NEAR EAST UNIVERSITY

Faculty of Engineering

Department of Electrical and Electronic Engineering

IMPLEMENTATION OF ECONOMIC DISPATCH FOR POWER SYSTEM USING MATLAB

Graduation Project

EE 400

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ABSTRACT

As the life getting more complex and more expensive, the people trying to get each of services and goods at the most economical and easier way to be served, here we consider the services as how to dispatch either generators or plants to get the maximum efficiency with lowest cost to meet the total demand load.

The power systems and their analysis have been developed at these days into many types; here we consider the economic dispatch type which is the most important thing in saving costs.

The economic dispatch calculations have many considerations in solving problems, we have considered the transmission losses, and without transmission losses including penalty factor using Lagrangian Relaxation technique, confirmed by using Matlab Programming.

Economic Dispatch is the process of allocating the required load demand between the available generation units such that the cost of operation is minimized.

We have assumed in our calculations the fossil-fuel units, with the areas of economic dispatch, which they are;

1- Optimal power flow
2- Economic dispatch in relation to AGC
3-Dynamic dispatch
4- Economic dispatch with non-conventional generation sources

LIST OF ACRONYMS

ACE	Area control error
AGC	Automatic generation control
Btu	British thermal unit
CEED	Combined economic emission dispatch
ECC	Energy control center
ED	Economic dispatch
EPD	Economic dispatch problem
IFC	Incremental fuel cost
UC	Unit commitment

TABLE OF CONTENTS

INTRODUCTION

The main target of electric power utilities is to provide high quality reliable supply to the consumers at the lowest possible cost while operating to meet the limits and constraints imposed on the generating units .This formulates the well known Economic Dispatch(ED) problem for finding the optimal combination of the output power of all online generating units that minimizes the total fuel cost, while satisfying all constraints. Line flows are calculated for the global optimal generator settings and are compared with MVA line flow limits of the corresponding rises to monitor overloading. The optimum economic dispatch may not be the best in terms of the environment criteria. Harmful ecological effects by the emission of gaseous pollutants from fossil fuel power plants can be reduced by proper load allocation among the various generating units of the plants. But this load allocation may lead to increase in the operating cost of the generating units. So, it is necessary to find out a solution which gives a balanced result between emission and cost. This is achieved by Combined Economic Emission Dispatch Problem. The classical lambda iteration method can be used to solve the CEED. [1]

Electrical power systems and their operation are among the most complex problems of engineering due to their highly nonlinear and computationally difficult environments, the objective of the economic dispatch problem (EDP) of electric power generation is to schedule the committed generating unit outputs so as to meet the required load demand at minimum operating cost while satisfying all unit and system equality and inequality constraints. [8]

Economic operation is very important for a power system to return a profit on the capital invested, Rates fixed by regulatory bodies and the importance of conservation of fuel place pressure on power companies to achieve maximum possible efficiency.

Maximum efficiency minimizes the cost of kilowatt hour to consumer and the cost to the company of delivering that kilowatt-hour in the face of constantly rising prices for fuel, labor, supplies, and maintenance. [2]

Operational economics involving power generation and delivery can be subdivided into two parts—one dealing with minimum cost of power production called "*economic dispatch*" and the other dealing with minimum-loss delivery of the generated power *to* the loads. For any specified load condition economic dispatch determines the power output of each plant (and each *generating unit* within be plant) which will minimize the overall cost of fuel needed to serve the system load. Thus, economic dispatch focuses upon coordinating the production costs at all power plants operating on the system and is the major emphasis of this project. [1]

The minimum-loss problem can assume many forms depending on how control the power flow in the system is exercised. The economic dispatch problem and also the minimum-loss problem can be solved by means of the *optimal power-flow* (OPF) program.

The OPF calculation can be viewed as a sequence of conventional Newton-Raphson power-flow calculations in which certain controllable parameters are automatically adjusted to satisfy the network constraints while minimizing a specified objective function. Here I consider the *classical* approach to economic dispatch.

In the second chapter, we will study first the most economic distribution of the output of a plant also applies to economic scheduling of plant outputs for a given loading of the system without consideration of transmission losses. Next we will express transmission loss as a function of the outputs of the various plants. Then, determine how the output of each of the plants of a system is scheduled to achieve the minimum cost of power delivered to the load theoretically. [2]

In the third chapter we will solve first, a problem which has four unit generators without transmission losses followed by matlab programming. Second problem will be including transmission losses and considering the penalty factor by formulation and followed by confirmed solution using matlab programming.

Because the total load of the power system varies throughout the day, coordinated control of the power plant outputs is necessary to ensure generation-to-load balance so that the system frequency will remain as close possible to the nominal operating value, usually 50 or 60 Hz.

Accordingly, the problem of *automatic generation control* (AGC) is developed from state viewpoint. Also, because of the daily load variation, the utility has to decide on the basis of economics which generators to start up, which to shut down, and in what order. The computational procedure for making such decisions called "*unit commitment*" (UC).

Economic Dispatch is the process of allocating the required load demand between the available generation units such that the cost of operation is minimized. There have been many algorithms proposed for economic dispatch: Merit Order Loading, Range Elimination, Binary Section, Secant Section, Graphical/Table Look-Up, Convex Simplex, Dantzig-Wolf Decomposition, Separable Convex Linear Programming, Reduced Gradient with Linear Constraints, Steepest Descent Gradient, First Order Gradient, Merit Order Reduced Gradient, etc. The close similarity of the above techniques can be shown if the solution steps are compared. These algorithms are well documented in the literature. We will use only the graphical (LaGrangian Relaxation) techniques. [2]

The types of power system units:

1. fossil-fuel
2. hydro
3. nuclear
4. pumped-storage hydro

In this project we will consider the fossil-fuel units.

1.1) what is economic dispatch?

The operation of generation facilities is to produce energy at the lowest cost to reliably serve consumers, recognizing any operational limits of generation and transmission facilities, and how the real power output of each controlled generating unit in an area is selected to meet a given load and to minimize the total operating costs in the area. [8]

1.2) the benefit of economic dispatch:

Economic dispatch benefits electricity users in a number of ways. By systematically seeking the lowest cost of energy production consistent with electricity demand, economic dispatch reduces total electricity costs. To minimize costs, economic dispatch typically increases the use of the more efficient generation units, which can lead to better fuel utilization, lower fuel usage, and reduced air emissions than would result from using less-efficient generation. As the geographic and electrical scope integrated under unified economic dispatch increases, additional cost savings result from pooled operating reserves, which allow an area to meet loads reliably using less total generation capacity than would be needed otherwise. Economic dispatch requires operators to pay close attention to system conditions and to maintain secure grid operation, thus increasing operational reliability without increasing costs.

Economic dispatch methods are also flexible enough to incorporate policy goals such as promoting fuel diversity or respecting demand as well as supply resources. Over the long term, economic dispatch can encourage new investment in generation as well as in transmission expansion and upgrades that enhance both reliability and cost savings.[8]

1.3) the fundamental components to economic dispatch

1-Planning for tomorrow's dispatch [7]

2-Dispatching the power system today [7]

1.3.1) Planning for Tomorrow's Dispatch

A-Scheduling generating units for each hour of the next day's dispatch

1-Based on forecast load for the next day

2-Select generating units to be running and available for dispatch the next day (operating day)

B-Recognize each generating unit's operating limit, including it's

1-Ramp rate (how quickly the generator's output can be changed).

2- Maximum and minimum generation levels.

3- Minimum amount of time the generator must run.

4- Minimum amount of time the generator must stay off once turned off.

C-Recognize generating unit characteristics including:

1-Cost of generating which depends on:

- Its efficiency (heat rate).
- Its variable operating costs (fuel and non-fuel).

2-Variable cost of environmental compliance.

3-Start-up costs.

-Next day scheduling is typically performed by a generation group or an independent market operator.

B- Reliability Assessment.

1-Analyze forecasted load and transmission conditions in the area to ensure that scheduled generation dispatch can meet load reliably.

2-If the scheduled dispatch is not feasible within the limits of the transmission system, revise it.

3-This reliability assessment is typically performed by a transmission operations group.

1.3.2) Dispatching the Power System Today:

A- Monitor load, generation and interchange (imports/exports) to ensure balance of supply and load:

1-Monitor and maintain system frequency at 60 Hz during dispatch using Automatic Generation Control (AGC) to change generation dispatch as needed.

2-Monitor hourly dispatch schedules to ensure that dispatch for the next hour will be in balance.

B- Monitor flows on transmission system:

1-Keep transmission flows within reliability limits.

2-Keep voltage levels within reliability ranges.

C-Take corrective action when needed by:

1- Limiting new power flow schedules.

2- Curtailing existing power flow schedules.

3- Changing the dispatch.

4- Shedding load.

1.4) Area Factors Limiting the Effectiveness of Dispatch in Minimizing Customer Costs:

1-Geographic area included:

The size of the geographic region over which the dispatch occurs affects the level of costs that is which generation resources and which transmission facilities are considered in planning and economic dispatch.

2-Generation resources included:

Which generation resources in the area are included in the planning and economic dispatch and whether they are included in the same manner affects the level of costs.

3-Transmission facilities included:

What transmission facilities are included in the planning and economic dispatch and how the reliability security limits of transmission facilities are incorporated into the economic dispatch?

1.5) Implementation Factors Limiting Effectiveness of Dispatch in Minimizing Customer Costs:

1-Frequency of the dispatch:

Performing an economic dispatch more frequently (e.g. 5 or 15 minutes rather than each hour) affects the level of costs.

2-Communication of information:

Generation operators, transmission owners, and load serving entities must provide accurate and current information to those performing the planning and dispatch functions.

Those performing planning and dispatch must provide accurate and current dispatch instructions to generation operators, transmission operators and load serving entities.

Inadequate or incomplete communications affects the level of costs of the economic dispatch.

3-Software tools for dispatch and information:

Reliable and secure computer software is essential for rapidly responding to system changes to maintain power system reliability, while selection the lowest cost generators to dispatch. Obsolete software affects the level of costs achieved by the economic dispatch.

4-Coordination of dispatch across regions:

Where there are multiple, independently performed, dispatches in a region, the effectiveness of coordination agreements and their implementation affect the level of costs of the economic dispatch.

1.6) THE AREAS OF ECONOMIC DISPATCH ARE AS FOLLOWING:

1- Optimal power flow
2- Economic dispatch in relation to AGC
3-Dynamic dispatch
4- Economic dispatch with non-conventional generation sources

1.6.1) OPTIMAL POWER FLOW (OPF)

Economic dispatch has one significant shortcoming which it ignores the limits imposed by the devices in the transmission system. Each transmission line and transformer has a limit on the amount of power that can be transmitted through it, with the limits arising because of thermal, voltage, or stability consideration.

Traditionally, the transmission system was designed so that when the generation was dispatched economically there would be no limit violations. Hence, just solving economic dispatch was usually sufficient. However, with the worldwide trend toward deregulation of electric utility industry, the transmission system is becoming increasingly constrained.

The solution to the problem of optimizing the generation while enforcing the transmission lines is to combine economic dispatch with the power flow. The result is known as optimal power flow (OPF).[1]

The optimal power flow procedure consists of methods of utilizing load flow techniques for the purpose of economic dispatch. While some authors have used the ac load flow model others have used the dc load flow model. The latter is based on the P-Q decomposition and then using known optimization techniques. The ac optimal load flow problem on the other hand consists of finding the active and reactive power output and the voltage magnitudes at any generator unit, in order to minimize the operating cost while meeting various security constraints. Security constrained dispatch involves those dispatch activities which are constrained to respect selected system security limits. In general, optimal power flow requires use of network modeling as well as resource modeling and naturally results in higher system costs. [3]

1.6.2) AUTOMATIC GENERATION CONTROL (AGC)

Almost all generating companies have tie-line interconnections to neighboring utilities. Tie lines allow the sharing of generation resources in emergencies and economic of power production under normal conditions of operation. For purposed of control the entire interconnected system is subdivided into *control areas* which usually conform to the boundaries of one or more companies. The *net interchange* of power over the tie lines of an area is the algebraic difference between area generation and area load (plus losses).
A schedule is prearranged with neighboring areas for such tie-line flows, and as long as an area maintains the interchange power on schedule, it is evidently fulfilling its primary responsibility to absorb its own load changes. But since each area shares in the benefits of interconnected operation, it is expected to share the responsibility to maintain system frequency. [4]

Frequency changes occur because system load varies randomly throughout the day so that an exact forecast of real power demand cannot be assured. The imbalance between real power generation and load demand (plus losses) throughout the daily load cycle causes kinetic energy of rotation to be either added to or taken from the on-line generating units, and frequency throughout the interconnected system varies as a result. Each control area has central facility called the *energy control center* (ECC), which monitors the system frequency and the actual power flows on its tie lines to neighboring areas. The deviation between desired and actual system frequency is then combined with the deviation from the scheduled net interchange to form a composite measure called the *area control error,* or simply (ACE).

To remove area control error, the energy control sends command signals to the generating units at the plants within its area to control the generator outputs so as to restore the net interchange power to scheduled values and to assist in restoring the system frequency to its desired value. The monitoring, telemetering, processing, and control functions are coordinated within the individual area by computer based automatic generation control (AGC) system at the energy control center.

The role of Automatic Generation Control (AGC) is to maintain desired megawatt output of a generator unit and control the system frequency. The AGC also helps to keep the set interchange of power between pool members at predetermined values. Highly differing response characteristics of units of various types, e.g., hydro, nuclear, fossil, etc. are used for the control. The AGC loop maintains control only during normal (small and slow) changes in load and frequency. Adequate control is not possible during emergency situations when large imbalances occur. [9]

1.6.3) DYNAMIC DISPATCH

Economic dispatch may sometimes be classified as a static optimization problem in which costs associated with the act of changing the outputs of generators are not considered. On the other hand, a dynamic dispatch is one that considers change related costs.

With the use of steady-state operating costs in the static optimization, poor transient behavior results when these solutions are incorporated in the feedback control of dynamic electric power networks. The dynamic dispatch method uses forecasts of system load to develop optimal generator output trajectories. Generators are driven along the optimal trajectories by the action of a feedback controller. [9]

1.6.4) DISPATCH WITH NON-CONVENTIONAL GENERATION SOURCES

Non-conventional generation sources, such as solar photovoltaic, solar thermal, wind, geothermal, storage battery, etc. can become attractive alternatives to fossil plants. Many utilities strongly feel that a number of these non-conventional sources of energy can ease the critical future problem of fuel cost and availability. Much of this optimism is delimitated by the fact that such generation sources are known to produce extraneous operating problems in the power system as a whole. The existing conventional generating units, through use of AGC, are capable of operating under the dynamic response required to supply the random variations in system load. Such is not the case with grid-connect photovoltaic or wind generation systems. Frequent weather changes may translate into extremely high variations in the power generation from these plants. If the plant is constantly connected to the distribution system, this causes operational problems like, load following, spinning reserve requirements, load frequency excursions, system stability, etc., which the conventional AGC is unable to handle. The following is a discussion of a part of the literature existing on this particular subject. [9]

1.7) UNIT COMMITMENT

Because the total load of power system varies throughout the day and reaches a different peak value from one day to another, the electric utility has to decide in advance which generators to start up and when to connect them to the network and the sequence in which the operating units should be shut down and for how long. The computational procedure for making such decisions is called *unit commitment.[1]*

THEORETICAL BACKGROUND

2.1) DISTRIBUTION OF LOAD BETWEEN UNITS WITHIN A PLANT

An early attempt at economic dispatch called for supplying power from only the most efficient plant at light loads. As load increased, power would be supplied by the most efficient plant until the point of maximum efficiency of that plant was reached. Then, for further increase in load the next most efficient plant would start to feed power to the system and a third plant would not be called upon until the point of maximum efficiency of the second plant was reached. [4]

Even with transmission losses neglected, this method fails to minimize cost. To determine the economic distribution of load between the various generating units (consisting of a turbine, generator, and steam supply), the variable operating costs of the unit must be expressed in terms of the power output. We base our discussion on the economics of fuel cost with realization that other costs which are a function of power output can be included in expression of fuel cost.

The fuel requirement for a given output is easily converted into dollars per megawatt hour. We will see the criterion for distribution of the load between any two units is based on whether increasing the load on one unit as the load is decreased on the other unit by same amount results in an increase or decrease in total cost. Thus, we are concerned with *incremental fuel cost* (IFC), which is determined by the slopes of the input-output curves of two units. If we assume the ordinates of input-output curve in dollars per hour and let

f_i = input to unit i, $(\$/h)$

P_{gi} = output of unit i, megawatts (MW)

The incremental fuel cost of the unit in dollars per megawatt hour$(\frac{df_i}{dP_{gi}})$, where the average fuel cost in the same units is$\frac{f_i}{P_{gi}}$. Hence, if the input-output curve of unit (i) is quadratic, so we write

$$fi = \frac{a_i}{2} P_{gi}2 + b_i P_{gi} + C_i \qquad \$/h \qquad (2.1)$$

And the unit has incremental fuel cost denoted by (λi) which is defined by

$$\lambda_i = \frac{df_i}{dP_{gi}} = a_i P_{gi} + b_i \qquad \frac{\$}{MWh} \qquad (2.2)$$

Where $ai, bi,$ and Ci are constants (the fuel cost coefficients). The approximate incremental fuel cost at any particular output is the additional cost in dollars per hour to increase the output by 1 MW. Actually, incremental cost is determined by measuring the slope of the input-output curve and multiplying by cost per Btu in the proper units.
Since mills (tenths of a cent) per kilowatt hour are equal to dollars per megawatt hour, and since a kilowatt is very small amount of power in comparison with the usual output of a unit of a steam plant, incremental fuel cost may be considered as the cost of fuel in *mills per hour* to supply an additional kilowatt output.

We now have the background to understand the principle of economic dispatch which guides distribution of load among the units within one or more plants of the system. For instance, suppose that the total output of particular plant is supplied by two units and that the division of load between these units is such that the incremental fuel cost of one is higher than that of the other. Now suppose that some of the load is transferred from the unit with the higher incremental cost to the unit with lower incremental cost, reducing the load on the unit with the higher incremental cost will result in a greater reduction of cost than the increase in cost for adding the same amount of load to the unit with the lower incremental cost. The transfer of load from one to the other can be continued with a reduction in total fuel cost until the incremental fuel costs of the two units are equal. [4]

The same reasoning can be extended to a plant with more than two units. Thus, for economical division of load between units within a plant, the criterion is that all units must operate at the same incremental fuel cost.

When the incremental fuel cost of each of the units in a plant is nearly linear with respect to power output over a range of operation under consideration, equations that represent incremental fuel costs as linear functions of power output will simplify the computations. An economic dispatch schedule for assigning loads to each unit in a plant can be prepared by:

1. Assuming various values of total plant output,
2. Calculating the corresponding incremental fuel cost λ of the plant,
3. Substituting the value of λ for λ_i in the equation for the incremental fuel cost of each unit to calculate its output.

   A curve of λ versus plant load establishes the value of λ at which each unit should operate for a given total plant load.

   For plant with two units operating under economic load distribution the λ of the plant equals λi of each unit, and so

$$\lambda = \frac{df_1}{dP_{g1}} = a_1 P_{g1} + b_1 \text{ and } \lambda = \frac{df_2}{dP_{g2}} = a_2 P_{g2} + b_2 \qquad (2.3)$$

Solving for P_{g1} and P_{g2}, we obtain

$$P_{g1} = \lambda - b_1 / a_1 \qquad \text{and} \qquad P_{g2} = \lambda - b_1 / a_2 \qquad (2.4)$$

Adding these results together and then solving for λ gives

$$\lambda = \left(\sum_{i=1}^{2} \frac{1}{a_i}\right)^{-1} \times \left(P_{g1} + P_{g2}\right) + \left(\sum_{i=1}^{2} \frac{1}{a_i}\right)^{-1} \times \left(\sum_{i=1}^{2} \frac{b_i}{a_i}\right) \qquad (2.5)$$

Or $$\lambda = a_T P_{gT} + b_T \qquad (2.6)$$

Where $$a_T = \left(\sum_{i=1}^{2} \frac{1}{a_i}\right)^{-1}$$

$$b_T = a_T\left(\sum_{i=1}^{2}\frac{b_i}{a_i}\right)$$

And $\quad P_{gT} = (P_{g1} + P_{g2})$ $\quad$ is the total plant output

Equation (2.6) is a closed-form solution for λ which applies to a plant with more than two units on economic dispatch when the appropriate number of terms is added to the summations of equation (2.5).

For instance, if the plant has K units operating on economic dispatch, when the coefficients of equation (2.6) are given by

$$a_T = \left(\sum_{i=1}^{K}\frac{1}{a_i}\right)^{-1} = \left(\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_K}\right)^{-1} \tag{2.7}$$

$$b_T = a_T\left(\sum_{i=1}^{K}\frac{b_i}{b_i}\right) = a_T\left(\frac{b_1}{a_1} + \frac{b_2}{a_2} + \cdots + \frac{b_K}{a_K}\right) \tag{2.8}$$

Where the total plant output $P_{gT} = (P_{g1} + P_{g2} + \cdots + p_{gK})$ (2.9)

The individual output of each of the K units is then calculated from the value of (λ) given by equation (2.6). If maximum and minimum loads are specified for each unit, some units will be unable to operate at the same incremental fuel cost as the other units and still remain within the limits specified for light and heavy loads.

Suppose that this occurs for K=4 and that the calculated value of *(P_{g4})* violates a specified limit of unit 4. We then discard the calculated outputs of all four units and set the operating value of P_{g4} equal to the violated limit of unit 4. We recalculate the coefficients a_T and b_T from equation *(2.6)* for the other three units and set the effective economic dispatch value of (P_{gT}) equal to the total plant load minus the limit value of P_{g4}. The resulting value of (λ) then governs the economic dispatch of units 1, 2, and 3 when the actual plant output is to be increased or decreased, so long as unit 4 remains as the only unit at a limit.

Example (2.1):

If the incremental fuel costs in dollars per megawatt hour for a plant consisting of two units are given by

$\lambda_1 = 0.0080P_{g1} + 8.0$ And $\lambda_2 = 0.0096P_{g2} + 6.4$

Assume all the units are operating all the times, that total load varies from (250 to 1250) MW, and that maximum and minimum loads on each unit are to be (625 and 100) MW, respectively.

Find the incremental fuel cost of the plant and the allocation of load between units for minimum cost of various total loads.[5]

Solution. At light loads unit 1 will have the higher incremental fuel cost and operates at its lower limit of 100MW for which (λ_1) is ($8.8/MWh). When the output of unit 2 is also 100MW, (λ_2) is ($7.36/MWh). Therefore, as plant output increases, the additional load should come from unit 2 until (λ_2) equals ($8.8/MWh). Until that point is reached, the incremental fuel cost (λ) of plant is determined by unit 2 alone. When the plant load is 250MW, unit 2 will supply 150MW with (λ_2) equal to ($7.84/MWh). When ($\lambda_2$) equals ($8.8/MWh),

$$0.0096P_{g2} + 6.4 = 8.8$$

$$P_{g2} = \frac{2.4}{0.0096} = 250\ MW$$

The total plant output P_{gT} is 350MW. From this point on the required output of each unit for economic load distribution is found by assuming various values of P_{gT}, calculating the corresponding plant (λ) in equation (2.6), and substituting the value of (λ) in equation (2.4) to compute each unit's output. Results are shown in Table (2.1). When P_{gT} is in the range from (350 to 1175) MW, the plant (λ) is determined by equation (2.6). At (λ=12.4) unit 2 is operating at its upper limit and additional load must come from unit 1, which then determines the plant (λ), which is shown in figure (2.1).

TABLE (2.1)
The plant λ and outputs of each unit for various values of total output PgT for example 2.1

plant		Unit1	Unit2
PgT MW	λ $/MWh	Pg1 MW	Pg2 MW
250	7.84	100 *	150
350	8.80	100 *	250
500	9.45	182	318
700	10.33	291	409
900	11.20	400	500
1100	12.07	509	591
1175	12.40	550	625*
1250	13.00	625	625*

* Indicates the output of each unit at its minimum or maximum limit and plant λ is then equal to the incremental fuel cost of unit not at limit.

We could plot the output of each individual unit versus plant output, as shown in Figure (2.2). The correct output of each of many units can be easily computed from equation (2.6) by requiring all unit incremental costs to be equal for any total plant output.

$$P_{gT} = (P_{g1} + P_{g2}) = 500\ MW$$

$$a_T = \left(\frac{1}{a_1} + \frac{1}{a_2}\right)^{-1} = \left(\frac{1}{0.0080} + \frac{1}{0.0096}\right)^{-1} = 4.363636 * 10^{-3}$$

$$b_T = a_T\left(\frac{b_1}{a_1}+\frac{b_2}{a_2}\right) = a_T\left(\frac{8}{0.0080}+\frac{6.4}{0.0096}\right) = 7.272727$$

and then we calculate for each unit

$$P_{g1} = \frac{\lambda - b_1}{a_1} = \frac{9.4545454 - 8.0}{0.0080} = 181.8182\,MW$$

$$P_{g2} = \frac{\lambda - b_2}{a_2} = \frac{9.454545 - 6.4}{0.0096} = 318.1818\,MW$$

Figure 2.1

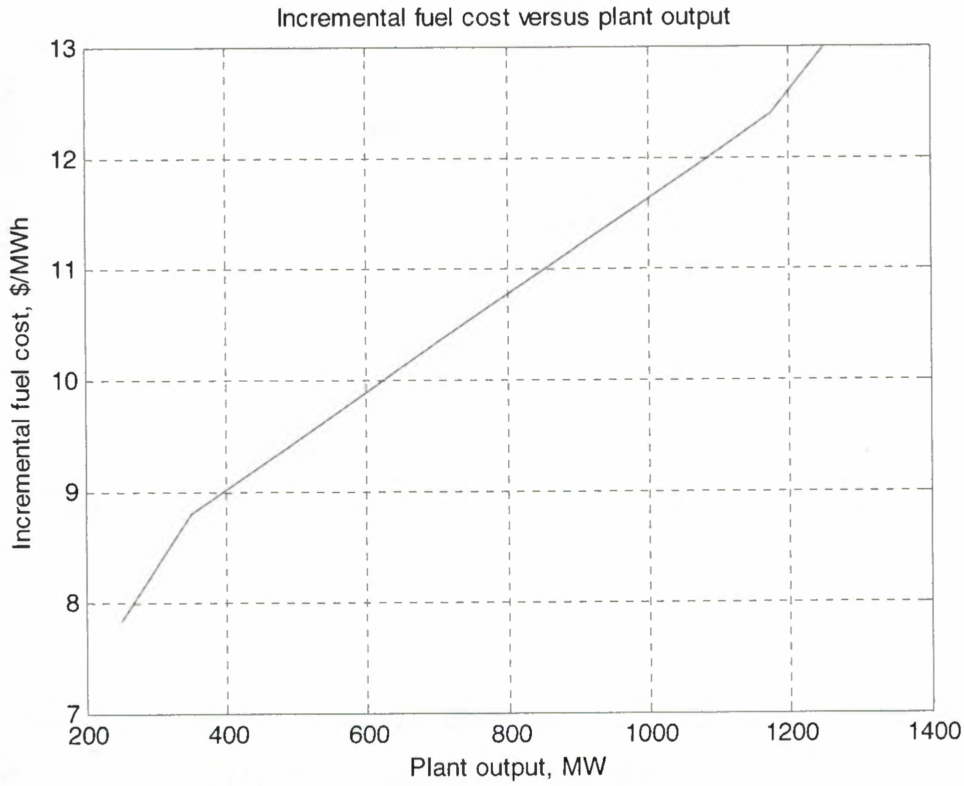
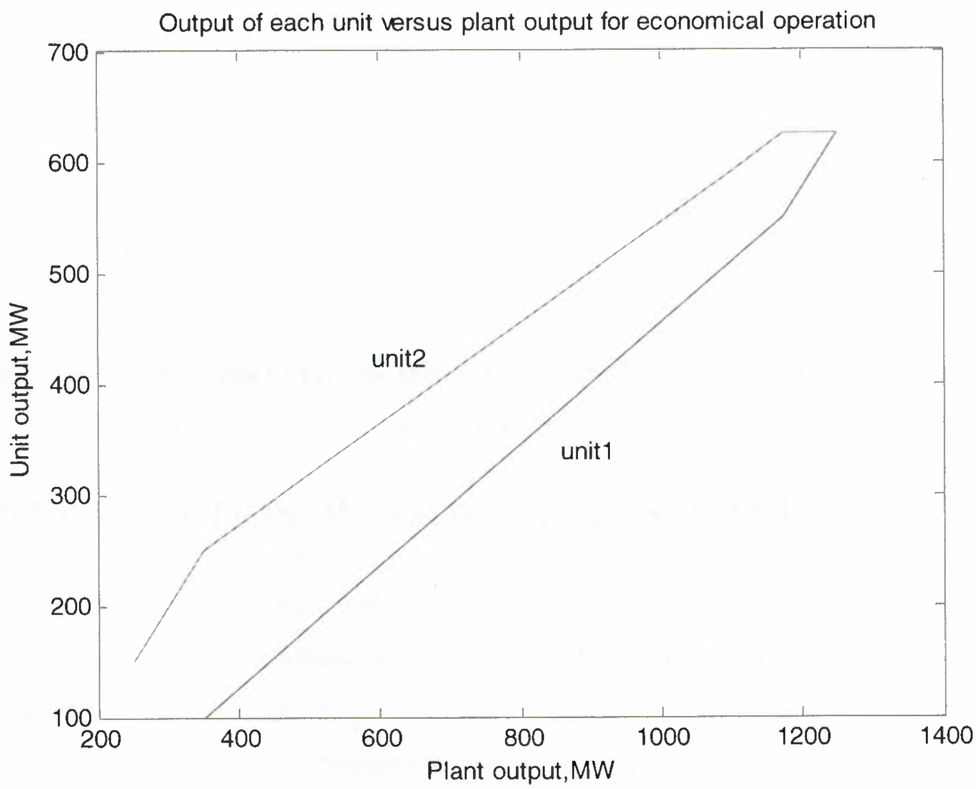


Figure 2.2



The savings effected by economic distribution of load rather than some arbitrary distribution can be found by integrating the expression for incremental fuel cost and by comparing increases and decreases of cost for the units as load is shifted from the most economical allocation.

Example (2.2): Determine the saving in fuel cost in dollars per hour for the economic distribution of a total load of a (900MW) between the two units of the plant described in previous example, Compared with equal distribution of the same total load. [5]

Solution: from table (2.1) it shows that Unit 1 should supply (400MW) and Unit 2 should supply (500MW). If each unit supplies (450MW), the increase in cost for Unit 1 is

$$\int_{400}^{450}(0.008P_{g1} + 8)dP_{g1} = (0.004P^2_{g1} + 8P_{g1} + C_1)|_{400}^{450} = 570\ \$\ per\ hour$$

The constant (C_1) is cancels when we evaluate at the two limits. Similarly, for Unit 2

$$\int_{500}^{450}(0.0096P_{g2} + 6.4)dP_{g2} = (0.0048P^2_{g2} + 6.4P_{g2} + C_2)|_{500}^{450} = -548\ \$\ per\ hour$$

The negative sign indicate a decrease in cost, as we expect for a decrease in output.

The net increase in cost is $\$570 - \$548 = \$22\ per\ hour$.

The saving seems small but, but this amount saved every hour for a year of continuous operation would reduce fuel cost by$\$22 \times 24hr \times days365 = \$192{,}720$ for the year.

2.2) DISTRIBUTION OF LOAD BETWEEN PLANTS

In determining the economic distribution of load between plants, we encounter the need to consider losses in transmission lines. Although the incremental fuel cost at one plant bus may be lower than that of another plant for a given distribution of load between the plants, the plant with the lower incremental cost at its bus may be much farther from the load center. The losses in transmission from the plant having the lower incremental cost may be so great that economy may dictate lowering the load at the plant with the lower incremental cost and increasing it at the plant with the higher incremental cost. Thus, we need to coordinate transmission loss into the scheduling of the output of each plant for maximum economy at a given level of system load. [5]

For a system with K generating units let

$$f = f_1 + f_2 + \dots + f_K = \sum_{i=1}^{K} f_i \tag{2.10}$$

Where (f) is the cost function giving the total cost of all the fuel for the entire system and is the sum of the fuel costs of the individual units $f_1, f_2, \dots, f_K$. The total megawatt power input to the network from all the units is the sum

$$P_{g1} + P_{g2} + \dots + P_{gK} = \sum_{i=1}^{K} P_{gi} \tag{2.11}$$

Where P_{g1}, P_{g2}, *and* P_{gK} are the individual outputs of the units injected into the network. The total fuel cost (f) of the system is a function of all the power plant outputs. The constraining equation on the minimum value of (f) is given by the power balance of equation

$$P_L + P_D - \sum_{i=1}^{k} P_{gi} = 0 \tag{2.12}$$

Where $P_D = \sum_{i=1}^{N} P_{di}$ are the total power received by the loads and P_L is the transmission loss of the system. Our objective is to obtain a minimum (f) for a fixed system load (P_D) subject to the power balance constraint of equation (2.12).

We now present the procedure for solving such minimization problems called *method of Lagrange multipliers.*

The new cost function (F) is formed by combining the total fuel cost and the equality constraint of equation (2.12) in the following manner

$$F = (f_1 + f_2 + \dots + f_K) + \lambda(P_L + P_D - \sum_{i=1}^{K} P_{gi}) \tag{2.13}$$

The augmented cost function (F) is often called the *Lagrangian*, and we shall see that the parameter(λ), which we call now the *Lagrange multiplier,* is the effective incremental fuel cost of the system when transmission line losses are taken into account.

When (f_i) is given in dollars per hour and (P) is in megawatts, F and λ are expressed in dollars per hour and dollars per megawatt hour, respectively. The original problem of minimizing (f) subject to equation (2.12) is transformed by means of equation (2.13) into an unconstrained problem in which it is required to minimize (F) with respect to (λ) and the generator outputs. Therefore for minimum cost we require the derivative of (F) with respect to each (P_{gi}) to equal zero, and so

$$\frac{\partial F}{\partial P_{gi}} = \frac{\partial}{\partial P_{gi}}\left[(f_1 + f_2 + \cdots + f_K) + \lambda\left(P_L + P_D - \sum_{i=1}^{K} P_{gi}\right)\right] = 0 \qquad (2.14)$$

Since (P_D) is fixed and the fuel cost of any one unit varies only if the power output of that unit is varied equation (2.14) yields

$$\frac{\partial F}{\partial P_{gi}} = \frac{\partial f_i}{\partial P_{gi}} + \lambda\left(\frac{\partial P_L}{\partial P_{gi}} - 1\right) = 0 \qquad (2.15)$$

For each of the generating unit outputs P_{g1}, P_{g2}... P_{gK}. Because (f_i) depends on only (f_{gi}), the partial derivative of (f_i) can be replaced by the full derivative, and equation (2.15) then gives

$$\lambda = \left(\frac{1}{1-\frac{\partial P_L}{\partial P_{gi}}}\right)\frac{df_i}{dP_{gi}} \qquad (2.16)$$

For every value of (i), this equation often rewritten in the form

$$\lambda = L_i \frac{df_i}{dP_{gi}} \qquad (2.17)$$

Where L_i is called the *penalty factor* of plant I and is given by

$$L_i = \frac{1}{1-\frac{\partial P_L}{\partial P_{gi}}} \qquad (2.18)$$

The result of equation (2.17) means that minimum fuel cost is obtained when the incremental fuel cost of each unit multiplied by its penalty factor is the same for all generating units in the system. The products $L_i(\frac{df_i}{dP_{gi}})$ are each equal to (λ), which is approximately the cost in dollars per hour to increase the total delivered load by (1MW). For a system of three units, not necessary in same power plant, equation (2.17) yields

$$\lambda = L_1 \frac{df_1}{dP_{g1}} = L_2 \frac{df_2}{dP_{g2}} = L_3 \frac{df_3}{dP_{g3}} \quad (2.19)$$

The penalty factor L_i depends on $\frac{\partial P_L}{\partial P_{gi}}$, which is a measure of the sensitivity of the transmission system losses to changes in P_{gi} alone. Generating units connected to the same bus within a particular power plant have equal access to the transmission system, and so the change in system losses must be the same for a small change in the output of any one of those units. This means that the penalty factors are the same for such unit located in the same power plant. Therefore, for a plant having, say, three generating units with outputs P_{g1}, P_{g2}, and P_{g3}, the penalty factors L_1, L_2, and L_3, are equal, and equation (2.19) then shows that

$$\frac{df_1}{dP_{g1}} = \frac{df_2}{dP_{g2}} = \frac{df_3}{dP_{g3}} \quad (2.20)$$

The general form of loss equation for any number of sources is

$$P_L = \sum_m \sum_n P_m B_{mn} P_n \quad (2.21)$$

Where $\sum_m$ and $\sum_n$ indicate independent summations to include all sources for instance, for three sources,

$$P_L = P^2{}_1 B_{11} + P^2{}_2 B_{22} + P^2{}_3 B_{33} + 2P_1P_2B_{12} + 2P_2P_3B_{23} + 2P_1P_3B_{13} \quad (2.22)$$

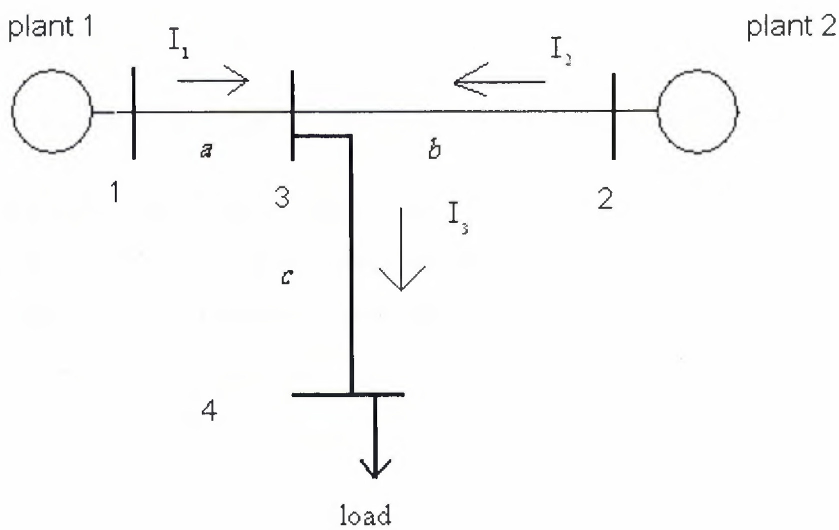
The matrix form of the transmission loss is

$$P_L = P^T B P \quad (2.23)$$

Where for a total of (3) sources

$$P = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} \quad \text{And} \quad B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$$

Example (2.3): for the system whose one-line diagram is shown in Figure (2.3), assume $I_1 = 1.0\angle 0^\circ$ per unit and $I_2 = 0.8\angle 0^\circ$ per unit. If the voltage at bus 3 is $V_3 = 1.0\angle 0^\circ$ per unit, find the loss coefficients and transmission losses. Line impedances are $0.04 + j0.16$ per unit, $0.03 + j0.12$ per unit, and $0.02 + j0.08$ per unit for section a, b, c, respectively. [5]



Solution: for this example, the bus voltages can be calculated from the data given:

$v_1 = 1.0 + (1.0 + j0)(0.04 + j0.16) = 1.04 + j0.16$ per unit

$v_2 = 1.0 + (0.8 + j0)(0.03 + j0.12) = 1.024 + j0.096$ per unit

Since all currents have phase angles of zero, the power factor at each source node is the cosine of the angle of the voltage at the node, and voltage magnitude time's power factor equals the real part of the complex expression for voltage. Therefore

$B_{11} = \frac{0.004 + 0.03}{1.04^2} = 0.0554$ Per unit

$B_{12} = \frac{0.02}{1.024 \times 1.04} = 0.0188$ Per unit

$B_{22} = \frac{0.03+0.02}{1.04^2} = 0.0477$ Per unit

The transmission losses

$P_1 = Re\{(1.0 + j0)(1.04 + j0.16)\} = 1.04$ Per unit

$P_2 = Re\{(0.8 + j0)(1.024 + j0.096)\} = 0.8192$ Per unit

$P_L = 1.04^2 \times 0.0554 + 2 \times 1.04 \times 0.8192 \times 0.0188 + 0.8192^2 \times 0.0477 = 0.124$Pu

Adding the loss in each section computed by I^2R yields

$P_L = 1.0^2 \times 0.04 + 1.8^2 \times 0.02 + 0.8^2 \times 0.03 = 0.124\ per\ unit$

Exact agreement between methods is expected since the loss coefficients were determined for the condition for which loss was calculated. The amount of error introduced by using the same loss coefficients for two other operating conditions may be seen by examining the results shown in Table2.2.

Table (2.2) Comparison of transmission loss calculated by loss coefficients and I^2R for data for example (2.2) with several operating conditions

All quantities are in per unit

I_1	I_2	P_1	P_2	P_L by loss coefficients	P_L by I^2R
1.0	0.8	1.04	0.819	0.124	0.124
0.5	0.4	0.51	0.405	0.030	0.031
0.5	1.3	0.51	1.351	0.128	0.126

Example (2.4): A system consists of two plants connected by a transmission line. The only load is located at plant 2. When 200 MW is transmitted from plant 1 to plant 2 power loss in the line is 16 MW. Find the required generation for each plant and the power received by the load when λ for the system is \$12.5 per megawatt hour. Assume that the incremental fuel costs can be approximated by the following equations: [5]

$\frac{dF_1}{dP_1} = 0.010P_1 + 8.5$ \$/MWh

$\frac{dF_2}{dP_2} = 0.015P_2 + 9.5$ \$/MWh

Solution: for a two plant system

$P_L = P^2{}_1B_{11} + 2P_1P_2B_{12} + P^2{}_2B_{22}$

Since all the load is at plant 2, varying P_2 cannot affect P_L, therefore

$B_{22} = 0$ and $B_{12} = 0$

So when $P_1 = 200$ MW and $P_L = 16$ MW

$16 = 200^2B_{11}$

$B_{11} = 0.0004\ \text{MW}^{-1}$

and $\frac{\partial P_L}{\partial P_1} = 2P_1B_{11} + 2P_2B_{12} = 0.0008P_1$

$$\frac{\partial P_L}{\partial P_1} = 2P_1B_{12} + 2P_2B_{22} = 0$$

Penalty factors are

$L_1 = \frac{1}{1-0.0008P_1}$ and $L_2 = 1$

For $\lambda = \$12.5$

$$\frac{0.010P_1 + 8.5}{1 - 0.0008P_1} = 12.5$$

$P_1 = 200$ MW

$0.015P_2 + 9.5 = 12.5$

$P_2 = 200$ MW

Economic load dispatching therefore requires equal division of load between the two plants for $\lambda = 12.5$. The power loss in transmission is

$P_L = 0.0004 \times 200^2 = 16$ MW

And the delivered load is

$P_R = P_1 + P_2 - P_L = 384$ MW

Now we will find the savings in dollars per hour obtained by coordinating rather than neglecting the transmission loss in determining the load of the plants.

*NOTE: if the transmission loss neglected, the incremental fuel costs at the two plants are equated to give

$0.010P_1 + 8.5 = 0.015P_2 + 9.5$

The power delivered to load is

$P_1 + P_2 - 0.0004P^2{}_1 = 384$

Solving these two equations for P_1 and P_2 gives the following values for plant generation with losses not coordinated:

$P_1 = 290.7$ MW and $P_2 = 127.1$ MW

The load on plant 1 is increased from 200 to 290.7 MW. The increase in fuel cost is

$$\int_{200}^{290.7} (0.010P_1 + 8.5)dP_1 = \left|\frac{0.010}{2}P^2{}_1 + 8.5P_1\right|_{200}^{290.7}$$

= 222.53+770.95=993.48

The load on plant 2 is decreased from 200 to 127.1 MW. The decrease (negative increase) in cost for plant 2 is

$$-\int_{200}^{127.1} (0.015P_2 + 9.5)dP_2 = \left|\frac{0.015}{2}P^2{}_2 + 9.5P_2\right|_{200}^{127.1}$$

=178.84+692.55=871.39

The net savings by accounting for transmission loss in scheduling the received load of 384MW is $993.48 - 871.39 = \$122.09\ per\ hour$

CASES-STUDY

Detailed example 3.1

The distribution of load between plants, the incremental fuel costs in $\$/_{MWh}$ for the four units in the plant are [8]

$$\lambda_1 = \frac{\partial f_1}{\partial P_{g1}} = 0.012P_{g1} + 9.0$$

$$\lambda_2 = \frac{\partial f_2}{\partial P_{g2}} = 0.0096P_{g2} + 6.0$$

$$\lambda_3 = \frac{\partial f_3}{\partial P_{g3}} = 0.0080P_{g3} + 8.0$$

$$\lambda_4 = \frac{\partial f_4}{\partial P_{g4}} = 0.0068P_{g4} + 10.0$$

The maximum load at each four units is

$P_{g1} = 200\ MW$

$P_{g2} = 400\ MW$

$P_{g3} = 270\ MW$

$P_{g4} = 300\ MW$

And the minimum load at each four units is

$P_{g1} = 50\ MW$

$P_{g2} = 100\ MW$

$P_{g3} = 80\ MW$

$P_{g4} = 110\ MW$

The total plant load is 800MW

Analysis:

1. Find the incremental fuel cost (λ)

2. Find the power output of each unit for economic dispatch in (MW)

3. We assume that all four units operate to meet the total plant load

Solution:

First to find the incremental fuel cost (λ), we have to find the fuel cost coefficients which yields

$$a_T = \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \frac{1}{a_4}\right)^{-1}$$

$$a_T = \left(\frac{1}{0.0120} + \frac{1}{0.0096} + \frac{1}{0.0080} + \frac{1}{0.0068}\right)^{-1} = 2.176 * 10^{-3}$$

$$b_T = a_T\left(\frac{b_1}{a_1} + \frac{b_2}{a_2} + \frac{b_3}{a_3} + \frac{b_4}{a_4}\right)$$

$$b_T = 2.176 * 10^{-3}\left(\frac{9}{0.012} + \frac{6}{0.0096} + \frac{8}{0.0080} + \frac{10}{0.0068}\right) = 8.368$$

So the incremental fuel cost is then equal

$$\lambda = a_T P_{gT} + b_T = (2.176 * 10^{-3} \times 800) + (8.368) = 10.1088\ \$/MWh$$

After finding the fuel incremental cost, we can find the output of each unit which yields

$$P_{g1} = \frac{\lambda - b_1}{a_1} = \frac{10.1088 - 9}{0.012} = 92.4\ MW$$

$$P_{g2} = \frac{\lambda - b_2}{a_2} = \frac{10.1088 - 6}{0.0096} = 428\ MW$$

$$P_{g3} = \frac{\lambda - b_3}{a_3} = \frac{10.1088 - 8}{0.0080} = 263.6\ MW$$

$$P_{g4} = \frac{\lambda - b_4}{a_4} = \frac{10.1088 - 10}{0.0068} = 16\ MW$$

we have noticed that unit (2) and unit (4) violates its upper and lower limits, respectively, therefore, first we assume that unit (2) operate at its upper limit (400 MW), so the total of the rest units must be (400 MW).

$$a_T = \left(\frac{1}{a_1} + \frac{1}{a_3} + \frac{1}{a_4}\right)^{-1}$$

$$a_T = \left(\frac{1}{0.012} + \frac{1}{0.0080} + \frac{1}{0.0068}\right)^{-1} = 2.813793 * 10^{-3}$$

$$b_T = a_T\left(\frac{b_1}{1} + \frac{b_3}{a_3} + \frac{b_4}{a_4}\right)$$

$$b_T = 2.813793 * 10^{-3}\left(\frac{9}{0.012} + \frac{8}{0.0080} + \frac{10}{0.0068}\right) = 9.062069$$

Now we can fine the new incremental fuel cost at that operated limit

$$\lambda = a_T P_{gT} + b_T$$

$$\lambda = (2.813793 * 10^{-3} \times 400) + (9.062069) = 10.187586\ \$/MWh$$

Again we have to recalculate the power output of the units to see if there is any violation occur

$$P_{g1}=\frac{\lambda-b_1}{a_1}=\frac{10.187586-9}{0.012}=98.9655\ MW$$

$$P_{g3}=\frac{\lambda-b_3}{a_3}=\frac{10.187586-8}{0.0080}=273.4483\ MW$$

$$P_{g4}=\frac{\lambda-b_4}{a_4}=\frac{10.187586-10}{0.0068}=27.5862\ MW$$

$$P_{g2}=400\ MW$$

Here also we have seen that units (3&4) are violates their respective upper and lower limits, we assume that the unit (4) is operating at lower limit (110MW), using the units (1, 2, 3) only.

The new incremental fuel cost (λ), can be calculated as follow:

$$a_T=\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right)^{-1}=\left(\frac{1}{0.012}+\frac{1}{0.0096}+\frac{1}{0.0080}\right)^{-1}=3.2*10^{-3}$$

$$b_T=a_T\left(\frac{b_1}{a_1}+\frac{b_2}{a_2}+\frac{b_3}{a_3}\right)=3.2*10^{-3}\left(\frac{9}{0.012}+\frac{6}{0.0096}+\frac{8}{0.0080}\right)=7.6$$

Since we assumed unit (4) operate at its lower limit (110MW), so the rest units (1, 2,3) should be P_{gT} = (690MW)

$$\lambda=a_TP_{gT}+b_T=(3.2*10^{-3}\times 690)+7.6=9.808\ \$/MWh$$

The new value of power output of each unit is

$$P_{g1} = \frac{\lambda - b_1}{a_1} = \frac{9.808 - 9}{0.012} = 67.3333\ MW$$

$$P_{g2} = \frac{\lambda - b_2}{a_2} = \frac{9.808 - 6}{0.0096} = 396.667\ MW$$

$$P_{g3} = \frac{\lambda - b_3}{a_3} = \frac{9.808 - 8}{0.0080} = 226\ MW$$

$$P_{g4} = 110\ MW$$

After we have calculated the whole values. We notice that there is no any violation, so the economic dispatch requires that output of unit (4) must be set at its lower limit (110 MW) and the outputs of the remaining units be those obtained above.And more illustration followed in Table 3.1.

Table 3.1.

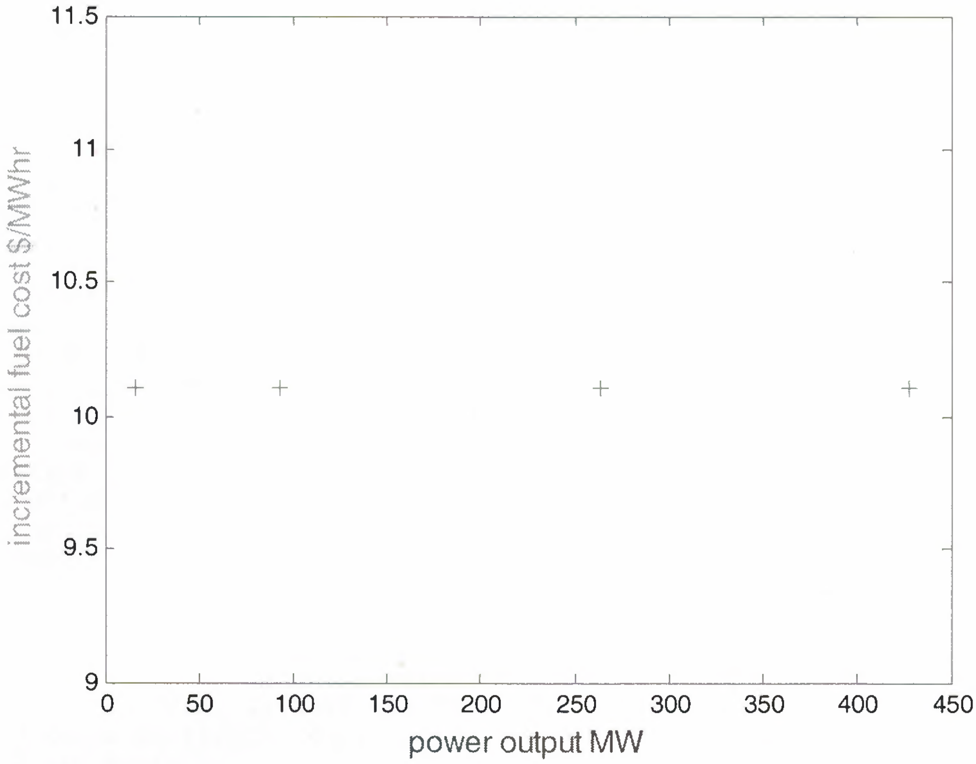
	Power output(MWh)	Incremental fuel cost ($/MWh)	Change ($/MWh)
	First iteration		
P1	92.4	10.1088	0
P2	428**	10.1088	0
P3	263.6	10.1088	0
P4	16*	10.1088	0
	TOTAL=800		
	Second iteration		
P1	98.9655	10.187586	-0.078786
P2	400	10.187586	-0.078786
P3	273.4483**	10.187586	-0.078786
P4	27.5862*	10.187586	-0.078786
	TOTAL=800		
	Third iteration		
P1	67.3333	9.808	+0.379586
P2	396.6667	9.808	+0.379586
P3	226	9.808	+0.379586
P4	110	9.808	+0.379586
	TOTAL=800		
		9.808	+0.379586

**,* indicates that it does violate its upper limit and lower limits respectively.

(+ and -) indicates that positive in change which means save money and negative change which means more expenses, respectively.

```
function [L1,L2,L3,L4,A,B,at,bt] = pout(P1,P2,P3,P4)
L1 = @ (P1) (0.012*P1+9); %the incremental fuel cost in $/MWh of unit1 at
economical dispatch required which it defined as anonymous function and
assumed that its variables already defined
L2 = @ (P2) (0.0096*P2+6); %the incremental fuel cost in $/MWh of unit2
at economical dispatch required which it defined as anonymous function
and assumed that its variables already defined
L3 = @ (P3) (0.008*P3+8); %the incremental fuel cost in $/MWh of unit3 at
economical dispatch required which it defined as anonymous function and
assumed that its variables already defined
L4 = @ (P4) (0.0068*P4+10); %the incremental fuel cost in $/MWh of unit4
at economical dispatch required which it defined as anonymous function
and assumed that its variables already defined
Pgt=800; %total output power demand on the plant
A= [0.012, 0.0096, 0.008, 0.0068]
B= [9, 6, 8, 10]
at=[1/A(1)+1/A(2)+1/A(3)+1/A(4)]^-1
bt=at*[B(1)/A(1)+B(2)/A(2)+B(3)/A(3)+B(4)/A(4)]
L=at*Pgt+bt %the economical value of incremental fuel cost which supposed
to all units to work with
Pt=[(L-B(1))/A(1)  ,  (L-B(2))/A(2)  ,  (L-B(3))/A(3)  ,  (L-B(4))/A(4)]%the
economical dispatch of output power required of each unit
Le= [L] % here called the anonymous function to compute their values
if Le>L % statement: to check if Le is bigger than L (economical inc.
cost of fuel),if its, then the value of Le directly will be zero which we
defined it as not good
Le=0
else
Plot (Pt, Le,'b+')
xlabel('power output MW','fontsize',12,'color','r') % operation: adds
text beside the X-axis on the current axis
ylabel('incremental fuel cost $/MWhr','fontsize', 12,'color','g') % adds
text beside the Y-axis on the current axis
end
end
```

```
L1 =

@ (P1) (0.012*P1+9)

L2 =

@ (P2) (0.0096*P2+6)

L3 =

@ (P3) (0.008*P3+8)

L4 =

@ (P4) (0.0068*P4+10)

Pgt = 800

A = [0.0120    0.0096    0.0080    0.0068]

B = [9     6     8     10]

at = 0.0022

bt = 8.3680

L = 10.1088

Pt = [92.4000 428.0000 263.6000   16.0000]

Le = 10.1088
```


11.5
11
10.5
10
9.5
9
0
50
100
150
200
250
300
350
400
450
incremental fuel cost $/MWhr
power output MW

```
function [L1,L2,L3,L4,A,B,at,bt,Lambda] = plantcost(P1,P2,P3,P4)
%the maximum load on each unit (200,400,270,300) MW respectively and
minimum
%load (50, 100, 80,110) MW
P1max=200;  P2max=400;  P3max=270;  P4max=300;  P1min=50;  P2min=100;
P3min=80; P4min=110;
% the incremental fuel costs in dollars per megawatt hour for a plant
consisting of four units
l1= inline ('0.012Pg1+9.0')
l2=inline ('0.0096Pg2+6.0')
l3=inline ('0.008Pg3+8.0')
l4=inline ('0.0068Pg4+10.0')
%the fuel cost coefficients
A= [0.012, 0.0096, 0.008, 0.0068]
B= [9, 6, 8, 10]
% here we assume that P2 is violates its upper and low limits, so assume
% P2 operates at its upper limit
at= [1/A(1)+1/A(3)+1/A(4)]^-1
bt= at*[B(1)/A(1)+B(3)/A(3)+B(4)/A(4)]
%since P2=400MW, so the total of rest units should be 400MW
Pgt=400
%here we will find new value of incremental fuel cost in dollars per
%megawatt hour
lambda=at*Pgt+bt;
%the summation of output power for each unit should be satisfy a 400 MW
Lambda= [10.1876, 10.1876, 10.1876, 10.1876]
P=[(lambda-B(1))/A(1),400,(lambda-B(3))/A(3),(lambda-B(4))/A(4)]
if P (4)>P4min
pgt= sum (P)-P4min
%here we see that P3&P4 are violate again their respective upper and
lower limits
%now assume P4 operate at lower limit, using P1, P2, and P3 only so we
%calculate lambda again
if P (4) <P4min
at_new= [1/A (1) +1/A (2) +1/A (3)] ^-1
bt_new=at_new*[B (1)/A (1) +B (2)/A (2) +B (3)/A (3)]
L=at_new*Pgt+bt_new
p=[(L-B(1))/A(1),(L-B(2))/A(2),(L-B(3))/A(3),sum(P)-(P(1)+P(2)+P(3))]
end
end
Pg= [98.9655, 400, 273.4483, 27.5862, 67.3333, 396.6667, 226, 110]
L= [10.1876, 10.1876, 10.1876, 10.1876, 9.8080, 9.8080, 9.8080, 9.8080]
bar (Pg, L,'g')
xlabel ('power output in MW/hr')
ylabel ('incremental fuel cost in $/MWhr')
gtext ('new inccost')
gtext ('old inccost')
```

l1 = Inline function:

l1(x) = 0.012Pg1+9.0

l2 = Inline function:

l2(x) = 0.0096Pg2+6.0

l3 = Inline function:

l3(x) = 0.008Pg3+8.0

l4 = Inline function:

l4(x) = 0.0068Pg4+10.0

A = [0.0120 0.0096 0.0080 0.0068]

B = [9 6 8 10]

at = 0.0028

bt = 9.0621

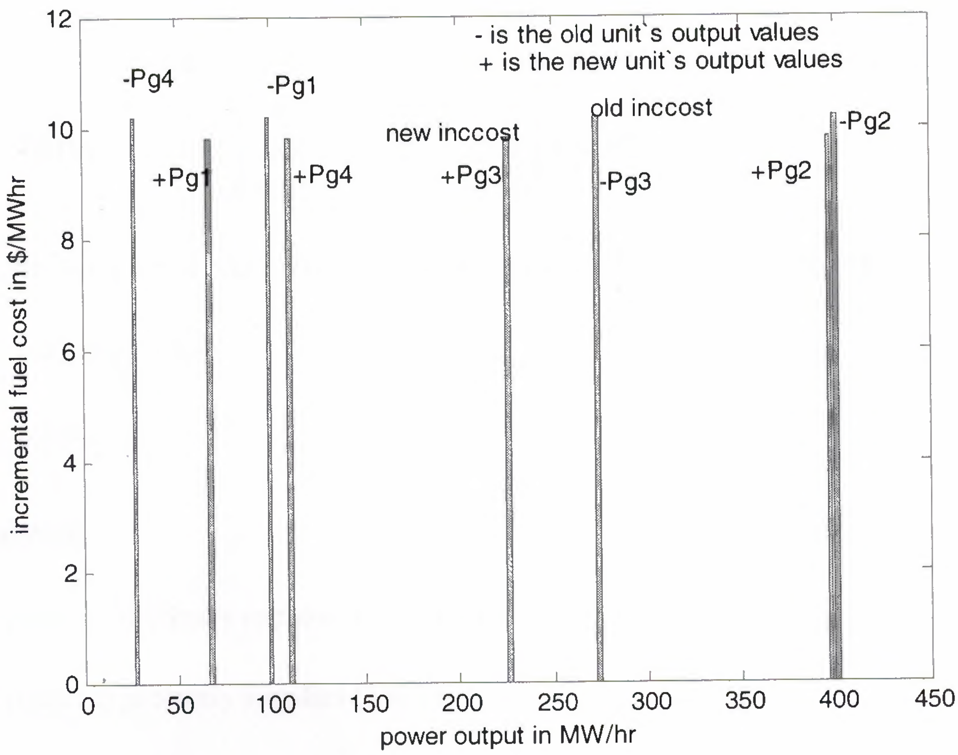
Pgt = 400

Lambda = [10.1876 10.1876 10.1876 10.1876]

P = [98.9655 400.0000 273.4483 27.5862]

Pg = [98.9655 400.0000 273.4483 27.5862 67.3333 396.6667 226.0000 110.0000]

L = [10.1876 10.1876 10.1876 10.1876 9.8080 9.8080 9.8080 9.8080]



- is the old unit`s output values
+ is the new unit`s output values
-Pg4
-Pg1
old inccost
new inccost
-Pg2
+Pg1
+Pg4
+Pg3
-Pg3
+Pg2
incremental fuel cost in $/MWhr
power output in MW/hr

Detailed example 3.2

The power system has two generating plants and B-coefficients which are given in per unit on (100MVA) base by $\begin{bmatrix} 5 & -0.03 & 0.15 \\ -0.03 & 8 & 0.2 \\ 0.15 & 0.2 & 0.06 \end{bmatrix} \times 10^{-3}$

And the incremental fuel costs in ($/MWh) of generating units at the two plants are: [8]

$\lambda_1 = 0.012P_{g1} + 6.6$

$\lambda_2 = 0.0096P_{g2} + 6$

Analysis

1. If plant (1) presently supplies (200 MW)

2. If plant (2) presently supplies (300 MW)

3. Find the penalty factors of each plant, is that present dispatch most economical? If not, which one should be decreased?

4. Find the incremental fuel costs of the plant buses

5. Find the incremental fuel costs when incorporated to penalty factors

Solution:

The power losses (P_L) is given by

$$P_L = [P_{g1} P_{g2} | 1] \begin{bmatrix} 5 * 10^{-3} & -0.03 * 10^{-3} & 0.15 * 10^{-3} \\ -0.03 * 10^{-3} & 8 * 10^{-3} & 0.2 * 10^{-3} \\ 0.15 * 10^{\wedge} - 3 & 0.2 * 10^{-3} & 0.06 * 10^{-3} \end{bmatrix} \begin{bmatrix} P_{g1} \\ P_{g2} \\ 1 \end{bmatrix}$$

$$P_L = (5 \times 10^{-3} P^2_{g1}) - 2(0.03 \times 10^{-3}) P_{g1} P_{g2} + (8 \times 10^{-3}) P^2_{g2} + (0.15 \times 10^{-3}) P_{g1} + (0.2 \times 10^{-3}) P_{g2} + (0.06 \times 10^{-3})$$

Where (P_{g1} *and* P_{g2}) are in per unit on (100 MVA) base, penalty factors are calculated as

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{g1}}} = \frac{1}{1 - \{2 \times (5 \times 10^{-3}) P_{g1} - 2(0.03 \times 10^{-3}) P_{g2} + 0.15 \times 10^{-3}\} |_{P_{g2}=3}^{P_{g1}=2}}$$

$$L_1 = 1.02037$$

$$L_2 = \frac{1}{1 - \frac{\partial p_L}{\partial P_{g2}}} = \frac{1}{1 - \{2 \times (8 * 10^{-3}) P_{g2} - 2(0.03 \times 10^{-3}) P_{g1} + 0.2 \times 10^{\wedge} - 3\} |_{P_{g2}=3}^{P_{g1}=2}}$$

$$L_2 = 1.05058$$

The incremental fuel costs at the two plants buses calculated as

$$\frac{\partial f_1}{\partial P_{g1}} |_{P_{g1}=200} = ((0.012 \times 200) + 6.6) = 9 \ \$/MWh$$

$$\frac{\partial f_1}{\partial p_{g2}} |_{P_{g2}=300} = ((0.0096 \times 300) + 6) = 8.88 \ \$/MWh$$

The incremental fuel costs when incorporated with penalty factors

$$L_1 \frac{\partial f_1}{\partial P_{g1}} = 1.02037 \times 9 = 9.1833921 \ \$/MWh$$

$$L_2 \frac{\partial f_1}{\partial P_{g1}} = 1.05058 \times 8.88 = 9.3285146 \ \$/MWh$$

*Since $L_1 \frac{\partial f_1}{\partial P_{g1}}$ is smaller than $L_2 \frac{\partial f_1}{\partial P_{g1}}$, the output of plant (1) should be increased while plant (2) should be decreased to achieve the economic dispatch.

```
function [L1,L2,lambda1,lambda2,PL]=translosses(P1,P2,dPL1,dPL2)
% the output power for the two generating plants in MWh
P= [200 300];
% we find the incremental fuel cost for each generator plant, in $/MWh
lambda1=0.012*P (1) + 6.6
lambda2=0.0096*P (2) + 6.0
% the power losses of generating plants
PL= inline('(5*10^-3*p1^2)-2*(0.03*10^-3)*p1*p2+(8*10^-3*p2^2)+(0.15*10^-3*p1)+(0.2*10^-3*p2)+(0.06*10^-3)');
% the value of each plant is given in per unit on the 100 MVA base
S1=100;
S2=100;
% we find the output power on base in per unit
p1base= P (1)/S1
p2base= P (2)/S2
% we find the derivative of the power losses of generating plant1
dPL1= ((2*5*10^-3*p1base)-2*(0.03*10^-3*p2base) + (0.15*10^-3));
% the penalty factor of plant1
L1= (1)/ (1-dPL1)
% we find the derivative of the power losses of generating plant2
dPL2=(2*8*10^-3*p2base)-2*(0.03*10^-3*p1base)+(0.2*10^-3);
% the penalty factor of plant2
L2= (1)/ (1-dPL2)
% Here when the penalty factor of two generating plants is incorporated with incremental
% fuel cost, we have its value in $/MWh
PFinccost= [L1*lambda1, L2*lambda2]
```

```
lambda1 =

     9


lambda2 =

    8.8800


p1base =

     2


p2base =

     3


L1 =

    1.0204


L2 =

    1.0505

PFinccost = [9.1834     9.3285]
```

CONCLUSION

In our project, we have discussed the economic dispatch solutions and how they are affected by the economic dispatch.

In the first detailed example, we considered it without transmission losses. We have noticed that some of the generator units have been violated either at its upper limit or lower limit. When some of the units violate it's upper or lower limit we have to consider its operation at the violated limit to meet the required economic output power with the lowest fuel cost.

The example took three iterations to meet the required output power economic dispatch with lowest fuel cost. In the first iteration, it was noticed that one violation was on an upper limit and the other was on a lower limit with some specific amount of fuel cost. In the second iteration, again the same violation occurred but here with increase in amount of fuel cost.

The third (last) iteration was the best since neither violation occurred and the fuel cost have been decreased to the lowest which is the wanted economic dispatch.

In the second detailed example, we considered it with transmission losses including penalty factor calculations. When the penalty factors are included in the calculations it should be incorporated into the incremental fuel cost, so the cost of fuel directly increases. We have noticed that the incorporated penalty factor into incremental fuel cost in plant one is smaller than that in plant two, so the output of plant one should be increased while that of plant two should be decreased to achieve the economic dispatch.

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