ABSTRACT

This work consists of definitions and basic properties of biorthogonal polynomials and some examples of biorthogonal polynomials family.

Biorthogonal polynomials are first introduced in 1965 by Konhauser and one pair of biorthogonal polynomials which are suggested by the Laguerre polynomials are called Konhauser polynomials. Another pair of biorthogonal polynomials are suggested by the Jacobi polynomials.

Several properties as generating functions, differential equations and recurrence relations for these biorthogonal polynomial families are obtained.

Key words: Orthogonal polynomials, biorthogonal polynomials, Laguerre polynomials, Jacobi polynomials, Konhauser polynomials, bilateral generating function, recurrence relation.

ÖZET

Bu çalışma biortogonal polinomların tanımları ve temel özellikleriyle bazı biortogonal polinom ailelerinin tanımlarımı içermektedir.

Biortogonal polinomlar ilk olarak 1965 yılında Konhauser tarafından çalışıldı. Bundan dolayı Konhauser tarafından bulunun ve Laguerre polinomları tarafından belirtilen biortogonal polinomlar olarak adlandırılan polinomlara Konhauser polinomları da denmektedir. Diğer bir biortogonal polinom ailesi de Jacobi polinomları tarafından belirtilen biortogonal polinomlardır.

Bu biortogonal polinom aileleri için doğurucu fonksiyon, diferensiyel denklem ve imdirgeme bağıntıları gibi birçok özellik elde edilmiştir.

Anahtar kelimeler: Ortogonal polinomlar, biortogonal polinomlar, Laguerre polinomları, Jacobi polinomları, Konhauser polinomları, bilateral doğurucu fonksiyon, indirgeme bağıntısı.

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