



# **NEAR EAST UNIVERSITY**

## **Faculty of Engineering**

### **Department of Electrical and Electronic Engineering**

#### **Pulse Width Modulation Techniques Used In Power Electronics**

#### **Graduation Project EE- 400**

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Finally we also wish to thank our parents that gave us this opportunitie to study in NEU and support us.

## LIST OF SYMBOLS :

$H(\phi)$	= Heaviside unit function
$M$	= modulation index
$P$	= PWM pulse number
$R$	= frequency ratio = $\omega_c / \omega_m$
$T$	= time period of carrier signal
$T_1, T_2$	= time instants
$V_a, V_b, V_c$	= inverter phase voltages
$V_d$	= direct-axis voltage
$V_q$	= quadrature-axis voltage
$V_n$	= nth-order harmonic voltage
$a$	= complex operator = $e^{j2\pi/3}$
$j, k, n$	= integers
$\text{mod}$	= $a \text{ mod } b$ is the remainder when $a$ is divided by $b$
$t$	= instantaneous time
$t_1, t_2$	= time instants
$t_p$	= width of modulated pulse
$y$	= vector of pulsed-wave levels
$\alpha_k$	= switching angle
$\alpha$	= vector of switching angles
$\phi$	= phase variable
$\omega_c$	= angular frequency of carrier signal
$\omega_m$	= angular frequency of modulating signal



## ABSTRACT

Power electronics is the technology that links the two major traditional divisions of Electrical Engineering, namely, electric power and electronics. Power electronics is popular for technical as well as economical reasons. Now a days, electronic power generation, transformation, transmission and distribution are in AC, but almost all the terminal equipment used in industries, laboratories, locomotion, agriculture and households require DC power. In order to satisfy these requirements, easy conversion of AC power to DC power is essential.

Voltage control in voltage-fed inverters has been a major area of interest regarding variable speed AC drives. Pulsewidth modulation methods aim to achieve control of the voltage output of an inverter over the maximum possible range and with minimum distortion. Also, the main features of these techniques regarding harmonic performance and implementation problems are also pointed out.

The increasing availability of digital computers now makes the computer-aided design of power electronic systems an attractive and cost-effective development tool. The detailed development of an extremely versatile PWM computer modelling package which can be used as a "stand-alone" package for harmonic analysis, or alternatively as a "building-block" for developing more complex systems. The capabilities of the package are demonstrated, using a number of examples of single-phase and 3-phase PWM inverters, which serve to highlight a number of important operational characteristics of PWM inverters. The validity and accuracy of the computer simulations are confirmed, using experimental results obtained from a microprocessor – controlled PWM inverter drive systems.

## **1. INTRODUCTION :**

### **1.1. Power Electronics History and Applications Areas :**

Power electronics is the technology that links the two major traditional divisions of electrical engineering, namely, electric power and electronics. It has shown rapid development in recent times, primarily because of the development of semiconductor power devices that can efficiently switch large currents at high voltages, and so can be used for the conversion and control of electrical energy at high power levels. The parallel development of functional integrated circuits for the controlled switching operation of power electronic converters for specific applications has also contributed to this development. Power electronic techniques are progressively replacing traditional methods of power conversion and control, causing what may be described as a technological revolution, in power areas such as regulated power supply systems, adjustable speed DC and AC electric motor drives, high voltage DC links between AC power networks, etc. The need to include power electronics in the undergraduate curriculum for electrical engineers is now well accepted.

The power semiconductor devices, such as the diode, thyristor, triac and power transistor are used in power applications as switching devices. The development of theory and application relies heavily on waveforms and transient responses, which distinguishes the subject of power electronics from many other engineering studies.

Generally speaking, electronics can assist the engineer in industry in the solution of two fundamental types of problems. The transformation of electrical energy and the execution of process analogues for functions, such as measuring, counting, sorting, etc. By the combination of the above two applications, various types of energy transformation and their control are possible.



The development of high-power semiconductor devices has facilitated electronic control techniques for electrical power control in a simple, economic and efficient manner. Thus, a new area of power electronics has now emerged and established its position firmly in the border area of electrical power and electronics. While electrical engineers can now conveniently replace the old and bulky methods of power control through the use of small electronic devices, for electronic engineers it has extended their field of trading and expertise into a world where currents, voltages and power are measured in macro-units rather than the usual micro-units.

Power electronics is popular for technical as well as economical reasons. Now a days, electronic power generation, transformation, transmission and distribution are in AC, but almost all the terminal equipment used in industries, laboratories, locomotion, agriculture and households require DC power. In order to satisfy these requirements, easy conversion of AC power to DC power is essential. The conversion of AC to DC power at different frequencies and DC to AC power can be effected through power electronics in a very dependable and economic manner.

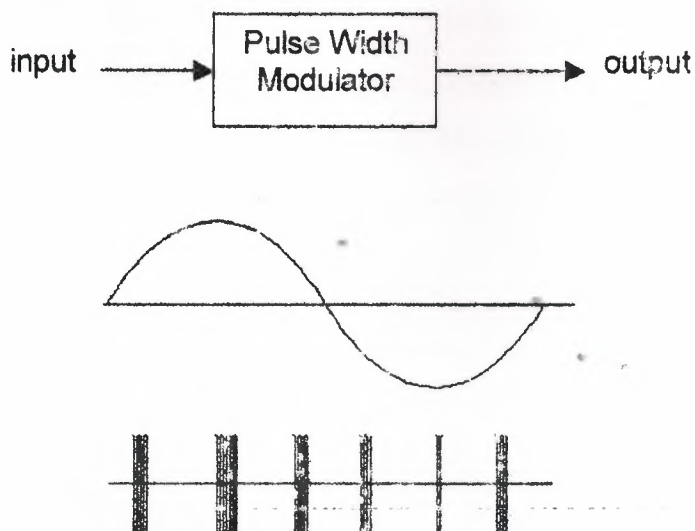
Power electronics occupies an indispensable position in the field of battery-charging uninterrupted power supply, electroplating, electrolysis, galvanisation and welding. It also plays an important role in all sorts of electric drives and lighting control. The techniques developed during the past few years enable improved and more efficient manufacturing methods, accurate control and regulation of almost every kind of process. By means of electronic control, mechanical drives can be given almost any desired speed-torque characteristics, the control apparatus being to all intents and purposes, inertialess and practically instantaneous in action. Feed drives of machine tools, multimotor drives in rolling mills, spinning machines, wire drawing mills, lifts and many other drives may be given the required characteristics by means of electronic control. Electronically generated high-frequency energy offers possibilities in the woodworking and plastic industries for economical

production of furniture, plywood and plastic articles, Hard-ening, soldering or smelting of metals by high-frequency energy increases the production of metal goods and contributes to improvement of quality of late, power electronics has assumed an extremely important role in modern main-line electric traction and power supply for urban transport systems as well as in High-voltage DC transmission.

## 1.2 Pulse Width Modulation and Application Areas :

Modulation means changing one characteristic of a voltage or current in response to changes that occur in another voltage or current. In pulse width modulation, the pulse widths of a rectangular waveform change as the amplitude of another waveform (the modulating voltage) changes. When the modulating voltage is large, the pulse widths are long, and as the modulating voltage decreases, the pulse widths become narrower.

Example of PWM;



**Fig. 1.1 Pulse Width Modulation**



Pulse Width Modulation is a form of analog control in which the duration of the conduction time of the output transistor or transistors in a switching-regulated power supply is varied by modulating the bias on the gate or base of the transistors in response to changes in the load. This keeps the output voltage of the power supply constant over varying operating conditions.

Pulse Width Modulation technique is a control within the inverter and is also known as variable-duty-cycle regulation. In the PWM control scheme the DC link voltage is obtained by an uncontrolled bridge rectifier, and the output voltage and frequency are controlled in the inverter itself. There are various PWM techniques but sinusoidal PWM is most widely used.

## 2. PULSE WIDTH MODULATION METHODS :

### 2.1 General Description :

In variable-speed AC drives which utilize voltage-fed inverters, control of the voltage and frequency output of the inverter feeding the AC motor is essential for torque and speed control of the motor. The classical approach has been to use a voltage-fed square-wave inverter fed by a variable DC voltage source. Variable DC voltage is required since the only way to change the fundamental voltage of a square-wave is to change its amplitude. The variable dc-link voltage is obtained from a phase-controlled rectifier, the output voltage of which is smoothed by an LC-filter. The dc-link square-wave inverter has a number of drawbacks. First, the rectifier output has a high ripple component, particularly at low voltage values, requiring a large filter capacitor for obtaining a relatively smooth voltage. The presence of this large filter capacitor causes the dynamic response of the system to be slow, thus creating stability problems. Secondly, the output of a square-wave inverter contains high-amplitude low frequency harmonics. Therefore, the losses of a motor due to the resulting harmonic currents tend to be high. Thirdly, if the inverter is a forced - commutated thyristor inverter, the current commutation capability is directly dependent on the dc-link voltage. Hence, at low dc input voltages, the commutation capability will be limited.

Pulse-width modulation techniques render possible both voltage and frequency control within the inverter itself. Hence, a variable voltage dc-link is not essential. A PWM inverter is usually fed by an uncontrolled diode bridge rectifier with a small filter at its output. The power factor presented to the AC supply is high and is independent of motor power factor.

Various PWM strategies have been devised for controlling inverters in variable speed drives. The common principle in all these strategies is to introduce notches

in the basic square-wave pole voltage, such that the resulting periodic waveform has the desired fundamental frequency and amplitude.

PWM techniques can be classified into the following categories

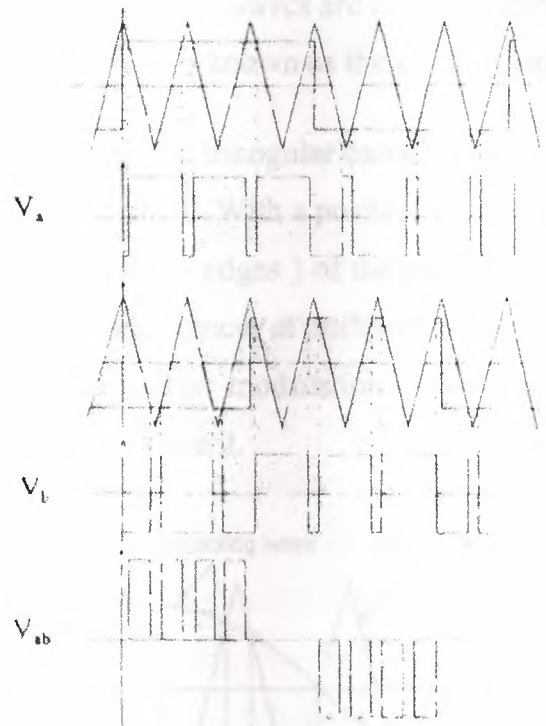
- (a) Square-wave modulation
- (b) The sampling method
- (c) Optimized PWM
- (d) Selected Harmonic Elimination
- (e) Delta Modulation
- (f) Space-vector based PWM

## 2.2 Square-wave Modulation :

In this modulation technique, a symmetrical triangular carrier wave is compared with a square-wave reference ( modulating ) wave. The switching instants of the half-bridge inverter switches are determined by the intersection points of the two waves, as shown in Fig.2.1. In three-phase inverter control, if the frequency ratio is chosen to be a multiple of three the carrier wave will have the same phase relationship with each of the three reference square waves. The resulting pole voltage waveforms will be identical with  $120^\circ$  phase relationships. The line-to-line voltage waveform depends on the phase relationship between the carrier and the modulating waves [1]. For the choice in Fig. 2.1 the line-to-line waveform has pulses of equal width, both in the positive and negative half-cycles. Square-wave PWM can also be obtained by modulating the square-wave pole voltage waveform during the middle  $60^\circ$  interval of each half-cycle [2]. This approach reduces the number of switchings per cycle of the inverter.



Harmonic analysis of square-wave PWM shows that the line-to-line waveform contains all the harmonics of the unmodulated six-step waveform with additional harmonics due to modulation by the high frequency carrier. The pole waveform contains a large harmonic at the carrier frequency, but since the frequency ratio is a multiple of three this harmonic does not appear in the line-to-line waveform.



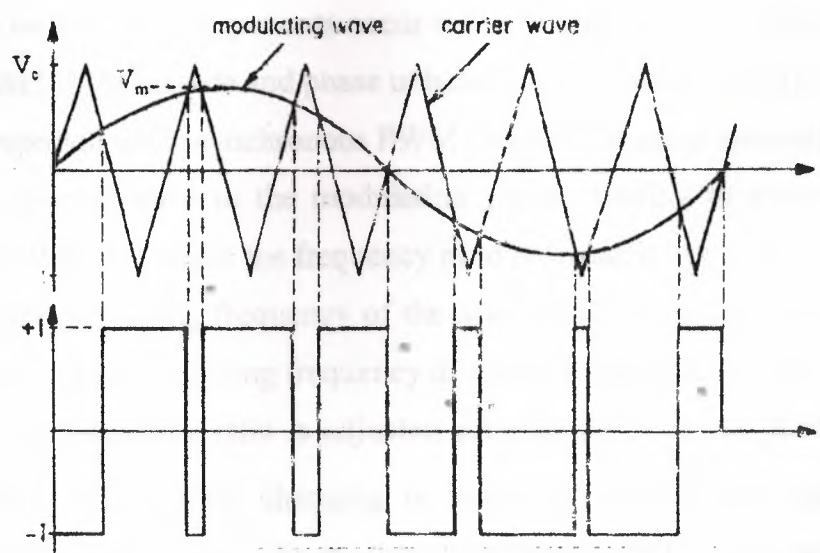
**Fig.2.1 Square-wave modulation**

### 2.3 The Sampling Method :

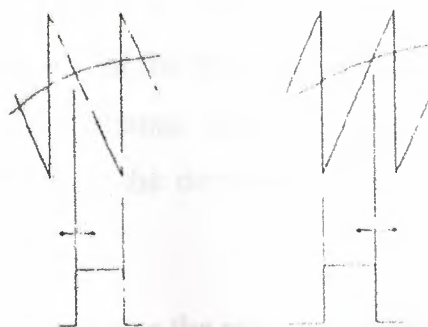
This modulation technique, also known as the subharmonic or the triangulation method [3,4,5,6], involves comparison between a sinusoidal modulating wave of fundamental frequency  $f_m$ , and a triangular carrier wave of much higher frequency ( $f_c$ ), as shown in Fig.2.2 . For a three-phase implementation a common carrier wave is used for all the three phases. This modulation process is

similar to the square-wave PWM described in the previous section, with the difference that sinusoidal waves are used as modulating waves, as should be the case since it is a sine-wave that the PWM output of an inverter is required to approximate. The reason why square-wave references have been considered in the past is related with implementation problems. Generation of three-phase sinusoidal references with conventional analog circuitry has drawbacks of offset and drift. On the other hand, square waves are easier to generate. The modulation process depicted in Fig.2.2 is also known as the natural sampling technique.

Depending on the shape of the triangular carrier, single-edge and double-edge modulated waves can be obtained. With a positive-ramp (negative-ramp) carrier wave the leading edges (trailing edges) of the pulses are modulated, while the trailing edges (leading edges) occur at uniformly spaced intervals (Fig.2.3). It can be shown that single-edge modulation produces significantly greater harmonic distortion in motor current.



**Fig.2.2** Subharmonic modulation



**Fig.2.3** (a)Leading-edge modulation. (b) Trailing-edge modulation.

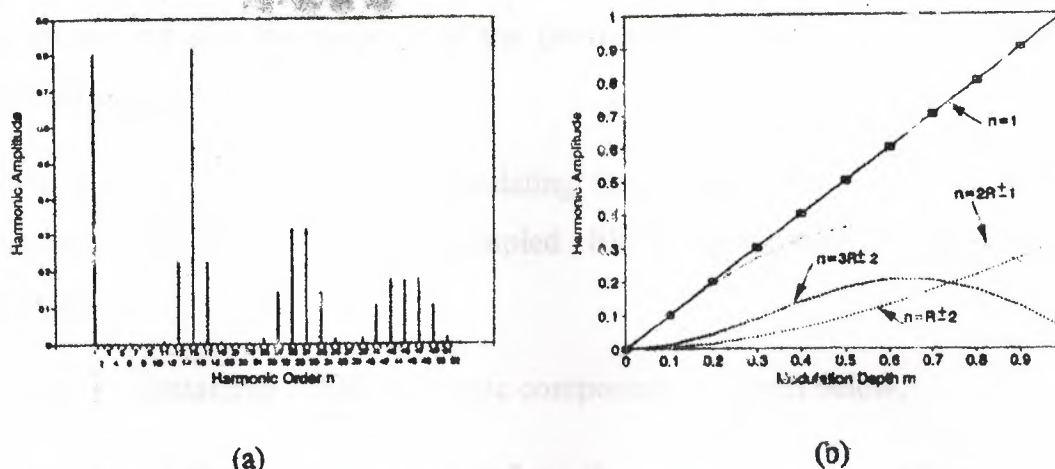
The fundamental voltage and frequency of the PWM waves are controlled by varying the amplitude and frequency of the modulating waves. In controlling the frequency, two alternatives exist on the choice of the carrier wave. The simplest approach is to fix the carrier frequency, in which case as the frequency of the modulating waves is varied, the frequency ratio, defined as  $R=f_c/f_m$ , varies and therefore is in general noninteger. PWM waveforms generated in this way are termed asynchronous. Harmonic analysis of such waveforms indicate that subharmonic as well as d.c. components occur in the output voltage of the inverter for  $R$  less than ten [3]. Amplitude and phase unbalance of the fundamental voltages have also been reported with asynchronous PWM control. The other alternative is to synchronize the carrier wave to the modulating waves, leading to synchronous natural sampled PWM, for which the frequency ratio is constant. As the modulating frequency is varied, switching frequency of the inverter also varies in proportion. Then, in order to keep the switching frequency in a narrow band, i.e. prevent a wide variation of  $f_c$ , the frequency ratio is adjusted accordingly at certain modulating frequencies. This type of ratio changing is sometimes called gear changing. Theoretical harmonic analysis of this modulation method, which involves double Fourier series in terms of Bessel functions [6], shows that the fundamental component ( $V_1$ ) of the PWM waveform is proportional to the modulation depth (index)  $m=V_m/V_c$  for  $m$  less than one. For  $m$  greater than one, overmodulation



occurs and the relationship between  $V_1$  and  $m$  becomes nonlinear. The harmonic amplitudes are almost independent of the frequency ratio  $R$ , provided  $R$  is greater than 9. Figure 2.4 shows the harmonic spectrum of a natural-sampled PWM waveform, and also the variation of the dominant harmonic components with the modulation depth.

Overmodulation is applied to increase the range of fundamental voltages that can be obtained and eventually make transition to quasi-square-wave ( six-step ) operation in the high frequency range. Pulses in the PWM waveform becoming shorter than the minimum commutation requirement of the inverter switches have to be dropped. This may result in large jumps in the fundamental voltage whenever pulses are dropped, particularly in thyristor inverters. Several techniques have been proposed to achieve a smooth transition from sinusoidal modulation to six-step operation [7,8].

Overmodulation also gives rise to low order harmonics in the output waveforms. Inverter control schemes based on the natural sampling PWM have been implemented using analog electronic techniques, with associated problems of drift and offset. These schemes directly attempt to realize the analog process of natural sampling by using electronic comparison of the reference and carrier waves. Implementation of natural sampling using digital hardware or microprocessor-based schemes are not very effective [9]. This stems from the fact that the pulse-widths are defined by transcendental equations which are difficult to solve on a microprocessor in real time.



**Fig.2.4** (a) Harmonic spectrum of natural-sampled PWM ( $R=15$ ,  $m=0.8$ ) (b) Variation of dominant harmonic amplitudes with modulation depth.

The regular ( uniform ) sampling technique [3] is a modified version of the classical natural sampling modulation. Regular sampling uses sample-and-hold forms of the modulating waves for comparison with the carrier. Symmetrical modulation is obtained when the modulating wave is sampled at time instants corresponding to positive peaks of the carrier, as shown in Fig.2.5. Asymmetrical modulation is obtained when sampling occurs at both the positive and negative peak instants.

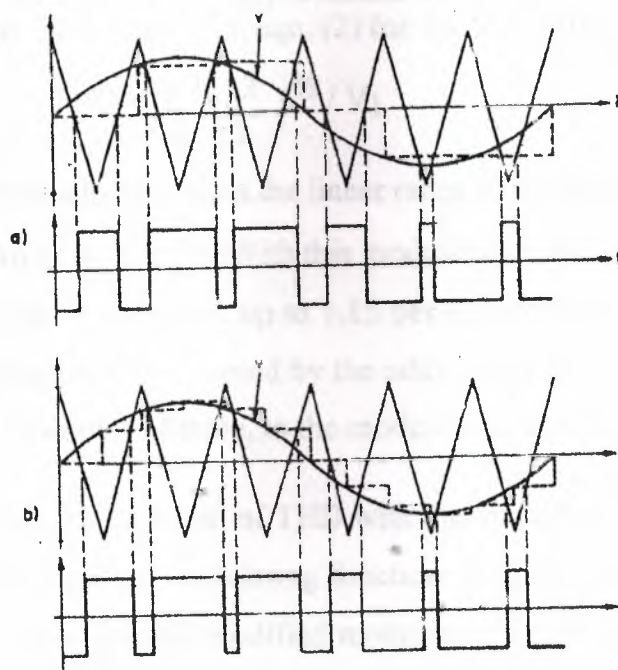
It has been shown that [6] regular sampling improves the harmonic spectrum by reducing the low order harmonics and suppressing the subharmonics at noninteger frequency ratios. On the other hand, the fundamental component of regular-sampled PWM is no longer directly proportional to the modulation depth. It is in fact a nonlinear function of both the modulation depth  $m$  and the frequency ratio  $R$ . However, the degree of nonlinearity is not significant and becomes negligible as  $R$  is increased. Regular sampling also introduces a phase shift between the fundamental component of the modulated wave and the reference wave, equal to a quarter cycle of the carrier wave for asymmetric sampling [10].

Figure 2.6 shows the harmonic spectrum of a regular asymmetric sampled PWM waveform and also the variation of the dominant harmonic components with the modulation depth.

The use of a nonsinusoidal modulating function has been found to improve harmonic distortion of regular-sampled PWM waveforms. In particular, a modulating

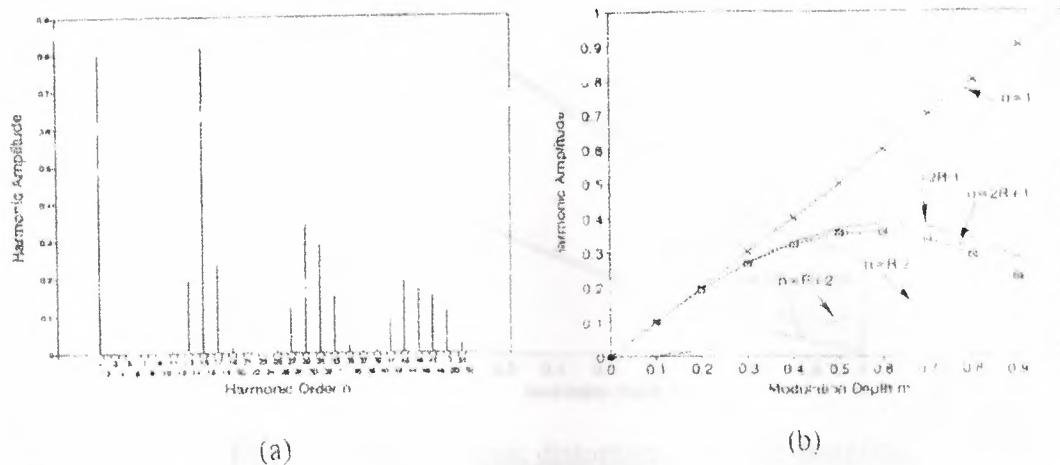
function containing a third harmonic components as given below,

$$v(t) = m[\sin \omega_m t + \alpha \sin 3\omega_m t] \quad (1)$$



**Fig.2.5** (a) Symmetric regular sampling. (b) Asymmetric regular sampling.





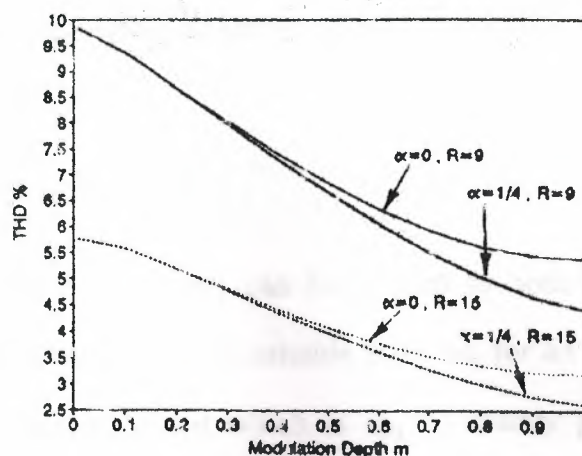
**Fig.2.6 (a) Harmonic spectrum of regular sampled PWM ( $R=15$ ,  $m=0.8$ ) (b) Variation of dominant harmonic amplitudes with  $m$ .**

has been shown to minimize the approximate total harmonic distortion (THD) of induction machine current given in eqn. (2) for  $\alpha=0.25$  [10,11,12].

$$THD = \left( \sum_{n=2}^{\infty} \frac{V_n^2}{V_1^2} \right)^{1/2} / V_1 \quad (2)$$

The value of  $\alpha$  which maximizes the linear range of the fundamental as a function of  $m$  can be shown to be  $\alpha = 1/6$ . With this modulating function, the fundamental of the PWM wave can be increased up to 1.15 per unit without overmodulation. The fundamental can further be increased by the addition of more harmonic terms, with orders which are multiples of three, to the modulating function in eqn.(1) [13].

Figure 2.7 shows the variation of THD with the modulation depth for the purely sinusoidal and the modified modulating function in eqn.(1) at two frequency ratios  $R=9,15$ . It is evident that the modified modulating function is more effective for fundamental levels above 0.5 per unit.



**Fig.2.7** Total harmonic distortion of regular sampling.

Implementation of the regular sampling technique can be efficiently achieved using digital hardware or microprocessor-based circuits. The pulse-widths in regular-sampled PWM waveforms can be obtained explicitly in terms of the carrier period and sampled values of the modulating functions. This makes it possible to develop software-based schemes for the real-time generation of these waveforms based on the on-line calculation of pulse-widths [14,15]. This approach would involve storing the sampled values of the modulating wave ( one phase only ) in memory. The three-phase modulating values corresponding to a certain carrier period are fetched from memory and are used to calculate the three-phase pulse-widths for the required modulation depth and frequency. The pulse-widths are then generated in real-time by hardware counters.

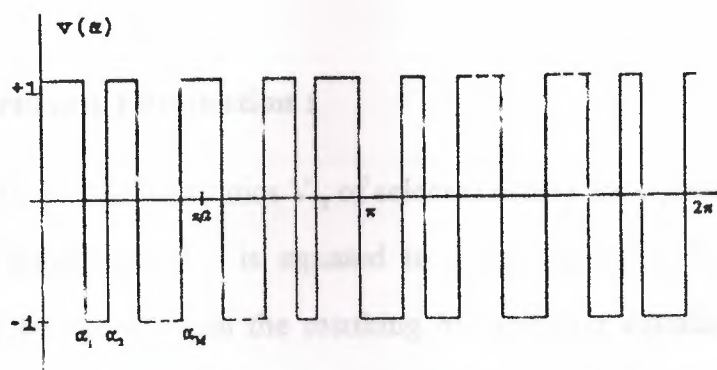
## 2.4 Optimized PWM :

PWM waveforms can be synthesized which optimize a suitable chosen criterion related to the performance of the drive system, such as total harmonic distortion ( THD ) of machine current or peak-to-peak torque pulsations [4,14,16]. In this approach, the PWM waveform is assumed to have quarter-wave symmetry with  $M$

switching angles per quarter-cycle, as shown in Fig.2.8. Voltage harmonics of such a waveform are given by

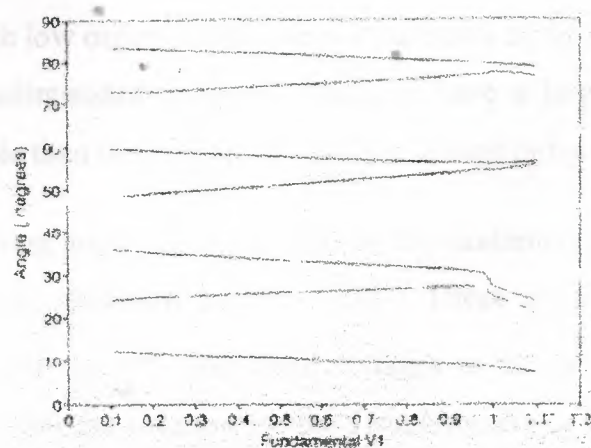
$$V_n = \frac{4}{n\pi} \left[ 1 + 2 \sum_{k=1}^M (-1)^k \cos(n\alpha_k) \right] \quad ; n=1,3,5,\dots \quad (3)$$

The switching angles  $\{ \alpha_1, \dots, \alpha_m \}$  can be chosen to optimize a performance criterion which is a function of  $V_n$ . A suitable criterion for an induction machine drive is the THD of motor current which is approximately proportional to the expression given in eqn.(2). The angles  $\{ \alpha_1, \dots, \alpha_m \}$  which minimize THD can be computed by using an optimization algorithm for various fundamental components  $V_1$ . Figure 2.9 shows the variation of the angles for  $M = 7$  as a function of the fundamental.



**Fig. 2.8** PWM wave with quarter-wave symmetry and  $M$  switching angles per quarter cycle

**Fig.2.9** Switching angles as a function of the fundamental for optimized PWM ( $M=7$ ).





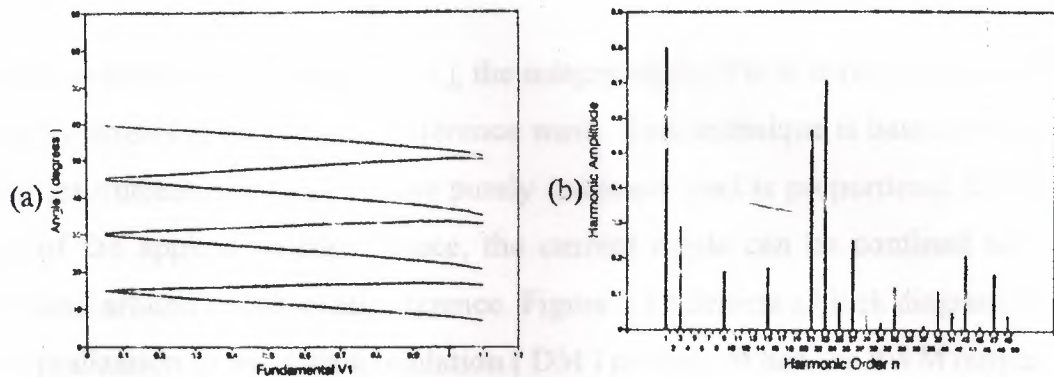
The optimization algorithm used to compute these angles in general requires several iterations before convergence to the minimum of the cost function ( eqn.2 ) is reached. This makes it almost impossible to implement such an algorithm on a microprocessor for on-line computation of the angles for real-time generation. Therefore, optimized PWM waveforms have been implemented by employing look-up tables of switching angles computed off-line on a mainframe computer and stored in microcomputer memory. A very large number of tables must be stored in memory in order to keep frequency and voltage resolution within acceptable limits. In another approach, the angles corresponding to predetermined pivot values of the fundamental are stored and then interpolation is used to compute on-line the angles at intermediate fundamental values [17]. In this way memory requirement can be drastically reduced and a quasicontinuous variation of the fundamental voltage can be achieved.

## 2.5 Selected Harmonic Elimination :

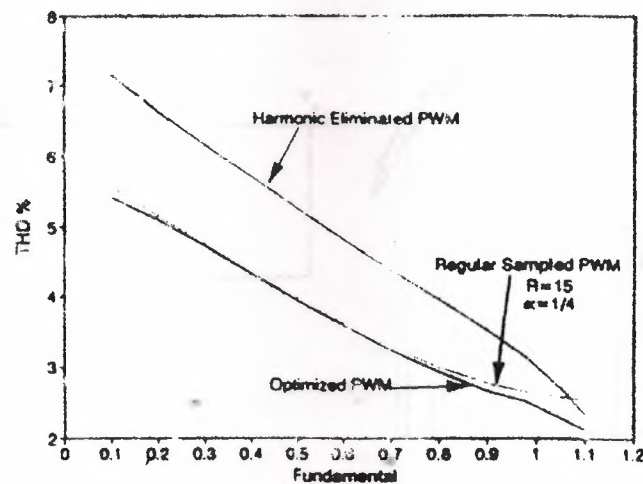
In eqn.3 if  $(M-1)$  of the harmonics  $V_n$  of selected orders are equated to zero and the fundamental component  $V_1$  is equated to a desired value  $V$ , then angles  $\{\alpha_1, \dots, \alpha_m\}$  can be solved from the resulting  $M$  nonlinear equations [18]. The range in which the fundamental  $V_1$  can be controlled is determined by the existence of a feasible solution to the nonlinear equations, i.e.  $\alpha_i \leq \alpha_{i+1}$ ,  $i \leq M$  and  $\alpha_M \leq \pi/2$ . In three-phase systems harmonics with low orders which are not multiples of three are eliminated. However, the first uneliminated harmonic tends to have a large amplitude, causing a larger current ripple than the optimized PWM described before.

Figure 2.10 shows computed switching angles as a function of the fundamental level  $V_1$  for  $M = 7$ , and the harmonic spectrum for  $V_1 = 0.8$ . These angles eliminate the 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, 13<sup>th</sup>, 17<sup>th</sup> and the 19<sup>th</sup> harmonic voltages in the pole voltage waveforms. It should be noted that the solution for the switching angles is

not unique and other sets of solutions may exist. The variation of THD with the fundamental  $V_1$  is shown in Fig. 2.11. The THDs of the regular asymmetric sampled PWM ( with  $\alpha = 1/4$  ) and of the optimized PWM with  $M = 7$  are also displayed here for comparison.



**Fig.2.10**(a)Switching angles for harmonic elimination. (b)Harmonic spectrum ( $V_1=0.8$ ).



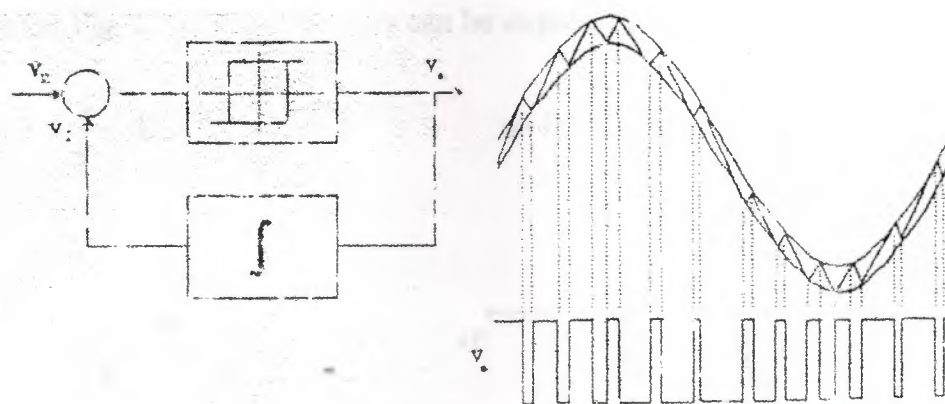
**Fig.2.11** THDs of the various PWM methods.

The harmonic elimination technique, as a special case of optimized PWM strategies, poses similar implementation difficulties. On the other hand, the switching angles in harmonic eliminated PWM have been observed to follow regular patterns, for odd values of  $M$  [19]. This property of the angles can be used

to derive generalized equations from which the angles can be approximately calculated given the fundamental voltage  $V_1$  and the number  $M$ . These equations can be implemented on a microprocessor for on-line calculation of the angles.

## 2.6 Delta Modulation :

In this modulation technique [20,21], the integral of the PWM waveform is kept within a hysteresis band around a reference wave. This technique is based on the fact that the current in a single-phase purely inductive load is proportional to the integral of the applied voltage. Hence, the current ripple can be confined to a narrow band around a sinusoidal reference. Figure 2.12 depicts a block diagram of a circuit realization of the delta modulation ( DM ) process. When the PWM output  $v_o$  of this circuit is used to control an inverter with a purely inductive load, the load current will have the same waveform as the analog signal  $v_i$ .



**Fig.2.12 The Delta Modulation Technique.**

An improved version of DM, called the Model Reference Adaptive ( MRA ) PWM technique [22], a reference wave consisting of a sinusoid with a triangular carrier wave superimposed on it is used in order to render the switching frequency almost constant. In both the DM and MRA PWM techniques, the generated PWM waves have the constant volts/hertz feature for variable frequency operation.



## 2.7 Space Vector Based PWM :

This method is for generating three-phase PWM waveforms by making use of space vectors ( or Park's vectors ) [23]. Given a set of three-phase voltages, the voltage space vector is defined as,

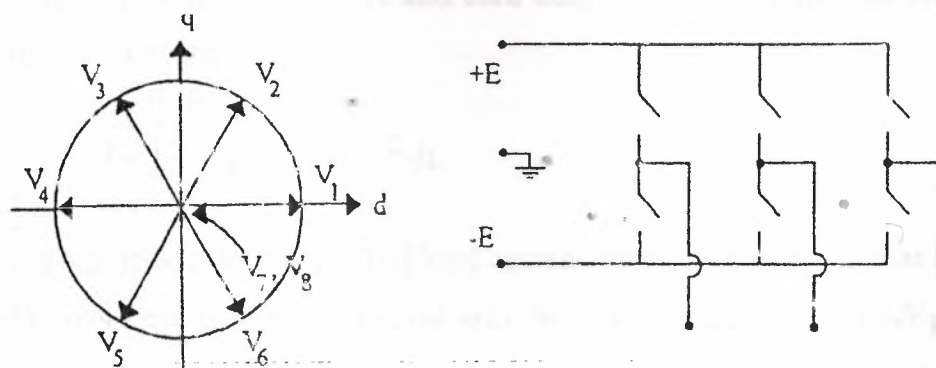
$$V(t) = \frac{2}{3} ( V_a(t) + a V_b(t) + a^2 V_c(t) ) \quad ; a = e^{j2\pi/3} \quad (4)$$

If the voltages are sinusoidal and balanced, then we have,

$$V(t) = V_m e^{j\omega t} \quad (5)$$

where  $V_m$  is the amplitude and  $\omega$  is the angular frequency of the phase voltages. The output phase voltages ( or pole voltages ) of a three-phase half-bridge inverter corresponding to a given switching state can also be represented by space vectors, as shown in Fig. 2.13. These vectors can be expressed as,

$$V_n = \frac{4}{3} E e^{j(n-1)\pi/3} \quad n=1, \dots, 6 \quad ; \quad V_7=V_8=0 \quad (6)$$



**Fig.2.13 Voltage of a three-phase half-bridge inverter.**

The zero vectors arise when either the upper or the lower switches are all closed. If the inverter feeds a three-phase purely inductive load, then the following vector equation can be written for the current space vector,

$$V_I(t) = L \frac{dI}{dt} \quad ; \quad I(t) = \frac{2}{3} (i_a + a i_b + a^2 i_c) \quad (7)$$

where  $V_I(t)$  is a sequence of inverter voltage vectors in eqn.(7). Integrating eqn.(6) in a time interval  $(t_k, t_{k+1})$  in which only one voltage vector  $V_k$  is applied,

$$I(t_{k+1}) = I(t_k) + \frac{1}{L} \int_{t_k}^{t_{k+1}} V_k dt = I(t_k) + \frac{\Delta t_k}{L} V_k \quad (8)$$

If the inverter voltage vectors and their durations are properly chosen, then the current vector can be made to track a reference vector  $I^*(t)$ , which is given as follows for balanced sinusoidal currents,

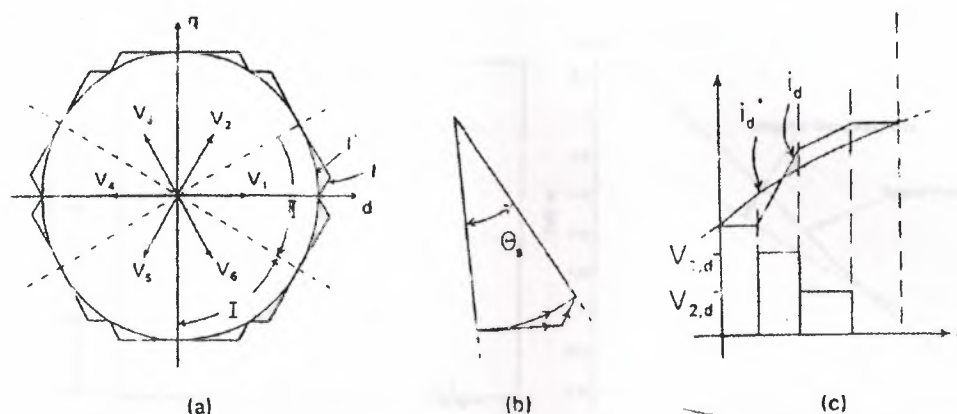
$$I^*(t) = I_m^* e^{j\omega t} \quad (9)$$

The quasi-circular locus method [12] is a well-defined process for the selection of the sequence of voltage vectors and their durations, to minimize the following performance criterion,

$$J = \int_0^{2\pi/\omega} |I(t) - I^*(t)|^2 dt \quad (10)$$

which is proportional to the THD of load current. In Fig. 2.14 the circular locus is divided into  $N$  (multiple of six) equal arcs. In sector I, the vectors  $(0, V_1, V_2, 0)$  are applied for appropriate durations [13], and in sector II the vectors  $(0, V_2, V_3, 0)$  are applied and so on. The zero vector is applied at the beginning and end of each interval. This is best illustrated on a timing diagram, as shown in Fig. 2.14c, where

only d-axis quantities are displayed. In Pulse Frequency Modulation [15], the arc angles  $\theta$ , are modulated to further reduce J, or to minimize torque pulsations of a machine.



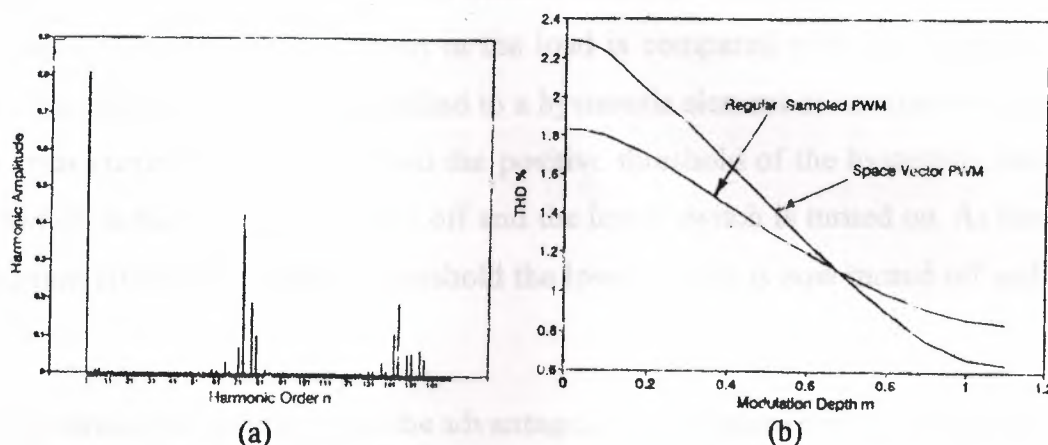
**Fig.2.14** The quasi-circular locus method. (a) Reference and actual current vectors. (b) Arc  $\theta_s$ . (c) Sequence of voltage vectors with zero vector at beginning and end of interval

An important advantage of space vector based PWM is its ability to reduce the average switching frequency compared with the sampling PWM. This is readily seen when the switching functions for the inverter switches are plotted for a complete output cycle. For this, one of the phases, e.g. phase A, is chosen and is not switched unless otherwise required by the sequence of the voltage vectors. The number of switchings in space vector PWM is 30 percent less than that in subharmonic modulation at the same carrier frequency ( the same number of line-to-line pulses ).

Theoretical harmonic analysis of space vector based PWM is extremely difficult because of the complicated nature of the algorithm involved in generating the waveforms. Figure 2.15(a) shows the computed harmonic spectrum of the line-to-line waveform with  $m=0.8$  and a sampling frequency of  $f_s = 3.6$  KHz ( 72 pulses in line voltage ). Figure 2.15(b) compares the THD variations of space vector PWM and optimized regular asymmetric sampled PWM ( $\alpha = 0.25$ ) as a function of the



modulation depth  $m$ . The carrier frequency in regular sampling has been chosen as  $f_c = 2.25$  kHz so as to produce the same number of switchings per output cycle.



**Fig. 2.15** (a) Harmonic spectrum of space vector based PWM. (b) THDs of space vector and regular sampled PWM.

The maximum fundamental phase voltage obtainable with the space vector PWM technique can be shown to  $1.15 E$ , which is the same as that in subharmonic PWM with modified reference ( $\alpha = 1/6$ ). Overmodulation in space vector PWM occurs when the duration of the zero vector applied in a sampling interval  $\theta_s$  comes out to be negative.

## 2.8 Closed-loop VFI Current Control Techniques :

In current control of voltage-fed inverters, the switching pattern of the inverter is determined in a feedback loop. The three-phase currents in the load of the inverter are measured and compared with three-phase reference currents. The error currents are then used to generate the PWM switching signals of the three-phase voltage-fed inverter. In a variable-speed AC drive system the reference currents are usually determined in an outer control loop. For instance, in high performance

vector-controlled induction drives, reference stator currents in stationary d-q frame are produced for decoupled torque and flux control of the machine.

Basically, two approaches exist for the current control of VFI, namely the hysteresis-band methods and predictive control methods. In the original hysteresis-band method [16], the actual current in the load is compared with the reference current. The error current is then applied to a hysteresis element of constant width. As the error current increases beyond the positive threshold of the hysteresis, the upper switch in the inverter is turned off and the lower switch is turned on. As the error current crosses the negative threshold the lower switch is now turned off and so on.

The hysteresis-band method has the advantages of fast response and peak current limiting. Its implementation is very simple and does not require any information about load parameters. However, a major disadvantage is that the switching frequency may vary widely during the output cycle and also with the operating conditions. In three-phase applications, due to the interaction between the phases the actual current error may become considerably greater than the hysteresis width. In the three-level hysteresis current control technique, the inverter voltages and the machine currents are represented by space vectors. A zero level is included in the hysteresis switching elements with the result that zero voltage are also selected whenever necessary. This has the effect of reducing the switching frequency of the three-phase inverter by minimizing the interaction between the phases. In the adaptive hysteresis-band method, the hysteresis width is programmed as a function of load parameters and operating conditions such that the switching frequency of the inverter is nearly constant.

In the predictive control techniques [19], the current error is sampled at a fixed rate and the voltage required to force the current to the reference is computed. In three-phase applications, the three-phase quantities are usually transformed to



stationary d-q frame. The space vector concept is particularly useful in formulating the predictive control algorithm. If the three-phase machine is modeled as an R-L impedance in series with a counter-emf per phase, then the following vector equation can be written for the load,

$$V(t) = E(t) + R I(t) + L \frac{dI(t)}{dt} \quad (11)$$

Where  $V(t)$  is the space vector of the three-phase voltages applied to the load,  $E(t)$  is the counter-emf space vector and  $I(t)$  is the machine current space vector. Equation 1 can be discretized as follows,

$$I(k+1) = e^{-RT/L} I(k) + \frac{1}{R} (1 - e^{-RT/L}) (V(k) - E(k)) \quad (12)$$

Where  $T$  is the sampling period. In eqn 12 if  $I(k+1)$  is equated to the reference current  $I^*(k+1)$  then the voltage vector required for zero current error at sampling instant  $(k+1)T$  is,

$$V(k) = E(k) + \left( \frac{R}{1 - e^{-RT/L}} \right) (I^*(k+1) - e^{-RT/L} I(k)) \quad (13)$$

Once the voltage vector is computed it can be synthesized with the discrete voltage vectors of the inverter in eqn. (6) as follows,

$$TV(k) = T_n V_n + T_m V_m + T_o V_o \quad ; T = T_n + T_m + T_o \quad (14)$$

Where  $V_n$  and  $V_m$  are the nearest vectors to  $V(k)$  and  $V_o$  is a zero vector. It should be noted that the current reference  $I^*(k+1)$  is unknown at the sampling instant  $kT$ , unless it is specified as a function of time such that it can be directly computed. But since the current reference is usually determined in an outer control loop then it must be predicted from previously acquired values. Another difficulty of this approach is that the counter emf  $E(t)$  cannot be easily measured and therefore must be estimated from measured quantities.



### 3. Pulse Width Modulation Inverter Systems :

#### 3.1 General Description :

It is generally recognised that PWM Inverters offer a number of advantages over rival convertor techniques. These advantages are usually gained at the expense of more complex control- and power-circuit configurations. It is expected, however, that in the future the cost and complexity of PWM inverter systems will significantly reduce with continuing developments in LSI technology, fast-switching thyristors, and power transistors. These developments should eliminate many of the practical limitations which have been experienced in the past, and allow the full potential and versatility of PWM control techniques to be realised.

The operational characteristics of PWM inverters depend intrinsically on quite complex modulation processes, and, for this reason, very few theoretical and experimental results have been published concerning the design techniques and operating limitations. This is in complete contrast to other types of converters, for example quasi-square-wave inverter systems which, because of their relatively simple operation, have been extensively analysed using both time- and frequency-domain techniques.

More recently, analysis techniques, based on Fourier-series methods, have been proposed and used to derive analytic expressions for the harmonic spectra of PWM inverter waveforms [1,2]. These expressions can provide the system designer with valuable insight into the harmonic structure of the PWM waveforms and highlight the relationships which exist between the various harmonics and the parameters of the modulation process. Unfortunately, in general, this approach can only be applied to well defined modulation processes, and usually requires quite complex and lengthy analysis to derive the harmonic spectra expressions. In addition,

because these harmonic spectra expressions involve Bessel function series, it is usually necessary to use a digital computer to calculate the magnitude of the individual harmonics. However, it is important to note that efficient computer methods for numerically evaluating these Bessel function expressions are available, and methods have recently been proposed which can significantly reduce both programming and computing times [3,4].

An alternative approach, which is more general and can in principle be used to investigate a wide range of PWM systems, uses the digital computer to model the PWM process, employing software simulation techniques. The computer model can then be used as the basis for computer investigations of a wide range of operating modes, using both time- and frequency- domain analysis techniques. For example, the PWM model can be combined with an electrical machine model to simulate variable-speed drive systems; or alternatively combined with a filter-load model and feedback control to simulate voltage regulating systems.

Using this approach, harmonic and transient analysis of the various systems can easily be performed by the computer, using numerical techniques. This facility considerably reduces the analytic effort required of the system designer, and allows extremely complex PWM inverter systems to be investigated.

Therefore concerned with the development of an extremely versatile PWM computer modelling package which can be used as a 'stand-alone' package for harmonic spectra analysis, or alternatively as a 'building-block' for developing more complex systems.

The next Section briefly reviews the concepts and principles associated with the various PWM methods, which have been used as the basis for formulating the computer modelling package presented in Section 3.3.



### 3.2 Survey of PWM techniques :

It is possible, by surveying the literature over the last decade [5], to trace the historical development of PWM inverter control techniques and relate these developments to the changes in technology. To clarify the current situation, it is helpful to recognise three distinct approaches currently in vogue to formulate the PWM switching strategy. The first, and the one which has been most widely used because of its ease of implementation using analogue techniques, is based on 'natural' sampling techniques [2,6,7]. More recently, a new switching strategy [1], referred to as 'regular' sampling, has been proposed which is considered to have a number of advantages when implemented using digital or microprocessor techniques [5]. The third approach uses the so called 'optimal' PWM switching strategies which are based on the minimisation of certain performance criteria [8-16]; for example, elimination or minimisation of particular harmonics, or the minimisation of harmonic current distortion, peak current, torque ripple etc. These optimised PWM control strategies are currently receiving considerable attention and, as a result of the developments in microprocessor technology, the feasibility of implementing these strategies has now become a real possibility [5, 14,16].

In the following section, the modulating principles associated with each PWM switching strategy will be outlined and used to derive equations which describe the PWM switching process. These equations form the basis for developing the computer models and associated algorithms presented in Section 3.3. Only sinusoidal modulation, which is commonly used in PWM schemes, will be considered, although, as will become evident with minor modification to the principles outlined, other types of modulation, such as trapezoidal, triangular, square etc. can equally be catered for.

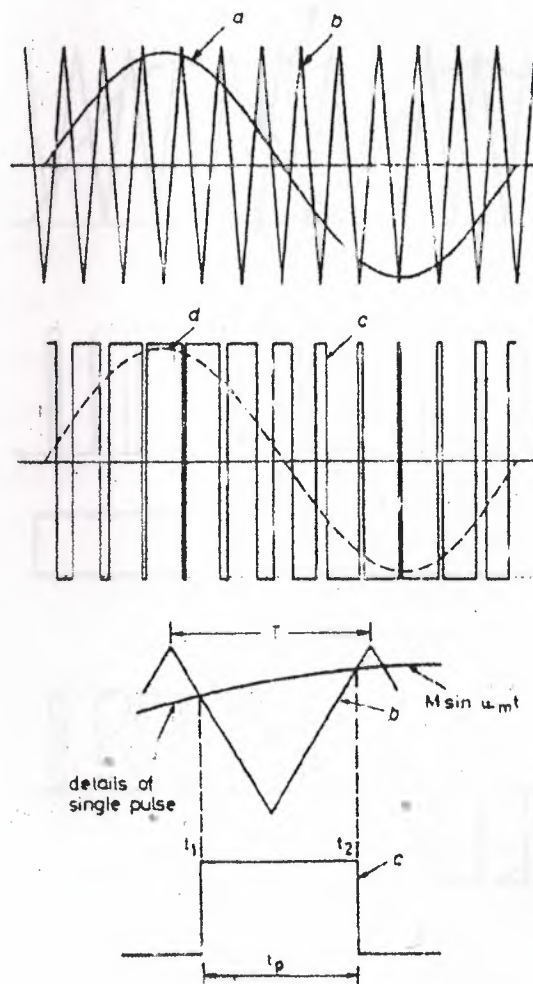


### 3.2.1 Natural sampled PWM:

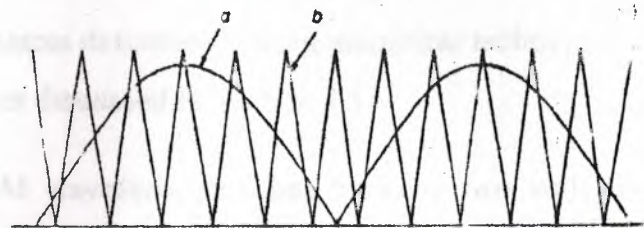
Most analogue implemented PWM inverter control schemes employ natural sampling techniques [2,6,7]. A practical implementation, showing the general features of this mode of sampling, is illustrated in Fig. 3.1. From the Figure, it can be seen that a triangular carrier wave (sampling signal) is compared directly with a sinusoidal modulating wave to determine the switching instants, and therefore the resultant pulse widths.

**Fig.3.1** 2-level natural sampled PWM

- a Reference modulating signal
- b Carrier signal
- c PWM voltage
- d Fundamental of PWM voltage

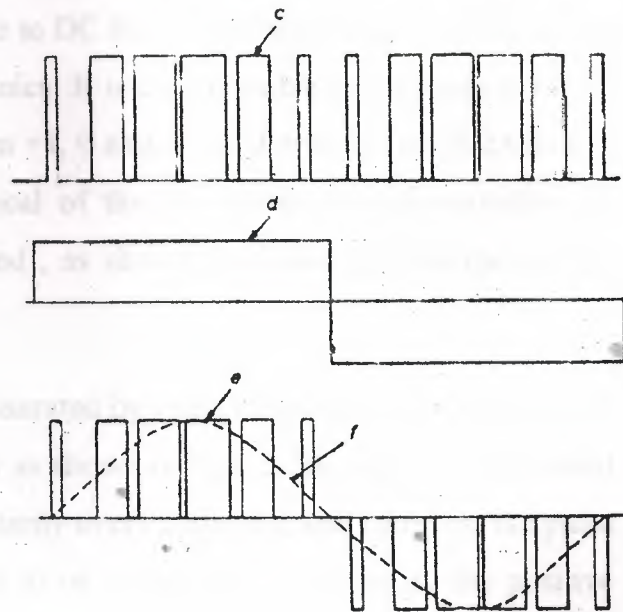


It is important to note that, because the switching edge of the width-modulated pulse is determined by the instantaneous intersection of the two waves, the resultant pulse width is proportional to the amplitude of the modulating wave at the instant that switching occurs. This has two important consequences: the first is that the centres of the pulses in the resultant PWM are not equidistant or uniformly spaced and, secondly, it is not possible to define the width of the pulses using analytic expressions.



**Fig.3.2A** 3-level natural sampled PWM

- a Reference modulating signal
- b Carrier signal
- c 2-level PWM control signal
- d Gating circuit polarity discriminator
- e 3-level PWM inverter voltage
- f Fundamental of PWM voltage



Indeed, it is possible to show [1,2] that the widths of the pulses can only be defined using a transcendental equation of the form

$$t_p = (T/2) [1 + (M/2) (\sin \omega_{mt1} + \sin \omega_{mt2})] \quad (15)$$

Because of the transcendental relationship existing between the switching times, it is not possible to calculate the widths of the modulated pulses directly. Indeed, it is possible to show [1, 2] that the widths of the modulated pulses can only be defined in terms of a series of Bessel functions.

To construct a computer model of the natural sampling process requires the analogue process illustrated in Fig.3.1 to be simulated directly in the computer software, and the PWM switching Instants determined using numerical techniques. The details of this approach are further discussed in Section 3.3.

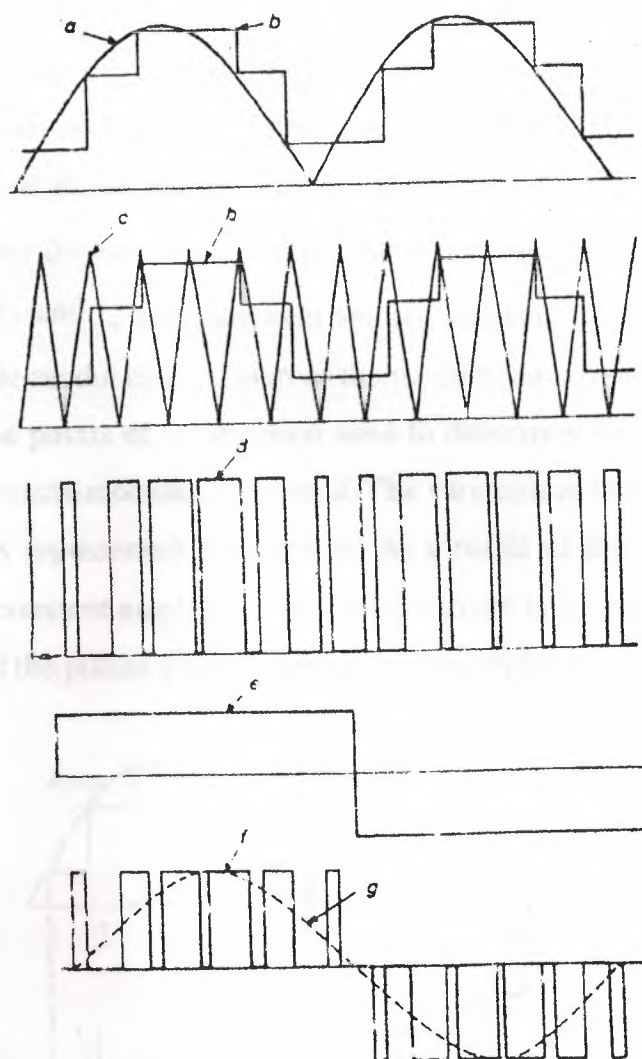
As illustrated in Fig.3.1, the PWM waveform switches between two voltage levels +1 and -1, and is therefore usually referred to as 2-level PWM. This waveform is typical of the inverter line to DC link centre-tap voltage, and as shown includes the carrier frequency harmonics. It is also possible to generate a 3-level PWM waveform by switching between +1, 0 and -1 as shown in Figs. 3.2A and B. This 3-level PWM waveform is typical of the line-to-line voltage waveform in single-phase and 3-phase inverters and, as shown, does not include the carrier-frequency harmonics.

The 3-level waveform can either generated by combining two suitably phased 2-level waveforms, or generated directly as shown in Figs. 3.2A and B. As illustrated in these Figures, the pulses change polarity every halfcycle, and therefore the pulse widths in each halfcycle are required to be modulated according to the positive halfcycle of the modulating wave. The polarity discriminator illustrated in Fig. 3.2A and B represents the function of the gating logic which is necessary to correctly apply the PWM gating sequence to the switching devices in the inverter power circuit.



**Fig.3.2B 3-level regular sampled PWM**

- a Reference modulating signal
- b Sampled-hold modulating signal
- c Carrier signal
- d 2-level PWM control signal
- e Gating circuit polarity discriminator
- f 3-level PWM inverter voltage
- g Fundamental of PWM voltage



Once computer models for 2-level and 3-level natural sampled PWM have been constructed, these can then be used as basic building blocks to construct a wide variety of single phase and multiphase PWM inverter systems.

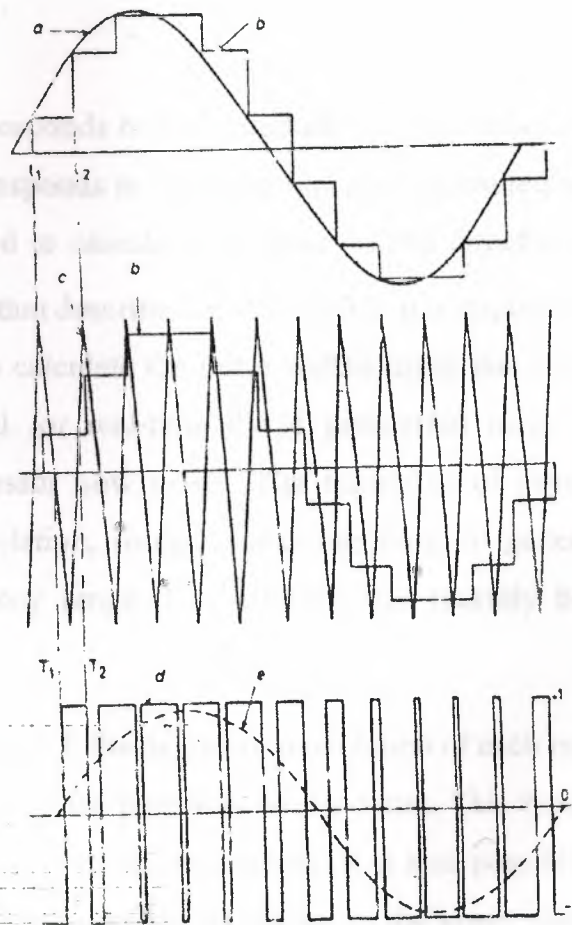
### **3.2.2 Regular sampled PWM:**

Regular sampled PWM inverter control is recognised to have certain advantages when implemented using digital or microprocessor techniques [1,5].

A practical implementation, illustrating the general features of this mode of sampling for 2-level PWM, is shown in Fig.3.3. In this mode of control, the amplitude of the modulating signal  $a$  at the sample instant  $f_1$  is stored by a sample and hold circuit (operated at the carrier frequency), and is maintained at a constant level during the intersample period  $t_1$  and  $t_2$  until the next sample is taken. This produces a sampled-hold, or amplitude-modulated, version of the modulating signal  $b$  with the carrier signal  $c$  defines the points of intersection used to determine the switching instants  $T_1$  and  $T_2$ , of the width-modulated pulses  $d$ . The variation of the fundamental of the PWM wave  $d$  is represented by Fig.3.3e. As a result of this process, the modulating wave has a constant amplitude while each sample is being taken, and consequently the widths of the pulses are proportional to the amplitude of

**Fig.19** 2-level regular sampled PWM

- a Reference modulating signal
- b Sampled-hold modulating signal
- c Carrier signal
- d PWM waveform
- e Fundamental of PWM waveform



the modulating wave at uniformly spaced sampling times; hence the terminology 'uniform' or 'regular' sampling.

It is an important characteristics of regular sampling that the sampling positions and sampled values can be defined unambiguously, such that the pulses produced are predictable both in width and position. It should be noted that this was not case in the natural-sampled process, as discussed previously in section 3.2.1.

Because of this ability to define precisely the pulse configuration, it is now possible to derive a simple trigonometric function to calculate the pulse widths.

With reference to Fig.3.4a, the width of a pulse may be defined in terms of the sampled value of the modulating wave taken at  $t_1$ . Thus

$$T_p = T/2 [ 1 + M \sin (\omega_m t_1) ] \quad (16)$$

The first term in this equation corresponds to be unmodulated carrier frequency pulse width, and the second term corresponds to the sinusoidal modulation required at time  $t_1$ . This equation can be used to calculate the pulse widths directly, and forms the basis of the computer algorithm described in Section 3.3. It is important to note that, as a result of being able to calculate the pulse widths using this simple trigonometric equation, the potential for real-time PWM generation using the computing ability of the microprocessor now exists. The feasibility of using a microprocessor software based calculation, using regular sampling, to generate PWM inverter control for a frequency range 0 to 100 Hz, has recently been demonstrated [5].

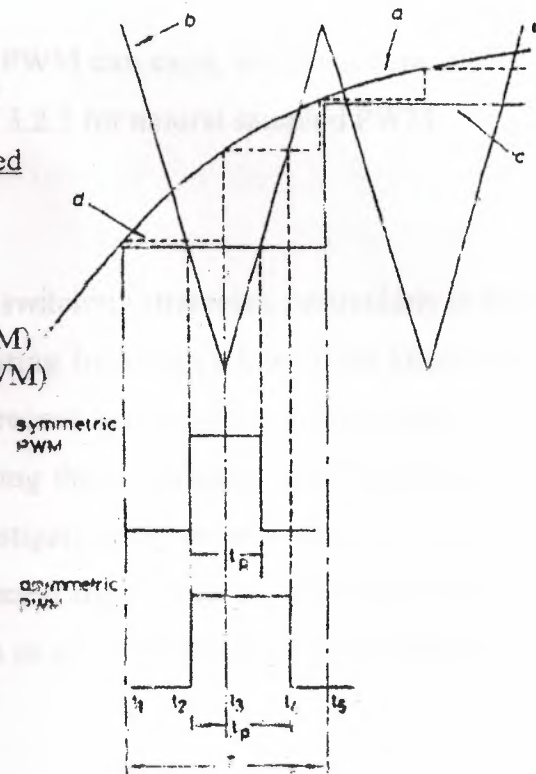
As illustrated in the upper part of Fig.3.4, the degree of modulation of each pulse edge, with respect to regularly spaced pulse positions, is the same. This type of modulation is usually referred to as 'symmetric' modulation. It is also possible to modulated each pulse edge by a different amount, as shown in the lower part of



Fig.3.4. In this case, the leading and trailing edges of each pulse are determined, using two different samples of the modulating wave, taken at time instants  $t_1$  and  $t_3$  respectively.

**Fig.3.4** Symmetric and asymmetric regular sampled PWM

- a Reference modulating signal
- b Carrier signal
- c Sampled-hold modulating signal (symmetric PWM)
- d Sampled-hold modulating signal (asymmetric PWM)



The width of the resulting 'asymmetrically' modulated pulse may be defined in terms of these sampling times; thus

$$t_p = T/2 [ 1 + (M/2) (\sin (\omega_m t_1) + \sin (\omega_m t_3)) ] \quad (17)$$

It is of interest to note that because more information about the modulating wave is contained in the asymmetric modulated PWM waveform, its harmonic spectrum is superior to that produced using symmetric modulation. It should be noted, however, that the number of calculations required to generate asymmetric PWM is double that

required for symmetric PWM. This can significantly extend the computation time required if a microprocessor software based calculation is used to generate the PWM inverter control waveform, and thereby reduce the maximum inverter output frequency [5].

Both 2-level and 3-level regular sampled PWM can exist, and be generated in a similar manner to those described in Section 3.2.1 for natural sampled PWM.

### **3.2.3 Optimised PWM :**

The advantages of using optimised PWM switching strategies, particularly at low frequency ratios (carrier frequency / modulating frequency), have been known for some time. However, it is only as a result of recent developments in microprocessor technology that the feasibility of implementing these strategies has now become a real possibility. A number of research investigations have recently been reported which confirm this [14-16], and it is expected that future investigations should produce efficient microprocessor algorithms to generate these optimised strategies 'on-line'.

Because of this continuing developments, it was considered important to include, in the PWM computer modelling package, facilities to cater for these optimised switching strategies.

These facilities are described in detail in Section 3.3. However, to provide some background to these facilities it is of value at this stage to briefly review the essential features of optimised strategies.

As discussed in previous Sections, both natural and regular sampled PWM are generated using practical circuit implementations based on well defined modulation processes. In contrast, it has been usual to generate optimised PWM by first defining a general PWM waveform in terms of a set of switching angles and then to



determine these switching angles using numerical methods and a mainframe computer. As a consequence of this approach, a knowledge of the optimised modulation processes involved, and associated practical circuit implementations, does not automatically emerge. This has presented practical implementation difficulties in the past which have now largely been overcome using microprocessor technology.

Typical optimised PWM waveforms are illustrated in Fig.3.5. Based on the switch angles defined in this Figure. It is possible to define the harmonic spectrum of each of the PWM waveforms. For example; if odd quarter-wave symmetry is assumed, then only odd harmonics exist and these can be defined by the equation

$$V_n = (4/n\pi) \left[ 1 + 2 \sum_{K=1}^m (-1)^K \cos n\alpha_K \right] \quad (18)$$

For 2-level PWM, and

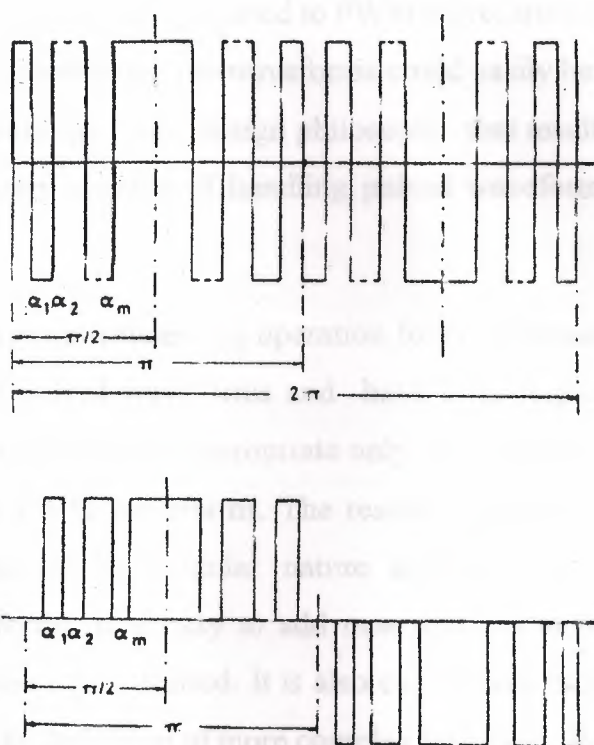
$$V_n = (4/n\pi) \sum_{K=1}^m (-1)^{K+1} \cos n\alpha_K \quad (19)$$

For 3-level PWM, where  $n$  corresponds to the harmonic order, and  $m$  equals the number of switching angles per quarter cycle of the PWM waveform.

These expressions can be used in a variety of ways to produce PWM waveforms which have been optimised with respect to performance criteria; for example, the elimination of low frequency harmonics, or the minimisation of harmonic current distortion or peak current. Using performance criteria of this kind results in a set of nonlinear equations in terms of the unknown switching angles. These equations can be solved, using the mainframe computer and numerical minimisation techniques to determine the optimised switching angles. These angles can subsequently be preprogrammed into the microprocessor memory and used to generate the PWM waveform in 'real-time'.



In the next Section, facilities will be described which allow these optimised switching angles to be used, in the PWM computer modelling package, to simulate various PWM inverter operating modes.



**Fig.3.5 Optimised 2-level and 3-level PWM**

### 3.3 Computer simulation of PWM systems :

To meet the needs of computer simulation of the transient and steady operation of electrical systems, machinery and components when fed with PWM waveforms, a library of computer subprograms has been designed and is in regular use in the Department of Electrical and Electronic Engineering at Bristol University. It was recognised that, if a versatile tool was to emerge, a unified way of handling PWM waveforms would be needed. Such a unified system was designed and is embodied in a library of Fortran subroutine and function modules designated, appropriately

enough, PWLIB. The library is written in strictly ANSI Fortran to ensure its portability.

During the design of the package, It became apparent that many of the features of the package were not naturally restricted to PWM waveforms, and, in particular, the unified system for representing the waveforms could easily be extended to a wider class of pulsed waveforms. The design philosophy that resulted aims to make the modules of the library capable of handling pulsed waveforms of a very general character.

Many of the waveform-processing operation found necessary are appropriate to this whole class of pulsed waveforms and have been implemented in this way. Other processing operations are appropriate only to restricted subclasses of pulsed waveform, such as PWM waveforms. The resulting package achieves the aim of versatility. Because of its modular nature and the well defined system of representing waveforms, it is easy to add new routines to the library when new waveforms or processes are desired. It is also easy to use routines in the library as building blocks in the definition of more complex processes. Because of the general nature of the basic waveforms considered, the package has considerable potential for aiding investigation of new types of pulsed waveform that might emerge in the future.

These are conveniently represented in the subroutine library by an integer variable and two 1-dimensional arrays of real variables. There is no limit, so far as the modules of PWLIB are concerned, to the number of switching angles that the reference wavelength of a waveform may contain.

In using this representation, it is implicit that the operations on pulsed waveforms deal with those waveforms in terms of phase angles measured in radians and related to the period of the waveform. In transient simulations, It is almost always desired



to work with a time or 'time-like' variable, and transformation of this variable into a phase angle through scaling to the frequency of the waveform is necessary. This is relatively simple to achieve in practice and it is believed that the use of a phase variable within the package is the simplest and best approach.

Subroutines of PWLIB operate on pulsed waveforms by referring to and usually altering their representation in terms of the set of characteristic parameters. The operations currently available fall naturally into three major categories, first the generation of pulsed waveforms according to predetermined formulae (e.g. the generation of the various pulse width modulated waveforms), secondly the processing in various ways of pulsed waveforms, and finally subroutines provided specifically to enable the pulsed waveforms to be interpreted and used as inputs to simulation programs (e.g. transient simulations of the action of filter or electrical machines ).

### 3.3.1 Generation of sampled PWM waveforms :

The available waveform generation options include five forms of sampled PWM wave. All this PWM forms switch between two levels, +1 and -1. PWM waves switching between any two desired levels can obviously be obtained by operation of amplification and zero shift on the PWM wave derived from those subroutines. All the sampled PWM waves are modulated with a sine wave which may be given any desired phase shift from the zero point of phase measurement. The carrier wave may also be given a phase shift; this facility is quite useful. PWM waveforms of any frequency ratio and modulation depth, including overmodulated waves, can be generated.



### 3.3.1.1 Naturally sampled PWM:

As noted above, there are no analytic expressions for the switching points of naturally sampled PWM waves. These waveforms are, by their nature, suited more to analogue than digital implementation. The necessity of simulating the operation of equipment using this form of PWM necessitates the inclusion of naturally sampling in PWLIB. Each switching point is implicitly defined via a nonlinear equation expressing the equality, at the switching point, of the carrier wave and the modulating sine wave.

The routine for generating natural PWM uses this technique to compute the switching angles of the waveform when frequency ratio and modulation depth are specified.

### 3.3.1.2 Regular symmetric sampled PWM:

In regular symmetric PWM, the switching angles can be analytically specified. It should be noted that there is an essential difference between the natural PWM wave created by the analogue technique and the regular symmetric (or equally regular asymmetric) one. In the case of natural sampling, the fundamental component of the Fourier breakdown of the PWM wave is in exact phase with the modulating wave. In both the regular cases, the fundamental component of the Fourier breakdown of the PWM is lagged from the modulating wave. This is a result of the sampled-and-hold system of analogue generation of the regular PWM waves. In the case of regular symmetric PWM, this lag is  $T/2$ . This lag need not occur if the PWM is generated digitally through the microprocessor-based systems. The package generates a PWM wave whose fundamental component is in phase with the modulating wave. The characteristics of an analogue PWM generating system can be simulated by the use of the phase-lagging option.

For modulation depths less than unity the switching angles lie strictly within successive intervals of length  $\pi/R$  in the phase variable. If the modulation depth exceeds unity, some switching points may spill over into neighbouring divisions and this gives rise to two types of regular sampled PWM. If this overspill is accepted, a 'nonlimited' pulse-width form of PWM wave is generated. Alternatively, the pulse may be limited to its nominal phase interval.

This is referred to a pulse-width limited regular sampled PWM. Pulse which switches off before it switches on! In these cases the pulse is suppressed altogether and a gap is left. For modulation depths less than unity, the number of pulses in the resulting - PWM wave is always equal to the frequency ratio  $R$ . For  $M > 1$ , the number of pulses may be less than  $R$  and, in these cases, the number of pulses is designated pulse number  $P$ . As modulation depth increases beyond unity, pulses are successively dropped and  $P$  decreases. For sufficiently large  $M$ , the pulse number becomes 1 and, finally, a square wave results.

### 3.3.1.3 Regular asymmetric sampled PWM:

In regular symmetric PWM, the phase angle representing leading and trailing edges of the pulses are both based on the same sampled value of the modulating wave. In regular asymmetric PWM, the trailing edge of the pulse is based on a sampled value of the modulating wave taken later in the cycle than that of the leading edge. Some of the advantages and disadvantages of this were mentioned previously. The switching angles are again definable by analytic expression. In the case of over modulation, the same distinction between nonlimited and pulse-width limited regular asymmetric PWM occurs, and both forms are provided by PWLIB.



### 3.3.2 Optimised waveforms :

Optimised is the generic description of pulse-width modulated waveforms generated not by any particular modulating wave by reference to some property of the resulting wave or of its interaction with the device or machine to which it is to be fed. The generation and use of such waveforms have been made possible by the introduction of microprocessors in machine and device control systems. Previously, when generation had to be achieved by analogue means, the use of optimised waveforms was not a practicable proposition[12,14].

For the purposes of simulation studies, it is necessary, as with practical implementation, to carry out studies by computation, or otherwise to determine the desired switching angles in the optimised case. It is then necessary to translate the desired wave into the standard form used by the PWLIB package. This is achieved by interrogation of the user in an interactive mode. The subroutine asks the user to specify the switching points and the levels of the waveform between switching points. The user may specify that the wave has no symmetry or may give it half- or quarter-wave symmetry. In half-wave case the switching points are symmetric about  $\phi = \pi$  and the levels asymmetric. In the quarter-wave case, both switching points and levels are symmetric about  $\phi = \pi/2$  in the first half-wave and the waveform is extended into the second half wave as a half-wave symmetric waveform. Obviously, the user could generate a PWM wave by specifying the appropriate symmetries and switching points, although there would be little point in this laborious process. The real purpose of the facility is to allow complete flexibility to the user to specify nonstandard waveforms when this is required. Although this subroutine is designed primarily for interactive use, with the user entering his responses from a terminal while the program is running, it is possible, should it be necessary, to run the systems in batch mode with the appropriate responses to the interrogations pre-entered in the correct order on the batch input stream.



### 3.3.3 Processing of PWM waveforms :

In the simulation of the operation of power-electronic equipment, such as inverters, it is often necessary to process the PWM waveforms generated by the modules of PWLIB described above. For instance, some designs of inverter use PWM waves on each of the three phases with a 2 level PWM wave imposed between line and DC link centre tap. If it is desired to simulate the operation of a delta or star wound 3-phase motor, the line-to-line voltage waveforms experienced by each winding must be found. This naturally leads to the necessity of providing a waveform differencing module. Some inverters might generate several waveforms with a designated phase difference between these waveforms. To allow such systems to be simulated in complete generality, a waveform lagging or advancing option is included. Again, with practical available components, certain constraints on the switching angles of waveforms might be imposed. Typical of this is the commutation limit of switching components such as thyristors or power transistors. To meet these limitations, practically implemented PWM waveforms must often have a designated minimum pulse width and the microprocessor software must include a trap to prevent attempts to produce pulses less than this minimum width. The PWLIB package contains a number of options designed to aid the simulation of systems implementing strategies such as this.

The waveform processing options currently fall into two groups - those that deal with the most general pulsed waveforms as specified above and those whose action is restricted to waveforms switching between two, and only two, distinct levels. In fact, the only waveforms currently being considered, which fall into this latter category are PWM forms but others could be incorporated in the future. Obviously, those processing operations which can be carried out on general pulsed waveforms can also be carried out on two level waveforms (such as PWM) as these constitute a strict subset of the general waveforms. In the first group are the operations of phase

shift, concatenation of waveforms, synthesis of difference waveforms and harmonic analysis. The shift operation, in keeping with the general philosophy of the package, shifts the switching points and then renormalises the resulting description of the wave, so that the switching points all lie with the reference wavelength,  $0 < \phi < 2\pi$ . There are two types of waveform differencing provided. The first produces the straightforward difference between two waveforms, which of course may have up to the sum of the number of switching points of the two differenced waves [12]. The output of the second differencing operation is the waveforms of the voltages,  $V_d$  and  $V_q$  of the equivalent, 2-axis representation of a star-connected 3 phase motor when general pulsed waveform voltages are applied between the three input terminals and the common reference terminal. Although this operation will deal with any pulsed waveforms, its normal use is with PWM waveforms.

In the second category are subroutines for suppressing short pulses and for symmetrising waveforms. Operations are provided in PWLIB to suppress any pulse shorter than a fixed Interval in the phase variable. Related to this are operations for suppressing the shortest pulse or shortest several pulses independently of their length. Such operators have been found useful in formulating overall PWM strategies. Another feature which has been found useful in developing practical strategies is 'symmetrisation'. It is sometimes desirable that a PWM waveform should have half-wave symmetry. Regular symmetric and regular asymmetric waveforms do not in general possess this property. A subroutine has been provided, however, to modify any PWM waveform to produce a half-wave symmetric waveform based on the positive half wave. A half wave symmetric form based on the negative half wave can be provided, simply by combining this operation with two half wave period shifts.



### 3.3.4 Harmonic analysis :

Experience of using the PWM package rapidly led to an enhanced appreciation of its utility in areas other than time- dependent simulation problems. Simulation studies carried out have included both transient and steady-state investigations. Where steady-state investigations of linear or linearisable devices are required, It is convenient to break down a pulsed waveform into its Fourier components and compute the response of the device by summing its responses to the individual components. To this end, an operation is included in PWLIB which yields the Fourier series sine and cosine coefficients of any pulsed waveform.

It is not possible, of course to compute all the harmonic coefficients in the infinite series. It is left to the user to specify to the harmonic analysis subroutine how many harmonics are required – the subroutine is written in such a way as to be able to compute however many harmonics are required to it. This process has, of course, myriad uses outside of steady state response calculations and is probably the most widely used operation in the package[19]. Its use is not, of course, restricted to any particular variety of PWM waveform; it can be applied to the most general pulsed waveforms considered by the package.

### 3.3.5 Use of the package in transient simulations :

When simulation of highly nonlinear devices or systems are required, or when the simulation of transient characteristics is needed, the simulation is carried out in the time domain. For nonlinear steady-state simulations, sufficient simulated time must have elapsed for the transient to have decayed to a negligible level and this can be computationally expensive. In both these types of simulations, it is usually necessary to compute the value of a PWM waveform at any desired time. As pointed out before, given the operating frequency of the system, it is an easy matter to transform the time variable to a phase one and a function is included in the



package to return the level of any waveform corresponding to any given phase angle (not necessarily in the base wavelength  $(0, 2\pi)$ ).

When simulations involve the integration of sets of differential equations in which driving terms and possibly coefficients involve pulsed voltage waveforms, say, it is important that time steps of the simulation do not straddle switching points. Large errors in the simulated behavior may be incurred if this is allowed to happen. A function in PWLIB can return the next switching angle after any given phase and this function is commonly used in simulations to check if a proposed time step will straddle a switching point. It is then a simple matter to take one or more shortened time steps to ensure that the switching point coincides with a step junction. The switching actions taken then ensure that the simulation adequately represents the simulated system.

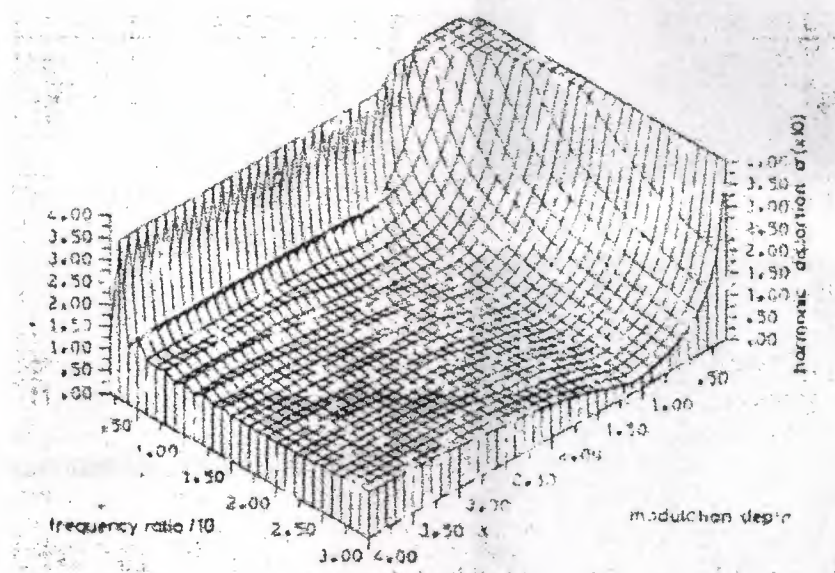
### 3.4 Synthesis of 3-Dimensional models :

It is possible, using PWLIB and its associated graphical output facilities, to generate 3-dimensional models which show the relationship between any three parameters. For example, using a performance criterion corresponding to the weighted harmonics voltage distortion of the PWM voltage, defined by the equation;

$$\alpha = \left( \sum_{n=2}^{30} \left[ \frac{V_n}{V_1} \right]^2 \right)^{1/2} / V_1 \quad (20)$$

A 3-dimensional model can be constructed, relating this performance index to the modulation depth and frequency ratio. Of course, any other desired performance criterion could equally be used, e.g. the amplitude of individual harmonics, current

distortion, torque ripple etc. These performance criteria can be related to specific operating conditions, e.g. particular volts/cycle characteristic with the effect of pulse dropping included, and displayed using a 3-dimensional model. These models may also be rotated to allow the designer to observe the relationships from the most appropriate angle.



**Fig.3.6 3-Dimensional Model**

As can be appreciated, these 3-dimensional models allow vast amounts of information to be condensed into a very compact visual display, thus allowing the system designer to appreciate the full implication of parameter changes on system performance.

### 3.5 Experimental results :

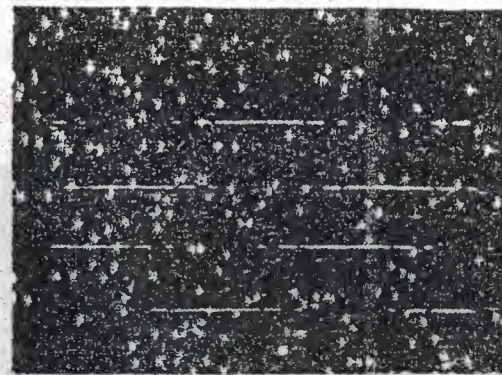
The results presented in this Section were obtained using an experimental microprocessor- controlled PWM inverter drive system [4]. A Zilog Z80



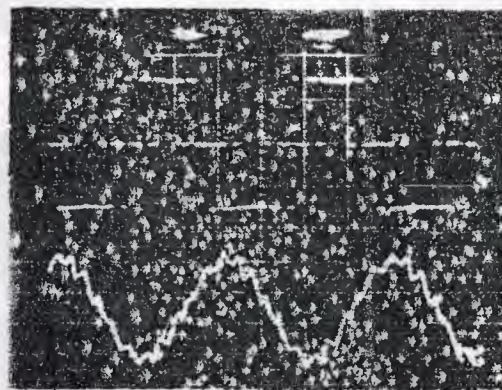
microprocessor was used to provide the PWM base-drive control for 3-phase transistor inverter supplying 1 hp induction motor.

**Fig.3.7** Experimental optimised PWM waveforms.

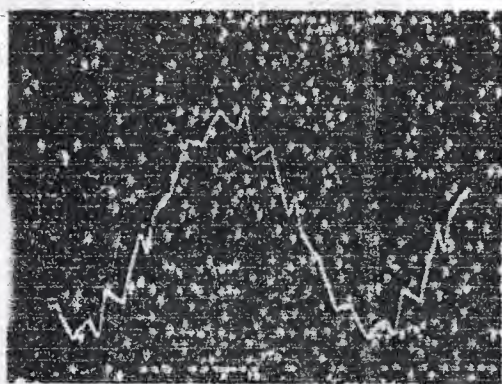
- (a) Two phase of microprocessor output
- (b) Machine line voltage and current
- (c) Machine current



*a*



*b*



*c*

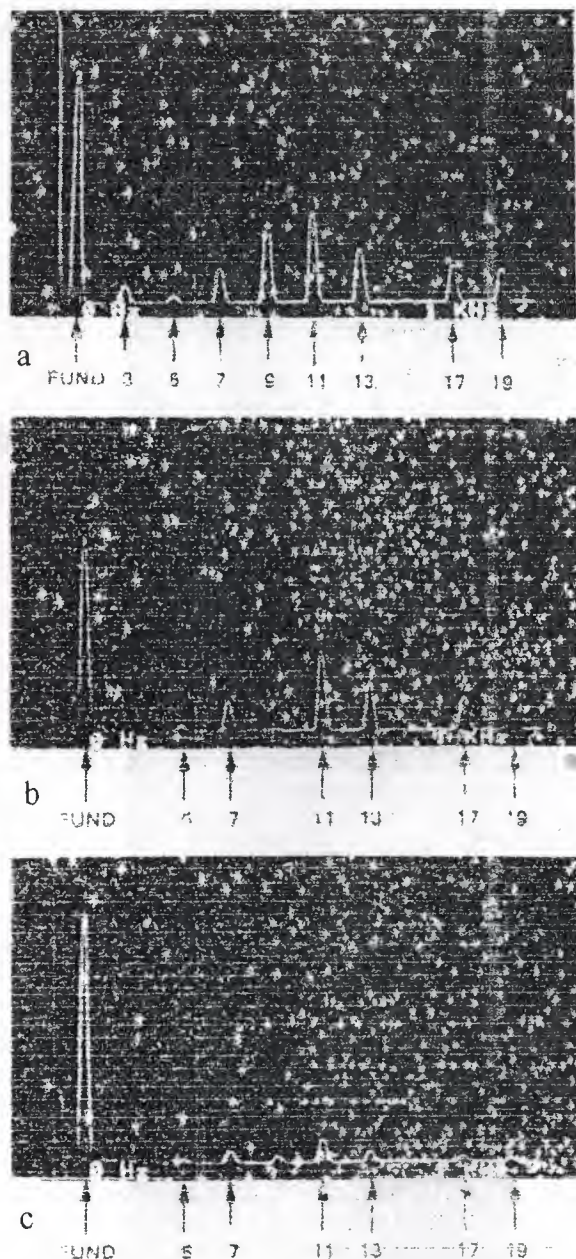


The experimental results have been chosen for comparison and confirmation of PWLIB computer simulation results.

Fig.3.7 shows experimental results when the optimised PWM switching strategy. Fig 3.7a shows two phases of the microprocessor 3-phase PWM output. These signals, when amplified, provide the base drive for the switching transistor in the

**Fig.3.8** Experimental optimised PWM spectra results.

- (a) Microprocessor output
- (b) Machine line voltage
- (c) Machine current



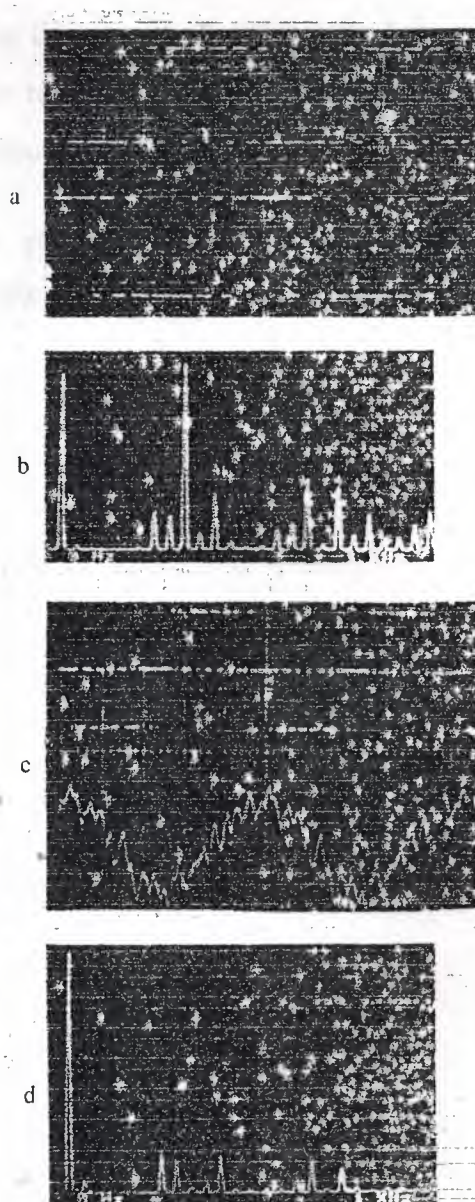
power stage of the inverter. The experimental harmonic spectrum corresponding to one of these phases is shown in Fig 3.8a. Comparisons of the experimental results shown in Fig.3.7a and 3.8a, with the corresponding computed results, demonstrate good agreement.

The experimental machine line voltage and current are shown in Fig.3.7b and c, and the corresponding harmonic spectra are shown in Fig.3.8b and c, respectively.

**Fig 3.9 Microprocessor and machine waveforms**

- (a) Two phases of microprocessor output
- (b) Spectrum of one phase of microprocessor output
- (c) Machine line voltage and current
- (d) Spectrum of machine current

(frequency ratio = 9, modulation depth = 0.9, inverter frequency = 40 Hz)





We see that good agreement exists between the computed and experimental results. It should be noted that the existence of a small fifth harmonic in the voltage spectrum is caused by the quantisation error introduced when programming the switching angles  $\alpha_1$  and  $\alpha_2$  on the microprocessor. The microprocessor switching angles were also used in the PWLIB simulations.

As a further demonstration, experimental results for symmetric regular sampling are shown in Fig. 3.9. These results should be compared with the computed results. Again, good agreement is demonstrated, noting that the small differences between the amplitudes of the higher harmonics are the result of the small time delays and quantisation error introduced by the microprocessor [13].

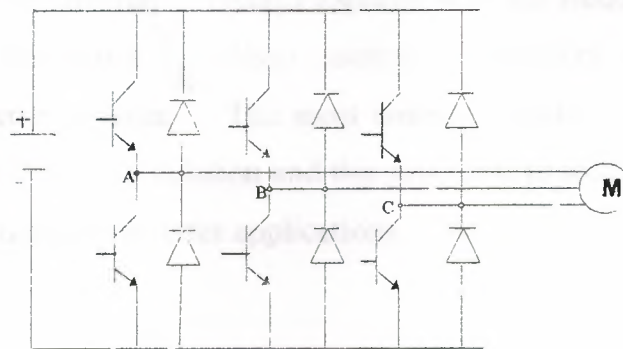
The results presented in this Section provide additional experimental confirmation of the accuracy and potential of PWLIB when used as a research and development tool.



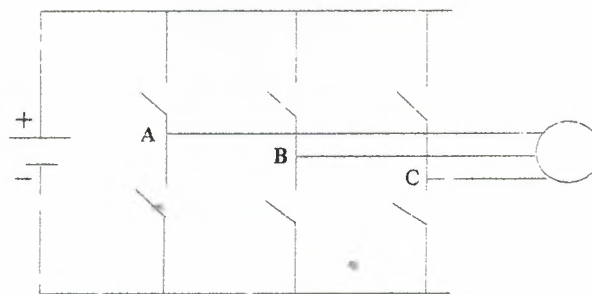


## CONCLUSION

Inverters are used to convert dc power to ac . Three phase inverters are normally used for high power applications . A general three phase bridge inverter is shown in figure (a) and its equivalent circuit using mechanical switches in figure (b) . Semiconductor devices with controlled turn off capability such as transistors , MOSFETs , GTOs , or Thyristors are used , depending on the voltage and current rating required to function as switches .



( a )



( b )

figure ( a ) A three-phase bridge inverter

( b ) Its equivalent circuit using mechanical switches

The output frequency of a static inverter is determined by the inverter control circuit by means of the rate at which it switches on and off the semiconductor devices . This control circuit provides an adjustable-frequency output . Nonsinusoidal voltage and current waveforms are produced as a result of basic switching functions of an inverter . These waveforms may adversely affect the motor performance . If the output frequency varies over a wide range filtering of harmonics is not possible . That's the reason why it is important to generate ac waveforms with low order harmonic content . In feeding an ac motor or transformer it is necessary to vary the output voltage together with the frequency to satisfy the proper magnetic conditions . Voltage control is therefore very important in adjustable - frequency systems . The most common method for minimizing the harmonics is Pulse Width Modulation and this project is to search various methods of PWM and their usage in inverter applications.

## References :

1. Murphy J.M.D., Turnbull F.G.(1989), "Power Electronic Control of AC Motors", Pergamon Press, UK.
2. P.C.Sen, G.Premchandran, (1984), "Improved PWM Control Strategy for Inverters and Induction Motor Drives" ,IEEE Trans. on Industrial Electronics.
3. M.G.Jayne, S.R.Bowes, B.M.Bird, (1975), "Developments in Sinusoidal PWM Inverters", IFAC Conf.Proc.
4. S.R.Bowes, R.R.Clements, (1982), "Computer-aided Design of PWM Inverter Systems", IEE Proc.
5. A.Balestrino, G.de Maria, L.Sciavicco, (1975), "On the Ordinary and Modified Subharmonic Control"
6. D.Grant, R.Seidner, (1982)"Technique for pulse elimination in pulsewidth-modulated inverters with no waveform discontinuity", IEE Proc.
7. R.M. Green, J.T.Bcys, (1982), "Implementation of Pulse width Modulated Inverter Modulation Strategies", IEEE Trans. on Ind. Applications.
8. S.R.Bowes, M.J.Mount, (1981), "Microprocessor Control of PWM Inverters".
9. O.Kukrier, H.B.Ertan, (1992), "Analytical Optimization of Regular-sampled PWM Waveforms", Aegean Inter.Conf.on Elec. Machines and Power Electronics.
10. S.R.Bowes, A.Midoun, (1985), "Suboptimal Switching Strategies for Microprocessor- controlled PWM Inverter Drives".



11. A.M.Trzynadlowski, (1989), "Nonsinusoidal Modulating Functions for Three-Phase Inverters", IEEE Trans.on Power Electronics.
12. S.R.Bowes, T.Davies, (1985), "Microprocessor-based Development System for PWM Variable-speed Drives", IEE Proc.
13. B.K.Bose, H.A.Sutherland, (1983), "A High Performance Pulse-width Modulator for an Inverter-fed Drive System using a Microcomputer", IEEE Trans. on Ind. Applications.
14. G.S.Buja, P.Fiorini, (1982), "Microcomputer Control of PWM Inverters", Ind. Electronics.
15. I.A.Taufiq, B.Melitt, C.J.Goodman, (1986), "Novel Algorithm for Generating Near Optimal PWM waveforms for AC Traction Drives", IEE Proc.
16. G.Joos, P.D.Ziogas, (1993), "On Maximizing Gain and Minimizing Switching Frequency of Delta Modulated Inverters", Ind. Electronics.
17. H.W.Van Der Broeck, H.C.Skudelny, G.V.Stanke, (1988), "Analysis and Realization of a Pulse Width Modulator Based on Voltage Space Vectors", Ind. Applications.
18. Y.Iwaji, S.Fukuda, (1992), "A Pulse Frequency Modulated PWM Inverter for Induction Motor Drives", Power Electronics.
19. M.P.Kazmierkowski, M.A.Dzieniakowski, W.Sulkowski, (1991) "Nevel Space Vector Based Current Conirollers for PWM Inverters".
20. B.K.Bose, (1990), "An Adaptive Hysteresis-Band Current Control Technique of a Voltage- Fed PWM Inverter for Machine Drive Systems", IEEE Trans.on Ind.Electronics.

21. Joseph Vithayathil, "Power Electronics Principles and Applications"  
International Edition.
22. Cyril W.Lander "Power Electronics"
23. P.C.Sen "Power Electronics"
24. John Marcus, Neil Sclater "Electronics Dictionary", Mc Graw-Hill

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