

NEAR EAST UNIVERSITY



**GRADUATE SCHOOL OF APPLIED
SCIENCES**

**INCREASING POWER SYSTEM LINE SECURITY
BY USING LOAD MANGEMENT AND
SENSITIVITY ANALYSIS**

Saffet AŞIKSOY

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Department of Electrical and Electronic Engineering

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ABSTRACT

In any power system today line flows play an integral part. System security is very much dependent on all the lines being healthy and able to transmit power from generating stations all the way to the consumers. As the conditions in the system changes the line flows also change and may lead to some lines being overloaded. One of the most important things that affect the line flows is the change in loading. Each load in the system has a different effect on each one of the line flows. The sensitivity of each line to each load can be calculated using power transfer distribution factors (PTDF). Sensitivity analysis of line flows to each load can help alleviate the line overflow problem. Once the most sensitive loads are determined, demand management techniques can be used to increase system security. In order to solve the congestion in power transmission lines, detecting the loads which cause congestion, load reduction methods can be used instead of expanding the infrastructure of lines.

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1 INTRODUCTION

In order to solve the congestion in power transmission lines detecting the loads which cause congestion becomes critical. Load reduction methods can be used as well as expanding the infrastructure of lines. Most effective loads to reduce congestion of lines can be found by doing a sensitivity analysis on the power system. In this thesis, the sensitivity of all loads to the line flows are examined. Distribution Power Transfer Factors (PTDF) help in this kind of analysis. As the power systems are usually very big, calculation of line flows requires the use of computers. In this thesis, a tool is developed by using MATLAB for doing power flow analysis and finding sensitivities of all loads to all lines. The program can get the data of a system from an IEEE common format file and do the analysis. All the unknown system parameters, all the line flows and the PTDF matrix can be calculated. This analysis helps in deciding how to use load management to reduce line congestion. Reducing congestion in lines will create a healthier power system.

In Chapter 2 a background analysis is done on the power flow analysis and different methods of solving the power flow problem is explained. These methods help in calculating the line flows in the system.

Chapter 3 explains how PTDF's are calculated and used. Once the tools are in place to analyze the system, Chapter 4 talks about how to achieve line security with demand (load) management.

Simulation results are summarized in Chapter 5, and then the conclusions are given after that.

2 BACKGROUND

2.1 Power Flow Analysis

Load flow is an important power system analysis component to ensure the system upgrades, future upgrades and present distribution equipment meeting the present and future requirements.

Load flow study in power system is the steady state solution of power system network. The information obtained from load flow solution is used for the continuous monitoring of current state of the system and for analyzing the effectiveness of future system expansion to meet increased load demand.

The main objective of the load flow is to find the voltage magnitude of each bus and its angle when the powers generated and loads are specified. Once the bus voltage magnitudes and their angles are computed using the load flow, the real and reactive power flow through each line can be computed.

AC power transmission systems are comprised of three phase circuits. However, under balanced conditions (the currents in all three phases are equal in magnitude and phase separated by 120°), we may analyze the three phase system using a per-phase equivalent circuit consisting of the a-phase and the neutral conductor. Per-unitization of a per-phase equivalent of a three phase, balanced system results in the per-unit circuit. It is the per-unitized, per-phase equivalent circuit of the power system that we use to formulate and solve the power flow problem. It is convenient to represent power system networks using the one-line diagram [1], which can be thought of as the circuit diagram of the per-phase equivalent. To show the power flow studies, consider the system with one line diagram shown in Figure 2.1

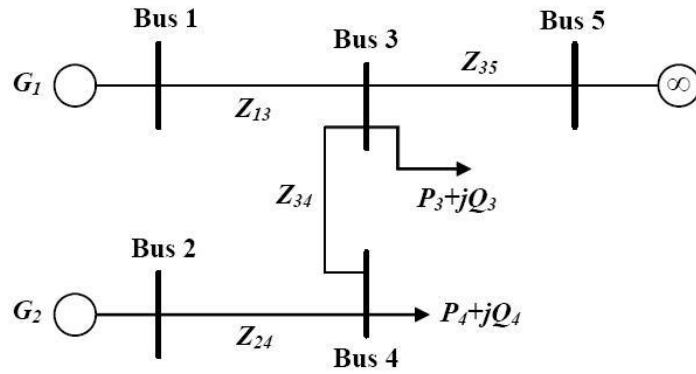


Fig 2.1 One Line Diagram

In this example system, the buses can be categorized as follows:

Buses 1 and 2 are categorized as PV buses since their power (P) and voltage (V) magnitudes can be specified. Most generator buses fall into this category, independent of whether it also has load; exceptions are buses that have reactive power injection. There are also special cases where a non-generator bus (i.e., either a bus with load or a bus with neither generation nor load) may be classified as type PV, and some examples of these special cases are buses having switched shunt capacitors or static var systems (SVCs).

Buses 3 and 4 are all type PQ buses. For these buses we know P and Q but not $|V|$ or angle $^\circ$. All load buses fall into this category, including buses that have neither load nor generation. The real power injections of the type PQ buses are chosen according to the loading conditions being modeled. The reactive power injections of the type PQ buses are chosen according to the expected power factor of the load.

Bus 5 can be referred to as the slack bus. Two other common terms for the slack bus type are swing bus and reference bus. In general, there is only one slack bus, and it can be designated by the engineer to be any generator bus in the system. For the slack bus, we know $|V|$ and $^\circ$. The fact that we know $^\circ$ is the reason why it is sometimes called the reference bus.

The generation must supply both the load and the losses on the circuits. Before solving the power flow problem, we will know all injections at PQ buses, but we will not know what the losses will be as losses are a function of the flows on the circuits which are yet to be computed. So we may set the real power injections for, at most, all but one of the generators.

The one generator for which we do not set the real power injection is the one modeled at the slack bus. Thus, this generator “swings” to compensate for the network losses. Therefore, rather than call this generator a $|V|$ -Delta, we choose the terminology “swing” or “slack” as it helps us to better remember its function. The voltage magnitude of the slack bus is chosen to correspond to the typical voltage setting of this generator. The voltage angle may be designated to be any angle, but normally it is designated as 0° . Because the real power injection of the slack bus is not set by the engineer but rather is an output of the power flow solution, it can take on mathematically tractable but physically impossible values. Therefore, the engineer must always check the slack bus generation level following a solution to ensure that it is within the physical limitations of the generator.

2.1.1. Classification of Buses

- Load Buses: In these buses no generators are connected and hence the generated real power P_{G_i} and reactive power Q_{G_i} are taken as zero.
- Voltage Controlled Buses: These are the buses where generators are connected.
- Slack or Swing Buses: Usually this bus is numbered 1 for the load flow studies. This bus sets the angular reference for all the other buses.

2.1.2 Bus Admittance Matrix (Y Bus)

First step in solving the power flow is to create what is known as the bus admittance matrix, often call the Ybus.

The bus admittance matrix Ybus is an important network description of the power system. In the bus admittance matrix representation, current injections at a bus are analogous to power injections. Current injections may be either positive (into the bus) or negative (out of the bus). Unlike current flowing through a branch is a branch quantity, a current injection is a nodal quantity. The admittance matrix, a fundamental network analysis tool that we shall use heavily, relates current injections at a bus to the bus voltages. Thus, the admittance matrix relates nodal quantities.

If we have n Buses then Ybus will be $n \times n$ matrix. The bus admittance matrix Y is then formed by inspection as follows :

The diagonal elements of $n \times n$ matrix will be summation of admittances connected to that bus. (Y_{ij} -where $i=j$)

The of diagonal elements of $n \times n$ matrix will be sum of admittances connected from bus i to bus j but it is negated. (Y_{ij})

Formation of Ybus plays a vital role in solving any load flow problem. With large systems Ybus is a sparse matrix (that is, most entries are zero)

2.1.3 Formulation of Power Flow Equations

In a large network we can describe current/voltage relations as follows;

$$\begin{bmatrix} \vec{I}_1 \\ \vdots \\ \vec{I}_n \end{bmatrix} = \begin{bmatrix} Y_{1,1} & \cdots & Y_{1,n} \\ \vdots & \ddots & \vdots \\ Y_{n,1} & \cdots & Y_{n,n} \end{bmatrix} \cdot \begin{bmatrix} \vec{V}_1 \\ \vdots \\ \vec{V}_n \end{bmatrix} \quad (2.1)$$

Instead of \vec{I} we can re-arrange the above formulation and find the complex power \mathbf{S} where;

$$S = P + jQ \quad (2.2)$$

$$S = \begin{bmatrix} V_1 & 0 & \cdots & 0 & 0 \\ 0 & V_2 & 0 & \cdots & 0 \\ \vdots & 0 & V_3 & 0 & \vdots \\ 0 & \vdots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & 0 & V_n \end{bmatrix} \cdot \vec{I}^* \quad (2.3)$$

If we use (2.1) at (2.3), we can write complex power as;

$$S = \begin{bmatrix} V_1 & 0 & \cdots & 0 & 0 \\ 0 & V_2 & 0 & \cdots & 0 \\ \vdots & 0 & V_3 & 0 & \vdots \\ 0 & \vdots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & 0 & V_n \end{bmatrix} \cdot Y_{bus}^* \cdot \underline{V}^* \quad (2.4)$$

Y_{bus} has two parts which are real and imaginary.

$$Y_{bus} = G_{ik} + jB_{ik} \quad (2.5)$$

G is the real part of Y_{bus} and known as Conductance.

B is the imaginary part of Y_{bus} and known as Susceptance.

and also;

$$\vec{V}_k = |V_k| . e^{j\delta_k} \quad (2.6)$$

So now **S** can be written as;

$$S_i = V_i e^{j\delta_i} \sum_{k=1}^n [(G_{ik} - jB_{ik})] V_k e^{-j\delta_k} \quad (2.7)$$

which is $S_k = P_k + jQ_k$

If we component wise this **S** equation;

The real part is;

$$P_i = Re \left\{ V_i e^{j\delta_i} \sum_{k=1}^n [(G_{ik} - jB_{ik})] V_k e^{-j\delta_k} \right\} \quad (2.8)$$

$$P_i = \sum_{k=1}^n |V_i| |V_k| (G_{ik} \cos(\delta_i - \delta_k) + B_{ik} \sin(\delta_i - \delta_k)) \quad (2.9)$$

and also the imaginary part is;

$$Q_i = \text{Im} \left\{ V_i e^{j\delta_i} \sum_{k=1}^n [(G_{ik} - jB_{ik})] V_k e^{-j\delta_k} \right\} \quad (2.10)$$

$$Q_i = \sum_{k=1}^n |V_i| |V_k| (G_{ik} \sin(\delta_i - \delta_k) - B_{ik} \cos(\delta_i - \delta_k)) \quad (2.11)$$

If we use KCL (Kirchhoff Current Law) for complex power balance at every bus, we will have 2n equations. And we have 4n variables which are P,Q,V and δ for every bus. Some of the variables are known and the others are unknown.

Bus type	Number of buses	Known	Unknown	Number of available equations
Slack i=1	1	$\delta_i, V_i $	P_i, Q_i	0
Generator (i=2,3,...Ng+1)	m-1	$P_i, V_i $	Δ_i, Q_i	m-1
Load (Ng+2,...N)	n-m	P_i, Q_i	Δ_i, V_i	2(n-m)
Total	n	2n	2n	2n-m-1

Figure 2.2

We have $m-1$ P equations for generator bus and $2(n-m)$ P and Q equations for load bus. So we have $2n-m-1$ equations for solving $n-1$ δ (angles) except at bus no 1 (slack bus) and $n-m$ V (voltages) for load buses. To solve these $2n-m-1$ unknown we use active power balance at buses $i = 2$ to n and reactive power balance at bus $i=m+1$ to n .

We can organize $2n-m-1$ unknowns and equations as vector.

$$\underline{X} = \begin{bmatrix} \delta_2 \\ \vdots \\ \delta_n \\ V_{m+1} \\ \vdots \\ V_n \end{bmatrix} \quad (2.12)$$

$$f(x) = \begin{bmatrix} \text{Active power mismatch at bus } 2(P_2 - P_2^o) \\ \vdots \\ \text{Active power mismatch at bus } n(P_n - P_n^o) \\ \text{Reactive power mismatch at bus } m+1(Q_{m+1} - Q_{m+1}^o) \\ \vdots \\ \text{Reactive power mismatch at bus } n(Q_n - Q_n^o) \end{bmatrix} \quad (2.13)$$

Then we must find a special x^* such that $\underline{f}(\underline{x}^*) = 0$

2.1.4 Methods for Solution of Power Flow Analysis

There are three main methods for solving power flow. In this thesis, all three are explained but at the program which solves the power flow analysis Newton Raphson

Method is used. Also, for formulation of PTDF matrix, Decoupled Load Flow Method is taken into consideration.

2.1.4.1 Gauss Seidel Method

The Gauss-Seidel algorithm is an iterative numerical procedure which attempts to find a solution to the system of linear equations by repeatedly solving the linear system until the iteration solution is within a predetermined acceptable bound of error. It is a robust and reliable load flow method that provides convergence to extremely complex power systems. With the bus admittance matrix, and applying Kirchhoff's current law, the following equation is derived. [2,16]

$$I = Y_{bus} V \quad (2.14)$$

where I is the current, V is the voltage and Y is the bus admittance matrix. The k'th (of N) nodal current is:

$$I_k = \sum_{n=1}^N Y_{kn} V_n \quad (2.15)$$

which can be expanded into the following form:

$$I_k = Y_{kk} V_k + \sum_{n=1, n \neq k}^N Y_{kn} V_n \quad (2.16)$$

Re-arranging Equation (2.16) yields:

$$V_k = I_k/Y_{kk} - \frac{1}{Y_{kk}} \sum_{n=1, n \neq k}^N Y_{kn} V_n \quad (2.17)$$

Current at the k'th bus:

$$I_k = \frac{P_k - jQ_k}{V_k^*} \quad (2.18)$$

With the complex power at a given node, Equation (2.17) will yield the following result:

$$V_k = \frac{1}{Y_{kk}} \left(\frac{P_k - jQ_k}{V_k^*} - \sum_{n=1, n \neq k}^N Y_{kn} V_n \right) \quad (2.19)$$

Since the Gauss-Seidel method is an iterative procedure, Equation (2.19) yields the following:

$$V_k^{(i+1)} = \frac{1}{Y_{kk}} \left(\frac{P_k - jQ_k}{(V_k^i)^*} - \sum_{n=1, n \neq k}^N Y_{kn} V_n^{(i)} \right) \quad (2.20)$$

From the equation (2.20), we can solve P_k and Q_k , where;

$$P_k^{(i+1)} = Re \left\{ (V_k^i)^* V_k^{(i)} \sum_{n=0}^N Y_{kn} - (V_k^i)^* \sum_{n=1}^N Y_{kn} V_n^{(i)} \right\} \quad (2.21)$$

where $n \neq k$

and

$$Q_k^{(i+1)} = -Im \left\{ (V_k^i)^* V_k^{(i)} \sum_{n=0}^N Y_{kn} - (V_k^i)^* \sum_{n=1}^N Y_{kn} V_n^{(i)} \right\} \quad (2.22)$$

$n \neq k$

where

2.1.4.2 Newton - Raphson (NR) Method

Newton-Raphson (NR) is *iterative root finding scheme*. Newton-Raphson is root finding scheme because it is geared towards solving equations like $f(x)=0$. The solution to such an equation, call it x^* , is clearly a root of the function $f(x)$.

Starts with an initial guess which is reasonably close to the true root, then the function is approximated by its tangent line (which can be computed using the tools of calculus), and one computes the x-intercept of this tangent line (which is easily done with elementary algebra). This x-intercept will typically be a better approximation to the function's root than the original guess, and the method can be iterated.

x^0 =initial guess

iteration :

$$x^{k+1} = x^k - \left[\frac{1}{\frac{df}{dx} \big|_{x^k}} \right] \cdot f(x^k) \quad (2.23)$$

We can generalize it. Suppose we have \underline{x}^0 as an initial guess we use Taylor Expansion to predict \underline{x}^* .

$$\underline{f}(\underline{x}^*) = \underline{f}(\underline{x}_k) + \frac{d\underline{f}}{d\underline{x}} \Big|_{\underline{x}^k} \cdot (\underline{x}^* - \underline{x}^k) + (h.o.t(ignored)) \quad (2.24)$$

We can rearrange the equation (2.24) that:

$$\underline{x}^* \equiv \underline{x}^k - \left[\frac{d\underline{f}}{d\underline{x}} \Big|_{\underline{x}^k} \right]^{-1} \cdot \underline{f}(\underline{x}^k) \quad (2.25)$$

Modifying this equation (2.25):

$$\underline{x}^{k+1} = \underline{x}^k - \left(\frac{d\underline{f}}{d\underline{x}} \right)^{-1} \Big|_{\underline{x}^k} \cdot \underline{f}(\underline{x}^k) \quad (2.26)$$

f and x are vectors so:

$$\frac{d\underline{f}}{d\underline{x}} = \begin{bmatrix} \frac{df_1}{dx_1} & \frac{df_1}{dx_2} & \dots & \frac{df_1}{dx_n} \\ \frac{df_2}{dx_1} & \frac{df_2}{dx_2} & \dots & \frac{df_2}{dx_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{df_n}{dx_1} & \frac{df_n}{dx_2} & \dots & \frac{df_n}{dx_n} \end{bmatrix} \quad (2.27)$$

For the power flow problems; from the equation (2.12) the x vector is:

$$\underline{x} = \begin{bmatrix} \delta_2 \\ \vdots \\ \delta_n \\ | \\ V_{m+1} \\ \vdots \\ V_n \end{bmatrix}$$

and $\left. \frac{df}{d\underline{x}} \right|_{x^k}$ is the Jacobean matrix **J**.

So Jacobean matrix **J** as;

$$J = \begin{bmatrix} \frac{dP}{d\delta} & \frac{dP}{dV} \\ \frac{dQ}{d\delta} & \frac{dQ}{dV} \end{bmatrix} \quad (2.28)$$

For the initial guess (\underline{x}^0) we will set all δ 's close to zero and all **V**'s close to 1.0 p.u.

- Consider a set of n non-linear algebraic equations

$$\begin{aligned} f_i(x_1, x_2, \dots, x_n) &= 0; & i=1, 2, 3, \dots, n \\ f_i(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \dots, x_n^0 + \Delta x_n^0) &= 0; \end{aligned}$$

- Taylor series expansion

$$\begin{aligned} f_i(x_1^0, x_2^0, \dots, x_n^0) + [(\partial f_i / \partial x_1)^0 \Delta x_1^0 + (\partial f_i / \partial x_2)^0 \Delta x_2^0 + \dots + (\partial f_i / \partial x_n)^0 \\ \Delta x_n^0] + \text{higher order terms} = 0 \end{aligned}$$

- Neglecting higher order terms we can write above equation in matrix form

$$\begin{bmatrix} f_1^0 \\ f_2^0 \\ \vdots \\ f_n^0 \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ \vdots & \vdots & \ddots & \vdots \\ \left(\frac{\partial f_n}{\partial x_1}\right)^0 & \left(\frac{\partial f_n}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^0 \end{bmatrix} \cdot \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (2.29)$$

Or in vector matrix form;

$$f^0 + J^0 \Delta x^0 = 0$$

J^0 is known as the Jacobian matrix.

the above Equation can be written as;

$$f^0 \approx [-J^0] \Delta x^0 \quad (2.30)$$

Update values of x are then

$$x^I = x^0 + \Delta x^0 \quad (2.31)$$

or, in general form of $x^{(r+I)}$ th iteration

$$x^{(r+I)} = x^{(r)} + \Delta x^{(r)} \quad (2.32)$$

Iterations are continued till equation is satisfied to any desired accuracy, i.e.;

$$f_i(x^{(r)}) < \varepsilon$$

where ε is a specified value;

Active and Reactive power for each bus for a Power System are;

$$P_i = |V_i|^2 G_{ii} + \sum_{k=1, k \neq i}^N |V_i V_k Y_{ik}| \cos(\Theta_{ik} + \delta_k - \delta_i) \quad (2.33)$$

and;

$$Q_i = -|V_i|^2 B_{ii} - \sum_{k=1, k \neq i}^N |V_i V_k Y_{ik}| \sin(\Theta_{ik} + \delta_k - \delta_i) \quad (2.34)$$

The power mismatches are;

$$f_{ip} = P_i (\text{specified}) - P_i (\text{calculated}) = \Delta P_i \quad (2.35)$$

$$f_{iq} = Q_i (\text{specified}) - Q_i (\text{calculated}) = \Delta Q_i \quad (2.36)$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{bmatrix}_J \cdot \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} \quad (2.37)$$

Where ΔP and ΔQ are the real and reactive power mismatch at each bus. \mathbf{J} is the jacobian matrix, \mathbf{J} represents the sensitivity measurement of the real and reactive power with respect to the bus voltage angle and magnitude. [3,4,17,18]

The Newton-Raphson (NR) approach incorporating ordered elimination is the most attractive general method it appears that many regular users of load-flow solutions are using the NR approach.

Many advantages are attributed to the NR approach. Its convergence characteristics are relatively powerful, compared with alternative processes, and very low computing times are achieved when the sparse network equations are solved by the technique of sparsity-programmed ordered elimination. [5,6,18]

Computer storage requirements are moderate and, like solution times, increase with problem size fairly linearly, thus making the NR approach particularly attractive for large networks. No acceleration factors have to be determined, the choice of slack bus is rarely critical, and network modifications require negligible computing effort. The NR formulation has great generality and flexibility, enabling a wide range of representational requirements to be included easily and efficiently.

2.1.4.3 Decoupled Load Flow Method

The decoupled power flow method is an approximate version of Newton-Raphson procedure [3,7]. The approximation of the Newton-Raphson procedure only affects the iteration approach it does not reduce the accuracy of the final solution.

The principle underlying the decoupled approach is based on two observations:

- Change in the voltage angle δ at a bus primarily affects the flow of real power P in the transmission lines and leaves the flow of reactive power Q relatively unchanged.
- Change in the voltage magnitude ($|V|$) at a bus primarily affects the flow of reactive power Q in the transmission lines and leaves the flow of real power relatively unchanged.

We know from the formula (2.37) that the power mismatches can be written as in matrix form as follow;

$$\begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial |V|} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial |V|} \end{bmatrix} \cdot \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

From the above equation we can assume that[8,19];

$$\frac{\partial P}{\partial |V|} \approx 0, \quad \frac{\partial Q}{\partial \delta} \approx 0$$

and

$$\frac{\partial P}{\partial \delta} \Delta \delta = \Delta P, \quad \frac{\partial Q}{\partial |V|} \Delta |V| = \Delta Q$$

A well designed a properly operated power transmission system:

- The angular differences ($\delta_i - \delta_k$) between typical buses of the system are usually so small that[8];

$$\cos(\delta_i - \delta_k) = 1$$

and

$$\sin(\delta_i - \delta_k) \approx (\delta_i - \delta_k)$$

The line susceptances B_{ik} are many times larger than the line conductance G_{ik} so that

$$G_{ik} \sin(\delta_i - \delta_k) \ll B_{ik} \cos(\delta_i - \delta_k)$$

The reactive power Q_i injected into any bus i of the system during normal operation is much less than the reactive power which would flow if all lines from that bus were short-circuited to reference.

That is

$$Q_i \ll |V_i|^2 B_{ii}$$

After simplifying:

$$\frac{\partial P_i}{\partial \delta_k} \approx |V_k| \frac{\partial Q_i}{\partial |V_k|} \approx -|V_i| |V_k| B_{ik} \quad (2.38)$$

and using the assumption that all V 's are close to each other;

$$\frac{\partial P_i}{\partial \delta_k} \approx -|V_i|^2 B_{ik} \quad (2.39)$$

The solution strategy

- Calculate the initial mismatch ΔP
- Solve for $\Delta \delta$
- Update the angles δ and use them to calculate mismatch ΔQ
- Solve for $\Delta |V|$ and update the magnitude $|V|$, and

Repeat the iteration until all mismatches are within specified tolerances.

2.1.5 Calculation of Power Flow of Each Line

If we consider figure 2.1. At Bus 1 we have a Voltage V_1 and at bus 3 we have a voltage V_3 . Between this two bus, we have Impedance Z_{13} or that say Admittance Y_{13} . Between this two buses, the current I_{13} flows. Actually, this I_{13} flows from the Bus 1. So we can say that, the power flows from Bus 1 to Bus 3 is;

$$S_{13} = V_1 \cdot (I_{13})^* \quad (2.40)$$

$$I_{13} = (V_1 - V_3) \cdot Y_{13} \quad (2.41)$$

$$(I_{13})^* = [(V_1 - V_3) \cdot Y_{13}]^* \quad (2.42)$$

$$S_{13} = V_1 \cdot [(V_1 - V_3) \cdot Y_{13}]^* \quad (2.43)$$

and the power flows from Bus 3 to Bus 1 is;

$$S_{31} = V_3 \cdot [(V_1 - V_3) \cdot Y_{13}]^* \quad (2.44)$$

if we subtract S_{31} from S_{13} ;

$$S_{13} - S_{31} = (V_1 - V_3)^2 \cdot Y_{13} \quad (2.45)$$

and this formula is obviously the power lost on the lines. So it means that the formulation of line flows (2.43) is clearly true.

if we generalizes this formula;

$$S_{jk} = V_j \cdot [(V_j - V_k) \cdot Y_{jk}]^* \quad (2.46)$$

3 PTDF- Power Transfer Distribution Factors

3.1 Available Transfer Capability

The market is fierce in power generation and supply. Population increase and so does the demand for electrical power. It is due to this demand that pushes us to find new ways of not only producing more efficient power, but also to look into ways of supplying it efficiently. This is where transfer capability steps in. Transfer capability is an indication of how much power a power system network can supply without compromising system security. Operators use transfer capability as judgment on the performance of a system. If the operators know the transfer capability, and know the present transfer values, it will be easy to find out the Available Transfer Capability.[20]

A transfer is specified by changes in power injections at buses in the network. For example, a point to point transfer from bus A to bus B is specified by increasing power at bus A and reducing power at bus B. In particular, if 100 MW are to be transferred from A to B, then power at bus B is reduced by 100 MW and power at bus A is increased by 100 MW plus an amount to cover the change in losses. In this case bus A is called a source of power and bus B is called a sink of power.

Available Transfer Capability (ATC) is a measure of the transfer capability remaining in the physical transmission network for further commercial activity over and above already committed uses. Mathematically, ATC is defined as the Total Transfer Capability (TTC) less the Transmission Reliability Margin (TRM), less the sum of existing transmission commitments (which includes retail customer service) and the Capacity Benefit Margin (CBM).[9,20,21]

$$ATC = TTC - TRM - \text{Existing Transmission Commitments (including CBM)}$$

Total Transfer Capability (TTC) is defined as the amount of electric power that can be transferred over the interconnected transmission network in a *reliable* manner while meeting *all* of a specific set of defined pre- and post-contingency system conditions.

Transmission Reliability Margin (TRM) is defined as that amount of transmission transfer capability necessary to ensure that the interconnected transmission network is secure under a reasonable range of uncertainties in system conditions.

Capacity Benefit Margin (CBM) is defined as that amount of transmission transfer capability reserved by load serving entities to ensure access to generation from interconnected systems to meet generation reliability requirements.[22]

Curtailed is defined as the right of a transmission provider to interrupt all or part of a transmission service due to constraints that reduce the capability of the transmission network to provide that transmission service. Transmission service is to be curtailed only in cases where system reliability is threatened or emergency conditions exist.

Recallability is defined as the right of a transmission provider to interrupt all or part of a transmission service for any reason, including economic, that is consistent with FERC policy and the transmission provider's transmission service tariffs or contract provisions.

Non-recallable ATC (NATC) is defined as $TTC - TRM - \text{non-recallable reserved transmission service (including CBM)}$.

Recallable ATC (RATC) is defined as $TTC - TRM - \text{recallable transmission service} - \text{nonrecallable transmission service (including CBM)}$. RATC must be considered differently in the planning and operating horizons. In the planning horizon, the only data available are recallable and non-recallable transmission service reservations, whereas in the operating horizon transmission schedules are known.

The ATC between two areas provides an indication of the amount of additional electric power that can be transferred from one area to another for a specific time frame for a specific set of conditions. ATC can be a very dynamic quantity because it is a function of variable and interdependent parameters. These parameters are highly dependent upon the conditions of the network. Consequently, ATC calculations may need to be periodically updated. Because of the influence of conditions throughout the network, the accuracy of the ATC calculation is highly dependent on the completeness and accuracy of available network data.

The system-limiting factors that limit a power system's ATC are many. Among them are the line current limits, voltage magnitude limit, generator reactive power limit, and voltage collapse limit, etc.

The line current limit usually is a line's thermal limit. Too much current flow in a line may cause a line to droop or damage nearby connected equipments. DC power flow has been widely used to calculate thermal limit with great speed. But DC power flow cannot deal with other limiting factors.

The bus voltage magnitudes also need to be kept within reasonable limits. Voltage over-limit may cause damage to system equipments, and reduce the power quality to the customers. Low voltage sometimes is also an indication that the system is near a voltage collapse. Both high voltage and low voltage are regulated by system circuit breakers and pose limits to the power transfer.

Generators have reactive power output limits. After a limit is reached, a generator will not be able to regulate its bus voltage. It is degraded from a PV bus into a PQ bus. This may cause voltage collapse or system instability.

The voltage collapse is the upper physical limit that a power system can function properly. Beyond this point, nonmathematical solution exists. This situation usually happens after a bus voltage has a significant drop or when a generator's var limit is reached. It is not considered here since the two factors that cause it has been considered [10,23].

Available Transfer Capability is important that where power system reliability meets electricity market efficiency. ATC can have a huge impact on market outcomes and system reliability, so the results of ATC are of great interest to all involved.

When electric power is transferred between two areas such as Area A to Area F in Figure A1, the entire network responds to the transaction. The power flow on each transmission path will change in proportion to the response of the path to the transfer. Similarly, the power flow on each path will change depending on network topology, generation dispatches, customer demand levels, other transactions through the area, and other transactions that the path responds to which may be scheduled between other areas.

Network conditions will vary over time, changing line loading conditions, and causing the ATC of the network to change. Also, the most limiting facility in determining the network's ATC can change from one time period to another, particularly in highly meshed networks. Therefore, the ATC of the network is time dependent.

3.2 Sensitivity

Bulk power transfers in electric power systems are limited by transmission network security. Transfer capability measures the maximum power transfer permissible under certain assumptions. Once a transfer capability has been computed for one set of assumptions, it is useful to quickly estimate the effect on the transfer capability of modifying those assumptions. Transfer capabilities must be quickly computed for various assumptions representing possible future system conditions and then recomputed as system conditions change. The usefulness of each computed transfer capability is enhanced if the sensitivity of the transfer capability is also computed.[9,20]

Estimates of transfer capabilities must be updated regularly as to avoid the combined effect of power transfers from causing an undue risk of system overloads, equipment

damage, or blackouts. However, being overly conservative over the estimates of transfer capability will unnecessarily limit the power transfers and would prove to be costly and an inefficient use of the network. If we have the sensitivity analysis of the system, the operators can easily calculate the available transfer capacity or making scenarios for most sensitivities line in case of the increase of a critical load.

3.3 Power Transfer Distribution Factors (PTDF) Calculations

The role of a transmission system is to transmit reliable electrical energy from generation facilities to distribution network. At the distribution network, loads are changes all the time. To understand the change of transmitted energy at transition system depends of changes load, we must calculate the Power Transfer Distribution Factors (PTDF) of the system [11,24,25].

$$P_i = Re \left\{ V_i e^{j\delta_i} \sum_{k=1}^n [(G_{ik} - jB_{ik})] V_k e^{-j\delta_k} \right\} \quad (3.1)$$

$$P_i = \sum_{k=1}^n |V_i| |V_k| (G_{ik} \cos(\delta_i - \delta_k) + B_{ik} \sin(\delta_i - \delta_k)) \quad (3.2)$$

By using Decoupled Method which mentioned in Chapter 2;

A well designed an properly operated power transmission system:

- The angular differences ($\delta_i - \delta_k$) between typical buses of the system are usually so small that

$$\cos(\delta_i - \delta_k) = 1 ; \quad \sin(\delta_i - \delta_k) \approx (\delta_i - \delta_k)$$

The line susceptances B_{ik} are many times larger than the line conductance G_{ik} so that

$$G_{ik} \sin(\delta_i - \delta_k) \ll B_{ik} \cos(\delta_i - \delta_k)$$

So, we can consider that, $G_{ik} \sin(\delta_i - \delta_k)$ and consider V 's as 1 p.u.

If we rearrange the formula we can get;

$$P_i = \sum_{k=1}^n B_{ik} (\delta_i - \delta_k) \quad (3.3)$$

If we take derivative of P_i with respect to δ_i ;

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{k=1}^n B_{ik} \quad (3.4)$$

so, we can rearrange the formula (3.4) as;

$$\Delta P_i = \sum_{k=1}^n B_{ik} \Delta \delta_k \quad (3.5)$$

Taking into consideration these assumption and the simplifications, the matrix equation for an n-bus system may be determined as ;

$$\begin{bmatrix} \Delta P_1 \\ \dots \\ \Delta P_n \end{bmatrix} = \begin{bmatrix} B_{11} & \dots & \dots & B_{1n} \\ \dots & \dots & \dots & \dots \\ B_{n1} & \dots & \dots & B_{nn} \end{bmatrix} \cdot \begin{bmatrix} \Delta \delta_1 \\ \dots \\ \Delta \delta_n \end{bmatrix} \quad (3.6)$$

So, we can say that;

$$\Delta \mathbf{P} = \mathbf{B} \cdot \Delta \boldsymbol{\delta} \quad (3.7)$$

Base on relation (3.7), it can be obtained:

$$\Delta \boldsymbol{\delta} = \mathbf{X} \cdot \Delta \mathbf{P} \quad (3.8)$$

Where \mathbf{X} represent the inverse of the \mathbf{B} matrix incremented with a row and column of zeros at the slack bus. The line and column corresponding to the slack bus of the power system are eliminated from the matrix and the n-1 x n-1 inverse matrix is computed.

$$\mathbf{X} = \left[\begin{array}{c|ccc} 0 & \dots & \dots & O_n \\ \hline - & - & - & - \\ \dots & & & \\ \dots & & B^{-1} & \\ O_n & & & \end{array} \right] \quad (3.9)$$

To finish the PTDFs calculation process, the equation of active power flow on the ik branch is needed [11,24].

$$P_{ik} = U_i^2 \cdot G_{ik} - U_i U_k \cdot [G_{ik} \cdot \cos(\delta_i - \delta_k) + B_{ik} \cdot \sin(\delta_i - \delta_k)] \quad (3.10)$$

Taking into consideration the simplifications which are;

$$G_{ik} \ll B_{ik}, G_{ik} \approx 0 \text{ and } \sin(\delta_i - \delta_k) = \delta_i - \delta_k$$

but to get better result we let U's as what they are;

$$P_{ik} = -U_i U_k \cdot B_{ik} (\delta_i - \delta_k) \quad (3.11)$$

and

The elements of the susceptance matrix **B** are computed based on the following relations:

The off diagonal elements:

$$B_{ik} = \frac{1}{x_{ik}} \quad (3.12)$$

When we use the (3.12) at (3.11), we can say that;

$$P_{ik} = -U_i U_k \cdot \frac{1}{x_{ik}} (\delta_i - \delta_k) \quad (3.13)$$

Now we can look deeply to the sensitivity of a line. Change on Power Flow at branch ℓ (from bus I to bus k) with respect to change on load at bus j:

$$P_{l,j} = \frac{dP_l}{dP_j} \quad (3.14)$$

re arrange this formula;

$$P_{lj} = \frac{dP_{ik}}{dP_j} = \frac{d}{dP_j} \left[-U_i U_k \frac{\delta_i - \delta_k}{x_{ik}} \right] \quad (3.15)$$

and,

$$P_{l,j} = -\frac{U_i U_k}{x_{ik}} \left[\frac{d\delta_i}{dP_j} - \frac{d\delta_k}{dP_j} \right] \quad (3.16)$$

if we use (3.8) we can say that; $\frac{\Delta\delta}{\Delta P} = X$

we can rearrange the formula (3.16):

$$P_{l,j} = -B_{ik} U_i U_k [X_{ij} - X_{kj}] \quad (3.17)$$

If we consider all V's as 1 p.u. than the formula is;

$$P_{l,j} = -B_{ik} [X_{ij} - X_{kj}] \quad (3.18)$$

4 Line Security with DSM

Power systems operation becomes more important as the load demand increases all over the world. This rapid increase in load demand forces power systems to operate near critical limits due to economical and environmental constraints. The objective in power systems operation is to serve energy with acceptable voltage and frequency to consumers at minimum cost.

Since generation and transmission units have to be operated at critical limits voltage stability problems may occur in power system when there is an increase in load demand. Voltage instability is one of the main problems in power systems. In voltage stability problem some or all buses voltages decrease due to insufficient power delivered to loads.

4.1 Definition of Voltage Stability

Voltage stability is the ability of a power system to maintain steady acceptable voltages at all buses in the system under normal operating conditions and after being subjected to a disturbance [12,26,27].

Loads are the main driving force of voltage instability, and that is why the voltage instability is also called as load instability. Exact modeling of load is a difficult task because the system loads are aggregation of many different devices. The heart of the problem is the identification of system load composition at a given time and aggregation of various loads. When a severe disturbance, such as fault, line tripping, etc., occurs, the voltage of some buses reduces drastically. Reduction of system voltage may cause to stall the heavily loaded induction motors and that may ultimately lead to voltage instability [13,26,27].

It is clear that load is one of the important variables in a power system. Controlling the load is important, but first we must know which loads are more important and most effective for a transmission system. To understand this, sensitivity analysis and demand side management can help us.

4.2 Sensitivity and Demand Side Management

Voltage stability can be attained by sufficient generation and transmission of energy. If the loads increase and assume that we have sufficient generation, the transmission system transfer capacity (transmission of energy) is the most important factor for keep the system alive. Increase of a load, effects all lines differently. Sensitivity analysis of a transmission system gives us this information. After this analysis, we can calculate the flow of lines after the increase of a certain load.

By using sensitivity, after increase of loads, we can see that some of line flows exceed their limits. This means that, increase of these loads make system collapse. By using Demand Side Management, we could have opportunity to reduce these loads and make the system secure.

Maximum customer benefit derives from minimum cost and sufficient supply availability. At times, lower cost implies risk of potential unavailability of enough power. Customers willing to share in “availability risk” can derive further benefit by way of participating in controlled outage programs. Specifically whenever utilities foresee dangerous loading patterns, there is a need for a rapid reduction in demand either system-wide or at specific locations. The utility needs to get some relief in order to solve its problems quickly and more efficiently without wasting energy. This relief can come from customers who agree to curtail their loads upon request in exchange for an incentive fee [14,28,29].

If we have sensitivities of all loads to all lines, it will be easier to calculate or estimate the cost of reduction of a load. If a load is most effective of a certain line which is in trouble (going to exceeds the limits of power transfer capability), we can say that this

load is most valuable for that line and worth to pay more for curtailing. If the utility has these information, it can be easily offer a contract to a customer whose load is valuable. Without these information, utility may pay more for less valuable loads.

4.3 Sensitivity of Line Flows to Individual Loads

The sensitivity of a line flow to each load in the system ($\Delta P_{km}/\Delta P_i$, where ΔP_{km} is the flow on the line from bus k to bus m and ΔP_i is the change in load at bus i) can be computed to help the utility relieve line overloads. When a transmission line gets overloaded it is important to know which load has the most impact on this line flow.[15,29]

Numerically, the sensitivity of a line flow to an individual load, can be practically calculated as;

- 1- Calculate the power flow of the line $P_{km-first}$ for a stable system.
- 2- Change the individual load and write ΔP_i as $P_{i-first} - P_{i-last}$
- 3- Re-calculate the power flow of line $P_{km-last}$ by using P_{i-last}
- 4- Calculate the change of power flow of the line as; $\Delta P_{km} = P_{km-first} - P_{km-last}$
- 5- Calculate the sensitivity of the line flow to the load as; $\Delta P_{km}/\Delta P_i$

At Power Transfer Distribution Factors (PTDF) Calculations, we create a formulation for calculating this sensitivities. When we will look deeply to the results we can see that there are some differences between the numerical sensitivities and the calculated (PTDF) sensitivities.

4.4 Congestion Management

Congestion is a situation where the demand for transmission capacity exceeds the transmission network capabilities, which might lead to a violation of network security limits, being thermal, voltage stability limits or a (N-1) contingency condition.

Therefore, in a physical sense congestion is merely an indication of a presence of transmission constraints.

One way to handle transmission contingency is load shading. But the important point is which load is the most effective for the mentioned transmission line. To know this, we can use sensitivity analysis for the power system. When we know the sensitivity of a line flow to a load, this means that, we know the rate of current decrease after curtailing a load.

$$\frac{\Delta P_{line-x}}{\Delta P_{load-n}}$$

Where ΔP_{line-x} is the change of power flow on transmission line x, and ΔP_{load-n} is the change of power of load at bus no n.

5 Simulation and Results

The power flow analysis method presented in Chapter 2 is applied to a 6-bus sample test system and then we calculate the sensitivities of all lines to all loads. For simulating and testing the system, there are some professional tools. Instead of using those tools, at this thesis, I preferred to write a program. This made the system and theoretical calculation more understandable.

This program has ability to compute power flow analysis, the flows on each line and the sensitivities of all line flows with respect to all loads of a system which has IEEE common format file. The number of lines, generators, loads or buses do not affect the running of the program. It can handle a power system with even thousands of buses which has a properly prepared common format file.

5.1 Six Bus Sample Test System

Sample test system consists of 6 buses, 3 generators, 11 transmission lines and 3 loads. The single line diagram of 6-bus test system is shown in Figure (fig 5.1). The IEEE common format data of this system can be seen below.

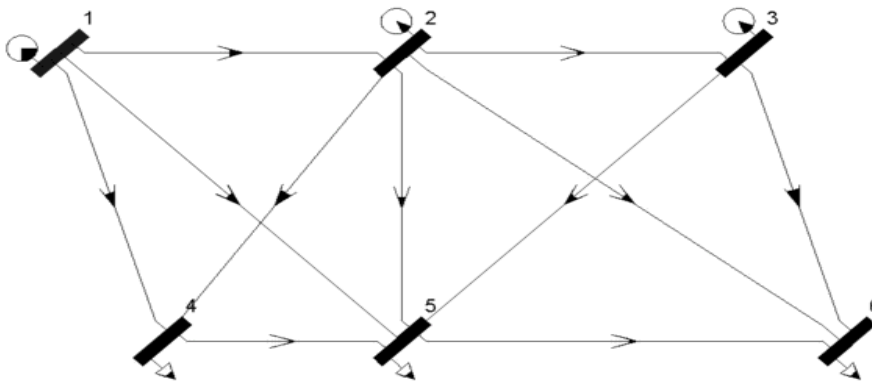


Figure 5.1

The IEEE common format data test case

BUS DATA FOLLOWS

1 BUS1	1	1	3	1.0000	0.00	0.00	0.00	100.00	.00	100.00
2 BUS2	1	1	2	1.0000	0.00	0.00	0.00	50.00	.00	100.00
3 BUS3	1	1	2	1.0000	0.00	0.00	0.00	60.00	.00	100.00
4 BUS4	1	1	0	1.0000	0.00	70.00	70.00	.00		
5 BUS5	1	1	0	1.0000	0.00	70.00	70.00	.00		
6 BUS6	1	1	0	1.0000	0.00	70.00	70.00	.00		

-999

BRANCH DATA FOLLOWS

1	2	1	1	1	0	.001000	.200000	.040000	100
1	4	1	1	1	0	.000500	.200000	.040000	100
1	5	1	1	1	0	.000800	.300000	.060000	100
2	3	1	1	1	0	.000500	.250000	.060000	100
2	4	1	1	1	0	.000500	.100000	.020000	100
2	5	1	1	1	0	.001000	.300000	.040000	100
2	6	1	1	1	0	.000700	.200000	.050000	100
3	5	1	1	1	0	.001200	.260000	.050000	100
3	6	1	1	1	0	.000200	.100000	.020000	100
4	5	1	1	1	0	.002000	.400000	.080000	100
5	6	1	1	1	0	.001000	.300000	.060000	100

-999

INTERCHANGE DATA FOLLOWS

-9

TIE LINE DATA FOLLOWS

-999

LOSS ZONE DATA FOLLOWS

-99

END OF DATA

5.2 Simulation Program

The simulation program was written in MATLAB.

It contains several phases which are;

a. Reading the common format file:

Program has the ability to read any common format file which is prepared for any High Voltage System. The size of the system does not cause a problem for the program.

b. Formation of Y-Bus of the system:

This part of the program forms the Y-Bus matrix of the system by directly reading bus and branch information from common format file.

c. Calculation of Active and Reactive power for each Bus.

At this part of the program, Active and Reactive Power mismatches are calculated by using the Newton Raphson Method.

d. Controlling the tolerance for ending the iterations

This part of the program checks the tolerance and when it is small enough stops the iteration. This is important because the tolerance has to be small enough for better results and big enough for ending iteration in a short time. And for a big power system, sometimes seconds are too big to make a decision.

e. Calculation of power flow each line

At this part of the program, all line flows are calculated. This formula is derived in section 2.1.5. (formula (2.26)).

f. Sensitivity Calculations:

Here, the sensitivity of all loads to all line flows are calculated. Two separate formulations are used here. The first one comes from the formula 3.18 and the second one comes from the formula 3.17. Two separate but very close results can be seen from these formulas.

g. Compare the results by using Matpower4 and numerical sensitivities.

This part of the program shows all sensitivity results and makes comparisons. Sensitivities come from the professional tool Matpower4, sensitivities from the numerical applications and two other sensitivities are calculated at the "sensitivity calculation" of the program. All differences and all differences in percentages between sensitivities can be calculated at this part.

5.3 Results

Program results have a lot of information for the power system;

From the results we can see the Y-Bus of the system. For 6-bus sample test system the result of Y-Bus calculated as 6x6 (nxn) matrix:

```
ybus =  
  
4.0063 -11.7479i -2.0000 + 4.0000i      0      -1.1765 + 4.7059i -0.8299 + 3.1120i      0  
  
-2.0000 + 4.0000i  9.3283 -23.1955i -0.7692 + 3.8462i -4.0000 + 8.0000i -1.0000 + 3.0000i -1.5590 + 4.4543i  
  
0      -0.7692 + 3.8462i  4.1557 -16.5673i      0      -1.4634 + 3.1707i -1.9231 + 9.6154i  
  
-1.1765 + 4.7059i -4.0000 + 8.0000i      0      6.1765 -14.6359i -1.0000 + 2.0000i      0  
  
-0.8299 + 3.1120i -1.0000 + 3.0000i -1.4634 + 3.1707i -1.0000 + 2.0000i  5.2933 -14.1378i -1.0000 + 3.0000i  
  
0      -1.5590 + 4.4543i -1.9231 + 9.6154i      0      -1.0000 + 3.0000i  4.4821 -17.0047i
```

After that, Voltages, Angles, Active Power and Reactive power for all bus, which is 6 for this test system, can be calculated as a 6x1 matrix.

V =		P =	
(1,1)	1.0500		1.0788
(2,1)	1.0500		0.4997
(3,1)	1.0700		0.6000
(4,1)	0.9893		-0.6998
(5,1)	0.9854		-0.7000
(6,1)	1.0044		-0.7000
del =		Q =	
			0.1597
(2,1)	-3.6720		0.7440
(3,1)	-4.2737		0.8963
(4,1)	-4.1950		-0.7005
(5,1)	-5.2766		-0.7000
(6,1)	-5.9479		-0.7000

At the end for calculation is the power flow for each line which has both active and reactive part and computation of sensitivity for all lines to all loads.

For the 6-bus sample test system the results for power flow as an nxn matrix are;

Powerflow =

0	0.2870 - 0.1322i	0	0.4358 + 0.2234i	0.3560 + 0.1456i	0
-0.2779 + 0.1503i	0	0.0293 - 0.0896i	0.3308 + 0.4719i	0.1551 + 0.1756i	0.2625 + 0.1516i
0	-0.0289 + 0.0916i	0	0	0.1912 + 0.2604i	0.4377 + 0.6187i
-0.4249 - 0.1799i	-0.3157 - 0.4418i	0	0	0.0408 - 0.0103i	0
-0.3453 - 0.1054i	-0.1501 - 0.1607i	-0.1802 - 0.2367i	-0.0405 + 0.0111i	0	0.0161 - 0.0675i
0	-0.2566 - 0.1349i	-0.4277 - 0.5685i	0	-0.0156 + 0.0690i	0

The elements of nxn matrix, for example P_{ij} , shows the flow of power from bus i to bus j.

When we look the results for sensitivity, we can see lxn matrix, which l is the number of transmission lines and n is the number of buses. The elements show us the sensitivity of line l with respect to load buses at n.

SensV =

0	-0.4844	-0.4205	-0.3263	-0.3372	-0.4243
0	-0.3107	-0.2947	-0.4902	-0.2704	-0.2957
0	-0.2122	-0.2991	-0.1787	-0.3970	-0.2943
0	0.0511	-0.3479	0.0135	-0.1104	-0.1966
0	0.3012	0.2116	-0.3590	0.1014	0.2168
0	0.0927	-0.0344	0.0267	-0.1848	-0.0272
0	0.0572	-0.2376	0.0150	-0.1241	-0.3976
0	0.0606	0.2761	0.0185	-0.1130	0.1471
0	-0.0072	0.3529	-0.0020	0.0147	-0.3272
0	-0.0049	-0.0745	0.1038	-0.1559	-0.0706
0	-0.0522	-0.1168	-0.0159	0.0979	-0.2244

If we consider all V's as 1 p.u. then the results change a little;

Sens =

0	-0.4844	-0.4205	-0.3263	-0.3372	-0.4243
0	-0.3263	-0.3094	-0.5148	-0.2839	-0.3105
0	-0.2248	-0.3168	-0.1893	-0.4205	-0.3117
0	0.0511	-0.3479	0.0135	-0.1104	-0.1966
0	0.3163	0.2222	-0.3770	0.1065	0.2276
0	0.0982	-0.0365	0.0283	-0.1958	-0.0288
0	0.0601	-0.2495	0.0158	-0.1304	-0.4176
0	0.0641	0.2925	0.0196	-0.1197	0.1558
0	-0.0075	0.3707	-0.0021	0.0154	-0.3437
0	-0.0054	-0.0829	0.1154	-0.1734	-0.0785
0	-0.0581	-0.1299	-0.0177	0.1089	-0.2496

Rather than the calculated sensitivities, we have also numerical application sensitivities which are mentioned in section 4.3. This was obtained by changing the loads one by one and looking at the power flow changes at each line.

SensReal =

0	-0.4850	-0.4170	-0.3260	-0.3330	-0.4210
0	-0.3080	-0.2890	-0.5010	-0.2650	-0.2900
0	-0.2080	-0.2950	-0.1750	-0.4040	-0.2900
0	0.0540	-0.3550	0.0160	-0.1110	-0.2000
0	0.3040	0.2140	-0.3880	0.1000	0.2190
0	0.0960	-0.0340	0.0290	-0.1970	-0.0260
0	0.0600	-0.2420	0.0180	-0.1250	-0.4140
0	0.0610	0.2830	0.0180	-0.1270	0.1510
0	-0.0070	0.3620	-0.0020	0.0150	-0.3510
0	-0.0030	-0.0740	0.1130	-0.1640	-0.0700
0	-0.0530	-0.1190	-0.0150	0.1100	-0.2350

After calculations, we have one more phase which is the comparison of the results with a professional tool called Matpower4. The program both shows the sensitivity (PTDF) matrix computed by Matpower4 and the difference between matrixes computed by my program.

H =

0	-0.4706	-0.4026	-0.3149	-0.3217	-0.4064
0	-0.3149	-0.2949	-0.5044	-0.2711	-0.2960
0	-0.2145	-0.3026	-0.1807	-0.4072	-0.2976
0	0.0544	-0.3416	0.0160	-0.1057	-0.1907
0	0.3115	0.2154	-0.3790	0.1013	0.2208
0	0.0993	-0.0342	0.0292	-0.1927	-0.0266
0	0.0642	-0.2422	0.0189	-0.1246	-0.4100
0	0.0622	0.2890	0.0183	-0.1207	0.1526
0	-0.0077	0.3695	-0.0023	0.0150	-0.3433
0	-0.0034	-0.0795	0.1166	-0.1698	-0.0752
0	-0.0565	-0.1273	-0.0166	0.1096	-0.2467

Now we have 4 different sensitivity matrixes. If we calculate the differences between Sensitivity Matrix which is calculated from the formula (3.17), and Numerical Sensitivity, and differences between Sensitivity Matrix which is calculated from the formula (3.18), and Numerical Sensitivity, we can see that using V's as what they are improve the results. Differences:

1- The differences in percentage between sensitivity matrix which use the formula (3.17) and Numerical sensitivity matrix is;

$$\text{perTDifV} = \% 3.7286$$

2- The differences in percentage between sensitivity matrix which use the formula (3.18) and Numerical sensitivity matrix is;

$$\text{perTDif} = \% 5.5087$$

3- The differences in percentage between sensitivity matrix which calculated by using Matpower and Numerical sensitivity matrix is;

$$\text{perTDifH} = \% 2.8438$$

5.4 Results for 14 Bus System and Analysis of these Results

To understand deeply the Sensitivity Analysis and what will happen after changing of load a 14 Bus System is used. For the 14 Bus system, by using the simulation program, all power flow calculations for each bus and load flows for branches and PTDF matrix are calculated. After that, load at bus 6 is doubled. First it was 11.2 MW. After all calculations, I changed the load at bus 6 as 22.4 MW and make all calculations by using the program.

This 14 Bus test system consists of 14 buses, 2 generators, 3 Synchronous Condensers which are regard as generator bus and 9 loads. Also this system has 20 transmission lines. The single line diagram of 14-bus test system is shown in Figure (fig 5.2).

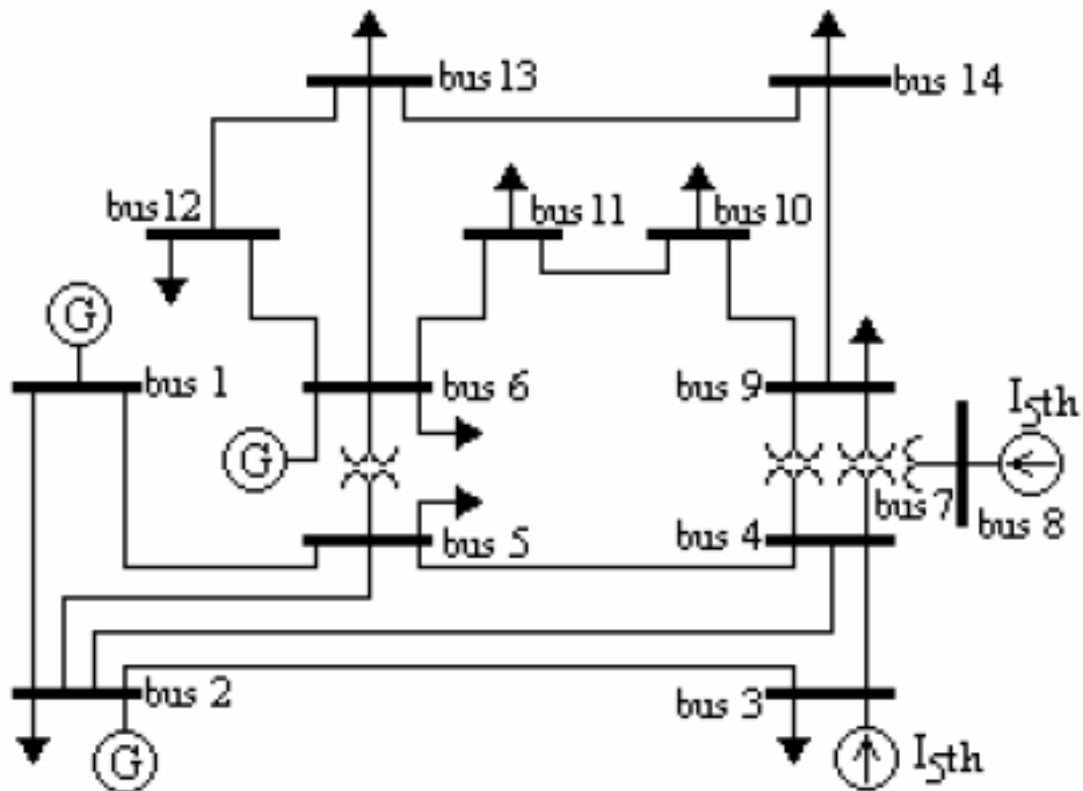


Figure 5.2

When we execute the program we can get the following results for each bus as a nodal value.

Bus No	Bus Type	Voltage		Generation		Load	
		Mag(pu)	Ang(rad)	P(MW)	Q(MVAr)	P(MW)	Q(MVAr)
1	slack	1,0600	0,00	232,39	-16,55	0,00	0,00
2	Generator	1,0450	-0,09	40,00	43,56	21,70	12,70
3	Generator	1,0100	-0,22	0,00	25,08	94,20	19,00
4	Load	1,0177	-0,18			47,80	-3,90
5	Load	1,0195	-0,15			7,60	1,60
6	Generator	1,0700	-0,25	0,00	12,73	11,20	7,50
7	Load	1,0615	-0,23			0,00	0,00
8	Generator	1,0900	-0,23	0,00	17,62	0,00	0,00
9	Load	1,0559	-0,26			29,50	16,60
10	Load	1,0510	-0,26			9,00	5,80
11	Load	1,0569	-0,26			3,50	1,80
12	Load	1,0552	-0,26			6,10	1,60
13	Load	1,0504	-0,26			13,50	5,80
14	Load	1,0355	-0,28			14,90	5,00

Table 5.1

Here at table 5.1, we can see all power generation and all loads of each bus. As mentioned at section 3.3, we need line flows to understand the sensitivities. As the sensitivity of a line with respect to any load is the power flow difference at this branch divided by the load change at the load bus.

After execution of the program, we can get all power flows for each branch as follows;

Branch No	From Bus	To Bus	From Bus Injection		To Bus Injection	
			P(MW)	Q(MVAr)	P(MW)	Q(MVAr)
1	1	2	156,88	-20,40	-152,59	27,68
2	1	5	75,51	3,85	-72,75	2,23

3	2	3	73,24	3,56	-70,91	1,60
4	2	4	56,13	-1,55	-54,45	3,02
5	2	5	41,52	1,17	-40,61	-2,10
6	3	4	-23,29	4,47	23,66	-4,84
7	4	5	-61,16	15,82	61,67	-14,20
8	4	7	28,07	-9,68	-28,07	11,38
9	4	9	16,08	-0,43	-16,08	1,73
10	5	6	44,09	12,47	-44,09	-8,05
11	6	11	7,35	3,56	-7,30	-3,44
12	6	12	7,79	2,50	-7,71	-2,35
13	6	13	17,75	7,22	-17,54	-6,80
14	7	8	0,00	-17,16	0,00	17,62
15	7	9	28,07	5,78	-28,07	-4,98
16	9	10	5,23	4,22	-5,21	-4,18
17	9	14	9,43	3,61	-9,31	-3,36
18	10	11	-3,79	-1,62	3,80	1,64
19	12	13	1,61	0,75	-1,61	-0,75
20	13	14	5,64	1,75	-5,59	-1,64

Table 5.2

Here at **Table 5.2**, we can see all power flow for every branch as both way. We can see both from bus injection and to bus injection. So it is now easy to calculate all losses at every branch. If we look the differences between power flow for from injection and to injections, this gives us the losses.

At Table 5.3, we can see all losses for each branch and also the total loss of power at the system. When we calculate all active powers at all bus at **Table 5.1**, we can see that it is 259 MW. So the total load of the system is 259 MW but the total generation is 272.39 MW. This means that, the total losses of the system also produce and this is generated by Slack Bus. So this is the proof of the explanation of the Slack Bus that pick up the losses of the system.

Branch No	From Bus	To Bus	Power Loss at Each Branch	
			P(MW)	Q(MVAr)
1	1	2	4,30	7,27
2	1	5	2,76	6,08
3	2	3	2,32	5,16
4	2	4	1,68	1,47
5	2	5	0,90	0,93
6	3	4	0,37	0,36
7	4	5	0,51	1,62
8	4	7	0,00	1,70
9	4	9	0,00	1,30
10	5	6	0,00	4,42
11	6	11	0,06	0,12
12	6	12	0,07	0,15
13	6	13	0,21	0,42
14	7	8	0,00	0,46
15	7	9	0,00	0,80
16	9	10	0,01	0,03
17	9	14	0,12	0,25
18	10	11	0,01	0,03
19	12	13	0,01	0,01
20	13	14	0,05	0,11
		TOTAL	13,39	32,70

Table 5.3

The last important result after this power flows is the Power Transfer Distribution Factors matrix. The elements of this matrix are the Sensitivities of line flows with respect to Loads. In any Power System, if we have l branches and n buses, this means that the Sensitivity matrix dimensions are $l \times n$.

Sensitivities (Power flow change at any branch wrt load	LOAD NO (BUS NO)													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0,0000	-0,8380	-0,7465	-0,6675	-0,6106	-0,6291	-0,6573	-0,6573	-0,6518	-0,6477	-0,6386	-0,6309	-0,6323	-0,6433
2	0,0000	-0,1620	-0,2535	-0,3325	-0,3894	-0,3709	-0,3427	-0,3427	-0,3482	-0,3523	-0,3614	-0,3691	-0,3677	-0,3567
3	0,0000	0,0273	-0,5320	-0,1513	-0,1031	-0,1188	-0,1427	-0,1427	-0,1380	-0,1346	-0,1269	-0,1204	-0,1215	-0,1308
4	0,0000	0,0572	-0,1434	-0,3167	-0,2158	-0,2487	-0,2986	-0,2986	-0,2888	-0,2817	-0,2655	-0,2519	-0,2543	-0,2738
5	0,0000	0,0774	-0,0711	-0,1994	-0,2917	-0,2616	-0,2160	-0,2160	-0,2249	-0,2314	-0,2463	-0,2587	-0,2564	-0,2387
6	0,0000	0,0273	0,4680	-0,1513	-0,1031	-0,1188	-0,1427	-0,1427	-0,1380	-0,1346	-0,1269	-0,1204	-0,1215	-0,1308
7	0,0000	0,0799	0,3067	0,5026	-0,3012	-0,0389	0,3584	0,3584	0,2808	0,2240	0,0948	-0,0137	0,0061	0,1607
8	0,0000	0,0030	0,0113	0,0186	-0,0111	-0,2075	-0,6338	-0,6338	-0,4469	-0,4043	-0,3076	-0,2264	-0,2412	-0,3569
9	0,0000	0,0017	0,0066	0,0108	-0,0065	-0,1211	-0,1658	-0,1658	-0,2608	-0,2360	-0,1795	-0,1321	-0,1408	-0,2083
10	0,0000	-0,0047	-0,0179	-0,0294	0,0176	-0,6714	-0,2004	-0,2004	-0,2924	-0,3597	-0,5128	-0,6415	-0,6181	-0,4348
11	0,0000	-0,0028	-0,0108	-0,0177	0,0106	0,1979	-0,1207	-0,1207	-0,1760	-0,2873	-0,5402	0,1683	0,1452	-0,0356
12	0,0000	-0,0004	-0,0016	-0,0026	0,0016	0,0291	-0,0177	-0,0177	-0,0259	-0,0161	0,0061	-0,5211	-0,1697	-0,0887
13	0,0000	-0,0014	-0,0056	-0,0091	0,0055	0,1017	-0,0620	-0,0620	-0,0904	-0,0563	0,0213	-0,2886	-0,5936	-0,3104
14	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
15	0,0000	0,0030	0,0113	0,0186	-0,0111	-0,2075	0,3662	0,3662	-0,4469	-0,4043	-0,3076	-0,2264	-0,2412	-0,3569
16	0,0000	0,0028	0,0108	0,0177	-0,0106	-0,1979	0,1207	0,1207	0,1760	-0,7127	-0,4598	-0,1683	-0,1452	0,0356
17	0,0000	0,0019	0,0071	0,0117	-0,0070	-0,1307	0,0797	0,0797	0,1163	0,0724	-0,0274	-0,1902	-0,2367	-0,6008
18	0,0000	0,0028	0,0108	0,0177	-0,0106	-0,1979	0,1207	0,1207	0,1760	0,2873	-0,4598	-0,1683	-0,1452	0,0356
19	0,0000	-0,0004	-0,0016	-0,0026	0,0016	0,0291	-0,0177	-0,0177	-0,0259	-0,0161	0,0061	0,4789	-0,1697	-0,0887
20	0,0000	-0,0019	-0,0071	-0,0117	0,0070	0,1307	-0,0797	-0,0797	-0,1163	-0,0724	0,0274	0,1902	0,2367	-0,3992
B R A N C H N O														

Table 5.4

Here at table 5.4, let say at colon 6, we have 20 different numbers. These numbers give us the sensitivities of each line (branch) with respect to load at bus no 6. The first number of this column is 0.6291. The meaning of this number is that, if we reduce this load by 1 MW the power flow at branch 1 (The line from bus 1 to bus 2) reduces by 0,6291 MW. All numbers at this column have the similar meaning. Second number gives us the ratio of change power flow at branch 2 with respect to change of load at bus 6. So, the l^{th} number at column 6 gives us the ratio of power flow change at branch l with respect to change of load at bus 6. Now we are able to calculate the new active power flow for each branch after this load at bus 6 change.

Suppose the load at bus 6 increases by 11.2 MW and reach 22.4 MW. Again we can assume that the load change at bus 6 as 11.2 MW. Also we know the previous active power flow for every bus. If we multiply these previous active power flows with this change of load and with the sensitivity of each line with respect to load change at bus 6, we will find the new active power flows for every branch.

Branch No	From Bus	To Bus	Previous Active Power Flow P(MW)	PTDF of each Line Flows wrt Load 6	Active Power Flow after Load 6 increase by 11.2 MW P(MW)
1	1	2	156,88	-0,6291	163,93
2	1	5	75,51	-0,3709	79,66
3	2	3	73,24	-0,1188	74,57
4	2	4	56,13	-0,2487	58,92
5	2	5	41,52	-0,2616	44,45
6	3	4	-23,29	-0,1188	-21,95
7	4	5	-61,16	-0,0389	-60,72
8	4	7	28,07	-0,2075	30,40
9	4	9	16,08	-0,1211	17,44
10	5	6	44,09	-0,6714	51,61
11	6	11	7,35	0,1979	5,14
12	6	12	7,79	0,0291	7,46
13	6	13	17,75	0,1017	16,61
14	7	8	0,00	0,0000	0,00
15	7	9	28,07	-0,2075	30,40

16	9	10	5,23	-0,1979	7,44
17	9	14	9,43	-0,1307	10,89
18	10	11	-3,79	-0,1979	-1,57
19	12	13	1,61	0,0291	1,29
20	13	14	5,64	0,1307	4,18

Table 5.5

Table 5.5 shows us the new calculated active power flow of each branch after the load at bus 6 increase by 11.2 MW. These values are calculated by using the sensitivities of all lines with respect to load 6. At the **section 4.3**, the method for calculation of numerical sensitivity explained. To see and prove this, at this 14 bus test system, load at bus 6 increased by 11.2 MW and the simulation program execute again. After this execution, we get new results for the system.

Bus No	Bus Type	Voltage		Generation		Load	
		Mag(pu)	Ang(rad)	P(MW)	Q(MVAr)	P(MW)	Q(MVAr)
1	slack	1,0600	0,00	244,71	-17,84	0,00	0,00
2	Generator	1,0450	-0,09	40,00	47,13	21,70	12,70
3	Generator	1,0100	-0,23	0,00	25,88	94,20	19,00
4	Load	1,0162	-0,19			47,80	-3,90
5	Load	1,0176	-0,16			7,60	1,60
6	Generator	1,0700	-0,27	0,00	15,67	22,40	7,50
7	Load	1,0614	-0,25			0,00	0,00
8	Generator	1,0900	-0,25	0,00	17,72	0,00	0,00
9	Load	1,0566	-0,28			29,50	16,60
10	Load	1,0516	-0,28			9,00	5,80
11	Load	1,0574	-0,28			3,50	1,80
12	Load	1,0551	-0,29			6,10	1,60
13	Load	1,0506	-0,29			13,50	5,80
14	Load	1,0360	-0,30			14,90	5,00

Table 5.6

We can see from Table 5.6 that the load at bus 6 is change. This load changing, change all system calculated data (or measured data in a Power System). The generation at

slack bus change. Some of the voltages and angle at the buses also change. So it is obvious that Power Flows are going to change. Now we can calculate these new power flows for every branch.

Branch	From	To	From Bus Injection		To Bus Injection	
No	Bus	Bus	P(MW)	Q(MVAr)	P(MW)	Q(MVAr)
1	1	2	164,71	-22,21	-159,97	30,85
2	1	5	80,00	4,38	-76,90	3,12
3	2	3	74,69	3,42	-72,28	2,13
4	2	4	59,02	-1,36	-57,17	3,37
5	2	5	44,55	1,52	-43,51	-2,02
6	3	4	-21,92	4,75	22,26	-5,21
7	4	5	-60,51	16,67	61,02	-15,07
8	4	7	30,28	-10,21	-30,28	12,19
9	4	9	17,34	-0,73	-17,34	2,25
10	5	6	51,79	12,37	-51,79	-6,38
11	6	11	5,21	4,32	-5,17	-4,24
12	6	12	7,53	2,64	-7,46	-2,50
13	6	13	16,65	7,58	-16,46	-7,20
14	7	8	0,00	-17,25	0,00	17,72
15	7	9	30,28	5,07	-30,28	-4,15
16	9	10	7,35	3,43	-7,34	-3,38
17	9	14	10,77	3,08	-10,62	-2,78
18	10	11	-1,66	-2,42	1,67	2,44
19	12	13	1,36	0,90	-1,35	-0,90
20	13	14	4,31	2,30	-4,28	-2,22

Table 5.7

Table 5.7 shows the new calculated power flows for each branch by using the data calculated from Newton Rapson Method at **Table 5.6**.

The differences between 2 methods which can be used for calculating new power flows after changing a load is:

a- To find the new power flows at **Table 5.5** we don't solve all the system again after changes. We use PTDF and old data and calculate simply the new values for each branch.

b- To find the new power flows at **Table 5.7** we directly calculate everything for the system at the beginning which explained at chapter 2 and find all power flows for every branch.

Now, for the next step, we can use these two different power flows and the two different load value at bus 6 and can calculate PTDF for all branch with respect to load 6.

Branch	From	To	From Bus Injection before Load 6 change	From Bus Injection after Load 6 change	PTDF of any branch wrt Load 6
No	Bus	Bus	P(MW)	P(MW)	$\Delta P_l / \Delta P_6$
1	1	2	156,88	164,71	-0,6987
2	1	5	75,51	80,00	-0,4010
3	2	3	73,24	74,69	-0,1299
4	2	4	56,13	59,02	-0,2579
5	2	5	41,52	44,55	-0,2711
6	3	4	-23,29	-21,92	-0,1217
7	4	5	-61,16	-60,51	-0,0576
8	4	7	28,07	30,28	-0,1969
9	4	9	16,08	17,34	-0,1127
10	5	6	44,09	51,79	-0,6876
11	6	11	7,35	5,21	0,1915
12	6	12	7,79	7,53	0,0231
13	6	13	17,75	16,65	0,0978
14	7	8	0,00	0,00	0,0000
15	7	9	28,07	30,28	-0,1969
16	9	10	5,23	7,35	-0,1899
17	9	14	9,43	10,77	-0,1197
18	10	11	-3,79	-1,66	-0,1894
19	12	13	1,61	1,36	0,0228
20	13	14	5,64	4,31	0,1188

Table 5.8

At **Table 5.8** we can see the calculated Power Transfer Distribution Factors by using the formula $\Delta P_l / \Delta P_6$. These PTDFs are a little different from the PTDFs which are calculated by using the formula (3.18). We calculate these differences at the simulation program and find the difference approximately %4 which are shown at **section 5.3**. There are some differences because when we use the method calculated at table 5.8, the changes of all active power of all lines affected by the new voltages and angles. As we see from the **Tables 5.1 and 5.6** the Voltages and the angles of all buses change and also the losses change.

Branch No	From Bus	To Bus	Power Loss at Each Branch	
1	1	2	4,74	8,63
2	1	5	3,10	7,49
3	2	3	2,42	5,55
4	2	4	1,85	2,01
5	2	5	1,04	0,50
6	3	4	0,33	0,46
7	4	5	0,51	1,61
8	4	7	0,00	1,98
9	4	9	0,00	1,52
10	5	6	0,00	5,99
11	6	11	0,04	0,08
12	6	12	0,07	0,14
13	6	13	0,19	0,38
14	7	8	0,00	0,47
15	7	9	0,00	0,92
16	9	10	0,02	0,05
17	9	14	0,14	0,30
18	10	11	0,01	0,02
19	12	13	0,01	0,00
20	13	14	0,04	0,08
		TOTAL	14,51	38,19

Table 5.9

At **Table 5.9** we can see that the losses increase. For the Active Power, the losses increase as 1,12 MW. This is why the Active Power at **Table 5.5 and 5.7** are different. The calculation method which is used for the Table 5.5 ignores the changes of losses.

At chapter 4, Demand Side Management by using Sensitivity Analysis is explained. If we look closer to **Table 5.4**, which has the data of PTDF, we can understand how this PTDF values can help any Utility to make contracts with customers and manage the Loads. Here the first element of the colon 2 is 0.838. This means that for branch no. 1, which is from bus 1 to bus 2, Load at bus 2 is the most valuable. Now for preventing congestion at branch 1, it is worth to pay more for curtailing this load at bus 2. Again if we look the value at colon 2 row 16, it is 0,0028. If we have congestion at branch no 16, it will not be worth to pay for curtailing the Load at bus 2. All values at this Table 5.4 help the utility about these contracts.

6 Conclusion

It is clear that the most effective method for calculation of PTDF matrix is the physical parameters of the lines of the network. But, if we try to use all physical parameters, it is almost impossible to solve it mathematically. The best way is making assumptions by using well known approximation methods and matrix manipulation techniques.

Using these PTDF matrices can help the system operator for making better plans. But every system and electricity market has its own dynamics and when we talk about load curtailing, we must consider some other things like ATC (Available Transfer Capacity). Sometimes, the most effective line for an individual load may have a large ATC and there is no meaning curtailing that load to relieve the network. But if we take into consideration sensitivity of all loads to all lines and if we have a line which is congested, now we can say that which load can be curtailed to help us relieve that line.

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