



ABSTRACT

Heat transfer mechanism divided into three main parts; conduction, convection and radiation. Conduction is an energy diffusion process in materials which does not contain molecular convection. Kinetic energy exchanged between molecules results in a net transfer between regions of different energy levels; these energy levels are commonly called temperature. Particularly, heat conduction in metals is mainly attributed to the motion of free electrons and in solid electrical insulators to the longitudinal oscillations of atoms. In fluids. The elastic impact of molecules is considered as heat conduction process.

By Working on the unsteady heat conduction theory to create a program by using Microsoft excel, this program can solve any one-dimensional unsteady heat conduction problem by finding:

$T(x, t)$: Temperature at any given distance & time.

Or

$t(x, T)$: Time at any given distance & temperature .

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List of symbols

h	Convection heat transfer coefficient, $\text{W}/\text{m}^2\cdot\text{K}$
L	Length; half thickness of a plane wall
R, r_0	Radius, m
t	Time, s
T	Temperature, $^{\circ}\text{C}$ or K
α	Thermal diffusivity
τ	Fourier number
θ	Dimensionless temperature
Bi	Biot number
K	Thermal conductivity
J	Bessel function

Chapter 1: INTRODUCTION

The most basic problem of time dependent conduction is the calculation of the temperature history inside a conducting body that is immersed suddenly in a bath of fluid at a different temperature. This problem finds application in many areas, for example, in the heat treating (e.g., quenching) of special alloys. The temperature of such a body, in general, varies with time as well as position. In rectangular coordinates, this variation is expressed as $T(x, y, z, t)$, where (x, y, z) indicates variation in the x , y and z directions, respectively, and t indicates variation with time. In the preceding chapter, we considered heat conduction under steady conditions, for which the temperature of a body at any point does not change with time. This certainly simplified the analysis, especially when the temperature varied in one direction only, and we were able to obtain analytical solutions. In this chapter, we consider the variation of temperature with time as well as position in one dimensional system.

Transient conduction occurs when the temperature within an object changes as a function of time. Analysis of transient systems is more complex and often calls for the application of approximation theories or numerical analysis by computer.

Moreover, there is a special case that we are going to show in the program where the system is lumped, where Bi is too small to actually make a difference in the values of the temperature in the system, thus, the temperature becomes uniform during the entire process.

Interior temperatures of some bodies remain essentially uniform at all times during a heat transfer process. The temperature of such bodies are only a function of time, $T = T(t)$. The heat transfer analysis based on this idealization is called lumped system analysis.

Chapter 2: TRANSIENT HEAT CONDUCTION

2-1 Transient Heat Conduction In Large Plane Walls, Long Cylinders, And Spheres With Spatial Effects

In general, however, the temperature within a body changes from point to point as well as with time. In this section, we consider the variation of temperature with time and position in one-dimensional problems such as those associated with a large plane wall, a long cylinder, and a sphere.

Consider a plane wall of thickness $2L$, a long cylinder of radius r_o , and a sphere of radius r_o initially at a uniform temperature T_i , as shown in Fig. 2-2. At time $t = 0$, each geometry is placed in a large medium that is at a constant temperature T_∞ and kept in that medium for $t > 0$. Heat transfer takes place between these bodies and their environments by convection with a uniform and constant heat transfer coefficient h . Note that all three cases possess geometric and thermal symmetry: the plane wall is symmetric about its center plane ($x = 0$), the cylinder is symmetric about its centerline ($r = 0$), and the sphere is symmetric about its center point ($r = 0$). We neglect radiation heat transfer between these bodies and their surrounding surfaces, or incorporate the radiation effect into the convection heat transfer coefficient h . The variation of the temperature profile with time in the plane wall is illustrated in Fig. 2-3. When the wall is first exposed to the surrounding medium at $T_\infty < T_i$ at $t = 0$, the entire wall is at its initial temperature T_i but the wall temperature at and near the surfaces starts to drop as a result of heat transfer from the wall to the surrounding medium. This creates a temperature gradient in the wall and initiates heat conduction from

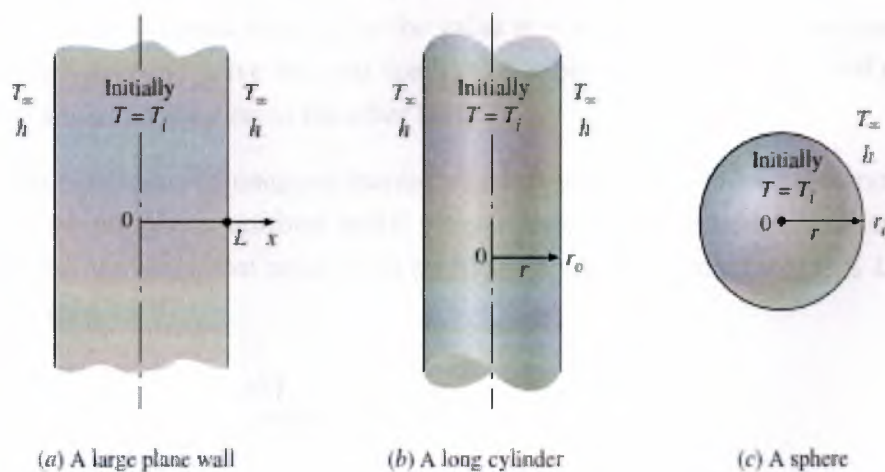


FIGURE 2-1: Schematic of the simple geometries in which heat transfer is one-dimensional.

the inner parts of the wall toward its outer surfaces. Note that the temperature at the center of the wall remains at T_i until $t = t_2$, and that the temperature profile within the wall remains symmetric at all times about the center plane. The temperature profile gets flatter and flatter as time passes as a result of heat transfer, and eventually becomes uniform at $T = T_\infty$. That is, the wall reaches thermal equilibrium with its surroundings. At that point, heat transfer stops since there is no longer a temperature difference. Similar discussions can be given for the long cylinder or sphere.

2.1.1 Non-Dimensionalized One-Dimensional Transient Conduction Problem

The formulation of heat conduction problems for the determination of the one-dimensional transient temperature distribution in a plane wall, a cylinder, or a sphere results in a partial differential equation whose solution typically involves infinite series and transcendental equations, which are inconvenient to use. But the analytical solution provides valuable insight to the physical problem, and thus it is important to go through the steps involved. Below we demonstrate the solution procedure for the case of plane wall.

Consider a plane wall of thickness $2L$ initially at a uniform temperature of T_i , as shown in Fig. 2-2a. At time $t = 0$, the wall is immersed in a fluid at temperature T_∞ and is subjected to convection heat transfer from both sides with a convection coefficient of h . The height and the width of the wall are large relative to its thickness, and thus heat conduction in the wall can be approximated to be one-dimensional. Also, there is thermal symmetry about the midplane passing through $x = 0$, and thus the temperature distribution must be symmetrical about the midplane. Therefore, the value of temperature at any $-x$ value in $-L \leq x \leq 0$ at any time t must be equal to the value at $+x$ in $0 \leq x \leq L$ at the same time. This means we can formulate and solve the heat conduction problem in the positive half domain $0 \leq x \leq L$, and then apply the solution to the other half.

Under the conditions of constant thermo physical properties, no heat generation, thermal symmetry about the midplane, uniform initial temperature, and constant convection coefficient, the one-dimensional transient heat conduction problem in the half-domain $0 \leq x \leq L$ of the plain wall can be expressed as

$$\text{Differential equations:} \quad \frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (2-1 a)$$

$$\text{Boundary conditions:} \quad \frac{\partial T(0, t)}{\partial x} = 0 \quad \text{and} \quad -k \frac{\partial T(L, t)}{\partial x} = h[T(L, t) - T_\infty] = h \quad (2-1 b)$$

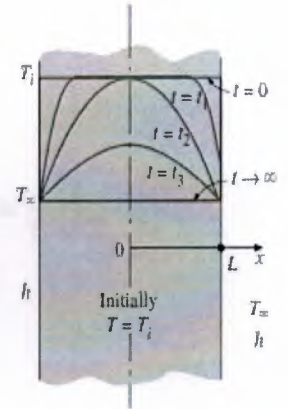


FIGURE 2- 2: transient temperature profiles in a plane wall exposed to convection from its surfaces for $T_i > T_\infty$.

Initial condition: $T(x, 0) = T_i$ (2-1 c)

Where the property $\alpha = k / \rho c_p$ is the thermal diffusivity of the material.

We now attempt to non-dimensionalize the problem by defining a dimensionless space variable $X = x/L$ and dimensionless temperature $\theta(x, \tau) = \frac{T(x, \tau) - T_\infty}{T_i - T_\infty}$. These are convenient choices since both X and θ vary between 0 and 1. However, there is no clear guidance for the proper form of the dimensionless time variable and the h/k ratio, so we will let the analysis indicate them. We note that

$$\frac{\partial \theta}{\partial X} = \frac{\partial T \theta}{\partial x/L} = \frac{L}{T_i - T_\infty} \frac{\partial T}{\partial x}, \quad \frac{\partial^2 \theta}{\partial X^2} = \frac{L^2}{T_i - T_\infty} \frac{\partial^2 T}{\partial x^2} \quad \text{and} \quad \frac{\partial \theta}{\partial \tau} = \frac{1}{T_i - T_\infty} \frac{\partial T}{\partial t}$$

Substituting into Eqs. 2-1a and 2-1b and rearranging give

$$\frac{\partial^2 \theta}{\partial X^2} = \frac{L^2}{\alpha} \frac{\partial \theta}{\partial \tau} \quad \text{and} \quad \frac{\partial \theta(1, \tau)}{\partial X} = \frac{hL}{k} \theta(1, \tau) \quad (2-2)$$

Therefore, the proper form of the dimensionless time is $\tau = \alpha t/L^2$, which is called the **Fourier number** Fo , and we recognize $Bi = k/hL$ as the Biot number. Then the formulation of the one dimensional transient heat conduction problem in a plane wall can be expressed in non-dimensional form as

Dimensionless differential equation: $\frac{\partial^2 \theta}{\partial X^2} = \frac{\partial \theta}{\partial \tau}$ (2-3 a)

Dimensionless BC's: $\frac{\partial \theta(0, \tau)}{\partial X} = 0$ and $\frac{\partial \theta(1, \tau)}{\partial X} = -Bi \theta(1, \tau)$ (2-3 b)

Dimensionless initial condition: $\theta(X, 0) = 1$ (2-3 c)

Where,

$$\theta(x, \tau) = \frac{T(x, \tau) - T_\infty}{T_i - T_\infty}$$

Dimensionless temperature

$$X = \frac{x}{L}$$

Dimensionless distance from the center

$$Bi = \frac{hL}{k}$$

Dimensionless heat transfer coefficient (Biot number)

$$\tau = \frac{\alpha t}{L^2}$$

Dimensionless time (Fourier number)

The heat conduction equation in cylindrical or spherical coordinates can be non-dimensionalized in a similar way. Note that non-dimensionalization reduces the number of independent variables and parameters from 8 to 3 from $x, L, t, k, \alpha, h, T_i$, and T_∞ to X, Bi , and Fo (Fig. 2-3).

$$\theta = f(x, Bi, \tau) \quad (2-4)$$

This makes it very practical to conduct parametric studies and to present results in graphical form. Recall that in the case of lumped system analysis, we had $\theta = f(Bi, Fo)$ with no space variable.

2.1.2 Exact Solution of One-Dimensional Transient Conduction Problem

The non-dimensionalized partial differential equation given in Eqs. 2-3 together with its boundary and initial conditions can be solved using several analytical and numerical techniques, including the Laplace or other transform methods, the method of separation of variables, the finite difference method, and the finite-element method. Here we use the method of **separation of variables** developed by J. Fourier in 1820s and is based on expanding an arbitrary function (including a constant) in terms of Fourier series. The method is applied by assuming the dependent variable to be a product of a number of functions, each being a function of a single independent variable. This reduces the partial differential equation to a system of ordinary differential equations, each being a function of a single independent variable. In the case of transient conduction in a plain wall, for example, the dependent variable is the solution function $\theta(X, \tau)$, which is expressed as $\theta(X, \tau) = F(X)G(\tau)$, and the application of the method results in two ordinary differential equation, one in X and the other in τ .

The method is applicable if (1) the geometry is simple and finite (such as a rectangular block, a cylinder, or a sphere) so that the boundary surfaces can be described by simple mathematical functions, and (2) the differential equation and the boundary and initial conditions in their most simplified form are linear (no terms that involve products of the dependent variable or its derivatives) and involve only one nonhomogeneous term (a term without the dependent variable or its derivatives). If the formulation involves a number of nonhomogeneous terms, the problem can be split up into an equal number of simpler problems each involving only one nonhomogeneous term, and then combining the solutions by superposition.

(a) Original heat conduction problem:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}, \quad T(x, 0) = T_i$$

$$\frac{\partial T(0, t)}{\partial x} = 0, \quad -k \frac{\partial T(L, t)}{\partial x} = h[T(L, t) - T_\infty]$$

$$T = F(x, L, t, k, \alpha, h, T_i)$$

(b) Nondimensionalized problem:

$$\frac{\partial^2 \theta}{\partial X^2} = \frac{\partial \theta}{\partial \tau}, \quad \theta(X, 0) = 1$$

$$\frac{\partial \theta(0, \tau)}{\partial X} = 0, \quad \frac{\partial \theta(1, \tau)}{\partial X} = -Bi\theta(1, \tau)$$

$$\theta = f(X, Bi, \tau)$$

FIGURE 2-3: non-dimensionalization reduces the number of independent variables in one-dimensional transient conduction problems from 8 to 3, offering great convenience in the presentation of results.

Now we demonstrate the use of the method of separation of variables by applying it to the one-dimensional transient heat conduction problem given in Eqs. 2–3. First, we express the dimensionless temperature function $\theta(X, \tau)$ as a product of a function of X only and a function of τ only as

$$\theta(X, \tau) = F(X)G(\tau) \quad (2-5)$$

Substituting Eq. 4–14 into Eq. 4–12a and dividing by the product FG gives

$$\frac{1}{F} \frac{d^2 F}{dX^2} = \frac{1}{G} \frac{dG}{d\tau} \quad (2-6)$$

Observe that all the terms that depend on X are on the left-hand side of the equation and all the terms that depend on τ are on the right-hand side. That is, the terms that are function of different variables are separated (and thus the name separation of variables). The left-hand side of this equation is a function of X only and the right-hand side is a function of only τ . Considering that both X and τ can be varied independently, the equality in Eq. 2–6 can hold for any value of X and τ only if Eq. 2–6 is equal to a constant. Further, it must be a negative constant that we will indicate by $-\lambda^2$ since a positive constant will cause the function $G(\tau)$ to increase indefinitely with time (to be infinite), which is unphysical, and a value of zero for the constant means no time dependence, which is again inconsistent with the physical problem. Setting Eq. 2–6 equal to $-\lambda^2$ gives

$$\frac{d^2 F}{dX^2} + \lambda^2 F = 0 \quad \text{and} \quad \frac{dG}{d\tau} + \lambda^2 G = 0 \quad (2-7)$$

whose general solutions are

$$F = c_1 \cos(\lambda X) + c_2 \sin(\lambda X) \quad \text{and} \quad G = c_3 e^{-\lambda^2 \tau} \quad (2-8)$$

And

$$\theta = FG = c_3 e^{-\lambda^2 \tau} [c_1 \cos(\lambda X) + c_2 \sin(\lambda X)] = e^{-\lambda^2 \tau} [A \cos(\lambda X) + B \sin(\lambda X)] \quad (2-9)$$

Where $A = C_1 C_3$ and $B = C_2 C_3$ are arbitrary constants. Note that we need to determine only A and B to obtain the solution of the problem.

$$\theta_n = A_n e^{-\lambda_n^2 \tau} \cos(\lambda_n X)$$

$$A_n = \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)}$$

$$\lambda_n \tan \lambda_n = \text{Bi}$$

For $\text{Bi} = 5$, $X = 1$, and $\tau = 0.2$:

n	λ_n	A_n	θ_n
1	1.3138	1.2402	0.22321
2	4.0336	-0.3442	0.00835
3	6.9096	0.1588	0.00001
4	9.8928	-0.876	0.00000

FIGURE 2-4: the term in the series solution of transient conduction problems decline rapidly as n and thus λ_n increases because of the exponential decay function with the exponent $-\lambda_n^2 \tau$

Applying the boundary conditions in Eq. 2-3b gives

$$\frac{d\theta(0,\tau)}{dx} = 0 \rightarrow -e^{-\lambda^2\tau}(A\lambda \sin 0 + B\lambda \cos 0) = 0 \rightarrow B = 0 \rightarrow \theta = Ae^{-\lambda^2\tau} \cos(\lambda X)$$

$$\frac{d\theta(1,\tau)}{dx} = -Bi\theta(1,\tau) \rightarrow -Ae^{-\lambda^2\tau}\lambda \sin \lambda = -BiAe^{-\lambda^2\tau} \cos \lambda \rightarrow \lambda \tan \lambda = Bi$$

But tangent is a periodic function with a period of π , and the equation $\lambda \tan \lambda = Bi$ has the root λ_1 between 0 and π , the root λ_2 between π and 2π , the root λ_n between $(n-1)\pi$ and $n\pi$, etc. To recognize that the transcendental equation $\lambda \tan \lambda = Bi$ has an infinite number of roots, it is expressed as

$$\lambda_n \tan \lambda_n = Bi \quad (2-10)$$

Eq. 2-10 is called the **characteristic equation** or **Eigen function**, and its roots are called the **characteristic values** or **eigenvalues**. The characteristic equation is implicit in this case, and thus the characteristic values need to be determined numerically. Then it follows that there are an infinite number of solutions of the form $Ae^{-\lambda^2\tau} \cos(\lambda X)$, and the solution of this linear heat conduction problem is a linear combination of them,

$$\theta = \sum_{n=1}^{\infty} A_n e^{-\lambda_n^2\tau} \cos(\lambda_n X) \quad (2-11)$$

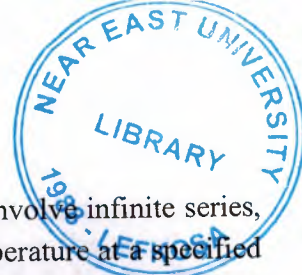
The constants A_n are determined from the initial condition, Eq. 2-3c,

$$\theta(X, 0) = 1 \rightarrow 1 = \sum_{n=1}^{\infty} A_n \cos(\lambda_n X) \quad (2-12)$$

This is a Fourier series expansion that expresses a constant in terms of an infinite series of cosine functions. Now we multiply both sides of Eq. 2-12 by $\cos \lambda_m X$, and integrate from $X = 0$ to $X = 1$. The right-hand side involves an infinite number of integrals of the form $\int_0^1 \cos(\lambda_m X) \cos(\lambda_n X) dx$. It can be shown that all of these integrals vanish except when $n = m$, and the coefficient A_n becomes

$$\int_0^1 \cos(\lambda_n X) dx = A_n \int_0^1 \cos^2(\lambda_n X) dx \rightarrow A_n = \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)} \quad (2-13)$$

This completes the analysis for the solution of one-dimensional transient heat conduction problem in a plane wall. Solutions in other geometries such as a long cylinder and a sphere can be determined using the same approach. The results for all three geometries are summarized in Table 2-1. The solution for the plane wall is also applicable for a plane wall of thickness L whose left surface at $x = 0$ is insulated and the right surface at $x = L$ is subjected to convection since this is precisely the mathematical problem we solved.



The analytical solutions of transient conduction problems typically involve infinite series, and thus the evaluation of an infinite number of terms to determine the temperature at a specified location and time. This may look intimidating at first, but there is no need to worry. As demonstrated in Fig. 2-5, the terms in the summation decline rapidly as n and thus λ_n increases because of the exponential decay function $e^{-\lambda_n^2 \tau}$. This is especially the case when the dimensionless time τ is large. Therefore, the evaluation of the first few terms of the infinite series (in this case just the first term) is usually adequate to determine the dimensionless temperature θ .

TABLE 2-1

Summary of the solutions for one-dimensional transient conduction in a plane wall of thickness $2L$, a cylinder of radius r_0 and a sphere of radius r_0 subjected to convection from all surfaces.*

Geometry	Solution	λ_n 's are the roots of
Plane wall	$\theta = \sum_{n=1}^{\infty} \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \cos(\lambda_n x/L)$	$\lambda_n \tan \lambda_n = \text{Bi}$
Cylinder	$\theta = \sum_{n=1}^{\infty} \frac{2 J_1(\lambda_n)}{\lambda_n J_0^2(\lambda_n) + J_1^2(\lambda_n)} e^{-\lambda_n^2 \tau} J_0(\lambda_n r/r_0)$	$\lambda_n \frac{J_1(\lambda_n)}{J_0(\lambda_n)} = \text{Bi}$
Sphere	$\theta = \sum_{n=1}^{\infty} \frac{4(\sin \lambda_n - \lambda_n \cos \lambda_n)}{2\lambda_n - \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \frac{\sin(\lambda_n x/L)}{\lambda_n x/L}$	$1 - \lambda_n \cot \lambda_n = \text{Bi}$

2.1.3 Approximate Analytical Solutions

The analytical solution obtained above for one-dimensional transient heat conduction in a plane wall involves infinite series and implicit equations, which are difficult to evaluate. Therefore, there is clear motivation to simplify the analytical solutions and to present the solutions in tabular or graphical form using simple relations.

The dimensionless quantities defined above for a plane wall can also be used for a cylinder or sphere by replacing the space variable x by r and the half-thickness L by the outer radius r_0 . Note that the characteristic length in the definition of the Biot number is taken to be the half-thickness L for the plane wall, and the radius r_0 for the long cylinder and sphere instead of V/A used in lumped system analysis.

We mentioned earlier that the terms in the series solutions in Table 4-1 converge rapidly with increasing time, and for $\tau > 0.2$, keeping the first term and neglecting all the remaining terms in the series results in an error under 2 percent. We are usually interested in the solution for times with $\tau > 0.2$, and thus it is very convenient to express the solution using this **one-term approximation**, given as

$$\text{Plane wall: } \theta_{\text{wall}} = \frac{T(x,t) - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \cos\left(\lambda_1 \frac{x}{L}\right) \quad (2-14)$$

$$\text{Cylinder: } \theta_{\text{cyl}} = \frac{T(r,t) - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} J_0\left(\lambda_1 \frac{r}{r_0}\right) \quad (2-15)$$

$$\text{Sphere: } \theta_{\text{sph}} = \frac{T(r,t) - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 \frac{r}{r_0})}{\lambda_1 \frac{r}{r_0}} \quad (2-16)$$

Where the constants A_1 and λ_1 are functions of the Bi number only, and their values are listed in Table 4-2 against the Bi number for all three geometries. The function J_0 is the zeroth-order Bessel function of the first kind, whose value can be determined from Table 4-3. Noting that $\cos(0) = J_0(0) = 1$ and the limit of $(\sin x)/x$ is also 1, these relations simplify to the next ones at the center of a plane wall, cylinder, or sphere[3] :

Plane wall:

$$\theta_{wall} = \frac{T(0)-T_{\infty}}{T_i-T_{\infty}} = A_1 e^{-\lambda_1^2 \tau}$$

(2-17)

Cylinder:

$$\theta_{cyl} = \frac{T(0)-T_{\infty}}{T_i-T_{\infty}} = A_1 e^{-\lambda_1^2 \tau}$$

(2-18)

Sphere:

$$\theta_{sph} = \frac{T(0)-T_{\infty}}{T_i-T_{\infty}} = A_1 e^{-\lambda_1^2 \tau}$$

(2-19)

TABLE 2-2

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres ($Bi = hL/k$ for a plane wall of thickness $2L$, and $Bi = hr_o/k$ for a cylinder or sphere of radius r_o)

Bi	Plane Wall		Cylinder		Sphere	
	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

TABLE 2-3

The zeroth- and first-order Bessel functions of the first kind

η	$J_0(\eta)$	$J_1(\eta)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	-0.0968	-0.4708
2.8	-0.1850	-0.4097
3.0	-0.2601	-0.3391
3.2	-0.3202	-0.2613

Chapter 3: THE PROGRAM

For 1D unsteady heat conduction simulation program we used the Microsoft excel to create the program along fourteen work sheets.

3.1 Flow Chart

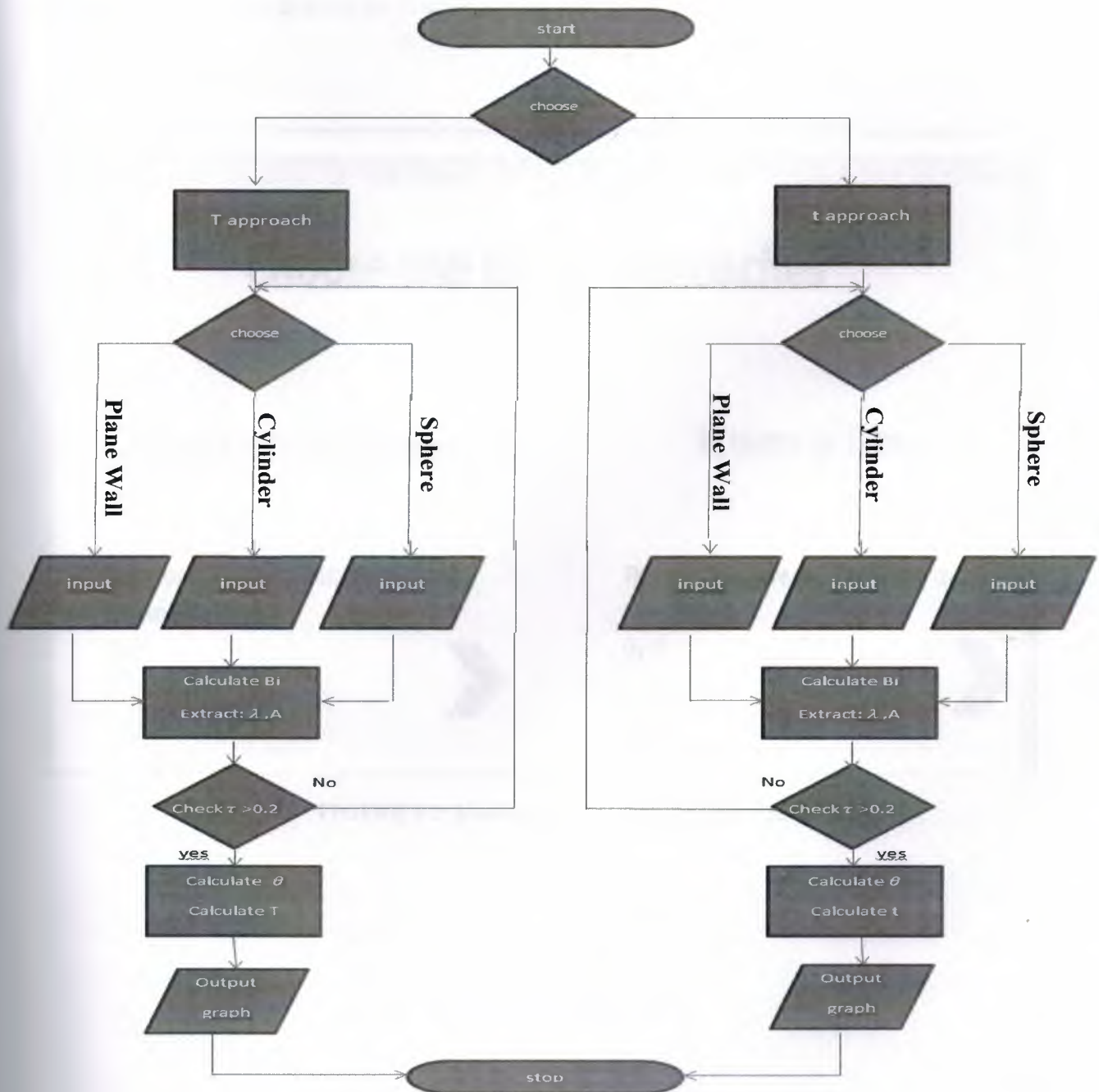


FIGURE 3-1: Flow chart for the program

3.2 Interface

In the first sheet we have the interface, where we can choose one of the two approaches:

_ Case 1: $T(x,t)$: T at any given distance & time

_ Case 2: $t(x,T)$: t at any given distance & temperature

Clicking the arrow moves us to the relative home sheet.

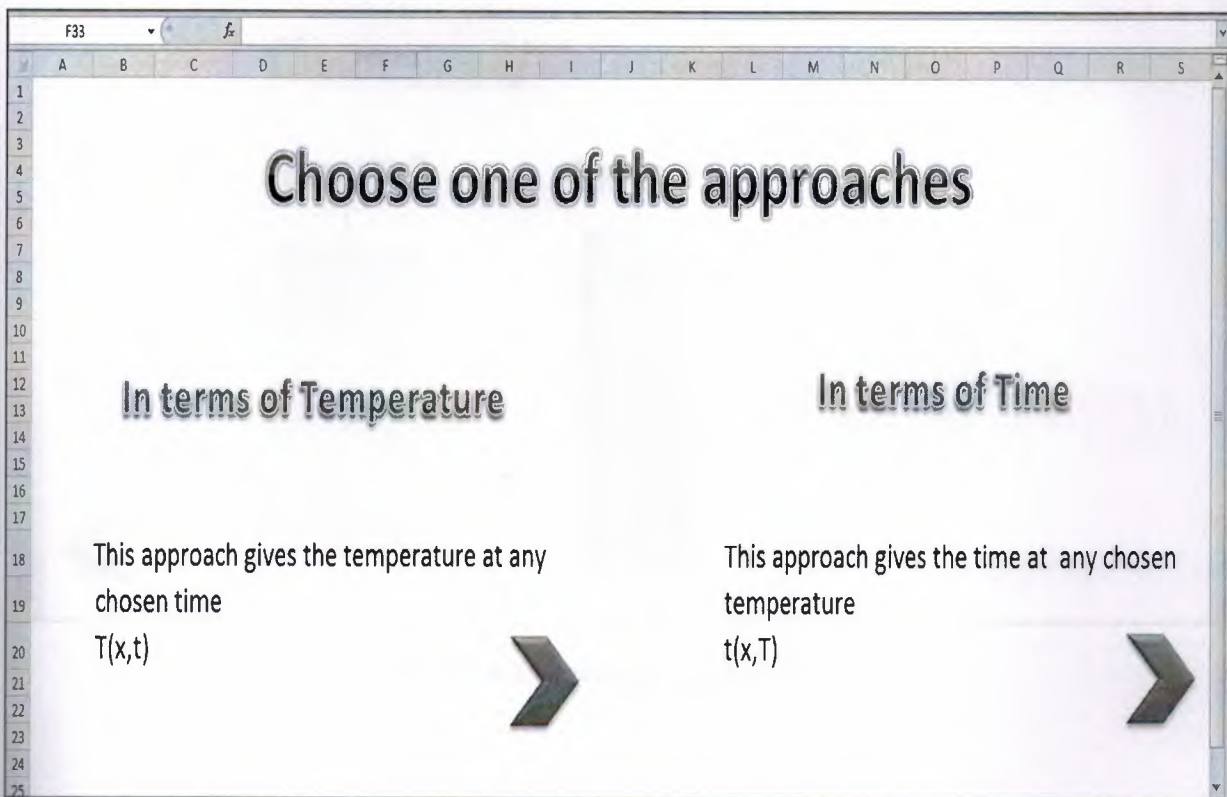


FIGURE 3-2: Interface sheet of the excel program

3.3 Home

We can choose one of the three geometries (plane wall, cylinder and sphere). By clicking on the geometry itself, it moves us to the relative input sheet.

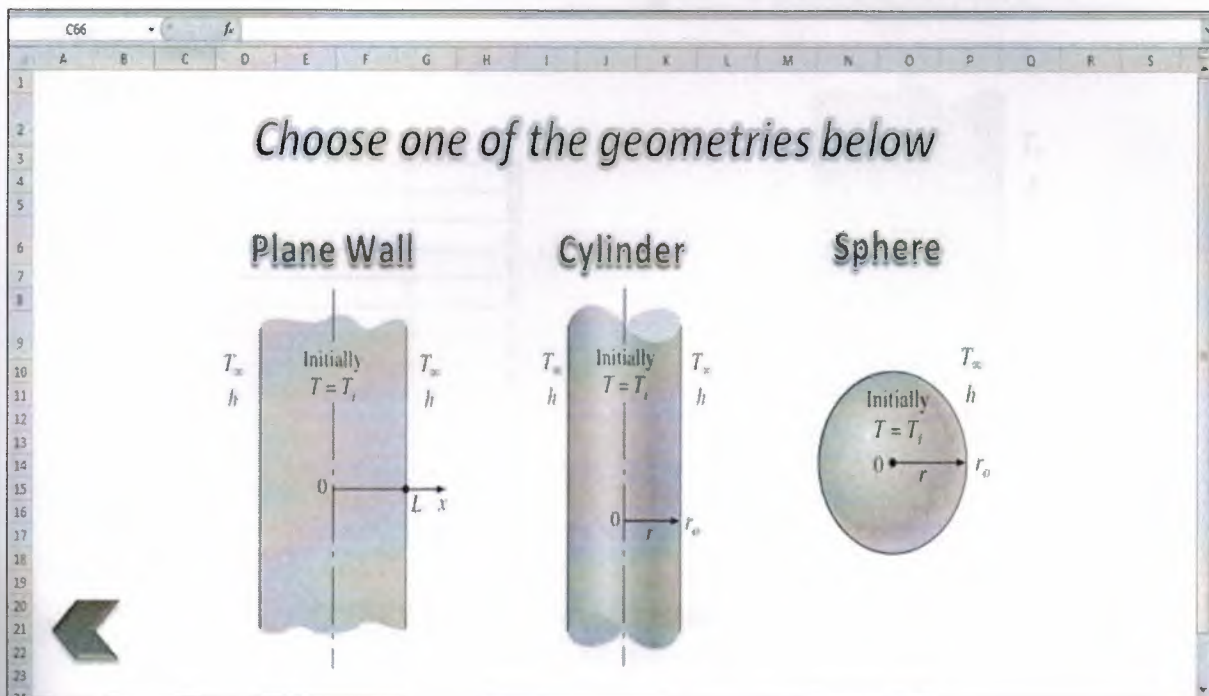


FIGURE 3-3: Geometry choosing sheet.

3.4 Case: 1: In terms of temperature

3.4.1 Input sheets

In this sheet the inputs of the program which are eight parameters ($h, k, x, L, \alpha, t, T_i, T_\infty$) are entered manually. And we have three different Input sheets for Plane wall, cylinder and sphere.

INPUT		
h [w/m ² k]	=	0
k [w/mk]	=	0
x [m]	=	0
L [m]	=	0
α [m ² /s]	=	0
t [s]	=	0
T_i [k]	=	0
T_∞ [k]	=	0

FIGURE 3-4: Input sheet of the plane wall case 1

INPUT		
h [w/m ² k]	=	0
k [w/mk]	=	0
r [m]	=	0
r_o [m]	=	0
α [m ² /s]	=	0
t [s]	=	0
T_i [k]	=	0
T_∞ [k]	=	0

FIGURE 3-5: Input sheet of the cylinder case 1

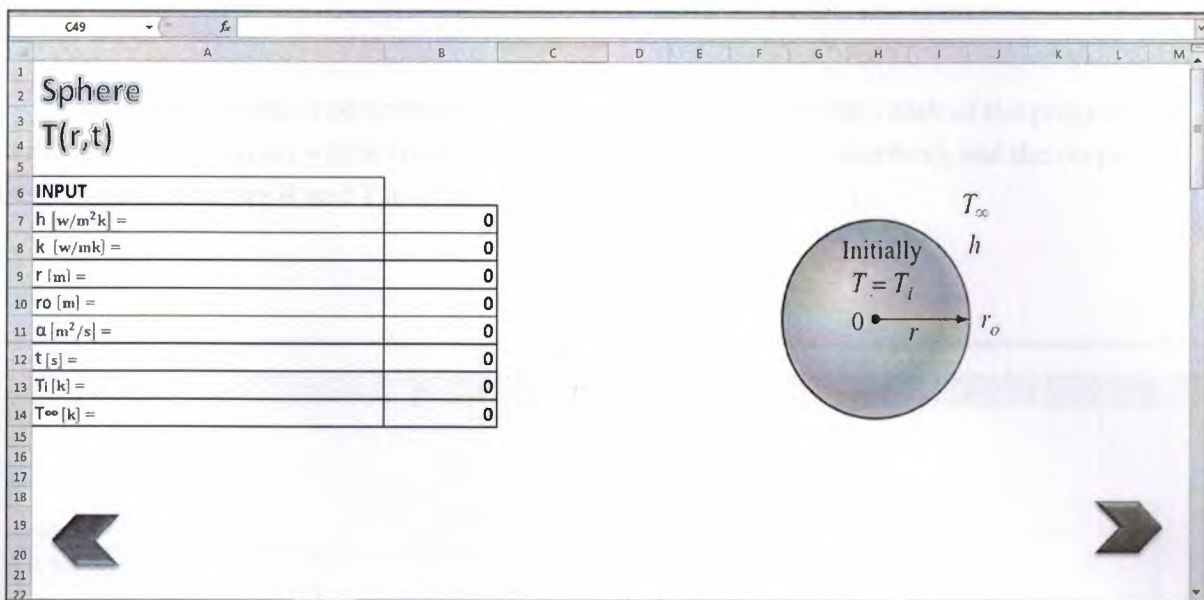


FIGURE 3-6: Input sheet of the sphere case 1

Where,

h : Convection heat transfer coefficient

k : Thermal conductivity

L : Thickness

x : Distance at any point

α : Thermal diffusivity

T_i : Initial temperature

T_∞ : Fluid temperature

t : Time at a given point & temperature

Remark: In the cylinder and sphere x and L inputs are replaced by r and r_0 .

Pressing the arrows related to the input sheet of the specified geometry, the program moves to the respective output sheet.

3.4.2 Output sheet of case 1

The output sheet is common for all the geometries. It shows the result of the program: the variables of the program which are Bi (Biot Number) and τ (Fourier number), and the outputs of the program which are θ and T and the graph (θ in as a function of τ).

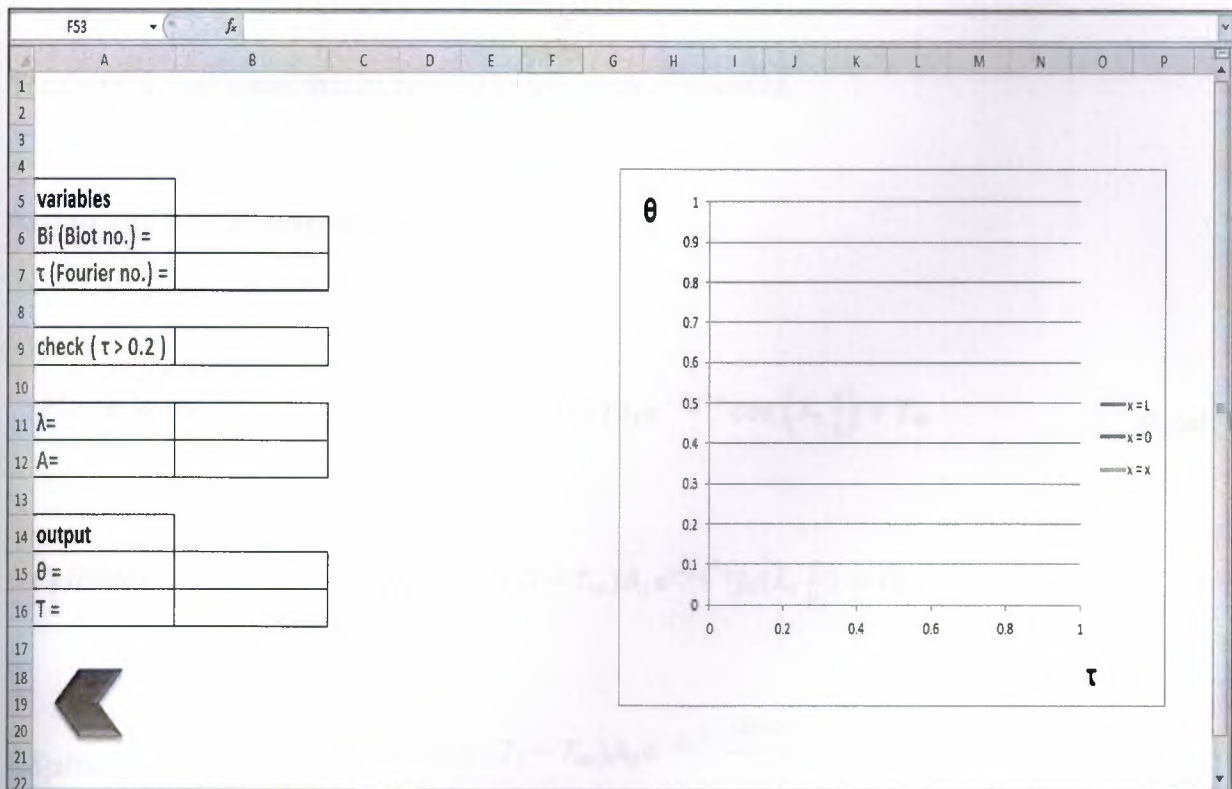


FIGURE 3-7: Output sheet of case 1

The above graph shows the three curves:

- $x = L$ (at the surface)
- $x = x$ (at any point)
- $x = 0$ (at the center)

3.4.3 Output Calculation

In this case we are looking for temperature and θ , we also need to find the variables before that

$$Bi = \frac{hL}{k}$$

We find Bi to extract the λ and A

$$\tau = \frac{\alpha t}{L^2}$$

We find τ to make sure that its applicable for one term equation

And we can find the temperature for

Plane wall
$$T(x, t) = (T_i - T_\infty)A_1 e^{-\lambda_1^2 \tau} \cos\left(\lambda_1 \frac{x}{L}\right) + T_\infty \quad (3-1a)$$

Cylinder
$$T(r, t) = (T_i - T_\infty)A_1 e^{-\lambda_1^2 \tau} J_0\left(\lambda_1 \frac{r}{r_0}\right) + T_\infty \quad (3-1b)$$

Sphere
$$T(r, t) = (T_i - T_\infty)A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 \frac{r}{r_0})}{\lambda_1 \frac{r}{r_0}} + T_\infty \quad (3-1c)$$

And for θ

$$\theta = \frac{T(x, t) - T_\infty}{T_i - T_\infty}$$

3.4.4 How the graph drawn

The program starts by calculating t initial and t final for (x=0, x=x and x=L)

The screenshot shows an Excel spreadsheet titled "Plane Wall" with columns A through R and rows 1 through 37. The spreadsheet is organized into three main sections for calculations at different positions: "at given x", "x=0", and "x=L". Each section contains a table of temperature (T) and time (t) values. Below these tables, there are summary rows for "Tini", "Tlast", and "T given". The "Tini" and "Tlast" rows show values of 0, while the "T given" row shows a value of 0.001. The "time at given B" row shows a value of 0.001. The "T" and "t" columns are labeled with "#DIV/0!" errors, indicating that the formulas are not yet entered or are incorrect.

FIGURE 3-8: Calculation sheet case 1

- t initial

$$\text{Eq} \quad t = 0.2 * \frac{L^2}{\alpha}$$

By use this code:

$$=(0.2*'wall plane'!B10*'wall plane'!B10)/'wall plane'!B11$$

- t final

$$\text{Eq} \quad t = -\frac{L^2}{\alpha \lambda_1^2} \ln \left[\frac{0.001}{(A_1 \cos(\lambda_1 \frac{x}{L}))} \right]$$

By using this code:

$$=-\text{LN}(0.001/(\text{output!}B12*\text{COS}(\text{output!}B11*'wall plane'!B9/'wall plane'!B10)))*('wall plane'!B10^2)/(\text{output!}B11^2*'wall plane'!B11)$$

And then we substitute the value of t for the first cell

and for the remaining cells we use this code

$$=(F17-E4)/9$$

Where,

F17: t last

E4: t initial

9: number of cells in table -1

Then we find τ table by this equation

$$\tau = \frac{\alpha t}{L^2}$$

and this code

```
=('wall plane'!B11*(D76)/('wall plane'!B10^2)
```

Then we find T table by this equation

$$Eq \quad T = (T_i - T_\infty)A_1 e^{-\lambda_1 \frac{2\alpha t}{L^2}} \cos\left(\lambda_1 \frac{x}{L}\right) + T_\infty$$

And we use this code

```
=('wall plane'!B14+(('wall plane'!B13-'wall plane'!B14)*output!B12*EXP(-(output!B11^2)*'wall  
plane'!B11*'ss3'!E4/('wall plane'!B10^2))*COS(output!B11*'wall plane'!B9/'wall plane'!B10))
```

Then we can find θ table by use equation

$$Eq \quad \theta = \frac{T(x,t) - T_\infty}{T_i - T_\infty}$$

And this is the code for the first cell in theta table

```
=(E76-'wall plane'!B14)/('wall plane'!B13-'wall plane'!B14)
```

And now after we fill θ and τ tables we can draw the graph

And it's in the same way for cylinder and sphere.

3.5 Case: 2: In terms of time

3.5.1 Input sheets

In this sheet the inputs of the program which are eight parameters ($h, k, x, L, \alpha, T, T_i, T_\infty$) are entered manually.

For case 2 we have the same geometry, by clicking on any geometry from the home sheet, the input sheet for second case appears.

INPUT	
$h [w/m^2k] =$	0
$k [w/mk] =$	0
$x [m] =$	0
$L [m] =$	0
$\alpha [m^2/s] =$	0
$T [k] =$	0
$T_i [k] =$	0
$T_\infty [k] =$	0

FIGURE 3-9: Input sheet of the plane wall case 2

INPUT	
$h [w/m^2k] =$	0
$k [w/mk] =$	0
$r [m] =$	0
$r_0 [m] =$	0
$\alpha [m^2/s] =$	0
$T [k] =$	0
$T_i [k] =$	0
$T_\infty [k] =$	0

FIGURE 3-10: Input sheet of the cylinder case 2

INPUT	
h [w/m ² k] =	0
k [w/mk] =	0
r [m] =	0
$r0$ [m] =	0
α [m ² /s] =	0
T [k] =	0
T_i [k] =	0
T_{∞} [k] =	0

Diagram of a sphere with radius r_0 , initial temperature T_i , and ambient temperature T_{∞} . The diagram also shows a radial coordinate r and a convection coefficient h .

FIGURE 3-11: Input sheet of the sphere case 2

Where,

h : convection heat transfer coefficient

k : thermal conductivity

L : thickness

x : distance at any point

α : thermal diffusivity

T_i : initial temperature

T_{∞} : fluid temperature

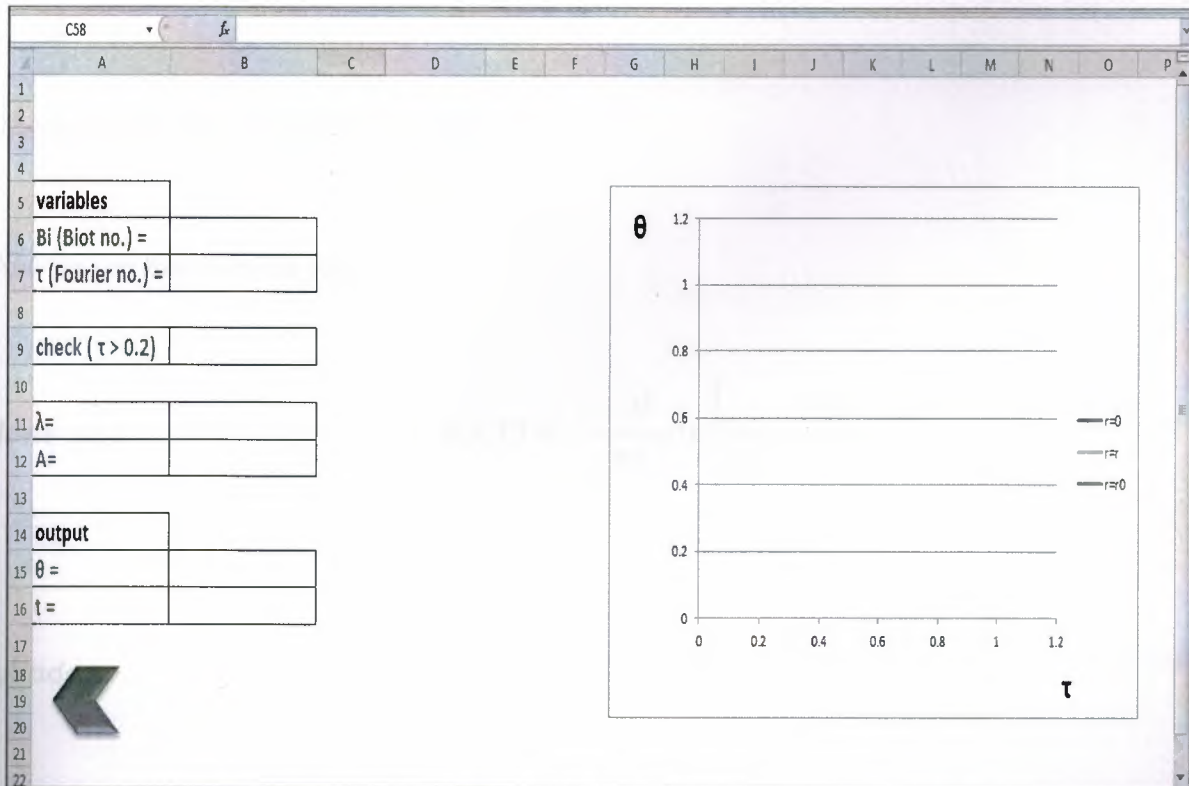
T : temperature at a given point & time

Remark: In the cylinder and sphere x and L inputs are replaced by r and r_0 .

Pressing on the arrows related to the input sheet of the specified geometry, the program moves to the output sheet.

3.5.2 Output sheet of case 2

The output sheet is again common for all the geometries It shows the result of the program: the variables of the program which are Bi (Biot Number) and τ (ferriour number), and the outputs of the program which are θ and t and the graph (θ as a function of τ).



FIGERE 3-12: Output sheet of case 2

The above graph shows three curves :

- $x = L$ (at the surface)
- $x = x$ (at any point)
- $x = 0$ (at the center)

3.5.3 Output Calculation

In this case we are looking for time and θ

But we need to find the variables before that

$$Bi = \frac{hL}{k}$$

We find bi to extract the λ and A

$$\tau = \frac{\alpha t}{L^2}$$

We find τ to find the time that we need it

And we can find the time for

Plane wall

$$t(x, T) = -\frac{L^2}{\alpha \lambda_1^2} \ln \left[\frac{(T_x - T_\infty)}{(T_i - T_\infty) A_1 \cos(\lambda_1 \frac{x}{L})} \right] \quad (3-2a)$$

Cylinder

$$t(r, T) = -\frac{L^2}{\alpha \lambda_1^2} \ln \left[\frac{(T_x - T_\infty)}{(T_i - T_\infty) A_1 J_0(\lambda_1 \frac{r}{r_0})} \right] \quad (3-2b)$$

Sphere

$$t(r, T) = -\frac{L^2}{\alpha \lambda_1^2} \ln \left[\frac{(T_x - T_\infty)}{(T_i - T_\infty) A_1 \frac{\sin(\lambda_1 \frac{r}{r_0})}{\lambda_1 \frac{r}{r_0}}} \right] \quad (3-2c)$$

And for θ

$$\theta = \frac{T(x, t) - T_\infty}{T_i - T_\infty}$$

3.5.4 How the graph drawn

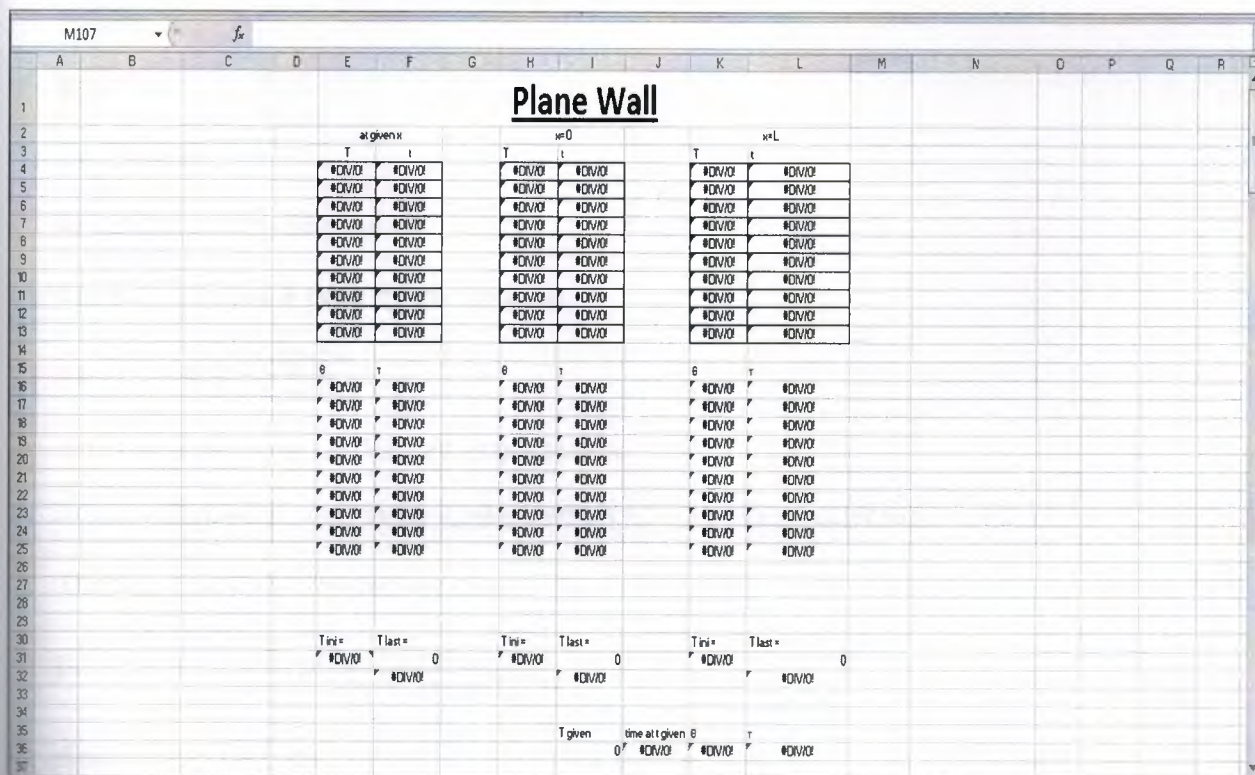


FIGURE 3-13: Calculation sheet case 2

The program starts by calculating T initial (where the program starts with this value) and T final (where the program ends with this value) for ($x=0$, $x=x$ and $x=L$)

- T initial

$$\text{Eq} \quad T = (T_i - T_\infty)A_1 e^{-\lambda_1^2 0.2} \cos\left(\lambda_1 \frac{x}{L}\right) + T_\infty$$

By use this code:

$$=((\text{'wall plane t'!B13}-\text{'wall plane t'!B14})*(\text{'output (2)'!B12})*(\text{EXP}(-\text{'output (2)'!B11}*\text{'output (2)'!B11}*0.2)))*\text{COS}(\text{'output (2)'!B11}*\text{'wall plane t'!B9}/\text{'wall plane t'!B10}+\text{'wall plane t'!B14})$$

- T final

$$\text{Eq} \quad T = (0.001 * (Ti - T_{\infty})) + T_{\infty}$$

By use this code:

$$=(0.001*(\text{'wall plane t'!B13}-\text{'wall plane t'!B14}))+\text{'wall plane t'!B14}$$

And then e substitute T initial in the first cell

and for the remaining cells we use this code

$$=(F31-E4)/9$$

Where,

F31 T last

E4 T initial

9: number of cells in table -1

Then we can find θ table by use equation

$$\text{Eq} \quad \theta = \frac{T(x,t)-T_{\infty}}{T_i-T_{\infty}}$$

And this is the code for first cell in theta table

$$=(E4-'wall plane t'!\$B\$14)/('wall plane t'!\$B\$13-'wall plane t'!\$B\$14)$$

Then we find t table by this equation

$$\text{Eq} \quad t = -\frac{L^2}{\alpha \lambda_1^2} \ln \left[\frac{(T_x-T_{\infty})}{(T_i-T_{\infty}) A_1 \cos(\lambda_1 \frac{x}{L})} \right]$$

And we use this code

$$=(-('wall plane t'!\$B\$10^2/('wall plane t'!\$B\$11*'output (2)'!\$B\$11^2))*(\text{LN}(E16/('output (2)'!\$B\$12*\text{COS}('output (2)'!\$B\$11*'wall plane t'!\$B\$9/'wall plane t'!\$B\$10))))$$

Then we find τ table by this equation

$$\tau = \frac{\alpha t}{L^2}$$

and this code

$$=F8*'wall plane t'!\$B\$11/('wall plane t'!\$B\$10^2)$$

And now after we fill θ and τ tables we can draw the graph

And it's in the same way for cylinder and sphere

3.6 Tables

M65																											
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA
	a1		A1			b1	A2	A2			b1						n	J ₀ (n)	J ₁ (n)								
20	0.01	0.0938	1.0017			0.01	0.1412	1.0025			0.01	0.173	1.003				0	1	0								
21	0.02	0.141	1.0033			0.02	0.1955	1.005			0.02	0.2445	1.006				0.1	0.9975	0.0499								
22	0.04	0.1967	1.0066			0.04	0.2694	1.0099			0.04	0.345	1.012				0.2	0.99	0.0995								
23	0.06	0.2425	1.0098			0.06	0.3438	1.0148			0.06	0.4217	1.0179				0.3	0.9775	0.1493								
24	0.08	0.2791	1.013			0.08	0.396	1.0197			0.08	0.486	1.0239				0.4	0.9604	0.196								
25	0.1	0.3111	1.0161			0.1	0.4417	1.0246			0.1	0.5423	1.0298				0.5	0.9385	0.2423								
26	0.2	0.4328	1.0311			0.2	0.617	1.0483			0.2	0.7593	1.0592				0.6	0.912	0.2867								
27	0.3	0.5218	1.045			0.3	0.7465	1.0712			0.3	0.9208	1.088				0.7	0.8812	0.329								
28	0.4	0.5932	1.059			0.4	0.8516	1.0931			0.4	1.0529	1.1164				0.8	0.8463	0.3698								
29	0.5	0.6533	1.0701			0.5	0.9408	1.1143			0.5	1.1656	1.1441				0.9	0.8075	0.4059								
30	0.6	0.7051	1.0814			0.6	1.0184	1.1345			0.6	1.2644	1.1713				1	0.7652	0.44								
31	0.7	0.7506	1.0918			0.7	1.0873	1.1539			0.7	1.3525	1.1978				1.1	0.7196	0.4709								
32	0.8	0.791	1.1016			0.8	1.149	1.1724			0.8	1.432	1.2236				1.2	0.6711	0.4983								
33	0.9	0.8274	1.1107			0.9	1.2048	1.1902			0.9	1.5044	1.2488				1.3	0.6201	0.522								
34	1	0.8603	1.1191			1	1.2558	1.2071			1	1.5708	1.2732				1.4	0.5663	0.5419								
35	2	1.0769	1.1785			2	1.5395	1.3084			2	2.0288	1.4793				1.5	0.5188	0.5579								
36	3	1.1825	1.2102			3	1.7887	1.4191			3	2.2889	1.6227				1.6	0.4554	0.5699								
37	4	1.2646	1.2287			4	1.9081	1.4636			4	2.4556	1.7202				1.7	0.398	0.5778								
38	5	1.3198	1.2403			5	1.9898	1.5029			5	2.5704	1.787				1.8	0.34	0.5815								
39	6	1.3496	1.2479			6	2.049	1.5253			6	2.6537	1.8338				1.9	0.2816	0.5812								
40	7	1.3768	1.2532			7	2.0937	1.5411			7	2.7165	1.8673				2	0.2239	0.5787								
41	8	1.3978	1.257			8	2.1286	1.5526			8	2.7654	1.892				2.1	0.1688	0.5683								
42	9	1.4149	1.2598			9	2.1566	1.5611			9	2.8044	1.9106				2.2	0.1104	0.556								
43	10	1.4289	1.262			10	2.1795	1.5677			10	2.8363	1.9249				2.3	0.0555	0.5399								
44	20	1.4961	1.2699			20	2.288	1.5919			20	2.9857	1.9781				2.4	0.0025	0.5202								
45	30	1.5202	1.2717			30	2.3261	1.5973			30	3.0372	1.9898				2.6	-0.968	-0.471								
46	40	1.5325	1.2723			40	2.3455	1.5983			40	3.0632	1.9942				2.8	-0.185	-0.41								
47	50	1.54	1.2727			50	2.3572	1.6002			50	3.0788	1.9962				3	-0.26	-0.339								
48	100	1.5552	1.2731			100	2.3809	1.6015			100	3.1102	1.999				3.2	-0.32	-0.261								
49	E+26	1.5708	1.2732			E+26	2.4048	1.6021			E+26	3.1416	2														

FIGURE 3-14: Tables for A, λ and J

In the above tables we introduced an equation to solve the interpolation needed to find the values of λ_1 and A_1 in the three different geometries depending on the values of B_i , and to solve the interpolation needed to find J_0 needed to solve the one term approximation of the cylinder as well.

Chapter 4: CASE STUDY

Here we are going to check the credibility and accuracy of the excel program by comparing one of the book's example's result to the program result.

Example 4-4 from Heat and Mass Transfer by Yunus Cengel .

EXAMPLE 4-4 Heating of Large Brass Plates in an Oven

In a production facility, large brass plates of 4 cm thickness that are initially at a uniform temperature of 20°C are heated by passing them through an oven that is maintained at 500°C (Fig. 4-20). The plates remain in the oven for a period of 7 min. Taking the combined convection and radiation heat transfer coefficient to be $h = 120 \text{ W/m}^2 \cdot ^\circ\text{C}$, determine the surface temperature of the plates when they come out of the oven.

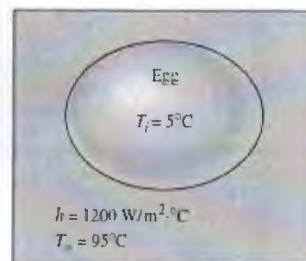


FIGURE 3-7: Schematic for Example 4-4

Assuming the egg as a sphere, the calculations and results obtained are:

Properties The water content of eggs is about 74 percent, and thus the thermal conductivity and diffusivity of eggs can be approximated by those of water at the average temperature of $(5 + 70)/2 = 37.5^\circ\text{C}$; $k = 0.627 \text{ W/m} \cdot ^\circ\text{C}$ and $\alpha = k/\rho c_p = 0.151 \times 10^{-6} \text{ m}^2/\text{s}$ (Table A-9).

Analysis The temperature within the egg varies with radial distance as well as time, and the temperature at a specified location at a given time can be determined from the Heisler charts or the one-term solutions. Here we use the latter to demonstrate their use. The Biot number for this problem is

$$\text{Bi} = \frac{hr_o}{k} = \frac{(1200 \text{ W/m}^2 \cdot ^\circ\text{C})(0.025 \text{ m})}{0.627 \text{ W/m} \cdot ^\circ\text{C}} = 47.8$$

which is much greater than 0.1, and thus the lumped system analysis is not applicable. The coefficients λ_1 and A_1 for a sphere corresponding to this Bi are, from Table 4-2,

$$\lambda_1 = 3.0754, \quad A_1 = 1.9958$$

Substituting these and other values into Eq. 4-28 and solving for τ gives

$$\frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{70 - 95}{5 - 95} = 1.9958 e^{-(3.0754)^2 \tau} \longrightarrow \tau = 0.209$$

which is greater than 0.2, and thus the one-term solution is applicable with an error of less than 2 percent. Then the cooking time is determined from the definition of the Fourier number to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.209)(0.025 \text{ m})^2}{0.151 \times 10^{-6} \text{ m}^2/\text{s}} = 865 \text{ s} \approx 14.4 \text{ min}$$

The result shows that the time needed for the center of the egg to reach 70 C is 865 sec

And now for the program's solution:

The input sheet of the same example with the same inputs:

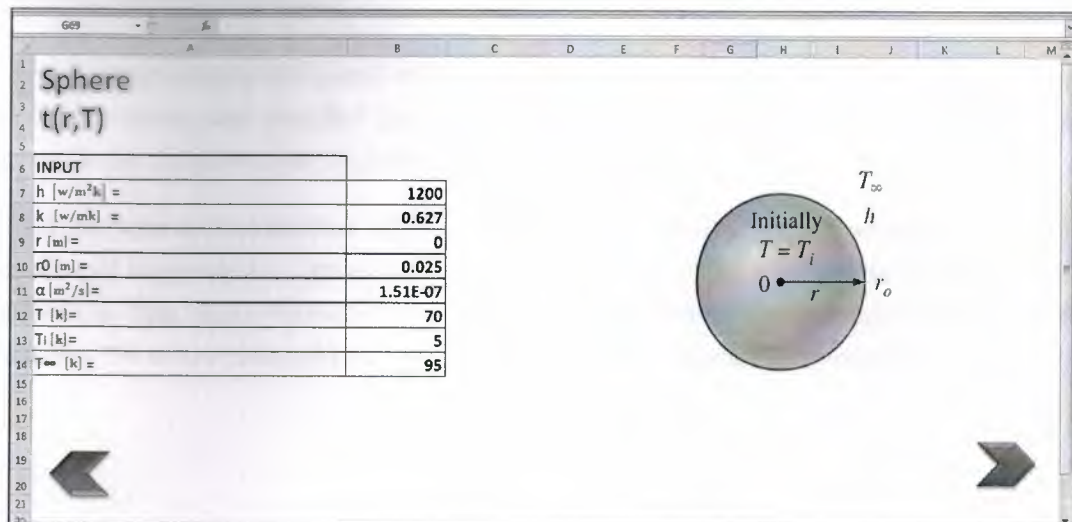


FIGURE 4-15: excel program's solution.

And the output sheet is:

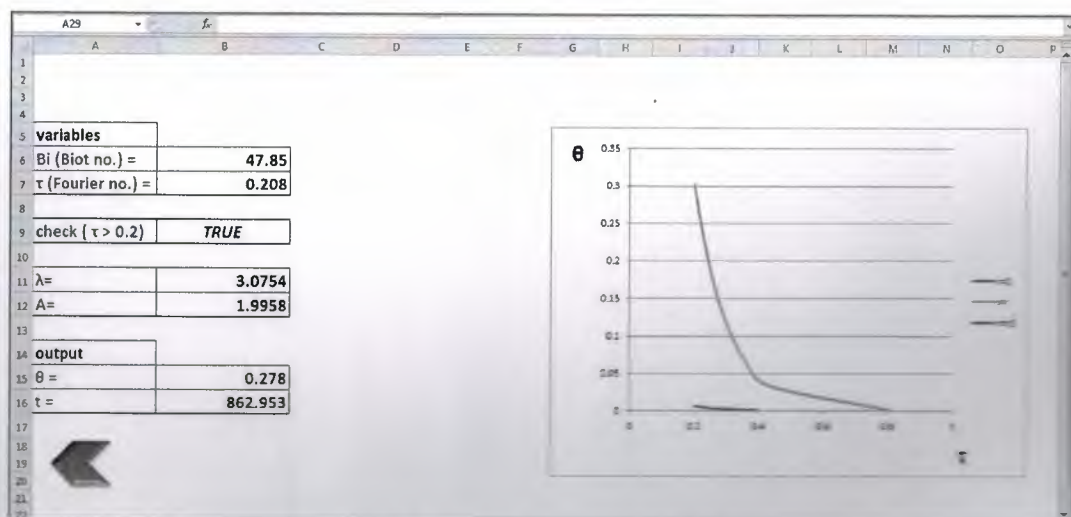


FIGURE 4-16: excel program's solution.

This sheet shows the results of the program for the same example where we obtained $t=862.953$ sec.

Thus the absolute error obtained is $\varepsilon = \left| \frac{865 - 862.953}{865} \right| 100\% = 0.23\%$

An absolute error of 0.23% is negligible, thus the program satisfies its credibility

Chapter 5: CONCLUSION

Instead of using dimensional equations and parameters we used dimensionless ones in order to decrease the dependency of the output on other terms, and thus to have more accurate results and easier graph to observe.

The program shows the graph of θ as a function of τ for three x values, and shows the outputs of the relative case specified from the approach sheet. And we checked the credibility of the program in the Case Study to find that the program works with a negligible error.

The one term approximation equations provide a convenient way of accuracy for influence of curvature and temperature- dependent thermal properties within a substance used for transient heat conduction. This small error arises due to the finite difference approximations are likely to be "represent less 1% of the inferred heat conduction for typical transient test conditions.

REFERENCES

- [1]: Heat and Mass Transfer 4th edition by Yunus Cengel.
- [2] : https://en.wikipedia.org/wiki/Heat_transfer
- [3]:<http://worldwidescience.org/topicpages/t/temperature+heat+transfer.html>
- [4]: H. Hillman. Kitchen Science. Mount Vernon, NY:Consumers Union, 1981
- [5]: P. J. Schneider. Conduction Heat Transfer. Reading, MA; Addison-Wesley, 1955.
- [6]: H. S. Carslaw and J. C Jaeger. Conduction of Heat in Solids. 2nd ed. London: oxford University Press, 1959.