



NEAR EAST UNIVERSITY

DEPARTMENT OF ELECTRICAL AND ELECTRONIC
ENGINEERING

GRADUATION PROJECT

PULSE-WIDTH-MODULATION INVERTERS

(EE-400)

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1-INTRODUCTION:-

DC to ac converters are known as inverter. The function of an inverter is to change a dc input voltage to a symmetrical ac output of desired magnitude and frequency. The output voltage could be fixed or variable at a fixed or variable frequency. A variable output voltage can be obtained by varying the input dc voltage and maintaining the gain of the inverter constant. On the other hand, if the dc input voltage is fixed and is not controllable, a variable output voltage can be obtained by varying the gain of the inverter, which is normally accomplished by pulse-width-modulation (PWM) control within the inverter. The inverter gain may be defined as the ratio of the ac output voltage to ac input voltage. The output voltage waveform of ideal inverters should be sinusoidal, however the wave forms of practical inverters are non sinusoidal and contain certain square-wave voltages may be acceptable: and for high-power applications, low distorted sinusoidal wave forms are required. With the availability of high speed power semiconductor devices, the harmonic contents of output voltage can be minimized or reduced significantly by switching techniques. Inverters are widely used in industrial applications (e.g., variable speed ac motor drives, induction heating, standby power supplies, uninterruptible power supplies). The input may be a battery, fuel cell, solar cell, or other dc source. The typical single-phase outputs are (120V at 60Hz, 220V at 50Hz, 115V at 400Hz). For high power three phase systems, typical outputs are 220/380V at 50Hz, 120/208V at 60Hz and 115/200V at 400Hz.

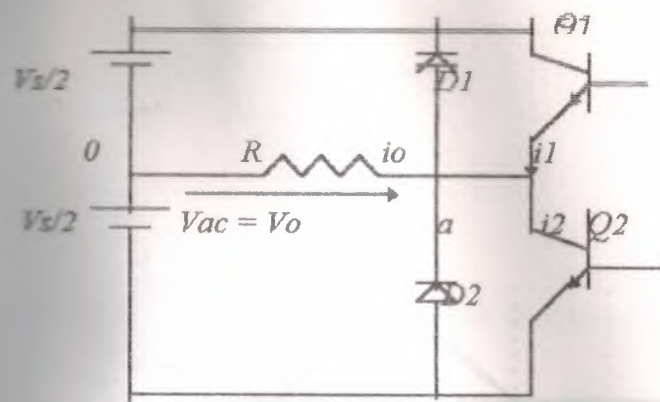
Inverters can be broadly classified into types: (1) single phase inverter, and (2) Three phase inverters . Each type can use controlled turn on and turn off devices (e.g. BJTs, MOSFETs, IGBTs, MCTs, SITs, GTOs) or forced commutated thyristors depending on applications. These inverters generally use PWM control signals for producing an ac output voltage. An inverter is called a voltage inverter (VFI) if the input voltage remains constant, a current fed inverter (CFI) if the input current is maintained constant, and a variable dc linked inverter if the voltage is controllable.

2.1-PRINCIPLE OF OPERATION:

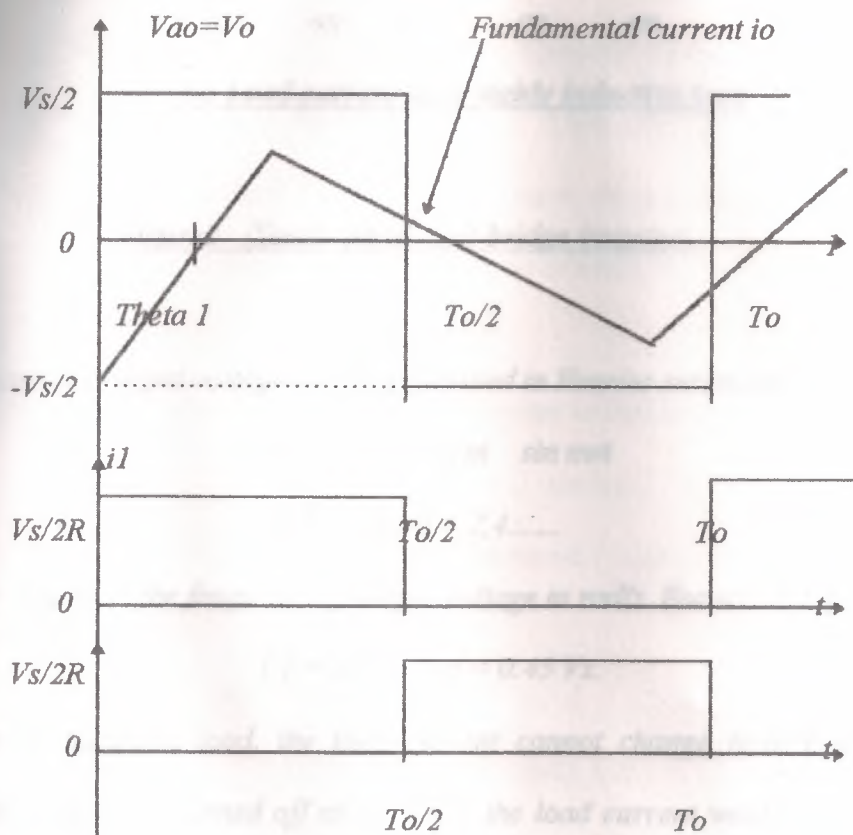
The principle of single phase inverters can be explained with fig.2-a . The inverter circuit consists of two choppers. When only transistor Q1 is turned on for a time $T_o/2$, The instantaneous voltage across the load v_o is $V_s/2$. If transistor Q2 only is turned on for a time $T_o/2$, $-V_s/2$ appears across the load. The logic circuit should be designed such that Q1 and Q2 are not turned on at the same time. Figure 2-b shows the wave forms for the output voltage and transistor currents with a resistive load . This inverter requires a three-wire dc source, and when a transistor is off, its reverse voltage is V_s instead of $V_s/2$. This inverter is known as half-bridge inverter.

The rms output voltage can be found from :

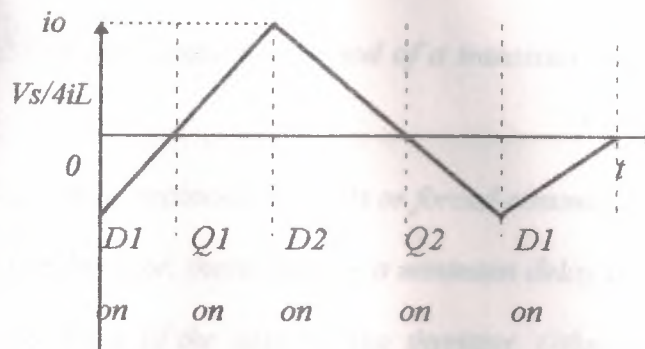
$$V_o = (2/T_o \int_0^{T_o/2} V_s/2 dt) = V_s / 2$$



(a) Circuit



(b) Waveforms with resistive load



(c) Load current with highly inductive load

Figure2 (Single-phase half-bridge inverter).

The instantaneous output voltage can be expressed in Fourier series as:

$$v_o = \frac{2V_s}{n\pi} \sin n\omega t$$

$$= 0 \quad \text{for } n = 2, 4, \dots$$

where $\omega = 2\pi f_o$ is the frequency of output voltage in rad/s. For $n = 1$, Eq.

$$V_1 = \frac{2V_s}{\pi} = 0.637 V_s.$$

For an inductive load, the load current cannot change immediately with the output voltage. If $Q1$ is turned off at $t = T_o/2$, the load current would continue to flow through $D2$, load, and the lower half of the dc source until the current falls to zero, and the upper half of the dc source. When diode $D1$ or $D2$ conducts, energy is fed back to the dc source and these diodes are known as feedback diodes. figure 1.2-c shows the load current and conduction intervals of devices for a purely inductive load. It can be noticed that for a purely inductive load, a transistor conducts only for $T_o/2$ (or 90°). Depending

on the load power factor, the conduction period of a transistor would vary from 90 to 180 degree.

The transistors can be replaced by GTOs or forced-commutated thyristors. If t_q is the turn-off time of a thyristor, there must be a minimum delay time of t_q between the outgoing thyristor and firing of the next coming thyristor. Other-wise, a short-circuit condition would result through the two thyristors. Therefore, the maximum conduction time of a thyristor would be $T_o/2 - t_q$. In practice, even the transistors require a certain turn on and turn-off time. For a successful operation of inverters, the logic circuit should take these into account.

For an RL load, the instantaneous load current i_o can be found from :

$$i_o = \frac{2V_s}{n\pi} \left[R + (\omega L) \sin(n\omega t - n) \right]$$

where $n = \tan^{-1}(\omega L/R)$. If I_{o1} is the rms fundamental load current, the fundamental output power (for $n = 1$) is :

$$\begin{aligned} P_{o1} &= V_1 I_{o1} \cos \phi = I_{o1}^2 R \\ &= \left[\frac{2V_s}{\pi} \right]^2 \frac{R}{R^2 + (\omega L)^2} \end{aligned}$$

3- PERFORMANCE PARAMETERS :

The output of practical inverters contain harmonics and the quality of an inverter is normally evaluated in terms of the following performance parameters.

Harmonic factor of nth harmonic, H_{fn} .

The harmonic factor (of the nth harmonic), which is a measure of individual harmonic contribution, is defined as :

$$H_{fn} = V_n / V_1$$

where V_1 is the rms value of the fundamental component and V_n is the rms value of nth harmonic component.

Total harmonic distortion THD .

The total harmonic distortion, which is a measure of closeness in shape between a wave form and its fundamental component, is defined as :

$$THD = 1/V_1 (\sum V_n)$$

Distortion factor DF.

THD gives the total harmonic content, but it does not indicate the level of each harmonic component. If a filter is used at output of inverter, the higher order harmonics should be attenuated more effectively, therefore, knowledge of both the frequency and magnitude of each harmonic is important. The distortion factor indicates the amount of harmonic distortion that remains in particular wave form after the harmonics of that wave form have been subjected to a second order attenuation. Thus DF is a measure of effectiveness

in reducing unwanted harmonics without having to specify the values of a second-order load filter and is defined as:

$$DF = 1/V_1 \left[\sum (V_n/n) \right]$$

The distortion factor of an individual harmonic component is defined as :

$$DF_n = V_n / V_{1n}$$

Lowest-order harmonic LOH.

The lowest order harmonic is that harmonic component whose frequency is closest to the fundamental one, and its amplitude is greater than or equal to three percent of the fundamental component.

SINGLE PHASE BRIDGE INVERTERS:

A single phase bridge inverter is shown in fig 4-a. It consists of four choppers. When transistors Q1 and Q2 are turned on simultaneously, the input voltage V_s appears across the load. If transistors Q3 and Q4 are turned on at the same time, the voltage across the load is reversed and is $-V_s$. The wave form for the output voltage is shown in fig 4-b.

The rms output voltage can be found from :

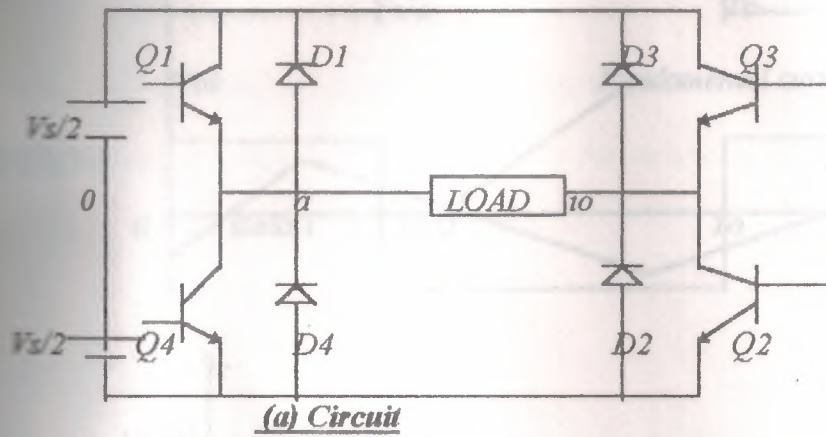
$$V_n = (2/T_o \int_0^{T_o/2} V_s dt) = V_s$$

The equation can be extended to express the instantaneous output voltage in a fourier

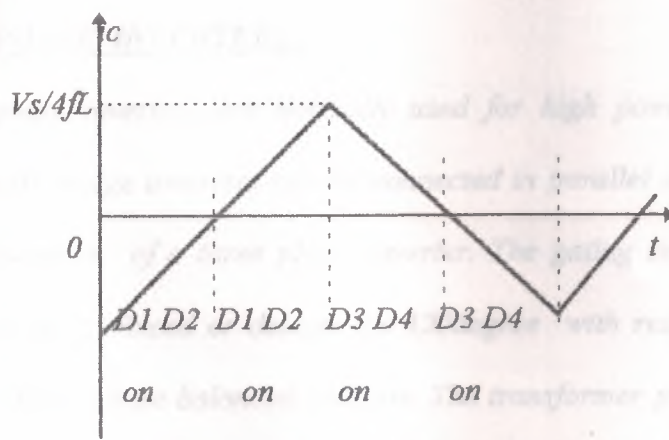
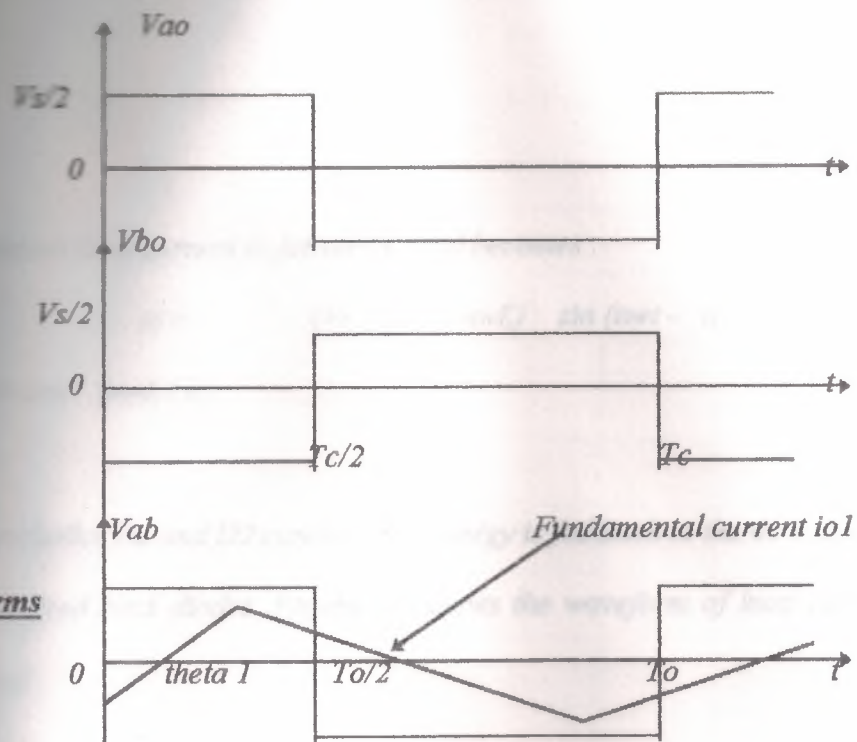
$$v_o = \frac{4V_s}{n} \sin n\omega t$$

and for $n = 1$ equation gives the rms value of fundamental component as :

$$V_1 = \frac{4V_s}{\sqrt{2} \pi} = 0.90V_s$$



Waveforms



(c) Load current with highly inductive load

Figure-4 (Single-phase full-bridge inverter).

the instantaneous load current i_o for an RL load becomes :

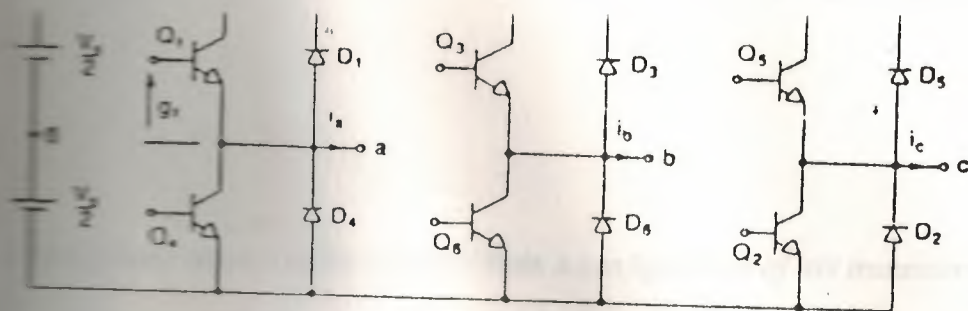
$$i_o = \frac{4V_s}{R} + (n\omega L) \sin(n\omega t - n)$$

where $n = \tan^{-1}(\omega L / R)$.

When diodes D1 and D2 conduct, the energy is fed back to the dc source and they are known as feed back diodes. Figure 2-c shows the waveform of load current for an inductive load.

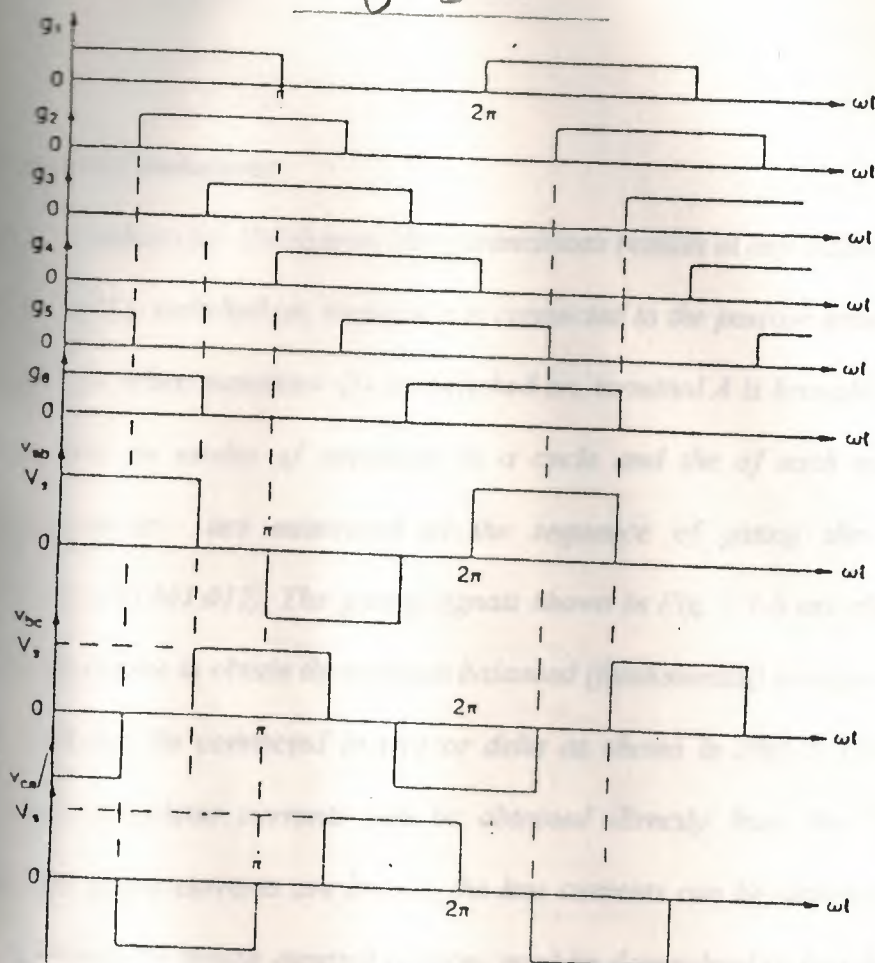
5- THREE PHASE INVERTERS :

Three phase inverters are normally used for high power applications. Three single phase half bridge inverters can be connected in parallel as shown in fig 5-a to form the configuration of a three phase inverter. The gating signals of single phase inverters should be advanced or delayed by 120 degree with respect to each other in order to obtain three phase balanced voltages. The transformer primary windings must be isolated from each other, while the secondary windings may be connected in wye or delta. The transformer secondary is normally connected in wye to eliminate triplen harmonics ($n = 3, 6, 9, \dots$) appearing on the output voltages and the circuit is shown in figure 5-b. This arrangement requires three single phase transformers, 12 transistors, and 12 diodes. If the output voltages of single phase inverters are not perfectly balanced in magnitudes and phase, the three phase output voltages will be unbalanced.



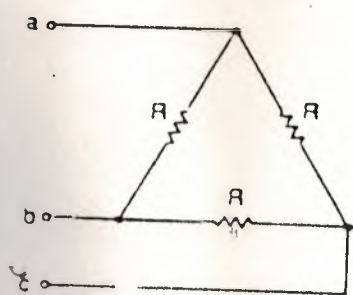
(a) Circuit

Fig: 5-a

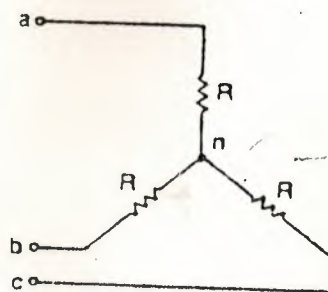


(b) Waveforms for 180° conduction

Fig-5-b



(a) Delta-connected



(b) Wye-connected

Fig-5-c

A three phase output can be obtained from a configuration of six transistors and six diodes as shown in fig 5.1-a. Two types of control signals can be applied to the transistors: 180degree conduction or 120degree conduction.

180-degree Conduction:

Each transistor conducts for 180 degree. Three transistors remain at any instant of time.

When transistor Q_1 is switched on, terminal a is connected to the positive terminal of the DC input voltage. When transistor Q_4 is switched on, terminal A is brought to the DC source. There are six modes of operation in a cycle and the of each mode is 60 degree. The transistors are numbered in the sequence of gating the transistor (e.g. 123.234.345.456.561.612). The gating signals shown in Fig. 5.1-b are shifted from each other by 60 degree to obtain three-phase balanced (fundamental) voltages.

The load may be connected in wye or delta as shown in Fig5.3. For a delta-connected load, the phase currents can be obtained directly from the line-to-line voltage. Once the phase currents are known, the line currents can be determined. For a wye-connected load, the line-to-neutral voltages must be determined to find the line (or phase) currents. There are three modes of operation in a half-cycle and the equivalent circuits are shown in Fig. 5.4-a for a wye-connected load.

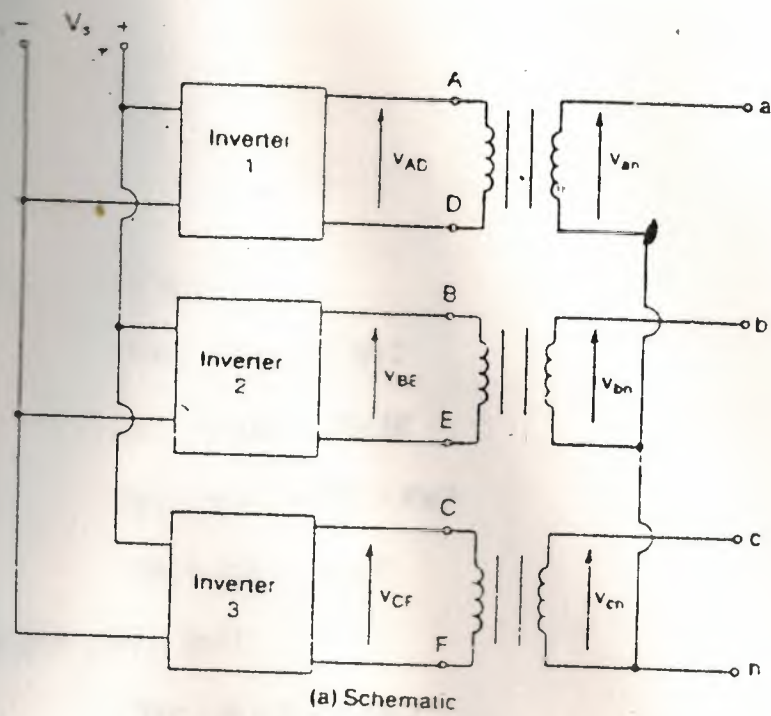


Fig: 5.1-a

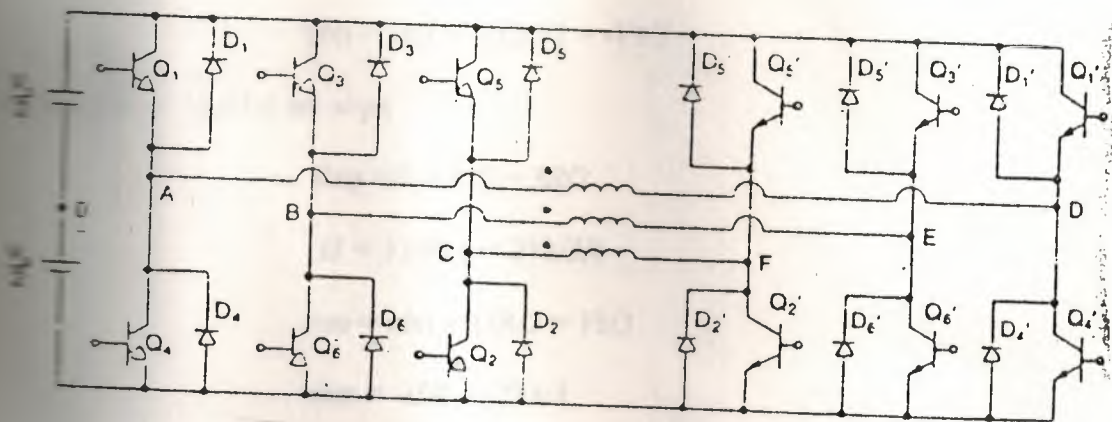
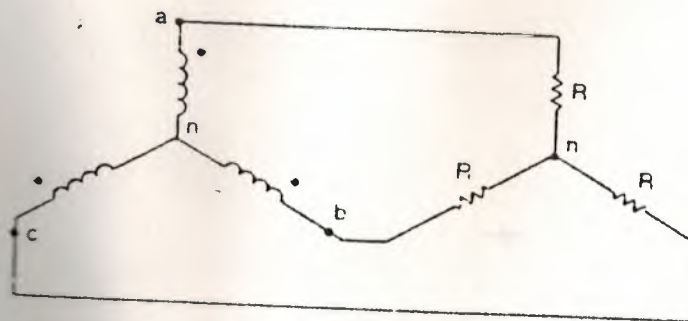


Fig: 5.1-b



(b) Circuit diagram

Fig: 5.3

During mode 1 for $0 \leq \omega t \leq \pi/3$,

$$R_{eq} = R + R/2 = 3R/2$$

$$i_1 = V_s/R_{eq} = 2V_s/3R$$

$$v_{an} = v_{cn} = i_1 R/2 = V_s/3$$

$$v_{bn} = -i_1 R = -2V_s/3$$

During mode 2 for $\pi/3 \leq \omega t \leq 2\pi/3$.

$$R_{eq} = R + R/2 = 3R/2$$

$$i_2 = V_s/R_{eq} = 2V_s/3R$$

$$v_{an} = i_2 R = 2V_s/3$$

$$v_{bn} = v_{cn} = -i_2 R/2 = -V_s/3$$

During mode 3 for $2\pi/3 \leq \omega t \leq \pi$,

$$R_{eq} = R + R/2 = 3R/2$$

$$i_3 = V_s/R_{eq} = 2V_s/3R$$

$$v_{an} = v_{bn} = i_3 R/2 = V_s/3$$

$$v_{cn} = -i_3 R = -2V_s/3$$

The line-to-neutral voltages are shown in figure 5.3-b. The instantaneous line-to-line voltages, v_{ab} , in figure 5.2-b can be expressed in a fourier series, recognizing that v_{ab} is shifted by $\pi/6$ and the even harmonics are zero.,

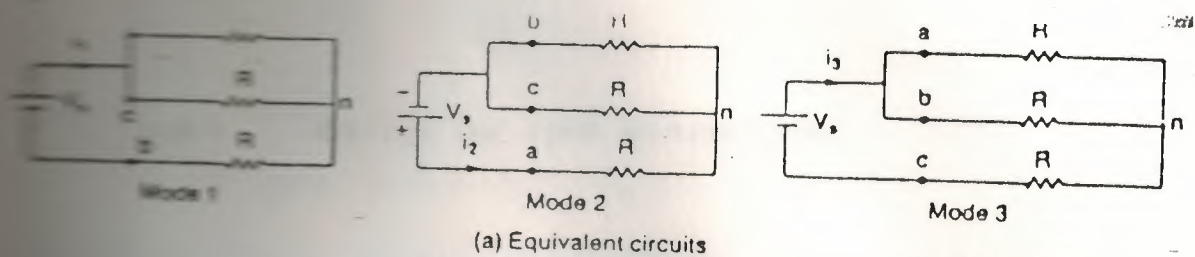
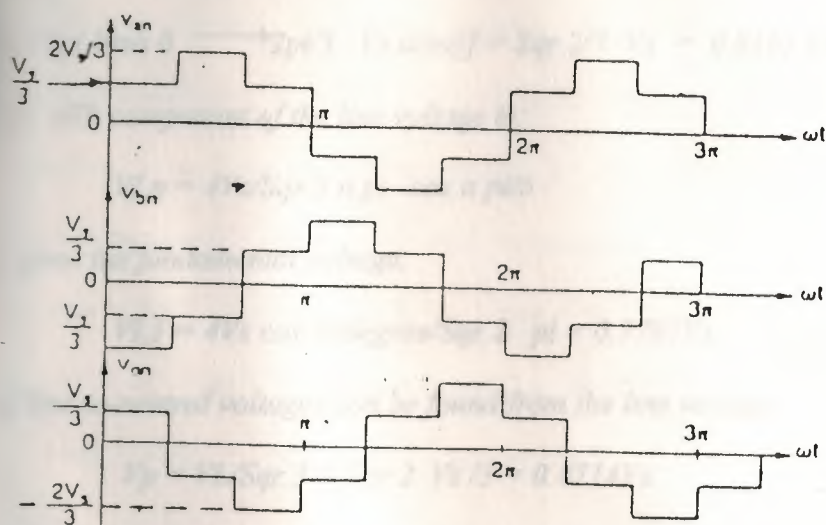


Fig: 5.3-a



(b) Phase voltages for 180° conduction

Fig: 5.3-b

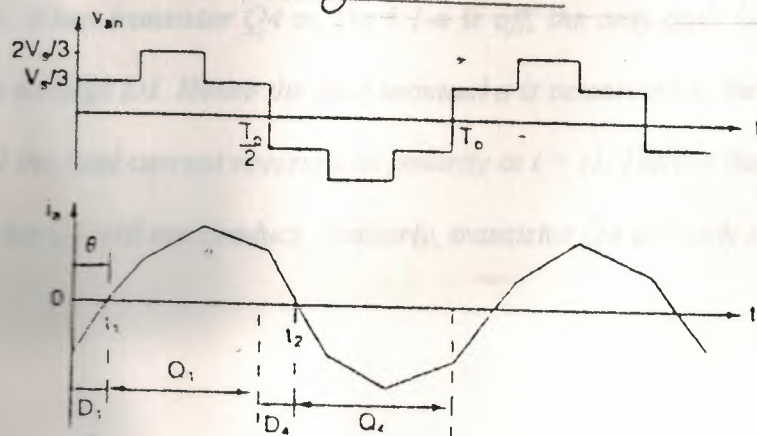


Fig: 5.4.1

$$v_{ab} = \frac{4V_s}{n\pi} \cos n\pi/6 \sin n(\omega t + \pi/6).$$

It can be found from above Eq. by phase shifting v_{ab} by 120degree and 240degree, respectively.

$$v_{bc} = \frac{4V_s}{n\pi} \cos n\pi/6 \sin n(\omega t - \pi/2)$$

$$v_{ca} = \frac{4V_s}{n\pi} \cos n\pi/6 \sin n(\omega t - 7\pi/6)$$

We can notice from Eqs. that the triplen harmonics ($n = 3, 9, 15, \dots$) would be zero in the line-to-line voltages.

The line-to-line rms voltage can be found from:

$$V_L = \left[\frac{2}{\pi} \int_0^{\pi/3} V_s d(\omega t) \right] = \frac{\sqrt{2}}{3} V_s = 0.8165 V_s.$$

From Eq. the rms n Th component of the line voltage is:

$$V_{Ln} = \frac{4V_s}{\sqrt{2} n\pi} \cos n\pi/6$$

which, for $n=1$, gives the fundamental voltage.

$$V_{L1} = \frac{4V_s \cos 30^\circ}{\sqrt{2} \pi} = 0.7797 V_s.$$

The rms value of line-to-neutral voltages can be found from the line voltage,

$$V_p = V_L / \sqrt{3} = \frac{\sqrt{2}}{3} V_s / \sqrt{3} = 0.4714 V_s.$$

With resistive loads, the diodes across the transistors have no functions. If the load is inductive, the current in each arm of the inverter would be delayed to its voltage as shown in Fig. 5.4. When transistor Q_4 in Fig 5.1-a is off, the only path for the negative line current I_a is through D_1 . Hence the load terminal a is connected to the DC source through D_1 until the load current reverses its polarity at $t = t_1$. During the period for $0 < t < t_1$, Transistor Q_1 will not conduct. Similarly, transistor Q_4 will only start to

conduct at $\omega t = 2$. The transistors must be continuously gated, since the conduction time of transistors and diodes depends on the load power factor.

For a wye-connected load, the phase voltage is $v_{an} = v_{ab}/\sqrt{3}$ with a delay of $\pi/6$ degree. Using Eq. , the line current I_a for an RL load is given by:

$$i_o = \left[\frac{4V_s}{\sqrt{3}} \sin \left(\frac{\pi}{6} + \frac{\omega t}{2} \right) \right] \cos(\omega t - \phi)$$

where $\phi = \tan^{-1}(\omega L/R)$.

120-Degree Conduction:

In this type of control, each transistor conducts for 120 degree. Only two transistors conduct at any instant of time. The gating signals are shown in Fig. 5.5. The conduction sequence of transistors is 61, 12, 23, 34, 45, 56, 61. There are three modes of operation in one half-cycle and the equivalent circuits for a wye-connected load are shown in Fig. 5.6-a. During mode 1 for $0 < \omega t < \pi/3$, transistors 1 to 6 conduct.

$$v_{an} = V_s/2, \quad v_{bn} = -V_s/2, \quad v_{cn} = 0.$$

During mode 2 for $\pi/3 < \omega t < 2\pi/3$, transistors 1 and 2 conduct,

$$v_{an} = V_s/2, \quad v_{bn} = 0, \quad v_{cn} = -V_s/2.$$

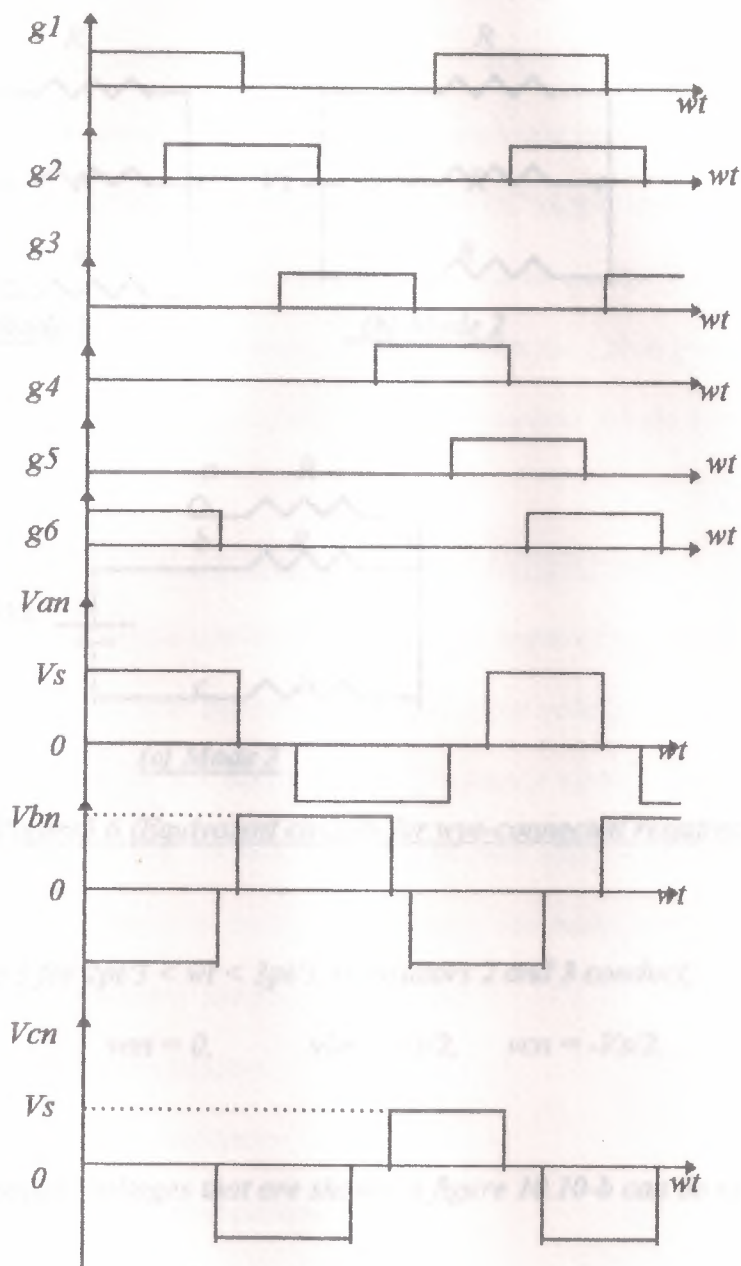
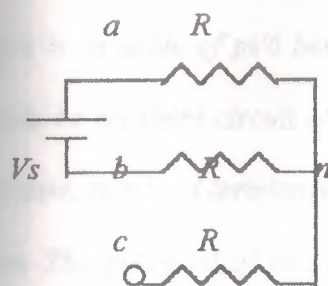
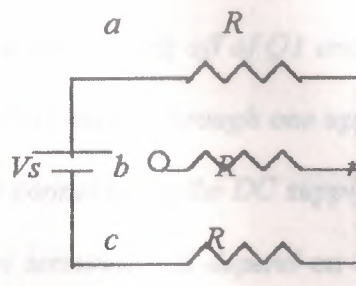


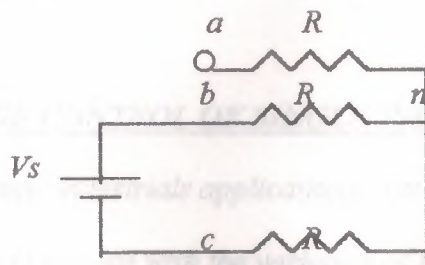
Figure 5.5 (Gating signals for 120 degree conduction.)



(a) Mode 1



(b) Mode 2



(c) Mode 3

Figure 5.6 (Equivalent circuits for wye-connected resistive load).

During mode 3 for $2\pi/3 < \omega t < 3\pi/3$, transistors 2 and 3 conduct,

$$v_{an} = 0, \quad v_{bn} = V_s/2, \quad v_{cn} = -V_s/2.$$

The line-to-neutral voltages that are shown in figure 10.10-b can be expressed in Fourier series as:

$$v_{an} = \frac{2V_s}{n\pi} \cos \frac{n\pi}{6} \sin n(\omega t + \pi/6)$$

$$v_{bn} = \frac{2V_s}{n\pi} \cos \frac{n\pi}{6} \sin n(\omega t - \pi/2)$$

$$v_{cn} = \frac{2V_s}{n\pi} \cos \frac{n\pi}{6} \sin n(\omega t - 7\pi/6)$$

The line a-to-b voltage is $v_{ab} = \sqrt{3} v_{an}$ with a phase advance of 30 degree.

There is a delay of $\pi/6$ between the turning off of Q1 and turning on Q4. Thus there should be no short circuit of the DC supply through one upper and lower transistors. At any time, two load terminals are connected to the DC supply and the third one remains open. The potential of this open terminal will depend on the load characteristics and would be unpredictable. Since one transistor conducts for 120 degree, the transistors are less utilized as compared to that of 180 degree conduction for the same load condition.

(6) VOLTAGE CONTROL OF SINGLE-PHASE INVERTERS:-

In many industrial applications, it is often required to control the output voltage of inverters (1) to cope with the variation of DC input voltage, (2) for voltage regulation of inverters, and (3) for the constant volts/frequency control requirement. There are various techniques to vary the inverter gain. The most efficient method of controlling the gain (and output voltage) is to incorporate pulse-width-modulation (PWM) control within the inverters. The commonly used techniques are:

1. Single-pulse-width modulation.
2. Multiple-pulse-width modulation.
3. Sinusoidal pulse-width modulation.
4. Modified sinusoidal pulse-width modulation.
5. Phase-displacement control.

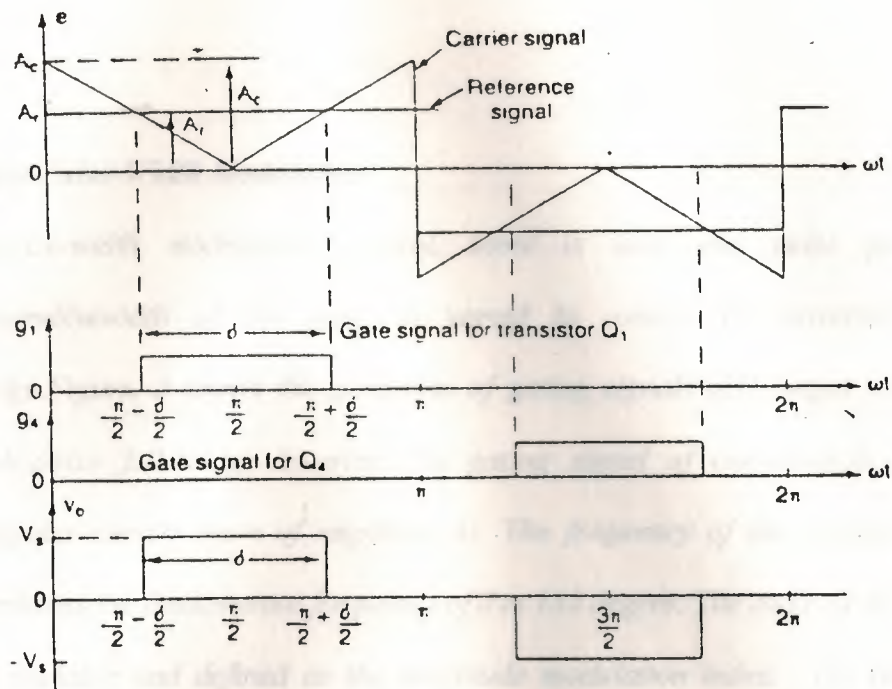


Fig : 6

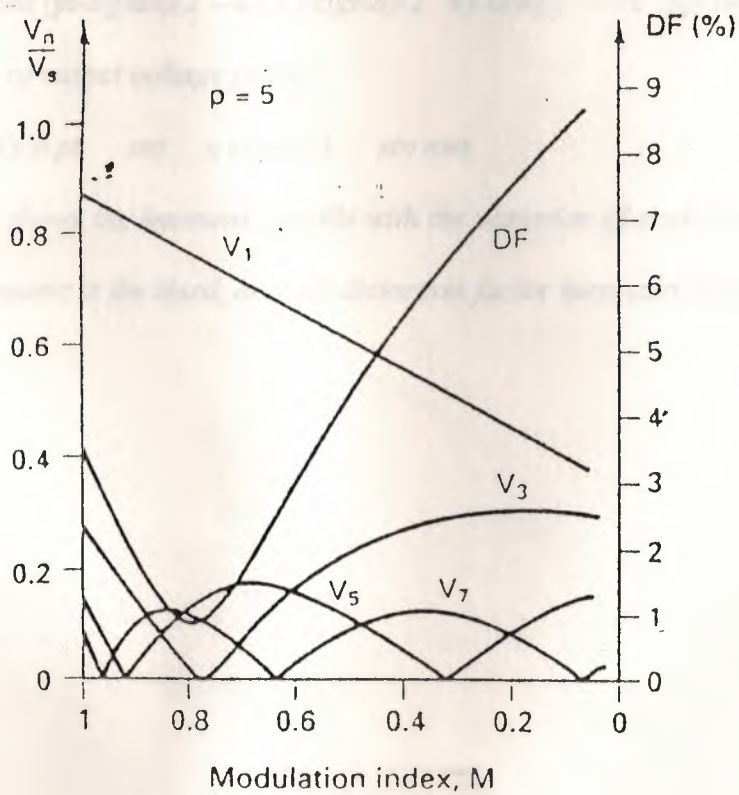


Fig : 6.1

Single-Pulse-Width Modulation:

In single-pulse-width modulation control, there is only one pulse per half-cycle and the width of the pulse is varied to control the inverter output voltage. Figure 6 shows the generation of gating signals and output voltage of single-pulse full-bridge inverters. The gating signal of amplitude, A_r , with a triangular carrier wave of amplitude, A_c . The frequency of the reference signal determines the fundamental frequency of 0 to 180 degree. The ratio of A_r to A_c is the control variable and defined as the amplitude modulation index. The amplitude modulation index, or simply modulation index:

$$M = A_r/A_c$$

The rms output voltage can be found from:

$$V_o = \left[\frac{2}{2\pi} \int_{(\pi-\sigma)/2}^{(\pi+\sigma)/2} V_s d(\omega t) \right] = V_s \sqrt{\sigma/\pi}$$

The Fourier series of output voltage yields:

$$V_o(t) = \frac{4V_s}{n\pi} \sin \frac{n\sigma}{2} \sin n\omega t$$

Figure 6.1 shows the harmonic profile with the variation of modulation index, M . The dominant harmonic is the third, and the distortion factor increases significantly at a low output voltage.

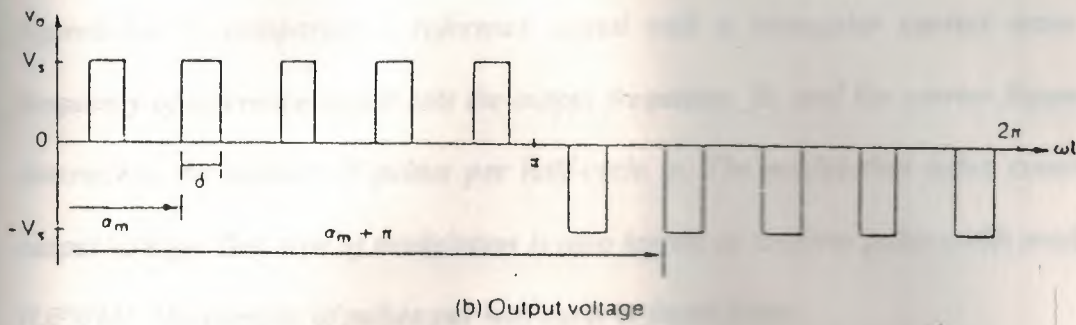
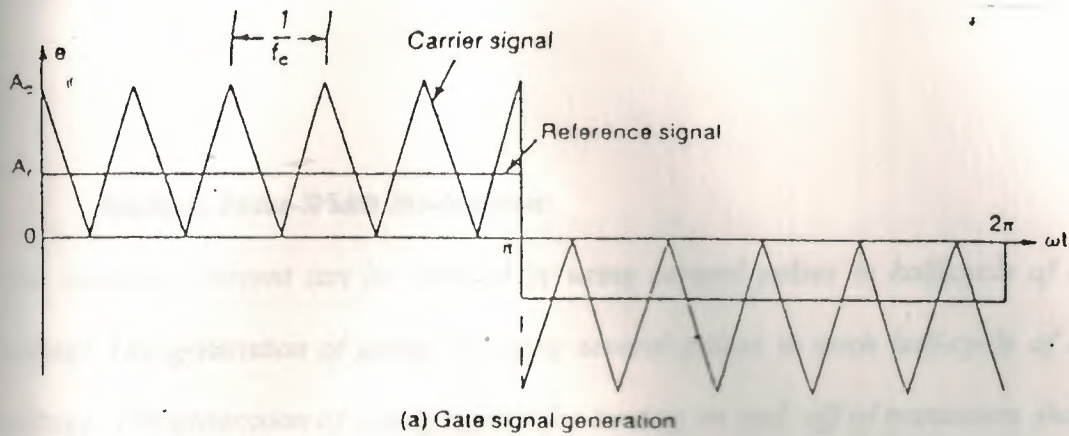
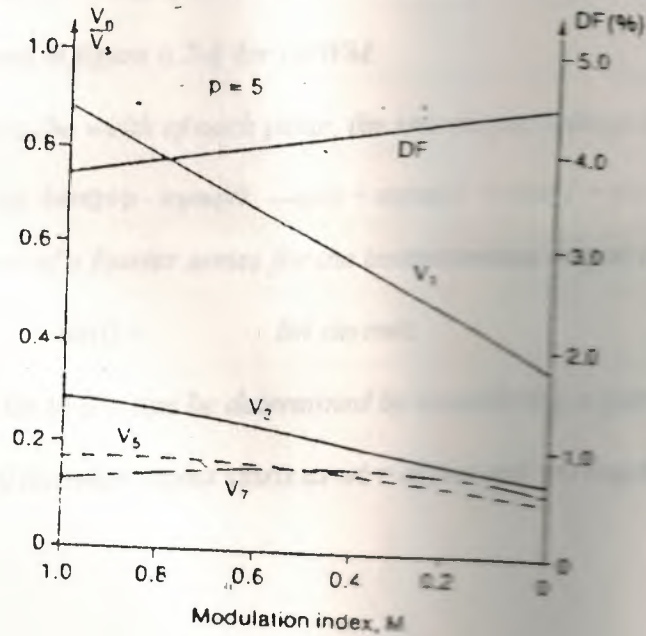


Fig: 6.2-a,b



Multiple-Pulse-Width Modulation:

The harmonic current can be reduced by using several pulses in half-cycle of output voltage. The generation of gating by using several pulses in each half-cycle of output voltage. The generation of gating signals for turning on and off of transistors shown in figure 6.2-a by comparing a reference signal with a triangular carrier wave. The frequency of reference signal sets the output frequency, f_o , and the carrier frequency, f_c , determines the number of pulses per half-cycle, p . The modulation index controls the output voltage. This type of modulation is also known as uniform pulse-width modulation (UPWM). The number of pulses per half-cycle is found from:

$$p = f_c / 2f_o = mf/2$$

where $mf = f_c/f_o$ is defined as the frequency modulation ratio.

The variation of modulation index M from 0 to 1 varies the pulse width from 0 to π/p and the output voltage from 0 to V_s . The output voltage for single-phase bridge inverters is shown in figure 6.2-b for UPWM.

If σ is the width of each pulse, the rms output voltage can be found from:

$$(V_o = [2p/2\pi \int_{(\pi/p - \sigma)/2}^{(\pi/p + \sigma)/2} V_s d(\omega t)] = V_s \sqrt{p \sigma / \pi})$$

The general form of a fourier series for the instantaneous output voltage is:

$$v_o(t) = \sum B_n \sin n\omega t.$$

The coefficient B_n in Eq. can be determined by considering a pair of pulses such that the positive pulse of duration σ starts at $\omega t = \alpha$ and the negative one of the same

width starts at $\omega t = \pi + \alpha$. This is shown in fig. 6.2-b. The effects of all pulses can be combined together to obtain the effective output voltage.

If the positive pulse of m th pairs starts at $\omega t = \alpha m$ and ends at $\omega t = \alpha m + \pi$, the Fourier coefficient for a pair of pulses is :

$$\begin{aligned} b_n &= \frac{1}{\pi} \left[\lim_{\omega t \rightarrow \alpha m + \pi} \cos n\omega t - \lim_{\omega t \rightarrow \alpha m} \cos n\omega t \right] \\ &= \frac{2V_s}{n\pi} \sin \frac{n\sigma}{2} \left[\sin n\left(\alpha m + \frac{\sigma}{2}\right) - \sin n\left(\alpha m + \frac{\sigma}{2}\right) \right] \end{aligned}$$

The coefficient B_n of Eq. can be found by adding the effects of all pulses.

$$B_n = \frac{2V_s}{n\pi} \sin \frac{n\sigma}{2} \left[\sin n\left(\alpha m + \frac{\sigma}{2}\right) - \sin n\left(\alpha m + \frac{\sigma}{2}\right) \right]$$

Figure.6.3 shows the harmonic profile against the variation of modulation index for five pulses per half-cycle. The order of harmonics is the same as that of single-pulse modulation. The distortion factor is reduced significantly compared to that of single-pulse modulation. However, due to large number of switching on and off process of power transistors, the switching losses would increase. With larger values of p , the amplitudes of lower-order harmonics would be lower, but the amplitudes of some higher-order harmonics would increase. However, such higher-order harmonics produce negligible ripple or can easily be filtered out.

Sinusoidal Pulse-Width Modulation:

Instead of maintaining the width of all pulses the same as in the case of multiple-pulse modulation, the width of each pulse is varied in proportion to the amplitude of a sine wave evaluated at the center of the same pulse. The distortion factor and lower-order harmonics are reduced significantly. The gating signals as shown in figure.6.4-a are generated by comparing a sinusoidal reference signal with a triangular carrier wave frequency, f_c . The type of modulation is commonly used in industrial applications and abbreviated as SPWM. The frequency of reference signal, f_r , determines the inverter output frequency, f_o , and its peak amplitude. A_r , controls the modulation index, M , and then turn the rms output voltage, V_o . The number of pulse per half-cycle depends on the carrier frequency. Within the constraint that two transistors of the same arm (Q1 and Q4) cannot conduct at the same time, the instantaneous output voltage is shown in Fig.6.5-a. The same gating signals can be generated by using unidirectional triangular carrier wave as shown in Fig.6.5-b.

The rms output voltage can be varied by varying the modulation index M . It can be observed that the area of each pulse corresponds approximately to the area under the sine wave between the adjacent midpoint of off periods on the gating signals. If σ_m is the width of m th pulse, Eq. can be extended to find the rms output voltage.

$$V_o = V_s \left(\frac{\sigma_m}{\pi} \right)$$

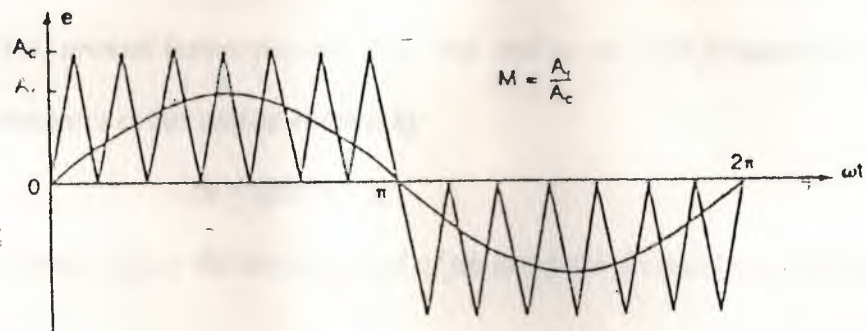
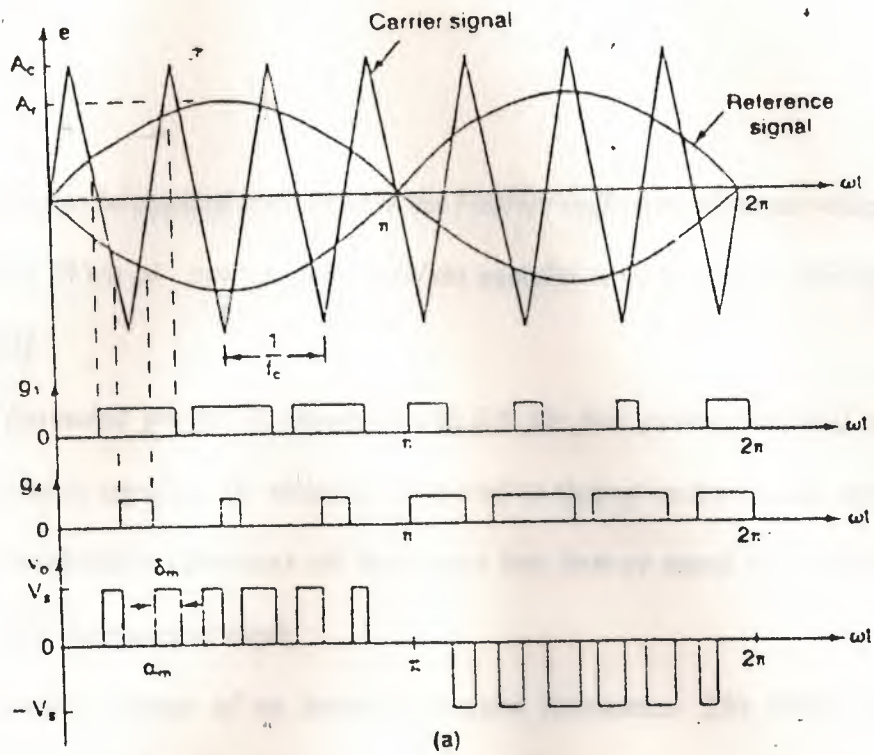


Fig: 6.5-a, b

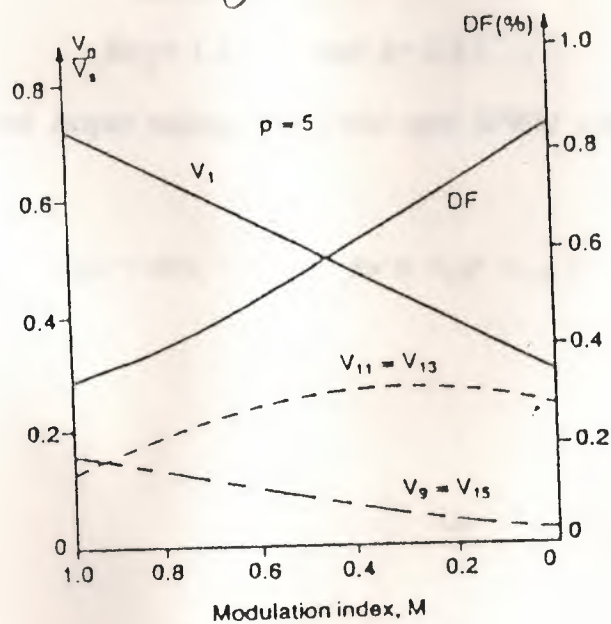


Fig: 6.6

Equation can also be applied to determine the Fourier coefficient of output voltage as.

$$B_n = \frac{2V_s}{n\pi} \sin n \frac{\delta}{2} [\sin n(\alpha + \frac{\delta}{2}) - \sin n(\pi + \alpha + \frac{\delta}{2})].$$

The harmonic profile is shown in Fig.6.6 for five pulses per half-cycle. The distortion factor is significantly reduced compared to that of multiple-pulse modulation. This type of modulation eliminates all harmonics less than or equal to $2p - 1$. For $p = 5$, the lowest-order harmonic is ninth.

The output voltage of an inverter contains harmonics. The PWM pushes the harmonics into a high-frequency range around the switching frequency f_c and its multiples, that is, around harmonics mf , $2mf$, $3mf$, and so on. The frequencies at which the voltage harmonics occur can be related by

$$f_n = (jmf + k)f_c$$

Where the harmonic equals the k th sideband of j th times the frequency-modulation ratio mf .

$$n = jmf + k$$

$$= 2jp + k \quad \text{for } j = 1, 2, 3, \dots \text{ and } k = 1, 3, 5, \dots$$

The peak fundamental output voltage for PWM and SPWM control can be found approximately from:

$$V_{m1} = dV_s, \quad \text{for } 0 \leq d \leq 1.0$$

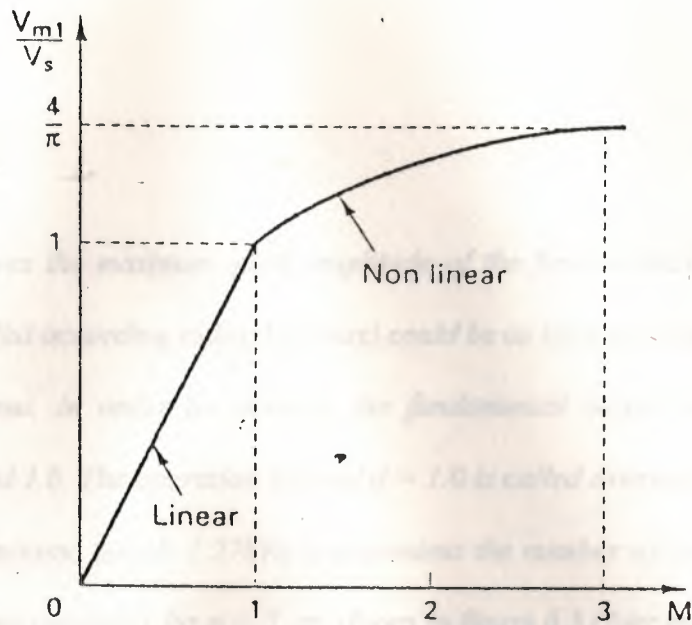


Fig: 6.5

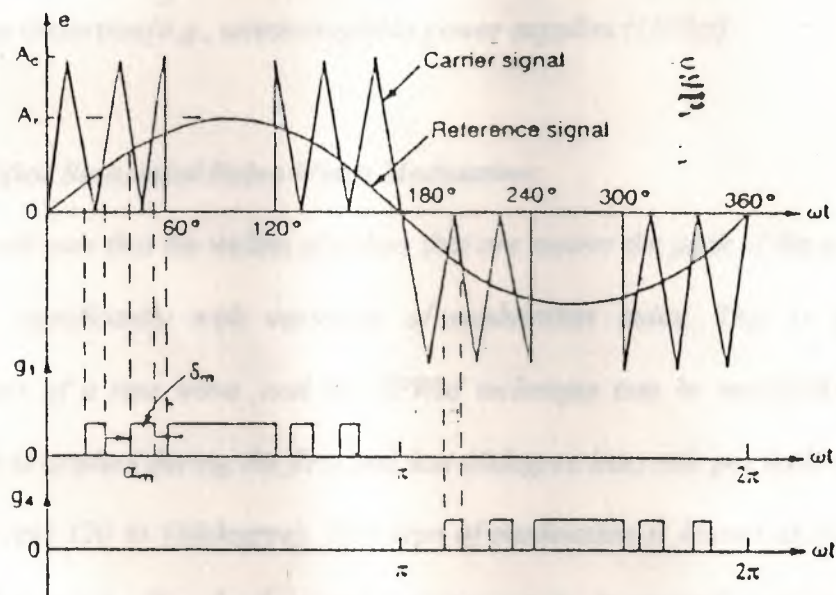


Fig: 6.8

For $d = 1$, Eq gives the maximum peak amplitude of the fundamental output voltage as $V_m(\max) = V_s$. But according to Eq, $V_m(\max)$ could be as high as $4V_s/\pi = 1.278V_s$ for a square-wave output. In order to increase the fundamental output voltage, d must be increased beyond 1.0. The operation beyond $d = 1.0$ is called overmodulation. The value of d at which $V_m(\max)$ equals $1.278V_s$ is dependent the number of pulses per half-cycle per p and is approximately 3 for $p = 7$, as shown in figure.6.5. Over modulation basically leads to a square-wave operation and adds more harmonics as compared to operation in the linear range (with $d < 1.0$). Over modulation is normally avoided in applications requiring low distortion [e.g., uninterruptible power supplies (UPSs)].

Modified Sinusoidal Pulse-Width Modulation:

Figure.6.5 indicates that the widths of pulses that are nearer the peak of the sine wave do not change significantly with variation of modulation index. This is due to the characteristics of a sine wave, and the SPWM technique can be modified so that the carrier wave is applied during the first and last 60 degree intervals per half-cycle (e.g., 0 to 60 degree and 120 to 180 degree). This type of modulation is known as MSPWM and shown in figure.6.8. The fundamental component is increased and its harmonic characteristics are improved. It reduces the number of switching of power devices and also reduces switching losses.

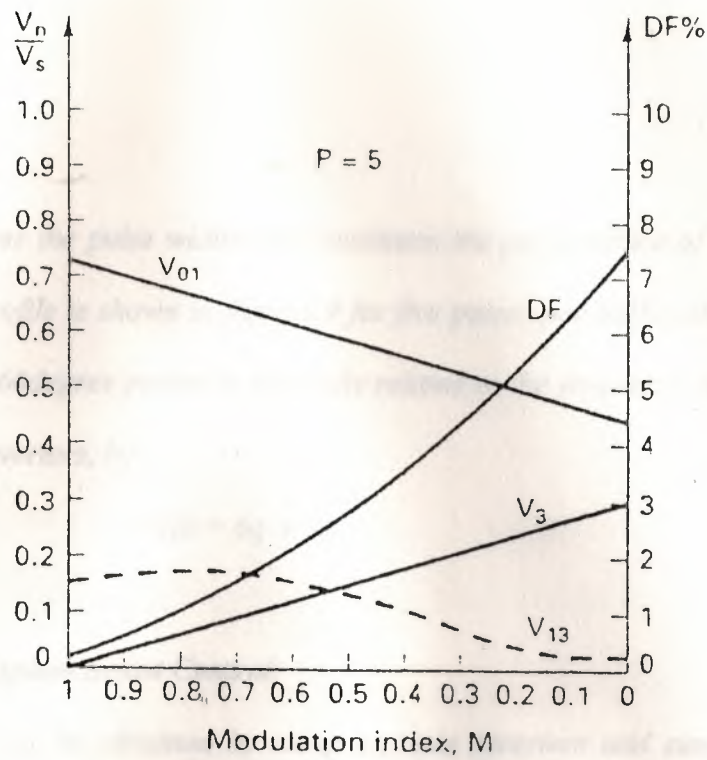


Fig: 6.9

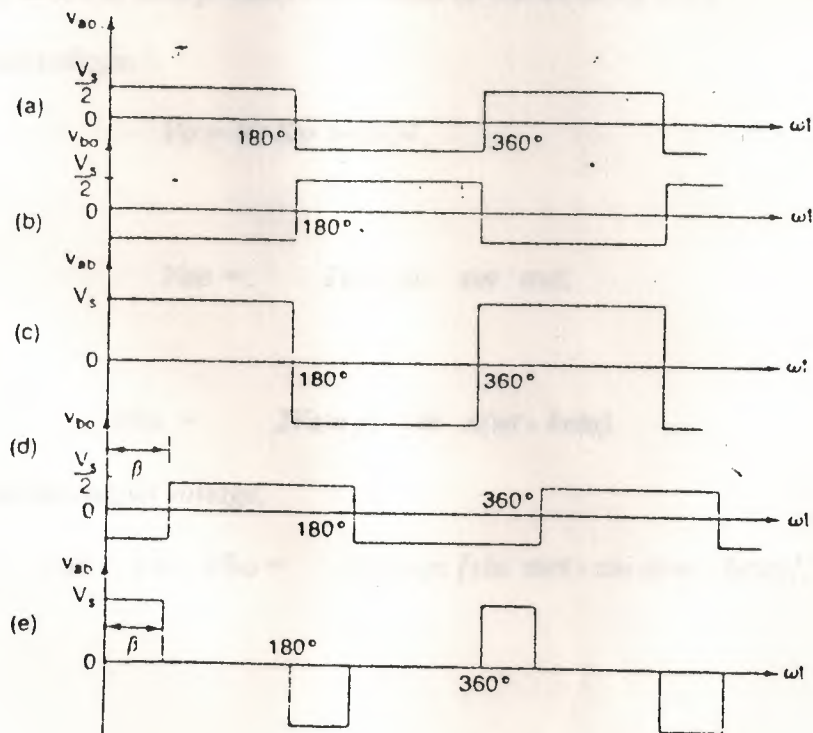


Fig: 6.4c, 6.5-e

Determines the pulse widths and evaluates the performance of modified SPWM.

The harmonic profile is shown in figure 6.9 for five pulses per half-cycle. The number of pulses, q , in the 60 degree period is normally related to the frequency ratio, particularly in three-phase inverters, by :

$$f_c/f_o = 6q + 3.$$

Phase-Displacement Control:

Voltage control can be obtained by using multiple inverters and summing the output voltages of individual inverters. A single-phase full-bridge inverter in fig. 6.2-a can be perceived as the sum of two half-bridge inverters in fig. 6.3-a . A 180 degree phase displacement produces an output voltage as shown in fig. 6.4-c, whereas a delay (or displacement) angle of β produces an output as shown in fig. 6.5-e.

The rms output voltage,

$$V_o = V_s \sqrt{\beta/\pi}.$$

if

$$V_{ao} = \frac{2V_s}{n\pi} \sin n\omega t.$$

then,

$$V_{bo} = \frac{2V_s}{n\pi} \sin n(\omega t - \beta).$$

The instantaneous output voltage,

$$V_{ab} = V_{ao} - V_{bo} = \frac{2V_s}{n\pi} [\sin n\omega t - \sin n(\omega t - \beta)].$$

since $\sin A - \sin B = 2\sin[(A - B)/2]\cos[(A+B)/2]$,

$$V_{ab} = \frac{4V_s}{n\pi} \sin \frac{n\beta}{2} \cos n(\omega t - \beta/2).$$

The rms value of the fundamental output voltage is:

$$V_1 = \frac{4V_s}{\sqrt{2}} \sin \frac{\beta}{2}.$$

Eq. indicates that the output voltage can be varied by varying the delay angle. This type of control is especially useful for high-power applications, requiring a large number of transistors in parallel.

(7) VOLTAGE CONTROL OF THREE-PHASE INVERTERS:-

A three-phase inverter may be considered as three single-phase inverters and the output of each single-phase inverter is shifted by 120degree. The voltage control techniques discussed in section 6 are applicable to three-phase inverters. As an example, the generations of gating signals with sinusoidal pulse-width modulation are shown in figure.7. There are three sinusoidal reference waves each shifted by 120degree. A carrier wave is compared with reference signal corresponding to a phase to generate the gating signals for the phase.

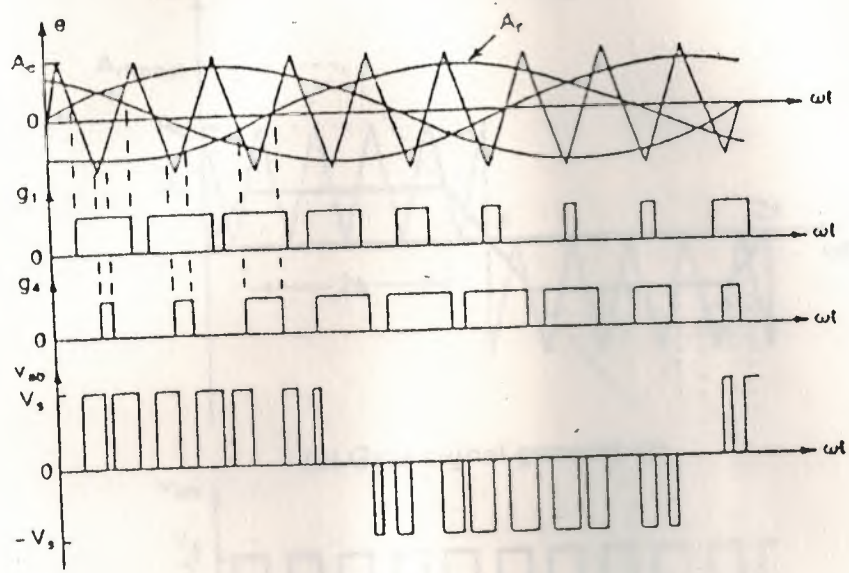
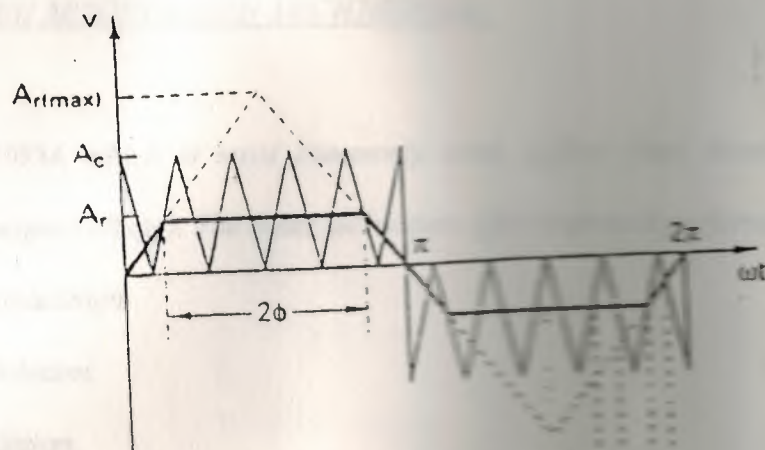
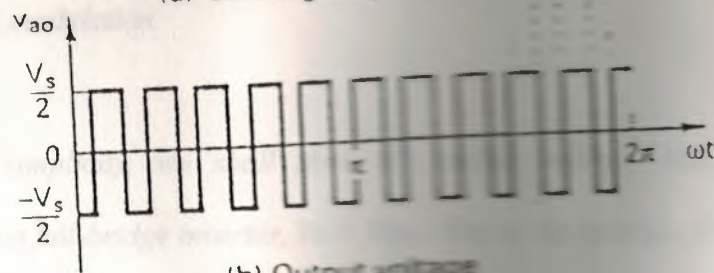


Fig: 7, 7.1



(a) Gate signal generation



(b) Output voltage

Graphical modulation:

Fig: 8

The output voltage as shown in figure.7.1, is generated by eliminating the condition that two switching devices in the same arm cannot conduct at the same time.

(8) ADVANCED MODULATION TECHNIQUES:-

The SPWM, which is most commonly used, suffers from drawbacks(e.g, low fundamental output voltage). The other techniques offer improved performances are:

Trapezoidal modulation.

Staircase modulation.

Stepped modulation.

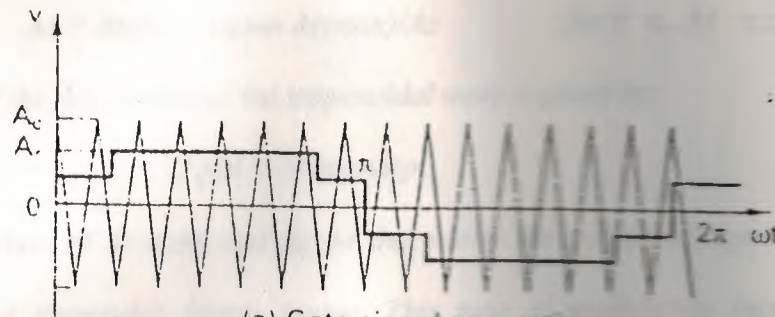
Harmonic injection modulation.

Delta modulation.

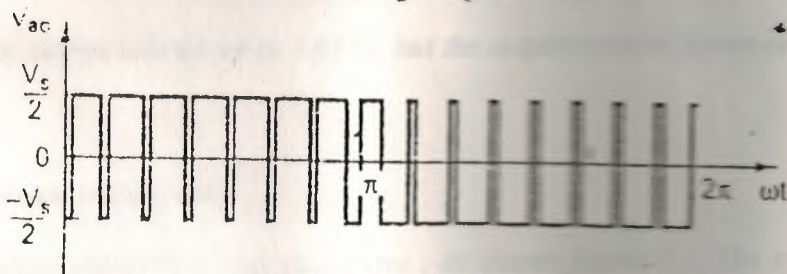
For the sake of simplicity, we shall show the output voltage, V_{ao} , for a half-bridge inverter. For a full-bridge inverter, $V_o = V_{ao} - V_{bo}$ is the inverse of V_{ao} .

Trapezoidal modulation:

The gating signals are generated by comparing a triangular carrier wave with a modulating trapezoidal wave[6] as shown in figure.8.



(a) Gate signal generation



(b) Output voltage

Fig: 8.1

The trapezoidal wave can be obtained from a triangular wave by limiting its magnitude to $\pm A_r$, which is related to the peak value $A_r(\max)$ by:

$$A_r = \sigma A_r(\max)$$

where σ is called the triangular factor, because the waveform becomes a triangular wave then $\sigma = 1$. The modulation index M is :

$$M = A_r/A_c = \sigma A_r(\max)/A_c \quad \text{for } 0 \leq M \leq 1$$

The angle of the flat portion of the trapezoidal wave is given by:

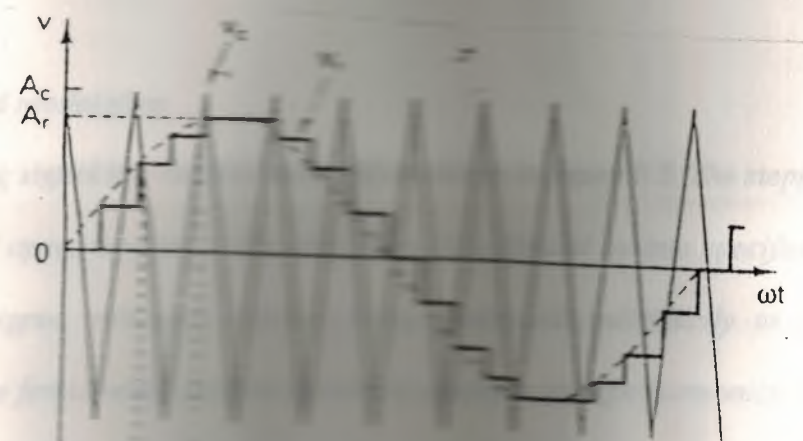
$$2\phi = (1-\sigma)\pi.$$

For fixed values of $A_r(\max)$ and A_c , M that varies the output voltage can be varied by changing the triangular factor, σ . This type of modulation increases the peak fundamental output voltage up to 1.05Vs, but the output contains lower-order harmonics.

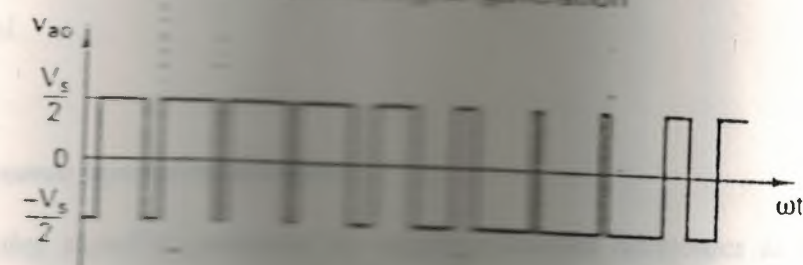
Staircase modulation:

The modulating signal is a staircase wave, as shown figure.8.1. The staircase is not a sampled approximation to the sine wave. The levels of the stairs are calculated to eliminate specific harmonics. The modulation frequency ratio m_f and the number of steps are chosen to obtain the desired quality of the output voltage. This is an optimized PWM and it is not recommended for fewer than 15 pulses in one cycle.

it has a low ripple, low harmonic distortion, low voltage and low distortion.
 (b) The ripple voltage is given by $V_r = \frac{V_m}{2fRC}$ for three levels and $V_r = \frac{V_m}{2fRC}$ for four levels. The ripple voltage is given by $V_r = \frac{V_m}{2fRC}$ for three levels and $V_r = \frac{V_m}{2fRC}$ for four levels.



(a) Gate signal generation



(b) Output voltage

$$F_g = 8.2$$

It has been shown [7] that for high fundamental output voltage and low distortion factor, the optimum number of pulses in one cycle is 15 for two levels, 21 for three levels and 27 for four levels. This type of control provides a high-quality output voltage with a fundamental values of up to 0.94Vs.

Stepped modulation:

The modulating signal is a stepped wave [8] as shown in figure.8.2. The stepped wave is not a sampled approximation to the sine wave. It is divided by into specified intervals, say with 20degree with each interval being controlled individually to control the magnitude of the fundamental component and to eliminate specific harmonics. This type of control low distortion, but a higher fundamental amplitude compared to that of normal PWM control.

Harmonic injected modulation:

The modulating signal is generated by injecting selected harmonics to the sine. The results in flat-topped wave form and reduces the amount of overmodulation. It provides the higher fundamental amplitude and low distortion of the output voltage. The modulating signal[9] is generally composed of:

$$V_r = 1.15 \sin \omega t + 0.27 \sin 3\omega t - 0.029 \sin 9\omega t.$$

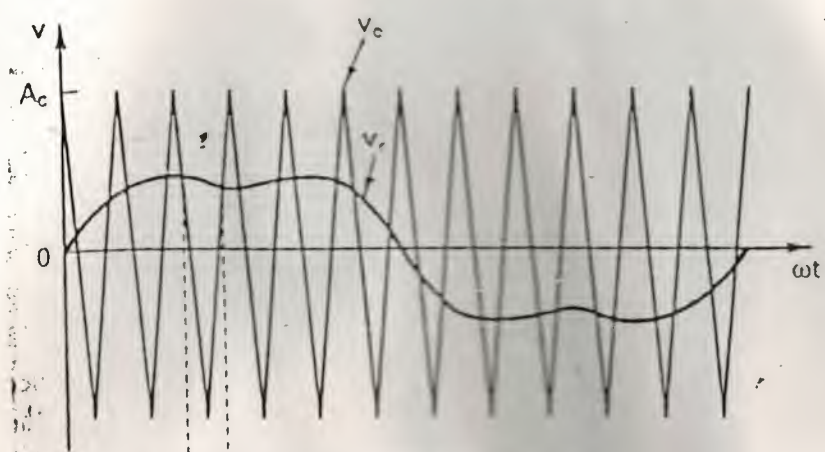
The modulating signal with third and ninth harmonic injections is shown in figure.8.3 it should be noted that the injection of 3rd harmonics will not effect the quality of the output voltage, because the output of the three-phase inverter does not contain triplen harmonics. If only third harmonic is injected, V_r is given by:

$$V_r = 1.15 \sin \omega t + 0.19 \sin 3\omega t$$

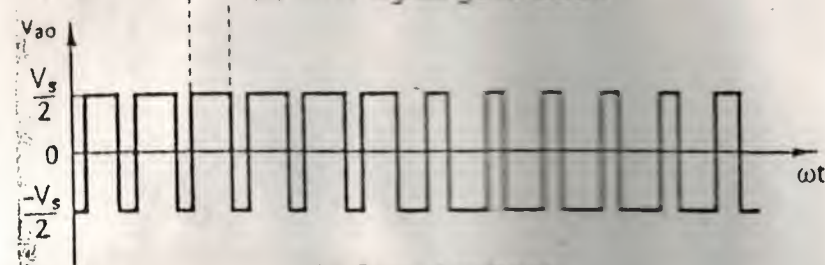
The modulating signal[10] can be generated from $2\pi/3$ segments of a sine wave as shown in figure.8.4. This is the as injecting 3rd harmonics to a sine wave. The line-to-line voltage is sinusoidal PWM and the amplitude of the fundamental component is approximately 15% more than that of a normal sinusoidal PWM. Since each arm is switched off for one-third of the period, the heating of the switching devices is reduced.

Delta modulation:

In delta modulation [11], a triangular wave is allowed to oscillate within a defined window ΔV above and below the reference sine wave V_r . The inverter switching function, which is identical to the output voltage V_o is generated from the vertices of the triangular wave V_c as shown in fig.8.5. It is also known as hysteresis modulation. If the frequency of the modulating wave is changed keeping the slope of the triangular wave constant, the number of pulses widths of the modulated wave would change.



(a) Gate signal generation



(b) Output voltage

Fig: 8.3

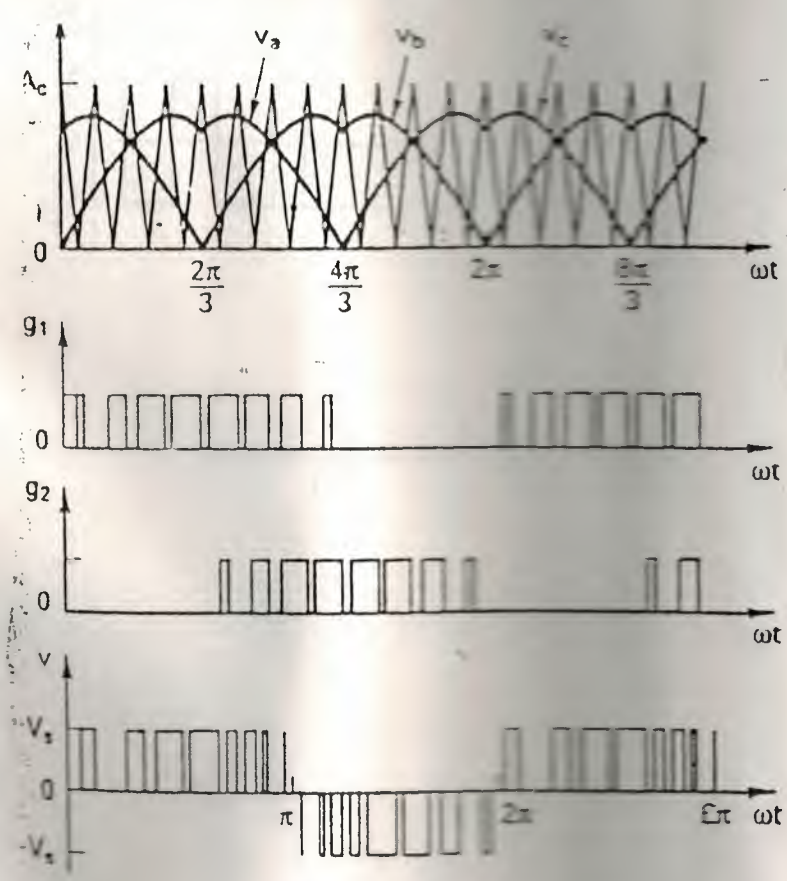


Fig: 8.4

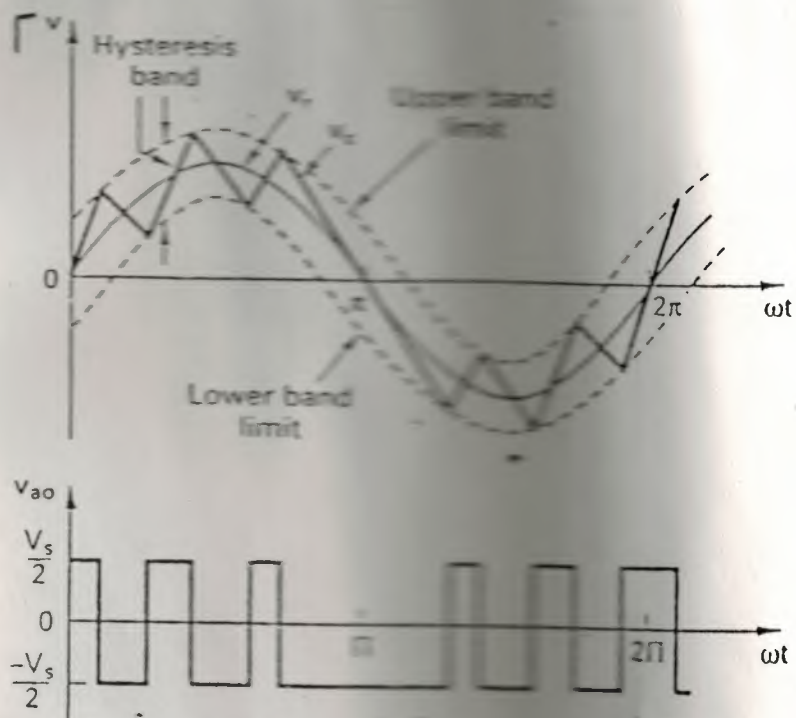


Fig: 35

The fundamental output voltage can be up to 1Vs and is dependent on the peak amplitude A_r and frequency f_r of the reference voltage. The delta modulation can control the ratio of voltage to frequenc, which is a desirable feature in ac motor control.