



NEAR EAST UNIVERSITY

Faculty of Engineering

**Department of Electrical and Electronic
Engineering**

**POWER FLOW PROGRAM IN MATLAB USING
NEWTON-RAPHSON METHOD**

**Graduation Project
EE- 400**

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ABSTRACT

Successful power system operation under normal balanced three-phase steady-state conditions requires that: the generation supplies the demands (load) plus losses, bus voltage magnitudes remain close to rated values, generators operate within specified real and reactive limits and transmission lines and transformers are not overloaded. To calculate the approximate values for these required conditions we use power flow analysis or load flow analysis. Power flow analysis computes the voltage magnitude and angle at each bus in a power system under three-phase steady-state conditions and real and reactive power flows for lines or transformers interconnecting the buses, as well as their losses.

The input data for power flow calculation normally given for loads and generators is in complex power form, the power flow problem is therefore formulated as a set of non-linear equations. For the solution of these types of equation, iterative methods are used, such as Gauss-Seidel and Newton-Raphson method. The equations of the power flow problem are complex and take iterations to converge. This cannot be simple to perform computation for a system which consists of large number of buses. This can be simplified by solving power flow equations with computer programs.

The aim of this project is to get familiar with the power flow analysis by the aid of computer based program such as MATLAB. The computer based algorithm of Newton-Raphson method to solve power flow problems is developed in MATLAB. For this purpose the basic concepts in electric power system which are necessary to understand power flow are briefly described, the formulation and mathematical calculations involved in power flow analysis are explained in detail with solved examples by hand and by computer.

INTRODUCTION

In power system, power flow analysis is used to obtain information on the current state and conditions of the system in terms of voltage magnitudes and angles as well as real and reactive power. From power flow analysis we can get information about what is happening in the system, how the power is flowing in the lines to the loads. For the solution of power flow problem we have to solve the non-linear equations for the system which may not have the exact solution so we apply numerical analysis

Usually Gauss-Seidel and Newton-Raphson iterative methods are used to solve these types of equations. The calculation involved in the power flow analysis is not very simple. For large systems it's even more complex and time consuming to solve it by hand, as done in the early days. Nowadays many computer simulation programs have been developed to perform power flow analysis for even larger systems in a short time with more accuracy.

In this project computer based program using Newton-Raphson method developed in MATLAB is considered for the solution of power flow equations. The project consists of introduction, 5 chapters and conclusion.

Chapter One describes the background and history related to power flow. The development and applications of power flow.

Chapter Two presents the history of power industry and short description of power system.

Chapter Three describes the basic quantities and relationships used in electric power system. Explains the theory of alternating current and voltage. The description of complex quantities such as reactance and impedance.

Chapter Four presents the definition of power in a.c. circuits. The description of real and reactive power. The description of losses, stability and results of instability in power system.

Chapter Five explains the theory of power flow analysis. The representation of power system by one-line diagram and per unit system. The description of buses in power system and their types. The description of variables used in power flow. The formulation of power flow equations and solutions. The calculation procedure of the

Newton-Raphson method. The solution of an example solved with Newton-Raphson method. The structure of the MATLAB program used to solve power flow equations.

Finally, the conclusion section presents the important results obtained within the project.

1. BACKGROUND

1.1. Introduction to Power Flow Analysis

In power engineering, the power flow study (also known as load-flow study) is an important tool involving numerical analysis applied to a power system. Unlike traditional circuit analysis, a power flow study usually uses simplified notation such as a one-line diagram and per-unit system, and focuses on various forms of AC power (i.e. reactive, real, and apparent) rather than voltage and current. It analyses the power systems in normal steady-state operation. There exist a number of software implementations of power flow studies.

In addition to a power flow study itself, sometimes called the base case, many software implementations perform other types of analysis, such as *short-circuit fault analysis* and *economic analysis*. In particular, some programs use linear programming to find the *optimal power flow (OPF)*, the conditions which give the lowest cost per kilowatt generated.

When solving large power systems, power flow studies are increasingly used for purposes, such as outage security assessment, and for more complicated calculations such as optimization and stability. The great importance of power flow or load-flow studies is in the planning the future expansion of power systems as well as in determining the best operation of existing systems.

The information obtained from power flow study are:

- the magnitude and phase angle of the voltage at each bus with reference to swing bus voltage
- real and reactive power flowing in each line
- current in rectangular or polar form

1.2. History of Power Flow

Improved economy and reliability were recognized well over half a century ago as benefit of using an interconnected network for the transport of electric power. But critical to its realization was (and still is) is the ability to predict voltages and flows on

network components. As the network evolved, the challenge was to develop a tool that would produce this critical information. The *load-flow (or power flow)*, as the tool came to be known, predicts all flows and voltages in the network when given the status of generators and load. It is the tool most heavily used by power engineers.

Early load-flows were solved using what were called calculator boards. These boards were a kind of analog computers, in that they emulated a specific system by using a physical lumped-parameter resistor-inductor-capacitor realization of the actual system, the components being connected using the same topology. For a realistic system, these boards filled several rooms, consumed substantial power, and had to be rewired when any modification was desired. As studies often required teams of engineers working in unison adjusting knobs and setting and reading out results aloud, the need for a flexible alternative was clear.

Enter the modern digital computer, which, in fact, owes as much of the impetus behind its original development to power engineers and their need for a better way to solve load-flows. In the early days of computing, electric power business was by far the largest commercial user (and even developer) of digital machines. It was not unusual for utility to spend several million dollars (not adjusted for inflation) on the development of digital hardware and software. While IBM corp. was advancing mainframe machine architectures, theorists were publishing the first papers on load-flow algorithms.

The earliest algorithms were based on the *Gauss-Seidel method*, which made it possible, for the first time, to solve the load-flow problem for relatively large systems. It suffered, however, from relatively poor convergence characteristics. Then the Newton algorithm was developed to improve the convergence of the Gauss-Seidel method, but was initially thought to be impractical for realistically sized systems because of computational problems with large networks. The underlying problem for the iterative Newton method is the solution of a matrix equation of large dimension.

In the 1960s Bill Tinney and his colleagues at the Bonneville Power Administration observed that, although the main system matrix was very large, it was also very sparse (meaning it had a very small proportion of nonzero values). Those observations gave rise to the development of sparse methods. The concept made it possible to apply the *Newton method* to systems of arbitrary size, to attain for the first time both speed and excellent convergence characteristics.

Since the '60s, numerous advances and extensions have been made in load-flow methods. In the early '70s came the *fast-decoupled* load-flow, which enhanced computational speed. Extensions to the load-flow itself included the representation of components such as high-voltage direct-currents (HVDC) transmission lines, better methods for loss calculations, solution of the optimum power flow and state estimation problems, the continuation power flow, and the determination of spot prices of electricity in the presence of constraints—plus, of course, the development of better ways of visualizing and presenting load-flow results.

1.3. Developments in Power Flow

As the world advanced to the computer generation, remarkable developments were also made by the implementation of computer programs in electric power industry. These computer programs also contained power flow programs which made handling of large network systems very easy.. To determine how power flows thorough a transmission network from generator to loads, it is necessary to calculate the real and reactive power flow in each and every transmission line or transformer, along with associated bus voltage. With networks containing tens of thousands of buses and branches, such calculation yields lot of numbers. Traditionally they were presented either in reams of tabular output showing the power flows at each bus or else as data in a static so-called one-line diagram. The visualization challenge is to make these concepts intuitive. One simple yet effective technique to depict the flow of power in electricity network is to use animated line flow. Dynamically sized pie charts are another visualization idea that has proven useful for quickly detecting overloads in a large network. On the one-line, the percentage fill in each pie chart indicates how close each transmission line is to its thermal limits. Visualization software packs a large amount of information into a single computer-generated image, enabling viewers to interpret the data more rapidly and more accurately than ever before. This visualization also provides a picture of the complex interaction between the grid and the power market, allowing market participants to respond more quickly to changing conditions.

The power flow computer program computes the voltage magnitude and angle at each bus in a power system under balanced three-phase steady-state conditions. It also computes real and reactive power flows for all equipment interconnecting the buses, as

well as equipment losses. Both existing power system and proposed changes including new generation and transmission to meet projected load growth are of interest.

Conventional nodal or loop analysis is not suitable for power-flow studies because the input data for loads are normally given in terms of power, not impedance. Also, generators are considered as power sources, not voltage or current sources. The power flow problem is therefore formulated as a set of nonlinear algebraic equations suitable for computer solution.

1.4. Applications and Optimal Power Flow

Power flow analysis is a fundamental and essential tool for operating a power system, as it answers the basic question, what happens to the state of the system if we do such-and-such? This question may be posed in the context of either day-to-day operations or longer-term planning.

In the short run, a key part of a system operator's responsibility is to approve generation schedules that have been prepared on the basis of some economic considerations, whether by central corporate planning or by competitive bidding, and scrutinize them for technical feasibility. This assessment hinges on power flow studies to predict the system's operating state under a proposed dispatch scenario, if the analysis shows that important constraints such as line loading limits would be violated, the schedule is deemed infeasible and must be changed.

Even with feasible schedules in hand, reality does not always conform to plans, requiring operators to monitor any changes and, if necessary, make adjustments to the system in real time. Power flow analysis is the only comprehensive way to predict the consequences of changes such as increasing or decreasing generation levels, increasing or decreasing loads, or switching transmission links and assessing whether they are safe or desirable for the system. Specifically, operators need to know impacts of any actions on voltage levels (are they within proper range?), line flows (are any thermal or stability limits violated?), line losses (are they excessive?) and security (is the operating state too vulnerable to individual equipment failures?). Similarly, power flow analysis is a fundamental tool in the planning context to evaluate changes to generation capacity or the transmission and distribution infrastructure.

Sometimes it is necessary to compare several hypothetical operating scenarios for the power system to guide operating and planning decisions. Specifically, one often

wishes to compare and evaluate different hypothetical dispatches of generation units that could meet a given loading condition. Such an evaluation is performed by an *optimal power flow (OPF)* program, whose objective is to identify the operating configuration or “solution” that best meets a particular set of evaluation criteria. These criteria may include the cost of generation, transmission line losses, and various requirements concerning the system’s security, or resilience with respect to disturbances.

An OPF algorithm consists of numerous power flow analysis runs, one for each hypothetical dispatch scenario that could meet the specified load demand without violating any constraints. This makes OPF more computation-intensive than basic power flow analysis. The output of each individual power flow run, which is a power flow solution in terms of bus voltage magnitudes and angles, is evaluated according to one or more criteria that can be wrapped into a single quantitative metric or objective function, for example, the sum of all line losses in megawatts, or the sum of all generating costs in dollars when line losses are included. The OPF program then devises another scenario with different real and reactive power contributions from the various generators and performs the power flow routine on it, then another, and so on until the scenarios do not get any better and one is identified as optimal with respect to the chosen metric. This winning configuration with real and reactive power dispatches constitutes the output of the OPF run. OPF solutions may then provide guidance for on-line operations as well as generation and transmission planning.

Especially for applications in a market environment, where planning and operating decisions may have sensitive economic or political implications for various parties, it is crucial to recognize the inherently subjective nature of OPF. Power flow analysis by itself basically answers a question of physics. By contrast, OPF answers a question about human preferences, coded in terms of quantitative measures. Thus, what is found to constitute an “optimal” operating configuration for the system depends on how the objective function is defined, which may include the assignment of prices, values, or trade-offs among different individual criteria. In short, “optimality” does not arise from a power system’s intrinsic technical properties, but derives from external considerations.

It is also important to understand that the translation of an OPF solution into actual planning and operating decisions is not clear-cut and has always involved some

level of human judgment. For example, the computer program may be too simplistic in its treatment of security constraints to allow for sensible trade-offs under dynamically changing conditions, which then calls for some engineering judgment in adapting the OPF recommendation in practice. At the same time, the computational process is already complex enough that different OPF program packages may not offer identical solutions to the same problem. Therefore, the output of power flow analysis including OPF constitutes advisory information rather than deterministic prescriptions. Indeed, the complexity of the power flow problem underscores the difficulty of managing power systems through static formulas and procedures that might some day lend themselves to automation, especially if a system is expected to perform near its physical limits.

2. ELECTRIC POWER INDUSTRY

2.1. History of Electric Power Industry

The electric utility industry can trace its beginnings to the early 1880s. During that period several companies were formed and installed water-power driven generation for the operation of arc lights for street lighting; the first real application for electricity in the United States. In 1882 Thomas Edison placed into operation the historic Pearl Street steam-electric plant and the pioneer direct current distribution system, by which electricity was supplied to the business offices of downtown New York. By the end of 1882, Edison's company was serving 500 customers that were using more than 10,000 electric lamps.

Satisfied with the financial and technical results of the New York City operation, licenses were issued by Edison to local businessmen in various communities to organize and operate electric lighting companies. By 1884 twenty companies were scattered in communities in Massachusetts, Pennsylvania, and Ohio; in 1885, 31; in 1886 48; and in 1887 62. These companies furnished energy for lighting incandescent lamps, and all operated under Edison patents.

Two other achievements occurred in 1882: a water-wheel-driven generator was installed in Appleton, Wisconsin; the first transmission line was built in Germany to operate at 2400 volts direct current over a distance of 37 miles (59 km). Motors were introduced and the use of incandescent lamps continued to increase. By 1886, the dc systems were experiencing limitations because they could deliver energy only a short distance from their stations since their voltage could not be increased or decreased as necessary. In 1885 a commercially practical transformer was developed that allowed the development of an ac system. A 4000 volt ac transmission line was installed between Oregon City and Portland, 13 miles away. A 112-mile, 12,000 volt three-phase line went into operation in 1891 in Germany. The first three-phase line in the United States (2300 volts and 7.5 miles) was installed in 1893 in California. In 1897, a 44,000-volt transmission line was built in Utah. In 1903, a 60,000-volt transmission line was energized in Mexico.

In this early ac period, frequency had not been standardized. In 1891 the desirability of a standard frequency was recognized and 60 Hz (cycles per second) was proposed. For many years 25, 50, and 60 Hz were standard frequencies in the United States. Much of the 25 Hz was railway electrification and has been retired over the years. The City of Los Angeles Department of Water and Power and the Southern California Edison Company both operated at 50 Hz, but converted to 60 Hz at the time that Hoover Dam power became available, with conversion completed in 1949. The Salt River Project was originally a 25 Hz system, but most of it was converted to 60 Hz by the end of 1954 and the balance by the end of 1973.

Over the first 90 years of its existence, until about 1970, the utility industry doubled about every ten years, a growth of about 7% per year. In the mid 1970s, due to increasing costs and serious national attention to energy conservation, the growth in the use of electricity dropped to almost zero. Today growth is forecasted at about 2% per year.

The growth in the utility industry has been related to technological improvements that have permitted larger generating units and larger transmission facilities to be built. In 1900 the largest turbine was rated at 1.5 MW. By 1930 the maximum size unit was 208 MW. This remained the largest size during the depression and war years. By 1958 a unit as large as 335 MW was installed, and two years later in 1960, a unit of 450 MW was installed. In 1963 the maximum size unit was 650 MW and in 1965, the first 1,000 MW unit was under construction.

Improved manufacturing techniques, better engineering, and improved materials allowed for an increase in transmission voltages in the United States to accompany the increases in generator size. The highest voltage operating in 1900 was 60 kV. In 1923 the first 220 kV facilities were installed. The industry started the construction of facilities at 345 kV in 1954, in 1964 500 kV was introduced, and 765 kV was put in operation in 1969. Larger generator stations required higher transmission voltages; higher transmission voltages made possible larger generators.

These technological improvements increased transmission and generation capacity at decreasing unit costs, accelerating the high degree of use of electricity in the United States. At the same time, the concentration of more capacity in single generating units, plants, and transmission lines had considerably increased the total investment required for such large projects, even though the cost per unit of electricity had come

down. Not all of the pioneering units at the next level of size and efficiency were successful. Sometimes modifications had to be made after they were placed in operation; units had to be derated because the technology was not adequate to provide reliable service at the level intended. Each of these steps involved a risk of considerable magnitude to the utility first to install a facility of a new type or a larger size or a higher transmission voltage. Creating the new technology required the investment of considerable capital that in some cases ended up being a penalty to the utility involved. To diversify these risks companies began to jointly own power plants and transmission lines so that each company would have a smaller share, and thus a smaller risk, in any one project. The sizes of generators and transmission voltages evolved together.

The need for improved technology continues. New materials are being sought in order that new facilities are more reliable and less costly. New technologies are required in order to minimize land use, water use, and impact on the environment. The manufacturers of electrical equipment continue to expend considerable sums to improve the quality and cost of their products.

2.2. Electric Power System

The electric power industry delivers electric energy to its customers which they, in turn, use for a variety of purposes. While power and energy are related, customers usually pay for the energy they receive and not for the power.

In electric power industry electric power system is the system which consists of components that transform other types of energy into electrical energy and transmit this energy to a consumer. The production and transmission of electricity is relatively efficient and inexpensive, although unlike other forms of energy, electricity is not easily stored and thus must generally be used as it is being produced.

A modern electric power system consists of six main components:

- the power station
- a set of transformers to raise the generated power to the high voltages used on the transmission lines
- the transmission lines
- the substations at which the power is stepped down to the voltage on the distribution lines
- the distribution lines

- the transformers that lower the distribution voltage to the level used by the consumer's equipment.

Taken together, all of the parts that are electrically connected or intertied operate in an electric balance. The technical term used to describe the balance is that the generators operate in synchronism with one another.

3. BASIC ELECTRIC POWER CONCEPTS

Before continue to discuss power flow analysis we should understand the basic concepts and terminologies related to electric power system. Especially the concepts related to alternating current and voltage because these are the most basic concepts on which power flow analysis depends. Basic quantities and relationships in electric power system are also need to be considered.

3.1. Basic Quantities and Relationship

Table 3.1 Basic electric relationships

Quantity	Name or Unit	Symbol	Relationships
Electric charge	Coulomb	q	
Time	Seconds, Hours	t	
Current	Amperes	I	$I = q / t = V / R$
Resistance	Ohms	R	$R = V / I$
Inductive Reactance	Ohms	X_L	$X_L = 2 * \pi * f * L$
Capacitive Reactance	Ohms	X_C	$X_C = 1 / (2 * \pi * f * C)$
Impedance	Ohms	Z	$Z = R + j (X_L + X_C)$
Voltage	Electromagnetic force (EMF), Volts, kilovolts	E, V, kV	$V = I * R$ $V = J / Q$
Power or Real Power	Watts, kilowatts, megawatts,	P	$P = V * I$ $P = I^2 * R$ $P = V^2 / R$
Reactive Power	VArS, kiloVArS, megaVArS	Q	$Q = I^2 * X_L$ $Q = I^2 * X_C$
Apparent Power		S	$S = P + j Q$
Energy	kilowatt-hours, megawatt hours, Joules	J	$J = V * I * t$ $J = I^2 * R * t$
Frequency	Hertz, cycles per second	f	

3.2. Alternating Current and Voltage

The first utility systems installed by Edison used direct current technology. The electrical energy in a direct current system is the same as found when a battery is used. If one looked at a picture of the voltage and the current, one would see that both had a constant, non-varying value. Not long after Edison installed his direct current system, others realized that the use of an alternating current system had advantages over the direct current. The concepts discussed heretofore apply to direct current systems.

Many of the important technical characteristics of power systems have to do with their use of alternating current (a.c.) instead of direct current (d.c.). In a d.c. circuit, the polarity always remains the same: the potential always stays positive on one side and negative on the other, and the current always flows in the same direction.

In an a.c. circuit, this polarity reverses and oscillates very rapidly. For power systems in the United States, the a.c. frequency is 60 hertz (Hz) or 60 cycles per second, meaning that the direction of voltage and current are reversed, and reversed back again, 60 times every second.

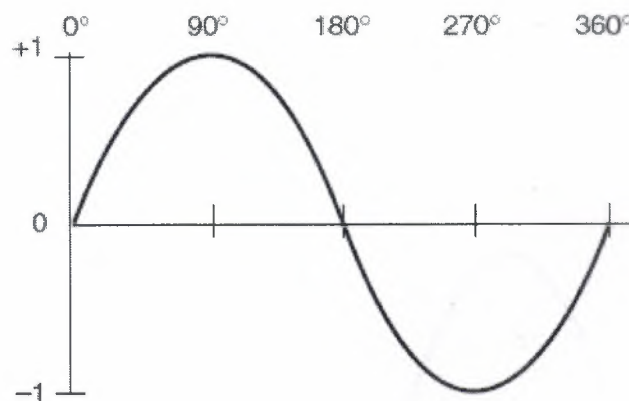


Figure 3.1 Sinusoidal shape of voltage or current

3.2.1. Mathematical Description

A sine wave represents the cyclical increase and decrease of a quantity over time. The oscillation of voltage and current in an a.c. system is modeled by a sinusoidal curve, meaning that it is mathematically described by the trigonometric functions of

sine or cosine. In these functions, time appears not in the accustomed units of seconds or minutes, but in terms of an angle.

A sinusoidal function is specified by three parameters: *amplitude*, *frequency*, and *phase*. The amplitude gives the maximum value or height of the curve, as measured from the neutral position. (The total distance from crest to trough is thus twice the amplitude.) The frequency gives the number of complete oscillations per unit time. Alternatively, one can specify the rate of oscillation in terms of the inverse of frequency, the period. The period is simply the duration of one complete cycle. The phase indicates the starting point of the sinusoid. In other words, the phase angle specifies an angle by which the curve is ahead or behind of where it would be, had it started at time zero. Graphically, we see the phase simply as a shift of the entire curve to the left or right. The phase angle is usually denoted by ϕ , the Greek lowercase phi.

The frequency of a sinusoidal function is often given in terms of radians per second, in which case it is called an *angular frequency*. Angular frequencies are usually denoted by ω , the Greek lowercase omega. The angular frequency corresponding to 60 cycles / s is

$$\omega = 60 \text{ cycles / s} * 2 \pi \text{ radians / cycle} = 277 \text{ rad / s}$$

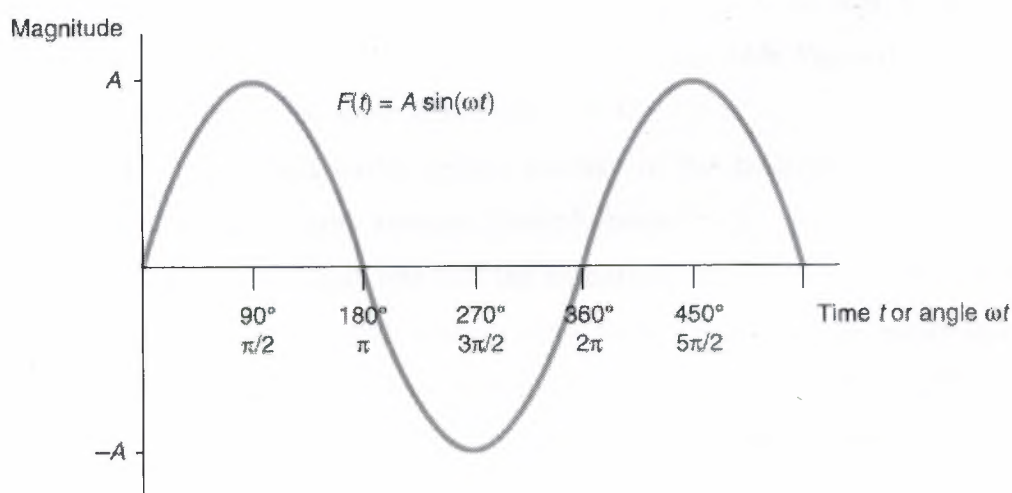


Figure 3.2 A sine function plotted against angle $f(t) = A \sin(\omega t)$

An alternating current as a function of time can be written as the following sinusoidal function:

$$I(t) = I_{\max} \sin(\omega t + \phi_I)$$

The same discussion as for current holds for the voltage, which is written

$$V(t) = V_{\max} \sin(\omega t + \phi_V)$$

The quantity I_{\max} and V_{\max} are the maximum value or amplitude of the current and voltage respectively.

The subscripts on the phase angles are there to indicate that current and voltage do not necessarily have the same phase, that is, their maximum values do not necessarily coincide in time.

3.2.2. The rms Value

For most applications, we are only interested in the overall magnitude of these functions. Specifically, we would like average values of current and voltage that yield the correct amount of power when multiplied. Such an average is readily computed, it is called the *root mean square (rms)* value.

The rms value is derived by first squaring the entire function, then taking the average (mean), and finally taking the square root of this mean. Squaring the curve eliminates the negative values, since the square of a negative number becomes positive. Figure 3.3 illustrates this process with the curves labeled $V(t)$ and $V^2(t)$. If we arbitrarily label the vertical axis in units such that the amplitude $V_{\max} = 1$, it is obvious that the squared wave has the same amplitude ($1^2 = 1$).

Because the squared curve resides entirely in the positive region, it is now possible to take a meaningful average. Indeed, because the curve is still perfectly symmetric, its average is simply one half the amplitude. The only counterintuitive step now consists of renormalizing this average value to the original curve before squaring, which is accomplished by taking the square root, basically, we are just going backwards and undoing the step that made the curve manageable for averaging purposes.

Since $1/2$ is less than 1, its square root is greater than itself, it comes to

$$\sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} = 0.707$$

Thus, the rms value of a sine curve is 0.707 of the original amplitude.

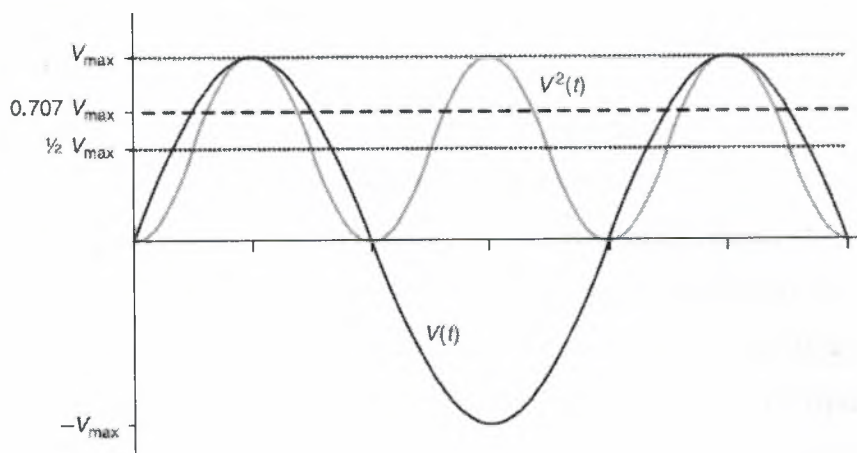


Figure 3.3 Derivation of the rms value

Utility voltages and currents are almost always given as rms values. For example, 120 V is the rms voltage for a residential outlet. Note that when the rms voltage and current are multiplied together, the product gives the correct amount of power transmitted.

The maximum instantaneous value of the voltage is also of interest because it determines the requirements for electrical insulation on the wires and other energized parts. In fact, one argument against a.c. in the early days was that it would be less economical due to its insulation requirements, which are greater by a factor of $\sqrt{2}$ than those for d.c. equipment transmitting the same amount of power. For current, the instantaneous maximum is relatively uninteresting because current limitations are related to resistive heating, which happens cumulatively over time.

3.3. Resistance

In an ac system, the voltage across a resistor and the current flowing through it are said to be in phase, that is, their zero value and their maximum values occur at the same times. There are two types of fields associated with an ac electric system; electric fields and magnetic fields. Electric fields relate to the voltage and magnetic fields relate to the current. The waveforms of the voltage and current associated with both of these

characteristics are not in phase, that is, the times of the maximum and zero values are not identical.

3.4. Reactance

Reactance is the property of a device to influence the relative timing of an alternating voltage and current. By doing so, it presents a sort of impediment of its own to the flow of alternating current, depending on the frequency. Reactance is related to the internal geometry of a device and is physically unrelated to the resistance. There are two types of reactance: inductive reactance, which is based on inductance, and capacitive reactance, based on capacitance. Finally, impedance is a descriptor that takes into account both resistance and reactance. Resistance, reactance, and impedance are all measured in ohms (Ω).

Both inductive reactance and capacitive reactance have an impact on the relationship between voltage and current in electric circuits. Although they are both measured in Ohms, they cannot be added to the resistance of the circuit since their impacts are quite different from that of resistance. In fact, their impacts differ one from the other. The current through an inductor leads the voltage by 90 degrees, while current through a capacitor lags the voltage by 90 degrees. Because of this difference, their effects will cancel one another. The convention is to consider the effect associated with the inductive reactance a positive value and that with the capacitive reactance a negative value and VARs as consumed by inductive reactance and supplied by capacitive reactance. A general term, *reactance*, is defined which represents the net effect of the capacitive reactance and inductive reactance. It is denoted by the capital letter X.

3.4.1. Inductive Reactance

An electric voltage is induced in a wire when a moving magnetic field “cuts” that wire. Similarly, a current varying with time (an alternating current) will produce a magnetic field around the wire carrying the current. Since the current is varying so will the magnetic field. This varying magnetic field “cuts” the conductor and a voltage is induced in the wire which acts to impede the originating current.

The relationship between the current and the induced voltage is defined by a quantity called the inductance. One Henry is the amount of inductance required to

induce one volt when the current is changing at the rate of one ampere per second. The letter L is used to represent the inductance in Henries.

The inductance, L , of one phase of a transmission or distribution line is calculated by considering the self-inductance of the individual phase conductor and the mutual inductance between that phase and all other nearby phases both of the same circuit/feeder and other nearby circuits/feeders. These quantities are calculated based on the physical dimensions of the wires and the distances between them. The induced voltage across an inductor will be a maximum when the rate of change of current is greatest. Because of the sinusoidal shape of the current, this occurs when the actual current is zero. Thus the induced voltage reaches its maximum value a quarter-cycle before the current does, the voltage across an inductor is said to lead the current by 90 degrees, or conversely, the current lags the voltage by 90 degrees.

The inductive reactance, X_L is a term defined to enable us to calculate the magnitude of the voltage drop across an inductor. The inductive reactance is measured in Ohms and it is equal to $2\pi f \times L$, where $2\pi f$ is the rotational speed in radians per second; π is called pi and its value is 3.1416, f = frequency in hertz and L = inductance in Henries. Inductances consume reactive power or VARs equal to $I^2 X_L$.

3.4.2. Capacitive Reactance

An electric field around the conductor results from a potential difference between the conductor and ground. There is also a potential difference between each conductor in a three phase circuit and with any other nearby transmission lines. The relationship between the charge and the potential difference is defined by a quantity called the capacitance. One Farad is the amount of capacitance present when one coulomb produces a potential difference of one volt. The letter C is used to represent the capacitance in Farads.

The capacitance C , depends on the dimensions of the conductor and the spacing between the adjacent lines and ground. Since the charge on a capacitor varies directly with the voltage, when an alternating voltage is impressed across a capacitor, the flow of charge (or current) will be greatest when the rate of change of voltage is at a maximum. This occurs when the voltage wave crosses the zero point. Thus in an alternating current system, the current across a capacitor reaches its maximum value a

quarter-cycle before the voltage does, the voltage is said to lag the current by 90 degrees, or conversely, the current leads the voltage by 90 degrees.

The capacitive reactance, X_C , is a term defined equal to $\frac{1}{2} \times \pi \times f \times C$, where C = capacitance in Farads. The unit of the capacitive reactance is Ohms. In a power system the capacitive reactance is viewed as a shunt connecting the conductor to ground. Capacitors supply reactive power or VARs equal to $I^2 X_C$. Figure 3.2 demonstrates the current and voltage relationships for a resistor, an inductor and a capacitor.

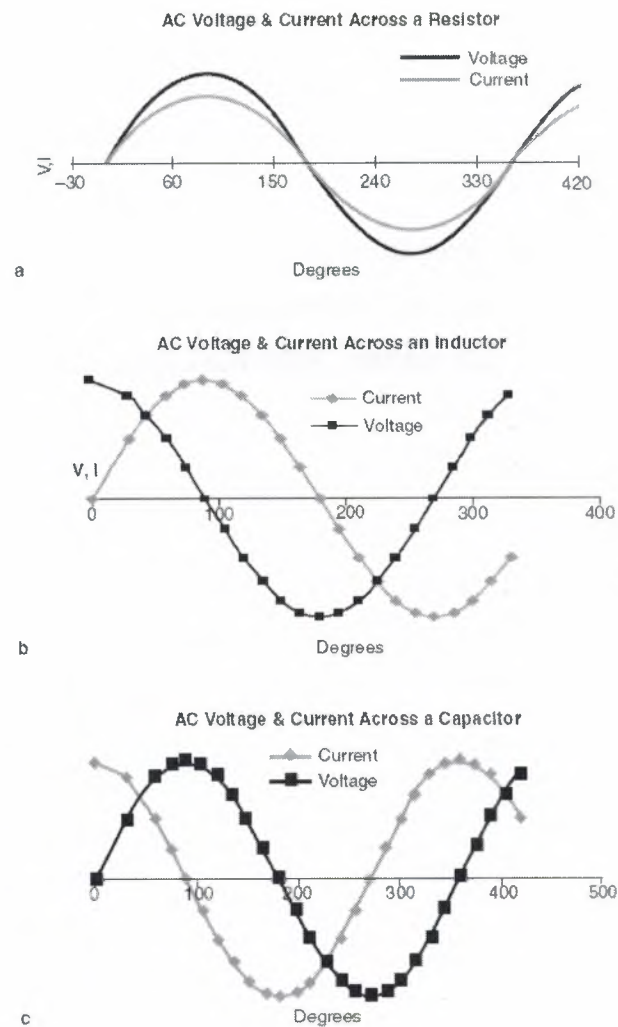


Figure 3.4 Current and voltage relationships for (a) a resistor, (b) an inductor and (c) a capacitor.

3.5. Impedance

The combination of reactance and resistance that describes the overall behavior of a device in a circuit is called the impedance, denoted by Z . However, Z is not a straightforward arithmetic sum of R and X . Mathematically speaking, Z is the vector sum of R and X in the complex plane. A boldface Z may be used to indicate a vector or complex number with a real and an imaginary component. As shown in Figure 3.5, the impedance Z is a complex number whose real part is the resistance and whose imaginary part is the reactance:

$$Z = R + jX$$

Any device found in an electric power system has impedance. For different devices and different circumstances, the resistive or reactive component may be negligible, but it is always correct to use Z .

Impedances can be combined according to the same rules for series and parallel combination for pure resistances. Qualitatively, we can note that inductive and capacitive reactance tends to cancel each other, whether they are combined in series or parallel.

When written in the polar format, the angle ϕ of the impedance has an important physical significance: it corresponds to the phase shift between current and voltage produced by this device. By convention, when the reactance is inductive and the current is lagging, ϕ is positive. When the reactance is capacitive and the current is leading, ϕ is negative. Thus, what appears as an angle in space in the triangle of Figure 3.5 can also be interpreted as an angle in time.

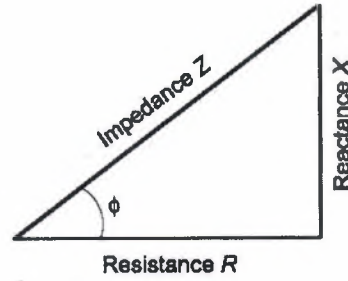


Figure 3.5 The complex impedance Z , with resistance R in the real direction and reactance X in the imaginary direction.

3.6. Admittance

The inverse of the complex impedance is called admittance, denoted by Y . The complex Y is decomposed into its real and imaginary parts, the conductance G and the susceptance B :

$$Y = G + jB$$

We have already encountered the conductance as the inverse of resistance, for the case of a pure resistor without inductance. In the complex case, however, it is not true that G is the inverse of R and B the inverse of X ; rather, we require that

$$Y = \frac{1}{Z}$$

Because Y and Z are complex (vector) quantities, this entails two things: first, that the magnitudes of Y and Z are reciprocal of each other,

$$Y = \frac{1}{Z}$$

and second, that the orientation (angle) in the complex plane remain the same. By performing some algebra, we can derive the magnitudes of G and B , respectively:

$$\begin{aligned}
 Y &= \frac{1}{Z} = \frac{1}{R + jX} = \frac{(R - jX)}{(R + jX)(R - jX)} = \frac{(R - jX)}{R^2 + jRX - jRX + X^2} \\
 &= \frac{(R - jX)}{R^2 + X^2} = \frac{(R - jX)}{Z^2} = \frac{R}{Z^2} - j \frac{X}{Z^2} = G + jB
 \end{aligned}$$

Where

$$G = \frac{R}{Z^2} \text{ And } B = -\frac{X}{Z^2}$$

This means that while a greater impedance is associated with a smaller admittance and vice versa, the relationship between G and B (considering only their magnitudes, not the negative sign) is directly proportional to the relationship between R and X . Thus, a device whose reactance outweighs its resistance also has a susceptance that outweighs its conductance.

4. POWER IN AC CIRCUITS

4.1. Definition of Electric Power

Power is a measure of energy per unit time. Power therefore gives the rate of energy consumption or production. The units for power are generally watts (W). For example, the watt rating of an appliance gives the rate at which it uses energy. The total amount of energy consumed by this appliance is the wattage multiplied by the amount of time during which it was used; this energy can be expressed in units of watt-hours (or, more commonly, kilowatt-hours).

The power dissipated by a circuit element whether an appliance or simply a wire is given by the product of its resistance and the square of the current through it:

$$P = I^2 R$$

The term “dissipated” indicates that the electric energy is being converted to heat. This heat may be part of the appliance’s intended function (as in any electric heating device), or it may be considered a loss (as in the resistive heating of transmission lines); the physical process is the same.

Another, more general way of calculating power is as the product of current and voltage: $P = IV$. For a resistive element, we can apply Ohm’s law ($V = IR$) to see that the formulas $P = I^2 R$ and $P = IV$ amount to the same thing:

$$P = IV = I (IR) = I^2 R$$

4.2. Complex Power

In a dc circuit, the power is equal to the voltage times the current, or $P = V \times I$. This is also true in an ac circuit when the current and voltage are in phase; that is, when the circuit is resistive. But, if the ac circuit contains reactance, there is a power component associated with the magnetic and/or electric fields. The power associated with these fields is not consumed as it is in a resistance, but rather stored and then discharged as the alternating electric current/voltage goes through its cycle. This leads to another definition:

*Apparent power = Real or True power (associated with a resistance)
+ Reactive power (associated with an inductance or capacitance).*

Using symbols:

$$S = P + jQ$$

4.2.1. Real Power

Real power is available to do work and is equal to the value of the resistance multiplied by the square of the current through the resistance. It is measured by a quantity called megawatts (mW) or kilowatts (kW).

$$P = I^2 R$$

4.2.2. Reactive Power

Reactive power neither consumes nor supplies energy. The reactive power associated with an inductive reactance is the value of the inductive reactance multiplied by the square of the current through it. The reactive power is measured by a quantity called volt-ampere reactive or VARs. As the length of a line increases, its inductive reactance increases, and the more capacitive reactive power needed to offset the effect and to maintain adequate voltage:

$$Q_L = I^2 \times X_L$$

The capacitive reactive power, Q_C , relates to the establishment of the electric field around a line. There are a number of ways to calculate this value, but the following offer insight into its effects on the transmission system.

$$Q_C = 3 \times \frac{V_{LN}^2}{X_C} \text{ or } = \sqrt{3} \times V_{LL} \times I_C$$

In a power system, under normal operations, the voltage level on any one line is kept more or less constant, so the reactive power associated with the capacitance of the line is also relatively constant. Charging current, I_C , is defined as the line to neutral voltage divided by the capacitive reactance

$$I_C = \frac{V_{LN}}{X_C}$$

If the charging current becomes too large, much of the line's current carrying capacity may be "eaten up" by charging current. This situation sets limits on the length of an overhead line or of a cable that can be operated without installing some intermediate measures to offset the capacitive current. It is useful to visualize the impact of various devices on the reactive power of a power system as follows:

Sources of reactive power which raise voltage:

- Generators
- Capacitors
- Lightly loaded transmission lines due to the capacitive charging effect

Sinks of reactive power which lower voltage:

- Inductors
- Transformers
- Most heavily loaded transmission lines due to the $I_L \times X_L$ effect
- Most customer load (due to the presence of induction motors and the supply to other electric fields)

A synchronous generator can be made to be either a source of reactive power or a sink by using the generator excitation system to vary the level of its dc field voltage. During peak load conditions generators are usually operated to supply reactive power to the grid. During light load conditions generators may be used to absorb excess reactive power from the grid, especially where there are long transmission lines or cables nearby.

A power system will not function properly and will not remain in operation unless sufficient reactive power is available equal to the reactive loads plus the large reactive losses on the system.

4.3. Losses

Distribution systems have two types of energy losses: losses in the conductors and feeders due to the magnitude of the current and transformer core losses that are independent of current. Current related losses are equal to the current squared time the resistance of the feeder or transformer (I^2R). Accompanying these losses are reactive losses which are given by (I^2X). The core losses result from the energy used in transformer cores as a result of hysteresis and eddy currents. These losses depend on the magnetic material used in the core. As voltages vary from the design level, core losses can vary by as much as V^3 to V^5 . Core losses in a power system can exceed 3% of the power generated constituting as much as 40% of the total loss on the system. The capacity of generation and reactive sources must be sufficient to supply these losses.

4.4. Stability

Stability refers to the ability of the generators in a power system to operate in synchronism both under normal conditions and following disturbances. Three categories of instability are:

- Steady-state instability
- Transient instability
- Dynamic instability

Steady-state instability refers to the condition where the equilibrium of the generators connected to the power system cannot accommodate increases in power requirements that occur relatively slowly or when a transmission line is removed from service for maintenance.

Since the power flow from one point to another is proportional to the sine of the angular difference between the voltages at the two points and inversely proportional to the total impedance of the circuits connecting the two points, there is a maximum level

of power flow, that is, the delivery level at which the angular difference is 90 degrees, and the sine is equal to one.

The fact that the power flow is dependent on the sine of the angular difference between the voltages has an important significance in that it defines the maximum amount of power that can be moved across the facilities connected by the impedance X_{12} . If the power required by the customers at bus 2 is greater than the amount that can be delivered at a 90 degrees separation in the voltages, the system is unworkable. A technical term to describe a situation where the customer load at bus 2 slowly increases and the angular spread responds until it reaches the 90 degree point and then goes beyond 90 degrees is that the system becomes unstable and will collapse.

If the net impedance is increased by removing a line, less power can be transmitted. If there are a number of lines connecting bus 1 with bus 2, the loss or outage of any one of them will increase the impedance between the two buses and the system can again become unstable. Conversely, if the net impedance is reduced, more power can be transmitted. The value of the net impedance can be reduced by:

- Building an additional line(s) in parallel
- Raising the design voltage of one or more of the existing lines
- Decreasing the impedance of any of the existing lines by inserting a capacitor in series (remember X_C cancels out X_L)

Transient instability refers to the condition where there is a disturbance on the system that causes a disruption in the synchronism or balance of the system. The disturbance can be a number of types of varying degrees of severity:

- The opening of a transmission line increasing the X_L of the system.
- The occurrence of a fault decreasing voltage on the system. (The voltage at the fault goes to zero, decreasing all system voltages in the area.)
- The loss of a generator disturbing the energy balance and requiring an increase in the angular separation as other generators adjust to make up the lost energy
- The loss of a large block of load in an exporting area

When there is a disturbance on the system, the energy balance of generators is disturbed. Under normal conditions, the mechanical energy input to the generator equals the net electrical energy output plus losses in the conversion process within the turbine generator and the power consumed in the power plant.

If a generator sees the electric demand at its terminal in excess of its mechanical energy input it will tend to slow down as rotational energy is removed from its rotor to supply the new increased demand. If a generator sees an electric demand at its terminal less than its mechanical energy input, it will tend to speed up due to the sudden energy imbalance. This initial reaction is called the inertial response.

Disturbances may also change the voltage at the generator's terminals. In response, the generator's automatic voltage regulating system will sense the change and adjust the generator's field excitation, either up or down, to compensate.

Transient stability or instability considers that period immediately after a disturbance, usually before the generator's governor and other control systems have a chance to operate. In all cases, the disturbance causes the generator angles to change automatically as they adjust to find a new stable operating point with respect to one another. In an unstable case, the angular separation between one generator or group of generators and another group keeps increasing. This type of instability happens so quickly, in a few seconds, that operator corrective action is impossible.

If stable conditions exist, the generator's speed governor system, sensing the beginning of change in speed, will then react to either admit more mechanical energy into the rotor to regain its speed or to reduce the energy input to reduce the speed. Directives may also be received by the generator from the company or area control center to adjust its scheduled output.

In addition to the measures noted to improve steady-state stability, other design measures available for selected disturbances to mitigate this type instability are:

- Improving the speed by which relays detect the fault and the speed by which circuit breakers operate to disconnect the faulted equipment sooner
- The use of dynamic braking resistors which, in the event of a fault, are automatically connected to the system near generators to reduce export from the generators

- The installation of fast-valving systems on turbines, allowing rapid reduction in the mechanical energy input to the turbine generator
- Automatic generator tripping
- Automatic load disconnection
- Special transmission line tripping schemes

Dynamic instability refers to a condition where the control systems of generators interreact in such a way as to produce oscillations between generators or groups of generators which increase in magnitude and result in instability, that is, there is insufficient damping of the oscillations. These conditions can occur either in normal operation or after a disturbance.

4.4.1. Results of instability

In cases of instability, as the generator angles separate, the voltage and current angular relationships at points on the system change drastically. Some of the protective line relays will detect these changes and react as if they were due to fault conditions causing the opening of many transmission lines. The resulting transmission system is usually segmented into two or more electrically isolated islands, some of which will have excess generation and some will be generation deficient. In excess generation pockets, the frequency will rise. In generation deficient pockets, the frequency will fall. If the frequency falls too far, generator auxiliary systems (motors, fans) will fail, causing generators to be automatically disconnected by their protective devices. Industry practice is to provide for situations where there is insufficient generation by installing under frequency-load-shedding relays. These relays, keyed to various levels of low frequency, will actuate the disconnection of blocks of customer load in an effort to restore the load-generation balance. In situations where the frequency rises because of excess generation, generators will be automatically removed from service by protective devices detecting an overspeed condition. If studies indicate potential excess generation pockets, special, selective generation disconnection controls can be installed.

5. POWER FLOW PROGRAM

5.1. Power Flow Analysis

Power flow analysis is concerned with describing the operating state of an entire power system, by which we mean a network of generators, transmission lines, and loads that could represent an area as small as a municipality or as large as several states. Given certain known quantities, typically the amount of power generated and consumed at different locations, power flow analysis allows one to determine other quantities. The most important of these quantities are the voltages at locations throughout the transmission system, which, for alternating current (a.c.), consist of both a magnitude and a time element or phase angle. Once the voltages are known, the currents flowing through every transmission link can be easily calculated. Thus the name *power flow* or *load flow*, as it is often called in the industry, given the amount of power delivered and where it comes from, power flow analysis tells us how it flows to its destination.

Owing mainly to the peculiarities of a.c., but also to the sheer size and complexity of a real power system, its elaborate topology with many nodes and links, and the large number of generators and loads, it turns out to be no mean feat to deduce what is happening in one part of the system from what is happening elsewhere, despite the fact that these happenings are intimately related through well-understood, deterministic laws of physics. Although we can readily calculate voltages and currents through the branches of small direct current (d.c.) circuits in terms of each other, even a small network of a handful of a.c. power sources and loads defies our ability to write down formulas for the relationships among all the variables, as a mathematician would say, the system cannot be solved analytically; there is no closed-form solution. We can only get at a numerical answer through a process of successive approximation or iteration. In order to find out what the voltage or current at any given point will be, we must in effect simulate the entire system.

Historically, such simulations were accomplished through an actual miniature d.c. model of the power system in use. Generators were represented by small power supplies, loads by resistors, and transmission lines by appropriately sized wires. The voltages and currents could be found empirically by direct measurement. To find out how much the current on line A would increase, for example, due to Generator X taking over power production from Generator Y, one would simply adjust the values on X and

Y and go read the ammeter on line A. The d.c. model does not exactly match the behavior of the a.c. system, but it gives an approximation that is close enough for most practical purposes. In the age of computers, we no longer need to physically build such models, but can create them mathematically. With plenty of computational power, we can not only represent a d.c. system, but the a.c. system itself in a way that accounts for the subtleties of a.c. Such a simulation constitutes *power flow analysis*.

Power flow answers the question, What is the present operating state of the system, given certain known quantities? To do this, it uses a mathematical algorithm of successive approximation by iteration, or the repeated application of calculation steps. These steps represent a process of trial and error that starts with assuming one array of numbers for the entire system, comparing the relationships among the numbers to the laws of physics, and then repeatedly adjusting the numbers until the entire array is consistent with both physical law and the conditions stipulated by the user. In practice, this looks like a computer program to which the operator gives certain input information about the power system, and which then provides output that completes the picture of what is happening in the system, that is, how the power is flowing.

There are variations on what types of information are chosen as input and output, and there are also different computational techniques used by different programs to produce the output. Beyond the straightforward power flow program that simply calculates the variables pertaining to a single, existing system condition, there are more involved programs that analyze a multitude of hypothetical situations or system conditions and rank them according to some desired criteria; such programs are known as *optimal power flow* (OPF).

5.2. Representation of a Power System

5.2.1. One-Line Diagram

In power engineering, one-line diagram is a simplified notation for representing a three-phase power system. In power flow studies one-line diagram has its largest application.

The theory of three-phase power systems tells us that as long as the loads on each of the three phases are balanced, we can consider each phase separately. In power engineering, this assumption is usually true (although an important exception is the asymmetric fault), and to consider all three phases requires more effort with very little

potential gain. One-line diagram is usually used along with other notational simplifications, such as the per-unit system. A secondary advantage to using a one-line diagram is that the simpler diagram leaves more space for non-electrical, such as economic, information to be included.

A power system is consists mainly of the following components:

- Generators
- Transformers
- Transmission lines

One-line diagram shows the main connection and the arrangement of the system parts. The data of the apparatus shown may be written on the diagram itself to show the constants of the parts and the connections in the circuit as well as their capacities.

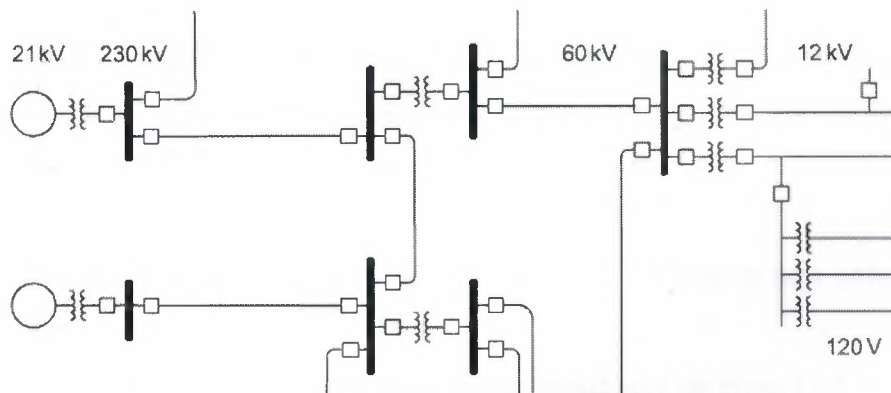


Figure 5.1 One-line diagram showing basic power system structure

5.2.2. Per-Unit System

The quantities involved in power system calculation are kVA, voltage, current and impedance of the equivalent circuits of the different components of the system. The equivalent circuits are at different voltages and connections. Each apparatus is rated in kVA and its impedance is given in actual ohms or in percentage values referred to its rated kVA and rated voltage. To facilitate the solution of the power system quickly, the

component ratings are expressed in values common to the same reference base, by expressing the quantities in 'per-unit' values or p.u. For a common representation, base kVA and base voltage are to be chosen. Then the base current and the base impedance can be expressed as follows:

$$\text{Base current} = \text{Base kVA} / \text{Base kV}$$

$$\begin{aligned} \text{Base impedance} &= \text{Base voltage in volts} / \text{Base current in amperes} \\ &= \text{Base kV}^2 / \text{Base MVA} \end{aligned}$$

$$\text{Per unit impedance, } Z = \text{Actual impedance} / \text{Base impedance}$$

$$\text{Base kW} = \text{Numerical value of base kVA}$$

The impedance of the system parts may be easily converted to per-unit values for solving the system operation problems. For a single-phase, phase-to-neutral voltage, kVA per phase, are taken as bases. In three-phase case, three-phase line-to-line voltage and three-phase kVA are used as bases. The kVA of the largest machine in the system is chosen as base kVA. This reduces the calculation work.

Per-unit impedance may be referred to the new kVA base or new voltage base.

$$\text{P.u. impedance on new kVA base} = \text{p.u. impedance on given kVA base} * (\text{New kVA base} / \text{given kVA base})$$

$$\begin{aligned} \text{P.u. impedance on new voltage base} &= \text{p.u. impedance on given voltage base} * \\ &[(\text{given base V})^2 / \text{New base V})^2] \end{aligned}$$

5.3. Buses

In order to analyze any circuit, we use as a reference those points that are electrically distinct, that is, there is some impedance between them, which can sustain a

potential difference. These reference points are called *nodes*. When representing a power system on a large scale, these nodes are called *buses*.

The term *bus* is very important in the analysis of power systems. Derived from the Latin *omnibus* (for all) the *busbar* is literally a bar of metal to which all the appropriate incoming and outgoing conductors are connected. To be more precise, the busbar consists of three separate bars, one for each phase. Called bus for short, it provides a reference point for measurements of voltage, current, and power flows.

In power flow analyses that encompass larger parts of the grid, buses constitute the critical points that must be characterized, while the detailed happenings “behind” the bus can be ignored from the system point of view. For a generator, voltage and current measurements at its bus are the definitive measure of how the generator is interacting with the grid.

A bus is electrically equivalent to a single point on a circuit, and it marks the location of one of two things: a generator that injects power, or a load that consumes power. At the degree of resolution generally desired on the larger scale of analysis, the load buses represent aggregations of loads (or very large individual industrial loads) at the location where they connect to the high-voltage transmission system. Such an aggregation may in reality be a transformer connection to a subtransmission system, which in turn branches out to a number of distribution substations; or it may be a single distribution substation from which originates a set of distribution feeders. In any case, whatever lies behind the bus is taken as a single load for purposes of the power flow analysis.

The buses in the system are connected by transmission lines. At this scale, one does not generally distinguish among the three phases of an a.c. transmission line. Based on the assumption that, to a good approximation, the same thing is happening on each phase, the three are condensed by the model into a single line, making a so-called one-line diagram. Indeed, a single line between two buses in the model may represent more than one three-phase circuit. Still, for this analysis, all the important characteristics of these conductors can be condensed into a single quantity, the impedance of the one line. Since the impedance is essentially determined by the physical characteristics of the conductors (such as their material composition, diameter, and length), it is taken to be constant. Note that this obviates the need for geographical

accuracy, since the distance between buses is already accounted for within the line impedance, and the lines are drawn in whatever way they best fit on the page.

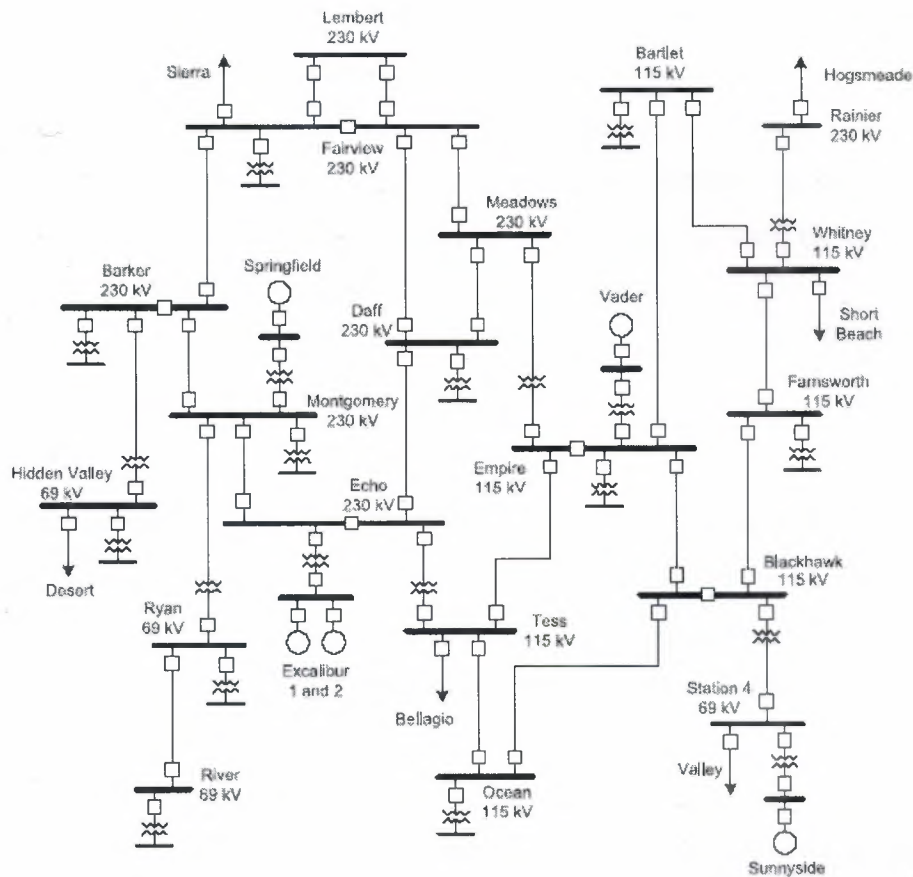


Figure 5.2 One-line diagram for a power system

Thus, the model so far represents the existing hardware of the power system, drawn as a network of buses connected by lines. An example of such a one-line diagram is shown in Figure 5.2.

5.3.1. Types of Buses

Let us now articulate which variables will actually be given for each bus as inputs to the analysis. Here we must distinguish between different types of buses based on their actual, practical operating constraints. The two main types are *generator buses* and *load buses*, for each of which it is appropriate to specify different information. At the load bus, we assume that the power consumption is given, determined by the

consumer and we specify two numbers, real and reactive power, for each load bus. Referring to the symbols P and Q for real and reactive power, load buses are referred to as P, Q buses in power flow analysis.

At the generator buses we could in principle also specify P and Q . Here we run into two problems, however, the first has to do with balancing the power needs of the system, and the second with the actual operational control of generators. As a result, it turns out to be convenient to specify P for all but one generator, the *slack bus*, and to use the generator bus voltage, V , instead of the reactive power, Q , as the second variable. Generator buses are therefore called P, V buses.

5.4. Data for Power Flow

5.4.1. Choice of Variables

So far, we are familiar with the notion of organizing the descriptive variables of the circuit into categories of “knowns” and “unknowns,” whose relationships can subsequently be expressed in terms of multiple equations. Given sufficient information, these equations can then be manipulated with various techniques so as to yield numerical results for the unknowns.

As we know in electric system there are two basic quantities that describe the flow of electricity, voltage and current. Recognizing these quantities in simple d.c. circuits we know that both voltage and current will vary from one location to another in a circuit, but they are everywhere related, the current through each circuit branch corresponds to the voltage or potential difference between the two nodes at either end, divided by the impedance of this branch. It is generally assumed that the impedances throughout the circuit are known, since these are more or less permanent properties of the hardware. Thus, if we are told the voltages at every node in the circuit, we can deduce from them the currents flowing through all the branches, and everything that is happening in the circuit is completely described. If one or more pieces of voltage information were missing, but we were given appropriate information about the current instead, we could still work backwards and solve the problem. In this sense, the number of variables in a circuit corresponds to the number of electrically distinct points in it: assuming we already know all the properties of the hardware, we need to be told one piece of information per node in order to figure out everything that's going on in a d.c. circuit.

For a.c. circuits, the situation is a bit more complicated, because we have introduced the dimension of time, unlike in d.c., where everything is essentially static (except for the instant at which a switch is thrown), with a.c. we are describing an ongoing oscillation or movement. Thus each of the two main variables, voltage and current, in an a.c. circuit really has two numerical components, a magnitude component and a time component. By convention, a.c. voltage and current magnitude are described in terms of *root-mean-squared (rms) values* and their timing in terms of a *phase angle*, which represents the shift of the wave with respect to a reference point in time. To fully describe the voltage at any given node in an a.c. circuit, we must therefore specify two numbers, a voltage magnitude and a voltage angle. Accordingly, when we solve for the currents in each branch, we will again obtain two numbers, a current magnitude and a current angle. And when we consider the amount of power transferred at any point of an a.c. circuit, we again have two numbers, a real and a reactive component. An a.c. circuit thus requires exactly two pieces of information per node in order to be completely determined. More than two, and they are either redundant or contradictory; fewer than two, and possibilities are left open so that the system cannot be solved.

Owing to the nonlinear nature of the power flow problem, it may be impossible to find one unique solution because more than one answer is mathematically consistent with the given configuration. However, it is usually straightforward in such cases to identify the “true” solution among the mathematical possibilities based on physical plausibility and common sense. Conversely, there may be no solution at all because the given information was hypothetical and does not correspond to any situation that is physically possible. Still, it is true in principle and most important for a general conceptual understanding that two variables per node are needed to determine everything that is happening in the system.

Having discussed voltage and current, each with magnitude and angle, as the basic electrical quantities, which are known and which are unknown? In practice, current is not known at all; the currents through the various circuit branches turn out to be the last thing that we calculate once we have completed the power flow analysis. Voltage is known explicitly for some buses but not for others. More typically, what is known is the amount of power going into or out of a bus. Power flow analysis consists of taking all the known real and reactive power flows at each bus, and those voltage magnitudes that are explicitly known, and from this information calculating the

remaining voltage magnitudes and all the voltage angles. This is the hard part. The easy part, finally, is to calculate the current magnitudes and angles from the voltages.

Power is basically the product of voltage and current, and the relative phase angle between voltage and current determines the respective contributions of real and reactive power. Conversely, we can deduce voltage or current magnitude and angle if real and reactive power are given, but it is far more difficult to work out mathematically in this direction. This is because each value of real and reactive power would be consistent with many different possible combinations of voltages and currents. In order to choose the correct ones, we have to check each node in relation to its neighboring nodes in the circuit and find a set of voltages and currents that are consistent all the way around the system. This is what power flow analysis does.

5.4.2. Variables for Balancing Real Power

Balancing the system means that all the generators in the system collectively must supply power in exactly the amount demanded by the load, plus the amount lost on transmission lines. This applies to both real and reactive power, but let us consider only real power first. If we tried to specify a system in which the sum of P generated did not match the P consumed, our analysis would yield no solution, reflecting the fact that in real life the system would lose synchronicity and crash. Therefore, for all situations corresponding to a stable operation of the system, and thus a viable solution of the power flow problem, we must require that real power generated and consumed matches up. Of course, we can vary the contributions from individual generators, that is, we can choose a different dispatch, so long as the sum of their P 's matches the amount demanded by the system. As mentioned earlier, this total P must not only match the load demand, it must actually exceed that amount in order to make up for the transmission losses, which are the resistive I^2R energy losses.

Now here rises a question that how are we supposed to know ahead of time what the transmission losses are going to be? Once we have completed the power flow analysis, we will know what the current flows through all the transmission lines are going to be, and combining this information with the known line impedances will give us the losses. But we cannot tell a priori the amount of losses. The exact amount will vary depending on the dispatch, or amount of power coming from each generator, because a different dispatch will result in a different distribution of current over the various transmission

paths, and not all transmission lines are the same. Therefore, if we were given a total P demanded at the load buses and attempted now to set the correct sum of P for all the generators, we could not do it.

The way to deal with this situation mathematically reflects the way it would be handled in actual operation. Knowing the total P demanded by the load, we begin by assuming a typical percentage of losses, say, 5%. We now dispatch all the generators in the system in some way so that the sum of their output approximately matches what we expect the total real power demand (load plus losses) to be, in this case, 105% of load demand. But since we do not yet know the exact value of the line losses for this particular dispatch, we will probably be off by a small amount. A different dispatch might, for example, result in 4.7% or 5.3% instead of 5% losses overall. We now make acceptable assumption that this uncertainty in the losses constitutes a sufficiently small amount of power that a single generator could readily provide it. So we choose one generator whose output we allow to adjust, depending on the system's needs, we allow it to "take up the slack" and generate more power if system losses are greater than expected, or less if they are smaller. In power flow analysis, this one generator bus is appropriately labeled the *slack bus*, or sometimes *swing bus*.

Thus, as the input information to our power flow analysis, we specify P for one less than the total number of buses. What takes the place of this piece of information for the last bus is the requirement that the system remain balanced. This requirement will be built into the equations used to solve the power flow and will ultimately determine what the as yet unknown P of the slack bus has got to be. The blank space among the initial specifications for the slack bus, where P is not given, will be filled by another quantity, the voltage angle.

5.4.3. Variables for Balancing Reactive Power

Analogous to real power, the total amount of reactive power generated throughout the system must match the amount of reactive power consumed by the loads. Whereas in the case of a mismatch of real power, the system loses synchronicity, a mismatch of reactive power leads to voltage collapse. Also analogous to real power transmission losses, there are *reactive power losses*. Reactive losses are defined simply as the difference between reactive power generated and reactive power consumed by the metered load.

Physically, these losses in Q reflect the fact that transmission lines have some reactance and thus tend to “consume” reactive power; in analogy to I^2R , we could call them I^2X losses. The term “consumption,” however, like the reactive power “consumption” by a load, does not directly imply an energy consumption in the sense of energy being withdrawn from the system. To be precise, the presence of reactive power does necessitate the shuttling around of additional current, which in turn is associated with some real I^2R losses “in transit” of a much smaller magnitude. The term “reactive losses” thus does not refer to any physical measure of something lost, but rather should be thought of as an accounting device. While real power losses represent physical heat lost to the environment and therefore always have to be positive, reactive losses on a given transmission link can be positive or negative, depending on whether inductive or capacitive reactance plays a dominant role.

In any case, what matters for both operation and power flow analysis is that Q , just like P , needs to be balanced at all time. Thus, just as for real power, all the generators in the system must generate enough reactive power to satisfy the load demand plus the amount that vanishes into the transmission lines.

This leaves us with the analogous problem of figuring out how much total Q our generators should produce, not knowing ahead of time what the total reactive losses for the system will turn out to be, as with real losses, the exact amount of reactive losses will depend on the dispatch. Operationally, though, the problem of balancing reactive power is considered in very different terms. When an individual generator is instructed to provide its share of reactive power, in practice this is not usually done by telling it to generate a certain number of MVAR. Instead, the generator is instructed to maintain a certain voltage magnitude at its bus. The voltage is continually and automatically adjusted through the generator’s field current, and is therefore a straightforward variable to control.

Their own bus voltage is in fact the one immediate measure available to the generators for determining whether the correct amount of reactive power is being generated, when the combined generation of reactive power by all the generators in the system matches the amount consumed, their bus voltage holds steady. Conversely, if there is a need to increase or decrease reactive power generation, adjusting the field current at one or more generators so as to return to the voltage set point will automatically accomplish this objective. The new value of MVAR produced by each generator can then be read off the dial for accounting purposes.

Conveniently for power flow analysis there is no need to know explicitly the total amount of Q required for the system. Specifying the voltage magnitude is essentially equivalent to requiring a balanced Q . In principle, we could specify P and Q for each generator bus, except for one slack bus assigned the voltage regulation (and thus the onus of taking up the slack of reactive power). For this “reactive slack” bus we would need to specify voltage magnitude V instead of Q , with the understanding that this generator would adjust its Q output as necessary to accommodate variations in reactive line losses. In practice, however, since voltage is already the explicit operational control variable, it is customary to specify V instead of Q for all generator buses, which are therefore called P, V buses. In a sense, this assignment implies that all generators share the “reactive slack,” in contrast to the real slack that is taken up by only a single generator.

5.4.4. The Slack Bus

For power flow analysis we have three categories of buses: P, Q buses, which are generally load buses, but could in principle also be generator buses; P, V buses, which are necessarily generator buses (since loads have no means of voltage control); and then there is the slack bus, for which we cannot specify P , only V . What takes the place of P for the slack bus?

As described before, real power balance clearly states the operationally as a steady frequency such as 60 Hz. A constant frequency is indicated by an unchanging voltage angle, which for this reason is also known as the *power angle*, at each generator. When more power is consumed than generated, the generators’ rotation slows down, their electrical frequency drops, and their voltage angles fall farther and farther behind. Conversely, if excess power is generated, frequency increases and the voltage angles move forward. While generators are explicitly dispatched to produce a certain number of megawatts, the necessary small adjustments to balance real power in real-time are made (by at least one or more *load-following* generator) through holding the generator frequency steady at a specified value. Not allowing the frequency to depart from this reference value is equivalent to not letting the voltage angle increase or decrease over time.

In power flow analysis, the slack bus is the one mathematically assigned to do the load following. Its instructions, as it were, are to do whatever is necessary to maintain real power balance in the system. Physically, this would mean holding the voltage angle constant. The place of P will therefore be taken by the *voltage angle*, which is the variable

that in effect represents real power balance. We can think of the voltage angle here as analogous to the voltage magnitude in the context of reactive power, where balance is achieved operationally by maintaining a certain voltage (magnitude) set point at the generator bus. Specifying that the bus voltage magnitude should be kept constant effectively amounts to saying that whatever is necessary should be done to keep the system reactive power balanced. Similarly, specifying a constant voltage angle at the generator bus amounts to saying that this generator should do whatever it takes to keep real power balanced.

We thus assign to the slack bus a voltage angle, which, in keeping with the conventional notation for the context of power flow analysis, we will call θ (lowercase Greek theta). This θ can be interpreted as the relative position of the slack bus voltage at time zero. Note that this θ is exactly the same thing that is elsewhere called the *power angle* and labeled as δ (delta).

What is important to understand here is that the actual numerical value of this angle has no physical meaning; what has physical meaning is the implication that this angle will not change as the system operates. The choice of a numerical value for θ is a matter of convenience. When we come to the output of the power flow analysis, we will discover a voltage angle θ for each of the other buses throughout the system, which is going to take on a different (constant) value for each bus depending on its relative contribution to real power. These numerical values only have meaning in relation to a reference, what matters is the difference between the voltage angle at one bus and another, which physically corresponds to the phase difference between the voltage curves, or the difference in the precise timing of the voltage maximum.

We now conveniently take advantage of the slack bus to establish a system wide reference for timing, and we might as well make things simple and call the reference point “zero.” This could be interpreted to mean that the alternating voltage at the slack bus has its maximum at the precise instant that we depress the “start” button of an imaginary stopwatch, which starts counting the milliseconds (in units of degrees within a complete cycle of 1/60th second) from time zero. In principle, we could pick any number between 0 and 360 degrees as the voltage angle for the slack bus, but 0° is the simple and conventional choice.

5.4.5. Summary of Variables

To summarize, our three types of buses in power flow analysis are P, Q (load bus), P, V (generator bus), and θ, V (slack bus). Given these two input variables per bus, and knowing all the fixed properties of the system (i.e., the impedances of all the transmission links, as well as the a.c. frequency), we now have all the information required to completely and unambiguously determine the operating state of the system. This means that we can find values for all the variables that were not originally specified for each bus θ and V for all the P, Q buses; θ and Q for the P, V buses; and P and Q for the slack bus. The known and unknown variables for each type of bus are tabulated later in the following table for easy reference.

Table 5.1 Variables in Power Flow Analysis

Type of Bus	Variables Given (Knowns)	Variables Found (Unknowns)
Generator	Real power (P)	Voltage angle (θ)
	Voltage magnitude (V)	Reactive power (Q)
Load or generator	Real power (P)	Voltage angle (θ)
	Reactive power (Q)	Voltage magnitude (V)
Slack	Voltage angle (θ)	Real power (P)
	Voltage magnitude (V)	Reactive power (Q)

Once we know θ and V , the voltage angle and magnitude, at every bus, we can very easily find the current through every transmission link; it becomes a simple matter of applying Ohm's law to each individual link. Depending on how the output of a power flow program is formatted, it may state only the basic output variables, as in Table 6.1, it may explicitly state the currents for all transmission links in amperes; or it may express the flow on each transmission link in terms of an amount of real and reactive power flowing, in megawatts (MW) and MVAR.

5.5. Example with Interpretation of Results

5.5.1. Six-Bus Case

Consider the six-bus example illustrated in Figure 5.3. This example is simple enough for us to observe in detail, yet too complex to predict its behavior without numerical power flow analysis.

Each of the six buses has a load, and four of the buses also have generators. Bus 1, keeping with convention, is the slack bus. Buses 2, 3, and 4, which have both generation and loads, are modeled as P, V buses; the local load is simply subtracted from the real and reactive generation at each. Buses 5 and 6, which have only loads, are modeled as P, Q buses.

The distribution of loads and the generation dispatch, for both real and reactive power, are completely determined somewhere outside the power flow program, whether in the real world or the program user's fantasy. The one exception is the generator at the slack bus, whose real power output varies so as to accommodate system wide losses. In addition to the MW and MVAR loads and the MW generation levels for every generator (except the slack), the user specifies the voltage magnitudes to be maintained at each generator bus. The program then computes the MVAR generation necessary to maintain this voltage at each bus.

By convention, the voltage angle at the slack bus is set to 0.00 degrees. The power flow program computes the voltage angle at each of the other five buses in relation to the slack bus. We may now begin to observe the relationship between real power and voltage angle, a more positive voltage angle generally corresponds to an injection of power into the system and a more negative voltage angle to a consumption of real power. Buses 2 and 4, which both have generation exceeding local load, have positive voltage angles of 2.77° and 1.03° , respectively. Bus 3, though it has a generator, is still a net consumer of real power, with 100 MW load and only 84 MW generated; its voltage angle is -3.68° . Buses 5 and 6 have loads only and voltage angles of -2.02° and -3.47° , respectively.

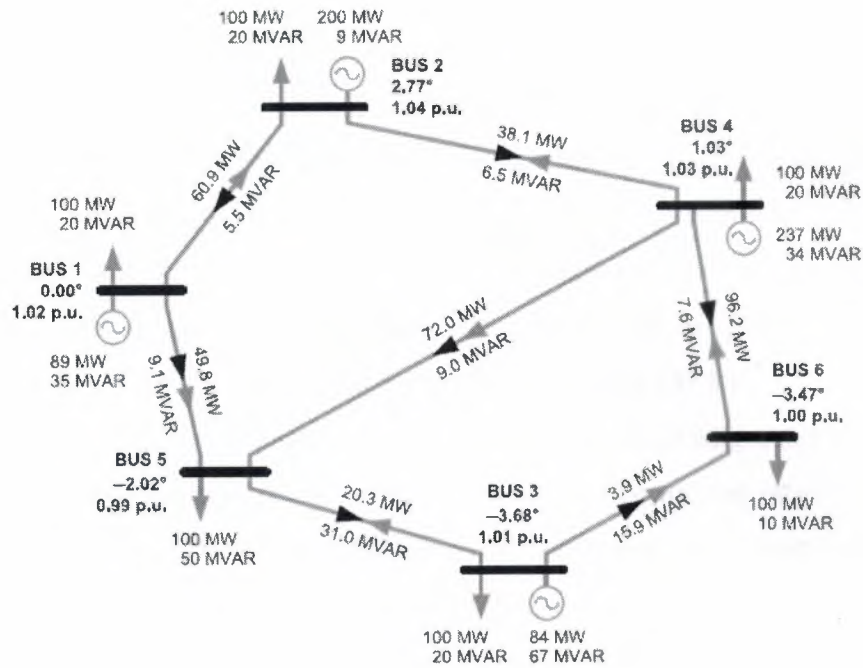


Figure 5.3 Six-bus power flow example

Note, however, that the voltage angles are not in hierarchical order depending on the amount of power injected or withdrawn at each individual bus. This is because we also must consider the location of each bus relative to the others in the system and the direction of power flow between them. For example, consider Buses 2 and 4. Net generation at Bus 4 is greater than at Bus 2 (137 MW compared to 100 MW), yet the voltage angle at Bus 2 is more positive. We can see that this is due to the location of these buses in the system: real power is generally flowing from north to south, that is, from Bus 2 to the neighborhood of Buses 5, 3, and 6 where there is more load and less generation. As indicated by the black arrow on the transmission link, real power is flowing from Bus 2 to Bus 4. As a rule, real power flows from a greater to a smaller voltage angle. This rule holds true for six of the seven links in this sample case, the exception is Link 3–6, where both the power flow and the difference in voltage angle are very small. The reader can verify that throughout this case, while power flow and voltage angle are not exactly proportional, a greater flow along a transmission link is associated with a greater angle difference.

We now turn to the relationship between reactive power and voltage magnitude, which is similar to that between real power and voltage angle. The nominal voltage of this hypothetical transmission system is 138 kV. However, just as the timing or angle of

the voltage differs by a small fraction of a cycle at different locations in the grid, the magnitude, too, has a profile across the system with different areas a few percent higher or lower than the nominal value. Because it is this percentage, not the absolute value in volts, that is most telling about the relationship among different places in the grid, it is conventional to express voltage magnitude in per-unit terms. Per-unit (p.u.) notation simply indicates the local value as a multiple of the nominal value; in this case, 138 kV equals 1.00 p.u. The voltage magnitude at Bus 1 is given as 1.02 p.u., which translates into 141 kV, at Bus 5, the voltage magnitude of 0.99 p.u. means 137 kV.

As a rule, reactive power tends to flow in the direction from greater to smaller voltage magnitude. In our example, this rule holds true only for the larger flows of MVAR, along Links 1–5, 3–5, 4–5, and 3–6. The reactive power flows along Links 1–2, 4–2, and 6–4 do not follow the rule, but they are comparatively small.

Note that real and reactive power do not necessarily flow in the same direction on a given link. This should not be surprising, because the “direction” of reactive power flow is based on an arbitrary definition of the generation or consumption of VARs, there is in fact no net transfer of energy in the direction of the gray arrow for Q . Also, note that having Q flow opposite P does not imply any “relief” or reduction in current. For example, on Link 3–5, the real power flow P is 20.3 MW and reactive flow Q is 31.0 MVAR. In combination, this gives apparent power S of 37.1 MVA (using $S^2 = P^2 + Q^2$), regardless of the direction of Q . As we know that MVA are the relevant units for thermal line loading limits, since total current depends on apparent power.

From Figure 5.3, it is possible to evaluate the total real and reactive system losses, simply by observing the difference between total generation and total load. The four generators are supplying 89, 200, 84, and 237 MW, respectively, for a total of 610 MW of real power generated. Subtracting the six loads of 100 MW each, the total real power losses throughout the transmission system for this particular scenario are therefore 10 MW. On the reactive side, total generation is 145 MVAR, while total reactive load is 140 MVAR, and system reactive losses amount to roughly 5 MVAR.

To account for losses we must record both the power (real or reactive) entering and exiting each link. In Figure 5.4, these data are given for real power (MW) in black and reactive power (MVAR) in gray. The numbers in parentheses represent the losses, which are the difference between power flows at either end. Bus power, line flows, and losses are rounded to different decimal places, but the numbers do add up correctly for each bus and each link.

The most significant losses tend to occur on links with the greatest power flow. In this case, Link 4–6 has the greatest power flow with 96.2 MW real and 7.6 MVAR reactive, yielding 96.5 MVA apparent, and the greatest losses. While the real line losses are all positive, as they should be, the negative signs on some of the reactive losses indicate negative losses; we might consider them “gains,” although nothing is actually gained. Reactive losses depend on operating conditions and impedance, where the model of a transmission link may incorporate reactive compensation such as capacitors. It is typical for system reactive losses to be positive overall, as they are in this example. Like real losses, reactive losses are related to the current and therefore apparent power flow. Thus, we also observe the greatest reactive losses in our example on Link 4–6. The real and reactive losses for every link can be totaled to confirm the estimated system losses.

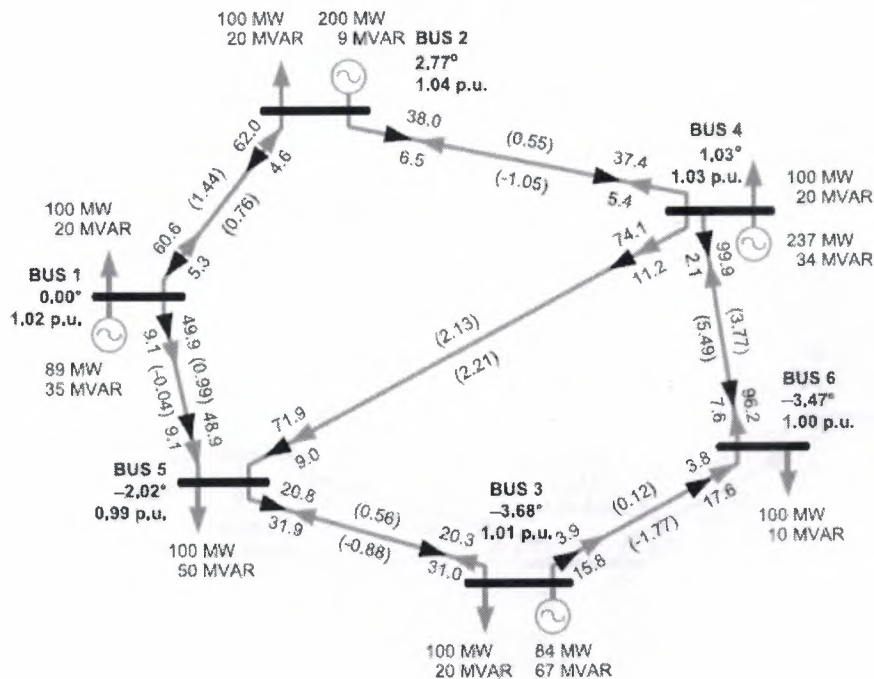


Figure 5.4 Six-bus power flow example with losses

5.5.2. Tweaking the Case

To gain a better sense of a power system's behavior and the information provided by power flow modeling, let us now make a small change to the operating state in the six-bus example and observe how the model responds. We simply increase the load at Bus 5 by 20% while maintaining the same power factor, thus changing it from 100 MW real and 50 MVAR reactive to 120 MW real and 60 MVAR reactive. This change is small enough for the generator at the slack bus to absorb, so we need not specify increased generation elsewhere. Indeed, generation at Bus 1 increases from 89 to 110 MW. Note that the difference amounts to 21, not 20 MW, as the increased load also entails some additional losses in the system. The new scenario is illustrated in Figure 5.5.

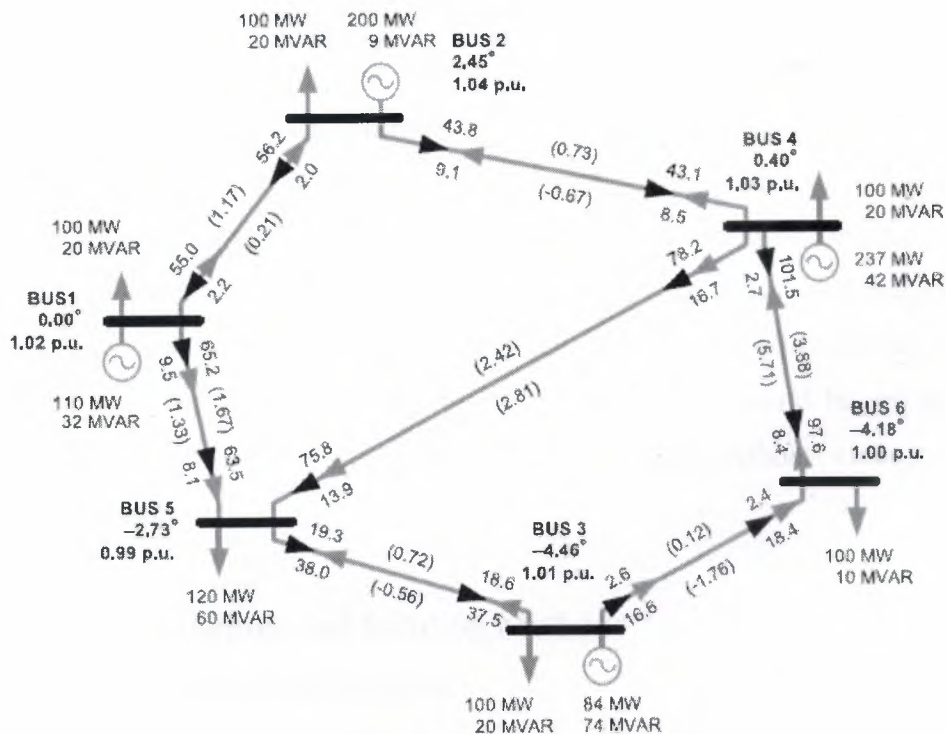


Figure 5.5 Modified six-bus power flow example

As we would expect, the line flows to Bus 5 increase by a total of 20 MW. The bulk (about 14.5 MW) of this additional power comes from Bus 1, about 4 MW from Bus 4, and the balance appears as a reduction of about 1.5 MW in the flow to Bus 3.

The changes do not stop here, however; they have repercussions for the remainder of the system. Three of the other buses are defined as P, V buses, and therefore have fixed voltage magnitudes. The voltage magnitude at Bus 6 (a P, Q bus) is affected slightly (from 0.9951 to 0.9950 p.u.), although the change does not show up after rounding. In order to maintain the preset voltages at the P, V buses, reactive power generation increases at Buses 4 and 6, as it does at the slack bus. Indeed, system reactive generation now totals 157 MVAR, which are needed to accommodate the additional reactive load introduced at Bus 5 as well as a substantial increase in system reactive losses of 2.33 MVAR (up almost 50% from 4.74 to 7.07 MVAR). While real power is fixed at all buses other than 1 and 5, their voltage angles change as a result of the changed power flow pattern.

The most serious influence in this example occurs on Link 4–6, which was fully loaded before the change to Bus 5 was made. The change at Bus 5 results in a slight increase in real power flow from Bus 4 to Bus 6. Link 4–6 now carries slightly more current and apparent power, going from 100 to 101.5 MVA. This is significant because, in this hypothetical scenario, each transmission link has a thermal limit of 100 MVA. The power flow program thus shows the line becoming overloaded as a result of the change at Bus 5, even though Buses 4 and 6 are located on the opposite geographic end of the system, and neither generation nor load levels there were affected. In reality, this violation would mean that the proposed change is inadmissible and other options would have to be pursued, specifically, a generator other than Bus 1 would be required to increase generation, in order to meet the additional load without violating any transmission constraints.

5.6. Power Flow Equations and Solution Methods

5.6.1. Formulation of Power Flow Equations

In previous section we stated the known and unknown variables for each of the different types of buses in power flow analysis. The power flow equations show explicitly how these variables are related to each other.

The complete set of power flow equations for a network consists of one equation for each node or branch point in this network, referred to as a bus, stating that the complex power injected or consumed at this bus is the product of the voltage at this bus and the current flowing into or out of the bus. Because each bus can have several

transmission links connecting it to other buses, we must consider the sum of power entering or leaving by all possible routes. To help with the accounting, we will use a summation index i to keep track of the bus for which we are writing down the power equation, and a second index k to keep track of all the buses connected to i .

We express power in complex notation, which takes into account the two-dimensionality, magnitude and time, of current and voltage in an a.c. system. As we know, complex power S can be written in shorthand notation as

$$S = VI^*$$

where all variables are complex quantities and the asterisk denotes the complex conjugate of the current.

The S represents the complex sum of real power P and reactive power Q , where P is the real and Q the imaginary component. While the real part represents a tangible physical measurement (the rate at which energy is transferred), the imaginary part can be thought of as the oscillating component. At different times it may be convenient to either refer to P and Q separately or simply to S as the combination.

In the most concise notation, the power flow equations can be stated as

$$S_i = V_i I_i^*$$

where the index i indicates the node of the network for which we are writing the equation. Thus, the full set of equations for a network with n nodes would look like

$$S_1 = V_1 I_1^*$$

$$S_2 = V_2 I_2^*$$

...

$$S_n = V_n I_n^*$$

We can choose to define power as positive either going into or coming out of that node, as long as we are consistent. Thus, if the power at load buses is positive, that at generator buses is negative.

So far, these equations are not very helpful, since we have no idea what the I_i are. In order to mold the power flow equations into something we can actually work

with, we must make use of the information we assumed have about the network itself. Specifically, we want to write down the impedances of all the transmission links between nodes. Then we can use Ohm's law to substitute known variables (voltages and impedances) for the unknowns (currents).

Written in the conventional form, Ohm's law is $V = IZ$ (where Z is the complex impedance). However, when solving for the current I , this would require putting Z in the denominator, $I = V/Z$. It is therefore better to use the inverse of the impedance, known as the admittance Y (where $Y = 1/Z$), so that Ohm's law becomes $I = VY$. Not only does this avoid the formatting issues of division, it also allows us to indicate the absence of a transmission link with a zero (for zero admittance) instead of infinity (for infinite impedance), which vastly improves the appearance of the imminent matrix algebra.

The complex admittance $Y = G + jB$, where G is the conductance and B the susceptance. The admittances of all the links in the network are summarized by way of a matrix Y called as *bus admittance matrix* Y_{bus} . The admittance matrix Y_{bus} is an $n \times n$ matrix and can be constructed from the line and transformer input data. The elements Y_{kn} of the bus admittance matrix Y_{bus} or simply Y are formed as follows:

Diagonal elements:

$$Y_{kk} = \text{sum of admittances connected to bus } k$$

$$\text{where } (k = 1, 2, \dots, n)$$

Off-diagonal elements:

$$Y_{kn} = -(\text{sum of admittances connected between buses } k \text{ and } n)$$

$$\text{where } (k \neq n)$$

The diagonal element Y_{kk} is called the *self-admittance* of the bus k , and the off-diagonal element Y_{kn} for $k \neq n$ called the *mutual admittance* between buses k and n . Since $Y_{kn} = Y_{nk}$, the matrix Y is symmetric. An example of an $n \times n$ admittance matrix is shown in Figure 5.6.

$$\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix}$$

Figure 5.6 Admittance matrix Y with n number of elements

The relationship $I = VY$ is what we wish to write down and substitute for every I_i that appears in the power flow equations. But now we face the next complication, the total current I_i coming out of any given node is in fact the sum of many different currents going between node i and all other nodes that physically connect to i . We will indicate these connected nodes with the index k , where k could theoretically include all nodes other than i in the network from 1 to n . In practice, fortunately, only a few of these will actually have links connecting to node i . For any node k that is not connected to i , $y_{ik} = 0$.

For the current between nodes i and k , we can now write

$$I_{ik} = V_k y_{ik}$$

or, expanding the admittance into its conductance and susceptance components,

$$I_{ik} = V_k (g_{ik} + b_{ik})$$

Note that if this equation stood by itself, we would want V_k to mean “the voltage difference between nodes i and k ” rather than simply “the voltage at node k .” However, we will see the voltage differences between nodes naturally arise out of the complex conjugate once we expand the terms, specifically, the voltage differences will appear in terms of differences between phase angles at each node.

To complete the statement $S_i = V_i I_i^*$ for node i , we must now sum over all nodes k that could possibly be connected to i . This summation over the index k means accounting for all the current that is entering or leaving this one particular node, i , by way of the various links it has to other nodes k . (For the complete system of power flow equations, we also have to consider every value of the index i so as to consider power flow for every node, but we do not need to write this out explicitly.) Thus,

$$S_i = V_i I_i^* = V_i \left(\sum_{k=1}^n y_{ik} V_k \right)^*$$

Now we expand the y 's into g 's and b 's, noting that the complex conjugate gives us a minus sign in front of the jb :

$$S_i = V_i \sum_{k=1}^n (g_{ik} - jb_{ik}) V_k^*$$

After changing the order of terms to look more organized, we write the voltage phasors out in longhand, first as exponentials and then broken up into sines and cosines:

$$\begin{aligned} S_i &= \sum_{k=1}^n |V_i| |V_k| e^{j(\theta_i - \theta_k)} (g_{ik} - jb_{ik}) \\ &= \sum_{k=1}^n |V_i| |V_k| [\cos(\theta_i - \theta_k) + j \sin(\theta_i - \theta_k)] (g_{ik} - jb_{ik}) \end{aligned}$$

The term $(\theta_i - \theta_k)$ in this equation denote the difference in voltage angle between nodes i and k , where the minus sign came from having used the complex conjugate of I initially, which gave us the complex conjugate of V_k . This angle difference is not the same as the instantaneous difference in magnitude between the two voltages, but it is the correct measure of separation between the nodes for the purpose of calculating the power transferred.

The equation we now have for S_i contains the product of two complex quantities, written out in terms of their real and imaginary components. By cross-multiplying all the real and imaginary terms, we can separate the real and imaginary parts of the result S , which will be the familiar P and Q . Remembering that j times j gives -1 , we obtain:

$$P_i = \sum_{k=1}^n |V_i| |V_k| [g_{ik} \cos(\theta_i - \theta_k) + b_{ik} \sin(\theta_i - \theta_k)]$$

$$Q_i = \sum_{k=1}^n |V_i| |V_k| [g_{ik} \sin(\theta_i - \theta_k) - b_{ik} \cos(\theta_i - \theta_k)]$$

The complete set of power flow equations for a network of n nodes contains n such equations for S_i , or pairs of equations for P_i and Q_i . This complete set will account for every node and its interaction with every other node in the network.

5.6.2. Solution Methods

There is no analytical, closed-form solution for the set of power flow equations given in the preceding section. In order to solve the system of equations, we must proceed by a numerical approximation that is essentially a sophisticated form of trial and error.

To begin with, we assume certain values for the unknown variables. For clarity, let us suppose that these unknowns are the voltage angles and magnitudes at every bus except the slack, making them all P, Q buses (it turns out that having some P, V buses eases the computational volume in practice, but it does not help the theoretical presentation). In the absence of any better information, we would probably choose a *flat start*, where we assume the initial values of all voltage angles to be zero (the same as the slack bus) and the voltage magnitudes to be 100% of the nominal value, or 1 p.u. In other words, for lack of a better guess, we suppose that the voltage magnitude and angle profile across the system is completely flat.

We then plug these values into the power flow equations. Of course, we know they do not describe the actual state of the system, which was supposed to be consistent with the known variables (P 's and Q 's). Essentially, this will produce a contradiction, based on the starting values, the power flow equations will predict a different set of P 's and Q 's than we stipulated at the beginning. Our objective is to make this contradiction go away by repeatedly inserting a better set of voltage magnitudes and angles. As our voltage profile matches reality more and more closely, the discrepancy between the P 's and Q 's, known as the *mismatch*, will shrink. Depending on our patience and the degree of precision we require, we can continue this process to reach some arbitrarily close approximation. This type of process is known as an *iterative* solution method, where "to iterate" means to repeatedly perform the same manipulation.

The heart of the iterative method is to know how to modify each guess in the right direction and by the right amount with each round of computation (iteration), so as

to arrive at the correct solution as quickly as possible. Specifically, we wish to gather information from our equations that tells us which value was too high, which was too low, and approximately by how much, so that we can prepare a well-informed next guess, rather than blindly groping around in the dark for a better set of numbers. There are several standard techniques for doing this; the ones most commonly used in power flow analysis are the Newton–Raphson, the Gauss, and the Gauss–Seidel iterations.

Regardless of which method is used, we will need to press our power flow equations for the crucial information about the direction of the error. We obtain this information by writing down the *partial derivatives* of the power flow equations, or their rates of change with respect to individual variables. Specifically, we need to know the rate of change of real and reactive power, each with respect to voltage angle or magnitude. This yields four possible combinations of partial derivatives. For example, $\partial P/\partial\theta$ is the partial derivative of real power with respect to voltage angle, and similarly there are $\partial P/\partial V$, $\partial Q/\partial\theta$, and $\partial Q/\partial V$. Each of these combinations is in fact a matrix known as a *Jacobian matrix* that, in turn, includes every bus combined with every other bus. The Jacobian matrix for four bus system with bus 1 as slack bus and bus 4 as P, V bus would look like this:

$$\begin{vmatrix} \Delta P_2 \\ \Delta Q_2 \\ \Delta P_3 \\ \Delta Q_3 \\ \Delta P_4 \end{vmatrix} = \begin{vmatrix} \partial P_2/\partial\theta_2 & \partial P_2/\partial V_2 & \partial P_2/\partial\theta_3 & \partial P_2/\partial V_3 & \partial P_2/\partial\theta_4 \\ \partial Q_2/\partial\theta_2 & \partial Q_2/\partial V_2 & \partial Q_2/\partial\theta_3 & \partial Q_2/\partial V_3 & \partial Q_2/\partial\theta_4 \\ \partial P_3/\partial\theta_2 & \partial P_3/\partial V_2 & \partial P_3/\partial\theta_3 & \partial P_3/\partial V_3 & \partial P_3/\partial\theta_4 \\ \partial Q_3/\partial\theta_2 & \partial Q_3/\partial V_2 & \partial Q_3/\partial\theta_3 & \partial Q_3/\partial V_3 & \partial Q_3/\partial\theta_4 \\ \partial P_4/\partial\theta_2 & \partial P_4/\partial V_2 & \partial P_4/\partial\theta_3 & \partial P_4/\partial V_3 & \partial P_4/\partial\theta_4 \end{vmatrix} \begin{vmatrix} \Delta\theta_2 \\ \Delta V_2 \\ \Delta\theta_3 \\ \Delta V_3 \\ \Delta\theta_4 \end{vmatrix}$$

Each of these four types of partial derivatives ($\partial P/\partial\theta$, $\partial P/\partial V$, $\partial Q/\partial\theta$, and $\partial Q/\partial V$) constitutes one partition of the big *Jacobian matrix* J . Within each partition there are rows for the θ or V and columns for the P or Q belonging to every bus (except the slack bus, so that the dimensionality of each partition is one less than the number of buses in the system). Note that when we write V we mean the magnitude of V , which would be more properly designated by $|V|$ except that the notation is already seems complicated enough without the vertical lines.

We must now combine the system of equations and its partial derivatives with our guess for the unknown variables in such a way that it suggests to us a helpful

modification of the unknowns, which will become our improved guess in the next iteration.

The Jacobian matrix is essentially a very large version of the derivative $f'(x)$ used in Newton's method, with multiple f 's and x 's tidily summarized into the single, bulky object labeled J . The function $f(x)$ itself appears essentially as the power flow equations $P(\theta, V)$ and $Q(\theta, V)$. In order to keep with the format of searching for $f(x) = 0$, we define $f(x)$ as the difference between the $P(\theta, V)$, $Q(\theta, V)$ computed and the actual values of P and Q at each of the buses, which are known at the outset. In other words, $f(x)$ represents the *mismatch*, which we want to get as close as is reasonable to zero.

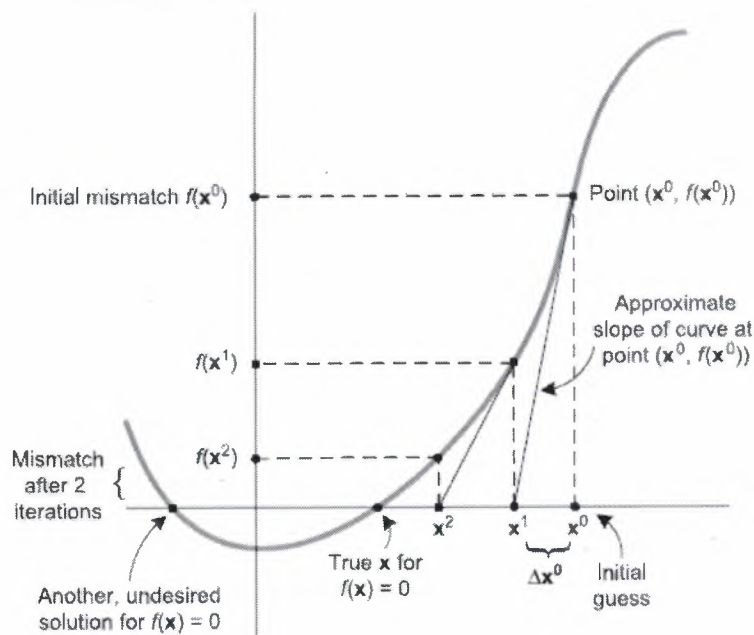


Figure 5.7 Iterative process of approximating $f(x) = 0$

In principle, we are now able to combine all the information at our disposal into improving our guess for the θ 's and V 's (the x 's). Let us label our initial guess for θ and V as x^0 , where the boldface x indicates the vector composed of one θ and one V for each bus (except the slack) and the superscript 0 indicates the zeroth iteration. Now we might simply adapt the expression for Δx from Newton's method.

$$\Delta x = -\frac{f(x)}{f'(x)}$$

and substitute our matrix and vector quantities

$$\Delta \mathbf{x} = \frac{-f(\mathbf{x})}{J}$$

or

$$\Delta \mathbf{x} = -f(\mathbf{x}) J^{-1}$$

or to rearrange the equation to read

$$J \Delta \mathbf{x} = -f(\mathbf{x})$$

Having thus identified the Δx , we simply add it to the old x and proceed to the next iteration. Rewriting the last equation

$$J^v \Delta \mathbf{x}^v = -f(\mathbf{x})^v$$

to indicate that we are on the v th (nuth) iteration. Eventually, we will find that $f(\mathbf{x})^v$ is close enough to zero to satisfy us.

When $f(\mathbf{x})^v = 0$, it means that there is no more mismatch, and the \mathbf{x}^v , that is, the θ 's and V 's from the v th iteration give us the operating state of the power system that is consistent with the P 's and Q 's we specified initially. We may need to verify, though, that \mathbf{x}^v is a realistic and true solution for our power system.

We have now found θ and V for each P, Q bus. For any P, V buses, we would have found θ and Q as part of our computational process. There are several steps left to produce the complete output of the power flow analysis. First, by writing the power flow equation for the slack bus, we determine the amount of real power generated there. This tells us how many MW of losses there are in the system, as we can now compare the total MW generated to the total MW of load demand. Also, by using Ohm's law for every transmission link, we solve explicitly for each line flow, in amperes or MVA. Through the θ 's and V 's at each bus, we have information about the real and reactive power both

going into and coming out of each link, and by subtracting we can specify the real and reactive losses on each link. Finally, we format the output and compare it to external constraints such as line flow limits so as to flag any violations. We have now completely described the system's operating state based on a given generation dispatch and combination of loads.

5.7. Calculation Procedure of Newton-Raphson Method

The procedure is summarized in the following steps:

- Bus admittance matrix is formed
- Initial values of voltages and phase angles are assumed for the load (P, Q) buses. Phase angles are assumed for P, V buses. Normally, the bus voltages are set equal to the slack bus voltage, and phase angles are assumed equal to 0° , i.e., a *flat* start
- Active and reactive powers, P and Q , are calculated for each load bus
- ΔP and ΔQ can, therefore, be calculated on the basis of the given power at the buses
- For P, V buses, the exact reactive power is not specified, but its limits are known. If the calculated value of the reactive power is within limits, only ΔP is calculated. If the calculated value of reactive power is beyond the specified limits, then an appropriate limit is imposed and Q is also calculated by subtracting the calculated value of the reactive power from the maximum specified limit. The bus under consideration is now treated as a P, Q (load) bus
- The elements of the Jacobian matrix are calculated
- This gives $\Delta\theta$ and ΔV
- Using the new values of $\Delta\theta$ and ΔV , the new values of voltages and phase angles are calculated.
- The next iteration is started with these new values of voltage magnitudes and phase angles.
- The procedure is continued until the required tolerance is achieved

5.7.1. Example solved by Newton-Raphson method

Consider a transmission system of two 138-kV lines, three buses, each line modeled by an equivalent Π network, as shown in Figure 5.8, with series and shunt admittances as shown. Bus 1 is the swing bus (voltage $1.02 \angle 0^\circ$), bus 2 is a P, Q bus with load demand of $0.25 + j0.25$ per unit, and bus 3 is a voltage-controlled bus with bus voltage of 1.02 and a load of $0.5 j0$ per unit all on 100 MVA base.

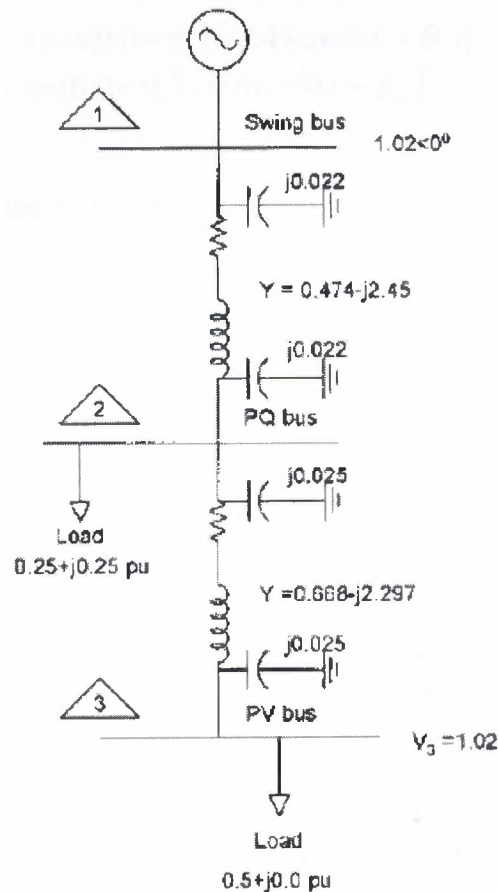


Figure 5.8 System example for load flow solution

Solution:

First, form a Y matrix as follows:

$$Y = \begin{bmatrix} 0.474 - j2.428 & -0.474 + j2.45 & 0 \\ -0.474 + j2.45 & 1.142 - j4.70 & -0.668 + j2.297 \\ 0 & -0.668 + j2.297 & 0.668 - j2.272 \end{bmatrix}$$

The active and reactive power at bus 1(swing bus) can be written as:

$$\begin{aligned} P_1 &= 1.02 \times 1.02 [0.474 \cos(0.0 - 0.0) + (-2.428) \sin(0.0 - 0.0)] \\ &+ 1.02 V_2 [(-0.474) \cos(0.0 - \theta_2) + 2.45 \sin(0.0 - \theta_2)] \\ &+ 1.02 \times 1.02 [0.0 \cos(0.0 - \theta_3) + 0.0 \sin(0.0 - \theta_3)] \end{aligned}$$

$$\begin{aligned} Q_1 &= 1.02 \times 1.02 [0.474 \sin(0.0 - 0.0) - (-2.428) \cos(0.0 - 0.0)] \\ &+ 1.02 V_2 [(-0.474) \sin(0.0 - \theta_2) - 2.45 \cos(0.0 - \theta_2)] \\ &+ 1.02 \times 1.02 [0.0 \sin(0.0 - \theta_3) - 0.0 \cos(0.0 - \theta_3)] \end{aligned}$$

These equations for the swing bus are not immediately required for load flow, but can be used to calculate the power flow from this bus, once the system voltages are calculated to the required tolerance.

Similarly, the active and reactive power at other buses are written:

$$\begin{aligned} P_2 &= V_2 \times 1.02 [-0.474 \cos(\theta_2 - 0.0) + (2.428) \sin(\theta_2 - 0.0)] \\ &+ V_2 \times V_2 [1.142 \cos(\theta_2 - \theta_2) + (-4.70) \sin(\theta_2 - \theta_2)] \\ &+ V_2 \times 1.02 [(-0.668) \cos(\theta_2 - \theta_3) + (2.297) \sin(\theta_2 - \theta_3)] \end{aligned}$$

Substituting the initial values ($V_2 = 1$, $\theta_2 = 0^\circ$), $P_2 = -0.0228$.

$$\begin{aligned} Q_2 &= V_2 \times 1.02 [-0.474 \sin(\theta_2 - 0.0) - 2.45 \cos(\theta_2 - 0.0)] \\ &+ V_2 \times V_2 [1.142 \sin(\theta_2 - \theta_2) - (-4.70) \cos(\theta_2 - \theta_2)] \\ &+ V_2 \times 1.02 [(-0.668) \sin(\theta_2 - \theta_3) - 2.297 \cos(\theta_2 - \theta_3)] \end{aligned}$$

Substituting the numerical values, $Q_2 = -0.142$.

$$\begin{aligned} P_3 &= 1.02 \times 1.02 [0.0 \cos(\theta_3 - 0.0) + 0.0 \sin(\theta_3 - 0.0)] \\ &+ 1.02 \times V_2 [(-0.668) \cos(\theta_3 - \theta_2) + 2.297 \sin(\theta_3 - \theta_2)] \\ &+ 1.02 \times 1.02 [0.668 \cos(\theta_3 - \theta_3) + (-2.047) \sin(\theta_3 - \theta_3)] \end{aligned}$$

Substituting the values, $P_3 = 0.0136$

$$\begin{aligned}
Q_3 &= 1.02 \times 1.02 [0.0 \sin(\theta_3 - 0.0) - 0.0 \cos(\theta_3 - 0.0)] \\
&+ 1.02 \times V_2 [(-0.668) \sin(\theta_3 - \theta_2) - 2.297 \cos(\theta_3 - \theta_2)] \\
&+ 1.02 \times 1.02 [0.668 \sin(\theta_3 - \theta_3) - (-2.272) \cos(\theta_3 - \theta_3)]
\end{aligned}$$

Substituting initial values, $Q_3 = -0.213$.

The Jacobian matrix is:

$$\begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \\ \Delta P_3 \end{bmatrix} = \begin{bmatrix} \partial P_2 / \partial \theta_2 & \partial P_2 / \partial V_2 & \partial P_2 / \partial \theta_3 \\ \partial Q_2 / \partial \theta_2 & \partial Q_2 / \partial V_2 & \partial Q_2 / \partial \theta_3 \\ \partial P_3 / \partial \theta_2 & \partial P_3 / \partial V_2 & \partial P_3 / \partial \theta_3 \end{bmatrix} \begin{bmatrix} \Delta \theta_2 \\ \Delta V_2 \\ \Delta \theta_3 \end{bmatrix}$$

The partial differentials are found by differentiating the equations for P_2 , Q_2 , P_3 , etc.

$$\begin{aligned}
\partial P_2 / \partial \theta_2 &= 1.02 [V_2 (0.474) \sin \theta_2 + 2.45 \cos \theta_2] + 1.02 \left[\begin{array}{l} V_2 (0.668) \sin(\theta_2 - \theta_3) \\ + V_2 (2.297) \cos(\theta_2 - \theta_3) \end{array} \right] \\
&= 4.842
\end{aligned}$$

$$\begin{aligned}
\partial P_2 / \partial \theta_3 &= V_2 \times 1.02 [(-0.668) \sin(\theta_2 - \theta_3) - 2.297 \cos(\theta_2 - \theta_3)] \\
&= -2.343
\end{aligned}$$

$$\begin{aligned}
\partial P_2 / \partial V_2 &= 1.02 [(-0.474) \cos \theta_2 + 2.45 \sin \theta_2] \\
&+ 2V_2 (1.142) + 1.02 [(-0.608) \cos(\theta_2 - \theta_3) + 2.297 \sin(\theta_2 - \theta_3)] \\
&= 1.119
\end{aligned}$$

$$\begin{aligned}
\partial Q_2 / \partial \theta_2 &= 1.02 - V_2 [(0.474) \cos \theta_2 + 2.45 \sin \theta_2] \\
&+ 1.02 [V_2 (-0.608) \cos(\theta_2 - \theta_3) + 2.297 \sin(\theta_2 - \theta_3)] \\
&= -1.1648
\end{aligned}$$

$$\begin{aligned}
\partial Q_2 / \partial V_2 &= 1.02 [-(0.474) \sin \theta_2 - 2.45 \cos \theta_2] \\
&+ 2V_2 (4.28) + 1.02 [(-0.608) \sin(\theta_2 - \theta_3) - 2.297 \cos(\theta_2 - \theta_3)] \\
&= 4.56
\end{aligned}$$

$$\begin{aligned}\partial Q_2 / \partial \theta_3 &= 1.02 V_2 [(0.668) \cos(\theta_2 - \theta_3) - 2.297 \sin(\theta_2 - \theta_3)] \\ &= 0.681\end{aligned}$$

$$\begin{aligned}\partial P_2 / \partial \theta_2 &= 1.02 V_2 [(0.668) \sin(\theta_3 - \theta_2) - 2.297 \cos(\theta_3 - \theta_2)] \\ &= 2.343\end{aligned}$$

$$\begin{aligned}\partial P_3 / \partial \theta_2 &= 1.02 [-(0.668) \cos(\theta_3 - \theta_2) + 2.297 \sin(\theta_3 - \theta_2)] \\ &= 0.681\end{aligned}$$

$$\begin{aligned}\partial P_2 / \partial \theta_3 &= 1.02 [(0.668) \sin(\theta_3 - \theta_2) + 2.297 \cos(\theta_3 - \theta_2)] \\ &= 2.343\end{aligned}$$

Therefore, the Jacobian is

$$J = \begin{vmatrix} 4.842 & 1.119 & -2.343 \\ -1.165 & 4.56 & 0.681 \\ -2.343 & 0.681 & 2.343 \end{vmatrix}$$

The system equations are

$$\begin{vmatrix} \Delta \theta_2^1 \\ \Delta V_2^1 \\ \Delta \theta_3^1 \end{vmatrix} = \begin{vmatrix} 4.842 & 1.119 & -2.343 \\ -1.165 & 4.56 & 0.681 \\ -2.343 & 0.681 & 2.343 \end{vmatrix}^{-1} \begin{vmatrix} -0.25 - (-0.0228) \\ 0.25 - (-0.142) \\ -0.5 - (0.0136) \end{vmatrix}$$

Inverting the Jacobian gives

$$\begin{vmatrix} \Delta \theta_2^1 \\ \Delta V_2^1 \\ \Delta \theta_3^1 \end{vmatrix} = \begin{vmatrix} 0.371 & -0.153 & 0.4152 \\ 0.041 & 0.212 & -0.021 \\ 0.359 & -0.215 & 0.848 \end{vmatrix} \begin{vmatrix} -0.2272 \\ 0.392 \\ -0.5136 \end{vmatrix} = \begin{vmatrix} -0.357 \\ -0.084 \\ -0.601 \end{vmatrix}$$

The new values of voltages and phase angles are

$$\begin{bmatrix} \theta_2^1 \\ V_2^1 \\ \theta_3^1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.357 \\ -0.084 \\ -0.601 \end{bmatrix} = \begin{bmatrix} -0.357 \\ 1.084 \\ -0.729 \end{bmatrix}$$

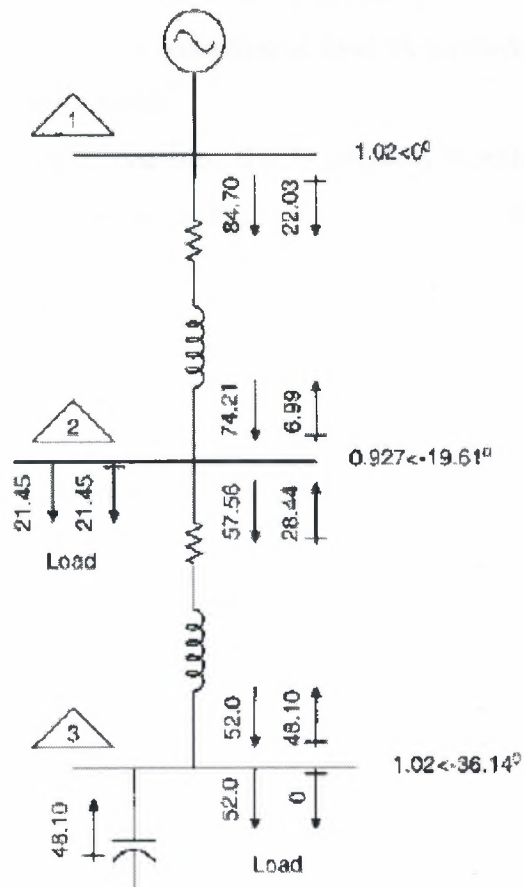


Figure 5.9 Final converged load-flow solution with reactive power injection at PV bus 3

This completes one iteration. Using the new bus voltages and phase angles the power flow is recalculated. Thus, at every iteration, the Jacobian matrix changes and has to be inverted.

In the first iteration, we see that the bus 2 voltage is 8.4% higher than the rated voltage; the angles are in radians. The first iteration is no indication of the final results. The hand calculations, even for a simple three-bus system, become unwieldy. The converged load flow is shown in Figure 5.9. A reactive power injection of 43 Mvar is required at bus 3 to maintain a voltage of 1.02 per unit and supply the required active

power of 0.5 per unit from the source. There is a reactive power loss of 48.65 Mvar in the transmission lines themselves, and the active power loss is 12.22 MW. The bus phase angles are high with respect to the swing bus, and the bus 2 operating voltage is 0.927 per unit, i.e., a voltage drop of 7.3% under load. Thus, even with voltages at the swing bus and bus 3 maintained above rated voltage, the power demand at bus 2 cannot be met and the voltage at this bus dips. The load demand on the system is too high, it is lossy, and requires augmentation or reduction of load. A reactive power injection at bus 2 will give an entirely different result.

If bus 3 is treated as a load bus, the Jacobian is modified by adding a fourth equation of the reactive power at bus 3. In this case the bus 3 voltage dips down to 0.78 per unit, i.e., a voltage drop of 22%; the converged load flow is shown in Figure 5.10. At this lower voltage of 0.78 per unit, bus 3 can support an active load of only 0.3 per unit. This is not an example of a practical system, but it illustrates the importance of reactive power injection, load modeling, and its effect on the bus voltages.

The load demand reduces proportionally with reduction in bus voltages. This is because we have considered a constant impedance type of load, i.e., the load current varies directly with the voltage as the load impedance is held constant. The load types are discussed further.

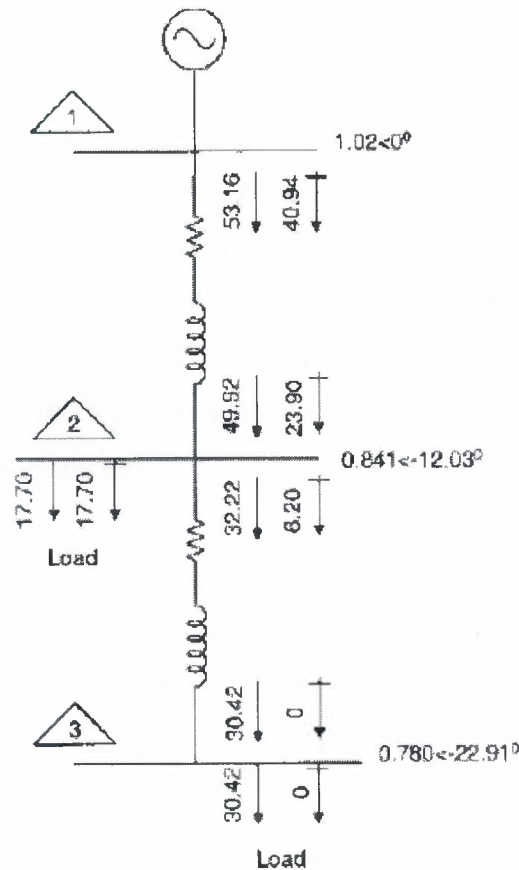


Figure 5.10 Converged load flow with bus 3 treated as a P, Q bus

5.8. Power Flow Program in MATLAB

As stated in previous section, the solution for the non-linear equations formulated in power flow analysis requires iterative methods such as Newton-Raphson iterative method. These methods are usually complex and take long time to compute by hand such as for large systems consisting of hundreds of buses. For convenience, we can write program for Newton-Raphson iterative method, in programming software such as MATLAB, to solve power flow equations precisely and in a short time.

5.8.1. Program Structure

The program is divided in several parts or *routines* and *subroutines* to solve the system. These routines contain necessary functions, commands and variables to solve equations mathematically. Reason for writing program in this way is because if we want to change the parameters of the system we don't have to edit the whole program, we just

edit the specific routine. These routines are written in M-files of MATLAB. The M-file used to start the power flow program is named as *gopflow.m*. The program is run by typing *gopflow* in the *command window*. The necessary data is then collected from other M-files and then computed and the final results are displayed in the command window.

The files that contain initial parameters such as number of buses, types of buses and values related to buses (i.e. load, voltage, real power of generators and line impedances) are *setswing.m*, *setline.m*, *setgen.m* and *setload.m*. The format of the input data to be written in the input matrix is defined in each file. The brief description of each of these files is described below:

- *setswing.m* is for setting the slack bus or swing bus, which is usually assigned as bus 1 and its voltage magnitude and voltage angle (which is usually chosen as 0°)
- *setline.m* is used to set each line's resistance and reactance interconnecting the buses
- *setgen.m* is to specify the generator buses or P, V buses and their appropriate values
- *setload.m* is used to define the load buses or P, Q and the value of load on them

To change the parameters of the system we can just change the required values we want to change or we can describe a new system by entering specific values for that system such as number of buses, number of generators, loads and line parameters etc.

5.8.2. Example solved by MATLAB program

The example solved by this MATLAB program is an IEEE 14 bus test case. The input data used for the system and the output computed by the program are shown below.

Number of devices in the case:

Buses: 14

Generators: 5

Loads: 9

Lines/Transformers: 20

Table 5.2 Bus data for IEEE 14 bus case

Bus Data						
Number	Voltage (p.u.)	Angle (Degrees)	Load MW	Load Mvar	Generator MW	Generator Mvar
1	1	0	0	0	0	0
2	1	0	0	0	55	0
3	1	0	0	0	50.8	0
4	1	0	40	-4	0	0
5	1	0	7	1	0	0
6	1	0	0	0	47	0
7	1	0	0	0	0	0
8	1	0	0	0	38	0
9	1	0	35	15	0	0
10	1	0	20	6	0	0
11	1	0	33	12	0	0
12	1	0	36	12	0	0
13	1	0	28	8	0	0
14	1	0	38	15	0	0

Table 5.3 Line data for IEEE 14 bus case

Line Data					
Line no.	From Bus	To Bus	R (p.u.)	X (p.u.)	B (p.u.)
1	2	1	0.0194	0.0592	0.0528
2	5	1	0.054	0.223	0.0492
3	3	2	0.047	0.198	0.0438
4	4	2	0.0581	0.1763	0.0340
5	5	2	0.0569	0.1739	0.0346
6	4	3	0.067	0.171	0.0128
7	5	4	0.0133	0.0421	0
8	7	4	0	0.2091	0
9	9	4	0	0.5562	0
10	6	5	0	0.252	0
11	11	6	0.095	0.1989	0
12	12	6	0.1229	0.2558	0
13	13	6	0.0661	0.1303	0
14	8	7	0.0756	0.0429	0
15	9	7	0	0.11	0
16	10	9	0.0318	0.0845	0
17	14	9	0.1271	0.2704	0
18	11	10	0.0821	0.1921	0
19	13	12	0.2209	0.1999	0
20	14	13	0.1709	0.348	0

The one-line diagram for the IEEE 14 bus system created by PowerWorldTM simulator is shown in Figure 5.11.

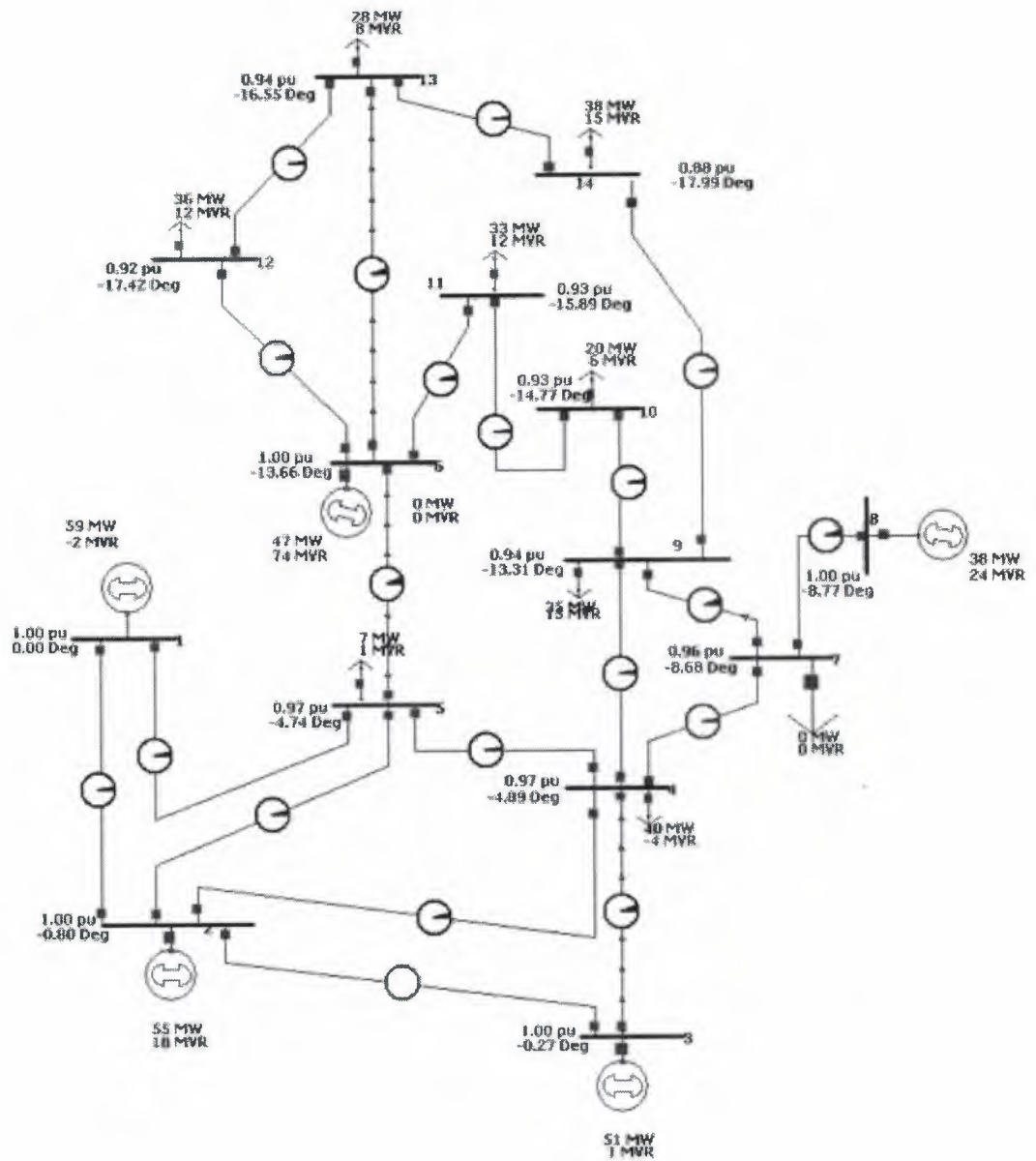


Figure 5.11 One-line diagram for IEEE 14 bus case

The program is solved after 3 iterations, the voltage magnitudes in per unit and voltage angles in degrees are given below as computed by the program.

Table 5.4 Solved data by the program for IEEE 14 bus case

Power Flow Solution						
Bus#	Generation				Load	
	V p.u.	Angle-deg	PG p.u.	QG p.u.	PL p.u.	QL p.u.
1	1.0000	0	0.5852	-0.0325	0	0
2	1.0000	-0.7987	0.5500	0.1531	0	0
3	1.0000	-0.2654	0.5080	-0.0017	0	0
4	0.9707	-4.9141	0	0	0.4000	-0.0400
5	0.9753	-4.7668	0	0	0.0700	0.0100
6	1.0000	-13.6623	0.4700	0.7283	0	0
7	0.9615	-8.6917	0	0	0	0
8	1.0000	-8.7444	0.3800	0.2273	0	0
9	0.9358	-13.3144	0	0	0.3500	0.1500
10	0.9264	-14.7763	0	0	0.2000	0.0600
11	0.9334	-15.8883	0	0	0.3300	0.1200
12	0.9249	-17.4217	0	0	0.3600	0.1200
13	0.9400	-16.5477	0	0	0.2800	0.0800
14	0.8785	-17.9880	0	0	0.3800	0.1500
TOTAL			2.4932	1.0745	2.3700	0.6500

Number of iterations: 3

CONCLUSION

In real or practical world, electric power systems are extremely large and complex consisting of hundreds of thousands of parts and equipments. For the efficient operation and control of power system, it is necessary to get information of the system instantly and accurately. Power flow analysis is the most efficient tool to acquire information of the system operating under normal balanced three-phase steady-state condition. The formulation of power flow equations were carried out in the project for better understanding.

The complexity of the calculation can be seen by the example solved by hand. The hand calculation of power flow equation takes long time and hard working. Using computer programs to simulate power flow save time and energy and give more accurate results. These programs are simple to understand and easy to use with the desire system configurations.

In this project a program for power flow using Newton-Raphson method, written in MATLAB, is considered and the results obtained were quit satisfactory. It is concluded that the use of computer programs for power flow analysis saves a lot of time and also it is quit accurate, reliable and efficient.

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