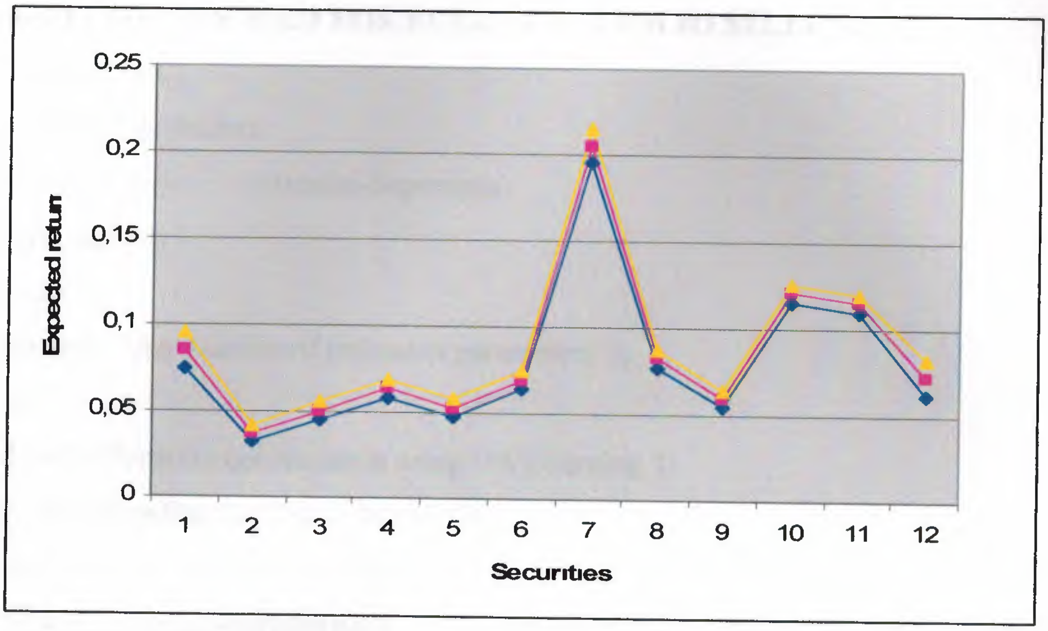
Return of securities X11 Over 12 month



Return of securities X12 Over 12 month



Plot of expected returns for twelve securities



APPENDIX C

MATLAB PROGRAM FOR FUZZY PORTFOLIO SELECTION

```
function []=...
   ga_opt_portrc(w)
% type_port= 1  - Mansini-Seperanza
type_port=1;
npar=12;
%input('Enter number of unknown parameters:');
clc
disp('1- Portfolio optimization using GA Learning ')
disp('2- Reading data from data file ')
disp('3- Saving data to data file ')
disp('4- Testing deterministic')
disp('5- Fuzzy portfolio optimization using GA')
disp('4- Testing ')
disp('6- Exit')
num=input('Select operation: ');
switch num
   case 1, regim=1;
        [w]=Galgorithm_port(regim,npar,type_port)
        ga_opt_port(w);
  case 2, fpt = fopen('fgaport1.dat','r');
     [w]=fscanf(fpt,'%f \n',[npar]);
     w=w';
     w
     pause
     ga_opt_port(w);
 case 3, fpt = fopen('fgaport1.dat','w');
     fprintf(fpt,'%f %f %f \n',w)
     fclose(fpt);
     pause
     ga_opt_port(w);
 case 4, regim=4;
    if(type_port==1)
```

```
        Fin_portfolio_mark(w);
     else
        [fobject,ert,x]=fin_port_msad(regim,npar,w)
     end
         pause
         ga_opt_port(w);

  case 5, regim=1;
         [w1,w2,w3]=Galgorithm_port_fuz(npar,type_port);
        %[w]=Galgorithm_port_fuz_cw(npar)

  case 6, %exit %quit
  otherwise disp('unknown')
end
function[wfind1,wfind2,wfind3]=...
   Galgorithm_port_fuz(npar,type_port)
endloop=5000;
cs1=15;  % ps=20;
ps=40;
cs=3*npar*cs1;                    % cs1/point    must be integer number
point=3;   pos=cs1/point            % cs1 is devided into 3 part
%pos=2*cs/(cs1);  % cs is divided into 2*cs1 parts
f_find=-10000;  max_err=0.1; reg_err=50; max_iter=70;
pc=0.8;  pm=0.2;
   beg1=0;   ent1=0.8;
   beg2=0;   ent2=0.9;
   beg3=0;   ent3=1;

if point==1
   regim=1;
else
   regim=2;
end
```

```
 for i=1:ps
    for j=1:cs
       v(i,j)=randint(1);
       if(j>3*cs1)
          v(i,j)=v(i,j-3*cs1);
       end
    end;
 end;
%v
%       Decimal
loop=1;
while(loop<=endloop)
   ftot=0;          % total fitness
   for ipop=1:ps
      for j=1:cs
         v1(j)=v(ipop,j);
      end;
      [w1,w2,w3]=decimal_opt_fuz(npar,beg1,ent1,beg2,ent2,beg3,ent3,cs1,v1);
      [w1,w2,w3]=reorder(w1,w2,w3,npar);


[fobject1,fobject2,fobject3,ert1,ert2,ert3,ww1,ww2,ww3]=fob_port_msad_fuz(npar,w1,w2
,w3,ent1,ent2,ent3);

      [fobject1 fobject2 fobject3]
      fi(ipop)=(0.75*fobject1+4.5*fobject2+0.75*fobject3)/6;         % fi(ipop)=fobject2;
      if(f_find<=fi(ipop))                                % MAXIMIZATION
         f_find=fi(ipop);
         f_find_fuz=[1/fobject1-1  1/fobject2-1  1/fobject3-1];
         ifind=ipop;
         for i=1:npar
            wfind1(i)=ww1(i); wfind2(i)=ww2(i);wfind3(i)=ww3(i);
         end
         for j=1:cs
```

```
            vfind(j)=v(ifind,j);
        end;
        ert_find1=ert1; ert_find2=ert2; ert_find3=ert3;
    end;
    ftot=ftot+fi(ipop);
    sprintf('%d, Func=%f  Total fitness=%f',ipop,fi(ipop),ftot);
end;
sprintf('Optimal function value')
f_find_fuz
wfind1
wfind2
wfind3

% Selection
for i=1:ps
    fqi(i)=0;
end
fqi(1)=fi(1)/ftot;
for i=2:ps
    fqi(i)=fqi(i-1)+fi(i)/ftot;
    %Probability of selection
end;
yr=rand(1,ps);
for i=1:ps
for i1=1:ps
    if ((yr(i)>fqi(i1))&(yr(i)<fqi(i1+1)))
        for j=1:cs
            v(i,j)=v(i1+1,j);
        end;
    end;
end;
end;
%   v
```

```
% Crossover
yr=rand(1,ps);
k=0;
for i=1:ps
   if(yr(i)<=pc)
      k=k+1;
      ind(k)=i;
   end
end
kk=k;
% ind

if(regim==1)
  % 1 point Crossover
  for k=1:2:kk-1
     for j=pos+1:cs
        x1(ind(k),j)=v(ind(k),j);
        v(ind(k),j)=v(ind(k+1),j);
        v(ind(k+1),j)=x1(ind(k),j);
     end;
  end
end

if(regim==2)
  %   Multipoint Crossover (if point=1  1-point,  point=2 2-point,
  %   point=3 3-point...)
  for k=1:2:kk-1
     num=1;
     while num<cs/pos
        for j=pos*num+1:(num+1)*pos
           x1(ind(k),j)=v(ind(k),j);
           v(ind(k),j)=v(ind(k+1),j);
           v(ind(k+1),j)=x1(ind(k),j);
        end;
```

```
            num=num+2;
          end
       end
    end

    if(regim==22)
     %    2 point Crossover
     for k=1:2:kk-1
         for j=pos+1:2*pos
             x1(ind(k),j)=v(ind(k),j);
             v(ind(k),j)=v(ind(k+1),j);
             v(ind(k+1),j)=x1(ind(k),j);
         end;
     end
    end
%   v
 %  pause

 %{ Mutation }
 for i=1:ps
     for j=1:cs
         if (rand(1)<pm)
             v(i,j);
             if (v(i,j)==0)
                 v(i,j)=1;
             else v(i,j)=0;
             end
             v(i,j);
         end;
    end
 end

 loop=loop+1
 if loop==endloop
```

```
    sprintf('Output results')
    f_find_fuz
    [wfind1 wfind2 wfind3]
    [ert_find1 ert_find2 ert_find3]
    pause
  end
end


function [w1,w2,w3]=decimal_opt_fuz(npar,beg1,ent1,beg2,ent2,beg3,ent3,cs1,v1)
%cs1 - 1st parameter in cs
ic=0;
for i=1:npar
   w1(i)=0; w2(i)=0; w3(i)=0;

   for j=ic+1:ic+cs1
      w1(i)=w1(i)+v1(j)*2^(ic+cs1-j);
   end;
   w1(i)=beg1+w1(i)*(ent1-beg1)/(2^cs1-1);
   ic=ic+cs1;

   for j=ic+1:ic+cs1
      w2(i)=w2(i)+v1(j)*2^(ic+cs1-j);
   end;
   w2(i)=beg2+w2(i)*(ent2-beg2)/(2^cs1-1);
   ic=ic+cs1;

   for j=ic+1:ic+cs1
      w3(i)=w3(i)+v1(j)*2^(ic+cs1-j);
   end;
   w3(i)=beg3+w3(i)*(ent3-beg3)/(2^cs1-1);
   ic=ic+cs1;
```

```
end;


function [fobject1,fobject2,fobject3,ert1,ert2,ert3,x1,x2,x3]=...
   fob_port_msad_fuz(npar,w1,w2,w3,ent1,ent2,ent3)
 %x=x/sum(x);
 if npar==5
   ret=[0.054 0.045 -0.030 -0.018 0.043 0.047 0.055 0.036 -0.039 -0.043 0.046 0.052
        0.032 0.055 -0.036 0.052 0.047 0.034 0.063 0.048 0.025 0.040 0.036 -0.017
 0.064 0.056 0.048 0.007 0.053 0.036 0.017 0.047 -0.059 0.047 0.040 0.032
 0.038 0.062 -0.037 0.050 0.065 -0.043 0.062 0.034 0.035 0.056 0.057 0.025
 0.049 0.067 -0.039 0.051 0.049 0.037 0.055 0.025 0.052 0.02 0.045 0.04];
   exret1=[0.0197 0.0306 0.0313 0.0327 0.0366];
   exret2=[0.0207 0.0316 0.0323 0.0337 0.0376];
   exret3=[0.0217 0.0326 0.0333 0.0347 0.0386];
 end
 if npar==6
  ret=[
   0.0667 0.2121 0.1647 0.0000 0.4167 0.2632 -0.0306 -0.0392 0.4366 0.1094 -
0.4386 0.3412
   0.3200 0.1300 0.1530 -0.1900 0.1560 0.1300 0.0100 -0.0780 0.6200 -0.0300 -
0.2100 0.0240
   0.1100 0.1030 0.0860 -0.0790 0.0 0.0350 0.3100 -0.0540 0.5900 0.1200 -
0.2700 0.0010
   0.156 0.45 0.40 -0.22 0.00 -0.096 0.00 -0.156 0.207 0.082 -0.25
0.052
   0.1600 0.3300 0.2700 -0.1580 0.0330 -0.0980 0.0100 -0.1140 0.4070 -0.0690 -
0.0417 0.0345
   0.0500 0.1700 0.3800 -0.3600 -0.1970 -0.1400 0.0760 0.0560 0.0870 0.3530 -
0.0730 0.1000];
   exret1=[0.1242 0.0852 0.0783 0.0511 0.0627 0.041];
   exret2=[0.1252 0.0862 0.0793 0.0521 0.0637 0.042];
   exret3=[0.1262 0.0872 0.0803 0.0531 0.0647 0.043];
end
```

```
if npar==8
   ret=[
    0.1613   -0.0159    0.2990   -0.0202   -0.0481    0.0833   -0.0303   -0.1281    0.8095  -
0.2588    0.1184    0.1692
    0.0787    0.0595    0.1667   -0.1724    0.0482    0.3387   -0.0159   -0.0597    0.2523
0.4658   -0.4786   -0.2391
    0.2405         0    0.2553    0.1325   -0.1170   -0.2656    0.3333   -0.0204    0.4627  -
0.0290   -0.3000   -0.0877
    0.2603    0.1587    0.1667   -0.1000       0.0   -0.0769    0.0947   -0.0686    0.4571  -
0.0139   -0.2577    0.1412
    0.2857    0.1667    0.2923   -0.3229   -0.0400    0.0204    0.1136   -0.0784    0.3247
0.1324   -0.2609         0
    0.1091    0.0377    0.3119   -0.1920    0.1111    0.0227    0.0233   -0.1224    0.6000
0.0533   -0.1758    0.1098
    0.1463    0.9680    0.0714   -0.4355    0.1481   -0.1290    0.6316    1.6389         0
0.2414   -0.5167   -0.1864
   -0.0149    0.0984    0.3708   -0.2054    0.3176   -0.2917    0.4286    0.0769    0.2188  -
0.0303   -0.0294    0.0462];

exret1=[0.0899 0.032   0.0454  0.0585  0.0478  0.0691 0.2098 0.0771];
exret2=[0.0949 0.037   0.0504  0.0635  0.0528  0.0741 0.2148 0.0821];
exret3=[0.0999 0.042   0.0554  0.0685  0.0578  0.0791 0.2198 0.0871];

end
if npar==12
ret=[0.1613   -0.0159    0.2990   -0.0202   -0.0481    0.0833   -0.0303   -0.1281    0.8095  -
0.2588    0.1184    0.1692
    0.0787    0.0595    0.1667   -0.1724    0.0482    0.3387   -0.0159   -0.0597    0.2523
0.4658   -0.4786   -0.2391
    0.2405         0    0.2553    0.1325   -0.1170   -0.2656    0.3333   -0.0204    0.4627  -
0.0290   -0.3000   -0.0877
    0.2603    0.1587    0.1667   -0.1000       0.0   -0.0769    0.0947   -0.0686    0.4571  -
0.0139   -0.2577    0.1412
```

```
   0.2857    0.1667    0.2923   -0.3229   -0.0400    0.0204    0.1136   -0.0784    0.3247
0.1324   -0.2609    0
   0.1091    0.0377    0.3119   -0.1920    0.1111    0.0227    0.0233   -0.1224    0.6000
0.0533   -0.1758    0.1098
   0.1463    0.9680    0.0714   -0.4355    0.1481   -0.1290    0.6316    1.6389         0
0.2414   -0.5167   -0.1864
  -0.0149    0.0984    0.3708   -0.2054    0.3176   -0.2917    0.4286    0.0769    0.2188   -
0.0303   -0.0294    0.0462
   0.0750    0.0811    0.3704   -0.2603   -0.0135    0.0571    0.0294   -0.1707    0.7083
0.0435   -0.0980   -0.0556
   0.4590   -0.0469    0.5422   -0.3712   -0.0959   -0.1310    0.0633    0.4630    0.4400
0.3636   -0.4149    0.1750
   0.5152    0.4143    0.5054   -0.2377   -0.0758   -0.0294    0.0099   -0.1140    0.4805
0.0132   -0.1648    0.0581
   0.1169    0.6383    0.9583   -0.2208   -0.0833   -0.2186    0.0238    0.0095    0.1798   -
0.0220   -0.3259   -0.0690];

  exret1=[0.0899  0.032   0.0454  0.0585  0.0478  0.0691 0.2098 0.0771 0.0589  0.1155
0.1096 0.0772];
  exret2=[0.0949  0.037   0.0504  0.0635  0.0528  0.0741 0.2148 0.0821 0.0639  0.1205
0.1146 0.0822];
  exret3=[0.0999  0.042   0.0554  0.0685  0.0578  0.0791 0.2198 0.0871 0.0689  0.1255
0.1196 0.0882];
  %expert perseption {1,6,7,9,12}
 % exret1=[0.0789  0.032   0.0454  0.0585  0.0478  0.0641 0.1948 0.0771 0.0549  0.1155
0.1096 0.0622];
 % exret2=[0.0869  0.037   0.0504  0.0635  0.0528  0.0691 0.2048 0.0821 0.0589  0.1205
0.1146 0.0722];
 % exret3=[0.0949  0.042   0.0554  0.0685  0.0578  0.0741 0.2148 0.0871 0.0639  0.1255
0.1196 0.0822];
end
n=npar;
m=12;
alfa1=0.095; alfa2=0.1; alfa3=0.105;
```

```
%alfa1=0.03; alfa2=0.035; alfa3=0.04;
[w1,w2,w3]=reorder(w1,w2,w3,n);
%---------------
sumw1=sum(w1);
sumw2=sum(w2);
sumw3=sum(w3);
x1=ent1*w1/sumw1;
x2=ent2*w2/sumw2;
x3=ent3*w3/sumw3;
[x1,x2,x3]=reorder(x1,x2,x3,n);
%sum(x1);sum(x2);sum(x3);
%pause
%--------------
ert1=0; ert2=0; ert3=0;
for i=1:n
   ert1=ert1+exret1(i)*x1(i);
   ert2=ert2+exret2(i)*x2(i);
   ert3=ert3+exret3(i)*x3(i);
end
%[ert1 ert2 ert3];
sumx1=sum(x1); sumx2=sum(x2);  sumx3=sum(x3);
%pause
if (ert2>=alfa2)   %  &(sumx2<=1)
   fun1=0; fun2=0; fun3=0;
   for t=1:m
      ym1(t)=0; ym2(t)=0; ym3(t)=0;
      for i=1:n
         ym1(t)=ym1(t)+(ret(i,t)-exret1(i))*x1(i);
         ym2(t)=ym2(t)+(ret(i,t)-exret2(i))*x2(i);
         ym3(t)=ym3(t)+(ret(i,t)-exret3(i))*x3(i);
      end
      if ym1(t)>0
         ym1(t)=0;
      end
```

```
    if ym2(t)>0
      ym2(t)=0;
    end
    if ym3(t)>0
      ym3(t)=0;
    end
    fun1=fun1+abs(ym1(t));
    fun2=fun2+abs(ym2(t));
    fun3=fun3+abs(ym3(t));
  end
 % ym
  fobject1=fun1/m; fobject2=fun2/m; fobject3=fun3/m;
else
  fobject1=-1000; fobject2=-1000; fobject3=-1000;
end
fobject1=1/(1+fobject1); fobject2=1/(1+fobject2); fobject3=1/(1+fobject3);
%pause
```

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Notes