NEAR EAST UNIVERSITY

Faculty of Engineering

Department of Electrical and Electronic Engineering

INVERSE KINEMATIC AUGMENTED 3-DOF STEWART PLATFORMS (ROBOTICS)

Graduation Project
EE400

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ACKNOWLEDGMENTS

" TO MY FATHER, MY MOTHER,

To MY BROTHER MOHAMMAD AND MY SISTERS."

" TO MY SUPERVISOR

ASSIST.PROF.DR.KADRI BURUNCUK

WHO GUID ME THROUGH THIS TOPIC."

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ABSTRACT

A 3-DOF Stewart Platform has upper and lower platforms with equilateral triangular shape. In the present article this platform is augmented by locating an extendible limb at the mass center of the upper platform along the direction of the unit normal vector. Previously obtained results for the kinematics of 3-DOF Stewart Platform is extended to study the kinematics of this augmented mechanism. In particular the problem that is studied is to determine the motion of all the limbs when the tip of the extendible limb is constrained to move in space from one point to another point along any function passing through these points with a suitable velocity and acceleration profiles. If the positions of the two points in space, and also the z component of position vector of the mass center for the upper platform are given then a written Matlab program supplies the rest of the information about the inverse kinematics of the mechanism. Through this program one can easily calculate the lengths, velocity and acceleration for each limb.

1. Introduction

Robotics is a field in which robots, or automated machines, are devised and created to perform a variety of tasks. These tasks can range from industrial-strength cleaning services, to patrolling a nuclear power plant. There are many aspects involved in creating a robot. A great deal of physics, engineering, electronics, as well as general construction techniques must be known before attempting to build a robot. Robots come in two flavors, the drone type controlled by humans, and the artificially intelligent type, which runs from its own programming. [1]

The word "robot' was coined by Karel Capek who wrote a play entitled "R.U.R." or "Rossum's Universal Robots" back in 1921. The base for this word comes from the Czech word 'robotnik' which means 'worker'. In his play, machines modeled after humans had great power but without common human failings. In the end these machines were used for war and eventually turned against their human creators. But even before that the Greeks made movable statues that were the beginnings of what we would call robots. For the most part, the word "Robot" today means any man-made machine that can perform work or other actions normally performed by humans. [14]

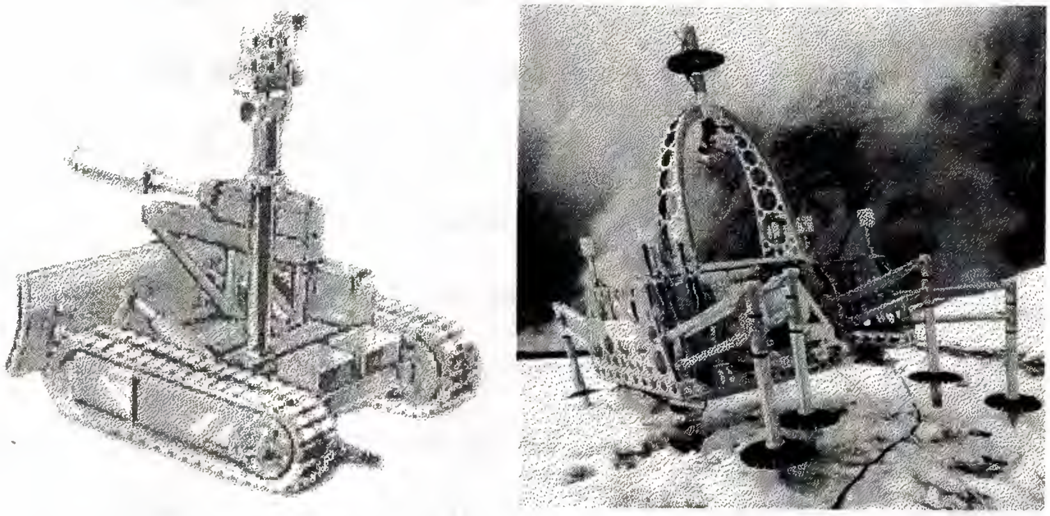


Figure 1.1 Example of what robots look like.

(Courtesy of Carnegie Mellon and NASA)

A beterdeti: Was “A robot is machine which can be programmed to do a variety of tasks, in the same way that a computer is an electronic circuit which can be programmed to do variety of tasks” (Mc Kerrow, 1986). [2]

the Robotics is a very broad topic and generally there is no accepted definition for robotics. Even when narrowed down to the category of "Industrial Robotics", no two sources will give the same definition of the term. In Japan, for example, any mechanical device that operates in a factory and performs a single, simple task, time and time again, is considered to be a robot and they defined a robot as any device, which replace human labor, (Soska, 1985). [2], whereas in America, such devices are considered "automation systems". Although automation systems have been around in factories for several decades, robotics is a fairly young topic in industrial society. In this history of industrial robotics, and (specifically in the automotive industry), we will distinguish between the newer, American idea of industrial robots, and its less versatile predecessor, namely automation systems. In America, an automation system, or device, is a mechanical component in a factory that replaces or partially replaces an employee that performed a simple, menial task several hundred times per day. Automation systems have been utilized by industrial society since the early twentieth century. They may be complex devices, and they may be controlled by software, but they are physically and electronically designed to do a specific task, and if the industry must make drastic changes to a product, or if a new product is to arrive on the production line, it is most likely that all automated machines on the line will have to be replaced. Automation systems are not, however, to be mistaken for robots. Robots are a newer addition to industry, which began (according to the American definition of the term) in the 1950's. A robot is a software-controlled mechanical device that also replaces, or partially replaces a worker in a factory, but, although not nearly as versatile or adaptable as a human, it is much more flexible and universal than its predecessor: the automated machine. A robot can be manipulated to adapt to changes in environment, product design, or to handle an entirely new product, simply by some reprogramming, or electronic alterations on the part of a computer engineer. In the very beginning stages of the development of industrial "robots" - when the word was first introduced into the English language - some people had visions of human - shaped mechanical "creatures". The inventor and designers of robots, hovever, knew then as they know now that robots need not resemble humans to perform the repetitive, tedious tasks that are required in a factory. Thus, it was perceived that, for example, senses such as taste and smell would not even need to be considered in the design of an industrial robot. As a result of these considerations, most early robots (as well as many robots of today) resemble a single human arm with a relatively unrestricted range of motion, and a clamp, or "hand" for grasping. They were quite versatile in that they could be reprogrammed to grasp different objects, to move in different directions or to grasp with different strengths. [13].

1.2 A history of robotics

In 1968 RS Moshes at general electric built a quadrupled walking machine this walking track was over 3 meters long. Weighted 1400Kg. And was powered by a 68 Kw petrol motor In 1976 first robot arm in space was used by NASA Viking probe to collect smalls Martians soil for analysis. [3]. In ancient times, human slaves were used to accomplish dangerous, unpleasant, and repetitive tasks. As time passed, machines began to replace human beings in carrying out tasks that people did not like doing. With the development of industrialization, machines were able to do more of the dangerous, tedious work. Although robots can't do every type of job, there are certain tasks robots do very well such as assembling products, handling dangerous materials, spraying finishes, inspecting parts, produce, and livestock, and cutting and polishing. In contemporary manufacturing, fewer people are doing these tasks as robots fill this niche.

1940s

Early work leading to today's industrial robots can be traced to the period immediately following world war- 2 [4]. According to K.S.Fu [5], R.C. Gonzalez, and C.S.G.Lee [6], during the late 1940s, research programs were started at the Oak Ridge and

Argonne National Laboratories to develop remotely controlled mechanical manipulators for handling radioactive materials. These systems were designed to reproduce hand and arm motions made by a human operator.

1950s

More sophisticated systems were designed in the mid-1950s. According to K.S.Fu [5], George C. Devol developed a device called a "programmed hand which could follow a sequence of motion steps determined by the instructions in the program. Further development by Devol and Joseph F. Engelberger led to the first industrial robot, introduced byUnimation Inc. [7], in 1959. These machines could be taught to carry out a variety of tasks automatically. In the same year, there was the development of the integrated circuit chip. According to William C. Burns, the development of IC chips came about because of the need for smaller and smaller circuits that used less power and were more durable.

1960s

While programmed robots offered a powerful manufacturing tool, it became evident in the 1960s that the flexibility of these machines could be enhanced significantly by the use of sensory [8] feedback. K.S. Fu states that the development of a computer-controlled [9] mechanical hand with tactile sensory [8] called MH-1 could feel blocks and use this information to control the hand so it stacked the blocks without operator assistance. This work is one of the first examples of a robot capable of adaptive behavior in a reasonably unstructured environment.

1970s

During the 1970s, a great deal of research work focused on the use of external sensory [8] to facilitate manipulative operations. By 1975, mass experimentation with digital circuitry was underway by both government and private industry, as well as by individual inventors who set up small labs in their basements and garages to tinker with electronic component applications. This led to fast progress in new technology that today makes it difficult to produce a generic re-programmable control unit that can be applied to a variety of tasks.

1980s

Modern industrial arms have increased in capability and performance through controller [9] andlanguage development [10] , improved mechanisms [11], sensing [8], and drive system [12]. In the early to mid 80s, the robot industry "grew very fast due to large investments by the automotive industry", says Stan Viet. The fast jump into the factory turned into a plunge when the integration and economic viability of efforts proved disastrous. The robot industry has only recently recovered to mid-1980s revenue levels. In the meantime, there has been an enormous shakeout in the robot industry.

1.3 What Do Robots Do?

Most robots today are used in factories to build products such as cars and electronics. Others are used to explore underwater and even on other planets. [14]

Some practical application of robots [2]:

a) Industry

- Arc welding.
- Assembly.
- Machining metals and there are eight ways of machining metals:
    1. Drilling.
    2. Milling.
    3. Grinding.
    4. Turing.
    5. Boring.
    6. Shaping.
    7. Planning.
    8. Slotting.

b) Laboratories

- Carrying the instrument tasks, like placing the test tubes into measuring instrument that's tedious work of laboratory technician.

c) Kinestatic manipulator

- In the recent years robots used for remote welding and pipes inspection in the high radiation area.

d) Agriculture

- Sheep shearing robots (in Australia).
- Transplanting of seeding.
- Pruning grape in France (1985).
- Picking apples.

e) Space:

- Robots has been used in space since 1976 when Viking 1 landed on Mars which was used to dig French in the Martian soil and scoop up the soil samples for analysis.

f) Submersible vehicles:

- Was used to find and recover the black boxes from the jetliners that had been crashed on the oceans.

g) Education:

- Educational programs using simulations of robot control as subset of Pascal, is used as an introductory programming language.
- Turtle robots in conjunction with the logo language to teach computer awareness.

h) Assisting the hand capped

- Automatic wheel chairs that carry the occupant around the hospitals.

1.4 What is Robots Made Of?

Robots have 3 main components:

- Brain - usually a computer
- Actuators and mechanical parts - motors, pistons, grippers, wheels, gears
- Sensors - vision, sound, temperature, motion, light, touch, etc.

With these three components, robots can interact and affect their environment to become useful. [14].

The Industrial robots have traditionally been used as general-purpose positioning devices and are anthropomorphic open-chain mechanisms, which generally have the links actuated series. The open kinematics chain manipulators usually have longer reach, larger workspace, and more dexterous maneuverability in reaching small space. However, the cantilever-like. Manipulator is inherently not very rigid and has poor dynamic performance at high-speed and high dynamic loading operating conditions. Due to several increasingly important classes of robot applications, especially automatic assembly, data-driven manufacturing and reconfigurable. Jigs and fixtures assembly for high-precision machining, significant effort has been directed towards finding techniques for improving the effective accuracy of the openchain manipulator with calibration methods, Recently, some effort has been directed towards the investigation of alternative manipulator designs based on the concepts of closed kinematics chain due to the following advantages as compared to the traditional open kinematics chain manipulators: more rigidity and accuracy due to the lack of cantilever-like structure, high force/torque capacity. for the number of actuators as the actuators are arranged in parallel rather than in series, "and relatively simpler inverse kinematics which is an advantage in real-time computer online control. The closed kinematic chain manipulators have potential applications Where the demand on workspace and maneuverability is low but the dynamic loading is severe and high speed and precision motion are of primary concerns Typical, examples of in-parallel mechanism are a camera tripod and a six-degrees-of-freedom Steward platform, which has been originally designed as an aircraft simulator. And later as a robot wrist various application of the Steward platform have been investigated for use in mechanized assembly and for use as a compliance device. Significant effort has been directed towards tendon actuated in-parallel manipulators, which have the advantages of high Force-to-weight ratio. The manipulation approach analyzed in this communication is based on an in-parallel actuated tripod-like manipulator, which has two degrees of orientation freedom and one degree of translator freedom.

2. STEWART PLATFORM (Regular and Inverted)

2.1. DESCRIPTION

A Stewart platform is a parallel robot manipulator with 6 degrees of freedom that Basically consists of two rigid bodies, the base and the upper platform, connected to Each other by six or three, limbs with variable lengths. The limbs are jointed to the base and the upper platform usually by spherical joints; sometimes universal joints are used also with equal capability at the base platform.

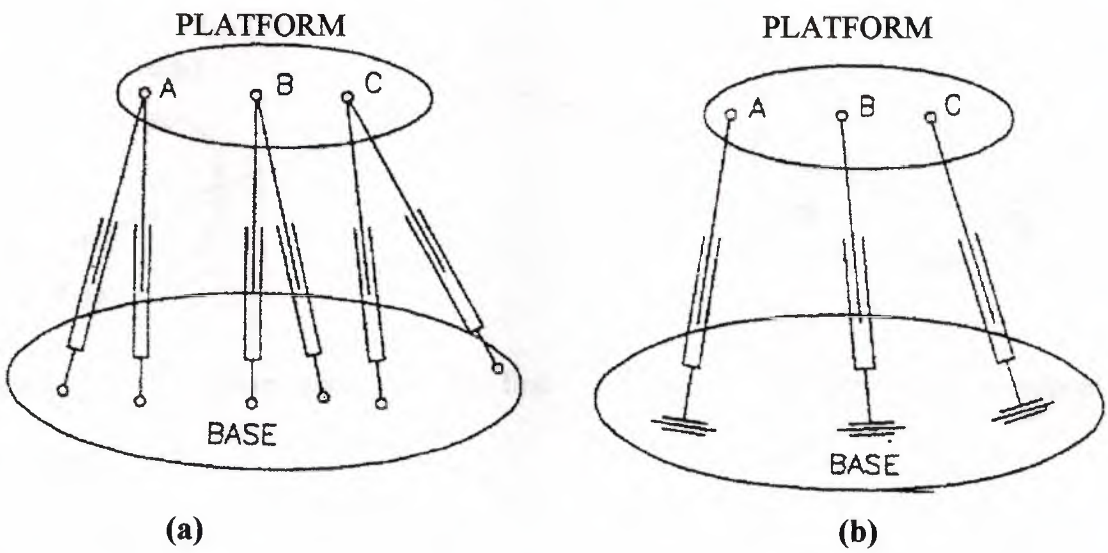


Figure 2.1

(a) A special Stewart platform with six extendable limbs.

(b) An equivalent platform where the joints on the base are cylindrical.

The Stewart Platform and the Inverted Stewart Platform parallel link manipulators are unique kinematically constrained work platforms. Both versions possess a control platform that can be manipulated through the six degrees of freedom (DOF) of x, y, z, pitch, roll, and yaw.

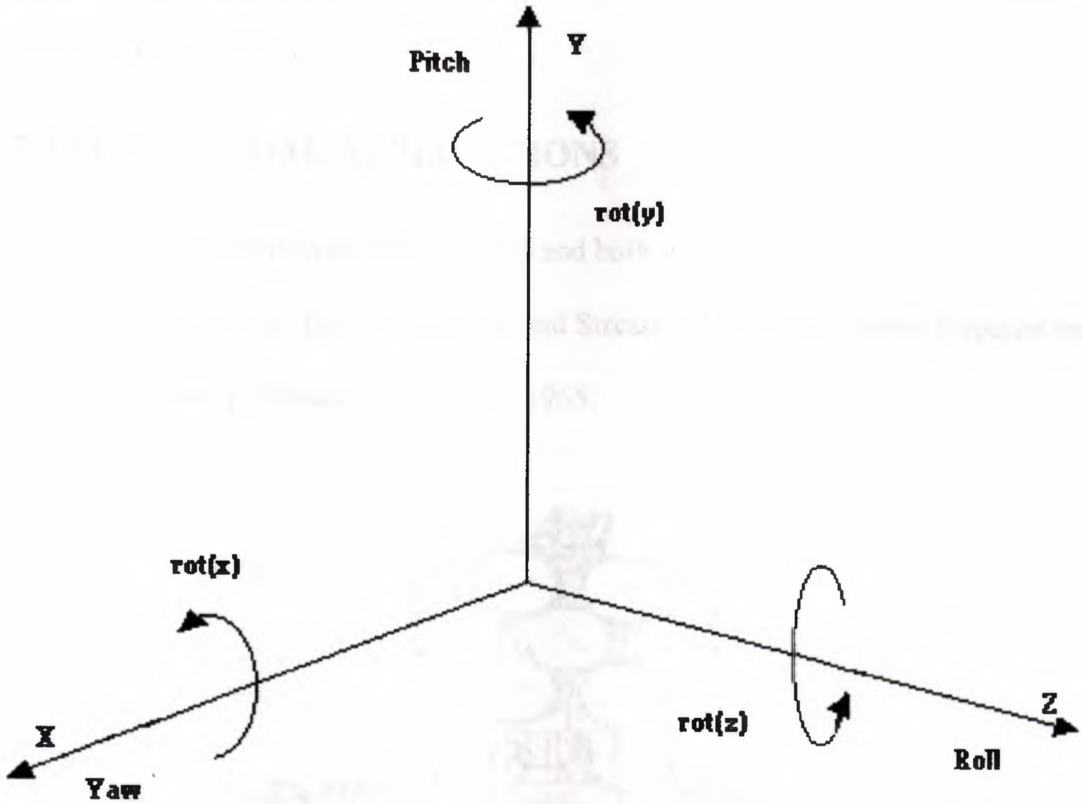


Figure 2.2 3D presentation of the first condition (41-a) with whole rotational workspace.

2.2. CONSTRUCTION

The Stewart Platform has applications in machine tool technology, crane technology, underwater research, air-to-sea rescue, flight simulation, and satellite dish positioning. The Inverted Stewart Platform is an equilateral triangle suspended in an octahedron frame by six actuators (cables or strings). Two strings are attached at each corner of the suspended platform triangle. Each pair of strings is run through pulleys attached to the vertices of the top triangle, down to the base of the octahedron frame and then fastened to the control devices (cranks). The control mechanism allows the operators to manipulate the platform through the 6 DOF. The National Institute of Standards and Technology (NIST) have conducted much of the research and development on the Stewart Platform. [15].

2.3 CONCEPTUAL APPLICATIONS

The Stewart Platform was first designed and built to [3, 19]

1. Test tires at the Performance and Stressing Department, over a decade before Stewart published his article in 1965.

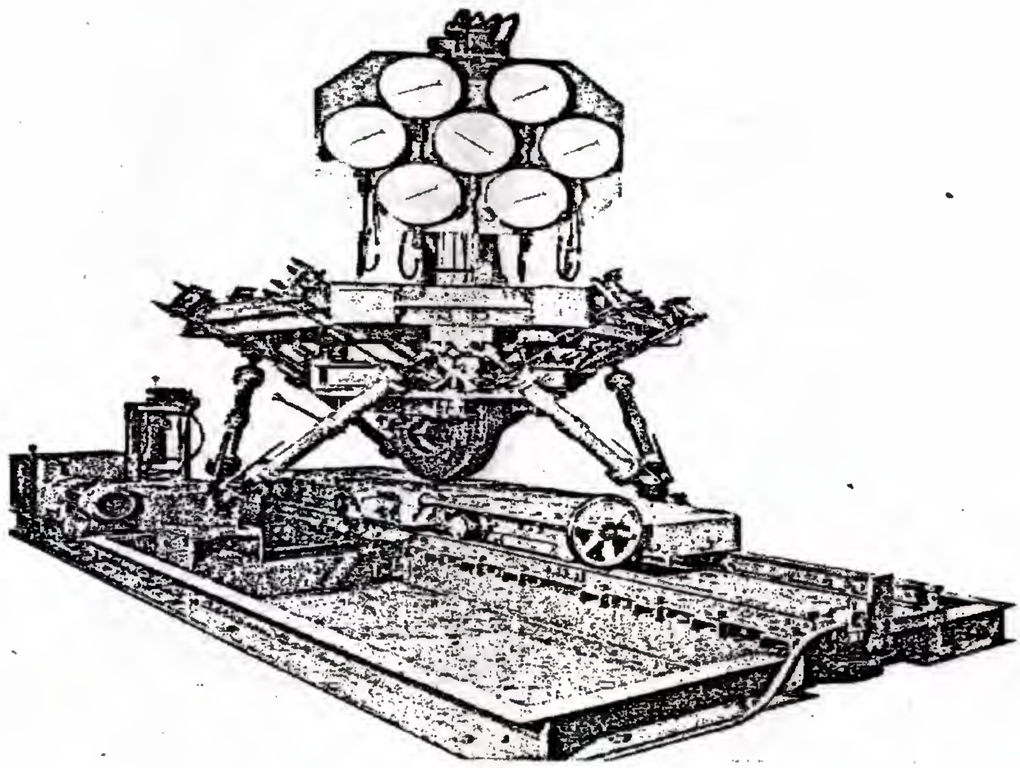


Figure 2.3 Testing tires machine

2.TRANSPORTATION: The regular Stewart Platform is used in flight simulators due to its unique six-degree of freedom positioning (x, y, z, pitch, roll, & yaw).

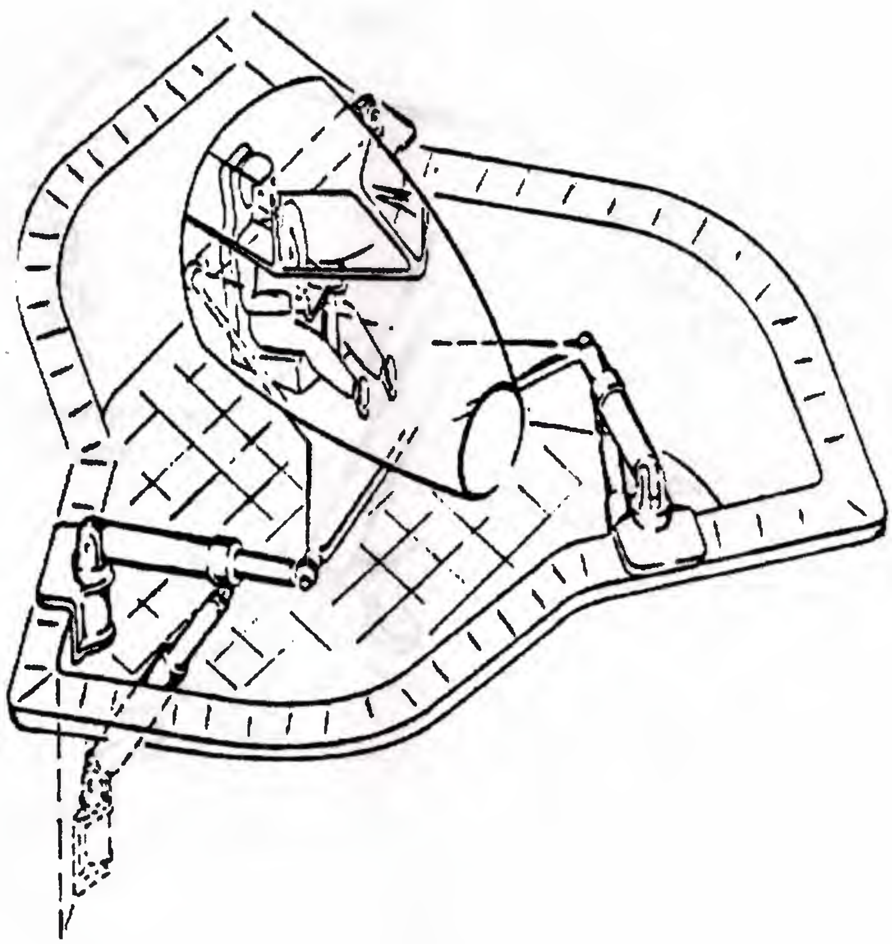


Figure 2.4 Pictorial view of Flight Simulator

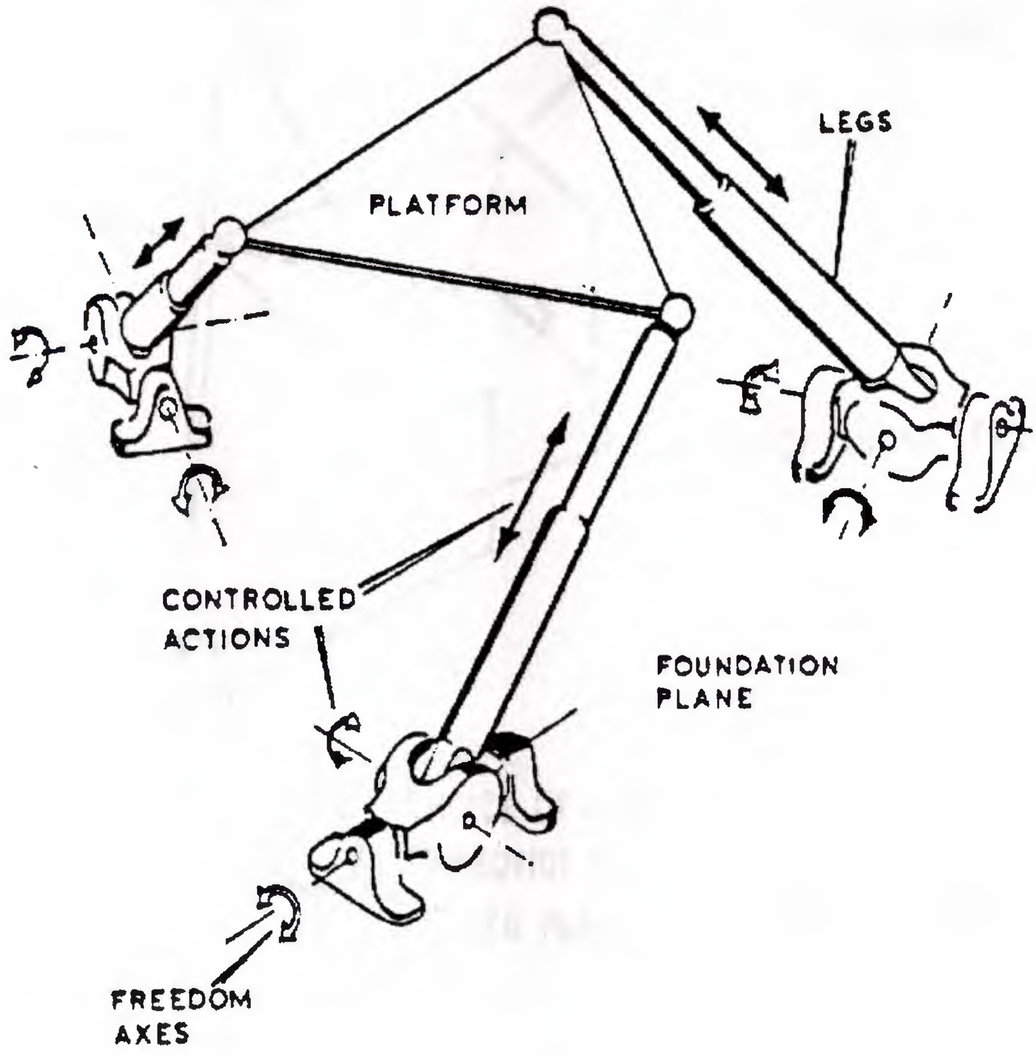


Figure 2.5 General arrangements of flight simulator

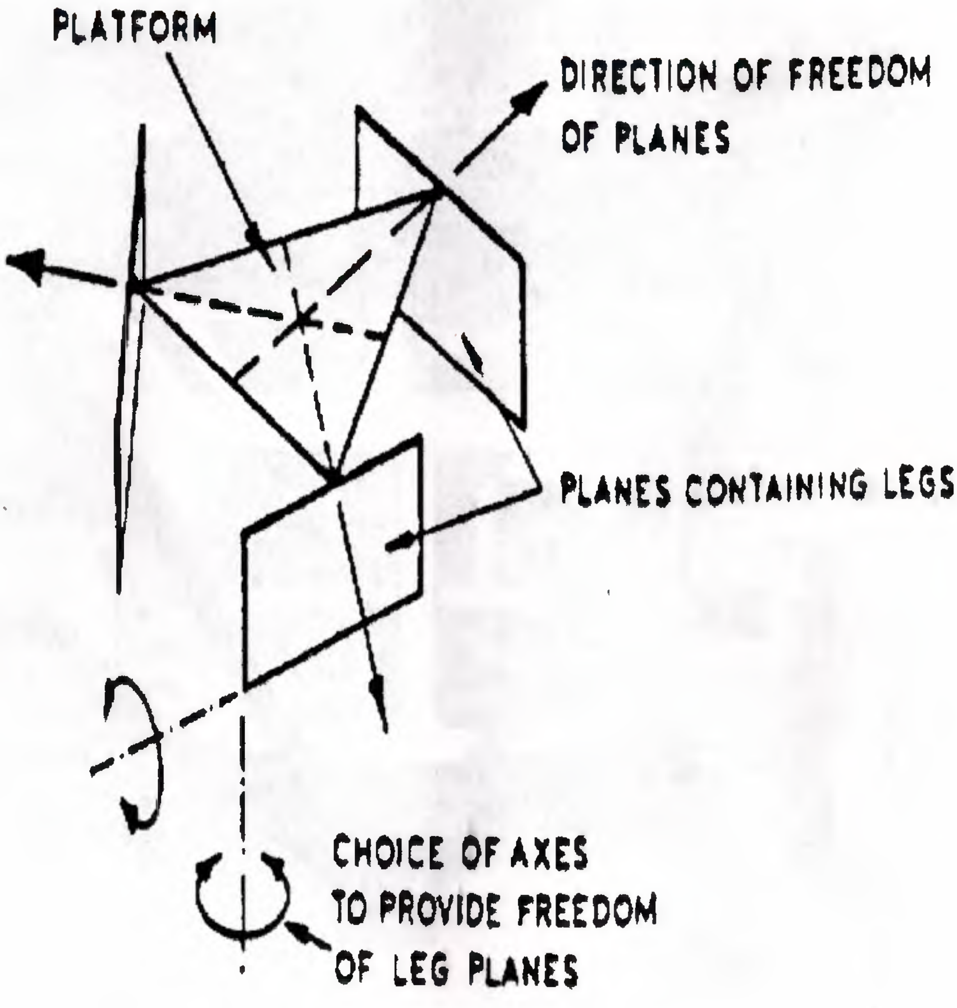


Figure 2.6 Diagram of leg planes of flight simulator

2. Machine tools, universal milling machine and oil drilling rigs. [3]

Figure 2.7 Impression of possible design of universal mill

3. ON LAND: The Inverted Stewart Platform can be used to replace conventional crane technology. The Inverted Stewart Platform crane provides the crane

operator with greater control of the crane hoist mechanism. The national Institute of Standards and Technology has developed a crane, known as ROBOCRANE, utilizing the Stewart Platform technology.

4. AT SEA: The Inverted Stewart Platform can be suspended from a single floating vessel or it can be suspended from three strategically located ships to place an underwater drilling platform, lay pipelines, or it can be used for underwater salvage.

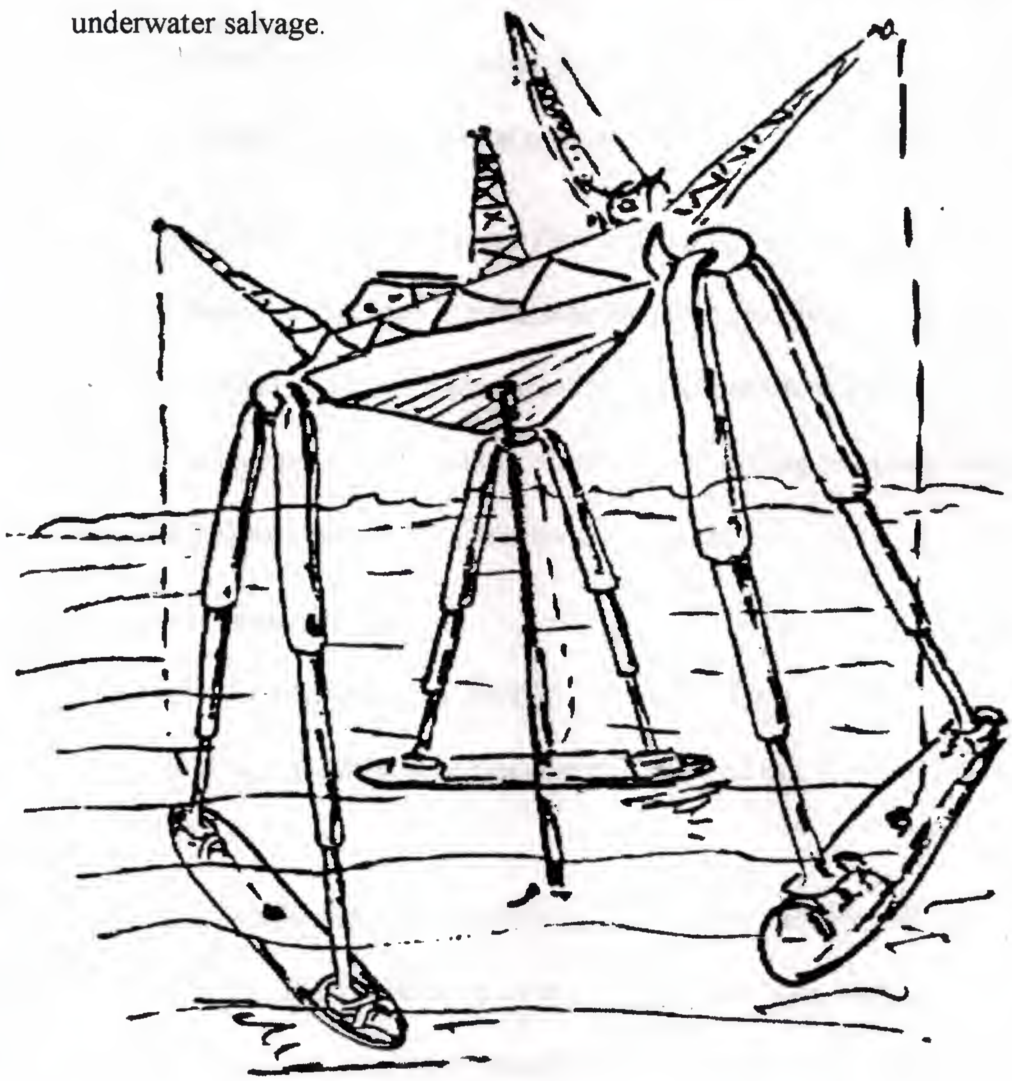


Figure 2.8 Impression of possible design for drilling rig

1. IN THE AIR: An Inverted Stewart Platform can replace the conventional single cable hoisting technology currently being used on helicopters for use as an air crane or in air-to-sea rescue.

2. IN SPACE: An Inverted Stewart Platform could be modified for use as a Lunar Rover. Equipped with the necessary wheels it could be used to lift heavy objects or it could serve as drilling platform.

3. COMMUNICATIONS: A Stewart Platform manipulator could be used for positioning satellite dishes on land or on a pitching and rolling ship at sea

4. MANUFACTURING: The Stewart Platform concept is currently being applied in machine tool technology in a machine manufactured by Ingersoll known as a Hexapod. The Hexapod is a unique milling machine which can be manipulated through six degrees of freedom. Conventional milling machines can only be operated in 3 to 4 axes (x, y, z, and rotational). [15].

The advantages of Stewart platform:

1) Stewart platform is a parallel link mechanism, which has the major mechanical differences from the typical serial link robot. [3]

2) The parallel link mechanism is to connect two platforms with a number of links, and to distribute the load between the legs, which is improving load-caring capabilities. As well as it's improving the accuracy and the repeatability, and errors will tend to average out rather than accumulating as in the case of serial link mechanisms. [3]

There are many different type of Stewart Platforms studied in the literature where shape of the base and upper platform are either assumed to be hexagonal or triangular. Also some of the platforms have either six legs or three legs. The connections of the legs are provided by either through ball joints or universal (cardan) joints. The 3-DOF Stewart Platform manipulator shown in Fig.2.9. is used as the basic part of the manipulator considered in this article. It consists of equilateral triangular upper and lower platformsas shown Fig. 2.10. This platform here is augmented by locating an extendible limb at the mass center of the upper platform along the direction of the unit normal vector. All the previously obtained results for 3-DOF Stewart Platform are used and extended for this 4-DOF manipulator, in particular the problem concerning the inverse kinematics of this manipulator is studied thoroughly. [3]

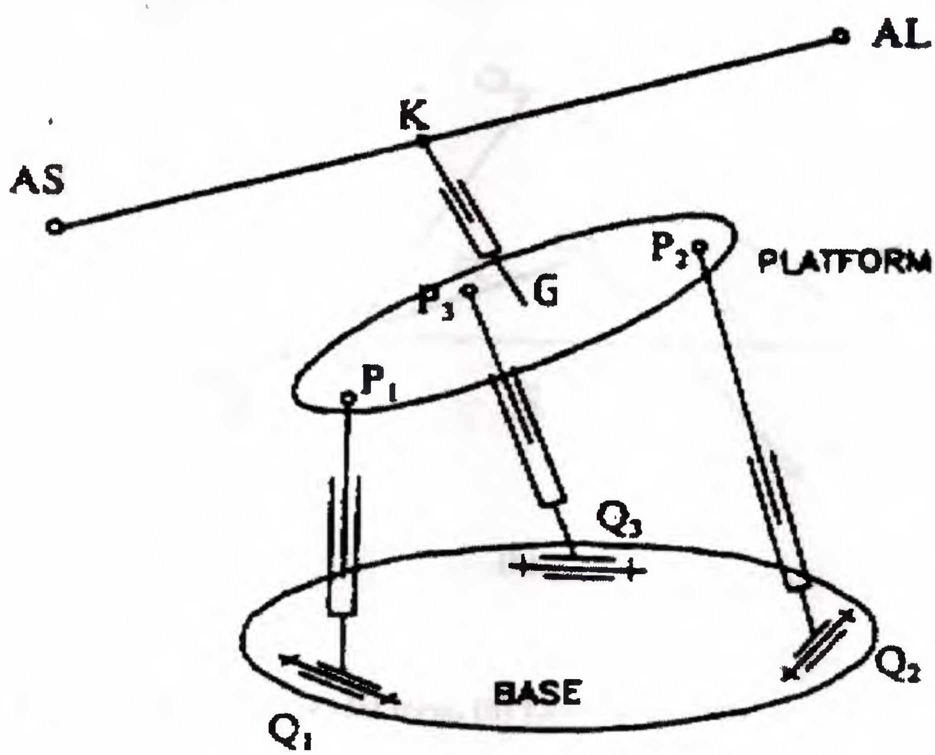
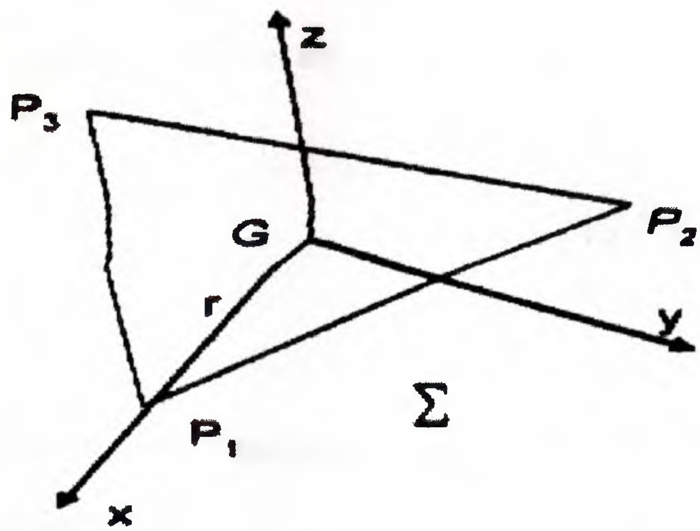
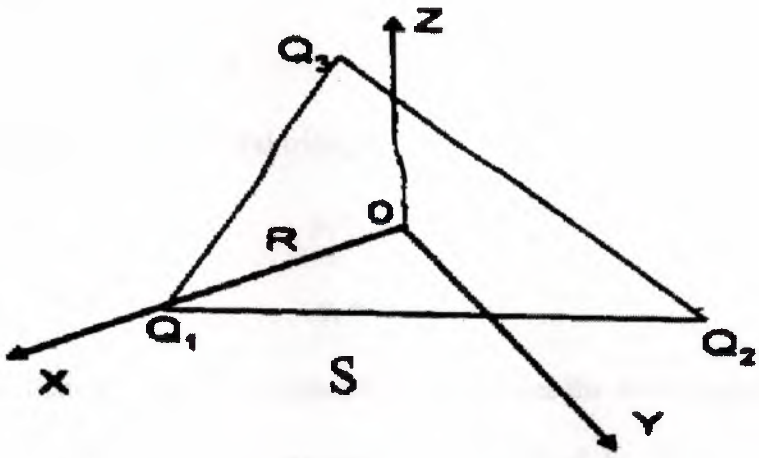


Figure 2.9 Augmented 3- Dof Stewart platform



(a)



(b)

Figure 2.10. (a) Upper plat form, (b) Lower platform.

2.4 3-DOF STEWART PLATFORMS

A 3-DOF platform with equilateral triangle base and upper platforms is shown in Figure 2. According to the selected coordinate frame S *(OXYZ)* for the base platform, the position vectors of the platform extremities are

$$q_1 = \begin{bmatrix} R \\ 0 \\ 0 \end{bmatrix} \qquad q_2 = \begin{bmatrix} -R/2 \\ \left(\sqrt{\frac{3}{2}}\right)R \\ 0 \end{bmatrix} \qquad q_3 = \begin{bmatrix} -R/2 \\ -\left(\sqrt{\frac{3}{2}}\right)R \\ 0 \end{bmatrix} \tag{1}$$

Similarly for the coordinate frame Σ *(Oxyz)* attached to the upper platform, the position vectors of the platform extremities are

$$p_1 = \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} \qquad p_2 = \begin{bmatrix} -r/2 \\ \left(\sqrt{\frac{3}{2}}\right)r \\ 0 \end{bmatrix} \qquad p_3 = \begin{bmatrix} -r/2 \\ -\left(\sqrt{\frac{3}{2}}\right)r \\ 0 \end{bmatrix} \tag{2}$$

Note that due to the property of equilateral triangle

$$p_1 + p_2 + p_3 \equiv 0 \tag{3}$$

$$q_1 + q_2 + q_3 \equiv 0 \tag{4}$$

Initially, we assume that Σ and S is coincident and shares the same origin *0*. After displacement of the upper platform let its origin be indicated by G (center of the equilateral triangle) and the corners indicates by $p_i = (i = 1,2,3)$ lie on the vertical planes Π_1, Π_2, and Π_3, respectively, that pass through the vertical *OZ* axis. Let the position vector of G be

$$\xi = [x_G \quad y_G \quad z_G] \tag{5}$$

Note that each limb has 2 degrees of freedom:

It can rotate about the revolute joint at $Qi(\ i=1,2,3)$ and also its length l_i may be changed, although the limb remains in the fixed vertical plane Π_i If $T=[t_1 \quad t_2 \quad t_3]$ represents the transformation matrix which transforms the initial position of the upper platform to its displaced position, we can write

$$\left.\begin{aligned} a &= Tp_1 + \xi \\ b &= Tp_2 + \xi \\ c &= Tp_3 + \xi \end{aligned}\right\} \qquad (6)$$

Where ξ gives the amount of translation of the rotated coordinate frame.

The Rodrigues formula is as following:

$$T(n,\theta) = [t_1 \quad t_2 \quad t_3] = \cos\theta\, I + (1-\cos\theta)nn^T + \sin\theta\, N \qquad (7)$$

$$T = \cos\theta \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + (1-\cos\theta) \begin{bmatrix} n_1^2 & n_1n_2 & n_1n_3 \\ n_2n_1 & n_2^2 & n_2n_3 \\ n_3n_1 & n_3n_2 & n_3^2 \end{bmatrix} + \sin\theta \begin{bmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{bmatrix} \qquad (8)$$

gives the explicit expression for the transformation matrix, where $n=[n_1 \quad n_2 \quad n_3]$ is the unit vector lying on the rotation axis, N is its skew-symmetric matrix representation, and θ is the rotation angle about the rotation axis.

Where $n=[n_1 \quad n_2 \quad n_3]^T$ is the unit vector lying on the rotation axis and N is its skew-symmetric matrix representation while θ is the rotation angle about the rotation axis.

Constraint equations of 3-DOF Stewart Platform Where C and S stand for cos(θ)and sin(θ), respectively. Third equation in (9) states that either sin (θ) =0 (which means that θ=k,r with k=0,± 1, ± 2...) or $n_3=0$. Where the occurrence of the second case is most probable. Therefore assuming that $n_3 = 0$ and hence, $n_1^2 + n_2^2 = 1$ the transformation matrix T in (7) takes the form given in (10).

$$\left.\begin{aligned} x_G &= -\frac{r}{2}(1-C)(n_2^2 - n_1^2) \\ y_G &= -r(1-C)n_1 n_2 \\ 0 &= Sn_3 \end{aligned}\right\} \tag{9}$$

$$T(n,\theta) = \begin{bmatrix} C+(1-C)n_1^2 & (1-C)n_1 n_2 & Sn_2 \\ (1-C)n_1 n_2 & C+(1-C)n_2^2 & -Sn_1 \\ -Sn_2 & Sn_1 & C \end{bmatrix} = \begin{bmatrix} t_1 & t_2 & t_3 \end{bmatrix} \tag{10}$$

From these expressions we understand the following :

The rotation axis and hence n lies in the OXY plane. The unit vector v = t_3 is perpendicular to both the upper platform giving its orientation and also to the rotation axis as shown in Fig. 2.14.

2.5. Forward Kinematics and its Need for Control and Measurement [18].

Forward and inverse kinematics are terms to describe mapping from the space of inputs to the space of outputs of a non-dynamic mechanical system as shown in Figure 2.11. Figure 2.12 shows a typical application of the forward and inverse kinematics algorithm. A trajectory for a device such as a milling machine or a welding system is to be followed. The controller, using the inverse kinematics algorithm (and calculating desired changes in position), computes the control signal given to the actuators. At the output of the platform we measure the lengths of the links controlled by the actuators. Then, the forward kinematics algorithm transforms those lengths into (platform) positions. The position signal is compared to the assigned position and added to the controller's input. Beacuse of the Stewart platform closed kinematic chain, the rigidity of the end effector, and the accuracy of the link length servos, a reasonably accurate feed-forward assignment is possible. Due to the complexity of older forward kinematics algorithms, most Stewart platforms are controlled without using feedback. The proposed algorithm will allow the calculation of the forward kinematics in real time. This opens an opportunity to increase the accuracy of the control algorithms and broaden the current uses of the platform.

There are other applications where the solution of forward kinematics is mandatory. These are the cases when the Stewart platform is used as a component of a measuring device. These cases include:

- camera mapping where a camera is mounted on top of a Stewart platform. For example, we let the camera roam until it finds a target, and then we ask where is the camera pointed. It is also useful when we have a fast control system to stabilize the camera and then have the forward kinematics algorithm tell where the camera is pointed (See Figure 2.13).

- positioning devices (6 degrees of freedom joysticks), and
- inspection systems mounted in Stewart platforms that could be used to touch the part to inspect it.

There are very few examples where the Stewart platform was used for any of these purposes in industry, although the uses of parallel manipulators for sensing are very common in nature (i.e. our necks and eyes). We can conjecture that these structural solutions did not emerge in industry because the previous forward kinematics algorithms were not fast enough to allow for real time operation of these devices.

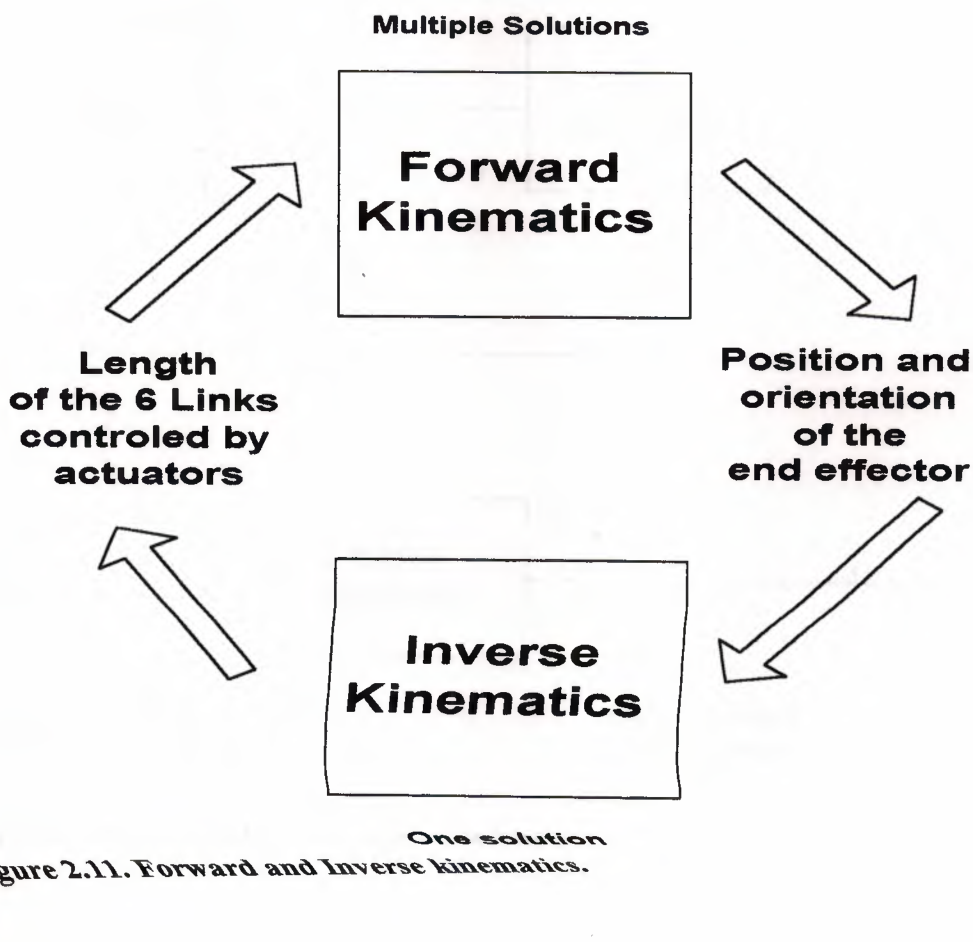


Figure 2.11. Forward and Inverse kinematics.

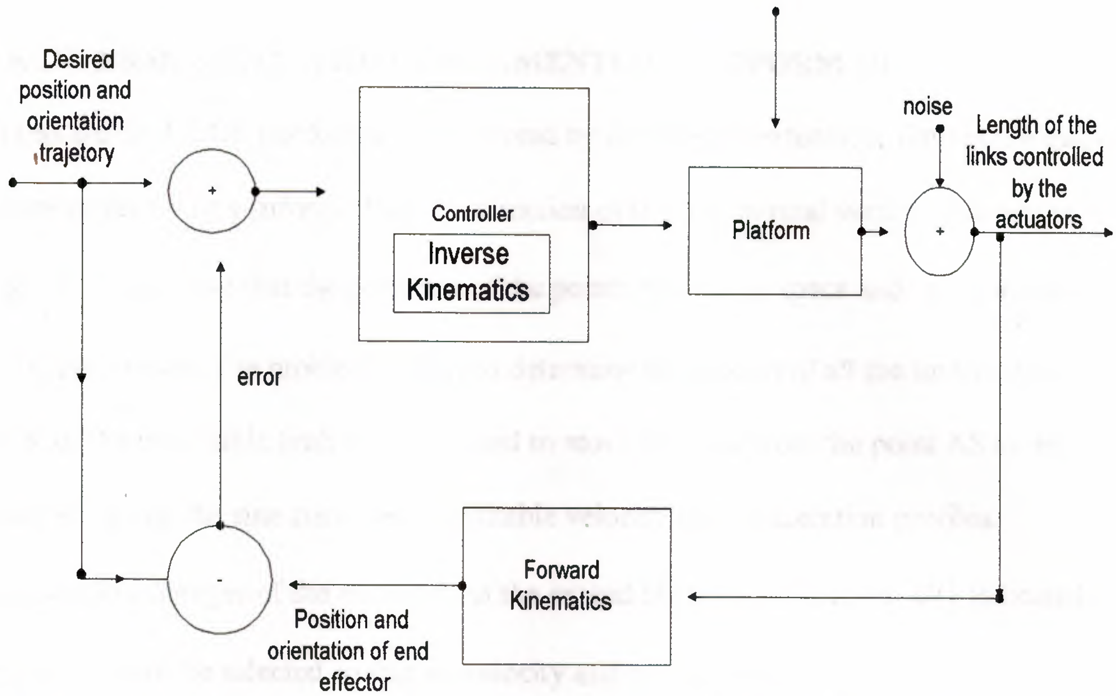


Figure 2.12. An example of using transformations of forward and inverse kinematics for control purposes.

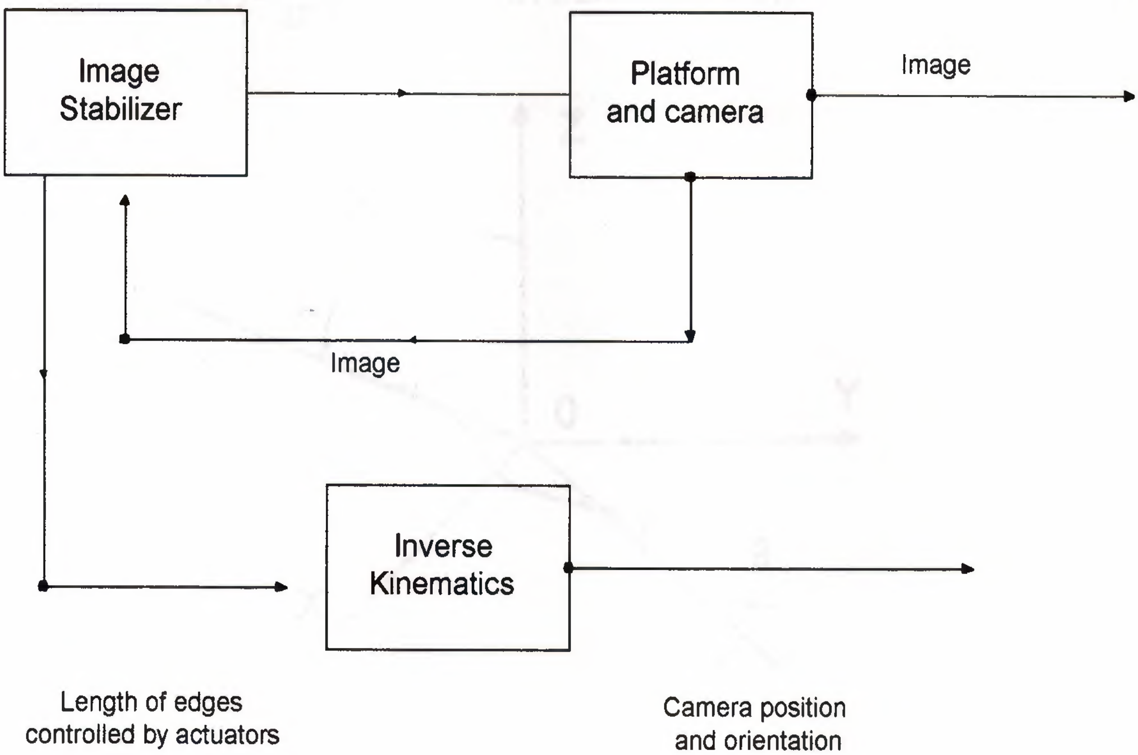


Figure 2.13. An example of using inverse kinematic for measuring purposes.

2.6. INVERSE KINEMATIC OF AUGMENTED PLATFORM [3]

In this article 3-DOF platforms is augmented by locating an extendible limb at the mass centre of the upper platform along the direction of the unit normal vector as shown in Fig. 10. We assume that the positions of the points AS, AL in space and z_G component in (3) are known. The problem here is to determine the motion of all the limbs when the tip K *of the extendible limb is constrained to move in space from the point AS to the* point AL along the sine curve with a suitable velocity and acceleration profiles.

For position changes of the end point of the extendible limb, a function $\lambda(t)$ indicated in Fig.15 must be selected so that its velocity and acceleration become zero at the beginning and at the end of the motion. To satisfy these conditions one way of selecting

$$\lambda(t) = \frac{1}{2\Pi}(t - \sin t) \qquad 0 \le t \le 2\Pi \qquad (0 \le \lambda \le 1) \tag{11}$$

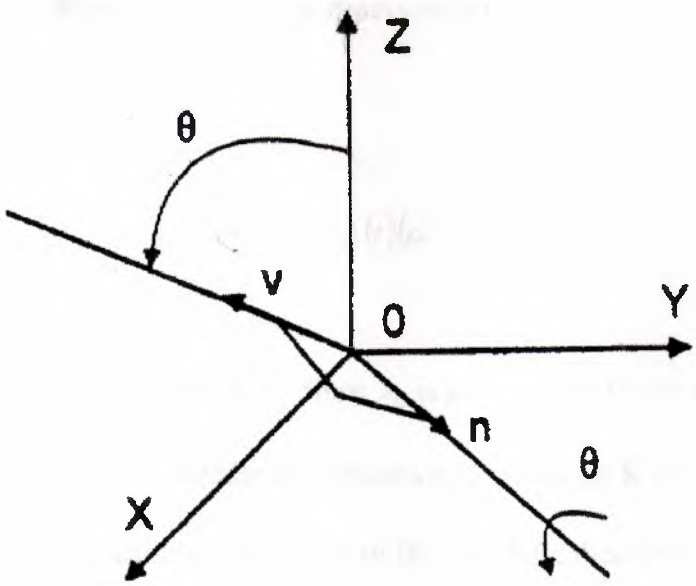


Figure 2.14. Interpretation of the transformation

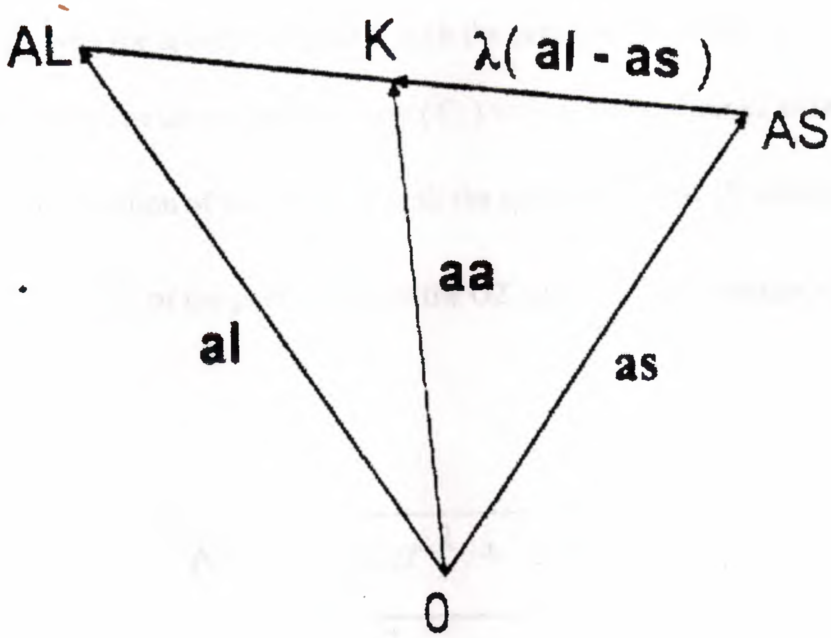


Figure 2.15. Vector representation for the path

From Fig. 2.15 one can write

$$aa = as + \lambda(t)(al - as) \tag{12}$$

To determine the limb lengths l_1, l_2, l_3 Since z_G is given, point G lies an the horizontal plane Π shown in Fig.17. Consider the projection K_1 of the tip K on Π flat a definite time. If L indicates the undermined length of the limb KG, then G is further constrained to lie on a circle (C_1) in H of radius

$$r_1 = (aa_z - z_G)\tan(\theta) = L\sin(\theta) \tag{13}$$

With center at K_1. In this expression θ, due to the **property** mentioned at the end of the previous section, is the same as the rotation angle. (C_1) Is actually is the intersection of the plane Π with the sphere of radius L with the center at K. On the other hand G may also be considered to lie on another circle (C_2) with an undetermined radius r_0. In fact (C_2) is the intersection of the plane Π with the sphere of radius $|\xi|$ centred at 0. For the distance $R_0 = |0_1 K_1|$ of the point K_1, from the OZ axis and for the radius r_0 we have the expressions

$$\left.\begin{aligned} R_0 &= \sqrt{aa_x^2 + aa_y^2} \\ r_0 &= \sqrt{x_G^2 + y_G^2} \end{aligned}\right\} \qquad (14)$$

Since both the circles (C_1) and (C_2) are lying in Π, their intersection points determine the location of G. In order to obtain a unique solution we shall consider the case where the circles are tangent to each other. However to keep G as close as possible to the OZ axis we consider the case indicated in Fig.4. Depending upon whether $R_0 - r_1$ is positive or negative we have $\overline{0_1 G} = k\overline{0_1 K_1}$, and from (12) and (13) we write

$$k = \frac{R_0 - r_1}{R_0} = 1 - \frac{(aa_z - z_G)}{\sqrt{aa_x^2 + aa_y^2}} \qquad (15)$$

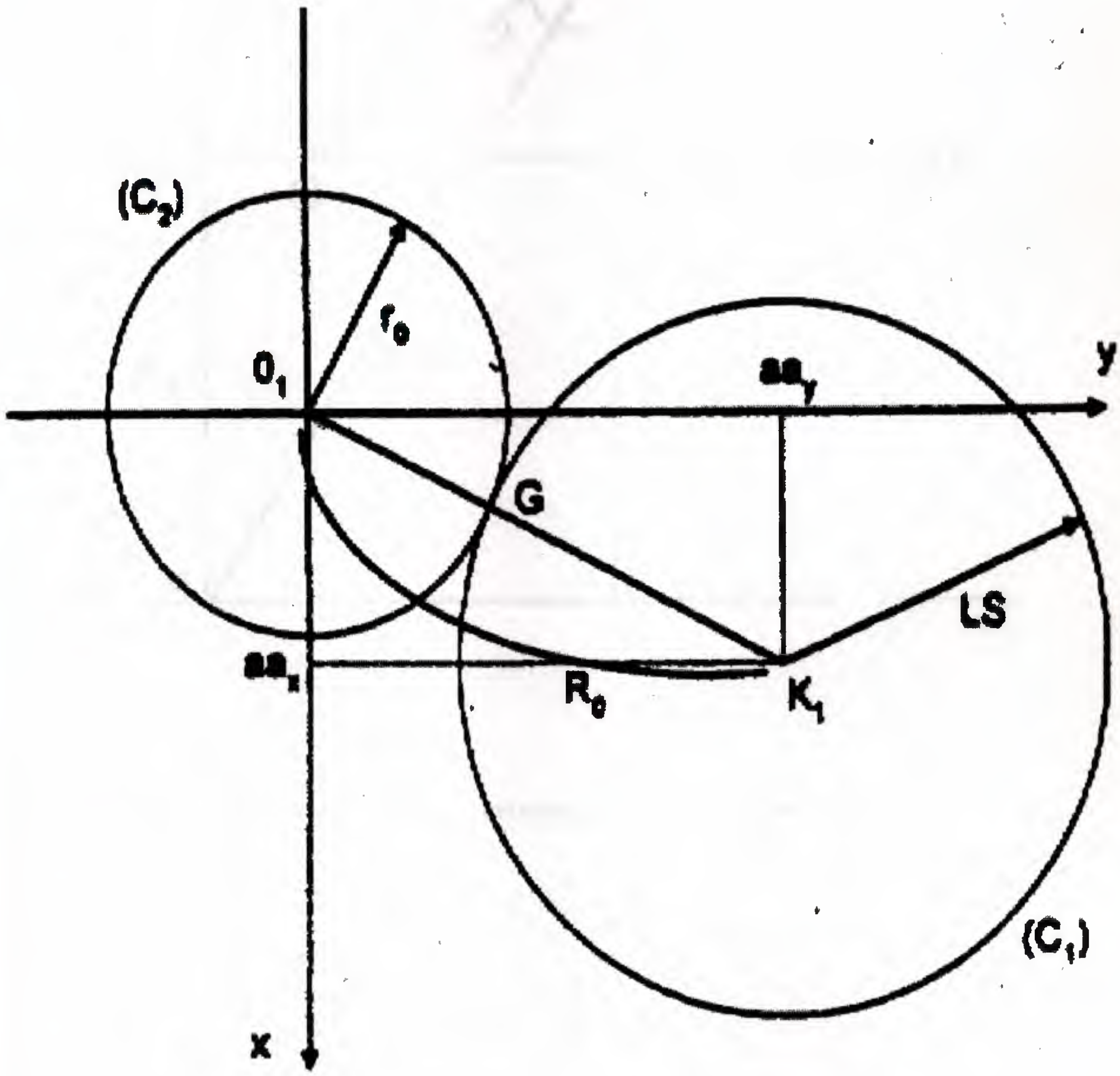


Fig.2.16: Geometry for the location of G in H plane

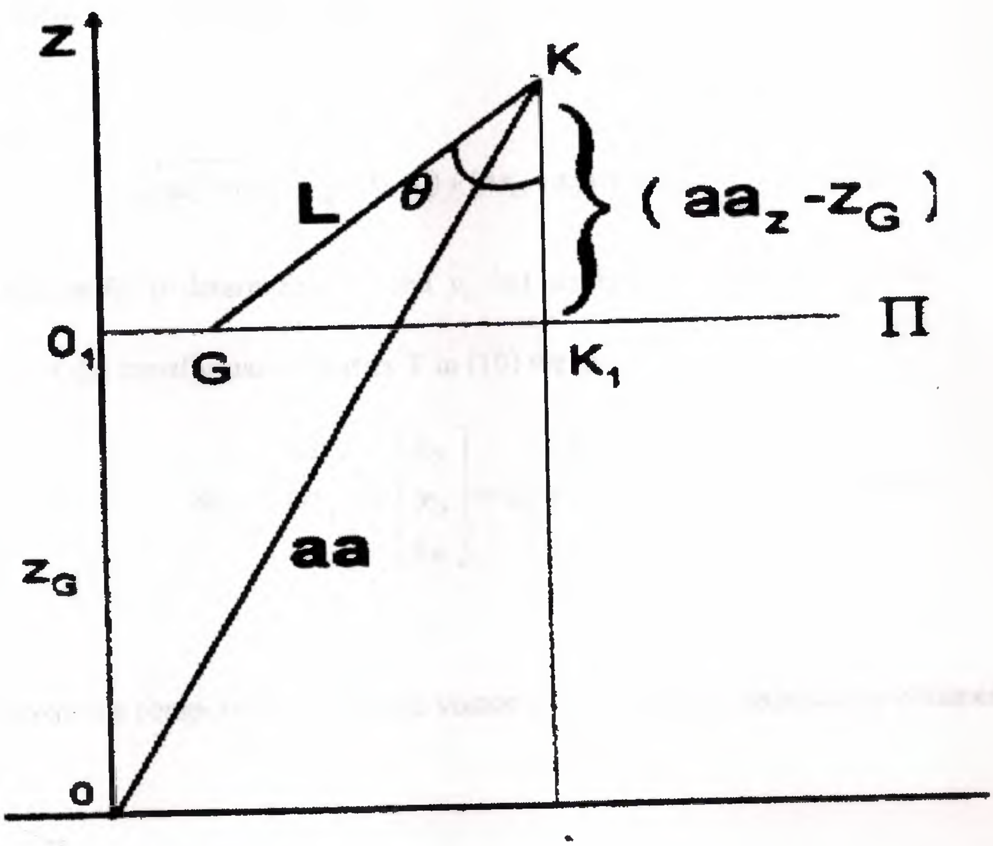


Fig.2.17. Vector representation on a vertical plane and rotation angle

Therefore:

$$\left.\begin{aligned} x_G &= kaa_x \\ y_G &= kaa_y \end{aligned}\right\} \qquad (16)$$

In these expressions θ is the only unknown parameter to be determined on the other hand from the constraint equations in (9), after squaring and adding the first two equations and considering the fact that $n_1^2 + n_2^2 = 1$, we have 17)

$$r_0 = \frac{1}{2} r(1 - C) \qquad (17)$$

Then the relation $r_0 = R_0 - r_1$ becomes:

$$\sqrt{aa_x^2 + aa_y^2} = \frac{1}{2} r(1-C) + (aa_z - z_G)\tan(\theta) \qquad (18)$$

And its solution for θ determines x_G and y_G in (16). From Fig.17 and from the expression of the transformation matrix **T** in (10) we has

$$aa = \begin{bmatrix} aa_x \\ aa_y \\ aa_z \end{bmatrix} = \begin{bmatrix} x_G \\ y_G \\ z_G \end{bmatrix} + L \begin{bmatrix} Sn_2 \\ -Sn_1 \\ C \end{bmatrix} \qquad (19)$$

and the unknown components of the unit vector **n** on the rotation axis can be obtained as:

$$n_2 = \frac{aa_x - x_G}{LS}$$
$$n_1 = \frac{y_G - aa_y}{LS} \qquad (20)$$

Note that the unit normal vector v is identical to t3 in (10)

$$v = [Sn_2 \quad -Sn_1 \quad C]^T \qquad (21)$$

With these information **T** is calculated from (10) or (7) and a, b, c from (9), all in turn give the limb lengths as

$$l_1 = |a - q_1| \quad , \quad l_2 = |b - q_2|$$
$$l_3 = |c - q_3| \qquad (22)$$

$$l_1' = \frac{l_1(t+\Delta t) - l_1(t)}{\Delta t} \qquad (23)$$

Similarly acceleration of the limb lengths can also be obtained. A written Matlab program given in appendix A calculates all these expressions. A close study yields that the results for the example considered in [3] are displayed in the second row of Appendix B [3], where the speed and acceleration of the legs are not quite zero at the beginning and end of the motion. This situation may be improved if the expression of λ. in (11) is used twice:

$$\lambda(lt) = \frac{1}{2\Pi}\left(\left[\frac{2\Pi}{Tt}lt - \sin(\frac{2\Pi}{Tt}lt)\right] - \sin\left[\frac{2\Pi}{Tt}lt - \sin(\frac{2\Pi}{Tt}lt\right]\right) \qquad (24)$$

Where **Tt** is the duration of time (It). The results for the same example corresponding to this new choice of λ.

The main aim in this project is to determine the motion of all the limbs when the tip K of the extendible limb is constrained to move in space from the point AS to the point AL along a any function with a suitable velocity and acceleration, and we assumed that the function will be an sine curve. The procedure that we are going to follow it is to divide the sine curve to equal small straight lines by dividing the curve to equal small intervals each interval started at AS and finishing at AL as shown in Fig. 2.18 so we will use the same calculations used in the last problem in [3], and the matlab program shown in appendix A1 will call the terminal point Al in the first interval to be AS to calculate the next interval by using two for loops.

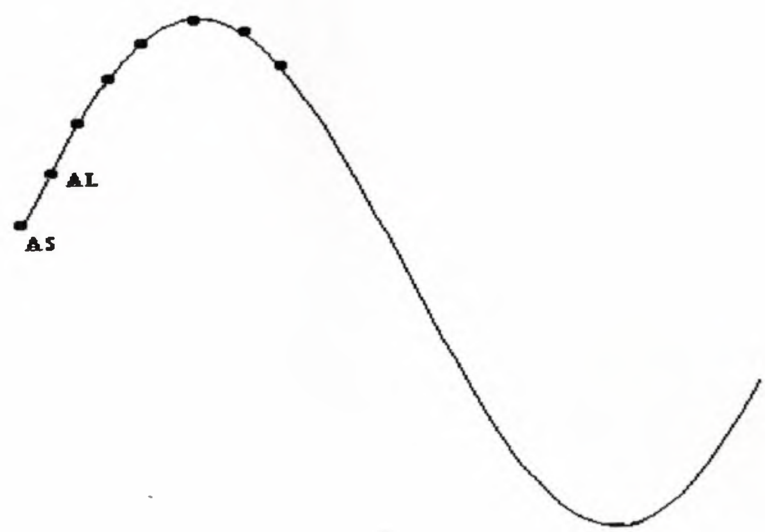


Figure 2.18. The sine curve.

Example 2.1:

For the above prcedure the MATLAB program in APPENDIX A1 is executed for the following constants.

r=7

R=20

as= [1 sin(1* pi /180) 24]

al= [2*pi sin(2*pi) 24]

functiomn is = sin (teta)

Ks(3) = 10

2.7. The Results of the MATLAB program:

Results of the MATLAB program are presented in the following figures in Figure 2.11 to Figure 2.13 you can see the length of the limbs .

From figure 2.14 to figure 2.16 the velocity of the limbs lengths are shown and the expected results are obtained in these figures, which is at the beginning and the end of the motion, Zero velocity are obtained .

Also from figure 2.17 to figure 2.19 the accelaration of the limbs lingths are presented and again expected results are obtained.

Finally in figure 3.20 to figure 2.22 you can see the velocity and the accelaration of the extendible limb. All therse results are are expected results.

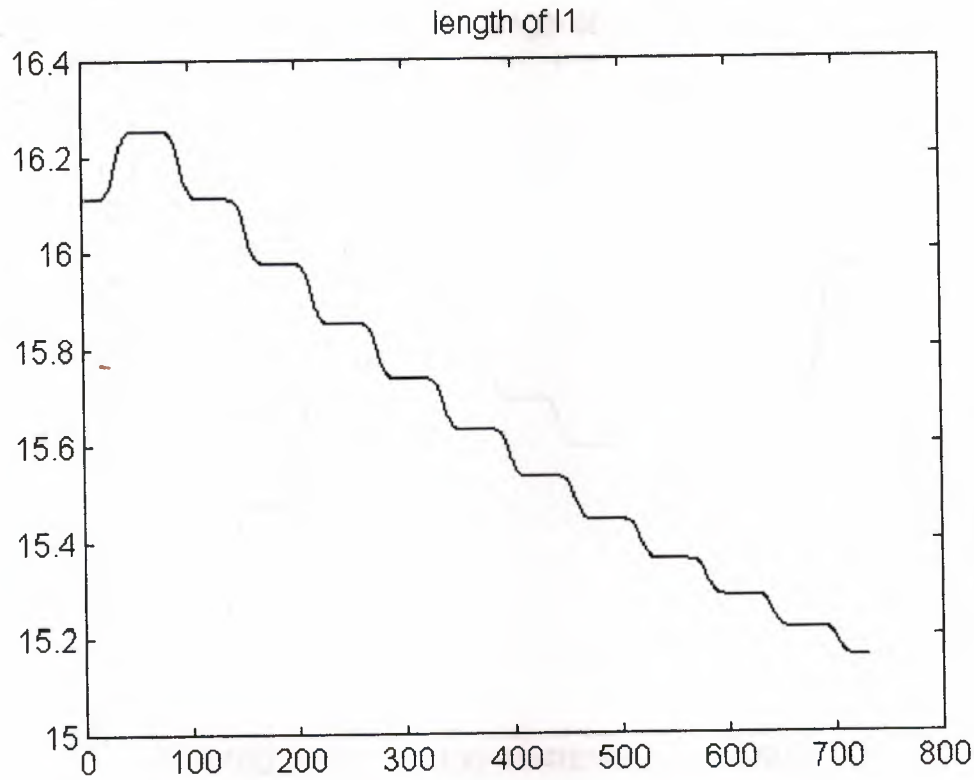


Figure 2.11. The length of the first limb (l1).

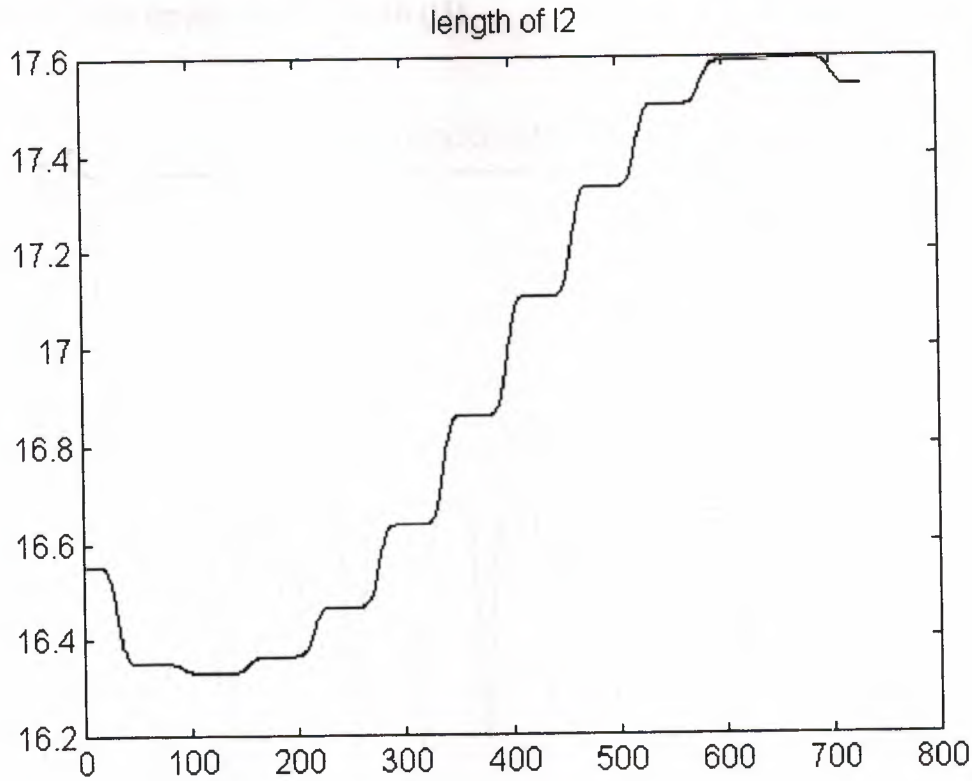


Figure 2.12. The length of the second limb (l2).

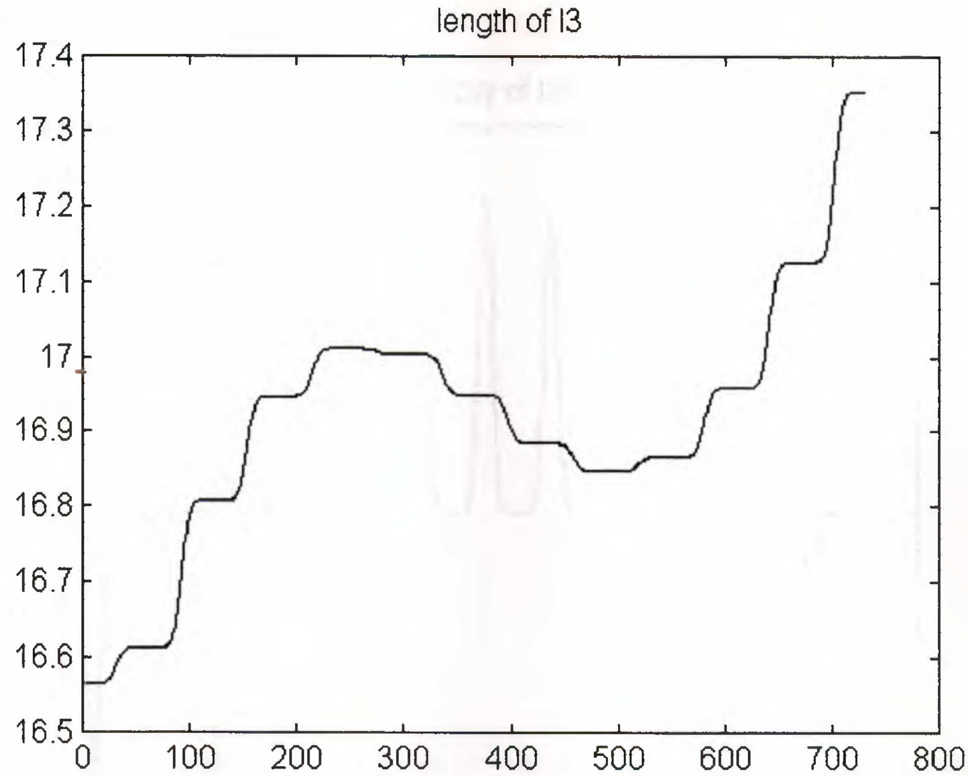


Figure 2.13. The length of third limb (l3).

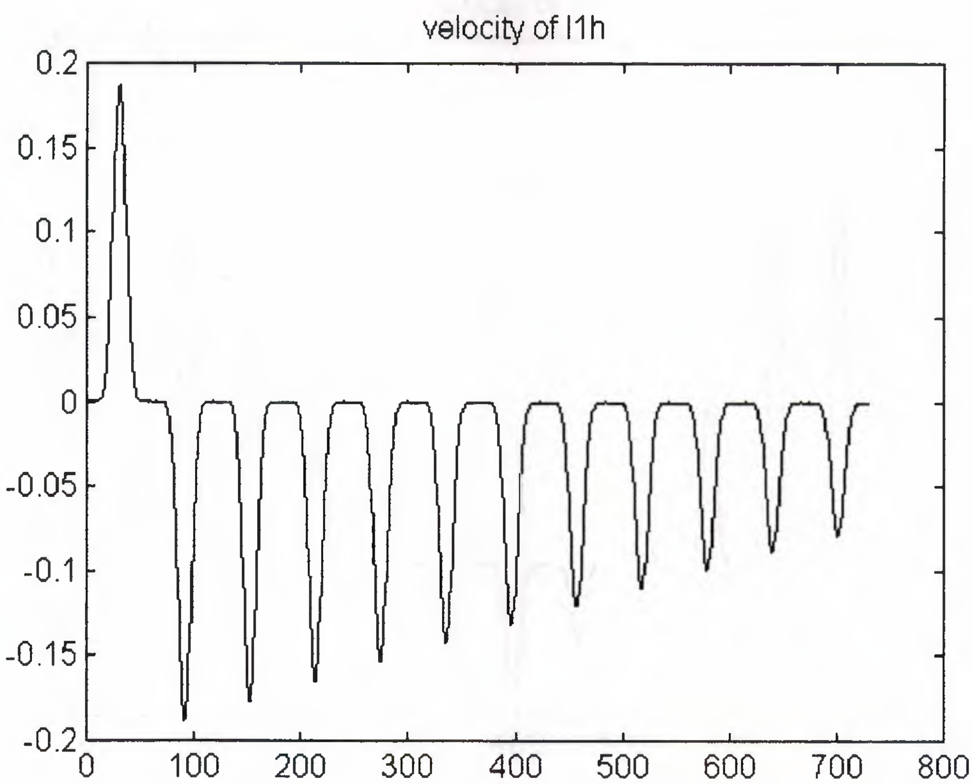


Figure 2.14. The velocity of the first limb (l1h) .

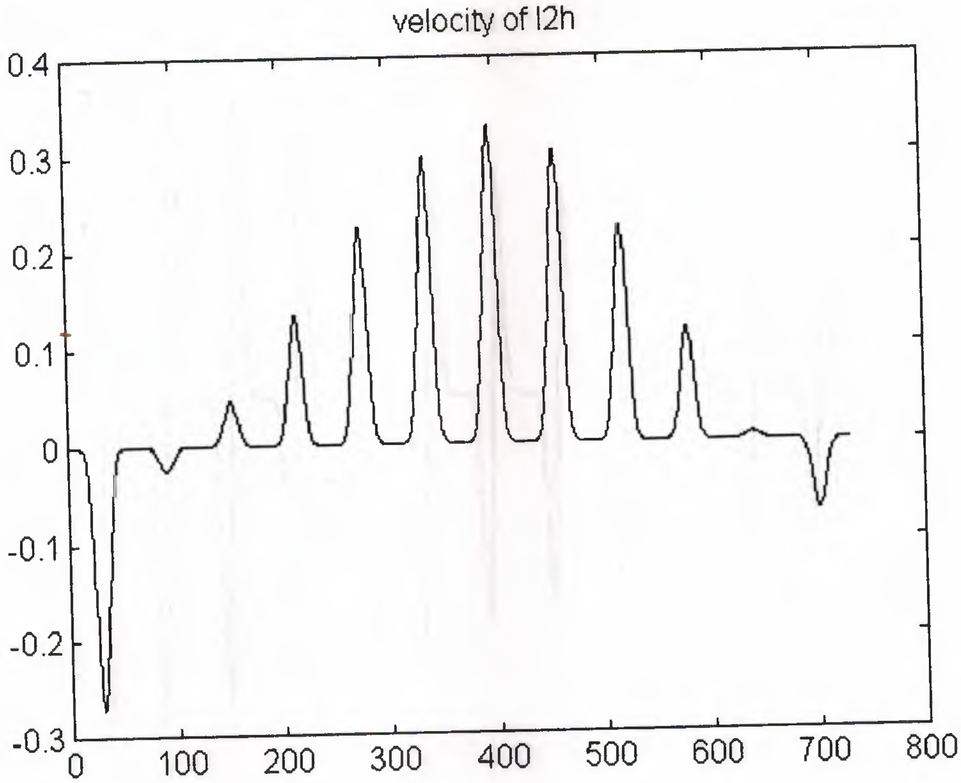


Figure 2.15 the velocity of the second limb (l2h).

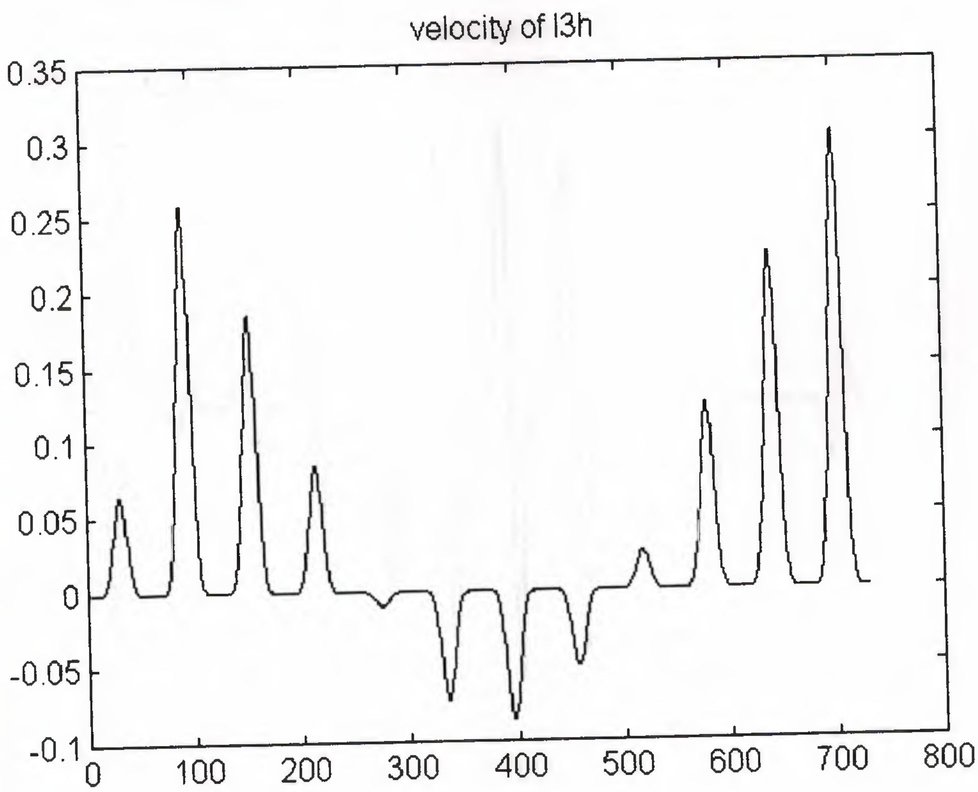


Figure 2.16. The velocity of third limb (l3h).

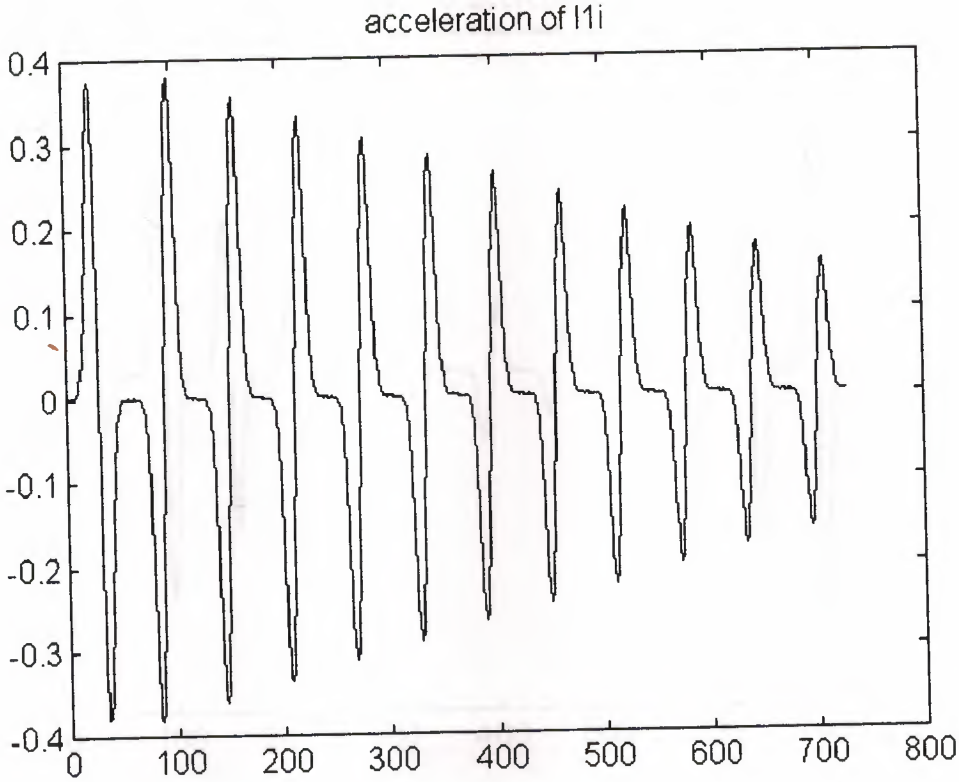


Figure 2.17. The acceleration of the first limb (l1i).

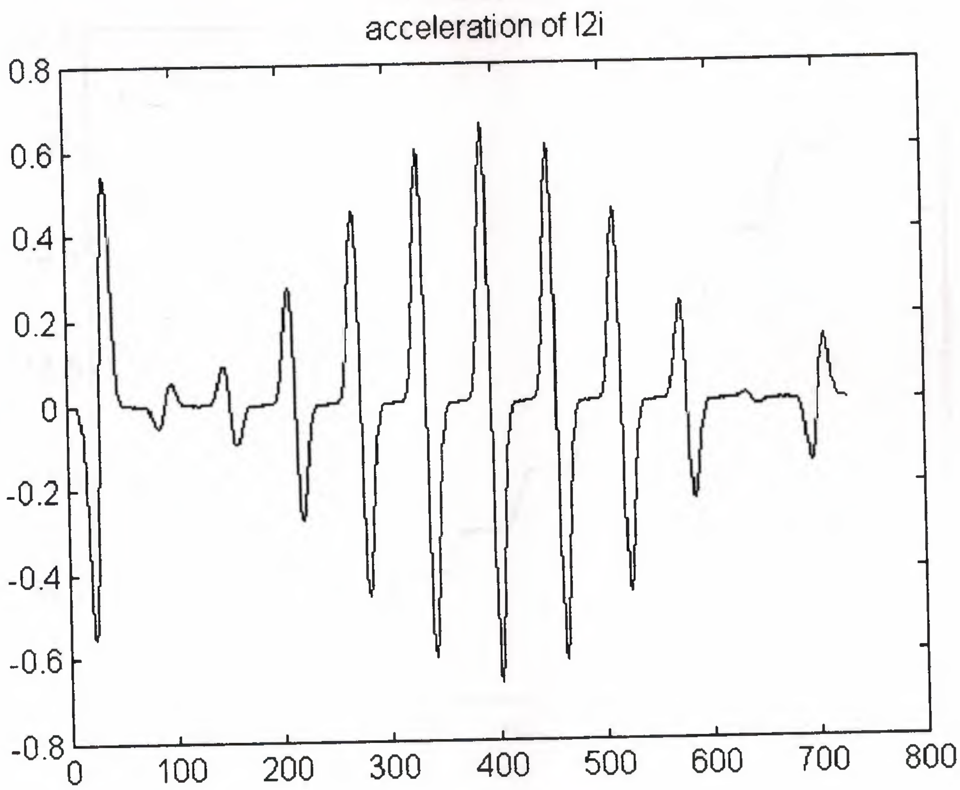


Figure 2.18. The acceleration of the second limb (l2i).

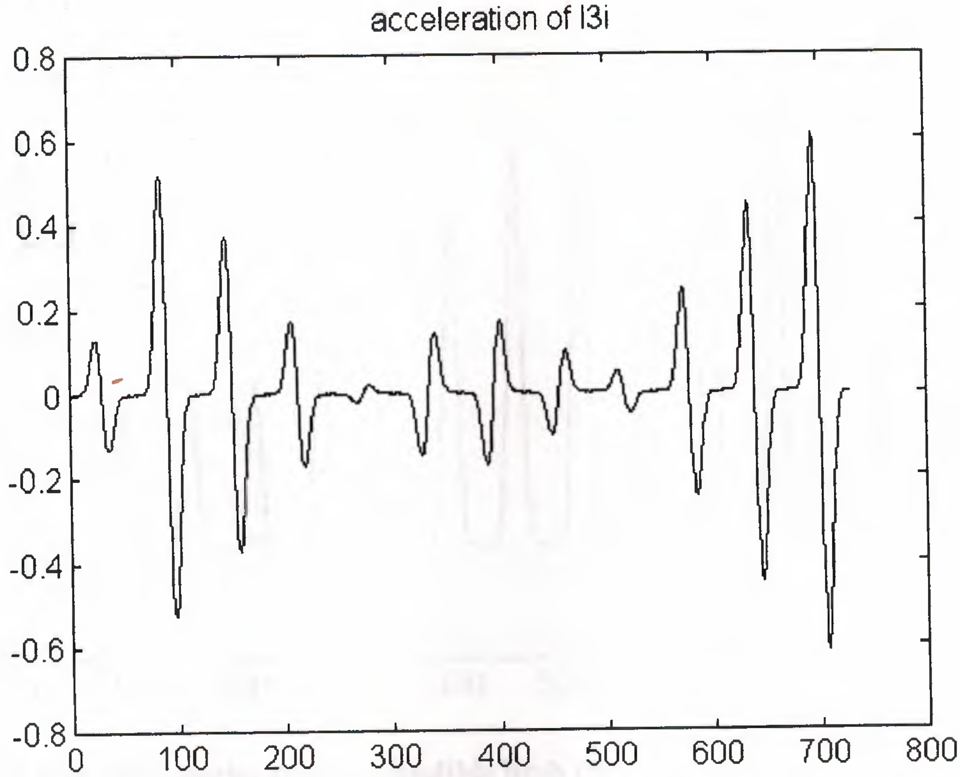


Figure 2.19. The acceleration of the third limb (l3i).

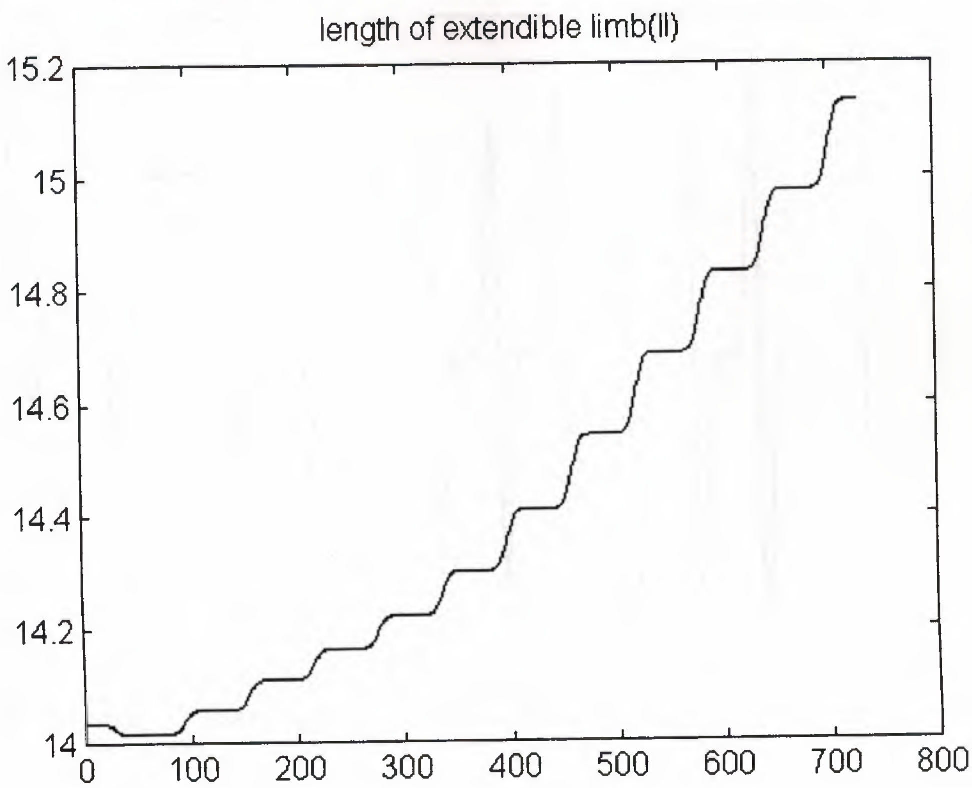


Figure 2.20. The length of the extendible limb (ll).

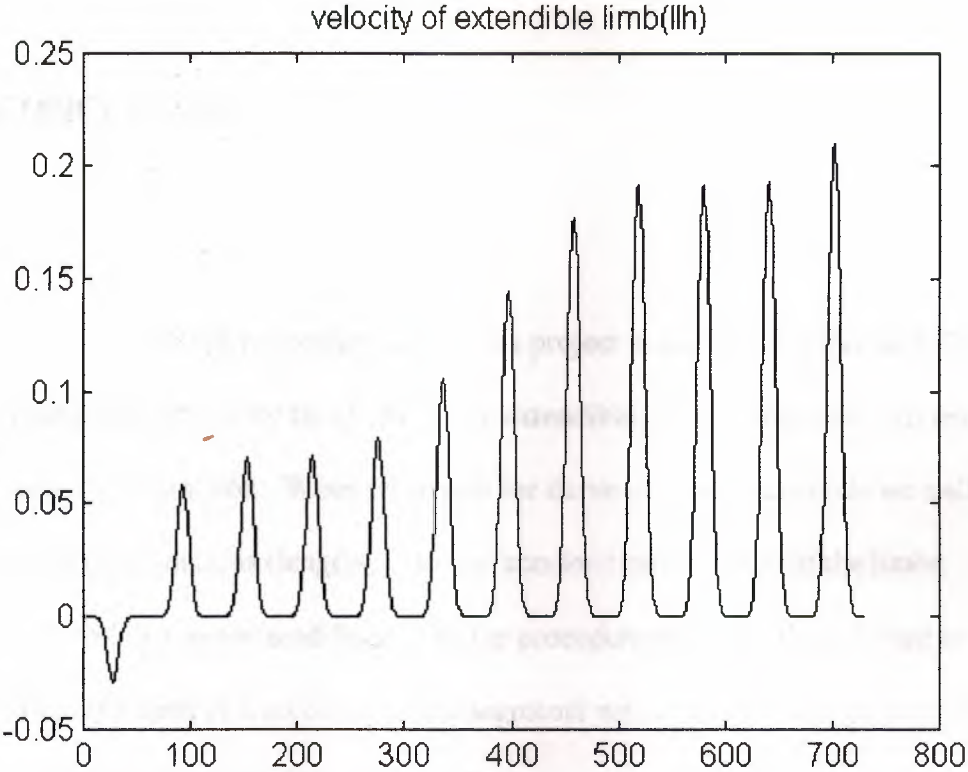


Figure 2.21. the velocity of the extendible limb (ll) .

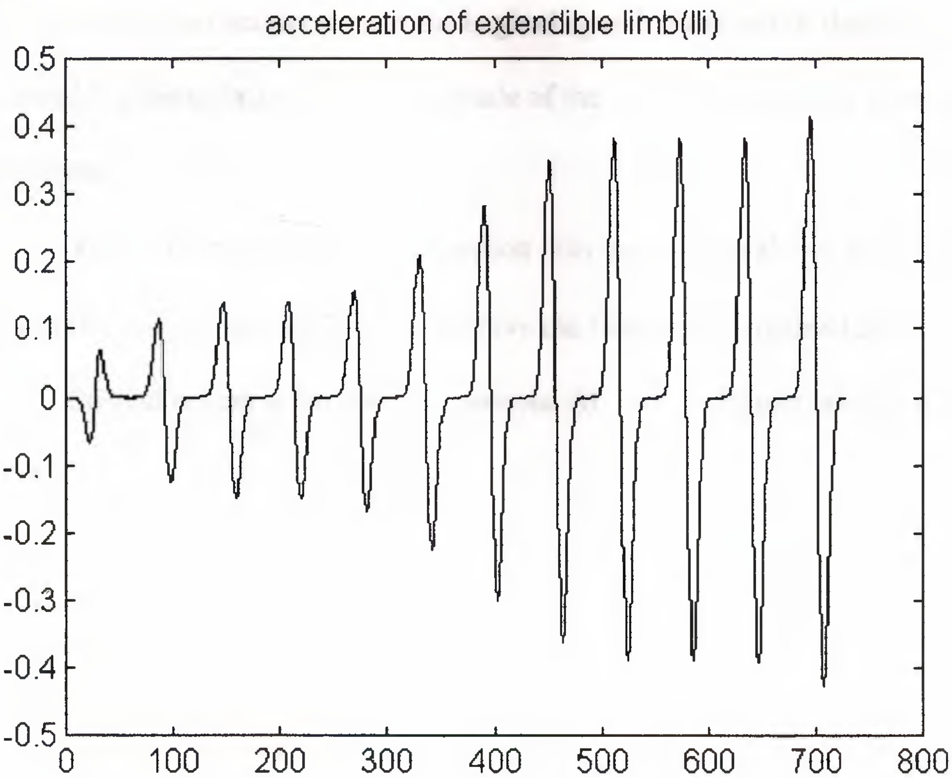


Figure 2. 22. The acceleration of the extindible limb (ll).

CONCLUSION

The procedure used in this project is applied to a special 4-Dof Manipulator where the tip of one of the extendible limbs is restrictied to move along a sine function . When we divide the curve to smaller intervals we will get more accurate calculations (lengths, velocity, acceleration) for each of the limbs.

With a minor modification to the procedure in [3] the tip is forced to follow a path in the form of a set of short line segments which may be thought as the pice wise linear approximation of the space curve.

Studing the figures in example 2.1 it can be seen that the restriction about having a zero velocity and accelaration at the beginning and at the end of the motion is obtained but the osilation in the magnitude of the velocity and accelaration graphs have two resons.

One is because of the $\lambda(t)$ function is in every interval, but this could be used only in the first and last intervals to acchive the restriction explained above.

Second reason is the distance between Al – As in all intervals because of the curve.

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APPENDIX A1

Main MATLAB program (f.m)

Fe400.m

```
%f.m
lear
p=0;
=0;
a=0;
a=0;
s=0;
q=0;
Xs(3)=10;

=7;
R=20;
1=[R;0;0];
2=[-R/2;(3^0.5/2)*R;0];
3=[-R/2;-(3^0.5/2)*R;0];
1=[r;0;0];
2=[-r/2;(3^0.5/2)*r;0];
3=[-r/2;-(3^0.5/2)*r;0];
AA1=sin(1*pi/180);
BB1=sin(pi);
dtteta=0.5;
s=[1 AA1 24];
z=0;
or TTETA=dtteta:dtteta:2*pi
ly=sin(TTETA);
l=[TTETA aly 24];
z=kz+1;
aman(kz)=TTETA;
```

```
Tt=3;
C=2*pi/Tt;
dt=dtteta/10;
for lt=0:dt:3;
 j=j+1;
 z(j)=0;
 ltt(j)=lt;
 zg(j)=Ks(3);
 A=sin(C*lt);
 B=cos(C*lt);
 D=C*lt-A;
 landa=(1/(2*pi))*(D-sin(D));
 aa=as+landa*(al-as);
 hh=((al-as)*(al-as)')^0.5;
 hiz(j)=(1/Tt)*(1-B)*(1-cos(D))*hh(1);
 ivme(j)=(2*pi)/((Tt)^2)*(A-A*cos(D)+(1-B)^2*sin(D))*hh(1);
 ww(j)=aa(1)/aa(2);
 aax(j)=aa(1);
 aay(j)=aa(2);
 aaz(j)=aa(3);
 f11
 ll(j)=(aa(3)-zg(j))/ct;
 if j>1;
  sa=sa+1;
  ltth(sa)=lt;
  llh(sa)=(ll(j)-ll(j-1))/dt;
  if sa>1;
   qa=qa+1;
   ltti(qa)=lt;
   lli(qa)=(llh(sa)-llh(sa-1))/dt;
  end;
 end;
```

```
K=(1/(2*aa(2)))*((r/2)*(1-ct))^2+(aa(1)^2+aa(2)^2)/(2*aa(2))-(aa(3)-
s(3))^2/(2*aa(2))*(st/ct)^2;
KK(j)=K;
ca=(aa(1)^2+aa(2)^2)/(aa(2)^2);
cd=(aa(1)/aa(2))*K;
x(j)=cd/ca;
y(j)=-(aa(1)/aa(2))*x(j)+K;
yy(j)=-ww(j)*x(j)+KK(j);
zzz=(ll(j)*st);
n2(j)=-(x(j)-aa(1))/zzz;
n1(j)=(y(j)-aa(2))/zzz;
n=[n1(j) n2(j) 0];
v=[st*n2(j) -st*n1(j) ct];
vx(j)=v(1);
vy(j)=v(2);
vz(j)=v(3);
Ks=[x(j);y(j);Ks(3)];
T=[ct+(1-ct)*n1(j)^2 (1-ct)*n1(j)*n2(j) v(1);(1-ct)*n1(j)*n2(j) ct+(1-ct)*n2(j)^2
(2);-v(1) -v(2) ct];
a=T*p1+Ks;
b=T*p2+Ks;
c=T*p3+Ks;
l11=-q1+a;
l22=-q2+b;
l33=-q3+c;
l1s=(l11'*l11);
l1(j)=l1s(1)^0.5;
l2s=(l22'*l22);
l2(j)=l2s(1)^0.5;
l3s=(l33'*l33);
l3(j)=l3s(1)^0.5;
if j>1;
   ss=ss+1;
```

```
(ss)=lt;
(ss)=(l1(j)-l1(j-1))/dt;
(ss)=(l2(j)-l2(j-1))/dt;
(ss)=(l3(j)-l3(j-1))/dt;
s>1;
q=qq+1;
ti(qq)=lt;
1i(qq)=(l1h(ss)-l1h(ss-1))/dt;
2i(qq)=(l2h(ss)-l2h(ss-1))/dt;
3i(qq)=(l3h(ss)-l3h(ss-1))/dt;
d;
```

rogram (f11.m)

```
.m
t long

);

 abs(ub)>0.0000001
teb;
tea;

s(teb*pi/180);
-ct^2)^0.5;
```

program (f12.m)

```
f12.m
=0;
=(t2-t1)/10;
r te=t1:ti:t2;
 i=i+1;
 t(i)=te;
 teta=te*pi/180;
 Ro=(aa(1)^2+aa(2)^2)^0.5;
 u(i)=Ro-(r/2*(1-cos(teta))+(aa(3)-zg(j))*tan(teta));
 if i>1;
   if sign(u(i))~=sign(u(i-1));
     tea=t(i);
     teb=t(i-1);
     ua=u(i);
     ub=u(i-1);
   end;
 end;
end;
```