NEAR EAST UNIVERSITY

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TRACKING RADAR

Graduation Project
EE - 400

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CHAPTER 1
INTRODUCTION

Multiple target tracking (MTT) is an essential requirement for surveillance systems employing one or more sensors, together with computer subsystems, to interpret the environment. Typical sensor systems, such as radar, infrared (IR), and sonar report measurements from diverse sources: targets of interest, background noise sources such as radar ground clutter, or internal error sources such as thermal noise.

The (MTT) objective is to partition the sensor data into sets of observations, or tracks, produced by the same source. Once tracks are formed and confirmed (so that background and other false targets are reduced), the number of targets can be estimated and quantities, such as target velocity, future predicted position and target classification characteristics, can be computed for, each track.

The earliest and probably still the best-known type of MTT system is the radar track-while-scan (TWS) system. The TWS system is a special case of an MTT system. The TWS system is a special case of an MTT system in which the data are received at regular intervals as the radar (or other sensor) regular intervals as the radar (or other sensor) regularly scans a predetermined search volume.

The project consists of the introduction, six chapters and conclusion.

The chapter 2 gives some of the required definitions, discussion of the MTT system design by introducing the basic elements contained in most systems. The most important element of an MTT system is data association (or correlation).

Thus, the chapter will illustrate in some detail how the related functions of gating and correlation are used.

Chapter 3 discusses filtering and prediction, which are the fundamental elements of any tracking system. An understanding of these elements is required in the discussion of other functions of a multiple-target tracking (MTT) system.

In chapter 4, a discrete formulation of the continuous state equations is presented and the considerations involved in choosing tracking coordinates are discussed several coordinate systems and sets of state variables that have been implemented for multiple target tracking (MTT) are presented.

Chapter 5, discusses how sensor design and measurement data processing relate to the overall MTT problem. The emphasis will be on radar system design, but the general techniques discussed for adaptive threshold setting and target resolution are also applicable to infrared (IR) devices.

The purpose of chapter six is to provide the tools and the frame work that can be used to design a detailed Monte Carlo MTT simulation. The chapter begins with some preliminaries concerning the generation and use of random numbers and the application of the random variables associated with radar. Topics include a suggested approach for documentation and check out, choice of evaluation statistics, and techniques for presenting results. A typical detailed design is illustrated using a simulation that has been developed for a radar TWS system.

Chapter 7 is concerned with the techniques for utilizing the powerful adaptive features of the electronically scanned antenna (ESA, agile beam or phased array radar). The ESA has the capability to perform adaptive sampling by directing the radar beam without inertia in any direction.

or radar Doppler (range rate), and measured attributes such as target type, identification number, length, or shape. Also, an observation should contain an estimate of the time at which the measurements were obtained.

2.2 Elements of a Basic MTT System

Fig 2.1 gives a representation of the functional elements of a simple recursive MTT system. There is considerable overlap of the functions of these elements, but this representation provides a convenient partitioning which will be used to introduce the typical functions required for an MTT system.

Let us assume recursive processing so that tracks have been formed on the previous scan. Now, input data are received from the sensor, and the processing loop described in fig.2.1 is to be performed. Incoming observations are first considered for the update of existing tracks. Gating tests determine which possible observation-to-track pairings are “reasonable," and a more refined correlation algorithm is used to determine final pairings. Observations not assigned to existing tracks can initiate new tentative tracks. A tentative track becomes confirmed when the number and quality of the observations included in the track satisfies confirmation criteria. Similarly, low quality tracks, as usually determined by the update history, are deleted. Finally, after inclusion of the new observations, tracks are predicted ahead to the arrival time for the next set of observations. Gates are placed around these predicted positions and the processing cycle repeats. Next, we shall discuss these elements in more detail.

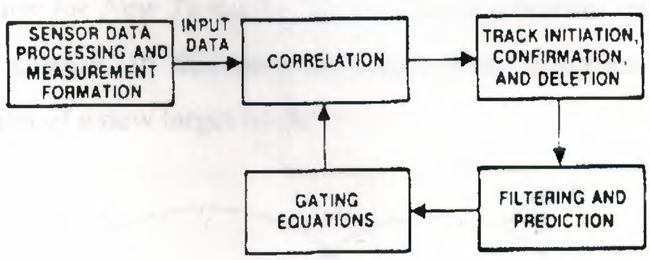

FIGURE 2.1 BASIC ELEMENTS OF A SIMPLE RECURSIVE MTT SYSTEM

2.2.1 Measurement Data Processing

For MTT the sensor will typically spend a limited amount of time on a single target because scanning is necessary in order to provide updated information on existing multiple target tracks and to search for new targets. One important sensor design consideration is the determination of a decision rule on the return, which is received during the time on target, so as to discriminate between returns from targets of interest and returns from extraneous sources, such as potential false alarms produced by noise and radar clutter.

The simplest approach to the decision process is to compare the incoming signal power to a threshold, which is set so that the probability of false alarm *(P_D)* remains constant. For a given threshold setting the probability of detection *(P_D)* generally will be a complicated function of the sensor capabilities, the target size and distance from the sensor and the environment (atmospheric attenuation, *et cetera)*

2.2.2 Gating

Gating is the first part of the correlation algorithm used to decide if an observation belongs to a previously established target track or to a new target. Gating is a coarse test that classifies an observation into one of two categories.

1. *Candidate for Track Update.* The observation may satisfy the gates of one or more existing tracks. In this case the observation becomes a candidate for association with that track. Note that more than one observation may satisfy the gate of a single track. Also, note that an observation ultimately might not be used to update the track, even if the gate is satisfied. Thus, it may be used to initiate a new track.

2. *Initial Observation for New Tentative Track.* The observation might not satisfy the gate of any existing track. In this case the observation becomes an immediate candidate for the initiation of a new target track.

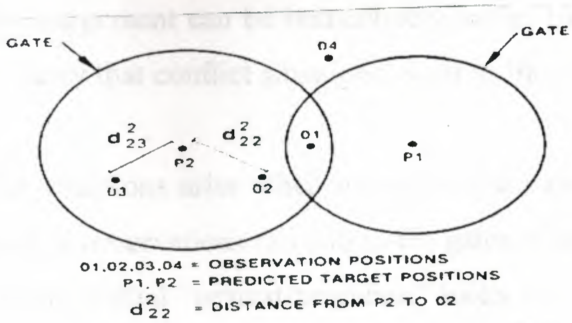

FIGURE 2.2 EXAMPLE OF GATING AND CORRELATION FOR TWO CLOSELY SPACED TRACKS

Fig2.2 illustrates gating for two closely spaced targets and four observations. Note that the gates may overlap for closely spaced targets. Gates are established, and gating is performed in the following general manner.

- Estimates are made of what the measured quantity should be at the time of the next observation.
- The difference between each measurement and its corresponding estimate is formed. It is often useful to form a total distance d_{ij}^2 from track i to observation j. Thus, a normalization process is required whereby the differences in each of the component measurements are squared, divided by the variances of the expected differences, and summed to form a total normalized distance. For example, if range *(R)* and angle (θ) are measured, the normalized distance is

$$d^2 = \frac{(R_p - R_o)^2}{\sigma_R^2} + \frac{(\theta_p - \theta_o)^2}{\sigma_o^2} \tag{2.1}$$

where $((R_p, \theta_p))$ is the predicted position, (R_o, θ_o) 'is the measured position, σ_R^2 is the variance of $R_p - R_o$, and is σ_o^2 the variance of $\theta_p - \theta_o$.

- A maximum error between estimate and measurement is formed for all measured quantities by using the estimate and measurement accuracy statistics. The computed differences are compared to the computed maximum allowable error. If the differences do not exceed the corresponding maximum allowable errors, the observation satisfies the gate.

2.2.3 Correlation

The correlation function takes the output of the gating function and makes final observation-to-track assignments. In the case where a single observation is within the gate of a single track, the assignment can be immediately made. However, for closely spaced targets, it is more likely that conflict situations, such as those shown in Fig. 2.2, will arise.

Correlation conflict situations arise when multiple observations fall within the same gate (or gates) and when observations fall within the gates of more than one track. The approach to this problem, called "nearest-neighbor," looks for a unique pairing so that at most one observation can be used to update a given track. Using this approach, the optimal solution is obtained by assigning observations to tracks in order to minimize the total summed distance from all observations to the tracks to which they are assigned. To illustrate one suboptimal solution, the example shown in Figure 2.2 is solved using the following rules:

1. *O*l is assigned to *T*l because *O*l is the only observation within the gates of *T*1 while *T*2 has other observations *(O2, O3)* within its gates.

2. *O*3 is assigned to *T*2 because *O*3 is closer than *O*2 $(d_{23}^2 < d_{22}^2)$.

3. *O*4 can, without question, be used to initiate a new track, but new track initiation using *O*2 may be restricted. This restriction is based upon the practical consideration that multiple observations within the gate of a single established track are often the result of a failure in the observation redundancy-elimination logic. Thus, this restriction serves to prevent initiation of extraneous tracks.

2.2.4 Track Initiation, Confirmation and Deletion

Observations not assigned to existing tracks are used to form new tentative tracks. Restrictions are sometimes used so that observations within gates of existing tracks may not be used to initiate new tentative tracks, even if, using nearest-neighbor correlation, the observations arc not assigned to an existing track. The problem of tentative track initiation becomes still more difficult using the all-neighbor approach. The author's experience with airborne radar systems using the nearest-neighbor approach indicates that in order to maintain accurate tracking it is best to initiate new tracks whenever initiation may be questionable, but then to make confirmation requirements more stringent.

Once a tentative track is formed, confirmation logic is usually required because the probability of a single observation being from an extraneous source is too high for immediate confirmation. Thus, it is usually required that at least one other observation be assigned to a tentative track before the track is considered to be confirmed. The gate size and the length of time allowed for that confirming observation can be chosen as functions of the confidence in the validity of the original observation.

A track that is not updated becomes degraded, and therefore must be deleted. If a sufficiently long time elapses without detection, the target probably will no longer be within the scan volume. Also, even if the lack of detections is consistent with an assumed low probability of detection, it might be best to delete a track just because of its low quality. A typical simple rule is to delete a track after N_D consecutive scans have produced no correlating observation. Alternatively, a test based upon the total elapsed time since track update may also be used.

2.2.5 Filtering and Prediction

The filtering step incorporates the correlating observations into the updated track parameter estimates. For those tracks that did not receive a correlating observation, the previous predicted estimates become the filtered estimates. Then, predictions are made to the time when the next data scans to be received. Thus, prediction quantities are of great importance because they define the center of the gated region discussed above. The size of the gate is also directly affected by the prediction uncertainty, which can be determined by the filter if Kalman filtering is used.

Fig 2.3 illustrates the prediction and gating processes. Note that as more observations are received, the predicted target position should approach the true target position unless the target performs a random maneuver. Also, as more data are received, the track gate sizes should decrease while remaining large enough to enclose a maneuvering target.

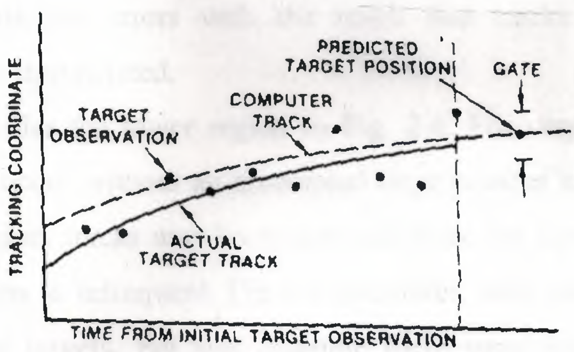

FIGURE 2.3 ILLUSTRATION OF PREDICTION AND GATING

2.3 Overview of Data Association Issues

This section introduces the observation-to-track correlation (or data association) problem, which is the key element of MTT. To begin, fig 2.4 gives a simplified, but instructive, interpretation of MTT data correlation. Under this interpretation there are basically three regions. These comprise a region of unambiguous correlation for widely spaced targets, an unstable region where highly inaccurate tracking may occur, and a region for closely spaced targets where miscorrelation occurs but tracking remains stable.

First, for sufficiently large target spacing unambiguous correlation occurs. Improving correlation techniques and detection performance can expand this region of unambiguous correlation. Also, for most cases, this region can be expanded by sampling at a faster rate (decreased sampling interval, T).

Next, an unstable region has been identified. Miscorrelation frequently occurs in this region. The result is erratic track performance and frequent premature track deletion, leading to a very inaccurate assessment of the target environment. The extent of the unstable region is also a function of the sampling rate and the probability of detection.

Fig 2.5 gives an example of the type of tracking that may occur in the unstable region. For this example, there were four targets at an approximately constant (as seen by the radar) azimuth angle separation of about three times the angular measurement-error standard deviation. Fig 2.5 shows the true targets' and the tracks' predicted azimuth angles as functions of time. Also, selected observations are denoted by dots, and the symbol D refers to the points of track deletion. This example shows how miscorrelation leads to large prediction errors with the result that tracks become "starved" for observations and are thus deleted.

Finally, consider the lower region in Fig. 2.4. For very closely spaced targets miscorrelation will occur without an associated large number of tracks being degraded and lost. In this region, tracks may be erratic and there are typically fewer tracks than targets, but track loss is infrequent. Fig 2.6 illustrates what happens in this region of very closely spaced targets. For this example there were four targets with angular separation of about 1.5 times the measurement-error standard deviation. We see that there are fewer tracks (three) than targets (four). The tracks tend to wander back and forth -and to cross, but none are lost.

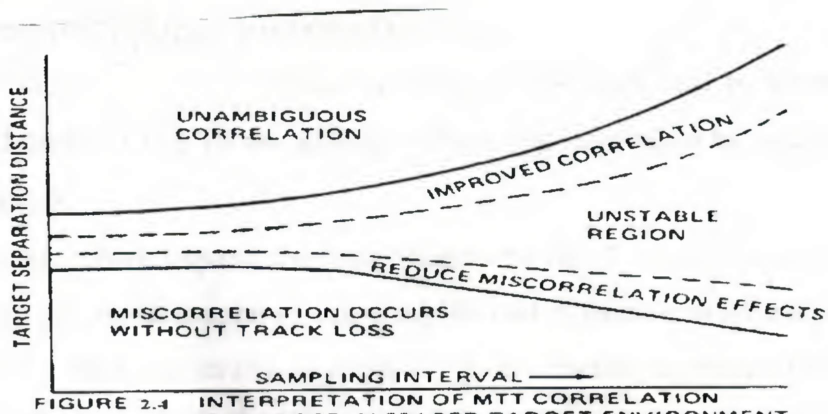

FIGURE 2.4 INTERPRETATION OF MTT CORRELATION IN A CLOSELY SPACED TARGET ENVIRONMENT

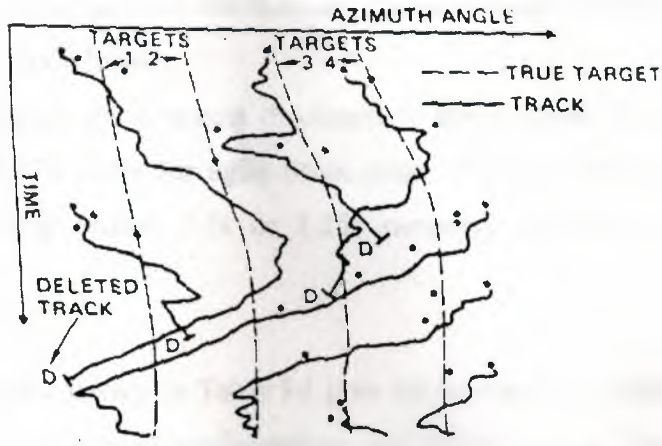

FIGURE 2.5 EXAMPLE OF DEGRADED TRACKING IN UNSTABLE REGION

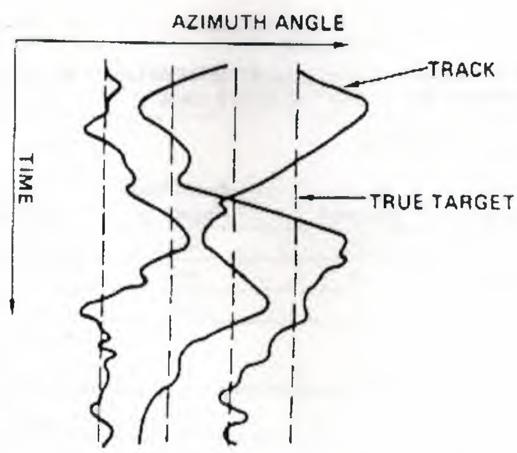

FIGURE 2.6 TYPICAL TRACKING RESULTS FOR CLOSELY SPACED TARGETS

2.3.1 Techniques for Reducing Unstable Tracking

Miscorrelation leading to unstable tracking can be decreased by increasing the probability of detection (P_D), by decreasing the sampling interval, or by using improved correlation methods.

Techniques, which expand the lower region of fig. 2.4, and thus decrease the unstable region, use an all-neighbors approach so that a single miscorrelation is less likely to degrade a track. Generally, in using the group tracking approach there will be no attempt to maintain individual tracks on closely spaced targets such as shown in fig. 2.6. Another approach for decreasing the unstable region in the presence of unavoidable miscorrelations is to increase the Kalman filter covariance matrix in order to reflect the uncertain correlation history.

To illustrate the concepts discussed in this section, Table 1.1 summarizes the results of an MTT study for agile beam radar. The agile beam radar system can use adaptive sampling (either 2.5s or 1.25s sampling interval) and enhance detection performance.

The results shown in Table I-I give the number of confirmed tracks' that were deleted for various system configurations and target spacing. The second column gives the target spacing (taken to be constant throughout a Monte Carlo run) as normalized with respect to the angular measurement-error standard deviation. The third column gives the total number of targets that were considered (using all Monte Carlo runs).

TABLE 2.1

NUMBER OF CONFIRMED TRACK DELETIONS FOR VARIOUS SPACINGS AND SYSTEM CONFIGURATIONS

			Nominal P_D (P_{D1})		Enhanced [1] P_D (P_{D2})	
Case	*Normalized Separation* [2]	*Total Number of Targets*	*Fixed Sampling*	*Adaptive Sampling*	*Fixed Sampling* [3]	*Adaptive Sampling* [3]
1	4.2	180	25	5	NE	NE
2	3.1	200	45	37	11	13
3	1.4	200	9	23	4	1

Note 1: $P_{D2} = 1 - (1 - P_{D1})^2$

Note 2: The target separation divided by the measurement-error standard deviation

Note 3: NE = not examined

CHAPTER 3

METHODS FOR FILTERING AND PREDICTION

This chapter discusses filtering and prediction, which are the fundamental elements of any tracking system. An understanding of these elements is required in the discussion of other functions of a multiple-target tracking (MTT) system.

This chapter initially discusses two commonly used approaches to filtering and prediction for multitarget tracking. The first is to use fixed tracking coefficients, and the second, Kalman filtering, generates time-variable tracking coefficients that are determined by a *priori* models for the statistics of measurement noise and target dynamics. The first approach has computational advantages for systems with large numbers of targets. However, with expanding computer capabilities, the high-accuracy tracking associated with Kalman filtering is becoming increasingly more appealing to the system designer.

Filtering and prediction methods are used to estimate present and future target kinematics quantities such as position, velocity, and acceleration. The introductory techniques presented in this chapter are most applicable when considering widely separated targets in a sparse false-alarm background so that the errors introduced by uncertain observation-to-track correlation can be ignored. Later chapters discuss modifications, which may be required in the presence of miscorrelation.

3.1 Fixed-Coefficient Filtering

3.1.1 The $\alpha-\beta$ Tracker

Fixed-coefficient filters have the advantage of simple implementation using fixed parameters for the filter gains. Probably the most extensively applied fixed-coefficient filter is the $\alpha-\beta$ tracker. This filter is used when only position measurement is available, and is defined by the following equations:

$$x_s(k)=\hat{x}(k|k)=x_p(k)+\alpha[x_o(k)-x_p(k)] \tag{3.1a}$$

$$v_s(k)=\hat{\dot{x}}(k|k)=v_s(k-1)+\beta/qT[x_o(k)-x_p(k)] \tag{3.1b}$$

$$x_p(k+1)=\hat{x}(k+1|k)=x_s(k)+Tv_s(k) \tag{3.1c}$$

$x_o(k) \overset{\Delta}{=}$ Observation received at k

$T \overset{\Delta}{=}$ Sampling interval

$\alpha, \beta =$ Fixed-coefficient filter parameters

The quantity q is normally defined to be unity, but in the case where missing observations occur its value may be taken as the number of scans since the last measurement. Finally, the usual initialization process is defined by

$$x_s(1) = x_p(2) = x_p(2) = x_o(1) \tag{3.2a}$$

$$v_s(1) = 0 \tag{3.2b}$$

$$v_s(2) = [x_o(2) - x_o(1)]/T \tag{3.2c}$$

3.1.2 The $\alpha - \beta - \gamma$ Tracker

The logical extension of the $\alpha - \beta$ tracker is the $\alpha - \beta - \gamma$ tracker, which includes an estimate of acceleration. The equations for the $\alpha - \beta - \gamma$ tracker are defined as:

$$x_s(k) = x_p(k) + \alpha[x_o(k) - x_p(k)] \tag{3.3a}$$

$$v_s(k) = v_s(k-1) + Ta_s(k-1) + \beta/qT\,[x_o(k) - x_p(k)] \tag{3.3b}$$

$$a_s(k) = a_s(k-1) + \gamma/(qT)^2[x_o(k) - x_p(k)] \tag{3.3c}$$

$$x_p(k+1) = x_s(k) + Tv_s(k) + T^2/2a_s(k) \tag{3.3d}$$

where the usual initialization is

$$x_s(1) = x_p(2) = x_o(1) \tag{3.4a}$$

$$v_s(1) = a_s(1) = a_s(2) = 0 \tag{3.4b}$$

$$v_s = \frac{[x_o(2) - x_o(1)]}{T} \tag{3.4c}$$

$$\frac{[x_o(3) + x_0(1) - 2x_o(2)]}{T^2} \tag{3.4d}$$

3.1.3 Choice of Fixed-Coefficient Gains

Equations (3.1) and (3.3) give examples of commonly used constant-coefficient filters. We next discuss how to determine the coefficient values. Decreasing coefficient values will lead to less responsive filters and as a result improved measures of performance for random noise input, while increasing coefficients leads to better performance *versus* dynamic inputs

Relationships between the coefficients for the $\alpha - \beta$ and $\alpha - \beta - \gamma$ trackers are derived. The relationships define gains that give a compromise between noise reduction and maneuver-following capability in the steady state. Considering a steady-state Kalman filter, Kalata shows the relationships to be

$$\beta = 2(2-\alpha) - 4\sqrt{1-\alpha} \tag{3.5}$$

$$\gamma = \beta^2 / 2\alpha \tag{3.6}$$

Equation (3.5) is valid for both filters, while (3.6) provides the additional relationship required for the $\alpha - \beta$ tracker.

The choice of gains for a constant-coefficient filter must reflect an overall compromise between noise and dynamic (maneuver) performance. One commonly proposed solution to this problem is to choose filter gains based on target behavior as determined by a maneuver detector. A final problem associated with the use of constant-coefficient filters occurs when P_D is less than one. For non-unity P_D, in order to improve tracking performance, the coefficients should be adjusted according to the input data detection sequence. Thus, although the simplicity of constant-coefficient filters is appealing, their inadequacy in many areas makes the choice of a variable gain sequence through Kalman filtering preferable when high accuracy is required.

3.2 Kalman Filtering

The Kalman filter is the general solution to the recursive, minimized mean-square estimation problem within the class of linear estimators. Use of the Kalman filter will minimize the mean-squared error as long as the target dynamics and the measurement noise are accurately modeled. In addition to minimizing the mean-squared error, the Kalman filter has a number of other advantages for application to MTT. These advantages include the following properties:

1. The gain sequence is chosen automatically, based on the assumed target maneuver and measurement noise models. This means that the same filter can be used for varying target and measurement environments by changing a few key parameters.

2. The Kalman gain sequence automatically adapts to changing detection histories. This includes a varying sampling interval as well as missed detections.

3. The Kalman filter provides a convenient measure of the estimation accuracy through the covariance matrix.

4. Through use of the Kalman filter it is possible to at least partially compensate for the effects of miscorrelation in the dense MTT environment.

3.3 Approximations and Simplifications of KALMAN Filtering

The computational requirements of the Kalman filter are sometimes considered to be beyond system capability. Each Kalman filter requires matrix multiplications of order $n \times n$, where n is the order of the state vector. Also, matrix inversions of order M, where M is the measurement dimension, are required. These c⁶mputations are usually required for at least three dimensions (such as the three Cartesian components x ,y ,z). However, the adaptivity and accuracy of the Kalman filter make it highly preferable from the performance viewpoint. Thus, considerable effort has been made to develop approximations and simplifications of the Kalman filter in an attempt to reduce computational requirements and loading without unacceptable degradation of overall tracking performance. This section discusses several of the more prominent schemes for approximating and simplifying the Kalman filter.

3.3.1 Constant Gain Filtering

Probably the simplest approximation of the Kalman filter is to use the constant gains that are obtained as the Kalman filter is allowed to reach a steady state. This approach is referred to as Weiner filtering, and, of course, it assumes that there is a steady state. This steady-state assumption may not be valid for many practical cases, such as angle tracking with changing range, or when data are missing. As an alternative to running the Kalman filter to steady state, it may be possible to find the steady-state Kalman gain and covariance from analytical expressions Another approach is to use the Kalman filter to determine a gain table that is computed a *priori,* stored and appropriately called upon. However, we take care that use of the table does not require more computational effort and storage than the Kalman filter computations which it replaces.

3.3.2 Simplified Generation of Kalman Gains

One major objection to the use of steady-state gains is the large errors, which may develop for the initial tracking phases. Kalata [3] has developed a computationally convenient means of circumventing this problem.

First, Kalata solves for the steady-state gains using the tracking index as defined by

$$\wedge = T^2 \sigma_w / \sigma_o \tag{3.7}$$

where σ_o and σ_w are the measurement noise and the system acceleration standard deviations, respectively. For example, for the $\alpha-\beta-\gamma$ tracker corresponding to the three-state Kalman filter, the relationship (again noting the factor of 1/2 that arises due to different definitions of the $\alpha-\beta-\gamma$ tracker):

$$\wedge^2 = \gamma^2 |1-\alpha \tag{3.8}$$

The gains for use prior to steady state are computed recursively using an exponential decay relationship.

The major problem with Kalata's method arises in the case of missing data (or a varying sampling interval). If an observation is missed (or the extrapolation period is extended), the Kalman gains will increase rather than exponentially decay to steady-state values. This is illustrated in fig. 3.1, where it may be noted that the gain K_1 always increases after missing data.

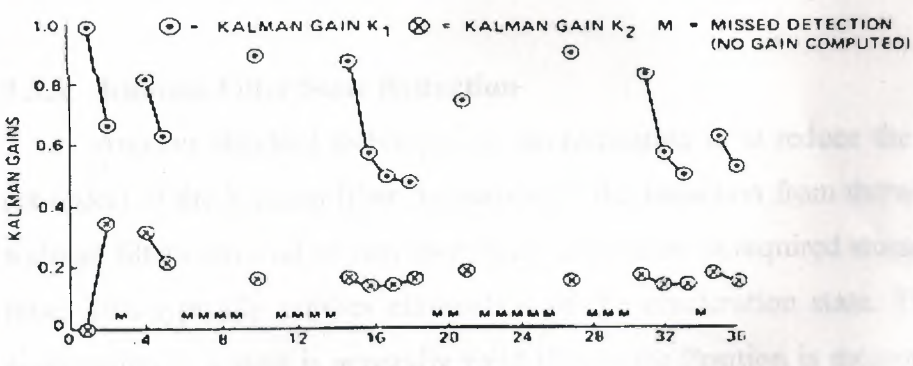

FIGURE 3.1 EXAMPLE OF KALMAN GAIN BEHAVIOR

Bridgewater presents a general algorithm for recursively computing Kalman gains With this method, changes in sampling interval, measurement noise variance, and assumed target-maneuver variance are readily incorporated into the gain calculations. This technique is applied to the $\alpha-\beta$ and $\alpha-\beta-\gamma$ trackers for random velocity and acceleration models. Steady-state expressions for $\alpha-\beta$ and $\alpha-\beta-\gamma$ trackers, which are applicable for various assumed target dynamic models, are also presented by Bridgewater.

The Kalata and Bridgewater methods both model the random input process noise as white, which differs from the more general correlated acceleration model of Singer. Thus, in general, a response similar to that found for a Kalman filter using Singer's model cannot be ensured. Use of Kalman filtering provides the covariance matrix whose terms are used for gating. Replacing the Kalman filter with a filter in which the gains are either constant or calculated recursively eliminates the necessity for the covariance matrix. However, a prediction variance (p_{11}) consistent with the gain can be simply computed by noting the relationships involved in the Kalman gain computation. For example, the position prediction variance consistent with the Kalman gain K is found through the relationship.

$$K_1 = p_{11} \Big/ p_{11} + \sigma_o^2 \tag{3.9}$$

So that

$$p_{11} = K_1 \sigma_o^2 \Big/ 1 - K_1 \tag{3.10}$$

3.3.3 Kalman Filter State Reduction

Another standard technique for simplification is to reduce the number of states (or order) of the Kalman filter. In particular, the reduction from three-state to two-state Kalman filters can lead to very significant reductions in required storage and processing time. This typically implies elimination of the acceleration state. The elimination of acceleration as a state is generally valid if only the Position is measured (no derivative measurement, such as range rate, is available) and if the sampling interval is a significant fraction of the maneuver time constant. One practical way to determine the utility of the acceleration state is to compare (through simulation) acceleration estimates in the presence of a typical target maneuver with the corresponding estimates that occur due to input noise alone for a non-maneuvering target. If random acceleration, or any other state, is eliminated as an estimated state, its effects must still be introduced into the system model. The usual technique for doing this is by introducing "state noise". Hutchinson develops a technique for determining the state noise which should be used in order to minimize the estimation error variance for reduced-order filters.

3.4 Maneuver Detection and Adaptive Filtering

The Kalman filter is determined by the assumed target kinematic model. Also, the choice of parameter for any approximate or simplified filter must inherently depend upon an assumed target maneuver model or capability. When the actual target kinematics differ from the model used for filter design, mean tracking errors will develop. The Kalman filter models the target dynamics through the use of continuous random variables statistically described by known parameters. The previously discussed Singer model in which a time-correlated acceleration is used to describe the target's dynamics exemplifies this. The Kalman filter will provide optimum estimates of target position and velocity only if this underlying target model is correct.

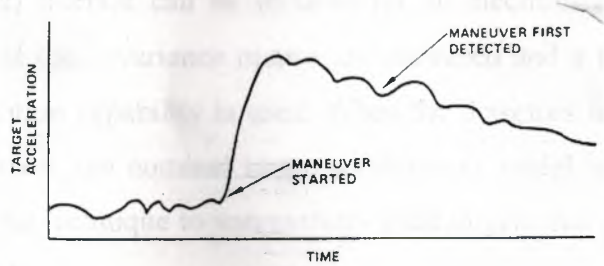

FIGURE 32 TYPICAL TARGET MANEUVER TIME HISTORY

assumption and constraints on target dynamics. Then, when the LM gate, but not the NM or SM gates, was satisfied, the filter was reinitiated using the last two data points. This is the simplest method for decreasing the effects of the time lag between the initiation and the detection of target maneuver. Fig.3.3 illustrates the form of the three gates.

Another method for maneuver detection is to examine the time history of the residual. With the use of Kalman filtering a generalized form of the distance function d^2, can be formed from the residual vector $\tilde{y}(k))$, and the residual covariance matrix S.

$$d^2(k) = \tilde{y}^T(k)s^{-1}\tilde{y}(k) \tag{3.11}$$

$$\tilde{y}(k) = y(k) - H\hat{x}(k|k-1) \tag{3.12}$$

$$S = S(k|k-1) = HP(k|k-1)H^T + R_C \tag{3.13}$$

$P(k|k-1) =$ One step prediction covariance matrix

$R_c =$ Measurement noise covariance matrix

The quantity $d^2(k)$ has a chi-square(χ^2_M)probability distribution with M degrees of freedom, where M is the measurement dimension. Thus,$d^2(k)$can be monitored in comparison to a threshold determined by the(χ^2_M) distribution.

For airborne radar tracking systems that have accurate Doppler (range rate) measurement capability, changes in target range rate may be used for maneuver detection. However, as discussed by Nelson, care must be taken in the presence of JEM.

A simple maneuver detection technique based upon the range rate residual and using the suboptimum maneuver detection method. Once maneuver is detected, the sampling (or update) interval can be reduced for an electronically scanned antenna. Also, the elements of the covariance matrix are increased and a filter model assuming larger target acceleration capability is used. When the detectors indicate that the target has ceased to maneuver, the nominal target acceleration model is restored. Simulation results have shown the technique to successfully track targets that performed maneuvers with acceleration up to 6*g*.

The problem associated with the maneuver adaptive filtering approach discussed above is that, as illustrated in Fig 3.2, there may be a significant lag between the time when the maneuver begins and when it is detected. Then, the adaptation to the maneuver through the choice of more responsive filter parameters may not occur until large tracking errors have already developed. Thus, other approaches have been proposed in which, in addition to adapting filter parameters, there is a correction of the past effects of the acceleration.

Chan defines a method for estimating the target input acceleration and updating the filter using the effects of the estimated acceleration. Thus, using this approach there may be a lag in detection, but, once detection occurs, an estimate of the cumulative effect of the acceleration is applied to the filter state estimates. Other techniques that effectively apply filter correction as well as adaptation are discussed below.

3.4.1 Maneuver Detection and State Augmentation

Bar-Shalom and Birmiwal have developed an algorithm in which a different state model is used by the filter upon maneuver detection. Before maneuver detection, an essentially constant velocity target model is assumed. After maneuver detection, tracking is performed with an augmented state model that uses an acceleration state.

This method requires the storage of previous measurements, which are used for filter reinitialization after maneuver detection. The augmented state model is used until the magnitude of the acceleration estimate is determined to be insignificant, when use of the initial state model is resumed. In addition to a full development of this method and comparative performance results, Bar-Shalom and Birmiwal also discuss other techniques and present an extensive list of references.

By storing past measurements, this method anticipates the effects of a delay between the time when a maneuver begins and when it is detected. Then, if a maneuver is detected at scan k the filter is reinitiated (using the past stored data) at scan $k - \Delta - 1$, where Δ is the effective lag time associated with the detector.

3.4.2 Use of Several Parallel Filters

Starting with Magill, a number of methods have been proposed based on the use of several target state models. The measurement data are used to decide upon the appropriate model or to obtain a "best" composite estimate based upon a statistical weighting. This approach typically requires the maintenance of several Kalman filters operating in parallel and the maintenance of concurrent *a posteriori* estimates of the relative validity of these filter models. These probability calculations are usually based upon application of Bayes' rule.

Fig 3.4 illustrates the method by showing a bank of N parallel Kalman filters. Each filter utilizes a different process model, and each filter operates simultaneously on the measurement sequence. Thus, there are effectively N hypothesized target state estimation vectors.

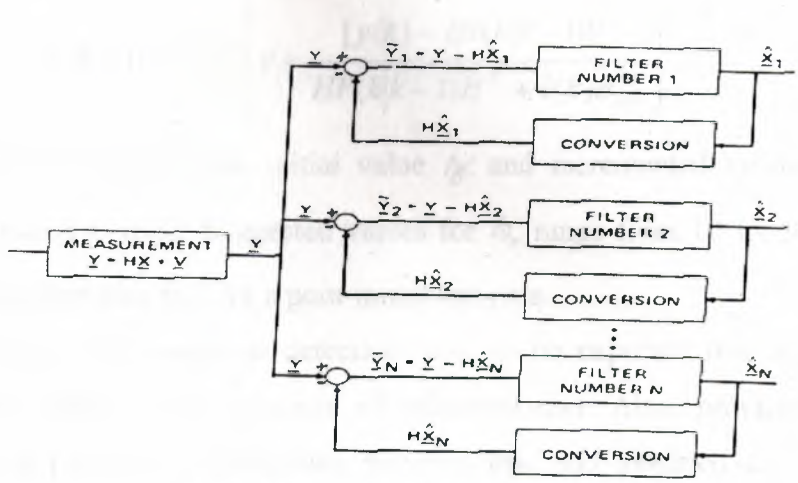

FIGURE 3.4 BANK OF N PARALLEL FILTERS

One implementation of parallel filters uses a different assumed maneuver model for each filter. For example, the simplest approach is to have just two filters in which one filter assumes essentially straight-line motion, while the other is matched to a worst case maneuver condition. The non-maneuver case is assumed until a detector is triggered and the switch is made to the filter with maneuver following ability. For an ESA system the sampling rate can also be increased at the same time that the more responsive filter is used. Thus, using this approach, upon maneuver detection, adaptation and correction are effectively applied together by switching to a more responsive filter whose output will be more representative of the true target state.

An alternative bank of N filters might be implemented to model an acceleration that could have been initiated at any one of N discrete times in the past. In this case, the maneuver characteristics are governed by the starting time of the maneuver and the N hypotheses refers to the times of maneuver initiation. McAulay and Den linger present a version of this method that operates on the Kalman filter residual sequence to determine the time at which the maneuver was initiated.

3.4.3 Adaptive Measurement Noise Estimation

A convenient method for adaptive estimation of the measurement noise variance has been developed. The technique involves the use of recursive equations that can be computed in conjunction with the Kalman filtering equations. With this method, an updated estimate of the observation variance is obtained at each measurement $\hat{\sigma}_o^2 = \hat{v}\sigma_{oN}^2$ where σ_{oN}^2 is the initial (nominal) estimate. Initially, $\hat{v}$ is chosen to be unity, and a new estimate of $\hat{v}$ is obtained at each measurement through the relationship (assuming one-dimensional observation y):

$$\hat{v}(k+1) = \frac{\hat{v}(k)}{v+1}\left[v + \frac{[y(k) - H\hat{x}(k|k-1)]^2}{HP(k|k-1)H^T + \hat{v}(k)\sigma_{oN}^2} \right] \tag{3.14}$$

The parameter v is set to some initial value Λ_D and incremented by one each time another observation is used. Suggested values for $\hat{\omega}_p$ range from 10 for a good initial estimate of noise variance to 2 for a poor initial estimate.

Finally, as with maneuver detection, it is to be expected that adaptive noise estimation will suffer in the presence of miscorrelation. Also, problems would be expected in the presence of mismatch between true and assumed target maneuver models.

CHAPTER 4

CHOICE OF TRACKING COORDINATE SYSTEM AND FILTERING STATE VARIABLE

In this chapter, a discrete formulation of the continuous state equations is presented and the considerations involved in choosing tracking coordinates are discussed. Several coordinate systems and sets of state variables that have been implemented for multiple-target tracking (MTT) are presented.

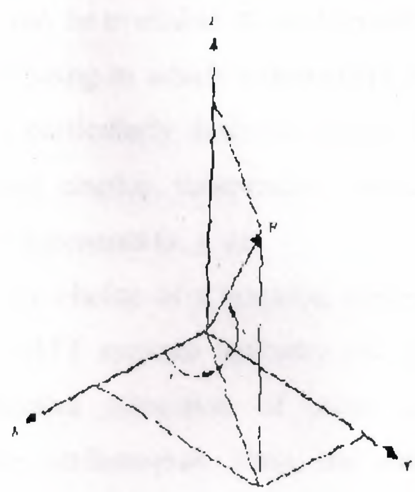

FIGURE 4.1 SPHERICAL POLAR AND CARTESIAN COORDINATE SYSTEMS

A number of coordinate Systems have been utilized for MTT. Two commonly used systems, spherical polar and Cartesian coordinates, are illustrated in fig. 4.1. The spherical coordinate system is defined by the range and two angles (azimuth (η) and elevation ($\in$)) with respect to the Cartesian (x, y, z) axes so that

$$x = R\cos\in\cos\eta, \tag{4.1}$$

$$y = R\cos\in\sin\eta, \tag{4.2}$$

$$z = R\sin\in, \tag{4.3}$$

An alternative type of polar coordinate system, defines the range vector with respect to the Cartesian coordinates by using the three direction cosines.

It is preferable to use a non-rotating (or inertial) coordinate system so that the multiple target tracks can be processed with respect to the same fixed reference. Thus, the transformation to the North-East-Down (NED) coordinate system that is convenient for airborne radar MTT systems. For airborne radar MTT applications the NED coordinate system can practically be considered inertial.

The use of Cartesian coordinates is convenient for target extrapolation, but the form of the radar measurement introduces coupling between filters.
Because of computational constraints, many MTT systems require relatively simple filtering schemes. This is particularly true for airborne radar systems, but such as not be the case for some other applications, such as sonar, where more time and more computational capabilities may be available. A requirement for simple filters implies the desirability of uncoupled filtering in which independent tracking is performed in each coordinate. For example, a particularly desirable simple filtering system f6r use with Cartesian coordinates would employ independent two-state (position and velocity) filters in each of the three components (x, y, z).

When considering the choice of a tracking system, note that experience with filtering and prediction in MTT systems indicates the primary source of error to be miscorrelation. This motivates allocation of most computing resources to the improvement of correlation performance. Thus, the discussion in this chapter will emphasize tracking coordinates for use with linear, uncoupled filters, which minimize computational requirements for the filtering and prediction functions of MTT.

4.1 North-East-Down (NED) Coordinate System

The NED coordinate system, shown in fig.4.2, is particularly useful for airborne systems, but it is also applicable for surface (ground or ship based) tracking system.

In fig. 4.2, the origin of an aircraft tracking system is the own-ship position. Then, the axes are determined by the north direction and the down direction pointing to the center of the earth. The east direction is the direction perpendicular to the north and down axes.

Note that the NED system is not strictly an inertial system for a moving platform because the platform axes are slowly changing their orientation in space as the vehicle moves over the earth's surface. However, except near the North Pole, the effects of the rotations are negligible, and the NED system is essentially inertial for aircraft platforms.

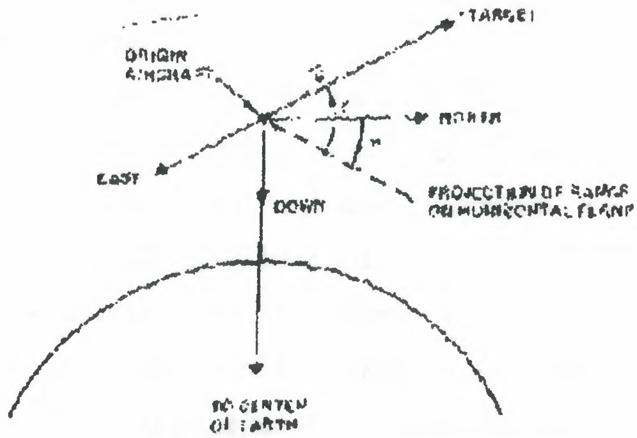

FIGURE 4.2 AIRCRAFT CENTERED INERTIAL (NED) COORDINATE SYSTEM

Radar measurements typically give the target range and angles with respect to the antenna boresight axes. However, the antenna boresight axes will be rotating as a result of changes in aircraft orientation.

4.2 Tracking in Cartesian Coordinates

Tracking in Cartesian coordinates has the advantage of allowing the use of linear target dynamic models for extrapolation. For example, given estimates of target velocity and acceleration, the Cartesian target x position prediction can be computed from the simple linear equation:

$$x(k+1) = x(k) + Tv_x(k) + {T^2}/{2}\, a_x(k) \tag{4.4}$$

The use of Cartesian coordinates has two major disadvantages. The first is that measured (or estimated) range must be available in order to transform the measurements to the Cartesian coordinates. However, measured range is not always available, as for the case of infrared (IR) sensors. This can present a problem for multiple-sensor systems using Cartesian coordinates. Also, the use of electronic countermeasures (ECM) may deny the radar range measurement.

A second disadvantage with Cartesian coordinate is that measurement errors are coupled.

The Cartesian coordinates *(x, y)* are not independent. In other words, the use of independent filters in the x and y coordinates may lead to less accurate filtering than would occur if a coupled four-state $(x, v_x y, v_y)$ or six-state filter were used.

4.3 Polar Coordinate Systems

Using polar coordinates allows tracking to be performed in the same system from which the radar measurements are obtained. Also, if; the measured range rate is available, it can be used directly in the range filter. There will be a range filter and, if azimuth and elevation angles are used, there will also be two angle filters. If direction cosines are used, there will be three angle filters. The range filter is defined in a straightforward manner, but the choice of angle filters is more difficult.

The most direct choice of polar coordinate angle filtering states would be angle, angle rate, and possibly angle acceleration. This set of states could be used for azimuth and elevation angles or for direction cosines. However, nonlinearities arise in the sense that a constant velocity target may not produce a constant angle rate (or even acceleration). Thus, in order for the filter to match the system dynamics, higher order derivatives are required in the system model, even for non-maneuvering targets. The introduction of these "pseudo-accelerations" makes accurate extrapolation difficult because estimates of these higher order derivatives are required.

An angle tracking filter using angle, angle rate, and angle acceleration can be derived by an adaptation of the Singer model, to the angular states. The adaptation is that the magnitude of the angular acceleration is range dependent. Unfortunately, results presented in the next section show that accuracy is degraded by the higher order derivatives which arise in this system.

To alleviate the problems associated with the system angular dynamics, two other sets of angular tracking states have been developed and will be discussed . The resulting angle tracking filters require the use of range and range rate estimates, which are taken from the range tracker and which are effectively assumed to be known constants, in the transition matrices. An alternative is to convert to Cartesian coordinates for extrapolation. The following discussion begins with range filtering, and then two angle tracking filters are presented.

4.3.1 Range, Range Rate Filtering

The range direction filter will use range, range rate, and usually range acceleration as states. The Singer model can be used for range acceleration. The derivation, leads to the Kalman filter defined by the state vector and transition matrix:

$$x_R = \begin{bmatrix} R \\ v_R \\ a_R \end{bmatrix}, \tag{4.5}$$

$$\Phi_R = \begin{bmatrix} 1+\omega_p^2 T/2 & T & T^2/2 \\ \omega_p^2 T & 1+\omega_p^2 T/2 & T\left(1-\beta_R T/2\right) \\ 0 & 0 & \rho_{aR} \end{bmatrix} \tag{4.6}$$

The random driving matrix Q for the Singer model, while the measurement matrix is

$$H = \begin{cases} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \end{cases} \tag{4.7}$$

Finally, the deterministic driving vector (f_R) expresses the contribution due to own-ship acceleration in the radial direction (a_{IR})

$$f_R = \begin{bmatrix} -T^2/2\, a_{IR} \\ -Ta_{IR} \\ 0 \end{bmatrix} \tag{4.8}$$

The polar coordinate systems discussed in this section eliminate the requirement for higher order derivatives in the prediction at the cost of introducing a coupling between range and angle that arises during the extrapolation. This coupling is shown by the term ω_p^2 in the range filter transition matrix, and it also arises in terms involving range and range rate.

4.3.2 The Direction Cosine, Velocity, Acceleration (ΛVA) Filter

The direction cosine (Λ) is defined in terms of a component of target position *(d)* in the north, east, or down direction, and the range (*R*):

$$\Lambda = d/R \tag{4.9}$$

In order to derive the state equation the complete the discrete representation for the target acceleration to give

$$f(k+1|k) = \begin{bmatrix} -a_1 T^2 / 2R \\ -a_1 T \\ 0 \end{bmatrix}, \tag{4.10}$$

$$\psi w(k) = \begin{bmatrix} 0 \\ 0 \\ \sqrt{1-\rho^2}\,\sigma_m r(k) \end{bmatrix} \overset{\Delta}{=} q(k) \tag{4.11}$$

Coupling with the range filter has been introduced through the range and range rate estimates used in Φ and f. The filtered estimates are used for prediction to the next $(k + 1)$ time frame. The initial estimates are the measured direction cosine and *v'1* respectively.

4.3.3 Use of Range Rate for Velocity Aiding in the Λ VA system

The accurate range rate estimate from the range rate filter can be used to improve the less accurate velocity estimates in the direction cosine filters This approach, although potentially less accurate avoid~ the complexity of a nonlinear filtering system that utilizes the range rate measurement more optimally. Farina and Pardini discuss the nonlinear filtering approach for a two-dimensional case.

Given estimates of own-ship velocity components $(\hat{v}_{IN}, \hat{v}_{IE}, \hat{v}_{ID})$ estimated range rate $(\hat{v}_R)$ and direction cosines $(\hat{\Lambda}_N, \hat{\Lambda}_E, \hat{\Lambda}_D)$, an estimate of the target component of range rate is obtained from the equation (since $\hat{v}_{TR} = \hat{v}_R + \hat{v}_{IR}$):

$$\hat{v}_{TR} = \hat{v}_R + \hat{\Lambda}_N \hat{v}_{IN} + \hat{\Lambda}_E \hat{v}_{IE} + \hat{\Lambda}_D \hat{v}_{ID} \tag{4.12}$$

Then, define

$$RES = \hat{v}_{TR} - \hat{v}_{TRA} \tag{4.13}$$

where $\hat{v}_{TRA}$ is the estimate of the target radial component of velocity formed by using quantities from the angle filters:

$$\hat{v}_{TRA} = \hat{\Lambda}_N \hat{v}_{TN} + \hat{\Lambda}_E \hat{v}_{TE} + \hat{\Lambda}_D \hat{v}_{TD} \tag{4.14}$$

$\hat{\Lambda}_N, \hat{\Lambda}_E, \hat{\Lambda}_D$ = Direction cosine estimates

$\hat{v}_{TN}, \hat{v}_{TE}, \hat{v}_{TD}$ = Estimates of the components of target velocity

The quantity *RES* represents the difference between the estimated target component of range rate $(\hat{v}_{TR})$, which is derived from the usually accurate range rate measurements, and the estimated target component of range rate ($\hat{v}_{TRA}$) that is derived from the angle filters that only use angle measurements. Because the range filter estimate should be more accurate, correction terms can be applied.

The use of range rate for velocity aiding has been found to improve velocity estimation. This technique is also directly applicable for establishing initial (k=1) velocity estimates in the angle filters using the initial measured range rate.

Finally, an estimate $\hat{\omega}_p$ of the magnitude of the line-of-sight rate vector is required for the range filter. This estimate is given by

$$\hat{\omega}_p = \hat{v}_p^2 / \hat{R}^2 \tag{4.15}$$

where, under the assumption that $\Lambda_D << 1$, valid for typical conditions in airborne radar tracking system, we write

$$\hat{v}_p^2 = (\hat{v}_N \hat{\Lambda}_E - \hat{v}_E \hat{\Lambda}_N)^2 + \hat{v}_D^2 \tag{4.16}$$

4.3.4 Azimuth, Elevation Angle Filtering System

Next, we present a pair of angle filters that uses the azimuth and elevation angles, as states. The azimuth angle filter uses the component of velocity (v_H) that is perpendicular to the line of sight and located in the horizontal plane as the second state. Similarly, the elevation angle filter uses the component of velocity (v_v) that is perpendicular to the line of sight and located in the vertical direction as the second state. The third states are accelerations (a_H, a_v) that are perpendicular to the line of sight and located in the horizontal plane and the vertical direction, respectively.

The polar coordinate system is a rotating coordinate system. Equation (4.1) gives the vector equations that relate target motion in this rotating coordinate system to motion observed in the inertial (NED) system:

$$\frac{d_x}{dt}\Big|_I = \frac{d_x}{dt}\Big|_r + \omega * x \tag{4.17}$$

$$\frac{dv}{dt}\Big|_I = \frac{dv}{dt}\Big|_r + \omega * v \tag{4.18}$$

where subscripts I and r refer to inertial and rotating, respectively;. Also, ω is the angular rate of the rotating coordinate system with respect t~) the inertial system.

Finally, **x** and **v** are the vector position and velocity.

The following unit vectors define the rotating coordinate system:

i_R =Unit vector in the direction of the range vector

i_H =Unit vector in the horizontal plane perpendicular to the range vector

i_v =Unit vector in the vertical plane perpendicular to the range vector (in the direction of $i_R * i_H$).

4.4 A Comparative Study of Angle Filtering Methods

A study was performed to compare angle-tracking performance of three filters using the angle tracking state models described above. This study used a two-dimensional Monte Carlo tracking simulation that examined the tracking of targets in the horizontal plane. The first filter used angle (η), angle rate ($\dot{\eta}$), and angle acceleration ($\ddot{\eta}$) as states. The second system used direction cosine filters (with states Λ VA) to estimate and then the estimate azimuth angle was computed from

$$\hat{\eta} = \tan^{-1}(\hat{\Lambda}_E / \hat{\Lambda}_N) \tag{4.19}$$

This method was evaluated with and without using range rate velocity aiding. The third method used the angle filter, defined in the previous section, with states (η, v_H, a_H). Table 4.1 summarizes the characteristics of the tracking methods.

TABLE 4.1
SUMMARY OF TRACKING METHODS EXAMINED

Method	Filtering Approach	Obtains Azimuth Angle Estimate (η)
One	One filter with states: ($\eta, \dot{\eta}, \ddot{\eta}$)	Directly
Two	(1) Two filters with states: (Λ_N, v_N, a_N) and (Λ_E, v_E, a_E) (2) May use $\dot{R}$ aiding to improve velocity estimation	$\hat{\eta} = \tan^{-1}(\hat{\Lambda}_E / \hat{\Lambda}_N)$
Three	One filter with states (η, v_H, a_H)	Directly

Tracking in the horizontal plane (ε =0) was considered. Two geometries were examined. For both geometries an initial range of 30nmi was assumed, and both the own-ship and the target velocity magnitudes were chosen to be flying to be 1000ft/sec. Also for both geometries, the own-ship was taken to be flying due north (v_I =1000 i_N).

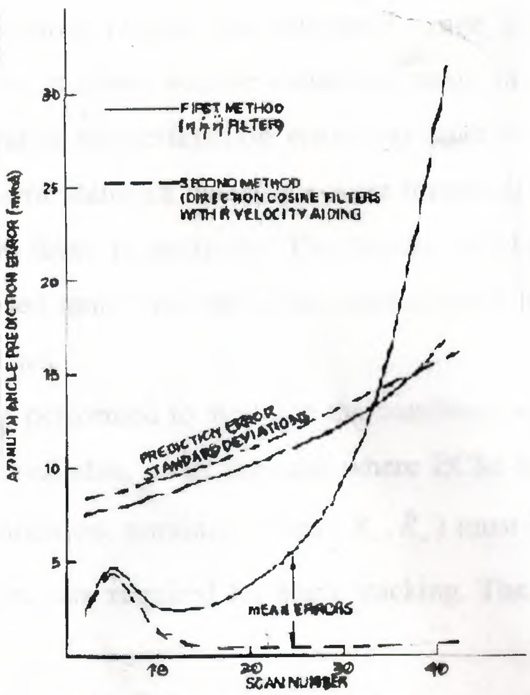

FIGURE 4.3 ANGLE TRACKING FOR CONSTANT OFFSET TARGET

the angle rate is only about 0.35 deg/sec. Thus, the inadequacy of the first method for target offset conditions where higher order angle derivatives are developed is apparent. For the second geometry the target is assumed to perform a 3*g* maneuver perpendicular to its velocity vector and in the horizontal plane. The maneuver begins after scan 15 (at time 30s) and lasts for 12s. The result is-that the target changes heading by about 66.5 degrees, so that the target velocity vector after the turn is

$$v_T = -400 i_N - 917 i_E \tag{4.20}$$

Figure 4.4 shows the mean azimuth prediction error for methods one and two, but this time we also consider the condition where range rate velocity aiding is not used for direction cosine filtering (method two). Again, angle, angle rate filtering (method one) is significantly less accurate. The use of range rate aiding shows somewhat better angle prediction accuracy, but the primary benefit was found to be in the estimation of the north component of velocity, which has little effect on azimuth angle estimation accuracy for this geometry. Again, methods two and three were indistinguishable if range rate velocity aiding was used for method two.

The angular prediction error standard deviations were about 12 mrad (0.012 rad) for all three methods for most of the encounter. The error standard deviation increased to a peak of about l5mrad at scan22 but returned to about I2mrad at scan 25.

Methods two and three require the estimated range and range rate in the transition matrix. All three methods require estimated range in the matrix Q. Thus, sensitivity to range and range rate estimation error was studied. First, the range and range rate measurement error standard deviations were increased from 1000 ft and 10 ft/sec to 20.000 ft and 200 ft/sec respectively. The increase in angular prediction error associated with the increased range and range rate measurement errors was found to be negligible for both geometries.

A second study was performed to simulate the condition where range and range rate measurements are unavailable, as in the case where ECM denies the radar range measurements. For this condition, nominal values ($R_n, \dot{R}_n$) must be used for the range and range rate estimates that are required for angle tracking. The nominal values were chosen to be

$$\dot{R}_n = 1000 \text{ ft /se}, \quad R_n = 8\text{nmi} \tag{4.21}$$

The values for R_n and $\dot{R}_n$, were chosen upon experimentation. It was found that a fairly small value for R_n was required in order to maintain acceptable dynamic response at shorter ranges. However, this led to larger random errors at ranges longer than R_n.

Figure 4.5 summarizes the results obtained for methods one and two. Again method three gave essentially the same results, as did method two, so that only the results for method two are presented.

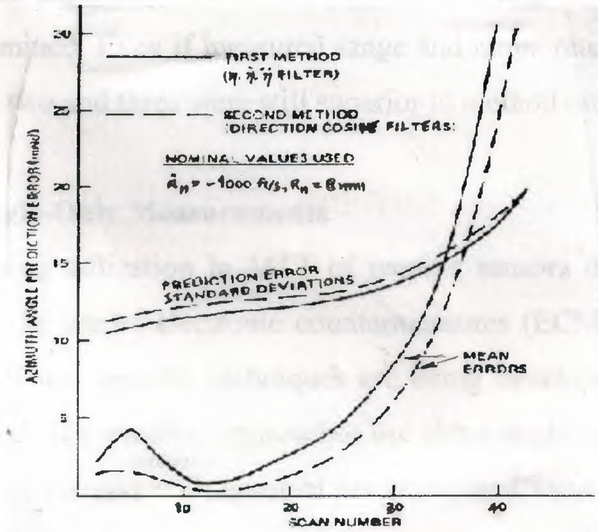

FIGURE 4.4 ANGLE TRACKING FOR CONSTANT OFFSET TARGET WITH NO MEASURED RANGE

Referring to the results in fig 4.5, method two now develops a large mean error, but the mean error for method one is about the same as was found when measured range was available. Note that method one only uses estimated range to form the Q matrix and does not use estimated range rate. Thus, the only significant effect of the lack of range and range rate information is that the gains will be somewhat too large at ranges beyond R_n. Using an even smaller value of R_n would decrease the mean error for method two, but at the cost of a larger prediction error standard deviation. As the prediction-error standard deviations are significantly larger for both methods when measured range and range rate are not available. The larger prediction-error standard deviations at ranges greater than R_n result from improper choice of Kalman gain. Using a nominal range that is smaller than the actual range makes the elements of Q too large with the eventual result that the gains are too large.

Tracking performance in the absence of range rate measurement was also examined for the second (maneuvering target) geometry. Results for this case also showed a significant increase in angular prediction-error standard deviation. However, the lack of measured range and range rate did not significantly affect the mean prediction error for this geometry.

Conclusions for the typical conditions considered are that the direction cosine and (η, v_H, a_H) angle tracking filters performed well as long as reasonably accurate range and range rate were available. When range rate velocity aiding was used with the direction cosine filtering method, the two methods gave essentially identical performances. However, the angle angle rate filter developed a large bias error for the offset target condition examined. Even if measured range and range rate were assumed to be unavailable, methods two and three were still superior to method one.

4.5 Tracking With Angle-Only Measurements

There is an increasing utilization in MTT of passive sensors that produce no range measurement. Also, the use of electronic countermeasures (ECM) can deny the radar range measurement. Thus, specific techniques arc being developed for tracking when only angle is measured. The simplest approaches use either angle and angle rate as states or the polar coordinate systems with nominal predetermined values for range and range rate. However, as shown in the previous section these simple methods can lead to poor tracking performance. Thus, a variety of other, more complex techniques have been developed.

Several sets of coordinate Systems have been proposed for angle-only filtering. However, in all cases complex filtering (usually involving the extended Kalman filter) and changes (or maneuvers) in own-ship motion are required. Lindgren and Gong discuss an application using the four Cartesian states (x, v_x, y, v_y) for tracking in the horizontal plane with angle-only measurements. The system is shown to be unobservable until the own-ship motion changes direction. Convergence may occur after own-ship maneuver, but angle-only tracking performance is dependent upon the choice of own-ship maneuver and upon the initialization method used.

Aidala and Hammel present a modified polar (MP) coordinate system that uses bearing (azimuth) angle, bearing rate, range rate divided by range, and the reciprocal of range as states. This system was derived in order to avoid the erratic performance which may be associated with the use of Cartesian coordinates. An extended Kalman filter is proposed, and again an own-ship maneuver is required.

CHAPTER 5

MEASUREMENT FORMATION AND PROCESSING FOR MULTIPLE-TARGET TRACKING

This chapter discusses how sensor design and measurement data processing relate to the overall MTT problem. The emphasis will be on radar system design but the general techniques discussed for adaptive threshold setting and target resolution arc also applicable to infrared (IR) devices.

An extremely important but difficult problem for MTT is multiple-target resolution. This problem presents a trade off issue between the determinations of the presence of multiple targets *versus* the false declaration of multiple targets given that only a single target is present

The radar signal return is frequently corrupted by spurious returns resulting from jet engine (or turbine) modulation (JEM). The resulting range rate measurement can be highly disruptive to the tracking process. The radar signal return may also be corrupted by electronic countermeasures (ECM), again leading to track disruption Thus, we can only present a brief overview of possible techniques for eliminating, or at least reducing, the effects of these corrupting signals.

5.1 Overview of Feedback Between Tracking and Detection Functions

The observation process and the rest of the tracking loop are often designed independently for MTT Systems. This dichotomy can also be seen in the design specializations where one group of analysts is concerned with sensor design and another group with tracking. However, the tracking and detection functions should be interrelated. Thus, fig. 5.1 shows how information (feedback) from the tracking loop can be incorporated into the detection process.

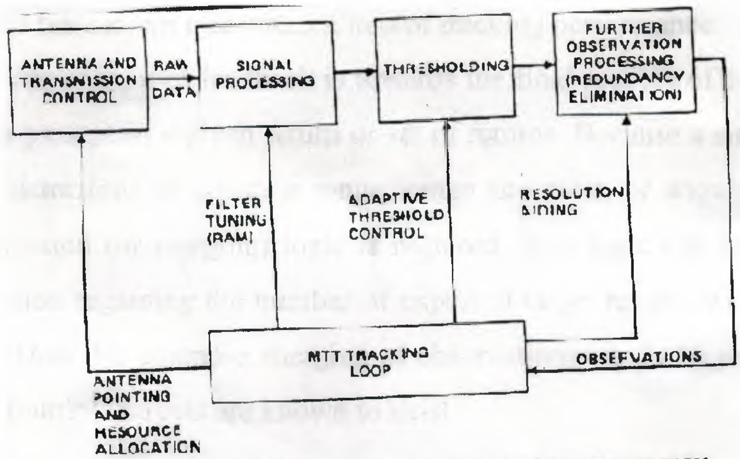

FIGURE 6-1 FEEDBACK FROM MTT LOOP TO OBSERVATION FORMATION PROCESS

Feedback can first be applied to determine antenna positioning, resource allocation, and transmission control. Basically, the approach is to sample important tracks more frequently and increase the time on target in order to improve the detection probability. In addition, the tactical situation may dictate that covertness he maintained. Thus, the transmitted power can be controlled so that tracks are maintained while minimizing the probability that the transmitted radar signal is detected by hostile aircraft or ground-based tracking systems. Possible techniques include limiting the system to intermittent search and adjusting the transmitted power according to the target range during track update illuminations.

A more complex, but potentially important, application of feedback is to affect signal processing. Special processing that cannot feasibly be done everywhere may be performed in the limited regions of expected target returns. One application in this area is to form finer Doppler (range rate) filters in the region of an expected target return. For example, in a radar raid assessment mode (RAM) the size of the Doppler filters can be reduced so that multiple targets traveling in formation can be resolved based on small differences in range rate. Another example is the performance of detailed signal processing for the purpose of determining target signature, thus indicating target type. However, typically, these techniques must also be accompanied by an increased time on target. Another application would be to apply special processing to recognize and reduce the effects of JEM returns in the vicinity of the expected target range rate.

Basically, the approach is to reduce the threshold in the region of expected target return (higher probabilities of detection (P_D) and false alarm (P_{FA}) and to increase it in regions of greater than average background (clutter) power (reduced P_{FA}, but also reduced P_D). Thus, by selective choice of the threshold it may be possible to obtain the required false alarm rate without loss of tracking performance.

Another application of feedback is towards the final process of determining the number of targets present in a given return or set of returns. Because a single target can generate several detections in adjacent range, range rite cells, or angular positions, a redundancy elimination (or merging) logic is required. This logic can be aided by the track-file information regarding the number of expected target returns within the region being processed. Thus, for example, merging of observations would be made less likely in regions where multiple targets are known to exist.

5.2 Adaptive Threshold for Enhanced Detection and Tracking Performance

Several techniques (discussed in this section) have been proposed for choosing the detection threshold based upon expected MTT performance. Burlage introduced the idea of "coached" detection; whereby the detection threshold was reduced in the region of an expected tar get return. This region was chosen using the filter covariance matrix to identify the detection cells in the vicinity of the expected target return. The lowered threshold in this region led to an increase in the probability of false alarm (P_{FA}) from $5x10^5 to 5x10^3$. However, a typical resulting increase in probability of detection (P_{FA}) was, at 17km, from about 0.5 to0.9. The conclusion was that tracking range was increased by about 10 to *15* percent as a result of the use of adaptive threshold.

5.2.1 Threshold Setting Based on Covariance Analysis

Fortmann uses covariance analysis to determine the appropriate threshold setting. Using this analysis, the effects of missed detections and miscorrelation are represented by an average contribution to the covariance iteration (Ricatti) equation. Then, the threshold setting is chosen to minimize tracking error given the constraint of the set of feasible values for $P_D versus P_{FA}$, as defined by the receiver operating characteristic (ROC) curve of the system.

5.2.2 Adaptive Threshold Setting Using One-Step Error Minimization

The approach presented by McLane is to vary the threshold setting adaptively so that the expected tracking error after the next detection attempt is minimized. The appropriate setting is based on target SNR and position uncertainty, and the derivation uses a standard detection model. A standard minimization procedure is performed and algebraic simplifications are made with the result that the desired threshold setting is

$$\lambda_M = \frac{1+SNR}{SNR}\ln\left\{(1+SNR)N_B\left[\frac{(1-\alpha)^2}{\alpha}+\frac{K_G^2}{3}\right]\right\} \tag{5.1}$$

Tracking error was examined for azimuth angle (η), range *(R)*, and range rate ($\dot{R}$). The results in table 5.1 were obtained using a combination of Monte Carlo and covariance methods. The results were derived using 100 Monte Carlo runs. The values given for tracking range (T_{90}) and tracking error standard deviation (σ_x) are normalized with respect to the values for the nominal setting.

TABLE 5-1
TRACKING PERFORMANCE AS FUNCTION OF THRESHOLD SETTING
(based upon 100 Monte Carlo runs)

					Normalized Values			
System	*Threshold Setting*	N_{TD}	N_{FC}	P_{CD}	T_{90}	σ_η	σ_R	$\sigma_{\dot{R}}$
1	Nominal (P_{FA} = 10^{-9})	99	6.2	0.994	1.0	1.0	1.0	1.0
	Fixed (P_{FA} = 10^{-5})	55	21.4	0.978	1.06	0.84	0.89	0.84
	Adaptive	53	14.4	0.987	1.09	0.82	0.88	0.81
2	Nominal (P_{FA} = 10^{-9})	170	4.5	0.996	1.0	1.0	1.0	1.0
	Fixed (P_{FA} = 10^{-5})	100	20.8	0.982	1.03	0.90	0.96	0.89
	Adaptive	98	16.9	0.988	1.06	0.87	0.93	0.87

Table 5.1 indicates that use of an appropriately chosen fixed threshold does nearly as well overall as the full adaptive method. However, P_{CD} is somewhat better for the adaptive threshold method. Also, a significant -increase in the expected number of false correlations results from use of the fixed ($P_{FA} = 10^5$) *versus* the full adaptive threshold method. Note that P_{CD} is highest (and N_{FC} is the lowest) for the nominal setting ($P_{FA} = 10^9$) because there are so few false alarms to mistake for true target returns. However, the nominal setting leads to many more track deletions and larger tracking error because of missed detections. Similarly conclusions were also obtained when an improved confirmation and deletion logic was used.

5.2.3 Combined Adaptive Thresholding and Branching

An appealing approach is to lower the threshold in the search area of an established track and then to apply branching for any questionable returns. What might be considered "questionable" could be made a function of available computer resources (empty track files). The lowered threshold will improve track maintenance and the branching can be used to maintain tracking accuracy in the presence of the inevitable increased number of false alarms. This approach is particularly applicable to maintaining tracks moving through patches of clutter.

Results given indicate that the combination of a lowered threshold and branching can lead to an improvement in track maintenance performance that is equivalent to an increase in signal power of about 2dB. It has been found that a 2dB increase in signal power is worth about an l0-percent increase in tracking range as measured by (T_{90}).

5.3 Measurement Processing for a Clutter Background

Ideally, the threshold setting within a given detection cell could be calculated from the returns in surrounding reference cells on the same scan. This approach is valid when the background interference is uncorrelated from scan-to-scan, and when the reference cells are independent and representative of the background. However, when the ground is illuminated, the same large amplitude clutter returns may persist over many scans and may be contained in several reference cells. In this case, more complex processing is required so that the processing of the return within a given detection cell will also be based on the returns found within the cell on previous scans. Using this approach, the goal is for the tracking system at least to have interclutter visibility, so that it can detect and track targets that are between large clutter returns.

The clutter background is nonhomogeneous and typically contains extraneous sources that produce returns which can easily be mistaken for true target returns. Fig 5.2 gives an overview of the combined detection and tracking processes that can be used for a clutter background. First, a constant false alarm rate (CFAR) logic is used to adjust the threshold according to the observed background signal level. One technique used is to estimate parameters of the background signal level for use in setting the threshold so that the required false alarm rate is achieved. Also, the threshold should be adjusted based on the number of threshold crossings. This is required in order to compensate for modeling errors, such as the occurrence of non-Rayleigh clutter statistics when the Rayleigh model is used.

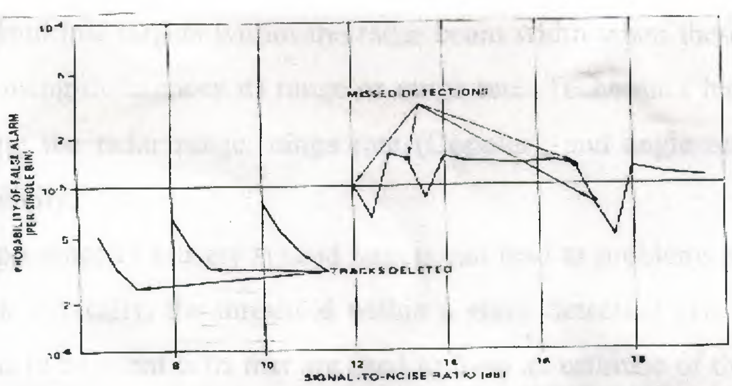

FIGURE 5-2 TYPICAL TIME HISTORY OF PFA FOR ADAPTIVE THRESHOLDING

In general, feedback from the MTT tracking loop to the activity control function will also exist. This is so that tentative tracks, which are later deleted (and thus declared to be false returns), can be used in the activity control statistics. Also, the number of stationary points, as determined by the logic discussed below, can be fed back and used to aid in estimation of the local clutter density.

The clutter point canceller (or clutter map) technique establishes tracks on persistent returns from stationary objects. Returns within a gated region of a stationary point track can be removed from further processing. Input returns that are not associated with either an existing stationary point or a regular moving track are used to initiate tentative stationary and moving tracks. Then, later data are used to determine which (if either) hypothesis is correct. If the return is a false alarm that is from neither a clutter point nor a new target, the tentative stationary and moving tracks will be deleted.

A convenient method for maintaining a stationary point track is to keep a counter with the track. The counter is incremented by amount γ whenever an observation associates with the stationary track and decremented by amount δ whenever there is a scan with no association. The track is deleted when the counter reaches zero. The counter is limited to maximum value M so that tracks can be deleted within a reasonable length of time. It may also he appropriate to vary the form of the correlation and tracking algorithms as a function of the clutter background.

5.4 Methods for Determing Target Multiplicity (Range/Range Rate Resolution)

The question of unresolved detections is one of the major issues in MTT. For the radar application, this first involves the problems of detecting and resolving multiple targets within adjacent detection elements. Also, it is important at least to determine the presence of multiple targets within the radar beam width when these targets cannot be resolved by using differences it) range or range rate. Techniques have been developed for processing the radar range, range rate (Doppler), and angle returns to determine target multiplicity.

The presence of closely spaced targets can lead to problems in detection as well as resolution. Typically, the threshold within a given detection cell will be determined by the returns in adjacent cells that are used to form an estimate of the local noise level. Thus, the presence of one or more interfering targets within the reference cells will lead to an increased threshold and a resulting loss of detectability within a given detection cell

The frequency discrimination capabilities of modern high pulse repetition frequency (HPRF) radars produce very accurate range rate estimates. Typical HPRF range rate measurement errors are on the order of only a few feet per second. This capability his led to the design of a raid assessment mode (RAM). This mode is periodically entered during the tracking process to determine if a given track represents a single target, or if it actually is a single track on several closely spiced targets.

A radar MTT system may have difficulty resolving a stream raid or a wave raid scenario. A stream raid or wave raid threat consists of several closely spaced aircraft or missiles traveling at nearly the same velocity. The raid assessment mode (RAM) is used to search for targets in a designated location in space, and to determine if a return represents a single target or a cluster of targets. The purpose of a RAM mode is to provide an estimate of the number of targets it the cluster. Resolution of the elements of the cluster is possible by utilizing longer time on target and frequency agility.

5.5 Target Multiplicity Detection Through Monopulse Angle Processing

The technique presented in this section addresses the important problem of detecting unresolved targets through angle processing. The basic technique is general and can also be applied to processing in Doppler (range rate) and potentially range as well.

To obtain a meaningful angle measurement in the case of multiple targets in the beam, an additional technique must be introduced. This is achieved by estimating the angular *extent* of the target collect ion as well as the angular *centroid.* Joint centroid-extent estimation can be accomplished because the presence of more than a single point target in the beam tends to produce extra degrees of freedom in the observable. From this an estimate can be obtained which corresponds to a measure of angular extent. If the individual targets themselves can be safely assumed to be of negligible angular extent, a large estimated value of extent serves to indicate the presence of multiple targets. In addition, the estimated value of extent is itself a measure of target separation (which can be used to ascertain individual target locations if relative target cross sections are know or can be assumed).

5.6 Measurement Degradation Due to Jet Engine Modulation and Electronic Countermeasures

Jet engine modulation (JEM) has been shown to have potentially degrading effects upon tracking performance. Also, it is known that a number of electronic countermeasures (ECM) techniques have been developed to corrupt the radar return, and thus disrupt tracking. This section briefly describes some of the most relevant characteristics of J EM and ECM and outlines approaches that may be helpful in reducing their effects for the MTT problem. However, these techniques typically require either extensive signal processing to recognize the corrupting effects during observation, or extensive additions to the MTT tracking logic in order to reduce the ultimate effects of the corrupting measurements.

5.6.1 MTT Modifications for JEM

The use of pulse Doppler radar provides a range rate measurement for tricking. However, in practice the returns from a single aircraft target often lead to multiple observations at the same range and angle, but with different range rates. Unfortunately, the correct range rate may not even be included in the observation set. The spurious range rate observations are the result of modulation produced by the motion of internal components of the jet engine. This modulation (JEM) tends to be of highest amplitude for head oil geometries and begins at about two-thirds of the usual tracking range. The modulation is also typically characterized by harmonic relationships among the returns.

The exact pattern of JEM returns is dependent upon the particular aircraft being observed and the range. However, the deviations of the JEM returns from the true target range rate (skin return) are often about the same as the expected deviations between true and expected ret urns during the conditions of target maneuver. Thus, in the absence of a skin return, a JEM return may correlate with the track and lead to a false indication of maneuver. Conversely, if the target does maneuver, a JEM return can appear closer to the predicted range rate than is the true skin return, and thus be correlated with the track ill question. In this case, the true target maneuver may go undetected. Finally, because multiple spurious returns are generated from the single target there is the potential to form multiple tracks on the same target.

There are basically two approaches to the JEM problem. The first approach is to examine the signature of the components using harmonic processing and amplitude information in order to identify returns that are likely to have been produced by JEM. Thus, once relative likelihoods of validity are determined, the correlation and track initiation processes are designed so that they heavily favor returns that have been designated as likely to be from the skin. However, this approach is highly complex.

An alternative (or complementary) approach is to modify the MTT logic to account for the potential presence of JEM. For example, one technique uses the principle that observations which satisfy gates with existing tracks in range and angle, but differ in range rate, can be identified as likely JEM returns. This logic reduces spurious track initiations. Under certain conditions of ambiguity, range rate observations either can be ignored or branch-mg type logic can be used to defer decisions on the true range rate measurement until further data are received. However, note that range and angle information can be used for track update (or new track initiation), even if the range rate observation is questionable.

As discussed by Nelson, a time-delay maneuver detector can be used so that consistency between the range rate measurements and the target flight path is established before the potentially spurious range rate is accepted. Other techniques include the use of more complex range-range rate gating procedures and the maintenance of tracks on the JEM lines in addition to the target skin return. Then, using the latter method, spurious returns corresponding to JEM line estimates can be identified, and it may also be possible to update estimated target range rate using a JEM return and the expected offset, even though the skin return is not present.

5.6.2 MTT Modifications for ECM

A wide variety of deception techniques have been developed for denying or confusing the measurements of range, range rate, or angle. The simplest technique is noise (or barrage) jamming, which can deny the measurement of range and range rate. The tracking coordinate system must be chosen so that track can be maintained in the presence of angle-only measurements. This constraint favors the use of polar coordinates over the Cartesian system. When range and range rate estimates are used in the angle tracking filters, it is necessary to provide "nominal" values for the cases where no direct range or range rate measurements are available. However, crude estimates of range can be obtained from the original range estimate (or measurements) and the line-of-sight rate. Then, eventually burn-through may occur so that a range measurement can be obtained. Finally, special filtering techniques have been developed for tracking with angle-only measurements.

Sophisticated ECM devices can corrupt the returning radar signal so that incorrect range, range rate, and angle information are received. Furthermore, these corrupting signals may be generated in a systematic manner with the intent of confusing the tracking system. For example, the RGPO (range gate pull-off) device can delay the returning radar pulses so that the tracking system "sees" the target as moving away.

A never-ending game exists between the designers of ECM techniques and those designing electronic counter-countermeasures (ECCM) techniques to counter ECM. A variety of ECCM techniques exist in order to design the transmitted radar signal and the processing techniques for the received signal to be resistant to ECM. However, the techniques developed by the MTT system designer represent the last line of defense for the total ECCM system.

For the MTT system to operate successfully in the ECM environment, it is necessary that models be developed to describe potential ECM returns. Then, as with JEM, measures of likelihood for the validity of returning observation can be used to aid the track initiation and correlation processes. Also, extensive consistency tests between the state estimates can be used to detect track degradation due to ECM before track is lost. For example, independent filters can be used to estimate range rate through measured range alone and through measured range rate alone. Consistency checks can then be used to determine the presence of range or range rate deception ECM. The important principle for the design of military systems is that ECCM (and JEM) logic should be an integral portion of the M'TT system design from inception.

CHAPTER 6

DESIGN OF A DETAILED MULTIPLE-TARGET TRACKING SIMULATION

The purpose of this chapter is to provide the tools and the framework that can be used to design a detailed Monte Carlo MTT simulation. The previous chapter discussed ways to obtain preliminary estimates of system performance. However, when evaluating a complex problem containing many random elements, such as an MTT system, it is often necessary to use Monte Carlo methods.

Briefly, the Monte Carlo approach is to examine the statistics of a random process by performing a large number of computer experiments and then compiling statistical results. Random number generators replace the random processes and nonrandom elements are simulated exactly.

6.1 Generation and use of Random Numbers

This section outlines methods for obtaining samples from commonly used probability densities. The process of random number generation usually begins by obtaining numbers from the uniform density. Computer systems typically allow the user convenient access to a subroutine that will generate uniform random numbers over the interval (0,1). Some discussion has indicated potential problems associated with nonrandom properties, such as periodicities, that may occur for certain of these uniform random number generators. However, experience with MTT simulations has shown that problems which at first may seem to be the result of "bad" uniform random numbers are later typically found to be attributable to faulty system design or to programming errors.

Perhaps the complexity of most MTT simulations and the resultant irregular manner in which random number generators arc called tends to mask inherent periodicities or similar problems. However, to be safe, some care should be exercised in the choice of a uniform number generator.

6.1.1 Conversion of Uniform to Other Random Numbers

Convenient transformations exist for the conversion of uniform random numbers to numbers from other probability distributions. Define u to be a random number from the uniform distribution over the interval (0,1). Also, define to be a number having the probability density $f(x)$, such that

$$\int_{-\infty}^{\infty} f(x)dx = 1 \tag{6.1}$$

Then, a number from distribution *f(x)* can (in theory) be generated using the relationship:

$$u = \int_{-\infty}^{\infty} f(z)dz = F(x) \tag{6.2}$$

and solving for x in terms of u.

Equation (6.2) is convenient for generating numbers from probability densities that have closed form integrals. For example, consider the exponential probability density:

$$f(s) = \frac{1}{\bar{s}} e^{-z/\bar{s}}, s \geq 0 \tag{6.3}$$

Then, a uniform random number u is conveniently converted to an exponential variable through the transformation:

$$u = \int_0^s 1/\bar{s}\, e^{-z/\bar{s}} dz = 1 - e^{-s/\bar{s}} \tag{6.4}$$

thus,

$$s = -\bar{s}\ln(1-u) = -\bar{s}\ln u' \tag{6.5}$$

Note that $u' \stackrel{\Delta}{=} 1-u$, used in (6.5), just defines another uniform random number that can be generated and used directly.

An MTT simulation will always require the generation of random measurement errors. Also, it is frequently desirable to generate random target accelerations. Both measurement error and target maneuver statistics are typically modeled through use of Gaussian random numbers.

Numbers from the Gaussian (or normal) probability density cannot be generated directly using (6.2). Box and Muller present a method which generates a pair of Gaussian variables (r_1, r_2) from a pair of independent uniform variables (u_1, u_2). The transformation is

$$r_1 = \sqrt{-2\ln u_1}\,\cos(2\pi u_2) \tag{6.6}$$

$$r_2 = \sqrt{-2\ln u_1}\,\sin(2\pi u_2) \tag{6.7}$$

The resultant r_1 and r_2 are independent Gaussian variables with 7.ero mean and unit standard deviation. The transformation from the zero-mean, unit standard deviation Gaussian r, to a generalized Gaussian x with mean μ_x, and standard deviation σ_x is

$$x = \mu_x + \sigma_x r \tag{6.8}$$

6.1.2 Generation of Time-Correlated Random Processes

It is often necessary to include the time-correlation properties of certain random variables. For example, a Gaussian measurement noise process may be correlated from one measurement frame to the next. There is no general solution to the problem of generating random numbers with arbitrary time-correlation properties while maintaining a specified probability distribution. However, there are simple algorithms for generating time-correlated random variables for the most important random processes that require modeling in radar detection and tracking simulation.

Probably the most important time-correlated random process to be modeled is the first-order Gaussian-Markov process. This process, used to model random target acceleration and correlated (colored) noise processes, is defined for quantity x by the recursive relationship

$$x(k+1) = \rho_x(k) + \sqrt{1-\rho_x^2}\,\sigma_x r(k) \tag{6.9}$$

The spectral density of the first-order Markov process is

$$S_x(\omega) = {}^{2\beta}\!\Big/_{\beta^2 + \omega^2} \qquad 6.10)$$

Because the autocorrelation function and the spectral density are Fourier transform pairs, the first-order Gaussian-Markov process can be defined by either (6.10). The spectral representation of (6.10) is sometimes most suitable for fitting to experimentally derived data. Then, after determining τ (or equivalently β) and σ_x, the process can be conveniently simulated using (6.9). The initial value, x (1), is generated as a zero-mean Gaussian with standard deviation σ_x.

6.2 Monte Carlo Simulation Design and Interpretation of Results

In this section, we outline the most basic principles of interpreting Monte Carlo simulation data that are valid for all types of system analysis.

6.2.1 Establishing Confidence Intervals

The primary outputs of a Monte Carlo simulation for MTT are estimates of such quantities as the mean tracking error, the probability of having established a valid track, etc. It is important to be able to place confidence intervals on these estimates such that the intervals will include the actual values of the quantities being estimated with a known degree of uncertainty.

Consider an estimate ($\hat{x}$) of the parameter x. Then, the confidence interval is defined by placing upper and lower limits so that

$$\Pr[f_L(\alpha,\hat{x}) \le x \le f_U(\alpha,\hat{x})] = 1-\alpha \qquad (6.11)$$

Some of the most basic outputs derived from a Monte Carlo simulation are estimates of the probabilities of occurrence of various events (such as false correlation). The unbiased estimate ($\hat{p}$) for the probability (p) of occurrence of an event that occurred n times in N opportunities is

$$\hat{p} = n/N \qquad (6.12)$$

The standard deviation of the error on the estimate $\hat{p}$ is given by

$$\sigma_{\hat{p}} = \sqrt{p(1-p)\big/N} \qquad (6.13)$$

Assume enough samples such that $n > 5$ for $\hat{p} \le 1/2$ or N-$n > 5$ for $\hat{p} > 1/2$. Then, the estimate $\hat{p}$ will have Gaussian distribution with mean p and we can replace p in (6.13) by $\hat{p}$ so that :

$$\sigma_p = \sqrt{\hat{p}(1-\hat{p})\big/N} \qquad (6.14)$$

Thus, the confidence limits of (6.6) are defined through the relationship:

$$\Pr[z_{1-\alpha/2} \le p-\hat{p}\big/\sigma_{\hat{p}} \le z_{\alpha/2}] = 1-\alpha \qquad (6.15)$$

6.2.2 Simulation Design

A great deal of study has gone into the development of methods for efficiently performing Monte Carlo simulation. The application of these methods to a problem with the complexity of MTT appears, however, still to be an open area for research.

One basic principle for efficient MTT Monte Carlo simulation is to maintain the same conditions, as much as possible, when the comparative performance of two methods is being evaluated. Thus, it would be desirable to maintain the same detection sequence and the same random measurement errors during the comparison of two correlation methods. However, this requires special effort because the order of the random number sequences readily changes as, for example, differing numbers of tracks are maintained by the different correlation methods so that different target illumination sequences occur.

One crucially important feature is to ensure repeatability so that ally given run (or runs) of a Monte Carlo simulation can be repeated with more detailed printout. This ensures that interesting, or anomalous, results can be examined in more detail without repeating the entire Monte Carlo experiment (for all runs). This can be accomplished by printing out the random number seeds (the first uniform number used) at the beginning of each run. Then, the run can be conveniently repeated for more detailed examination by initiating the simulation with the appropriate random number seed.

6.3 Selection of Evaluation Statistics

First, it is extremely helpful in the interpretation of results to plot time histories of the true target positions, the observations, and the resulting tracks that are formed. The plot will give either angle or range as a function of time.

In addition to track plots, there are three main categories of statistics that should be compiled in summary tables and, whenever possible, plotted as functions of time. The three categories, discussed below, are track maintenance, correlation, and kinematic estimation accuracy.

6.3.1 Track Maintenance Statistics

The three most important track maintenance statistics are the probabilities of having an initiated track, a confirmed track, and a confirmed track that will not later be deleted. The probabilities of having at least N confirmed tracks for a particular geometry with four closely spaced targets. Ideally, we would like to have exactly $N = 4$ confirmed tracks at all times. However, for the particular system being evaluated, confirmation of the fourth track is delayed. Later in the ruin, spurious tracks are maintained as a result of miscorrelation and the associated poor tracking performance. Finally, for the same case, fig. 6.1 shows the probability of having N confirmed tracks that will not later be deleted. Fig 6.1 indicates that a substantial number of tracks were confirmed and then were later deleted.

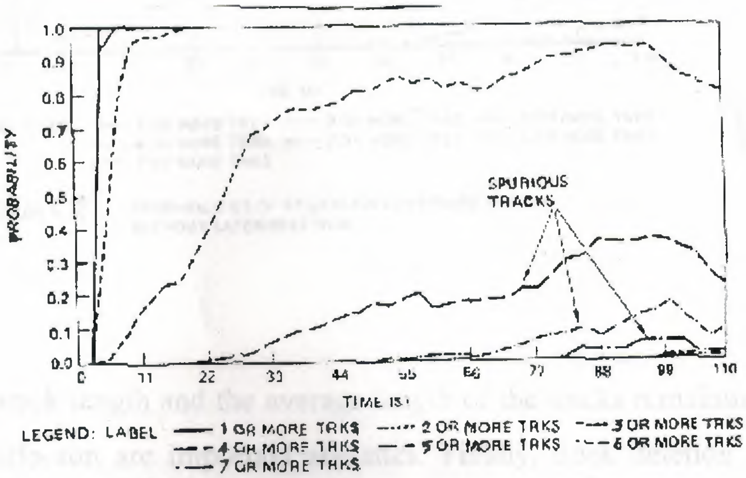

Figure6.1 PROBABILITIES OF AT LEAST N CONFIRMED TRACKS FOR A PARTICULAR CLOSELY SPACED FOUR TARGET ENCOUNTER

Another pair of interesting track maintenance statistics are the expected number of tracks and the number of targets in track. For the latter statistic, a track is assigned to the target in track which produced the last observation included in the track. Ideally, both of these numbers should equal the number of true targets. However, divergences from the ideal are shown, for example, as multiple tracks are formed on the same target. Fig 6.2 shows these statistics for the same k)ur-target case that produced fig 6.2. Note that during most of the encounter more than four tracks are expected, but the expected number of targets in track is less than 3.5.

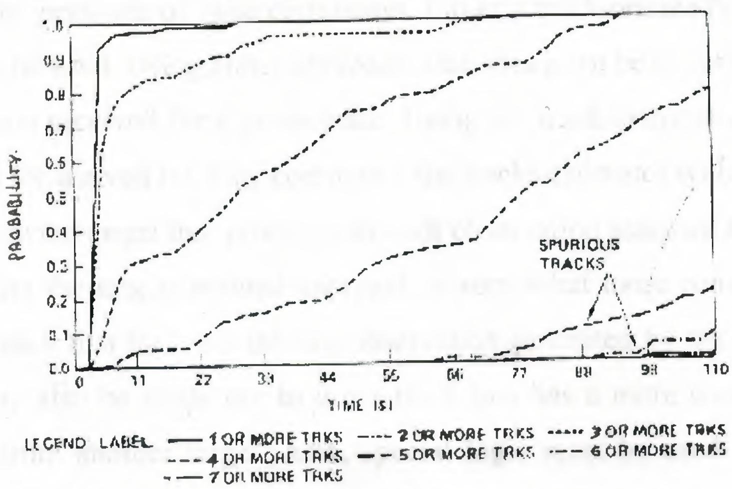

Figure 6..2 PROBABILITIES OF AT LEAST N CONFIRMED TRACKS WITHOUT LATER DELETION

Expected track length and the average length of the tracks remaining at the end of each Monte Carlo run are important statistics. Finally, track deletion statistics of interest are the average time required to delete a track on a target that leaves the scan volume and the number of premature track deletions that occur when the target in track is still present.

6.3.2 Correlation Statistics

Probably the most important correlation statistics are the probability of correct correlation *(P_{CC})*, the probability of false correlation *(P_{FC})*, the probability of correct decision *(P_{CD})*, and the probability that the gate will include a true target observation *(P_G)*. The expected number of false correlations per scan also provides a useful plot. Another interesting statistic is track depth. Track depth is defined for a given track as the number of observations which we can go back from the most recent observation before the actual target identity of an observation changes, or before the first observation of the track is reached.

6.3.3 Kinematic Statistics

The usual kinematic statistics are tile means and standard deviations of the tracking errors for such measures of position and velocity as range, range rate, angle; and target velocity components. However, problems can arise in the compilation of the statistics in the presence of false correlation. Either a track-oriented or a target-oriented approach can be used. Using either approach, statistics must be compiled based upon the last observation received for a given track. Using the track oriented approach, the error are compiled for a given track by comparing the tracks estimates with the true quantities associated with the target that produced the last observation assigned to the track.

Using the target-oriented approach is somewhat more complicated. Here, we can use the track that includes the last observation generated by the target. However, a provision may also be made not to use a track that has a more recent update with an observation from another target. Also, special logic must be used to account for the condition where tracks are dropped so that there is no longer a track on a given target.

6.3.4 Other Statistics

In addition to the statistics discussed previously, it may be desirable to compile statistics related to computational requirements. For example, the number of tracks (and hypotheses if multiple hypothesis tracking is used) formed, or, when using an electronically scanned antenna (ESA) system, the number of required track updates may be of interest. Finally, for conditions where the input measurement process is so complex that a well-defined statistical model is not available, it may be desirable to compile statistics on the input observations.

6.4.5 A Single Measure of Effectiveness

Using a variety of statistics may lead to contradictory conclusions. For example, one method may have smaller tracking errors and prematurely delete fewer tracks, while another competing method may have a closer match between the expected number of targets and the number of targets. Thus, the following general measure of effectiveness (MOE) is presented.

An effectiveness measure is computed at each scan (k) such that

$$\text{MOE}(k) = \frac{1}{N_o} \sum_{j=1}^{N_o} s_j(k) \tag{6.16}$$

Finally, to evaluate tracking and correlation performance, a rule is defined such that a target corresponds to an existing track only if the last observation generated (on a scan prior to k) by the target is included in that track and if that observation is the last observation in the track. Otherwise, the target belongs to no track and any correlation with observations from that target are taken to be false. Thus, using this rule, the condition where multiple tracks are formed on the same target is penalized.

TABLE 6.1
OBSERVATION SCORING (VALUES FOR $s_j(k)$)

	Description of Observation		
Correlation Result	False Alarm or Initial Target Detection	Observation from Target without Existing Track*	Observation from Target with Existing Track
New (Tentative) Track Formed	1.0	0.3	0
Correct Correlation with Tentative Track	Not Applicable (N/A)	N/A	0.6
Correct Correlation with Confirmed Track	N/A	N/A	1.0
Incorrect Correlation	0	0	0

* A previous detection has been received from the target in question.

Table 6.1 presents representative effectiveness values that may be assigned to various correlation events for an observation. The values range from zero (for incorrect correlation) to unity. The highest values (1.0) are assigned to correct correlation with a confirmed track and to the correct recognition of a new source. Correct establishment of a new track and correlation with an existing tentative track lead to intermediate score values (between zero and unity) if the target has been previously detected. The reasoning for this last rule is that an ideal system would have previously established a confirmed track and thus, for an ideal system, the observation in question would be correlated with a confirmed track.

6.6 Simulation Development

This section outlines the steps involved in the actual development, documentation, and verification of a detailed Monte Carlo MTT simulation. The recommended approach is based upon the development of several TWS simulations for major airborne radar tracking systems. However, the general methods are applicable' to any detailed MTT simulation.

Development of an MTT simulation typically takes about one year. Once developed, the simulation may be used for many years. It is important to remember that numerous modifications and changes in personnel involved with the simulation are to be expected. In addition, a high degree of credibility in the analysis resulting from use of the simulation requires a thorough testing of the simulation, a clear understanding of how all functions are modeled, and how these junctions influence the results obtained. Thus, emphasis on modular design and extensive documentation are worthwhile (and probably necessary) investments in time. Attempts to shortcut these processes may lead to temporary gains in development time, but in the long run such would be costly.

In order to complete any large development in a reasonable period of time, it is necessary to divide the task into smaller tasks, which can be handled independently by different people. This can lead to difficulties when the pieces are later collected to form the whole. Minimization of this type of problem requires good communication between the members of the development team and good coordination of the team. This is probably best handled through the use of a coordination focal point - the team leader. It is also the team leader's job to see that all procedures set forth for the development are complied with and to understand the design and development of the various parts of the simulation. This understanding is necessary for the coordination of the various interfaces between modules and as a check on the clarity of the documentation.

A convenient breakdown for Monte Carlo radar TWS simulation is shown in fig 6.3.

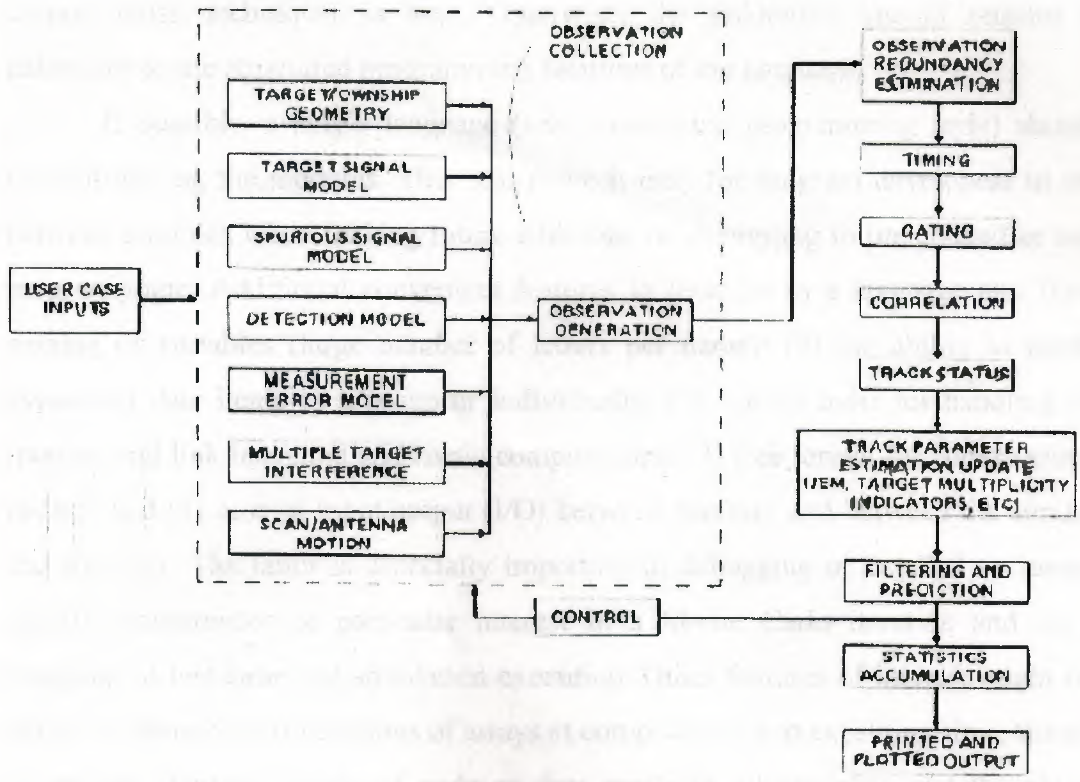

Figure 6.3 TRACK WHILE SCAN (TWS) SIMULATION OVERVIEW

The case of modification and flexibility of use of each module is enhanced by breaking down each high-level module into a module driver and several subroutines (one such breakdown of the track initiation/deletion function. However, distributing development of the subroutines among too many individuals can create so many interface points that the integration of the entire simulation becomes difficult. Although proposed methods for defining a structured program differ considerably, the following is generally accepted by many interested in software design 'hid serves as an example of a technique used with success,

1 Guidelines

At the initiation of the development, some ground rules (programming standards and practices) should he set forth as a guide to how the development will proceed and to ensure a reliable product at completion.

2 Language

An appropriate choice of programming language is essential. The most important feature of the language is the availability of structured programming controls. Without these controls, the clarity of code and ease of future modification will be limited. A language that insists on adherence to structured programming techniques (e.g., if-then-else. go-to-less/top down flow control) and does not afford many pathways around those techniques is best. Otherwise, the guidelines should require strict adherence to the structured programming facilities of the language.

If possible, a single language (and a restricted programming style) should be used in writing the modules. This will make it easy for program developers to switch between modules when making future additions or attempting to understand or correct program logic. Additional convenient features to look for in a language are: (I) open naming of variables (large number of letters per name): (2) the ability to reference associated data items as a group or individually: (3) special tools for handling arrays (pointer and link lists) and arithmetic computations: (4) free format for commenting and coding: and (5) ease of input/output (I/O) between routines and between the simulation and the user. The latter is especially important in debugging or detailed reviewing of specific occurrences of particular interest in a Monte Carlo iteration and for easy changing of test cases for simulation execution. Other features of interest might be the ability to change the dimensions of arrays at compilation or at execution time, the ability to include identical blocks of code or data easily in all modules, and the ability to restrict variables as to input or output use only. Finally, if a change in the computer on which the simulation will reside is possible during its life, consideration should be given to a language that is transportable (that is, available on many machines).

3 Module Design

Before actually programming any functions, a flow of the process should be developed as a guide for the coding. This flow should be documented in an easy to read form (avoiding use of variable names and describing the process in terms of the ideas involved) that mimics the programming language. It will later serve as both a high-level description of the process and as a valuable aid in quickly understanding the process, which will enable the reader to avoid becoming bogged down in the minute details that often obscure the larger picture.

4 Programming Style

Having chosen a language with the necessary capabilities, a set of: rules governing the use of the language should be proposed to reduce the potential for error. For example, the restriction of all communication between modules to the argument list of the call will ensure a well controlled and easily followed interface. Some global common for the entire simulation might be useful for data such as physical constants which should be the same wherever they appear in the simulation. Requirements for alphabetizing and identifying all arguments as to input or output and for defining all arguments and local variables of each routine as to type, size, and function insure against misuse of the variables.

Comments should precede each small block of code (several lines involvcd in a process) with additional comments to the side of those procedures that arc riot immediately clear otherwise. Also, each routine might be preceded by a general description of the purpose and function of the routine, noting any important features of the routine. Routines should be held to small size for ease in reading and maintenance, with subroutines used for all definable functions larger than a predetermined number of lines of code.

5 Debugging and Detailed Analysis

The option to list inputs and outputs of each module and its routines, specified at run time, should be included so as to debug the modules properly as well as understand what each function is doing under the unusual conditions which sometimes arise in one Monte Carlo iteration or another. Also, the general ability to follow the gross functioning of the processing (accomplished by listing such items as observed signal data, observation-to-track association data, and track filtering data), and the ability to repeat any Monte Carlo run in a series (accomplished by making the initial random number seed of each Monte Carlo run available) are necessary. These capabilities are required so that the designer may fully understand, and thus potentially improve upon, algorithm performance.

6 Testing

Testing is the next important part in the development procedure, and it serves to generate a great deal of the future credibility of the simulation. Once a module is developed, an extensive test plan should be formulated and documented. This test should consider the module in isolation and exercise all logic branches, so that the details of the processing may be thoroughly verified in a way which would be difficult within the larger simulation. The test documentation also serves as a demonstration of what the module is expected to do and helps to clarify the functioning of the module further to someone unfamiliar with the code. A test driver for the stand-alone module is written to execute the test plan. The test driver is then saved to allow for further detailed investigation of the function in an environment that is isolated from the larger system. The test driver will also serve as an example of how the module interface is to be handled.

7 Modification of Routines

As routines in the simulation are modified over time to add capabilities, to improve algorithms, a history of the nature of the changes made to each routine should become a part of the general description of the routine. In addition, the name of the routine should incorporate an identifier for the version of the routine. The above provisions help to ensure that the latest version of module is being used and help to trace the capabilities used in older versions of the simulation when making comparisons between recent and part simulation results Finally, very specific (non-general) processes should be isolated (as in module drivers) wherever possible for ease of future modification.

8 Simulation Output Data

The proper presentation of the data generated by the simulation is of utmost importance because these data form the basis of the decisions which will be made as a result of using the simulation.

The most convenient form for review and analysis of statistical data is plotted output. Thus, access to a good plotting package is desirable. This package should be able to label all plots adequately and plot easily readable multiple curves on a single grid.

In addition to ensemble Monte Carlo statistics, regular summary output indicating track quality for each Monte Carlo iteration (along with the Monte Carlo starting seed) is a desirable option. By examining individual runs in detail, the designer can determine the source of irregularities that appear in the ensemble statistics. Such output typically consists of a list of the tracks formed during each iteration, including track information such as the number of track drops, the time of track miscorrelations, the target originating the track, the time of track initiation, and the probability of updating the rack arid detecting the target for the entire iteration.

The collection of statistics to provide meaningful data for a multiple target multiple-track Monte Carlo simulation is riot straightforward. A major complication is that various tracks will represent different targets on different iterations and even on the same iteration if a track drop and restart occurs. In addition, an active track may be assigned to different targets during its life. One resolution of this problem is to assign a key (e.g., target identifier) to each track when it is first initiated, and then to accumulate (in a single group) all data for tracks having the same key.

Another problem source is that various processes may lead to the same statistical representation so that interpretation of the results may be misleading unless a significant variety of output representations is provided. For example, a plot of the mean and standard deviation for all tracks on a given target may not distinguish whether many tracks had a short period of large tracking error or just a few tracks had long periods of large tracking error. As another example, if we want to know how often spurious tracks are generated, simply counting the number of tracks and comparing that number to the number of targets is insufficient (because some targets may not be tracked at all, while other targets may produce many spurious tracks).

Another deleterious effect on data interpretation of tracking error is the combining of tracks at various points in their histories into the same statistics point, thus obscuring the true track response over time. For example, a time history of the tracking error for a given target may, when averaged over many Monte Carlo runs, include statistics on tentative as well as confirmed tracks, and thus will not give valid statistics for either track state. In order to control -this situation, restrictions on the data collection may be imposed. For example, the compilation of tracking statistics may be restricted to confirmed tracks.

Another situation to look out for is the accumulation of data by specific number, such as the probability of having a given number of tracks at a given time. The number changes with each Monte Carlo iteration and varies considerably from time to time, leading to a jumble of data (criss-crossed lines). An easy way to view such data is by having the lines represent "*N* or more" occurrences for each number of interest, or by showing the expected number of occurrences at each time.

Another situation to keep in mind is that a data processing period may require more time in one Monte Carlo than in another. It is, therefore insufficient to accumulate statistics on a processing period basis because these may represent widely varying times (for example, collection of data on a scan basis when the scan time may vary). A solution for this problem is to collect statistics on a time-bin (window in time) by time-bin basis, making sure to show each event in all time bins covered by its duration if a discrete count of events is desired, (such as number of track drops), care should be taken to show each event *at least once*, but *only* once. Finally, because the interpretation of data depends on sample size, the number of samples used to produce the statistics must be included along with the statistics.

CHAPTER 7

APPLICATIONS OF THE RADAR ELECTRONICALLY SCANNED ANTENNA TO MULTIPLE-TARGET TRACKING

Techniques for utilizing the powerful adaptive features of the electronically scanned antenna (ESA, agile beam or phased array radar). The ESA has the capability to perform adaptive sampling by directing the radar beam without inertia in any direction. This property gives the ESA the potential to achieve MTT performance that is significantly improved over that obtainable with the conventional mechanically scanned antenna (MSA). However, efficiently utilizing this capability requires a considerable departure from the previous track-while-scan (TWS) type of system design used with the MSA. The correlation logic becomes more complex and new problems, such as specifying adaptive illumination logic, arc introduced.

Because the MSA is mechanically gimbaled, it is almost always con-strained to a set of predetermined fixed scan patterns. These patterns can be changed periodically, such as to increase elevation coverage at the cost of azimuth. However, the inertia of the moving antenna severely limits the pointing flexibility of the MSA. On the other hand, the ESA can be repositioned within a few microseconds, using electronic phase shifting rather than mechanical gambling.

The MSA scan constraints naturally lead to the fixed sampling rate TWS system. Using the TWS approach, all targets within the scan volume are illuminated during the scan interval (or frame) and the observations are saved for processing at the end of the scan interval. Thus, for the TWS system, illumination for both search and track update is done simultaneously. Then, at the end of the scan interval, all observations received during the scan are correlated with the existing tracks. This fixed schedule greatly reduces timing and other computational complexities.

It is desirable to exploit the ESA capability for a fast update rate for selected individual targets, such as those determined to be maneuvering. However, the illumination of individual target tracks, without illumination of neighboring tracks. can lead to correlation problems for closely spaced targets. This means that if nearest-neighbor (NN) sequential correlation techniques are used. care must be taken to illuminate all members of a group before correlation is performed. Also, timing problems become more difficult as tracks are illuminated at different rates and search is intermixed with track update illumination. Thus, although the ESA offers great potential for performance improvement there are many practical problems involved in the implementation.

7.1 Enhancing Radar Detection With The ESA

Tracking and correlation performance in a multiple-target environment is very sensitive to detection performance. If observations can be obtained from all targets, the probability of miscorrelation (and, thus, of degraded tracking performance) can be significantly decreased.

There are three main techniques for enhancing detection performance with the ESA. These techniques theoretically could also be used with an MSA, but the agile beam capabilities of the ESA make them much more practical for ESA application. The first method varies the time spent during target illumination at a particular beam position (hereafter referred to as the time on target). This can be achieved by varying the number of integrated pulses used for detection.

A second technique for improving the detection performance of high PRF radar is to transmit two pulse trains with different pulse repetition frequencies (PRFs) at each beam position (known as PRF agility). Finally, a similar strategy uses several radar frequencies (known as RF frequency agility) in order to use the effects of radar target cross section scintillation to enhance detection performance. Using two or more radar frequencies enhances the probability that the radar cross-section variation will be favorable on at least one of the frequencies. The particular is a radar design question that is based upon the expected target signal-to-noise ratio *(SNR)* and the radar capabilities.

A third processing technique follows, an initial detection by a second, confirming update. The same PRF and RF, which provided the initial detection, are used for the confirming dwell because it can be assumed that these are appropriate choices insofar as they produced the initial detection.

The potential for improved detection associated with the use of the ESA. This figure compares the probability of detection that was obtained for a typical ESA system with and without use of the methods discussed above for enhancing detection. The enhanced system (as compared with the nominal) used a combination of longer time on target and RF and PRF agility.

The enhanced detection is reflected in the probability of having a track that will not later be deleted. The time on target required during a single dwell for the enhanced system was increased by a factor of over 3.5 as compared to the nominal system. However, from results, derived using the Markov chain method the total illumination time required during the entire run for the enhanced system was only about double that required for the nominal. This is because the enhanced detection scheme is more efficient in the sense that fewer misses occur and thus fewer repeated update attempts are required after unsuccessful looks.

7.2 Adaptive Sampling With The ESA

The ability to adaptively vary the update sample rate is probably the most important feature of the agile beam (ESA) radar. It is generally accepted that the sampling rate should be chosen to match target priority, expected target dynamics, and the density of the multiple-target environment. However, there are several approaches to determining the required adaptive sampling logic.

7.2.1 Use of Several Sampling Rates

One approach is to choose adaptively between three sampling rates. First, low priority or non-maneuvering targets are updated at the search scan rate. No special update illumination is required for low priority targets because the search illumination rate is sufficient. Then, two levels of faster sampling are employed for more important or more dynamic targets.

The Monte Carlo simulation considered a single target performing an S-shaped weave intermixed with segments of straight-line flight. Four levels of target maneuver were examined. The first three cases used $5g$, $2.5g$, and $1g$ target accelerations normal to the target velocity vector during the S-turns while the fourth used a randomized maneuver history based on the Singer model. Comparative results were obtained for the ESA system with adaptive sampling and for a system with fixed (2.5 s) sampling interval. Relative performance was evaluated by examining the number of tracks lost (failed to correlate within at least a maneuver gate) and the average number of updates required for those tracks that Were not lost.

The results show that better tracking performance with fewer required samples for the adaptive system. The adaptive system required fewer samples because the longest sampling interval was chosen during the periods of target straight-line motion. Finally, note that the effective allocation of update illuminations (demonstrated here) for a given track is important so that the remaining radar resources can be used to update other tracks and to search for new targets.

7.2.2 Relating Sampling Rate to Tracking Accuracy

The approach previously discussed used a simple predetermined sampling schedule. Next, several more sophisticated methods for choosing a variable sampling rate are discussed. These methods do not, however, consider the multiple-target density and the potential for miscorrelation.

Van Keuk presents an empirical expression for relating the sampling interval to the prediction error. First, define the one-step prediction error variance (σ_p^2) in terms of the first diagonal element of the Kalman filter,

$$\sigma_p^2 = p_{11}(k+1|k) = v_o^2\sigma_o^2 \tag{7.1}$$

Then, the relationship between the sampling interval, T, and the resulting prediction error can be expressed as

$$T \cong 0.4\left(\frac{\sigma_o\sqrt{\tau_m}}{\sigma_m}\right)^{0.4}\frac{v_o^{2.4}}{1+0.5v_o^2} \tag{7.2}$$

However, for non-unity P_D the relationship should be modified by the multiplicative factor P_D giving

$$T = 0.4P_D\left(\frac{\sigma_o\sqrt{\tau_m}}{\sigma_m}\right)^{0.4}\frac{v_o^{2.4}}{1+0.5v_o^2} \tag{7.3}$$

7.2.3 Allocation of ESA Between Search and Track Update

However, there always exists the trade-off between using the ESA for the update of existing tracks or for the search for new targets. Thus, the next question that logically arises is the manner for allocation of the ESA sensor between search for new targets and illumination of existing target tracks.

One approach is to maintain a fixed scan rate, but to allow enough "time slots" so that search can be interrupted regularly for track update, if required. This is, in effect, the approach of Reference [1), where it was assumed that a fixed number of pulses were available to allocate for track update. The allocation between targets was based upon a precomputed allocation schedule that would adapt to the sensed environment. Presumably, this approach could be extended so that the overall number of pulses devoted to track update of confirmed tracks could be precomputed as a function of the environment (number of targets, maneuver level of the targets, *el cetera).*

Methods, which assign a fixed number of pulses for track update, or which determine sampling interval in order to maintain certain tracking error requirements, only consider existing track quality. A method which simultaneously compares both the expected benefits from updating existing tracks and the benefits from search is preferable. Utility theory provides a convenient structure under which this comparison can be made. Also, a significant difference between the utility the6iy approach, discussed next, to track update scheduling and the previously discussed approaches is that the former controls the actual prediction error, whereas the latter only controls the average prediction error.

A utility theory based allocation method, was used to determine when to employ the ESA for search and when to perform track update. We define the utility for search based upon the expected number of undetected targets, and thus it is a function of the time since the particular search segment being considered was last scanned. The utility for track update is based upon the assumed target importance and the ratio of the prediction-error standard deviations (as supplied by the Kalman filter covariance matrix) to the desired (or assumed acceptable) estimation-error standard deviations. Utility calculations are made for the options to update each existing track and to search for new targets. Then, whenever the ESA becomes available for reallocation, the option with the highest expected utility gain (marginal utility) is chosen.

The time history starts with the conclusion of a search on bar 2. Updates on tracks I and 2 are commanded for the next two intervals. During the next search, target 3 is detected (with track 3 being initiated) and immediate update is called for track 3 after the search is completed. Thereafter, the antenna alternates between search (on bars I through 4) and track update. Insofar as target track I is given a higher importance weighting and because it is assumed to require greater accuracy, it receives more frequent updates and thus maintains a lower estimation error. The estimation error is presented on the scale as a normalized ratio of the estimation-error standard deviation (taken from the Kalman filter Co-variance matrix) to the input measurement-noise standard deviation.

As radar resources are allocated for track update illumination, there must be some degradation in search performance. For example, this degradation can be measured by the range at which an initial detection is received from a new target entering the scan volume. Results have shown, however, that by efficient allocation it is possible to achieve significant gains in tracking performance (over that achieved with a fixed update rate) with minimal loss in search performance.

7.3 ESA Techniques For Improving Nearest-Neighbor Correlation Performance

The adaptive update rate and the enhanced probability of detection features of the ESA can be used to improve correlation performance against closely spaced targets. For simplicity, we assume nearest-neighbor correlation techniques. However, the capability of the ESA to improve the information presented to any type of tracker-correlator should lead to comparable improvements for all methods.

The simple model to be examined can only give a preliminary indication of expected correlation performance because only a single correlation event is considered. To obtain a more complete picture of how false correlation occurs and how this leads to track degradation, it is necessary to examine an entire encounter history for a group of closely spaced targets. This must be done through Monte Carlo simulation. Thus, correlation performance derived using Monte Carlo simulation will also be presented.

7.3.1 Correlation Results from a Simple Two-Target Model

Assume there are two closely spaced targets with separation (Δx) in the single dimension x. The standard deviation of the prediction error can be defined by

$$\sigma_P = v_o \sigma_o \tag{7.4}$$

Taking P_D to be 0.7, typical values for v_o, were found to be 0.6, 0.85, and 1.25 corresponding to sampling intervals 0.5, 1.0, and 2.0 s, respectively. Define D to be the ratio of the target separation to the observation standard deviation ($D = \Delta x / x_0$). Finally, again for simplicity, assume that the measurement and prediction errors are the same for both target tracks and that there are no false alarms nor new targets detected. Next, we consider correlation performance using this simple model and these parameters.

Even for the simple one-dimensional example described above, the analytical expressions for the probability of false correlation are complex. However, Monte Carlo simulation is very simple for this case. in general,

$$P_{FC} = 2(1 - P_D)P_D P_{FC1} + P_D^2 P_{FC2} \tag{7.5}$$

In general there appears to be a complex interrelationship involved in determining the relative merits of sampling more often *versus* spending more time on target to insure detection for a given sample. This conclusion is also apparent from the tracking results of and the Monte Carlo correlation results presented in the next section. Results given here for the two target case favor decreased sampling intervals, while the Monte Carlo results to be presented next, derived using encounter geometries with three and four targets, favor enhanced detection.

7.3.2 Monte Carlo Correlation Performance Evaluation

A Monte Carlo simulation was used to compare correlation performance of tracking systems in a closely spaced target environment. The simulation included a conventional (MSA) system and an ESA system with enhanced detection and adaptive sampling capability. The enhanced detection was again modeled by using two independent looks at the target so that the new probability of detection. The adaptive sampling logic chose between sampling intervals of 5.0, 2.5, and 1.25 s. The shortest sampling interval was chosen by this logic for closely spaced targets. The MSA system used a fixed sampling interval *(T=2.5* s) and the standard single-look probability of detection.

Define D to be the ratio of the target angular separation to the angular measurement-error standard deviation. For spacings in order of D=3.5 or more, the adaptive sampling feature of the ESA was found to significantly improve tracking performance. For example, consider a head-on geometry in which three targets flying with spacing D=4.2 are approaching the tracking radar. Compare performance for a conventional (MSA) system with an ESA system that only utilizes the adaptive sampling feature. The first criterion is the probability of false correlation (P_{FC}). A false correlation is defined to occur when the return associated with *a* particular track is not from the same target that produced the previous correlating return. The second criterion is the number of tracks that became degraded as the result of false correlation, and therefore resulted in deletion. Tracks that were deleted simply as the result of missed observations were not included in this sum. The final criterion is the normalized standard deviation of the estimation error in the component of target velocity normal to the true velocity vector and in the horizontal plane (denoted the target cross velocity). The error is normalized with respect to the value found for the conventional (MSA) system.

The proportion of false correlations is small (one percent or less) for both Systems. However, the ESA system has significantly fewer deleted tracks. Also, the velocity estimation error is smaller for the ESA system.

Enhanced detection was again defined to be the application of a second independent detection attempt. This is an approximation of what can be effectively achieved by an ESA system through the use of adaptive RF and PRF selection. Then, upon applying enhanced detection, the number of deletions is significantly reduced even without adaptive sampling. No further correlation improvement is noted when both enhanced detection and adaptive sampling are applied to this case.

7.4 Implementation of Multiple-Target Tracking Logic For an ESA System

Previous sections have detailed the potential benefits of using the ESA for MTT. These benefits are also well documented in the tracking literature. However, the potential problems associated with using the ESA for MTT are typically not mentioned. Thus, this section discusses various implementation issues (and problems) and outlines approaches for their solution. The choice of techniques remains an open issue and is highly application dependent.

For most applications the sequential nearest-neighbor (NN) correlation approach seems most direct. However, as indicated in the discussion to follow, the logic required to perform sequential NN correlation, while efficiently using the properties of ESA, can become quite complex.

The problem inherent with the use of sequential NN correlation is the uncertainty associated with making correlation decisions under difficult conditions with insufficient information.

7.4.1 Sequential NN Logic for ESA MTT

An important feature of the ESA in its application to MTT is that the functions of search and track update can be decoupled. First, priorities are established to determine if the antenna should search for new targets or update existing target tracks. Then, detection threshold settings should be set separately for these two functions.

As the ESA searches for new targets it may also receive returns from targets already in track. Thus, as search observations are received they should be compared, using gating relationships, with the predicted positions of all existing tracks. For this comparison to be accurate the prediction times must either be variable (performed as observations are received) or all track predictions should be made in several short steps throughout the scan interval. Observations that do not satisfy the gates of any existing tracks can immediately be used to initiate new tracks, which should be updated as soon as possible thereafter. However, those observations that do satisfy gates of existing tracks present other problems.

Except in the most clear-cut situations, there are potential problems associated with assigning search observations to tracks before all search observations are received. As previously discussed, a simple example illustrating why all targets should be illuminated before assignments are made. Thus, the immediate assignment of search observations to existing tracks should only be made if the search observations satisfy the gate of a single track.

One solution to conflict resolution with search observations is to save all conflicting observations until the end of the search scan and then to perform correlation using the assignment matrix approach. This solution, in effect, mimics the MSA TWS solution, but can lead to the problem of track update with "stale" search observations. This can occur when a track being considered for update with a search observation is updated with an observation received during track illumination. If the track illumination occurred after the search observation was received, it may be best not to use the search observation for track update.

Next, considering track update illumination, one approach to MTT with an ESA is to treat the problem as a set of independent single-target tracking systems interleaved with the search for new targets. For example, existing tracks could receive update illuminations at a fixed update rate with each track being illuminated individually. However, the difficulties will arise if this approach is used for closely spaced targets. Thus, in order to reduce false correlation, it is necessary to identify interacting target tracks and to illuminate these tracks as a group.

Target tracks are defined to be interacting if a return from one target has a significant probability of correlating with a track from another target. This condition can be identified by performing a proximity test (whereby all predicted target positions arc compared), or by noting when observations fall within the gates of multiple target tracks. Then, when interacting target tracks are recognized they should be illuminated as a group.

Whenever possible, it is desirable to sample at a sufficiently high rate so that the uncertainty in predicted target position does not lead to the true position being outside the beam width of the initial commanded antenna position. However, due to the requirements for search and update of other tracks or due to missed detections, this condition cannot always be satisfied. Thus, a local search may be required even for the update illumination of a single isolated track. Then, because overlapping beams arc used, there will again be the necessity for observation redundancy elimination.

Once observations are received, the use of multiple gating tests is appropriate in order to determine subsequent data processing and illumination requirements. For example, using a standard gate (SG) and a maneuver gate (MG) can lead to immediate observation-to-track assignment whenever an observation satisfying the SG is received. However, if only the MG is satisfied, it would be appropriate to attempt immediately to obtain a confirming second observation. Then, if a maneuver is confirmed, it is desirable to increase the filter covariance matrix and to sample at a faster rate as long as the target is determined to be still maneuvering.

7.4.2 Multiple Hypothesis Approach to ESA MTT

Using the measurement-oriented MHT method, the full capability of the ESA theoretically can be achieved. New hypotheses are formed as observations are received and the ultimate correct correlation of observations-to-tracks will be less dependent upon the simultaneous illumination of all target tracks. However, it is expected that it will be necessary to maintain fewer hypotheses if closely spaced targets can be illuminated as a group.

Sensor allocation is less direct when the MHT approach is used. Specifically, there may be many more tracks contained in the multiple hypotheses than would be formed using the sequential NN method. Thus, the question arises regarding which tracks should be given update illumination. One approach is to illuminate the tracks within the most likely hypothesis. An alternative approach is to illuminate the tracks according to their probability of validity (as computed using all hypotheses).

Finally, it should be noted that efficient use of the ESA ought to make the MHT method more effective. Adaptive sampling can be used to obtain data in order to resolve difficult correlation decisions quickly, and thus reduce the number of required hypotheses. This also leads to the requirement for efficient sensor allocation.

CHAPTER 8
CONCLUSION

During target tracking the waveform parameters are typically matched to the target characteristics. In any search application the waveform parameters are typically dictated by the need to search a given amount of space in a given amount of time and the processing implication of that goal.

The generic PRF categories were discussed including low PRF(HPRF).LPRF is well suited for most ground-based application. Some airborne systems use LPRF to detect aircraft targets, especially at the tower RFs, as well as to detect surface targets below X-band. HPRF is better suited for most clutter-limited search applications, such as faced by an airborne tactical X-band radar. MPRF provides better detection in side lobe clutter and relaxes hardware requirements relation to HPRF.

The choice of coordinate system is a complex design question that depends on the application and the computational resources available. To provide a convenient framework for discussion, two simplifications were made. First the discussion was limited to Cartesian coordinate systems and, second it was assumed that computational considerations were not a major factor in selecting the coordinate system. Relative to these two restrictions, a reasonable complete discussion was presented of the coordinate systems needed fir efficient three-dimensional radar tracking. Error can occur due to the difference in units.

The choice of methods for filtering and prediction is usually the first tasks facing the designer of an MTT system, and an over whelming variety of approaches exists. Experience with airborne radar systems has shown the versatility of Kalman filters to be almost indispensable when dealing with the problems presented by missing data, variable measurement noise statistics, and maneuvering targets with the variable dynamic capabilities. If possible, reduced-state Kalman filters should be used. For example, before a designer proposes a three-state Kalman filter, including an acceleration state, he should ascertain that the data rate is high enough to allow accurate estimate acceleration.

Fixed-coefficient filters may be required as a result of computational limitations if the sampling internal is short or if many targets must be handled problems associated with transient response and missing data must be expected unless some adaptive gain calculation performed. However, the closed-form expressions for tracking performance with $\alpha-\beta$ and $\alpha-\beta-\gamma$ trackers are often useful in preliminary design filters and performance prediction, even if Kalman filters are eventually used.

Tacking performance with even the best-designed filter may become very degraded in the presence of miscorrelation. The effects of miscorrelation can completely invalidate the Kalman filter. Covariance and lead to divergence. Thus for tracking in dense multiple-target environment, the emphasis should be on developing correlation logic. The filtering techniques should be kept as simple as possible in order to accommodate the computational requirements of data association.

The observation-to-track correlation fir data association problem is the key element of MTT. There are basically three regions considering with MTT data correlation. These comprise a region of unambiguous correlation for widely spaced targets, an unstable region where highly inaccurate tracking may occur, and a region for closely spaced targets. Where miscorrelation occurs but tracking remains stable.

First, for sufficiently large target spacing unambiguous correlation occurs. This region of unambiguous correlation can be expanded by improving correlation techniques and detection performance. Also, for most cases, sampling at a faster rate can expand this region. Next, an unstable region has been identified. Miscorrelation frequently occurs in this region. The result is erratic track performance and frequent premature track deletion leading to a very inaccurate assessment of the target environment. Results show that this region may occurs for target angular separations of about two or five times the angular measurement-error standard deviation. The extent of the unstable region is also a function of the sampling rate and the probability of detection. Faster sampling decreases the size of the unstable region.

Finally, the lower region. For very closely spaced targets miscorrelation will occur without an associated large number of tracks being degraded and lost. Miscorrelation leading to unstable tracking can be decreased by increasing the probability of detection, by decreasing the sampling interval, or by using improved correlation methods.

Since, tracking and correlation performance in a multiple-target environment is ry sensitive to detection performance, if observation can be obtained from all targets, e probability of miscorrelation can be significantly decreased. Thus, enhanced tection can be achieved by the ESA.

There are three main techniques for enhancing detection performance with the SA. The first method varies the time spent during target illumination at a particular ean position. A second technique for improving the detection performance of a high RF radar is to transmit two plus trains with different pulse repetition frequencies PRFs) at each beam position. A third processing technique follows an initial detection y a second confirming update. The same PRF and RF which provide d the initial letection are used for the confirming dwell because if can be assumed that these are ppropriate choice insofar as they produced the initial detection.

The immediate advantage of the ESA is enhanced quality tracking and limination of mechanical errors. A secondary advantage is that search can be more fficiently implemented since turn around times are eliminated, more exotic scan patterns are possible, and alert/confirm logic can be employed to lower the FAR at no expense to that scan rate or detection performance. The ESA also allows the detection, resolution, measurement and confirmation processes to be independently optimized and thus performance is improved. The disadvantages of the ESA include loss of effective aperture, cost, weight, and (for an airborne radar) clutter broadcasting.

REFERENCES

[1] Samuel S. Blackman, *Multiple-Target Tracking with Radar Application,* Artech House, Inc.NY, 1986.

[2] Philip L. Bogler, *Radar Principles with Applications to Tracking System,* John Wiley & Sons, Inc.NY, 1990.

[3] George Biernson, *Optimal Radar Tracking System,* John Wiely & Sons, Inc.NY.1990.

[4] Benedict T. and g. Bordner, *Synthesis of an Optimal Set of Radar Track-While – Scan Smoothing equations,* IRE Transactions on Automatic Control, AC-7, July 1962.

[5] Papoulis A., *Probability, Random Variables and Stochastic Processes,* NY, McGraw-Hill, 1965.

[6] Hovanessian S.A., *Radar System Design and Analysis,* Dedham MA, Artech House, 1984.

[7] Associate Prof Dr Sameer Ikhdair, Notes of Electromagnetic 1,2. Notes of Radar Systems 1999,2001