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## Faculty of Engineering

## Department of Electrical and Electronic Engineering

## TRANSFORMERS

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#### Abstract

The sampling and reconstruction of transformers is one of the most important aspects of information theory, upon which many of technological advancements in transformers.

The main objective of this thesis is to device which more than any other has made possible the long distance, large- scale transmission of electric power and the modern power grid.

For this purpose which supplies thousands of square kilometers with power at a wide variety of different voltages is the transformer.

The underling principle of which the current and the voltage levels characterizing a given power flow in a circuit are readily traded off against each other.


## INTRODUCTION

The device which more than any other has made possible the long distance, large-scale transmission of electric power and the modern power grid which supplies thousands of square kilometers with power at a wide variety of different voltages is the transformer by means of which the current and voltage levels characterizing a given power flow in a circuit are readily traded off against each other. Thus, a given amount of energy can be shipped at high voltage and low current (low ohmic loss s along the line) and used at low voltage and high current (high consumer convenience).

It is just this flexibility provided by the transformer which gave alternating current its big initial edge over direct current, for only an alternating voltage will work in a transformer. It was the engineering realities of transformer action.

Union Electric here in St. Louis offers, for example, the following standard hookups.Local:120/240 141, 240 delta, 120/208 wye, 480 delta, 277/480 wye. Primary service: 4160 delta (mostly inside Lindbergh), 12470 deltas (mostly outside Lindbergh). Sub transmission: 34.5 kV delta. Transmission: 138 kV delta. Which were the principal determinant $s$ of the world's choice of power line frequencies $(50 \mathrm{~Hz}$ or 60 Hz$)$. And it is the existence and utility of the transformer principle which has made induction motors the end users of over half the electric power generated in the United States. In short, if you don't understand transformers you don't understand electric power.

Unfortunately, within the understanding of transformers, "There are more layers than in an onion's skin; and if you peel them all away in an effort to get at the heart of the matter you end up with lots of individual isolated bits of information and little grasp of the whole. We shall therefore proceed as simply as possible to discuss the physics of magnets (leaving details to a course on electromagnetic theory), the special properties of iron (leaving details to
a course on magnetic materials), the basic equivalent circuits for the transformer (leaving details to a course in network analysis), and the fundamentals of transformer losses (leaving details to a course in solid state physics) so that we can describe some few practical properties of and tests upon real power transformers. Naturally, we can not begin to offer even a comprehensive introduction to the arcana of real power transformers. But we can perhaps (indeed, it is our goal.) provide enough background to enable the student to meet simple challenges on his own and to delve successfully into the more advanced literature including that bible of the transformer world the J\&P Transformer Book (Stigant and Franklin, 1973).

## 1. Physics of Coupled Networks

Consider the physical realization of a single mesh circuit of arbitrary geometric form. And in particular let it be assumed that all conducting regions (wires, plates, etc.) Are uninsulated (i.e., not embedded in a dielectric). Then dip the mesh into a solution of suitable properties (e.g. soapy) so that, when it is withdrawn, a visible film forms defining a surface S bounded by the conducting mesh s ; let n be the right-handed normal to S • When this film covered mesh is placed in a time varying magnetic field $B\left[\mathrm{w} / \mathrm{m}^{2}\right]$, it is possible to define a quantity $\Phi[\mathrm{wb}]$ called the "flux" such that

$$
\begin{equation*}
\phi=\int_{S} \tilde{n} \cdot \vec{B} d s \tag{2,1}
\end{equation*}
$$

Where ds $\left[\mathrm{m}^{2}\right]$ is an element of area in S. Faraday's Iaw then states that the time rate of change of the flux induces a voltage $V_{\text {ind }}[v]$ in the mesh such that (c.f. King, 1963) , at sufficiently low frequencies,

$$
\begin{equation*}
v_{\text {ind }}=-\frac{\partial \phi}{\partial t} \tag{2,2}
\end{equation*}
$$

Where $v_{\text {ind }}$ measured by circling the mesh in a counter clock-wise direction.

However, since the electric scalar potential of a network is a singlevalued function of position, a counterclockwise circling of the mesh can yield no net voltage change and $v_{\text {ind }}$ must therefore be balanced by a compensating voltage drop due, for example, to resistance. That is vind will cause a current to flow in the mesh.

Consider, on the other hand, a closed contour $s$ of arbitrary geometric form winding about in a region of current density J . If the H -vector in the region (related to $S$ by way of magnetization $M[A / m]$ ) is given by

$$
\begin{equation*}
\vec{B}=\frac{1}{\mu_{0}} \text { 官 }-\vec{M} \text {, } \tag{2.3}
\end{equation*}
$$

Where $\mathrm{m}_{0}\left[=4 \Pi \times 10^{-7} \mathrm{H} / \mathrm{m}\right]$ is the permeability of free space, then the low frequency form of Ampere's Law becomes approximately (of. King, 1963)

$$
\begin{equation*}
\oint_{s} \text { 出 } \cdot d \vec{s}=\int_{s} \hat{n} \cdot \overrightarrow{\}} d, \tag{2.4}
\end{equation*}
$$

Where ds is a counterclockwise-directed infinitesimal along s. What this in effect says is that current flow can produce magnetic field.

To examine these concepts more deeply, consider a planar loop of some simple shape carrying a uniform counterclockwise current $i(t)[A]$ and assume that the loop is embedded in free the magnetic B-vector is sometimes called the flux density, and the magnetic H -vector is sometimes called the field strength.

Space, if one then applies to an element of this loop the consideration of Eq. $(2,4)$ in the
form of the right hand rule, it follows that within the plane of the loop

$$
\begin{equation*}
\vec{B}_{100 p}=\hat{z a}_{0} \dot{i}(t) s \tag{2.5}
\end{equation*}
$$

Where the positive quantity $\mathrm{B}\left[\mathrm{m}^{\wedge}-1\right]$ is a function of (i) loop geometry and (ii) position within the loop. Thus, by Eqs. (2.1) and (2.5),

$$
\begin{equation*}
\phi_{\text {loop }}=L_{100 p} i(t) \tag{2.6}
\end{equation*}
$$

Where L, [a] is a positive constant called the inductance which depends solely upon loop geometry. Thus, the flow of the current $i(t)$ will setup an induced voltage, .given by eg (2.2). So that Kirchhoff's current law yields

$$
\begin{equation*}
0=v_{g e n}(t)-L_{100 p} \frac{d i}{d t}-i_{R_{100 p}} \tag{2,7}
\end{equation*}
$$

Where $\mathrm{v}_{\mathrm{gen}}[\mathrm{v}]$ is the voltage of the generator driving i and Rloop [ohm] is the loop resistance. Clearly, the net effect is for the magnetic field generated by the current i to oppose changes in i .

Further insight can be gained by considering two arbitrary coils, each of which is carrying a current. The net induced

Voltage in coil-1 will be

$$
\begin{align*}
v_{\text {ind, } 1} & =-\frac{\partial \Phi_{1}}{\partial t}=-\left[\frac{\partial \Phi_{11}}{\partial t}+\frac{\partial \Phi_{12}}{\partial t}\right]  \tag{2.8a}\\
& =-\left[I_{11} \frac{\partial i_{1}}{\partial t}+I_{12} \frac{\partial i_{2}}{\partial t}\right]
\end{align*}
$$

And that in coil- 2 will be

$$
\begin{equation*}
v_{\text {ind }, 2}=-\left[L_{21} \frac{\partial \dot{j}_{1}}{\partial t}+L_{22} \frac{\partial \dot{L}_{2}}{\partial t}\right] \tag{2.8b}
\end{equation*}
$$

That is, spatially adjacent circuits can interact by virtue of the magnetic fields which their currents establish.

## 2. Magnetic Materials

In the derivation leading to Eqs. (2.8) a most important difficulty was evaded: the Ampere relation of Eq, (2.4) explained the fashion in which the magnetizing current density gave rise to the magnetic H -vector, whereas the Faraday relation of Eq. (2.2) explained the induced voltage in terms of the magnetic B-vector; and, as Eq. (2.3) reveals.

Magnetization can be explained as follows. What sets up a magnetic field is the circulation of current. Clearly, each atom has, by virtue of its structure, a circulating current associated with it. This current may be due either to orbital electron motion or to electron spin with electron spin dominating strongly in the ferrous materials used in electric power applications.

Each such miniscule current contributes little to the magnetic-H-vector however it is possible for groups of a djacent a toms to a lign to form extensive regions (called domains) where the contributions to M add and the domain becomes in effect a small magnet.

Consider now the case of an unmagnetized piece of material which is placed in
the region of influence of a current carrying wire. An H will be set up in the material. This field will in turn align to a degree the magnetic movements of that material's domains so that not only' H but also 6 . Will be monotone functions of the current. That is, from Eq. (2.5)

$$
\begin{equation*}
\vec{B}=\mu_{0}\left[\frac{2}{1}+\vec{M}\right] \tag{2.9}
\end{equation*}
$$

And $M=x H$ where $x$ [dimensionless] is the susceptibility. Now $M$ is due to alignment of intrinsic magnetic moments in the $m$ aterial; a nd, as there are only so $m$ any of these, there is a maximum magnetization such that

$$
\begin{equation*}
\lim _{|\overrightarrow{\mathrm{H}}|+\infty \infty}^{\vec{M}}=M \operatorname{sat} \frac{\vec{H}}{|\vec{H}|} \tag{2,10}
\end{equation*}
$$

Hence, as shown in Fig.2.1, at first, rises linearly in H and eventually rolls off into the asymptotic regime $\mathrm{dB} / \mathrm{dh}=\mathrm{M}_{0} ;$ a
quantity called the permeability of the medium is commonly defined as $\mu=[\mathrm{dB} / \mathrm{dH}] \mathrm{H}>,(2.11)$ and clearly varies with field strength. Suppose next that the H -vector is reduced. It is generally found, as illustrated in Fig. 2.1, that the 'normal" or "virgin" curve (i.e., initial path) is not retraced but that instead the domains resist reorientation and a so-called hysteresis loop 3 results
(cf. Olsen, 1966; Watson, 1980); it must be emphasized that the hysteresis loop is not a wnique entity but that as i Hi is -swept slowly from 0 to some $\mathrm{H}+$ and then back to some
H. and is then cycled between H. and $\mathrm{H}+$, a different loop is generated for each ordered pair (H.,H.+).

Moreover, the qualitative shape of the hysteresis loop can and does vary greatly from one

Material to another, as the curves of Fig. 2.2 illustrate. Some few descriptive terms may be Useful:
(a) A loop shaped like that of Fig. 2.1 is called "normal". If the loop is of small relative area
(say $\mathrm{BrHc} \ll(\mu 0 \mathrm{Msat}) \mathrm{H} \sigma$ ), the material is said to be "soft";
The term hysteresis is derived from the Greek vatepๆoıo (a coming late or a delay) and can be traced back to
the work of J. A. Ewing in the 1880's (cf. Heck, 1974).


If $\mathrm{H}_{\mathrm{c}}=\mathrm{H}_{\mathrm{c}}$ and $\mathrm{B}_{\mathrm{r}}=M_{0} M_{\text {sat }}$, the material is said. to have a square loop, (y) If

$$
\text { If }|\vec{B}| / \partial|\vec{H}| \equiv \text { constant for }|\vec{H}|<H_{\sigma} \text { and if } B_{r} \ll
$$

then the sides of the loop are straight and parallel and the loop is said to be "isoperm". It will be seen that the soft isoperm loop is highly desirable in power transformers.

B. Metglas

## METGLAS' magnetic roperties



## 3. A Networks Approach

Consider now, two inductors embedded near one another in some soft, isoperm, but not necessarily homogeneous material.


It then should follow that the field of one inductor influences current flow in the other so that an experience akin to Eq. (2.8) obtains and

$$
\begin{align*}
& v_{1}=L_{1} \frac{d i_{2}}{d t} \pm M_{12} \frac{d i_{2}}{d t}  \tag{2.12a}\\
& v_{2}= \pm M_{21} \frac{d i_{1}}{d t}+L_{2} \frac{d i_{2}}{d t} \tag{2,12b}
\end{align*}
$$

Where $\mathrm{M} .-=\mathrm{M}-, \mathrm{s}=\mathrm{M}>0$ by "reciprocity considerations'. The sign ambiguity can be resolved in one of two ways:
(i) Formal (The DOT convention)

Each inductor as placed in the circuit is provided with a single dot $(\cdot)$ located at one of its two terminals, The dots are placed so that, if one current enters at a dotted end and the other at an undotted end, the minus ( - ) sign is used; otherwise the plus $(+)$

In fact, M-- may not equal M-- if saturable media are al
lowed, and M is positive primarily because one has choice over the + and
Sign is used. To appreciate physically what this means, consider the situation of Fig. 2, 3 in which the two inductors are wound in the same toroidal core. The total flux seen by

Coil 1 will be (using Eqs. (2.1), (2.5))

$$
\begin{align*}
(2.6))_{1} & =N_{1} A_{1}+N_{2} A B_{2}=\left(N_{1} A\right)\left(N_{1} \mu_{0} i_{1} B_{1}\right)+\left(N_{2} A\right)\left(\mu_{0} N_{2} i_{2} B_{2}\right) \\
& =L_{11} i_{1}+L_{12} i_{2}=L_{1} i_{1}+M_{2},
\end{align*}
$$

Where the plus $(+)$ sign obtains since the fluxes produced by the two currents reinforce one another when sgn $\mathrm{i} 1=$ sgn i 2 and where. $\mathrm{L} 1=\mathrm{N} 1$ and $\mathrm{M} \infty$ N1M1Therefore, the dots must be placed at alfa and y or, equivalently, beta andgama). (ii) Informal (Trial and error)

Choose the plus sign. Then, if the circuit doesn't work as anticipated, it will just by $m$ aking the coupling inductance $M$ negative. Alternatively, one can - in lieu of rectifying one's expectations for the circuit - rectify the circuit by unsoldering, reversing, and resoldering the leads on one of the inductors. Before passing on, it should be noted that the time domain representation of Eqs. (2.12) becomes in the frequency domain just;


$$
\begin{align*}
& V_{1}=I_{1}\left[j \omega L_{1}\right]+I_{2}[ \pm j \omega M]  \tag{2.14a}\\
& V_{2}=I_{1}[ \pm j \omega M]+I_{2}\left[j \omega L_{2}\right] \tag{2.14b}
\end{align*}
$$

Consider next $\mathrm{E}(\mathrm{e}) \mathrm{in}$ ], than total energy defined to tvc coupled induction Clearly/ by Eqs, (2.12), it is

$$
\begin{align*}
E(t) & =\int_{\infty}^{t}\left[v_{1}\left(t^{\prime}\right) i_{1}\left(t^{\prime}\right)+v_{2}\left(t^{\prime}\right) \dot{s}_{2}\left(t^{\prime}\right)\right] d t^{\prime} \\
& =\frac{1}{2} L_{1} \dot{I}_{1}^{2}+\frac{1}{2} L_{2} i_{2}^{2} \pm M \int_{\infty}^{t}\left[i_{1} \frac{d i_{2}}{d t^{\prime}}+i_{2} \frac{d i}{d t^{\prime}}\right] d t^{\prime} \\
& =\frac{1}{2}\left[L_{1} i_{1}^{2}+L_{2} i_{2}^{2} \pm 2 M i_{1} i_{2}\right] \geq 0 \tag{2.15}
\end{align*}
$$

Suppose that at an instant one time $t, i=i ; i$ where is a dimensionless constant. At the instant considered

$$
\begin{equation*}
E(t)=\frac{1}{2} i_{1}^{2}\left[L_{1}+L_{2} r^{2} \pm 2 M r\right] \geq 0 \tag{2.151}
\end{equation*}
$$

$$
\pm \zeta_{e x}--\frac{M}{L_{2}}
$$

which corresponds to

$$
\begin{equation*}
E\left(t ; t_{e x}\right)=\frac{1}{2} i_{1}^{2}\left[L_{1}-\frac{M^{2}}{L_{2}}\right] \geq 0 \tag{2.17}
\end{equation*}
$$

And the inequality of Eq. (2.17) will be satisfied if and only if

$$
\begin{equation*}
\mathrm{I}_{1} \mathrm{~L}_{2} \geq \mathrm{m}^{2} \tag{2.18}
\end{equation*}
$$

or

$$
\begin{align*}
& M=k \sqrt{L_{1} L_{2}}  \tag{2,19a}\\
& 0 \leq k \leq 1 \tag{2.19b}
\end{align*}
$$

$$
\frac{v_{2}}{v_{1}}=\frac{L_{2} \frac{d i_{2}}{d t} \pm k \sqrt{L_{1} L_{2}} \frac{d i_{1}}{d t}}{L_{1} \frac{d i_{1}}{d t} \pm k \sqrt{L_{1} L_{2}} \frac{d i_{2}}{d t}}= \pm \sqrt{\frac{L_{2}}{L_{1}}} \frac{L_{2} \frac{d i_{2}}{d t} \pm k \sqrt{L_{1} L_{2}} \frac{d i_{2}}{d t}}{ \pm \sqrt{L_{1} L_{2}} \frac{d 1}{d t}+k L_{2} \frac{d L_{2}}{d t}}
$$

As $\mathrm{k}-\gg 1$, Eq. (2.20) simplifies to the PERFECT TRANSFORMER limit

$$
\begin{align*}
& \mathrm{L}_{1} * \mathrm{~N}_{1}^{2}  \tag{2.22a}\\
& \mathrm{~L}_{2} \propto \mathrm{~N}_{2}^{2} \tag{2.22b}
\end{align*}
$$

so that, with a turns ratio

But, by considerations akin to those which lead to Eq, (2.13),

$$
\begin{equation*}
n=\frac{N_{2}}{N_{1}}=\sqrt{\frac{L_{2}}{L_{1}}}, \tag{2.23}
\end{equation*}
$$

we have

$$
\begin{equation*}
\left.\frac{v_{2}}{v_{1}}\right]_{k=1}= \pm n \tag{2.24a}
\end{equation*}
$$

$$
\begin{equation*}
\left.\left\lvert\, \frac{v_{2}}{v_{1}}\right.\right]_{x=1} \mid=n . \tag{2.24D}
\end{equation*}
$$

In the $k=1$ limit, Eq. (2.12b) implies

$$
\begin{aligned}
& \qquad i_{2}(t)=\mp \frac{1}{\eta} i_{1}(t)+\frac{1}{L_{2}} \int_{-\infty}^{t} v_{2}(t \cdot) d t \\
& \text { If now } L_{2} \rightarrow \infty \text { with } n=\sqrt{L_{2} / L_{1}} \text { Eixed, the LDEAL TRANSFORMER is } \\
& \text { achieved, and Eq. }(2.25) \text { reauces to }
\end{aligned}
$$

$$
\left.\frac{i_{2}}{i_{1}}\right]_{k=1}^{L_{2}=\infty} \left\lvert\,=\mp \frac{1}{\eta}\right.
$$

IDEAL TRINSFORMER (2.26)

Both perfect and ideal transformers are lossless, as can be
seen from Eq. (2.15) by considering the limit $i,(t)=i 2(t)=0$

Further, all ideal transformers are perfect. But only the $L 2>$ infininity subset of perfect transformers will be ideal. These considerations are summed up in the equivalent circuits of Fig. 2.4. The current imag in this diagram is the so called magnetizing;

## Lossless Transformer



$$
\begin{aligned}
& v_{1}=L_{2} \frac{d i_{1}}{d t} \pm M \frac{d i_{2}}{d t} \\
& v_{2}= \pm M \frac{d i_{2}}{d t}+L_{2} \frac{d i_{2}}{d t}
\end{aligned}
$$

## Ideal Transformex



$$
\begin{aligned}
& v_{2}= \pm n v_{1} \\
& i_{2}=\mp \frac{1}{\eta} i_{1} \\
& \text { primary }
\end{aligned}
$$

secondary

## Perfect Transformer



## 4. Cove Loss and Jordan-type Lobs Coefficients

Of course, real transformers are not lossless, if for no other reasons than that (i) there will be ohmic losses in the wire of the inductor coils (copper lose) and (ii) there will be losses associated with the material upon which the coils of the primary and secondary are wound (core toes). Copper loss is seemingly straight-forward, although it may in fact contain some arcane points. Core loss is commonly split into three parts (Heck, 1974, p 35ff; Olsen, 1966, ch 48): (i) eddy-current;
(ii) hysteresis loss; and (iii) residual loss.

The eddy-current loss is due to the a lternating magnetic field which induces a voltage in the material of the core. This voltage then produces a current which in turn gives rise to an ohmic loss. For a core made up of thin high permeability sheets stacked closely together but not electrically in contact,the eddy current loss [w] can be shown to approximate

$$
\begin{equation*}
p_{e}=k_{e} f^{2} \hat{B}^{2} \delta^{2} \sigma V_{c} \tag{2.27}
\end{equation*}
$$

where K [dimensionless] is a constant which depends upon the shape and material of the core, $\mathrm{f}[\mathrm{Hz}]$ the applied frequency, $\mathrm{B} \mathrm{Wb} / \mathrm{m}$ the peak flux density, $\mathrm{f}[\mathrm{m}]$ the thickness of the sheet, $o[\mathrm{~s} / \mathrm{m}]$ the conductivity of the sheet, and V m the volume of the core.

The hysteresis loss is due to collision-like effects as magnetic domains are reoriented. It is most frequently given by the empirical allometric relationship

$$
\begin{equation*}
\mathrm{p}_{\mathrm{h}}=\mathrm{k}_{\mathrm{h}} \dot{\mathscr{B}} \hat{\mathrm{~B}}^{\alpha} \mathrm{v}_{\mathrm{c}} \tag{2.27}
\end{equation*}
$$

Where K is a constant which depends on the shape and

$$
1.6<\mathrm{m}<20 \text { material of the core and } 1,6<\mathrm{a}<2.0 \text { is a constant dependent }
$$

upon the material .
The residual loss, whose origin is obscure* is normally given by the empirical relationship

$$
\begin{equation*}
P_{r}=K_{Y} f \hat{B}^{2} \tag{2,28}
\end{equation*}
$$

where $\mathrm{K} \mid \mathrm{J} / \mathrm{T}$ is a constant independent of both flux density and core geometry (Olsen, 1966, pg 59),

Suppose now (of. Heck, 1974, p 35ff) that a set of measurements of total loss P, is taken as a function of frequency for fixed $B$ and the dc baseline found by extrapolating back to zero frequency. This baseline p . [w] is just the dc copper loss. The total core loss is then by definition taken to be Eq, (2.27) was presented by Steinmetz specifically to treat high induction levels in power transformers with steel cores. A lower H and with less saturated materials, the allometric relationship $\mathrm{P}, \mathrm{cc} \mathrm{H}$ obtains.

Observe that ac copper loss may, as a result of proximity and skin effects, be higher,

$$
\begin{equation*}
P_{\text {core }}=P_{e}+P_{h}+P_{Y}=P_{\text {loss }}-P_{d c} . \tag{2.29}
\end{equation*}
$$

The key idea of Jordan was that the three core terms behave differently in the (f, B) plane and can thereby be told apart.

Transformers are seldom B driven, it being more common to employ instead the sinuisoidal voltage drive provided by the power company's "infinite busbar". Clearly, however, the laws of magnetic induction require that

$$
\begin{equation*}
B=v \tag{2.30}
\end{equation*}
$$

so that Eqs. (2.27)-(2.28) can be used directly with an independent variable $\left(\mathrm{V}_{\mathrm{rms}}\right)$ readily measured in the laboratory.

## 5.The Equivalent Circuit of a Real Transformer at Low

## frequencies.

We come now to the problem of modeling a real transformer. We take as our basic variables the input line current I a (id) and input line voltage $V$ « (u). This current unequivocally, physically passes through the primary winding. Hence, the actual voltage drop due to the transformer acting as a transformer will be

$$
\begin{equation*}
V_{I}(\omega)=V_{g}(\omega)-R_{1} I_{\ell}(\omega) \tag{2,31}
\end{equation*}
$$

where $R$, is the resistance due to copper in the primary.
We can abstract the I R loss of the windings as resistors in series with the primary and secondary windings,

We will ignore core loss for the moment. Considering the two coupled coils of page 2-13, we can show that a variety of circuit representations are possible. So as to avoid the use of negative circuit elements? it is advantageous to use an ideal transformer (ref. page 2-18) as part of the network. One such network is shown in fig, 2-5a. Many others are possible. Figure $2-5 \mathrm{~b}$ shows fig, 2-5a with the effects of winding resistance
included while fig. 2-5c has a shunt resistance added to include the effects of core loss. It should be noted that since core loss depends on applied voltage in a non-linear way, the resistor, R, can only be an approximation to the actual situation. Note that the turn's ratio of our ideal transformer, which would also be the approximate primary/secondary voltage ratio of our actual transformer, depends on the primary/secondary .voltage ratio and also on the coefficient of coulpling. This is certainly reasonable since if $k$ were zero no secondary output could be had.

The inductor L1 ( $1-\mathrm{k}$ ) is non-zero because of imperfect coupling and so is called the 'leakage inductance', L, of the leans' former. In the shunt arm the inductor $\mathrm{k}^{2} \mathrm{~L} 1$ is the magnetizing inductance', L Generally, for power transformers L is small, giving a voltage drop at rated current that is only a few percent of rated voltage; while at rated voltage, L is large, and will -have--a current through-it of only a few percent of cated-curreat,

Since Lm a nd Rc , are p arallel branches it is common to refer to them as a susceptance, Bm , and a conductance, Gc. in practice little error is incurred by abstracting $B$ and $G$ - to the line terminals giving the equivalent network of fig. 2-5d.


Ideal
Eigure 2-5a


Figure $2-5 b$


Figure 2-5c


Figure 2-5d

## 6. Experimental Tests

## OPEN CIRCUIT

The equivalent network will be as shown in Fig. 2-6a.
Usually we can neglect R1 and Xe for this case and have
$\mathrm{POC}=R E\{\mathrm{VLI} * \mathrm{LOC}\}=\mathrm{GC}(\mathrm{VL})^{\wedge} 2(2.32)$
$\mathrm{ILOC}=(\mathrm{Gc}+\mathrm{jBm}) \mathrm{VL}(2.33)$
So, measurement of power input, line current, and line voltage to the transformer with secondary open-circuited will yield the network elements Gc and Bm . Additionally, the transformation ratio, $\mathrm{a}=\mathrm{k} \mathrm{L1/L2}$ can be approximately obtained by measuring the voltage appearing across the secondary.

## SHORT CIRCUIT

Figure 2-6b gives the equivalent network in this case. Sufficient accuracy can often be achieved by ignoring the shunt branch, $\mathrm{Gc}+\mathrm{jBm}$ in which case we have $\mathrm{PSC}=(\mathrm{ILSC})^{\wedge} 2 \mathrm{R} 1+(\mathrm{I} 2)^{\wedge} 2 \mathrm{R} 2,(2.34)$ or, using the transformation ratio, a , we have $\mathrm{PSC}=(\mathrm{ILSC})^{\wedge} 2[\mathrm{R} 1+\mathrm{a} 2 \mathrm{R} 2] \cdot(2.35)$

Additionally, $\mathrm{VL}=(\mathrm{R} 1+\mathrm{a} 2 \mathrm{R} 22+\mathrm{jXe}) \mathrm{ILSC},(2.36)$
so the quantities $[R 1+a 2 R 2]$ and Xe can be inferred from measurements with the transformer secondary shorted. R1 and R2 can be separated by a DC ohmmeter measurement of R1.


Figure 2-6a

nigure 2-6bn

## 7. Three Phase Transformers

For the benefit of those students whose polyphase theory is somewhat rusty, a brief review is in order. This will be based primarily on the peerless treatise by A. T. Dover (1947).

Consider a system of three conductors 1,2,3 arranged clockwise in space and having with respect to some imagined reference point the potentials $T$ (jw) defines

$$
\begin{align*}
& v_{A}=T_{2}-T_{1}=v  \tag{2.44a}\\
& v_{B}=T_{3}-T_{2}=v e^{-j \frac{2 \pi}{3} \delta} \\
& v_{C}=T_{1}-T_{3}=v e^{-j \frac{4 \pi}{3} \delta} \tag{2.44c}
\end{align*}
$$

where $\mathrm{V}[\mathrm{v}]$ is an rms voltage and 6 is a dimensionless constant being +1 for the positive phase (clockwise) sequence and -1 for the negative (counterclockwise) phase sequence. These three wires can be connected in either of two fashions as shown in Fig. 2.7. Fig. 2.7A shows the delta (or ring) configuration. Fig. 2.7B the wye (or star) configuration; if M (the neutral or star point) is taken to be at zero potential, then

$$
\begin{align*}
& T_{1}=\nabla_{1 N}=v_{1}=\frac{v}{\sqrt{3}} e^{j 8 \frac{5 \pi}{6}}  \tag{2.45a}\\
& r_{2}=v_{2 N}=v_{2}=\frac{V}{\sqrt{3}} e^{j 0 \frac{\pi}{6}}  \tag{2,45b}\\
& r_{3}=v_{3 N}=v_{3}=\frac{v}{\sqrt{3}} e^{-j 0 \frac{\pi}{2}}, \tag{2.45c}
\end{align*}
$$

As can readily be shown (You should, as an exercise, do so.) if the sure of the $\mathrm{T}^{*} \mathrm{a}$ is constrained to be zero. This brings us to the first great three phase result:

rms line-neutrat voltage $=1 / \sqrt{3}$ (rme tine-tine voltage)

Since a three phase system can be thought of as just an ordinary network with three generators phase locked with respect to each other, ordinary mesh and nodal analysis will suffice to solve three phase systems. However, if it is assumed that the- $\qquad$ system is loaded so that either (delta) $\mathrm{Z} 1=\mathrm{Z} 2=\mathrm{Z} 3=\mathrm{Zw}$ or (wye) $\mathrm{Za}=\mathrm{Zb}=\mathrm{Zc}=2 \mathrm{v}$ the system is said to be balanced, and the analysis becomes simpler. For example, in a balanced delta, the line currents I1, I2 and I3, are related to the load (phase) currents 1a, Ib , and Ic by the equations (see Fig. 2.7A)

$$
\begin{gather*}
I_{1}=I_{C}-I_{A}=\frac{V}{Z_{A}}-j \frac{4 \pi}{3} \hat{\delta}-\frac{V}{Z_{\Delta}}=\frac{V}{Z_{A}} \sqrt{3} e^{j \frac{5 \pi}{6} \delta}  \tag{2.46a}\\
I_{2}=I_{A}-I_{B}=\frac{V}{Z_{A}}-\frac{V}{Z_{\Delta}} e^{-j \frac{2 \pi}{3} \delta}=\frac{V}{Z_{\Delta}} \sqrt{3} e^{+\frac{j}{6}} \delta  \tag{2.46b}\\
I_{3}=I_{B}-I_{C}=\frac{V}{Z_{\Delta}} e^{-j \frac{2 \pi}{3} \delta}-\frac{V}{Z_{A}} e^{-j \frac{4 \pi}{3} \delta}=\frac{V}{Z_{\Delta}} \sqrt{3} e^{-j \frac{\pi}{2} \delta} \tag{2.46c}
\end{gather*}
$$

And this yields the second great three phase result:

For a balanced delta loads the rme line current is / J
times the rme phase current. The power for a balanced load can be computed in terms of the line-line voltage V and the rms line current I . For a balanced star,

$$
\begin{align*}
& s=p+j \quad Q=V_{1 N^{I}}{ }^{*}+V_{2 N} I_{2}^{*}+V_{3 N^{I}}{ }^{*} \\
& =\frac{V_{1 N} V_{1 N}^{*}}{z_{W}^{*}}+\frac{V_{2 N} V_{2 N}^{*}}{z_{W}^{*}}+\frac{V_{3 N} V_{3 N}^{*}}{z_{N}^{*}} \\
& =\sqrt{3} \text { VI } e^{j \arg } \mathrm{~N} \text {; } \tag{2.47a}
\end{align*}
$$

whereas for a balanced delta,

$$
\begin{align*}
S=P+j Q & =V_{A} I_{A}^{*}+V_{B} I_{B}^{*}+V_{C_{C}} I_{C}^{*} \\
& =\frac{V_{A} V_{A}^{*}}{2_{\Delta}^{*}}+\frac{V_{B} V_{B}^{*}}{z_{\Delta}^{*}}+\frac{V_{C} V_{C}^{*}}{z_{A}^{*}} \\
& =\sqrt{3} V_{A} e^{j} \arg Z_{A} \tag{2.17b}
\end{align*}
$$

These formulas are essentially the same. In terms of the power factor

$$
\begin{equation*}
\cos \theta=\arctan \frac{x}{R} \tag{2.48}
\end{equation*}
$$

the power is just

$$
\begin{equation*}
P=\sqrt{3} \text { VI } \operatorname{RE}\left\{e^{j 0}\right\}=\sqrt{3} V I \cos \theta \tag{2.49}
\end{equation*}
$$

Since each phase of a balanced three phase load draws the same power, the estimation of power is readily achieved by emplacing an ordinary wattmeter. If unbalance is the rule, this solution clearly will not work. However, while it may be conceptually leasing to use three wattmeters, two frequently suffice, as can be seen by considering (for example) an unbalanced three wire delta as in Fig. 2.7A.

$$
\begin{equation*}
P=R E\left\{V_{A} I_{A}^{*}+V_{B} I_{B}^{*}+V_{C} I_{C}^{*}\right\} \tag{2.50}
\end{equation*}
$$

By Kirchhoff's current law

$$
\begin{align*}
& I_{2}=I_{A}[1]+I_{B}[-1]+I_{C}[0]  \tag{2.51a}\\
& I_{3}=I_{A}[0]+I_{B}[1]+I_{C}[-1]  \tag{2.51b}\\
& 0=I_{A}[1]+I_{B}[1]+I_{C}[1]
\end{align*}
$$

and thus

$$
\begin{align*}
& I_{A}=\frac{1}{3}\left[2 I_{2}+I_{3}\right]  \tag{2,52a}\\
& I_{B}=\frac{1}{3}\left[-I_{2}+I_{3}\right]  \tag{2.52b}\\
& I_{C}=\frac{1}{3}\left[-I_{2}-2 I_{3}\right] \tag{2.52c}
\end{align*}
$$

Moreover, by Kirchhoff's voltage law.

$$
\begin{equation*}
V_{B}=-\left(v_{A}+v_{C}\right) \tag{2.53}
\end{equation*}
$$

Hence

$$
\begin{equation*}
P=R E\left\{V_{A} I_{2}^{*}-V_{C^{\prime}} I^{*}\right\} \tag{2.54}
\end{equation*}
$$

Note that line 1 is the common reference in this instance. Observe also that it will be simple to mix up one's wires and hitch things backwards. Finally, as you may see in the lab, it is possible to put the two meter movements in one case and put on but a single pointer scale.

We now almost as an afterthought come to the question of the three phase transformer. One could, of course, use three single phase units and connect them into a suitable star or ring, but this is seldom done. Instead a laminated iron yoke of the form shown in Pig. 2.7C is used and one primary and one secondary wound on each yoke. The only caution is that the secondaries be correctly connected together. By way of illustrating this, consider a wye secondary with

$$
\begin{align*}
& v_{1 N}=v  \tag{2.55a}\\
& v_{2 N}=v e^{-j \delta \frac{2 \pi}{3}}  \tag{2,55b}\\
& v_{3 N}=-v e^{-j \delta \frac{4 \pi}{3}} \tag{2.55c}
\end{align*}
$$

The 3 N phase is in backwords, and the result is

$$
\begin{align*}
& \left|v_{A}\right|=\left|v_{2}-v_{1}\right|=\sqrt{3} v  \tag{2.56a}\\
& \left|v_{B}\right|=\left|v_{3}-v_{2}\right|=v  \tag{2,56b}\\
& \left|v_{C}\right|=\left|v_{1}-v_{3}\right|=v \tag{2.56c}
\end{align*}
$$

## 8. Autotransformers

In the transformers connected to date, the primaries and secondaries were electrically isolated from each other. They need not be. In fact, one can define a circuit of the form shown in Fig. 2.8A Its lossless idealization as a coupled circuit is shown in Fig. 2.8B and the ideal transformer limit in Fig. 2.8C. Fig. 2.8C is readily solved to yield

$$
\begin{equation*}
v_{0}=\frac{n}{1+n} v_{g} \tag{2.57}
\end{equation*}
$$

A.

B.

c.


Now suppose that the primary has $\mathrm{N}(1-6)$ turns and the secondary 8 turns. This implies

$$
V_{0}=B V_{g},
$$

In practice such an effect is often achieved by connecting the high point of the secondary to a wiper arm and running it along the primary to give a variable autotransformer; such devices are commonly called Variacs after the model produced by Genrad (General Radio).

## 9. Practical Considerations

Even if you have now mastered faultlessly the above material, you are still not prepared to deal with specifying and installing a transformer because there is a plethora of practical information still missing in your background, and this short introduction can do no more than indicate where the more major deficiencies lie. Books such as that of Stigant and Franklin (1973) must be consulted for details about the niceties of grounding practice, cooling, regulation, performance under surge, insulation life, packaging, etc.

You should, however, be aware that transformers are generally specified by
(i) Primary voltage and secondary voltage. (ii)

Complex power put in (kVA).
In practice o ne would seek a family of transformers which had the required terminal voltages and then go up the family until a beefy enough one was reached. The reason that kVA rather than kW is specified is that line voltage is normally set by the power company's infinite busbar whereas load is set by the user. Hence, core loss (proportional roughly to the square of the terminal voltage) is fixed while the copper loss (proportional roughly to the square of the line voltage) can vary greatly. Thus, a highly reactive load could pull many kVAR (and kVA ) without drawing all that many kW ; and, therefore, a kW specification could lead to excessive copper loss and
transformer burn-out.


## 10.Standard (ANSI C67.12.90-1973) for Electrical Teats

There are three ways of testing a transformer: the right way/ the wrong way, and the official way. The official way is described by the a bove standard which is a lso known as IEEE Std 262-1973.

This standard describes (i) resistance measurements, (ii) dielectric tests, (iii) efficiency, loss, and impedance tests, (iv) ratio and regulation tests, (v) temperature rise tests, (vi) insulation power factor tests, and (vii) polarity and phase relation tests. The aim seems to be practical utility in industrial settings rather than deep understanding in the university laboratory. Nevertheless, you should be aware of this standard and refer to it once you enter the real world.

## CONCLUSION

Finally, I want to say one thing; if you do not understand transformers you do not understand electric power. There are more layers than in an onions' skin; and if you peel them all away in an effort to get at the heart of the matter you end up with lots of individual isolated bits of information and little grasp of whole.

Network approach was analyzed two inductors embedded near one another in some soft, isoperm, but not necessarily homogeneous material. It then should follow that the field of one inductor influences current flow in the other other so that an experience a kin to obtains.

In future, proposed approached can be developed to reconstruct of the transformers from its samples by adaptive sampling.

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## THE MAIN CALCULATION

$\mathrm{N}_{2}=50 \mathrm{VA}$

## $B=12,000$ Gauss.

$\mathrm{Id}=2,5 \mathrm{~A} / \mathrm{mm}^{2}$
$\mathfrak{J}=\% 95$
$\mathrm{F}=50 \mathrm{~Hz}$
$g \cdot d=\% 5$
$C=1.2$
$\mathrm{Sfe}=\mathrm{C} \cdot \sqrt{N}$

$$
=1.2 \cdot \sqrt{50}=8.48 \mathrm{~cm}^{2}
$$

$a=2.8 \mathrm{~cm}$
$\mathrm{b}=\frac{8.48 \mathrm{~cm}^{2}}{2.8 \mathrm{~cm}}=3.03 \mathrm{~cm}$
Lamination $=\frac{b}{l a \min \text { ationthick }}=\frac{3.03 \mathrm{~cm}}{0.5 \mathrm{~nm}}=60.6 \cong 61$
$\varnothing=$ B.sfe $=12000 \cdot 8,48 \mathrm{~cm}^{2}=101,760$ gauss $/ \mathrm{cm}^{2}$
$\mathrm{n}_{1}=\frac{N .10^{8}}{4.44 x \oslash x F}=\frac{220.10^{8}}{4.44 \times \varnothing x F}=\frac{220.10^{8}}{4.44 \times 101760 \times 50}=973$ sipir
$\mathrm{n}_{2}=\frac{U_{2} \cdot 10^{8}}{4 \cdot 44 \cdot \varnothing \cdot F}=\frac{12 \cdot 10^{8}}{4.44 \times 101760.50}=53$ sipir
$\mathrm{S}_{1}=\frac{N_{2}}{U_{1} \cdot I d \cdot \Im}=\frac{50}{240.2,5 \cdot 0,95}=0,08772$
$d_{1}=\sqrt{\frac{S_{1}}{0,785}}=0,33 \mathrm{~mm}$
$\mathrm{S}_{2}=\frac{N_{2}}{U_{2} I d}=\frac{50}{12,05}=1,66$
$D_{2}=\sqrt{\frac{1,66}{0,785}}=1,45 \mathrm{~mm}$

