NEAR EAST UNIVERSITY

Faculty of Engineering

Department of Electrical and Electronic Engineering

ADAPTIVE EQUALIZER

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EE- 400

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ABSTRACT

Any electrical communication system consists of a transmitter, a channel and a receiver. Transmitters and receivers may include filtering for various purposes. Bandlimited channels also act like a filter causing the received pulses overlap each other. As a result, the tail of one pulse smears into adjacent pulse. This is called intersymbol interference.

Intersymbol interference disturbs the transmitted signal. Various ways exist for combating this interference and are investigated in this report.

One way is to use raised cosine shape filter. This is not practically realizable therefore the use of channel equalizer is preferred. Zero forcing equalizer is a linear equalizer that attempts to reduce the effects of intersymbol interference. It has the disadvantage that it also amplifies the noise present in the channel. Another linear equalizer is an adaptive equalizer using the least mean squares algorithm. This report simulates a communication system using an adaptive equalizer with the least mean squares algorithm in Matlab and examines the effects of various parameters, such as the equalizer step size. Nonlinear equalizers such as the decision feed back equalizer also exist and are generally better performing than linear equalizers against intersymbol interference.

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INTRODUCTION

Bandlimited communication channels and wireless channels suffer from the effects of intersymbol interference. Intersymbol interference occurs when transmitted pulses are distorted in the channel when they interfere with each other. A few ways exist for eliminating intersymbol interference including pulse shaping and channel equalization.

This report discusses various types of equalizers. It particularly investigates the linear adaptive equalizer using the least mean squares algorithm. Using MATLAB simulation, the effects of different parameters on the equalizer adaptation is analyzed. It is shown that reducing the equalizer step size reduces the mean square error but it increases the time it takes to converge. Changing the number of iterations change the amount of variation of mean square error during convergence. Using a channel with more severe intersymbol interference increases the mean square error.

The first chapter describes the elements of a digital communication system, and the noise within the system.

Chapter two describes digital transmission through bandlimited additive white Gaussian noise channels. It explains digital pulse amplitude modulated transmission through bandlimited baseband channels and also digital transmission through bandlimited bandpass channels.

Chapter three details intersymbol interference, pulse shaping to reduce intersymbol interference, and briefly introduces channel equalization.

Chapter four explains the three kinds of equalizers, the linear zero-forcing-equalizer, the adaptive equalizer, and the feedback equalizer.

Chapter five gives the results obtained through simulation. The MATLAB code is included in the appendix section.

CHAPTER 1

BASICS OF AN ELECTRICAL COMMUNICATION SYSTEM

1.1 Elements of An Electrical Communication System

Electrical communication systems are designed to send messages or information from a source that generates the messages to one or more destinations. In general, a communication system can be represented by the functional block diagram shown in Figure 1.1.

The information generated by the source may be of the form of voice (speech source), a picture (image source), or plain text in some particular language, such as English, Japanese, German, French, etc. An essential feature of any source that generates information is that its output is described in probabilistic terms; that is, the output of a source is not deterministic. Otherwise, there would be no need to transmit the message.

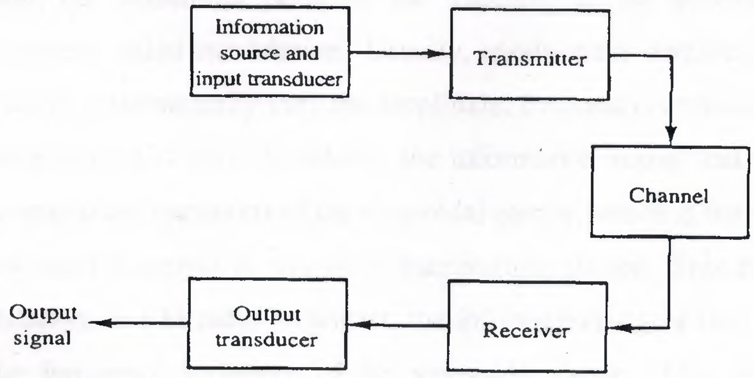


FIGURE 1.1. Functional block diagram of a communication system.

A transducer is usually required to convert the output of a source into an electrical signal that is suitable for transmission. For example, a microphone serves as the transducer that converts an acoustic speech signal into an electrical signal, and a video camera converts an image into an electrical signal. At the destination, a similar transducer is required to convert electrical signals that are received into a form that is suitable for the user; for example, acoustic signals, images, etc.

The heart of the communication system consists of three basic parts, namely, t he transmitter, the channel, and the receiver. The functions performed by these three elements are described below.

1.1.1 The Transmitter.

The transmitter converts the electrical signal into a form that is suitable for transmission through the physical channel or transmission medium. For example, in radio and TV broadcast, the Federal Communications Commission (FCC) specifies the frequency range for each transmitting station. Hence, the transmitter must translate the information signal to be transmitted into the appropriate frequency range that matches the frequency allocation assigned to the transmitter. Thus, signals transmitted by multiple radio stations do not interfere with one another. Similar functions are performed in telephone communication systems, where the electrical speech signals from many users are transmitted over the same wire.

In general, the transmitter performs the matching of the message signal to the channel by a process called modulation. Usually, modulation involves the use of the information signal to systematically vary the amplitude, frequency, or phase of a sinusoidal carrier. For example, in AM radio broadcast, the information signal that is transmitted is contained in the amplitude variations of the sinusoidal carrier, which is the center frequency in the frequency band allocated to the radio transmitting station. This is an example of amplitude modulation. In FM radio broadcast, the information signal that is transmitted is contained in the frequency variations of the sinusoidal carrier. This is an example of frequency modulation. Phase modulation (PM) is yet a third method for impressing the information signal on a sinusoidal carrier.

In general, carrier modulation such as AM, FM, and PM is performed at the transmitter, as indicated above, to convert the information signal to a form that matches the characteristics of the channel. Thus, through the process of modulation, the information signal is translated in frequency to match the allocation of the channel. The choice of the type of modulation is based on several factors, such as the amount of bandwidth allocated, the types of noise and interference that the signal encounters in transmission over the channel, and the electronic devices that are available for signal amplification prior to transmission. In any case, the modulation process makes it possible to accommodate the transmission of multiple messages from many users over the same physical channel.

In addition to modulation, other functions that are usually performed at the transmitter are filtering of the information-bearing signal, amplification of the modulated signal, and in the case of wireless transmission, radiation of the signal by means of a transmitting antenna.

1.1.2 The Channel

The communications channel is the physical medium that is used to send the signal from the transmitter to the receiver. In wireless transmission, the channel is usually the atmosphere (free space). On the other hand, telephone channels usually employ a variety of physical media, including wirelines, optical fiber cables, and wireless (microwave radio). Whatever the physical medium for signal transmission, the essential feature is that the transmitted signal is corrupted in a random manner by a variety of possible mechanisms. The most common form of signal degradation comes in the form of additive noise, which is generated at the front end of the receiver, where signal amplification is performed. This noise is often called thermal noise. In wireless transmission, additional additive disturbances are man-made noise and atmospheric noise picked up by a receiving antenna. Automobile ignition noise is an example of man-made noise, and electrical lightning discharges from thunderstorms is an example of atmospheric noise. Interference from other users of the channel is another form of additive noise that often arises in both wireless and wireline communication systems.

In some radio communication channels, such as the ionospheric channel that is used for long-range, short-wave radio transmission, another form of signal degradation is multipath propagation. Such signal distortion is characterized as a nonadditive signal disturbance which manifests itself as time variations in the signal amplitude, usually called fading.

Both additive and nonadditive signal distortions are usually characterized as random phenomena and described in statistical terms. The effect of these signal distortions must be taken into account in the design of the communication system.

The system designer works with mathematical models that statistically characterize the signal distortion encountered on physical channels. Often, the statistical description that is used in a mathematical model is a result of actual empirical measurements obtained from experiments involving signal transmission over such channels. In such case, there is a physical justification for the mathematical model used in the design of communication systems. On the other hand, in some communication system designs, the statistical characteristics of the channel may vary significantly with time. In such cases, the system designer may design a communication system that is robust to the variety of signal distortions. This can be accomplished by having the system adapt some of its parameters to the channel distortion encountered.

1.1.3 The Receiver

The function of the receiver is to recover the message signal contained in the received signal. If the message signal is transmitted by carrier modulation, the receiver performs carrier demodulation to extract the message from the sinusoidal carrier. Since the signal demodulation is performed in the presence of additive noise and possibly other signal distortion, the demodulated message signal is generally degraded to some extent by the presence of these distortions in the received signal. As we shall see, the fidelity of the received message signal is a function of the type of modulation, the strength of the additive noise, the type and strength of any other additive interference, and the type of any nonadditive interference.

Besides performing the primary function of signal demodulation, the receiver also performs a number of peripheral functions, including signal filtering and noise suppression.

1.2 Digital Communication System

Up to this point, it was described that if an electrical communication system is in rather broad terms based on the implicit assumption that the message signal is a continuous time-varying waveform, such continuous-time signal waveforms are referred as analog signals and the corresponding information sources that produce such signals are referred as analog sources. Analog signals can be transmitted directly via carrier modulation over the communication channel and demodulated accordingly at the receiver. Such a communication system is called an analog communication system.

Alternatively, an analog source output may be converted into a digital form and the message can be transmitted via digital modulation and demodulated as a digital signal at the receiver. There are some potential advantages to transmit an analog signal by means of digital modulation. The most important reason is that signal fidelity is better controlled through digital transmission than analog transmission. In particular, digital transmission allows us to regenerate the digital signal in long-distance transmission, thus eliminating effects of noise at each regeneration point. In contrast, the noise added in analog transmission is amplified along with the signal when amplifiers are used periodically to boost the signal level in long-distance transmission. Another reason for choosing digital transmission over analog is that the analog message signal may be highly redundant. With digital processing, redundancy may be removed prior to modulation, thus conserving channel bandwidth. Yet a third reason may be that digital communication systems are often cheaper to implement.

In some applications, the information to be transmitted is inherently digital, e.g., in the form of English text, computer data, etc. In such cases, the information source that generates the data is called a discrete (digital) source.

In a digital communication system, the functional operations performed at the transmitter and receiver must be expanded to include message signal discretization at the transmitter and message signal synthesis or interpolation at the receiver. Additional functions include redundancy removal, and channel coding and decoding.

Figure 1.2 illustrates the functional diagram and the basic elements of a digital communication system. The source output may be either an analog signal, such as audio or video signal, or a digital signal, such as the output of a teletype machine which is discrete in time and has a finite number of output characters.

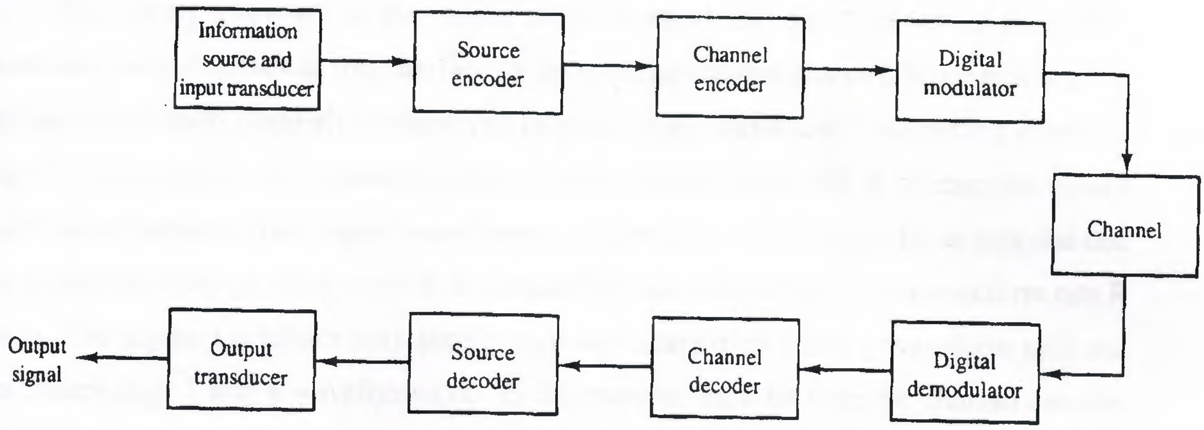


FIGURE 1.2. Basic elementes of a digital communication system.

In a digital communication system, the messages produced by the source are usually converted into a sequence of binary digits. Ideally, it is more likely to represent the source output (message) by as few binary digits as possible. In other words, communication engineers seek an efficient representation of the source output that results in little or no redundancy. The process of efficiently converting the output of either an analog or a digital source into a sequence of binary digits is called source encoding or data compression. The sequence of binary digits from the source encoder, which known as the information sequence, is passed to the channel encoder. The purpose of the channel encoder is to introduce in a controlled manner some redundancy in the binary information sequence which can be used at the receiver to overcome the effects of noise and interference encountered in the transmission of the signal through the channel. Thus, the added redundancy serves to increase the reliability of the received data and improves the fidelity of the received signal. In effect, redundancy in the information sequence aids the receiver in decoding the desired information sequence. For example, a (trivial) form of encoding of the binary information sequence is simply to repeat each binary digit m times, where m is some positive integer. More sophisticated (nontrivial) encoding involves taking k information bits at a time and mapping each k-bit sequence into a unique n-bit sequence, called a code word. The amount of redundancy introduced by encoding the data in this manner is measured by the ratio n/k. The reciprocal of this ratio, namely, k/n, is called the rate of the code or, simply, the code rate.

The binary sequence at the output of the channel encoder is passed to the digital modulator, which serves as the interface to the communications channel. Since nearly all of the communication channels encountered in practice are capable of transmitting electrical signals (waveforms), the primary purpose of the digital modulator is to map the binary information sequence into signal waveforms. To elaborate on this point, let us suppose that the coded information sequence is to be transmitted one bit at a time at some uniform rate R bits/s. The digital modulator may simply map the binary digit 0 into a waveform $s_0(t)$ and the binary digit 1 into a waveform $s_1(t)$. In this manner, each bit from the channel encoder is transmitted separately. We call this binary modulation. Alternatively, the modulator may transmit b coded information bits at a time by using $M = 2^b$ distinct waveforms $s_i(t)$, $i = 0, 1, ..., M - 1$, one waveform for each of the 2^b possible b-bit sequences. This is called M-ary modulation ($M > 2$). Note that a new b-bit sequence enters the modulator every b/R seconds. Hence, when the channel bit rate R is fixed, the amount of time available to transmit one of the M waveforms corresponding to a b-bit sequence is b times the time period in a system that uses binary modulation.

At the receiving end of a digital communications system, the digital demodulator processes the channel-corrupted transmitted waveform and reduces each waveform to a single number that represents an estimate of the transmitted data symbol (binary or M-ary). For example, when binary modulation is used, the demodulator may process the received waveform and decide on whether the transmitted bit is a 0 or 1. In such a case, this is said to be the demodulator has made a binary decision. As one alternative, the demodulator may make a ternary decision; that is, it decides that the transmitted bit is either a 0 or 1 or it makes no decision at all, depending on the apparent quality of the received signal. When no decision is made on a particular bit, it is called that the demodulator has inserted an erasure in the demodulated data. Using the redundancy in the transmitted data, the decoder attempts to fill in the positions where erasures occurred. Viewing the decision process performed by the demodulator as a form of quantization, we observe that binary and ternary decisions are special cases of a demodulator that quantizes to Q levels, where $Q \geq 2$. In general, if the digital communications system employs M-ary modulation, where $m = 0, 1, ..., M - 1$

represent the M possible transmitted symbols, each corresponding to $k = \log_2 M$ bits, the demodulator may make a Q-ary decision, where $Q \geq M$. In the extreme case where no quantization is performed, $Q = \infty$.

When there is no redundancy in the transmitted information, the demodulator must decide which of the M waveforms was transmitted in any given time interval. Consequently, $Q = M$, and since there is no redundancy in the transmitted information, no discrete channel decoder is used following the demodulator. On the other hand, when there is redundancy introduced by a discrete channel encoder at the transmitter, the Q-ary output from the demodulator occurring every k/R seconds is fed to the decoder, which attempts to reconstruct the original information sequence from knowledge of the code used by the channel encoder and the redundancy contained in the received data.

A measure of how well the demodulator and encoder perform is the frequency with which errors occur in the decoded sequence. More precisely, the average probability of a bit-error at the output of the decoder is a measure of the performance of the demodulator-decoder combination. In general, the probability of error is a function of the code characteristics, the types of waveforms used to transmit the information over the channel, the transmitter power, the characteristics of the channel (i.e., the amount of noise), the nature of the interference, etc., and the method of demodulation and decoding.

As a final step, when an analog output is desired, the source decoder accepts the output sequence from the channel decoder, and from knowledge of the source encoding method used, attempts to reconstruct the original signal from the source. Due to channel decoding errors and possible distortion introduced by the source encoder and, perhaps, the source decoder, the signal at the output of the source decoder is an approximation to the original source output. The difference or some function of the difference between the original signal and the reconstructed signal is a measure of the distortion introduced by the digital communications system.

1.3 Noise in Communication Systems

The term noise refers to unwanted electrical signals that are always present in electrical systems. The presence of noise superimposed on a signal tends to obscure or mask the signal; it limits the receiver's ability to make correct symbol decisions, and thereby limits the rate of information transmission. Noise arises from a variety of sources, both man-made and natural. Man-made noise includes such sources as spark-plug ignition noise, switching transients, and other radiating electromagnetic signals. Natural noise includes electrical circuit and component noise, atmospheric disturbances, and galactic sources.

Good engineering design can eliminate much of the noise or its undesirable effect through filtering, shielding, the choice of modulation, and the selection of an optimum receiver site. For example, sensitive radio astronomy measurements are typically located at remote desert locations, far from man-made noise sources. However, there is one natural source of noise, called thermal or Johnson noise, that cannot be eliminated. Thermal noise is caused by the thermal motion of electrons in all dissipative components-resistors, wires, and so on. The same electrons that are responsible for electrical conduction are also responsible for thermal noise.

We can describe thermal noise as a zero-mean Gaussian random process. A Gaussian process, n(t), is a random function whose value, n, at any arbitrary time, t, is statistically characterized by the Gaussian probability density function, $p(n)$:

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{n}{\sigma}\right)^2\right] \tag{1.1}$$

where σ^2 is the variance of n. The normalized or standardized Gaussian density function of a zero-mean process is obtained by assuming that $\sigma = 1$. This normalized pdf is shown sketched in Figure 1.3

We will often represent a random signal as the sum of a Gaussian noise rand om variable and a dc signal:

$$z = a + n$$

where z is the random signal, a the dc component, and n the Gaussian noise random variable. The pdf p(z) is then expressed as

$$p(z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z-a}{\sigma}\right)^2\right] \tag{1.2}$$

where, as before, σ^2 is the variance of n. The Gaussian distribution is often used as the system noise model because of a theorem, called the central limit theorem [1], which states that under very general conditions the probability distribution of the sum of j statistically independent random variables approaches the Gaussian distribution as $j \rightarrow \infty$, no matter what the individual distribution functions may be. Therefore, even though individual noise mechanisms might have other than

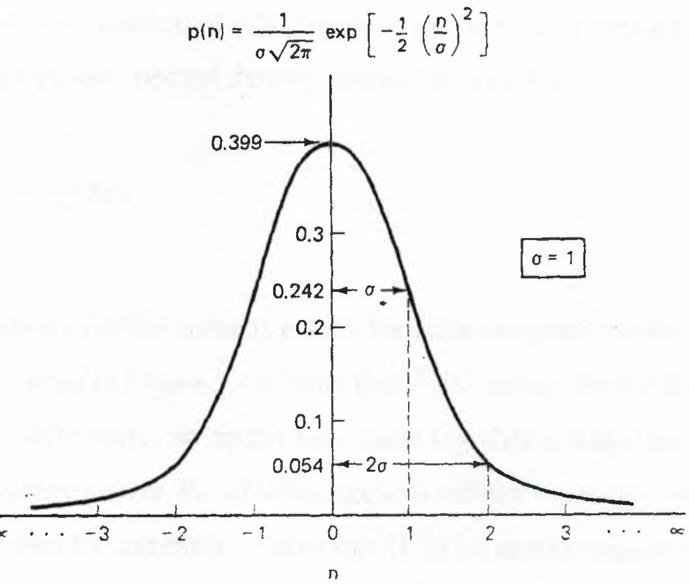


Figure 1.3. Normalized ($\sigma = 1$) Gaussian probability density function.

Gaussian distributions, the aggregate of any such mechanisms will tend toward the Gaussian distribution.

White Noise:

The primary spectral characteristic of thermal noise is that its power spectral density is the same for all frequencies of interest in most communication systems; in other words, a thermal noise source emanates an equal amount of noise power per unit bandwidth at all frequencies-from dc to about 10^{12} Hz. Therefore, a simple model for thermal noise assumes that its power spectral density $G_n(f)$ is flat for all frequencies, as shown in Figure 1.4(a), and is denoted as follows:

$$G_n(f) = \frac{N_0}{2} \quad \text{watts/hertz} \tag{1.3}$$

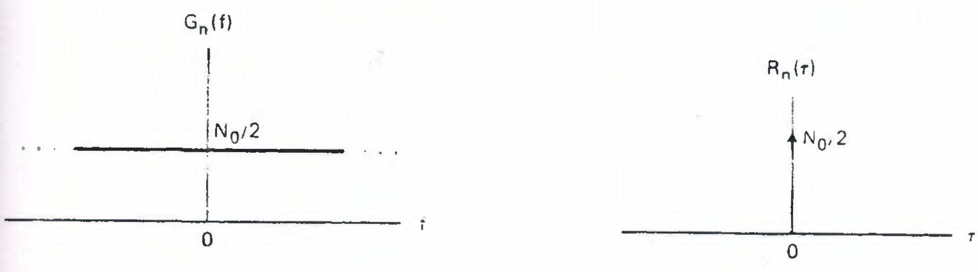
where the factor of 2 is included to indicate that G(f) is a two-sided power spectral density. When the noise power has such a uniform spectral density, we refer to it as white noise.The adjective "white" is used in the sense that white light contains equal amounts of all frequencies within the visible band of electromagnetic radiation.

The autocorrelation function of white noise is given by the inverse Fourier transform of the noise power spectral density denoted as follows:

$$R_n(\tau) = \mathcal{F}^{-1}\{G_n(f)\} = \frac{N_0}{2}\,\delta(\tau) \tag{1.4}$$

Thus the autocorrelation of white noise is a delta function weighted by the factor $N_0/2$ and occurring at $\tau = 0$, as seen in Figure 1.4b. Note that $R_n(\tau)$ is zero for $\tau \neq 0$ that is, any two different samples of white noise, no matter how close together in time they are taken, are uncorrelated. The average power, P_n, of white noise is infinite because its bandwidth is infinite. This can be seen by combining Equations (1.3) by another equation to yield.

$$P_n = \int_{-\infty}^{\infty} \frac{N_0}{2}\, df = \infty \tag{1.5}$$



(a) (b)

Figure 2.4 a) Power spectral density of white noise. b) Autocorrelation function of white noise.

Although white noise is a useful abstraction, no noise process can truly be white. However, the noise encountered in many real systems can be assumed to be approximately white. We can only observe such noise after it has passed through a real system which will have a finite bandwidth. Thus, as long as the bandwidth of the noise is appreciably larger than that of the system, the noise can he considered to have an infinite bandwidth.

The delta function in Equation (1.4), means that the noise signal, n(t). is totally decorrelated from its time-shifted version. for any $\tau > 0$. Equation (1.4) indicates that any two different samples of a white noise process are uncorrelated. Since thermal noise is a Gaussian process and the samples are uncorrelated. The noise samples are also independent [1]. Therefore, the effect on the detection process of a channel with additive white Gaussian noise(AWGN) is that the noise affects each transmitter sample independently. Such a channel is called a memoryless channel. The term 'additive' means that the noise is simply superimposed or added to the signal—that there are multiplicative mechanisms at work.

Since thermal noise is present in all communication systems and is the prominent noise source for most systems. the thermal noise characteristics—additive, white, and Gaussian—are most often used to model the noise in communication systems. Since zero-mean Gaussian noise is completely characterized by its variance, this model is particularly simple to use in the detection of signals and in the design of optimum receivers. In this report we shall assume, unless otherwise stated, that the system is corrupted by additive zero-mean white Gaussian noise, even though this is sometimes an oversimplification.

CHAPTER 2

TRANSMISSION THROUGH BANDLIMITED AWGN CHANNELS

2.1 Digital Transmission Through Bandlimited AWGN Channels

Digital communication over a bandlimited channel is modeled as a linear filter with a bandwidth limitation. Bandlimited channels most frequently encountered in practice are telephone channels, microwave LOS radio channels, satellite channels, and underwater acoustic channels.

In general, a linear filter channel imposes more stringent requirements on the design of modulation signals. Specifically, the transmitted signals must be designed to satisfy the bandwidth constraint imposed by the channel. The bandwidth constraint generally precludes the use of rectangular pulses at the output of the modulator. Instead, the transmitted signals must be shaped to restrict their bandwidth to that available on the channel. The design of bandlimited signals is one of the topics treated in this section. We will see that a linear filter channel distorts the transmitted signal. The channel distortion results in intersymbol interference at the output of the demodulator and leads to an increase in the probability of error at the detector. Devices or methods for correcting or undoing the channel distortion, called channel equalizers, are then described.

2.2 Digital Transmission Through Bandlimited Channels

A bandlimited channel such as a telephone wireline is characterized as a linear filter with impulse response c(t) and frequency response C(f) where

$$C(f) = \int_{-\infty}^{\infty} c(t)e^{-j2\pi ft}\,dt \tag{2.1}$$

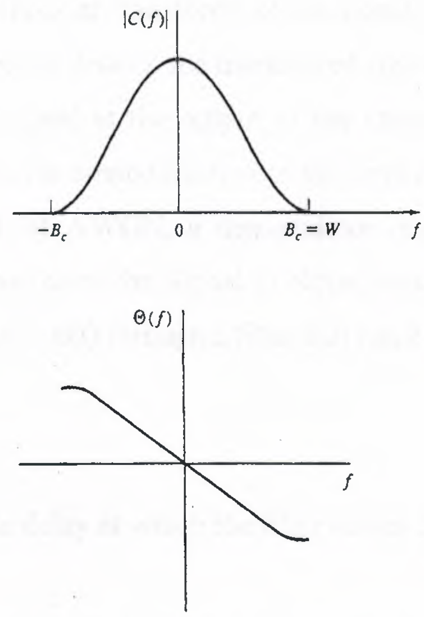


Figure 2.1. Magnitude and phase responses of bandlimited channel.

If the channel is a baseband channel that is bandlimited to B Hz, then C(f) = 0 for $|f| > B_c$. Any frequency components at the input to the channel that are higher than B Hz will not be passed by the channel. For this reason, we consider the design of signals for transmission through the channel that are bandlimited to $W = B_c$ Hz, as shown in Figure 2.1. Henceforth, We will denote the bandwidth limitation of the signal and the channel.

Now, suppose that the input to a bandlimited channel is a signal waveform $g_T(t)$. Then, the response of the channel is the convolution of $g_T(t)$ with c(t); i.e.,

$$h(t) = \int_{-\infty}^{\infty} c(\tau) g_T(t - \tau)\, d\tau = c(t) \star g_T(t) \tag{2.2}$$

or, when expressed in the frequency domain, we have

$$H(f) = C(f) G_T(f) \tag{2.3}$$

where $G_T(f)$ is the spectrum (Fourier transform) of the signal g(t) and H(f) is the spectrum of h(t). Thus, the channel alters or distorts the transmitted signal $g_T(t)$.

Let us assume that the signal at the output of the channel is corrupted by AWGN. Then, the signal at the input to the demodulator is of the form h(t) + n(t), where n(t) denotes the AWGN. In the presence of AWGN, a demodulator that employs a filter which is matched to the signal h(t) maximizes the Signal to Noise Ratio at its output. Therefore, let us pass the received signal h(t) + n(t) through a filter that has a frequency response

$$G_R(f) = H^*(f)\, e^{-j2\pi f t_0} \tag{2.4}$$

where t_0 is some nominal time delay at which the filter output is sampled.

2.3 Digital PAM Transmission Through Bandlimited Baseband Channels

Let us consider the baseband Pulse Amplitude Modulated communication system illustrated by the functional block diagram in Figure 2.2. The system consists of a transmitting filter having an impulse response g(t), the linear filter channel with AWGN, a receiving filter with impulse response $g_R(t)$, a sampler that periodically samples the output of the receiving filter, and a symbol detector. The sampler requires the extraction of a timing signal from the received signal. This timing signal serves as a clock that specifies the appropriate time instants for sampling the output of the receiving filter.

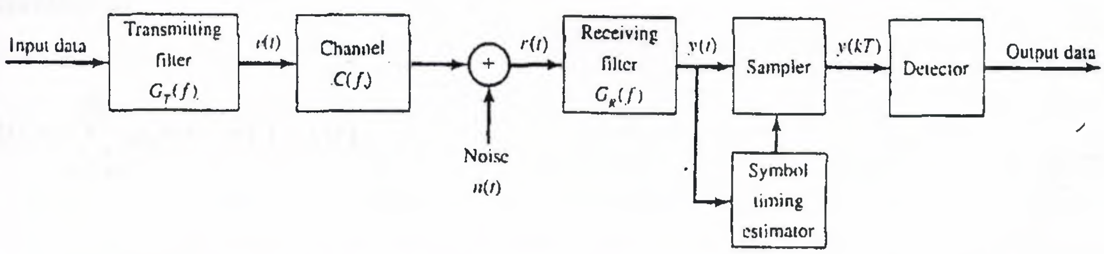


Figure 2.2. Block diagram of digital PAM system.

First we consider digital communications by means of M-ary PAM. Hence, the input binary data sequence is subdivided into k-bit symbols and each symbol is mapped into a corresponding amplitude level that amplitude modulates the output of the transmitting filter..The baseband signal at the output of the transmitting filter (the input to the channel) may be expressed as

$$v(t) = \sum_{n=-\infty}^{\infty} a_n g_T(t - nT) \tag{2.5}$$

where $T = k/R_b$ is the symbol interval ($1/T = R_b/k$ is the symbol rate), R_b is the bit rate, and $\{a_n\}$ is a sequence of amplitude levels corresponding to the sequence of k-bit blocks of information bits.

The channel output, which is the received signal at the demodulator, may be expressed as

$$r(t) = \sum_{n=-\infty}^{\infty} a_n h(t - nT) + n(t) \tag{2.6}$$

where h(t) is the impulse response of the cascade of the transmitting filter and the channel; i.e., $h(t) = c(t) * g_T(t)$, c(t) is the impulse response of the channel, and n(t) represents the AWGN.

The received signal is passed through a linear receiving filter with impulse response $g_R(t)$ and frequency response $G_R(f)$. If $g_R(t)$ is matched to h(t) then its output SNR is a maximum at the proper sampling instant. The output of the receiving filter may be expressed as

$$y(t) = \sum_{n=-\infty}^{\infty} a_n x(t - nT) + v(t) \tag{2.7}$$

where $x(t) = h(t) * g_R(t) = g_T(t) * c(t) * g_R(t)$ and $v(t) = n(t)*g(t)$ denotes the additive noise at the output of the receiving filter.

To recover the information symbols $\{a_n\}$, the output of the receiving filter is sampled periodically, every T seconds. Thus, the sampler produces

$$y(mT) = \sum_{n=-\infty}^{\infty} a_n x(mT - nT) + v(mT) \tag{2.8}$$

or, equivalently,

$$\begin{aligned} y_m &= \sum_{n=-\infty}^{\infty} a_n x_{m-n} + v_m \\ &= x_0 a_m + \sum_{n \neq m} a_n x_{m-n} + v_m \end{aligned} \tag{2.9}$$

where $x_m = x(mT)$, $v_m = v(mT)$, and m= 0, ±1, ±2. A timing signal extracted from the received signal as described in Section 7.8 is used as a clock for sampling the received signal.

The first term on the right-hand side of Equation (2.9) is the desired symbol a_n scaled by the gain parameter x_0. When the receiving filter is matched to the received signal h(t), the scale factor is

$$\begin{aligned} x_0 &= \int_{-\infty}^{\infty} h^2(t)\,dt = \int_{-\infty}^{\infty} |H(f)|^2\,df \\ &= \int_{-W}^{W} |G_T(f)|^2 |C(f)|^2\,df \equiv \mathcal{E}_h \end{aligned} \tag{2.10}$$

as indicated by the development of Equation (2.4), The second term on the RHS of Equation (2.9) represents the effect of the other symbols at the sampling instant t = mT, called the intersymbol interference(ISI). In general, ISI causes a degradation in the performance of the digital communication system. Finally, the third term, v_m, that represents the additive noise, is a zero-mean Gaussian random variable with variance $\sigma_v^2 = N_0\mathcal{E}_h/2$,

By appropriate design of the transmitting and receiving filters, it is possible to satisfy the condition $x_n = 0$ for $n \neq 0$, so that the ISI term vanishes. In the case, the only term that can cause errors in the received digital sequenceis the additive noise.

2.4 Digital Transmission Through Bandlimited Bandpass Channels

The development given in the last Section for baseband PAM is easily extended to carrier modulation via PAM, Quadrature Amplitude Modulation, and Phase Shift Keying. In a carrier-amplitude modulated signal, the baseband PAM given by v(t) in Equation (2.5) modulates the carrier, so that the transmitted signal u(t) is simply

$$u(t) = v(t)\cos 2\pi f_c t \tag{2.11}$$

thus, the baseband signal v(t) is shifted in frequency by f_c. A QAM signal is a bandpass signal which, in its simplest form, may be viewed as two amplitude-modulated carrier signals in phase quadrature. That is, the QAM signal may be expressed as

$$u(t) = v_c(t)\cos 2\pi f_c t + v_s(t)\sin 2\pi f_c t \tag{2.12}$$

where

$$\begin{aligned} v_c(t) &= \sum_{n=-\infty}^{\infty} a_{nc} g_T(t - nT) \\ v_s(t) &= \sum_{n=-\infty}^{\infty} a_{ns} g_T(t - nT) \end{aligned} \tag{2.13}$$

and $\{a_{nc}\}$ and $\{a_{ns}\}$ are the two sequences of amplitudes carried on the two quadrature carriers. A more compact mathematical representation of the baseband signal is the equivalent complex-valued baseband signal

$$\begin{aligned} v(t) &= v_c(t) - j v_s(t) \\ &= \sum_{n=-\infty}^{\infty} (a_{nc} - j a_{ns}) g_T(t - nT) \end{aligned}$$

$$= \sum_{n=-\infty}^{\infty} a_n g_T(t - nT) \tag{2.14}$$

where the sequence $\{a_n = a_{nc} - j\ a_{ns})$ is now a complex-valued sequence representing the signal points from the QAM signal constellation. The corresponding bandpass QAM signal u (t) may also be represented as

$$u(t) = \mathrm{Re}[v(t)e^{j2\pi f_c t}] \tag{2.15}$$

In a similar manner, we can represent a digital carrier-phase modulated signal as in Equation (2.15), where the equivalent baseband signal is

$$v(t) = \sum_{n=-\infty}^{\infty} a_n g_T(t - nT) \tag{2.16}$$

and the sequence {a} takes the value from the set of possible (phase) values $\{e^{-j2\pi m/M}$, m = 0, 1,..., M -1}. Thus, all three carrier-modulated signals, PAM, QAM, and PSK can be represented as in Equations (2.15) and (2.16), where the only difference is in the values taken by the transmitted sequence $\{a_n\}$.

The signal v(t) given by Equation (2.16) is called the equivalent lowpass signal. In the case of QAM and PSK, this equivalent Iowpass signal is a baseband signal which is complex-valued because the information-bearing sequence $\{a_n\}$ is complex-valued. In the case of PAM, v(t) is a real-valued baseband signal.

When transmitted through the bandpass channel, the received bandpass signal may be represented as

$$w(t) = \mathrm{Re}[r(t)e^{j2\pi f_c t}] \tag{2.17}$$

where r(t) is the equivalent lowpass (baseband) signal, which may be expressed as

$$r(t) = \sum_{n=-\infty}^{\infty} a_n h(t - nT) + n(t) \tag{2.18}$$

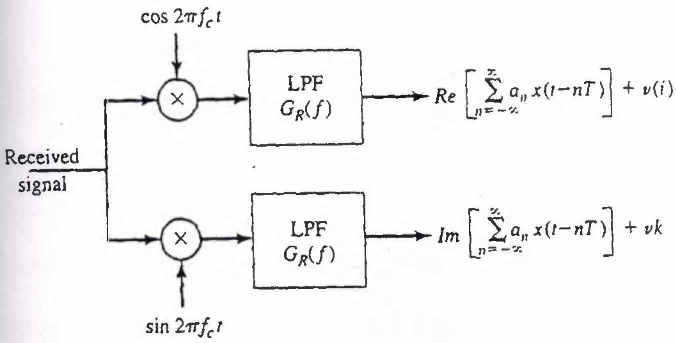


Figure 2.3. Conversion of the bandpass received signal to baseband.

and where, as in the case of baseband transmission, h(t) is the impulse response of the cascade of the transmitting filter and the channel; i.e., $h(t) = c(t) * g_T(t)$, where c(t) is the impulse response of the equivalent lowpass channel and n(t) represents the additive Gaussian noise expressed as an equivalent lowpass (baseband) noise.

The received bandpass signal can be converted to a baseband signal by multiplying w(t) with the quadrature carrier signals cos $(2\pi f_c t)$ and sin$(2\pi f_c t)$ and eliminating the double frequency terms by passing the two quadrature components through separate lowpass filters, as shown in Figure 2.3. Each one of the lowpass filters is assumed to have an impulse response $g_R(t)$. Hence, we can represent the two quadrature components at the outputs of these lowpass filters as an equivalent complex-valued signal of the form

$$y(t) = \sum_{n=-\infty}^{\infty} a_n x(t - nT) + \nu(t) \tag{2.19}$$

which is identical to the form given by Equation (2.7) for the real baseband signal. Consequently, the signal design problem for bandpass signals is basically the same as that described in the last Section for baseband signals.

CHAPTER 3

CHANNEL DISTORTION AND INTERSYMBOL INTERFERENCE

3.1 Intersymbol Interference

Figure 3.1a highlights the major filtering aspects of a typical baseband digital system. There are circuit reactances throughout the system-in the transmitter, in the receiver, and in the channel. The pulses at the input might be impulse-like samples, or flat-top samples. In either case, they are low-pass filtered at the transmitter to confine them to some desired bandwidth. Channel reactances can cause amplitude and phase variations that distort the pulses. The receiving filter, called the equalizing filter, should be configured to compensate for the distortion caused by the transmitter and the channel [2]. In a binary system with a

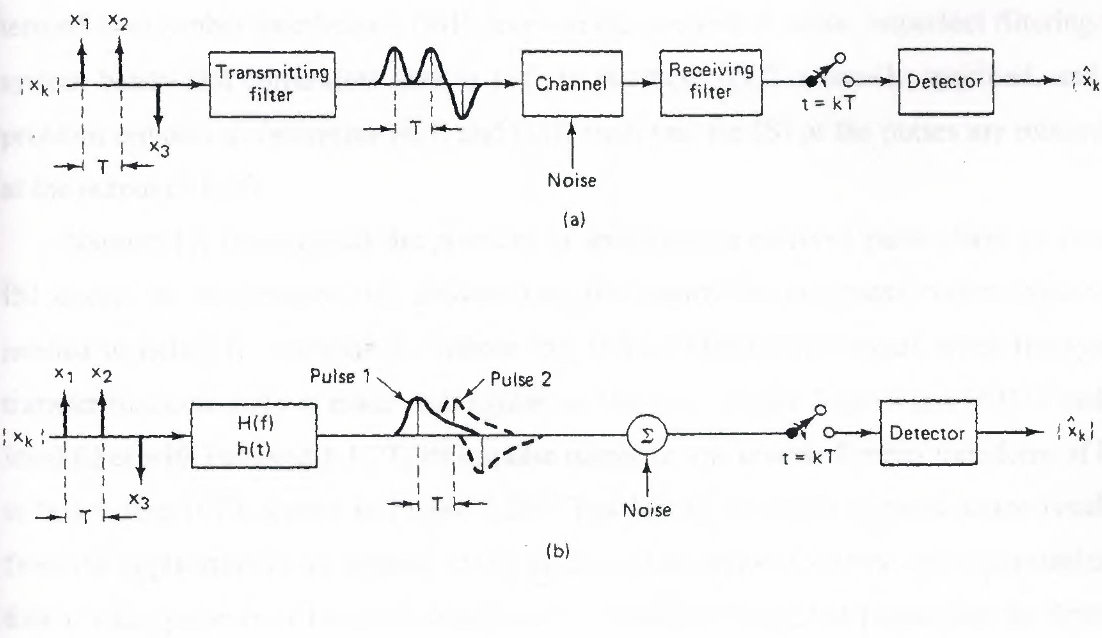


Figure 3.1 Intersymbol interference in the detection process. (a) Typical baseband digital system. (b) Equivalent model.

commonly used pulse code modulation format, such as Non Return to Zero-Level. the detector makes symbol decisions by comparing the received bipolar pulses to a threshold; for example, the detector decides that a binary one was sent if the received pulse is positive, and that a binary zero was sent if the received pulse is negative. Figure 3.1b illustrates a convenient model or the system, lumping all the filtering effects into one overall equipment system transfer function, H(f):

$$H(f) = H_t(f)H_c(f)H_r(f) \tag{3.1}$$

where $H_t(f)$ characterizes the transmitting filter, H(f) the filtering within the channel, and $H_r(f)$ the receiving or equalizing filter. The characteristic H(f), then, represents the composite system transfer function due to all of the filtering at various locations throughout the transmitter/channel/receiver chain. Due to the effects of system filtering, the received pulses overlap one another as shown in Figure 3.1b; the tail of one pulse "smears" into adjacent symbol intervals so as to interfere with the detection process; such interference is termed intersymbol interference (ISI). Even in the absence of noise, imperfect filtering and system bandwidth constraints lead to IST. In practice, $H_c(f)$ is usually specified, and the problem remains to determine $H_t(f)$ and $H_r(f)$ such that the ISI of the pulses are minimized at the output of $H_r(f)$.

Nyquist [3] investigated the problem of specifying a received pulse shape so that no ISI occurs at the detector. He showed that the theoretical minimum system bandwidth needed to detect R_s symbols/s, without ISI, is $R_s/2$ Hertz. This occurs when the system transfer function, H(f), is made rectangular, as shown in Figure 3.2a. When H(f) is such an ideal filter with bandwidth 1/2T, its impulse response, the inverse Fourier transform of H(f) is h(t) = sinc (t/T), shown in Figure 3.2b. Thus h(t) is the received pulse shape resulting from the application of an impulse at the input of such an ideal system. Nyquist established that if each pulse of a received sequence is of the form h(t), the pulses can be detected without ISI. The bandwidth required to detect l/T such pulses (symbols) per second is equal to l/2T; in other words, a system with bandwidth W = l/2T = $R_s/2$ hertz can support a maximum transmission rate of 2W = 1/T = R_s symbols/s (Nyquist bandwidth constraint) without ISI.

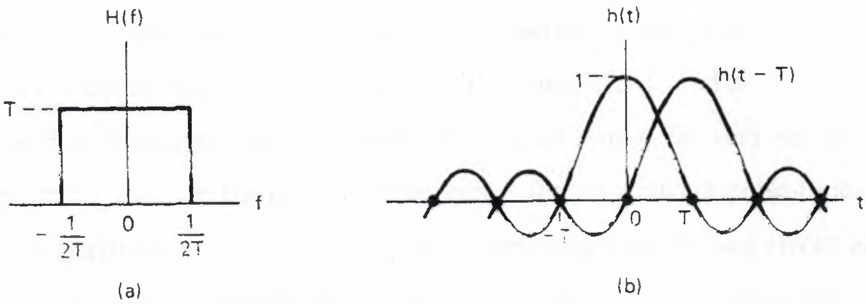


Figure 3.2 Nyquist channels for zero ISI. (a) Rectangular system transfer function H(f). (b) Received pulse shape h(t) = sinc (t /T)

Figure 3.2b illustrates how ISI is avoided. The figure shows two successive received pulses, h(t) and h(t - T), Even though h(t) has a long tail, it passes through zero at the instant that h(t - T) is sampled (at t = T) and therefore causes no degradation to the detection process. With such an ideal received pulse shape, the maximum possible symbol transmission rate per Hertz, called the symbol-rate packing, is 2 symbols/s/Hz, without ISI.

What does the Nyquist bandwidth constraint say about the maximum number of bits/s/Hz that can be received without ISI? It says nothing about bits, directly. The constraint deal only with pulses or symbols, and the ability to detect their amplitude values without distortion from other pulses. The assignment of how many hits each symbol represents is a separate issue. In theory. each symbol can represent M levels or k bits ($M = 2^k$): as k or M increases in value, so does the complexity of the system. For example, when k= 6 bits/symbol, each symbol represents M = 64 levels. The number of bits/s/Hz that a system can support is referred to as the bandwidth efficiency of the system.

For most communication systems the goal is to reduce the required system bandwidth as much as possible; Nyquist has provided us with a basic limitation to such bandwidth reduction. What would happen if we tried to force a system to operate at smaller bandwidths than the constraint dictates? We would find that restricting the bandwidth would spread the pulses in time; this would degrade the system's error performance, due to the increase in ISI.

3.2 Pulse Shaping to Reduce ISI

The Nyquist requirement for a sinc (t/T) received pulse shape is not physically realizable since it dictates a rectangular bandwidth characteristic and an infinite time delay. Also, with such a characteristic, the detection process would be very sensitive to small timing errors. In Figure 3.2b the pulse h(t) has zero value in adjacent pulse times only when the sampling is performed at exactly the correct sampling time; timing errors will produce ISI. Therefore, we cannot implement systems using the Nyquist bandwidth; we need to provide some "excess bandwidth" beyond the theoretical minimum. One frequently used system transfer function, H(f), is called the raised cosine filter. It can be expressed as

$$H(f) = \begin{cases} 1 & \text{for } |f| < 2W_0 - W \\ \cos^2\left(\dfrac{\pi}{4}\dfrac{|f| + W - 2W_0}{W - W_0}\right) & \text{for } 2W_0 - W < |f| < W \\ 0 & \text{for } |f| > W \end{cases} \tag{3.2}$$

where W is the absolute bandwidth, and $W_0 = 1/2T$ represents the minimum Nyquist bandwidth for the rectangular spectrum and the - 6-dB bandwidth (i.e., the fractional access bandwidth). For a given W_0, r specifies the required excess bandwidth (as a fraction of W_0) and characterizes the steepness of the filter roll-off. The raised cosine characteristic is illustrated in Figure 3.3a for roll-off values of r = 0, r = 0.5, and r = 1.0. The r = 0 roll-off is the Nyquist minimum-bandwidth case. Notice that when r = 1.0, the required excess bandwidth is 100%; a system with such an overall spectral characteristic can provide a symbol rate of R_s symbols/s using a bandwidth of R_s Hertz (twice the Nyquist bandwidth), thus yielding a symbol rate packing of 1 symbol/s/Hz. The corresponding impulse response for the H(f) of Equation (3.2) is

$$h(t) = 2W_0(\text{sinc } 2W_0 t)\frac{\cos[2\pi(W - W_0)t]}{1 - 4(W - W_0)t^2} \tag{3.3}$$

The impulse response is shown in Figure 3.3b for r = 0, r = 0.5, and r = 1.0.

Recall that for zero ISI, it is better to choose the system received pulse shape to be equal to h(t); we can only do this approximately, since strictly speaking, the raised cosine pulse is not precisely physically relizable. A realizable frequency characteristic must have a time response that is zero prior to the pulse turn-on time, which is not the case for the family of raised cosine characteristics.

These unrealizable filters are noncausal (the filter impulse response begins at time $t = -\infty$). However, a delayed version of h(t), say $h(t - t_o)$, may be approximately generated by real filters if the delay t_0 is chosen such that $h(t - t_o) = 0$, for $t < 0$. Notice in figure 3.3b that timing errors will still result in some ISI degradation for r = 1. However, the problem is not as serious as it is for r = 0, because the tails of the h(t) waveform are of much smaller amplitude for r = 1 than they are for r = 0.

The Nyquist bandwidth constraint states that the theoretical minimum required system bandwidth, W, for a symbol rate of R_s symbols/s without ISI, is $R_s/2$ hertz. A more general relationship between required bandwidth and symbol transmission rate involves the filter roll-off factor r, and can be stated as

$$W = \tfrac{1}{2}(1 + r)R_s \tag{3.4}$$

Thus with r = 0, Equation (3.4) describes the required bandwidth for ideal rectangular filtering, also referred to as Nyquist filtering. Bandpass-modulated signals (baseband signals that have been shifted in frequency) such as amplitude shift keying (ASK) and phase shift keying (PSK), require twice the transmission bandwidth of the equivalent baseband signals. Such frequency- translated signals, occupying twice their baseband bandwidth, are often called double-sideband (DSB) signals. Therefore, for ASK- and PSK-modulated signals, the relationship between the required DSB bandwidth, W_{DSB}, and the symbol transmission rate, R_s, is

$$W_{DSB} = (1 + r)R_s \tag{3.5}$$

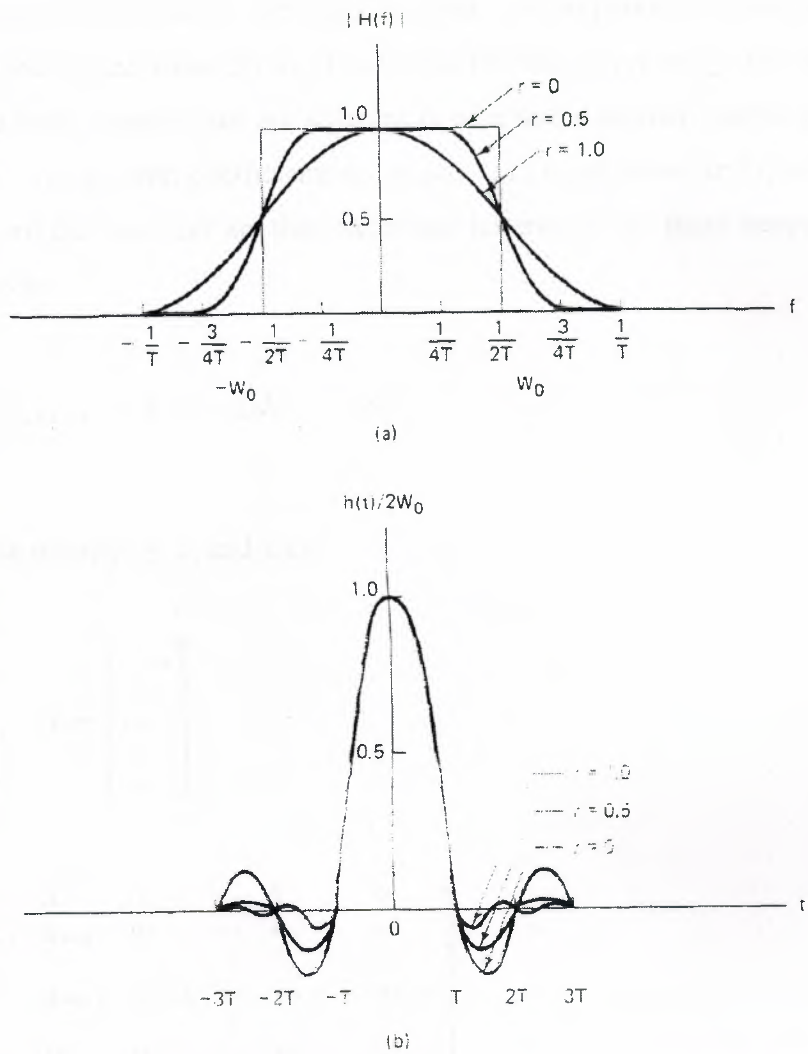


Figure 3.3 Raised cosine filter characteristics. (a) System transfer function. (b) System impulse response.

3.3 Equalization

In practical systems, the frequency response of the channel is not known with sufficient precision to allow for a receiver design that will compensate for the intersymbol interference (ISI) for all time. In practice, the filter for handling ISI at the receiver contains various parameters that are adjusted on the basis of measurements of the channel characteristics. The process of thus correcting the channel-induced distortion is called equalization. A transversal filter-a delay line with T-second taps (where T is the symbol duration)-is a common choice for the equalizer filter. The outputs of the taps are amplified, summed, and fed to a decision device. The tap coefficients, c_n are set to subtract the effects of interference from symbols that are adjacent in time to the desired symbol. Consider that there are (2N + 1) taps with coefficients $c_{-N}, c_{-N+1}, \ldots, c_N$ as shown in Figure 3.4. Output samples, $\{y_k\}$, of the equalizer are then expressed in terms of the input samples, $\{x_k\}$, and tap coefficients as

$$y_k = \sum_{n=-N}^{N} c_n x_{k-n} \qquad k = -2N, \ldots, 2N \tag{3.6}$$

By defining the matrices y, c, and x as

$$\mathbf{y} = \begin{bmatrix} y_{-2N} \\ \vdots \\ y_0 \\ \vdots \\ y_{2N} \end{bmatrix} \qquad \mathbf{c} = \begin{bmatrix} c_{-N} \\ \vdots \\ c_0 \\ \vdots \\ c_N \end{bmatrix} \tag{3.7}$$

$$\mathbf{x} = \begin{bmatrix} x_{-N} & 0 & 0 & \cdots & 0 & 0 \\ x_{-N+1} & x_{-N} & 0 & \cdots & \cdots & \cdots \\ \vdots & & & \vdots & & \vdots \\ x_N & x_{N-1} & x_{N-2} & \cdots & x_{-N+1} & x_{-N} \\ \vdots & & & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & x_N & x_{N-1} \\ 0 & 0 & 0 & \cdots & 0 & x_N \end{bmatrix} \tag{3.8}$$

we can simplify the computation for $\{Y_k\}$ as follows:

$$\mathbf{y} = \mathbf{xc} \tag{3.9}$$

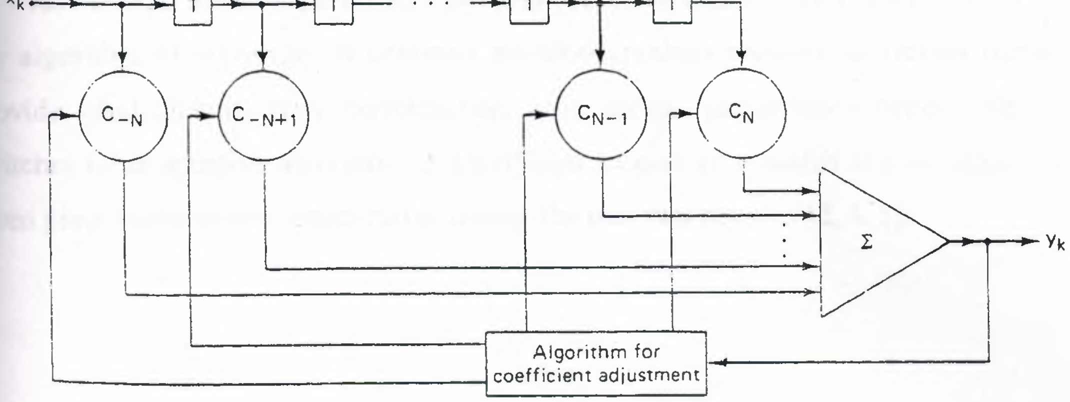


Figure 2.37 Transversal filter.

The criterion for selecting the c_n coefficients is typically based on the minimization of either peak distortion or mean-square distortion. Minimizing peak distortion can be accomplished by selecting the c_n coefficients so that the equalizer output is forced to zero at N sample points on either side of the desired pulse.
That is,

$$y_k = \begin{cases} 1 & \text{for } k = 0 \\ 0 & \text{for } k = \pm 1, \pm 2, \ldots, \pm N \end{cases} \tag{3.10}$$

then solving for c_n by combining Equations (3.7) to (3.9) and solving $2N + 1$ simultaneous equations. Minimizing the mean-square distortion similar results in $2N + 1$ simultaneous equations.

There are two general types of automatic equalization. The first, preset equalization, transmits a training sequence that is compared at the receiver with a locally generated sequence. The differences between the two sequences are used to set the coefficients c_n. With the second method, adaptive equalization, the coefficients are continually and automatically adjusted directly from the transmitted data A. Disadvantage of preset equalization is that it requires an initial training session, which must be repeated after any break in transmission. Also, a time-varying channel can degrade in ISI since the coefficients are fixed. Adaptive equalization can perform well if the channel error performance is satisfactory.

However, if the error performance is poor, received channel errors may not allow the algorithm to converge. A common solution employs preset equalization initially to provide good channel error performance; once normal transmission begins, the system switches to an adaptive algorithm. A significant amount of research and development has taken place in the area of equalization during the past two decades [2, 4, 5].

CHAPTER 4
CHANNEL EQUALIZATION

4.1 Introduction to Channel Equalization

In Section 2.4, we described the design of transmitting and receiving filters for digital PAM transmission when the frequency response characteristics of the channel are known. Our objective was to design these filters for zero ISI at the sampling instants. This design methodology is appropriate when the channel is precisely known and its characteristics do not change with time.

In practice we often encounter channels whose frequency response characteristics are either unknown or change with time. For example, in data transmission over the dial-up telephone network, the communication channel will be different every time we dial a number, because the channel route will be different. Once a connection is made, however, the channel will be time-invariant for a relatively long period of time. This is an example of a channel whose characteristics are unknown a priori. Examples of time-varying channels are radio channels, such as ionospheric propagation channels. These channels are characterized by time-varying frequency response characteristics. These types of channels are examples where the optimization of the transmitting and receiving filters is not possible.

Under these circumstances, we may design the transmitting filter to have a square-root raised cosine frequency response; i.e.,

$$G_T(f) = \begin{cases} \sqrt{X_{\mathrm{rc}}(f)}\, e^{-j2\pi f t_0}, & |f| \leq W \\ 0, & |f| > W \end{cases} \tag{4.1}$$

and the receiving filter, with frequency response $G_R(f)$, to be matched to $G_T(f)$. Therefore,

$$|G_T(f)||G_R(f)| = X_{\mathrm{rc}}(f) \tag{4.2}$$

Then; due to channel distortion, the output of the receiving filter is

$$y(t) = \sum_{n=-\infty}^{\infty} a_n x(t - nT) + v(t) \tag{4.3}$$

where x (t) = g (t) * c(t) * g (t). The filter output may be sampled periodically to produce the sequence

$$\begin{aligned} y_m &= \sum_{n=-\infty}^{\infty} a_n x_{m-n} + v_m \\ &= x_0 a_m + \sum_{\substack{n=-\infty \\ n \neq m}}^{+\infty} a_n x_{m-n} + v_m \end{aligned} \tag{4.4}$$

where x_n = x(nT), n =0, ±1, ±2 The middle term on the right-hand side of Equation (4.4) represents the ISI.

In any practical system, it is reasonable to assume that the ISI affects a finite number of symbols. Hence, we may assume that $x_n = 0$ for n < -L1 and n > L2, where L1 and L2 are finite, positive integers. Consequently, the ISI observed at the output of the receiving filter may be viewed as being generated by passing the data sequence $\{a_m\}$ through an FIR filter with coefficients $\{x_n,\ -L1 \leq n \leq L2\}$, as shown in Figure 4.1. This filter is called the equivalent discrete-time channel filter. Since its input is the discrete information sequence (binary or M-ary), the output of the discrete-time channel filter may be characterized as the output of a finite-state machine corrupted by additive Gaussian noise. Hence, the noise-free output of the filter is described by a trellis having M^L states where L = L1 + L2.

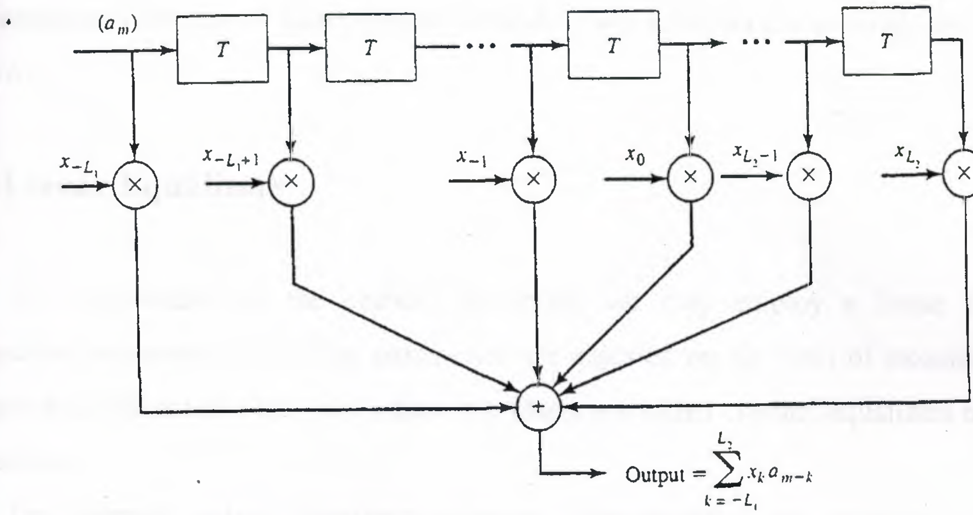


Figure 4.1 Equivalent discrete-time channel filter.

4.1.1 Maximum-Likelihood Sequence Detection.

The optimum detector for the information sequence {a_m) based on the observation of the received sequence (y_m), given by Equation (4.4), is a **ML** sequence detector. The detector is akin to the **ML** sequence detector described in the context of detecting partial response signals which have controlled ISI. The Viterbi algorithm provides a method for searching through the trellis for the **ML** signal path. To accomplish this search, the equivalent channel filter coefficients {x_n} must be known or measured by some method. At each stage of the trellis search, there are $\mathbf{M}^L$ surviving sequences with $\mathbf{M}^L$ corresponding Euclidean distance path metrics.

Due to the exponential increase in the computational complexity of the Viterbi algorithm with the span (length L) of the ISI, this type of detection is practical only when **M** and **L** are small. For example in mobile cellular telephone systems which employ digital transmission of speech signals, **M** is usually selected to be small; e.g., M = 2 or 4, and 2 < L < 5. In this case, the ML sequence detector may be implemented with reasonable complexity. However, when **M** and **L** are large, the **ML** sequence detector becomes impractical. In such a case other more practical but suboptimum methods are used to detect the information sequence (a_m} in the presence of ISI. Nevertheless, the performance of the ML sequence detector for a channel with ISI serves as a benchmark for comparing its performance with that of suboptimum methods. Two suboptimum methods are described below.

4.2 Linear Equalizers.

To compensate for the channel distortion, we may employ a linear filter with adjustable parameters. The filter parameters are adjusted on the basis of measurements of the channel characteristics These adjustable filters are called channel equalizers or, simply, equalizers.

On channels whose frequency-response characteristics are unknown, but time-invariant, we may measure the channel characteristics, adjust the parameters of the equalizer, and once adjusted, the parameters remain fixed during the transmission of data. Such equalizers are called *preset equalizers*. On the other hand, adaptive equalizers update their parameters on a periodic basis during the transmission of data.

First, we consider the design characteristics for a linear equalizer from a frequency domain viewpoint. Figure 4.2 shows a block diagram of a system that employs a linear filter as a channel equalizer.

The demodulator consists of a receiving filter with frequency response G_R (f) in cascade with a channel equalizing filter that has a. frequency response $G_E(f)$. Since

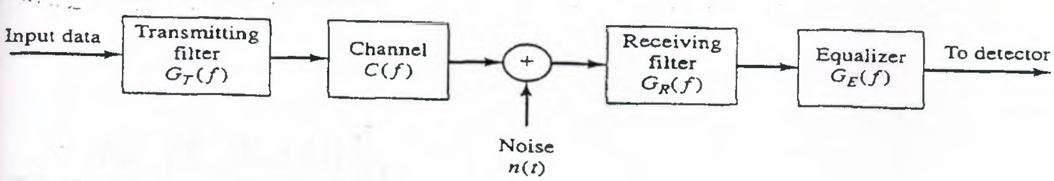


Figure 4.2 Block diagram of a system with an equalizer.

$G_R(f)$ is matched to $G_T(f)$ and they are designed so that their product satisfies Equation(4.2) $|G_E(f)|$ must compensate for the channel distortion. Hence, the equalizer frequency response must equal the inverse of the channel response; i.e.,

$$G_E(f) = \frac{1}{C(f)} = \frac{1}{|C(f)|} e^{-j\Theta_c(f)} |f| \leq W \tag{4.5}$$

where $|G_E(f)| = 1/|C(f)|$ and the equalizer phase characteristic $\Theta_E(f) = -\Theta_c(f)$. In this case, the equalizer is said to be the inverse channel filter to the channel response.

We note that the inverse channel filter completely eliminates ISI caused by the channel. Since it forces the ISI to be zero at the sampling times $t = nT$, the equalizer is called a zero-forcing equalizer. Hence, the input to the detector is of the form

$$\mathbf{y}_m = \mathbf{a}_m + \mathbf{v}_m$$

where v_m is the noise component, which is zero-mean Gaussian with a variance

$$\begin{aligned} \sigma_v^2 &= \int_{-\infty}^{\infty} S_n(f)|G_R(f)|^2|G_E(f)|^2\,df \\ &= \int_{-W}^{W} \frac{S_n(f)|X_{rc}(f)|}{|C(f)|^2}\,df \end{aligned} \tag{4.6}$$

where $S_n(f)$ is the power-spectral density of the noise. When the noise is white, $S_n(f) = N_0/2$ and the variance becomes

$$\sigma_v^2 = \frac{N_0}{2}\int_{-W}^{W} \frac{|X_{rc}(f)|}{|C(f)|^2}\,df \tag{4.7}$$

In general, the noise variance at the output of the zero-forcing equalizer is higher than the noise variance at the output of the optimum receiving filter $|G_R(f)|$.

Let us now consider the design of a linear equalizer from a time-domain view-point. We noted previously that in real channels, the ISI is limited to a finite number of samples, say L samples. As a consequence, in practice the channel equalizer is approximated by a finite. duration impulse response (FIR) filter, or transversal filter, with adjustable tap coefficients $\{c_n\}$, as illustrated in Figure 4.3. The time delay τ between adjacent taps may be selected as large as T, the symbol interval, in which case the FIR equalizer is called a symbol-spaced equalizer. In this case the input to the equalizer is the sampled sequence given by Equation (4.4). However, we note that when $1/T < 2W$, frequencies in the received signal above the folding frequency $1/T$

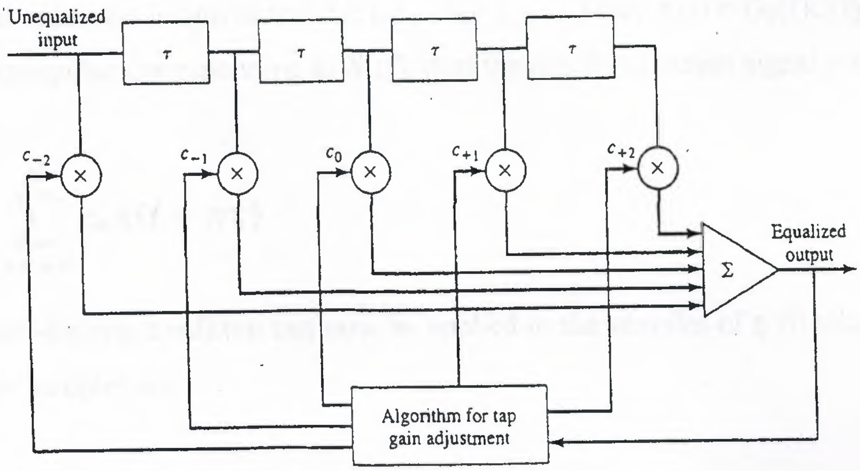


Figure 4.3 Linear transversal filter.

are aliased into frequencies below $1/T$. In this case, the equalizer compensates for the aliased channel-distorted signal. On the other hand, when the time delay τ between adjacent taps is selected such that $1/\tau \geq 2W > 1/T$, no aliasing occurs and, hence, the inverse channel equalizer compensates for the true channel distortion. Since $\tau < T$, the channel equalizer is said to have fractionally spaced taps and it is called a fractionally spaced equalizer. In practice, τ is often selected as $\tau = T/2$. Notice that, in this case, the sampling rate at the output of the filter $G_R(f)$ is $T/2$.

The impulse response of the FIR equalizer is

$$g_E(t) = \sum_{n=-N}^{N} c_n \delta(t - n\tau) \tag{4.8}$$

and the corresponding frequency response is

$$G_E(f) = \sum_{n=-N}^{N} c_n e^{-j2\pi f n\tau} \tag{4.11}$$

where $\{c_n\}$ are the (2N + 1) equalizer coefficients, and N is chosen sufficiently large so that the equalizer spans the length of the ISI; i.e., 2N + I ≥ L. Since X(f) = $G_T(f)C(f)G_R(f)$ and x (t) is the signal pulse corresponding to X (f), then the equalized output signal pulse is

$$q(t) = \sum_{n=-N}^{N} c_n x(t - n\tau) \tag{4.9}$$

The zero-forcing condition can now be applied to the samples of q (t) taken at times t = mT. These samples are

$$q(mT) = \sum_{n=-N}^{N} c_n x(mT - n\tau), \qquad m = 0, \pm 1, \ldots, \pm N \tag{4.11}$$

Since there are 2N + 1 equalizer coefficients, we can control only 2N + 1 sampled values of q (t). Specifically, we may force the conditions

$$q(mT) = \sum_{n=-N}^{N} c_n x(mT - n\tau) = \begin{cases} 1, & m = 0 \\ 0, & m = \pm 1, \pm 2, \ldots, \pm N \end{cases} \tag{4.12}$$

which may be expressed in matrix form as **Xc** = **q**, where **X** is a (2N + 1) x (2N+1) matrix with elements {x(mT - nτ)}, **c** is the (2N + 1) coefficient vector and **q** is the (2N + 1) column vector with one nonzero element. Thus we obtain a set of 2N + 1 linear equations for the coefficients of the zero-forcing equalizer.

We should emphasize that the FIR zero-forcing equalizer does not completely eliminate ISI because it has a finite length. However, as **N** is increased the residual ISI can be reduced and in the limit as $N \rightarrow \infty$, the ISI is completely eliminated.

One drawback to the zero-forcing equalizer is that it ignores the presence of additive noise. As a consequence, its use may result in significant noise enhancement. This is easily seen by noting that in a frequency range where C(f) is small, the channel equalizer $G_E(f) = 1/C(f)$ compensates by placing a large gain in that frequency range. Consequently, the noise in that frequency range is greatly enhanced. An alternative is to relax the zero ISI condition and select the channel equalizer characteristic such that the combined power in the residual ISI and the additive noise at the output of the equalizer is minimized. A channel equalizer that is optimized based on the minimum mean-squar-error (MMSE) criterion accomplishes the desired goal.

To elaborate, let us consider the noise-corrupted output of the FIR equalizer which is

$$z(t) = \sum_{n=-N}^{N} c_n y(t - n\tau) \tag{4.13}$$

where y(t) is the input to the equalizer, given by Equation (4.3). The output is sampled at times t = mT. Thus, we obtain

$$z(mT) = \sum_{n=-N}^{N} c_n y(mT - n\tau) \tag{4.14}$$

The desired response sample at the output of the equalizer at t = mT is the transmitted symbol $\mathbf{a_m}$ The error is defined as the difference between a_m and z(mT). Then, the man-square-error (MSE) between the actual output sample z(mT) and the desired values $\mathbf{a_m}$ is

$$\begin{aligned}
\text{MSE} &= E[z(mT) - a_m]^2 \\
&= E\left[\sum_{n=-N}^{N} c_n y(mT - n\tau) - a_m\right]^2 \\
&= \sum_{n=-N}^{N}\sum_{k=-N}^{N} c_n c_k R_Y(n-k) - 2\sum_{k=-N}^{N} c_k R_{AY}(k)
\end{aligned} \tag{4.15}$$

where the correlations are defined as

$$\begin{aligned}
R_Y(n-k) &= E[y(mT - n\tau)y(mT - k\tau)] \\
R_{AY}(k) &= E[y(mT - k\tau)a_m]
\end{aligned} \tag{4.16}$$

and the expectation is taken with respect to the random information sequence $\{a_m\}$, and the additive noise.

The MMSE solution is obtained by differentiating Equation (4.15) with respect to the equalizer coefficients $\{c_n\}$. Thus, we obtain the necessary conditions for the MMSE as

$$\sum_{n=-N}^{N} c_n R_Y(n-k) = R_{YA}(k), \qquad k = 0, \pm 1, \pm 2, \ldots, \pm N \tag{4.17}$$

These are (2N + 1) linear equations for the equalizer coefficients. In contrast to the zero-forcing solution described previously, these equations depend on the statistical properties (the autocorrelation) of the noise as well as the ISI through the autocorrelation $R_Y(n)$.

In practice, we would not normally know the autocorrelation are y(n) and the cross correlation $R_{ay}(n)$. However, this correlation can be estimated by transmitting a test sigal over the channel and using the time average estimates

$$\hat{R}_Y(n) = \frac{1}{K}\sum_{k=1}^{K} y(kT - n\tau)y(kT)$$

$$\hat{R}_{AY}(n) = \frac{1}{K}\sum_{k=1}^{K} y(kT - n\tau)\, a_k \tag{4.18}$$

In place of the ensemble average to solve for the equalizer coefficient given by Equation (4.17).

4.3 Adaptive Equalizers

We have shown that the tap coefficients of a linear equalizer can be determined by solving a set of linear equations. In the zero-forcing optimization criterion, the linear equations are given by Equation(4.12). On the other hand, if the optimizationcriterion is based on minimizing the MSE, the optimum equalizer coefficients are determined by solving the set of linear equations given by Equation(4.17).

In both cases, we may express the set of linear equations in the general matrix form

$$\mathbf{Bc} = \mathbf{d} \tag{4.19}$$

Where **B** is a (2N + 1) × (2N + 1) matrix, **c** is a column vector representing the 2N + 1 equalizer coefficients, and d is a (2N + 1) – dimensional column vector. The solution of Equation (4.19) yields

$$\mathbf{c}_{\text{opt}} = \mathbf{B}^{-1}\mathbf{d} \tag{4.20}$$

in practical implementations of equalizers, the solution of Equation (4.19) for the optimum coefficient vector is usually obtained by an iterative procedure that avoids the explicit computation of the inverse of the matrix **B**. The simplest iterative procedure is the method of steepest descent, in which one begins by choosing arbitrarily the coefficient vector **c**, say c_0. This initial choice of coefficients corresponds to a point on the criterion function that is begin optimized. For example, in the case of the MSE criterion, the initial guess c_0 corresponds to a point on the quadratic MSE surface in which is the derivative of the MSE with respect to the 2N + 1 filter coefficients, is then computed at this point on the criterion surface and each tap coefficient is changed in the direction opposite to its corresponding gradient component. The change in the j^{th} tap coefficient is proportional to the size of the j^{th} gradient component.

For example, the gradient vector, denoted as $\mathbf{g}_K$, for the MSE criterion, found by taking the derivative of the MSE with respect to each of the 2N + 1 coefficients, is

$$\mathbf{g}_k = \mathbf{B}\mathbf{c}_k - \mathbf{d} \qquad k=0,1,2,... \tag{4.21}$$

Then the coefficient vector c_k is updated according to the relation

$$\mathbf{c}_{k+1} = \mathbf{c}_k - \Delta \mathbf{g}_k \tag{4.22}$$

where Δ is the step-size parameter for the iterative procedure. To ensure convergence of the iterative procedure, Δ is chosen to be a small positive number. In such a case, the

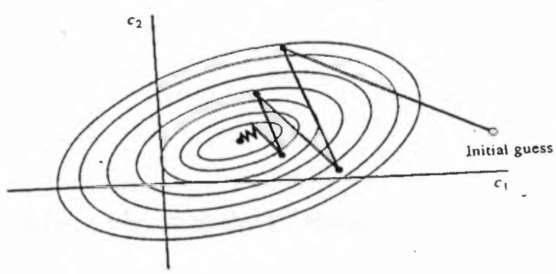


Figure 4.4 Example of convergence characteristics of a gradient algorithm.

gradient vector $\mathbf{g}_k$ converge toward zero; i.e., $g_k \rightarrow 0$ as $k \rightarrow \infty$, and the coefficient vector $\mathbf{c}_K \rightarrow \mathbf{c}_{opt}$ as illustrated in Figure (4.4) based on two –dimensional optimization. In general, convergence of the equalization tap coefficients to $\mathbf{c}_{opt}$ can not be attained in a finite number if iterations with the steepest – descent method. However, the optimum solution $\mathbf{c}_{opt}$ can be approached as closely as desired in a few hundred iterations. In digital communication systems that employ channel equalizers, each iteration corresponds to a time interval for sending one symbol and, hence, a few hundred iterations to achieve convergence to $\mathbf{c}_{opt}$ corresponds to a fraction of a second.

Adaptive channel equalization is required for channels whose characteristics change with time. In such a case, the ISI varies with time. The channel equalizer must track such time variations in the channel response and adapt its coefficients to reduce the ISI. In the context of the above discussion, the optimum coefficient vectoir $\mathbf{c}_{opt}$ varies with time due to time variations in the matrix **B** and, for the case of the MSE criterion, time variations in the vector **d**. Under these conditions, the iterative method described above can be modified to use estimates of the gradient components. Thus, the algorithm for adjusting the equalizer tap coefficient may be expressed as

$$\hat{\mathbf{c}}_{k+1} = \hat{\mathbf{c}}_k - \Delta\hat{\mathbf{g}}_k \tag{4.23}$$

where $\hat{\mathbf{g}}_k$ denotes an estimate of the gradient vector $\mathbf{g}_K$ and $\hat{\mathbf{c}}_k$ denotes the estimate of the tap coefficient vector.

In the case of the MSE criterion, the gradient vector $\mathbf{g}_K$ given by Equation (4.21) may also be expressed as

$$\mathbf{g}_k = -E(e_k\mathbf{y}_k) \tag{4.24}$$

An estimate $\hat{\mathbf{g}}_k$ of the gradient vector at the k[th] iteration is computed as

$$\hat{\mathbf{g}}_k = -e_k\mathbf{y}_k \tag{4.25}$$

where $\mathbf{e}_k$ denotes the difference between the desired output from the equalizer at the k^{th} time instant and the actual output z(kT), and $\mathbf{y}_k$ denotes the column vector of 2N + 1 received signal values contained in the equalizer at time instant k. The error signal $\mathbf{e}_k$ is expressed as

$$e_k = a_k - z_k \tag{4.26}$$

where z_k = z(kT) is the equalizer output given by Equation (4.16) and $\mathbf{a}_k$ is the desired symbol. Hence, by substituting an equation for Δ into Equation (4.23), the result will be the adaptive algorithm for optimizing the tap coefficients (based on the MSE criterion) as

$$\hat{\mathbf{c}}_{k+1} = \hat{\mathbf{c}}_k + \Delta e_k \mathbf{y}_k \tag{4.27}$$

Since an estimate of the gradient vector is used in Equation (4.27) the algorithm is called a stochastic gradient algorithm. It is also known as the LMS algorithm.

A block diagram of an adaptive equalizer that adapts its tap coefficient according to Equation (4.27) is illustrated in Figure (4.5). Note that the difference between the desired output $\mathbf{a}_k$ and the actual output $\mathbf{z}_k$ from the equalizer is used to form the error signal $\mathbf{e}_k$. This error is scaled by the step-size parameter **Δ**, and the scaled error signal $\Delta\mathbf{e}_k$ multiplies the received signal values {y(kT - nτ)} at 2N + 1 taps. The products $\Delta e_k y(kT - n\tau)$ at the (2N + 1) taps are then added to the previous values of the tap computation is priviated for each received symbol. Thus, the equalizer coefficient are updated at the symbol rate.

Initially, the adaptive equalizer is trained by the transmission of the known pseudo-random sequence $\{a_m\}$ over the channel. At the demodulator, the equalizer employs the known sequence to adjust its coefficients. Upon initial adjustment,

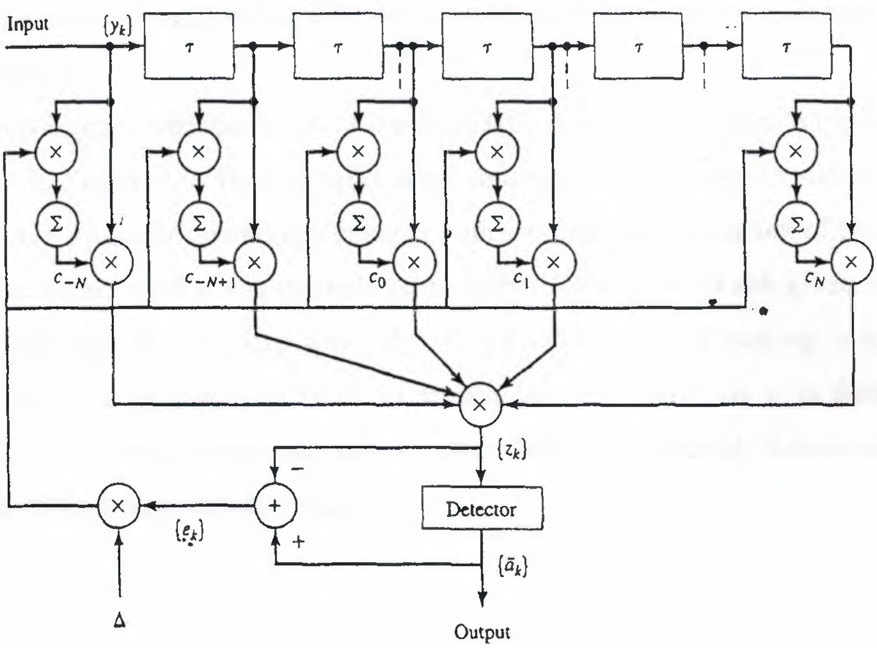


Figure 4.5 Linear adaptive equalizer based on the MSE criterion.

the adaptive equalizer switches from a training mode to a decision-directed mode, in which case the decisions at the output of the detector are sufficiently reliable so that the error signal is formed by computing the difference between the detector output and the equalizer output; i.e.,

$$e_k = \tilde{a}_k - z_k \tag{4.28}$$

where $\tilde{a}_k$ is the output of the detector. In general, decision errors at the output of the detector occur infrequently and, consequently, such errors have little effect on the performance of the tracking algorithm given by Equation (4.27).

A rule of thumb for selecting the step-size parameter so as to ensure convergence and good tracking capabilities in slowly varying channels is

$$\Delta = \frac{1}{5(2N+1)P_R} \tag{4.29}$$

where $\mathbf{P_R}$ denotes the received signal-plus-noise power, which can be estimated from the received signal.

The convergence characteristics of the stochastic gradient algorithm in Equation (4.27) is illustrated in Figure 4.6. These graphs were obtained from a computer simulation of an 11-tap adaptive equalizer operating a channel with a rather modest amount of ISI. The input signal-plus-noise power $\mathbf{P_R}$ was normalized to unity. The rule of thumb given in Equation (4.29) for selecting the step size gives $\Delta = 0.018$. The effect of making Δ too large is illustrated by the large jumps in MSE as shown for $\Delta = 0.115$. As Δ is decreased, the convergence is slowed somewhat, but a lower MSE is achieved, indicating that the estimated coefficients are closer to $\mathbf{c}_{opt}$.

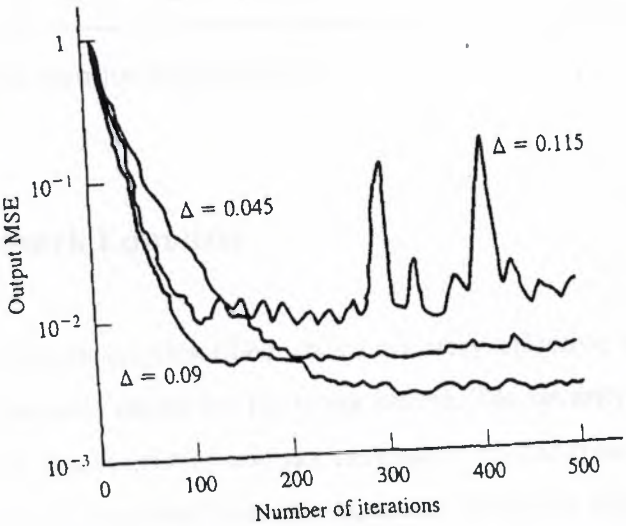


Figure 4.6 Initial convergence characteristics of the LMS algorithm with different step sizes.

Although we have described in some detail the operation of an adaptive equalizer which is optimized on the basis of the MSE criterion, the operation of an adaptive equalizer based on the zero-forcing method is very similar. The major difference lies in the method of estimating the gradient vectors $\mathbf{g}_k$ at each iteration. A block diagram of an adaptive zero-forcing equalizer is shown in Figure 4.7.

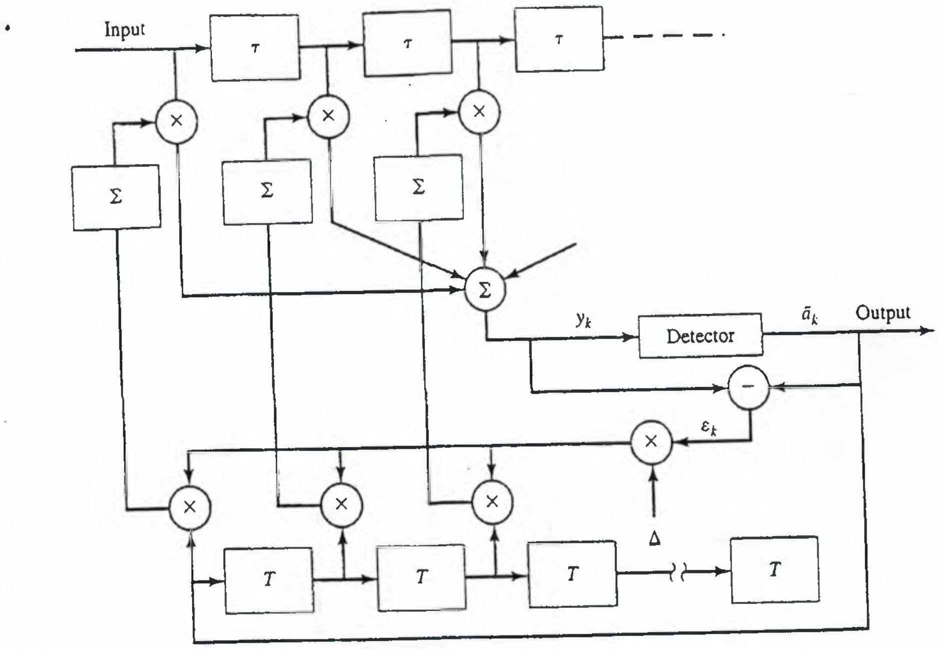
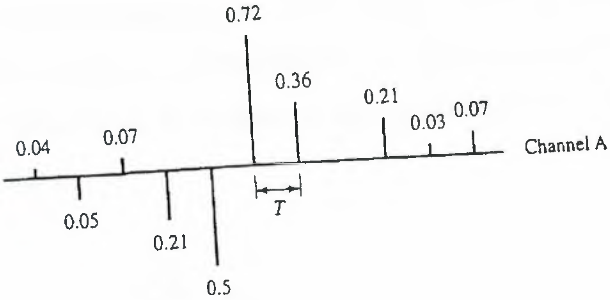


Figure 4.7 An adaptive zero-forcing equalizer.

4.4 Decision-Feedback Equalizer

The linear filter equalizers described above are very effective on channels, such as wireline telephone channels, where the ISI is not severe. The severity of the ISI is directly related to the spectral characteristics and not necessarily to the time span of the ISI. For example, consider the ISI resulting from two channels which are illustrated in Figure 4.8. The time span for the ISI in Channel A is 5 symbol intervals on each side of the desired signal component, which has a value of 0.72. On the other hand, the time span for the ISI in Channel B is one symbol interval on each side of the desired signal component, which has a value of 0.815. The energy of the total response is normalized to unity for both channels.

In spite of the shorter ISI span, Channel B results in more severe ISI. This is evidenced in the frequency response characteristics of these channels. We observe that Channel B has a spectral null (the frequency response C(f) = 0 for some frequencies in the band $|f| \leq W$) at f = 1/2T, whereas this does



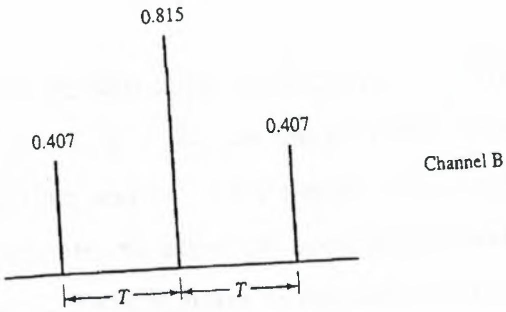


Figure 4.8 Two channels with ISI.

not occur in the case of Channel A. Consequently, a linear equalizer will introduce a large gain in its frequency response to compensate for the channel null. Thus, the noise in Channel B will be enhanced much more than in Channel A. This implies that the performance of the linear equalizer for Channel B will be significantly poorer than that for Channel A. This fact is borne out by the computer simulation results for the performance of the linear equalizer for the two channels. Hence, the basic limitation of a linear equalizer is that it performs poorly on channels having spectral nulls. Such channels are often encountered in radio communications, such as ionospheric transmission at frequencies below 30 MHz and mobile radio channels, such as those used for cellular radio communications.

A decision-feedback equalizer (DFE) is a nonlinear equalizer that employs previous decisions to eliminate the ISI caused by previously detected symbols on the current symbol to be detected. A simple block diagram for a DFE is shown in Figure 4.9. The DFE consists of two filters. The first filter is called a feedforward filter and it is generally a fractionally spaced FIR filter with adjustable tap coefficients. This filter is identical in form to the linear equalizer described above. Its input is the received filtered signal y(t). The second filter is a feedback filter. It is implemented as an FIR filter with symbol-spaced taps having adjustable coefficients. Its input is the set of previously detected symbols. The output of the feedback filter is subtracted from the output of the feedforward filter to form the input to the detector. Thus, we have

$$z_m = \sum_{n=1}^{N_1} c_n y(mT - n\tau) - \sum_{n=1}^{N_2} b_n \tilde{a}_{m-n} \tag{4.30}$$

where $\{c_n\}$ and $\{b_n\}$ are the adjustable coefficients of the feedforward and feedback filters, respectively, a_{m-n}, n = 1, 2, . . , N_2 are the previously detected symbols, N_1 is the length of the feedforward filter and N2 is the length of the feedback filter. Based on the input z_m, the detector determines which of the possible transmitted symbols is closest in distance to the input signal z_m. Thus, it makes its decision and outputs a_m.

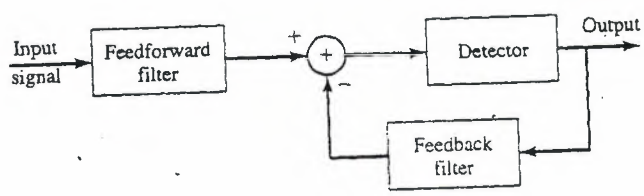


Figure 4.9 Block diagram of DFE.

What makes the DFE nonlinear is the nonlinear characteristic of the detector which provides the input to the feedback filter.

The tap coefficients of the feedforward and feedback filters are selected to optimize some desired perforthance measure. For mathematical simplicity, the MSE criterion is usually applied and a stochastic gradient algorithm is commonly used to implement an adaptive DFE. Figure 4.10 illustrates the block diagram of an adaptive DFE whose tap coefficients are adjusted by means of the LMS stochastic gradient algorithm.

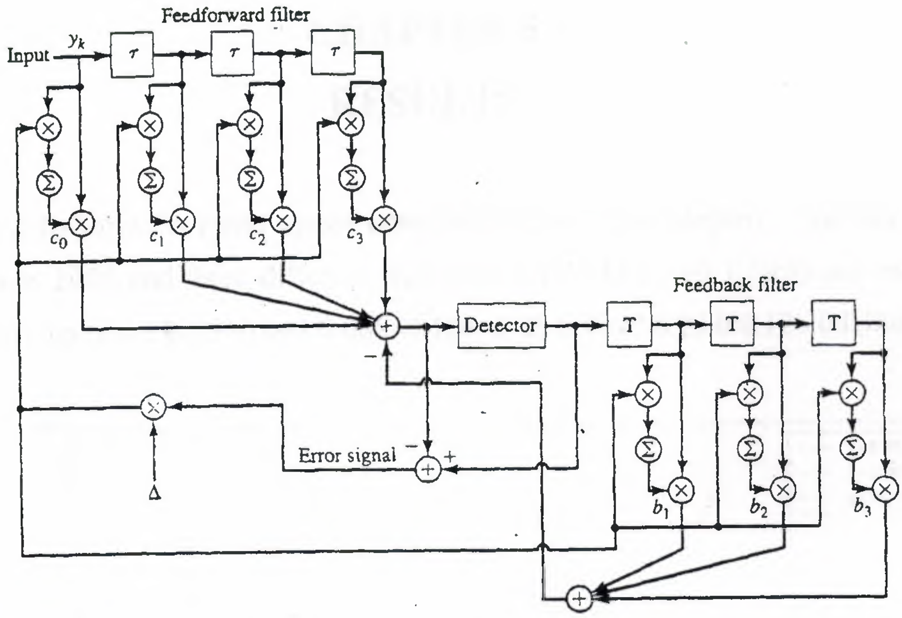


Figure 4.10 Adaptive DFE.

CHAPTER 5
RESULTS

Figure 5.1 shows the mean square error (MSE) plot of an adaptive equalizer. Number of iterations is 1000 and three different step sizes (Δ=[0.115 0.09 0.045]) are used. The channel coefficients are [0.05 -0.063 0.088 -0.126 -0.25 0.9047 0.25 0 0.126 0.0380.088].

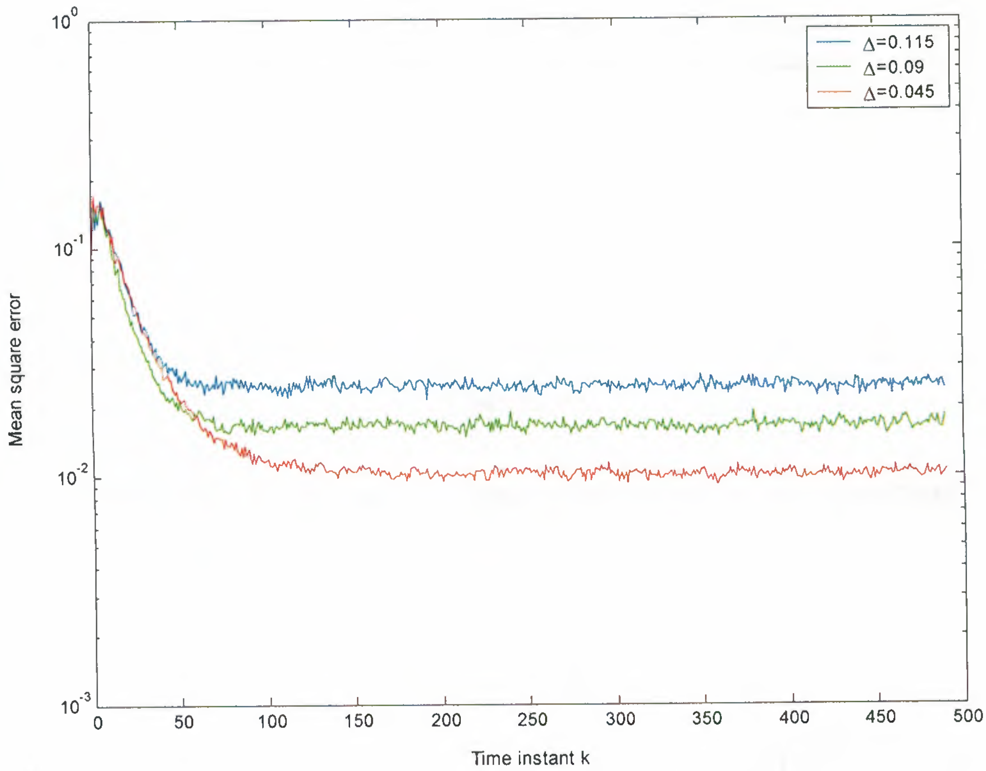


Figure 5.1 Convergence of adaptive equalizer using 1000 iterations

It can be seen from the figure that the smaller the step size, the longer it takes to reach to convergence and the smaller the mean square error.

Figure 5.2 and 5.3 show the plots for same configurations as before but with 5000 and 100 iterations respectively. It is observed that increasing the number of iterations provide more accurate convergence results.

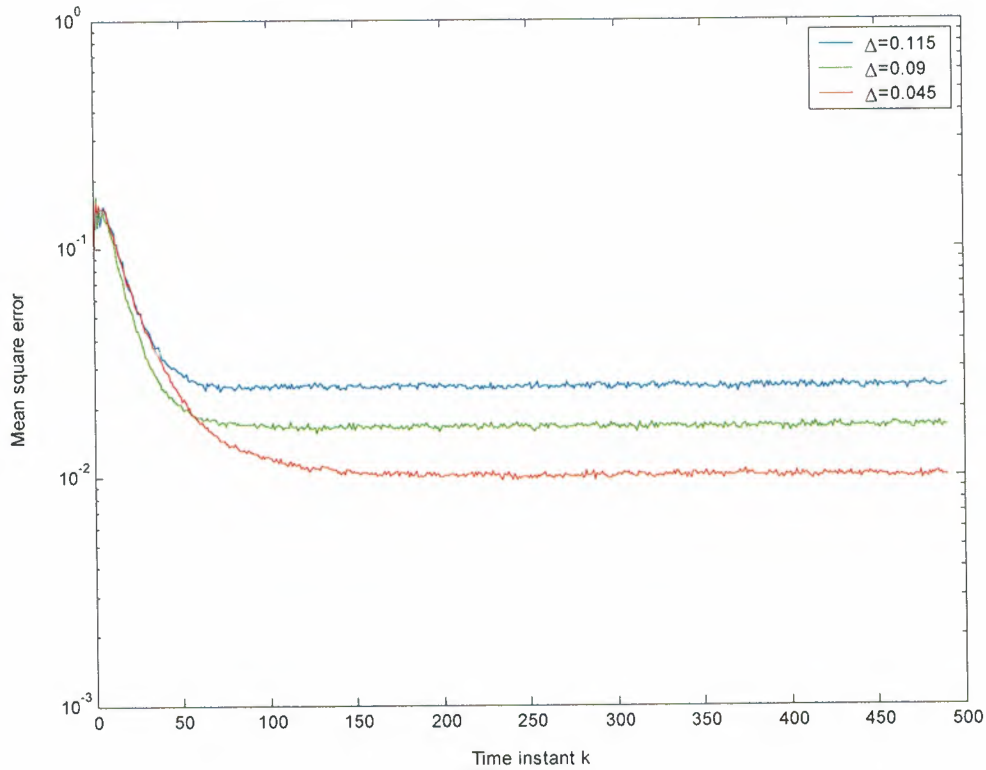


Figure 5.2 convergence of adaptive equalizer using 5000 iterations.

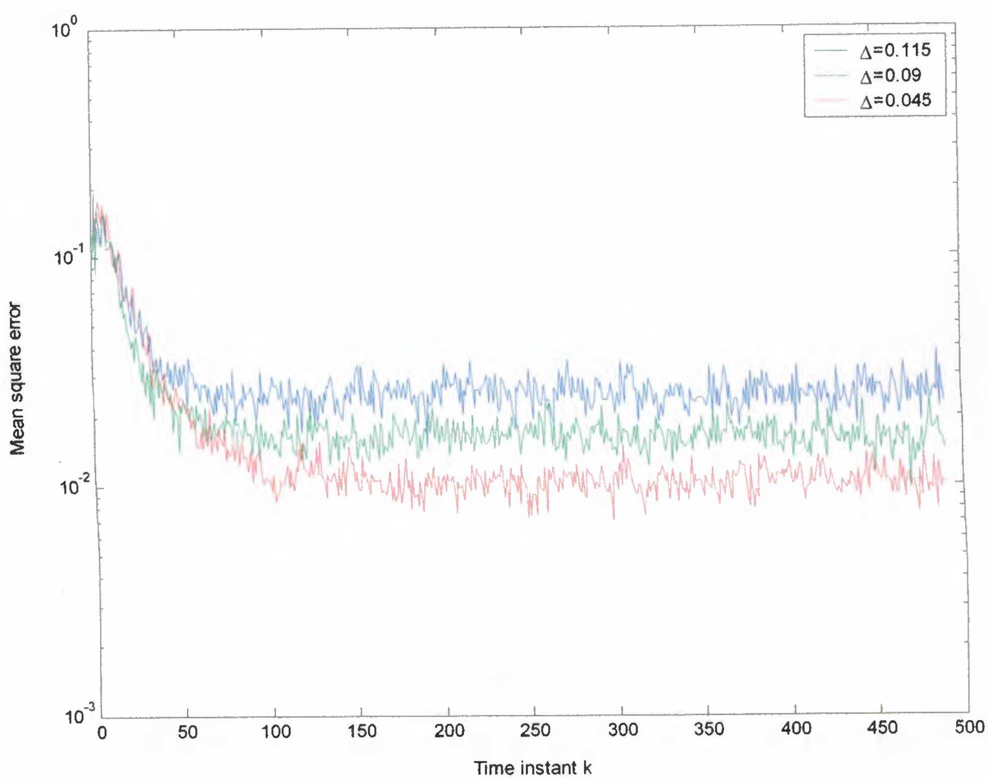
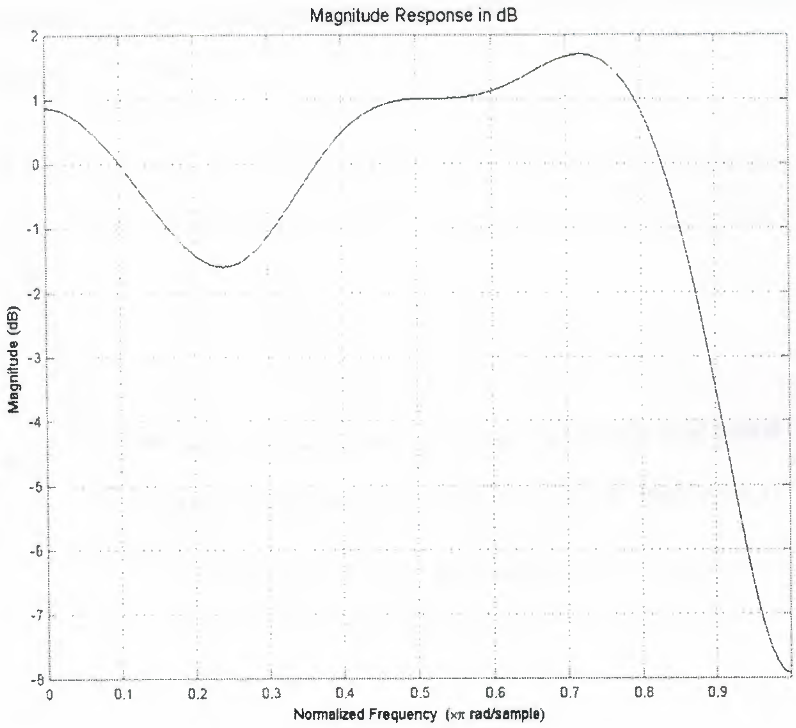


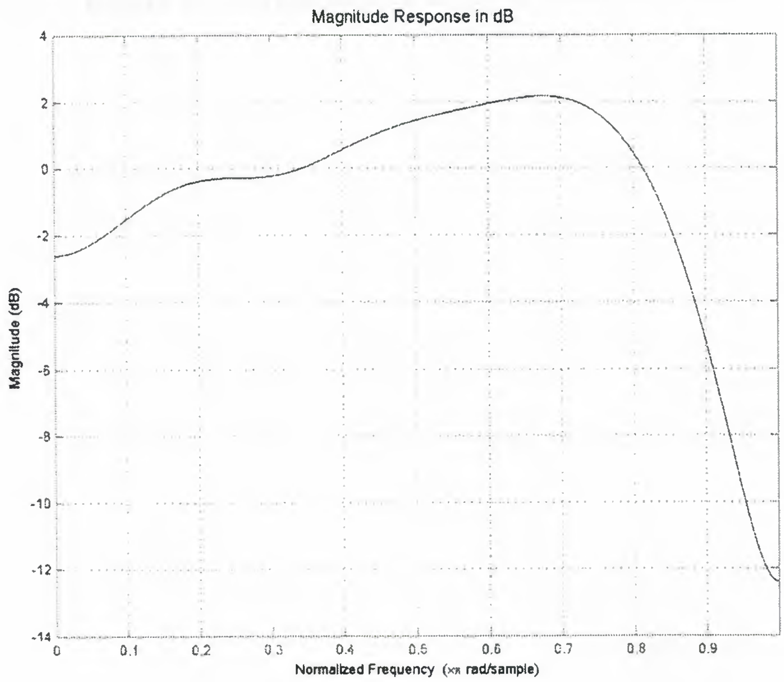
Figure 5.3 convergence of adaptive equalizer using 100 iterations

as may be seen clearly from the previous figures, as the number of iterations increase the more accurate the performance could be.

Figure 5.4 (a) shows the amplitude spectrum of the channel described earlier, named channel A. also in figure 5.4 (b) it is shown the spectrum of channel B which has more severe intersymbol interference with channel coefficients = [0.04 -0.05 0.07 -0.21 -0.5 0.72 0.36 0 0.21 0.03 0.07]. figure 5.5 shows the plot of convergence of the adaptive equalizer mean square error in channel B. 1000 iterations is used. It can be seen from the figure that a channel with more severe interference results in higher mean square error for equalizer convergence.



(a) channel A, coefficients =[0.05 -0.063 0.088 -0.126 -0.25 0.9047 0.25 0 0.126 0.038 0.088]



(b) channel B, coefficients = [0.04 -0.05 0.07 -0.21 -0.5 0.72 0.36 0 0.21 0.03 0.07]

Figure 5.4 amplitude spectra of channel A and B and the performance of the adaptive equalizer with channel B with 1000 iterations is illustrated in figure 5.5 below.

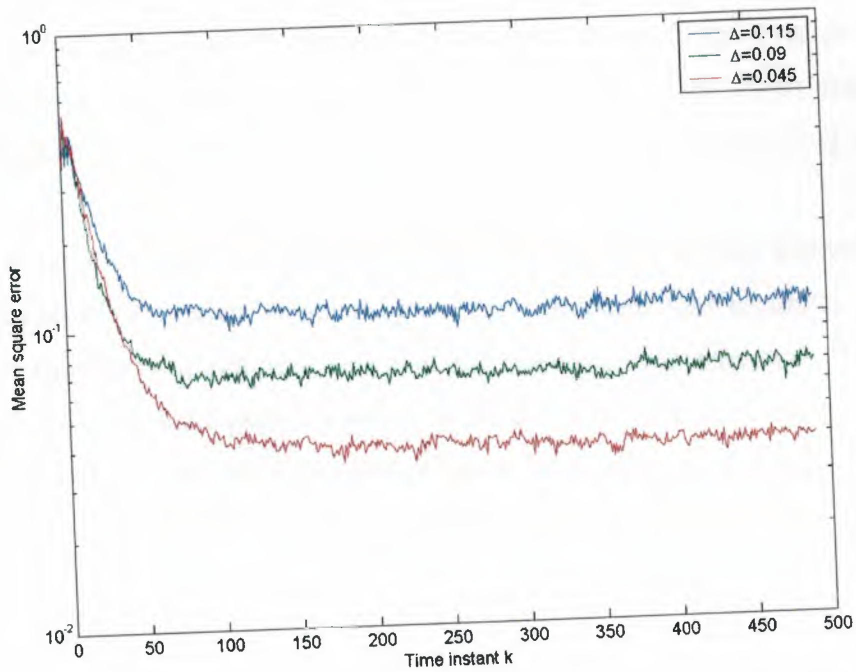


Figure 5.5 convergence of adaptive equalizer in channel B

CONCLUSION

Channel equalization is used to eliminate intersymbol interference that occurs in bandlimited and wireless channels. Adaptive equalizers are preferred over zero forcing equalizers that have the disadvantage of amplifying the noise. This report analyzes and reports the results of simulating the convergence of an adaptive equalizer using least mean squares algorithm.

It is shown that as the step size decreases, the mean square error also decreases. If the number of iterations is increased, the mean square error becomes more accurate.

Adaptive equalizers do not deal well with bad intersymbol interference. Therefore, in the future decision feedback equalizers should be investigated as an alternative. In addition the enhancements of noise in linear equalizers have to be examined thoroughly.

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APPENDIX A

MATLAB CODE OF ADAPTIVE EQUALIZER WITH CHANNEL A. 1000 ITERATIONS IS USED

```
N=500;                          % length of the information sequence
K=5;
actual_isi=[0.05 -0.063 0.088 -0.126 -0.25 0.9047 0.25 0 0.126 0.038 0.088];
sigma=0.01;
delta=[0.115 0.09 0.045];
Num_of_realizations=1000;
mse_av=zeros(3,N-2*K);
for t=1:3
for j=1:Num_of_realizations,
  for i=1:N,
    if (rand<0.5),
       info(i)=-1;
    else
      info(i)=1;
    end;
  end
  y=filter(actual_isi,1,info);
 %  1234
 %  y
  for i=1:N,
     noise(i)=sigma*randn;
  end;
  y=y+noise;
 %  22345
 %  y
  estimated_c=[0 0 0 0 0 1 0 0 0 0 0];
```

```
for k=1:N-2*K,
   y_k=y(k:k+2*K);
   z_k(k)=estimated_c*y_k';
   e_k=info(k)-z_k(k);
   estimated_c=estimated_c+delta(t)*e_k*y_k;
   mse(k)=e_k^2;
  end;
  mse_av(t,:)=mse_av(t,:)+mse;
end;
mse_av(t,:)=mse_av(t,:)/Num_of_realizations;
end
semilogy(1:490,mse_av(1,:),1:490,mse_av(2,:),1:490,mse_av(3,:))
ylabel('Mean square error'),xlabel('Time instant k')
legend('\Delta=0.115','\Delta=0.09','\Delta=0.045')
```

APPENDIX B

MATLAB CODE OF ADAPTIVE EQUALIZER WITH CHANNEL A. 5000 ITERATIONS IS USED

```
N=500;                          % length of the information sequence
K=5;
actual_isi=[0.05 -0.063 0.088 -0.126 -0.25 0.9047 0.25 0 0.126 0.038 0.088];
sigma=0.01;
delta=[0.115 0.09 0.045];
Num_of_realizations=5000;
mse_av=zeros(3,N-2*K);
for t=1:3
for j=1:Num_of_realizations,
  for i=1:N,
    if (rand<0.5),
       info(i)=-1;
    else
      info(i)=1;
    end;
  end
  y=filter(actual_isi,1,info);
 %  1234
 %  y
  for i=1:N,
     noise(i)=sigma*randn;
  end;
  y=y+noise;
 %  22345
 %  y
  estimated_c=[0 0 0 0 0 1 0 0 0 0 0];
  for k=1:N-2*K,
```

```
. y_k=y(k:k+2*K);
   z_k(k)=estimated_c*y_k';
   e_k=info(k)-z_k(k);
   estimated_c=estimated_c+delta(t)*e_k*y_k;
   mse(k)=e_k^2;
  end;
  mse_av(t,:)=mse_av(t,:)+mse;
end;
mse_av(t,:)=mse_av(t,:)/Num_of_realizations;
end
semilogy(1:490,mse_av(1,:),1:490,mse_av(2,:),1:490,mse_av(3,:))
ylabel('Mean square error'),xlabel('Time instant k')
legend('\Delta=0.115','\Delta=0.09','\Delta=0.045')
```

APPENDIX C

MATLAB CODE WITH CHANNEL A. 100 ITERATIONS IS USED

```
N=500;                          % length of the information sequence
K=5;
actual_isi=[0.05 -0.063 0.088 -0.126 -0.25 0.9047 0.25 0 0.126 0.038 0.088];
sigma=0.01;
delta=[0.115 0.09 0.045];
Num_of_realizations=100;
mse_av=zeros(3,N-2*K);
for t=1:3
for j=1:Num_of_realizations,
 for i=1:N,
   if (rand<0.5),
     info(i)=-1;
   else
     info(i)=1;
   end;
 end
 y=filter(actual_isi,1,info);
% 1234
% y
 for i=1:N,
     noise(i)=sigma*randn;
 end;
 y=y+noise;
% 22345
% y
```

```
estimated_c=[0 0 0 0 0 1 0 0 0 0 0];
 for k=1:N-2*K,
  y_k=y(k:k+2*K);
  z_k(k)=estimated_c*y_k';
  e_k=info(k)-z_k(k);
  estimated_c=estimated_c+delta(t)*e_k*y_k;
  mse(k)=e_k^2;
 end;
 mse_av(t,:)=mse_av(t,:)+mse;
end;
mse_av(t,:)=mse_av(t,:)/Num_of_realizations;
end
semilogy(1:490,mse_av(1,:),1:490,mse_av(2,:),1:490,mse_av(3,:))
ylabel('Mean square error'),xlabel('Time instant k')
legend('\Delta=0.115','\Delta=0.09','\Delta=0.045')
```

APPENDIX D

MATLAB CODE OF ADAPTIVE EQUALIZER WITH CHANNEL B. 1000 ITERATIONS IS USED.

```
N=500;                          % length of the information sequence
K=5;
actual_isi= [0.04 -0.05 0.07 -0.21 -0.5 0.72 0.36 0 0.21 0.03 0.07];
sigma=0.01;
delta=[0.115 0.09 0.045];
Num_of_realizations=1000;
mse_av=zeros(3,N-2*K);
for t=1:3
for j=1:Num_of_realizations,
 for i=1:N,
  if (rand<0.5),
     info(i)=-1;
  else
    info(i)=1;
  end;
 end
 y=filter(actual_isi,1,info);
%  1234
%  y
 for i=1:N,
    noise(i)=sigma*randn;
 end;
 y=y+noise;
%  22345
%  y
 estimated_c=[0 0 0 0 0 1 0 0 0 0 0];
 for k=1:N-2*K,
```

```
.y_k=y(k:k+2*K);
  z_k(k)=estimated_c*y_k';
  e_k=info(k)-z_k(k);
  estimated_c=estimated_c+delta(t)*e_k*y_k;
  mse(k)=e_k^2;
 end;
 mse_av(t,:)=mse_av(t,:)+mse;
end;
mse_av(t,:)=mse_av(t,:)/Num_of_realizations;
end
semilogy(1:490,mse_av(1,:),1:490,mse_av(2,:),1:490,mse_av(3,:))
ylabel('Mean square error'),xlabel('Time instant k')
legend('\Delta=0.115','\Delta=0.09','\Delta=0.045')
```