INTEGRATING FAHP AND TOPSIS FOR QUALITY AND USER SATISFACTION EVALUATION OF MOBILE LEARNING APPLICATIONS FOR MATHEMATICS

A THESIS SUBMITTED TO THE GRADUATE SCHOOL OF APPLIED SCIENCES OF NEAR EAST UNIVERSITY

By
HARUNA YUNUSA

In Partial Fulfillment of the Requirements for the Degree of Master of Science in Computer Information Systems

NICOSIA, 2017
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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name:

Signature:

Date:
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To my family...
ABSTRACT

Mobile learning application for mathematics (MLAM) has drastically changed the way individuals learn mathematics in recent years, however the manual selection of these applications had been tedious, time consuming and in most instances effectuated premature selection by many individuals. The aim of this study is twofold; firstly to propose a framework for evaluating both the quality and user satisfaction of MLAM and secondly to select the most suitable MLAM by integrating FAHP and TOPSIS multi-criteria methods. The criteria were defined based on the combination of technical and non-technical aspects of the applications. The ISO 9126 model was used for the technical aspect while user satisfaction was proposed for the non-technical aspect. The weight of each criterion identified in the framework was determined through FAHP method while MLAM’s were ranked based on preference with TOPSIS methods respectively. Mathsway, Malmaths, Cymaths, Mathematics and Mathspapa were chosen as sampled MLAM’s from Google Play Store based on top 5 highest user ratings. According to this study ranking result by TOPSIS method, Mathematics was ranked first, then Cymaths, Mathsway, Malmaths and lastly Mathspapa. Combining FAHP and TOPSIS methods yields less time consuming and more effective optimized result in selecting the most suitable MLAM. The integration of these methods could significantly improve the evaluation of MLAM’s by minimizing the manual expert evaluation. The adoption of FAHP and TOPSIS approach provides less time consuming and optimizes result in selecting MLAM’s.

Keywords: FAHP; ISO 9126 model; mobile learning application for mathematics; user satisfaction; TOPSIS
ÖZET


Anahtar Kelimeler: FAHP; ISO 9126 modeli; kullanıcı memnuniyeti; matematik için mobil öğrenme uygulaması; TOPSIS
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<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>API:</td>
<td>Application Programming Interface</td>
</tr>
<tr>
<td>DEMATEL:</td>
<td>Decision Making Trial and Evaluation Laboratory.</td>
</tr>
<tr>
<td>ELECTRE:</td>
<td>Elimination and Choice Expressing Reality.</td>
</tr>
<tr>
<td>FTOPSIS:</td>
<td>Fuzzy Technique for Order of Preference by Similarity to Ideal Solution.</td>
</tr>
<tr>
<td>ICT:</td>
<td>Information Communication Technology.</td>
</tr>
<tr>
<td>iOS:</td>
<td>iPhone Operating System.</td>
</tr>
<tr>
<td>IT:</td>
<td>Information Technology.</td>
</tr>
<tr>
<td>MCDM:</td>
<td>Multi Criteria Decision Making</td>
</tr>
<tr>
<td>MLAM:</td>
<td>Mobile Learning Application for Mathematics.</td>
</tr>
<tr>
<td>MLE:</td>
<td>Mobile Learning Environment.</td>
</tr>
<tr>
<td>NCTM:</td>
<td>National Council of Teachers of Mathematics.</td>
</tr>
<tr>
<td>PACMAD:</td>
<td>People At the Center of Mobile Application Development.</td>
</tr>
<tr>
<td>PC:</td>
<td>Personal Computer.</td>
</tr>
<tr>
<td>PDA:</td>
<td>Personal Device Assistant.</td>
</tr>
<tr>
<td>SERVQUAL:</td>
<td>Service Quality Model.</td>
</tr>
<tr>
<td>SITEQUAL:</td>
<td>Website Quality Model.</td>
</tr>
<tr>
<td>SMART:</td>
<td>Specific Measurable Achievable Responsible Time-related.</td>
</tr>
<tr>
<td>SRF:</td>
<td>Self-Regulated Framework.</td>
</tr>
</tbody>
</table>
SRL: Self-Regulated Learning.
TFN: Triangular Fuzzy Number.
TOPSIS: Technique for Order of Preference by Similarity to Ideal Solution.
UI: User Interface.
US: United States.
WEBQUAL: Website Quality Model.
CHAPTER 1
INTRODUCTION

This chapter discusses the general introduction to the integration of FAHP and TOPSIS methods for quality and user satisfaction evaluation of MLAM, the problem, the study importance, the aim of the study, study objectives, the limitations and also the overview.

1.1 Introduction

Mobile technology has been predominantly gaining attention since its inception worldwide thus transforming the values of different societies (Baller, Dutta & Lanvin, 2016) and turning into a necessary part of people lives everywhere throughout the world. These mobile devices such as smart phones, tablets have changed the way people access and share information (Drigas & Pappas, 2015) thus transforming the conventional way of learning by utilizing mobile devices, internet and mobile applications that are primarily designed for acquiring knowledge which are referred to as mobile learning applications. The invention of these mobile learning applications seems like a coincidence in the educational world due to some natures of the present educational requirements of quick access to learning materials and tenacious needs for communication (Hanafi & Samsudin, 2012). Since the invention of mobile learning application, the field has developed rapidly and also considered the future for learning (Trifonova, 2003).

Willacy and Calder (2017) defined mobile learning applications as mobile applications that make it possible for students or individuals to exercise learning in a versatile position. It establishes an anytime, anywhere learning environment (Martin, McGill & Sudweeks, 2013; Peng, Su, Chou, & Tsai, 2009; Uzunboylu, Cavus & Ercag, 2009). This method of learning gives more flexibility and freedom to the learner which as a result had foster high adoption by many individuals and learning institutions. Recently, researchers designed a mobile learning application which support teaching mathematics in the different areas such as mathematical analysis, geometry, statistics, algebra, and other areas of its discipline (Drigas & Pappas,
2015). However, mobile mathematical applications lets its users evaluate mathematical functions, giving graphical abilities and offer some sorts of mobile calculators. There are applications intended to handle estimation undertakings and educational applications for rehearsing on numerical and mathematical aptitudes. Technologies that give support to mathematical science in mobile devices have likewise been expanding in the course of the most recent decade (Bjerede, Atkins & Dede, 2010). The mobile educational apparatuses for mathematics can help students in critical thinking or problem solving, upgrade appreciation of mathematical ideas, give powerfully representations of thoughts and energize general meta-cognitive capacities (Pierce, Stacey & Barkatsas, 2007). The continuous utilization of mobile technologies over the span of mathematics would energize the change of mobile learning applications and also help students to enhance their learning objectives. However, Skillen (2015) reported that there are over 4,000 mobile applications for learning mathematics to select from which had craved ways for myriads options for making selection on what mobile learning environment to adopt. This scenario has led many individuals making a premature selection of mobile learning applications for mathematics because making an efficient selection from over 4,000 applications seems tedious and time consuming, thus making right selection is crucial to enhancing its continuous usage and developments. Hence, to address this problem a software quality model to provide a roadmap on making a reliable selection of mobile learning application for mathematics from different but conflicting options is inevitable.

At the time of conducting this study there was no software quality model purposely for evaluating MLAM’s. However, there are many models designed for evaluating software qualities in general such as McCall (1977), Boehm (1978), FURPS, Dromey, ISO-9126, ISO-25010, Bertoa, Gecuamo Model, Alvaro Model, Rawashdeh Model (Miguel, Mauricio and Rodríguez, 2014; Alvaro and de Almeida & Meira, 2007; Morisio, Stamelos and Tsoukias, 2002). According to a software quality models reviewed by Miguel et al. (2014), they stated that the ISO-9126 model was optimal and most widely used for evaluating software qualities, for these reasons and that MLAM’s are regarded utterly as software applications that are designed typically to run on smart phones (Pocatilu & Vetrici, 2009) MLAM’s could be evaluated using the ISO 9126 software quality characteristics (Pocatilu & Boja, 2009).
However, the selected ISO 9126 quality model for this study to evaluate MLAM’s were based on the software technical perspectives. The ISO 9126 does not include the non-technical aspects of the applications. Shee and Wang (2008) stressed the importance of the non-technical aspects when evaluating e-learning systems and named it user satisfaction by claiming the technical quality characteristics alone cannot guarantee a user satisfaction. For this reason this study adopts a combination of the ISO 9126 quality model based on the technical aspect and proposed user satisfaction based on non-technical aspect. For the technical perspective, ISO 9126 quality model includes accessibility, functionality, usability, efficiency, maintainability, portability and quality in use as its criteria. While the non-technical aspect which refers to the user satisfaction related criteria were based on ease of use, loading time, quality of content and conformance with learning goal as its criteria. These criteria were proposed because they were adopted by (Kurilovas & Vinogradova, 2015; Boja, Bătăgan & Vișoiu, 2011). These aforementioned criteria were used for achieving the study goal through decision making mechanisms which are grouped into two as single criterion and multiple criteria decision making. Since the study was based on numerous criteria, the most appropriate method is the MCDM.

MCDM is a mechanism used for selecting the best option out of multiple alternatives in most cases conflicting decision criteria (Pomerol & Barba, 2012). There are numerous MCDM techniques which are used for decision making such as AHP, ANP, FAHP, SMART, TOPSIS, ELECTRE, PROMETHEE, etc. In order to enhance the process of decision making normally different methods are merged together. According to Saaty and Vargas (2012) they stated that one of the most exceptional MCDM methods is the FAHP approach which can be used alongside TOPSIS in mostly complex decision making.

Analytic Hierarchy Process (AHP) approach is one of the most well-known and as of late most utilized approach for making decisions, in the process of making decisions out of different choices which their rankings are based on numerous attributes that are expressed utilizing distinctive scales. The AHP approach gives flexibility in the process of making decision and also assists decision makers to assign priority which helps in making good decisions by taking qualitative and quantitative aspects into considering (Forman & Gass, 2001). However, there
is an extension to AHP method which provides more flexibility and dynamism in weighting criteria called Fuzzy Hierarchy Process (FAHP).

Fuzzy Analytic Hierarchy Process (FAHP) is an expansion, also an upgrade of AHP technique that utilizes fuzzy logic, numbers and sets. It utilizes fuzzy numbers rather than assigning particular numbers to determine criteria weighting, these fuzzy sets provided more flexibility during assigning weights to the criteria. Also the method utilized fuzzy numbers to decide the advantages of single criteria to its other (Shaout & Yousif, 2014; Sun, Lin & Tzeng, 2009). Specifically the MCDM complex problems were much simpler with FAHP strategy with managing and understanding qualitative and quantitative data during utilization. This method determined criteria weights by pair wise comparison and the weight were used for ranking the alternative by using the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method.

TOPSIS approach is utilized for the final ranking of MLAM which was based on multiple criteria, whose significance is resolved utilizing a FAHP strategy. The best alternate MLAM is determined by checking one that is nearest to the positive ideal solution and the most remote from the negative ideal solution with this approach (Barrios, De Felice, Negrete, Romero, Arenas & Petrillo, 2016).

The aim of this study is twofold; firstly to propose a framework for evaluating both the quality and user satisfaction of MLAM and secondly to select the most suitable MLAM by applying FAHP and TOPSIS multi-criteria methods together. Conclusively, this study adopted the ISO 9126 quality model and proposed a user satisfaction model for its framework. The resulting framework was used to evaluate the selected five sampled MLAM’s using the FAHP method for determining criteria weights and ranking the conflicting alternatives using the TOPSIS method.

1.2 The Problem

At the time of conducting this study with regards to the quality assessment of MLAM using MCDM methods there was no framework solely dedicated for evaluating MLAM, therefore this situation needs to be addressed by proposing a framework for this purpose. In addition,
making a manual selection amongst numerous MLAM seems quite tedious, time consuming and has led many individuals making an inept selection therefore there is a need to select and evaluate MLAM in an autonomous way. For this purpose, utilizing MCDM approaches in the selection and evaluation of MLAM provides less time consuming and optimum result.

1.3 Aim of the Study

The aim of this study is to develop a framework for assessing the quality of mobile learning application for mathematics.

In addition, the study also aims to identify an optimal mobile learning tool for mathematics with respect to the proposed framework criteria through applying MCDM techniques (FAHP & TOPSIS).

1.3.1 Research Objectives

- To develop a framework for assessing the quality of mobile learning application for mathematics.
- To identify the optimal MLAM with respect to the proposed framework through applying MCDM techniques (FAHP and TOPSIS)

1.4 Significance of the Study

So far the quality evaluation based on technical aspects are widely used however including non-technical aspects are novel and distinct from the former studies.

This research can serve MLAM users in decision making during the selections of MLAM in such an autonomous time saving manner and on the other hand optimizing the decision results during the selection process. Thus, the developed framework will also help in making an efficient quality assessment of MLAM.

This study can serve the future researchers as their reference in making a related research in this subject area, which can provide them with ideas in order to make their overall work a lot easier.
1.5 The Limitations of the Study

Although this research will achieve its aims but there are some limitations attached to it due to time and logistics.

- The study targets only MLAMs for adult users.
- The total number of MLAMs for selection and evaluation are five.
- Evaluating the MLAMs depends on the ISO 9126 model and user satisfaction (Kurilovas & Vinogradova, 2015; Boja & Batagan, 2011).

1.6 Overview of the Study

The whole thesis contains six chapters.

Chapter 1 is the introductory part of the thesis and explains the problem definition, importance of study, aim of the study.

Chapter 2 explains the related research work on smart mobile phones, mobile applications, mobile learning applications, features of mobile learning, mobile learning for mathematics, limitations of MLAM, MCDM, AHP, FAHP and TOPSIS.

Chapter 3 introduces the conceptual framework.

Chapter 4 explains the methodology used in selecting, evaluation and writing thesis.

Chapter 5 is the empirical study and also discussed the result.

Chapter 6 concludes the study by making recommendations and suggestions for future studies.
This chapter discusses literature review of previously published studies on the subject area of the research by examining their findings and also the study gaps.

2.1 Introduction to the Study Related Research

In order to develop a framework for evaluating the quality and user satisfaction of MLAM using the FAHP and TOPSIS methods, a subset of different previous literatures below were selected for review because they fall within the subject area of the study; the list is as follows:

- Smart mobile phones
- Mobile applications
- Mobile learning applications
- Features of mobile learning
- Mobile learning application for mathematics
- Limitations of mobile learning application for mathematics
- Multi Criteria Decision Making (MCDM)
- FAHP and TOPSIS

However, the literature review concentrated on the information introduced in the peer articles, journals and other related reports, with the expectations that the findings were accurate and reliable.

2.2 Smart Mobile Phones

Boja and Batagan (2011) investigated the characteristics of mobile technologies used for learning activities, they reported that the rapid growth of mobile devices had opened ways for easy accessibility, collaboration, portability and also motivated users. They also stated that students are faced with problems such as mobility, informality and ownership of learning devices. Conclusively, they reported that developers should focus on the device quality
capabilities such as memory, display and processing power so that the developed application will run in optimal condition.

Chae and Kim (2004) investigated the screen size attribute of mobile learning devices to determine its effects on user satisfaction. They reported that mobile devices with smaller screen size requires lots of content scroll which makes the process of learning tedious and in turn does not offer a user satisfaction.

Han, Kim, Yun, Hong and Kim (2004) investigated mobile devices attributes that are essential to user satisfaction, they identified size, color, shape and material at its design attributes. They stated that user satisfaction will help in identifying some essential features of mobile device during design.

Ziefle and Bay (2006) reiterated the importance of considering some critical technological ability of mobile devices when designing a mobile application. They recommended that mobile applications developer should concentrate more on the device sub characteristics in other to attain a user satisfaction.

2.3 Mobile Applications

Sohn, Min and Kim (2015) conducted a study on the selection and the quality evaluation of open source mobile applications; they created a system that eases the selection of mobile application based on the ISO/IEC 2500 quality framework.

Moumane, Idri & Abran (2016) evaluated mobile applications running on different mobile OS regarding its usability which was based on combined frameworks ISO 9241 and ISO 25062. However they adopted the ISO 9126 standard for determining the usability characteristics of the applications. To improve the mobile applications usability they analyzed all the problems that come both with the hardware and software.

Harrison, Flood & Duce (2013) reviewed some studies based on different mobile applications usability models and they discovered that satisfaction, effectiveness and efficiency are the main attributes for measuring usability. They proposed a model called People At the Center of
Mobile Application Development (PACMAD), the model ignores some ISO 9126 model key attributes.

Nayebi, Desharnais & Abran (2012) evaluated different model used for measuring the quality of mobile applications usability models such as ISO 9241, ISO 9126, ISO 13407 and ISO 25010 standards, they stated that usability of mobile applications are different because their characteristics are also different.

2.4 Mobile Learning Applications

Regarding the evaluation of mobile learning environment Vavoula and Sharples (2009) documented that evaluation should be delayed unless an easy and intuitive easy to use environment is developed and additionally Vavoula and Sharples et al. (2009) reported that mobile learning is controlled by its environment more than the class room instruction, reiterating the importance of learning environment and its complexity. Customary measures for surveying learner’s fulfillment with information systems and for evaluating learner fulfillment with regards to classroom teaching are not reasonable for mobile based learning system (Shee & Wang, 2008). Reported by Kurilovas & Vinogradova (2009) that modern mobile learning utilizes tablets in view of critical thinking, personalization, joint effort, and flipped class are more adaptable than conventional ones, they have more potential for feedbacks.

Sha, Looi, Chen, Seow and Wong (2012) investigated learning in a mobile learning application through measuring and recognizing self-regulation in these technologies. They acknowledged that more research is needed to investigate the outside components and inside variables (e.g., earlier information, inspiration) fundamental students focused versatile taking in, the present concentrate theoretically and exactly investigates how the speculations and approaches of self-regulated learning (SRL) can help to break down and comprehend the procedures of portable learning. They observed information gathered from two rudimentary science classes in Singapore which showed that the explanatory SRL model of versatile learning proposed in their study lighted up the connections between three parts of portable taking in: students' self-reports of mental procedures, examples of internet learning conducted in the mobile learning environment (MLE), and learning accomplishment. They also
documented that factual investigations created three principle discoveries. They stated, understudy inspiration for this situation can represent whether and to what degree the students can effectively take part in versatile learning exercises meta cognitively, motivationally, and behaviorally. Secondly, the impact of students’ self-reported inspiration on their learning accomplishment is interceded by their behavioral engagement in a pre-outlined action in the MLE. Thirdly, students’ impression of parental self-sufficiency support is not just connected with their inspiration in school adapting, additionally connected with their real practices in self-regulating their learning.

Alexander (2004) investigated the mobile learning applications in higher education. He blended the wireless technology and mobile technology which brings about rising changes of the instructive world. He answered the question, about how wireless, mobile technology has influenced the learning environment, the teaching method, and grounds life. He evaluated the present situation, studying digital culture all-inclusive and generally. He also considered the United States just incidentally, since it falls behind different parts of the world in a few key patterns. Also, he found that wireless mobile learning knowledge had quickly developed, doing the best to get a handle on new patterns.

2.5 Features of Mobile Learning

Mobile learning means establishing learning across different scenario with content and social interactions with the use of mobile devices such as smart phones, tablets PC and so on via a wireless network which creates an anywhere, anytime, learning and teaching spaces (Crompton, 2013). It’s a type of distance education whereby users learn at their own comfort time (Crescente & Lee, 2011). Mobile learning is aimed at giving the user mobility (Trentin & Repetto, 2013) and portability substituting books with mobile devices loaded with customized learning contents (Jacob & Isaac, 2014). However, mobile learning comes with loads of glaring features which has led to many educational institutions widely integrating mobile learning into their institutions or organizations (Jacob & Isaac, 2014). However, Vayuvegula (2012) stated that mobile learning is defined by five features which are stated below:
2.5.1 Convenience and easy access

Mobile learning applications gives simple access to learning at the time and place helpful to learners. Since learners ordinarily have their mobile devices with them more often than not, they can access their courses or learning devices whenever and anyplace.

2.5.2 Just in time learning

Mobile learning application takes into account the hasty need of learners to get information. Today, what will you do to discover bearings to a place or discover the motion picture that is playing in the theater near your home? You simply go online for information. Mobile learning application gives comparative offices to representatives to get information relating to their employments at the snap of a catch.

2.5.3 Bite sized modules

Modules created purposely for mobile learning application are for a brief period of time, normally around five minutes. Henceforth, one module will concentrate on only one primary thought or idea.

2.5.4 Contextual and informal

Mobile learning application content is normally relevant, that is the preparation alludes to a particular circumstance or a setting that learners wind up in. The setting can be the area, movement or learning objectives and is employment particular in an association.

2.5.5 Application-oriented

Mobile learning solutions concentrate on sharing learning that finds prompt application. Learning is without a moment to spare i.e. at the time the learner requires that particular information and gets the opportunity to apply the same quickly to the specific circumstance.
Drijvers (2015) stated that for more than 20 years the capability of advanced technology for mathematics has been highlighted by partners as having numerous conceivable outcomes. But many instructors, researchers and educators are confronted with questions regarding the integration of this technology into learning. This claim was ascertained by Handal, El-Khoury, Campbell and Cavanagh (2013) who stated that the integration of mobile learning technologies into education has brought both new conceivable outcomes and difficulties to instructors. However, Melhuish and Falloon (2010) together with Falconer and Littlejohn (2007) depicted mobile learning technology as pervasive in nature, remote, profoundly convenient and supplied with mixed media abilities conveying another measurement to educational modules conveyance. Regarding the integration of education into ICT in the area of mathematics Hoyles and Lagrange (2010) confirmed these discoveries by stating that digital technology had turned out to be always pervasive and their impact touching most of the educational systems.
While in its initial stages, studies regarding the utilization of mobile technology with a specific concentration on mathematical education has quickly developed as a field of intrigue. This intrigues is created as portable gadgets and applications turned out to be all the more effectively opened and well known among students. Highfield and Goodwin (2013) confirmed the examination of the adequacy of mobile applications just like another developing field of research. Till now the vast majority of the exploration on mobile applications has been directed by extensive media organizations with an attention on showcasing and the trial of gadgets. Larkin (2013) states that mobile applications for hand-held gadgets, for example, smart phones and iPads, are in awesome supply. A hefty portion of these emphasis on mathematics. A late findings uncovered more than 4000 applications mainly for the field of mathematics education. Notwithstanding, there is restricted investigation in mathematical education regarding the utilization of mobile applications and accompanied by teaching methods. With the expansion of gadgets into classrooms and related mobile applications, instructors and teachers need to consider their adequacy in supporting learning, especially in connection to mathematics.

According to the U.S. National Council of Teachers of Mathematics (NCTM) ascertained that mobile technology is a crucial instrument for learning mathematics in the 21st century, and all educational institutions must make sure that every one of their have accessibility to this technology (NCTM, 2008).

Roberts, Spencer, Vänskä and Eskelinen (2015) examined the challenges of Nokia mobile learning for mathematics in South Africa by researching the measuring effects of these environments. Their study gave an account of the examination of the deliberate take-up and utilization of the Nokia Mobile Mathematics benefited by 3,957 Grade 10 learners. They quantified the impact of the administration on the school mathematics fulfillment of 1,950 of these learners more than one academic session. Their study uncovered that 21% of the Grade 10 mathematics learners intentionally and consistently made utilization of this versatile learning asset outside of educational time, with little contribution from their educators. They found that over the gathering of 1,950 learners, there was a normal decrease in mathematics achievement from Grade 9 to Grade 10 of 15 rate focuses.
Botzer and Yerushalmy (2007) investigated the learning encounters inside a mobile learning setting, also inspected how socio-social and arranged learning angles are reflected in these encounters. The paper showed a pilot contextual analysis planned to look at how socio-social and arranged learning angles are reflected in learning encounters inside a novel mobile learning environment, Math4Mobile, a mobile application for learning mathematics. The contextual investigation concentrated on four students in an arithmetic techniques course who were occupied with a mathematical project in view of the smart phone applications. They found that utilization of the mobile environment upgraded the members' engagement in the demonstrating of genuine situations and added to coordinated effort between members.

Drigas and Pappas (2015) reviewed some recent investigations which were carried out in the field of mobile learning applications for mathematics, the reasons for the study was to look at the most illustrative investigations of late years, including on the web and versatile applications and instruments for mathematics and also their impact in the instructive procedure. They introduced some enhanced applications which were tended to by kindergarten youngsters to college students. They found out that these applications could be utilized to enhance arithmetic abilities, numerosity for charts' representation, geometrical items constriction, variable based math critical thinking and numerical programming. The after effects of the studies uncovered that on the web and versatile learning applications roused the students, making science course more charming and intuitive than the common educating hones. Their work gave designers guarantees for additionally learning devices sooner rather than later, making another instructive model.

Skillen (2015) documented the impacts on mathematics education in mobile learning, stating the advances in technological changing procedures in education, it should be perceived that learning is a piece of another scene, mobile learning gives numerous chances to individual, casual and unconstrained, arranged learning. Using cell phones, students of all age levels and foundations will have the capacity to team up, draw in and learn in various ways.
2.7 Limitations of Mobile Learning Application for Mathematics

Messinger and Marino (2010) reported that the limitations of mobile learning applications for mathematics when considering implementing the technology are grouped into: institutional, situational and dispositional.

- **Situational limitations:** They elaborated this limitation as lack of time, funding and that teachers are lacking technological knowledge and additionally how to adequately coordinate it into the school educational program. However, Bolton (2014) ascertained that the absence of teacher knowledge and expertise is accounted for as a noteworthy limitation as often as possible noted in the literature. Palak, Walls and Wells (2006) also supported their claim by referring to a few issues with instructor utilization of technology. They observed that educators tended to utilize online devices as an electronic library and to recover data, as opposed to convey and work together. This has a stream on impact to the classroom when working with students, if an instructor is not sure and able, in the successful utilize and displaying of the utilization of smart phones and related mobile technologies for learning and educating circumstances.

- **Institutional limitations:** Ng and Nicholas (2009) identified that the absence of technological support from professionals, lack of funding, lack of expert development, and proceeding support to advance sustainability.

- **Dispositional limitations:** The main limitation regarding this context is the teacher reluctances according to Cumming (2014) when implementing mobile learning applications in an educational environment.

2.8 Multi-Criteria Decision Making (MCDM)

Sun et al. (2009) proposed a model based on MCDM approach. The AHP and the fuzzy fundamental approach are utilized for engineered utility as per subjective perceptive environment. They stated that internet advancement has influenced and quickened mobile learning development. Because of cost, time, or adaptability for originator courses and learners, mobile learning has been received by enterprises as an option preparing technique.

Peranginangin, Shieh and Chen (2013) proposed an outline assessment system for portable learning applications and assessing the plan of two versatile learning applications by applying
assigned criteria. They utilized the novel hybrid multi-criteria decision making which consolidates Decision Making Trial and Evaluation Laboratory Model (DEMATEL) and the Analytical Network Process (ANP), they explored the structure of versatile learning application in term of relationship among properties and also inspected the esteem that learners' see as to assigned portable learning application.

Cavus (2011) described how to evaluate Learning Management Software’s (LMS) using a multi-criteria decision making technique. In addition, the author developed a web based program which is based on an MCDM algorithm. However, the study tried to help instructors on selecting an optimal LMS in order to save time and cost.

İşiklar and Büyüközkan (2007) evaluated mobile phones using the multi-criteria decision making techniques. They adopted the combination of AHP and TOPSIS method in selecting the best out of many alternatives available in the market stores. The study helps potential buyers in making a wise selection out of conflicting alternatives to save cost and time. The AHP was used to weight the criteria while TOPSIS was used to make the ranking.

2.9 Fuzzy Analytic Hierarchy Process (AHP) and the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)

Volarić, Brajković and Sjekavica (2014) conducted a study on integrating FAHP and TOPSIS when making a suitable selection of learning applications based on the revised blooms taxonomy framework. Initially they utilized FAHP approach for deciding the weights of the criteria with their respective estimated values for the applications. Afterwards, they utilized triangular fuzzy number as a part of FAHP approach for deciding the advantages of one criteria to another. At that point the TOPSIS approach was utilized to decide the last positioning of the applications. They stated that the mix of FAHP and TOPSIS techniques empowers instructor to effectively choose a more appropriate interactive media applications for learning.

Ballı and Korukoğlu (2009) developed a fuzzy framework for the selection of most suitable operating system based on the FAHP and TOPSIS approach. The weights were first determined and then the selection of the most appropriate operating system was done using the FAHP and TOPSIS approach respectively.
Torfi, Farahani and Rezapour (2010) they proposed a fuzzy MCDM model to evaluate user alternate choices with regards to its preference. They combined Fuzzy Analytic Hierarchy Process (FAHP) and Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS) the model involves weighting the criteria and ranking the alternatives respectively.

Brajković, Sjekavica and Volarić (2015) proposed a model based on the fuzzy decision making techniques for selecting an optimal wireless network across different choices for students. They combined the FAHP method and TOPSIS method in achieving their goal. The FAHP method served as the tool to weight the criteria while the TOPSIS method served as the tool to make the final ranking. Their study helps students in making decision on selecting the optimal wireless network.

Zolfani et al. (2014) conducted a study about the selection and evaluation of advertisement strategy based on the products life cycle using the Iranian food industries as case study. They combined FAHP and TOPSIS methods. They used FAHP method to weight the five proposed criteria and later used the TOPSIS method for selection. They showed how to adopt two MCDM techniques to obtain an optimum result. And they also stated that they used the fuzzy in AHP to better the decision making.

Mikaeil et al. (2009) developed a hierarchical model, the model was used in making an optimal selection by combining FAHP and TOPSIS methods. The FAHP was used to weight the criteria while the TOPSIS was used in making the final ranking. The study shows how to making decision making by taking subjective judgements of decision makers into considerations

2.10 Summary of Related Research

With regards to the so far discussed literature reviews at the time of conducting this study, there was no framework for evaluating the qualities of MLAM from the technical and non-technical perspective point of view and there was no earlier quality evaluation conducted on MLAM’s. For these reasons this study exploited the research gaps by proposing a framework for evaluating both the technical and non-technical aspects of MLAM and secondly to selected the most suitable MLAM by applying FAHP and TOPSIS multi-criteria methods together.
CHAPTER 3
CONCEPTUAL FRAMEWORK

This chapter discusses a newly proposed framework for selecting and evaluating mobile learning applications for mathematics. The framework is based on two components. First, it’s the technical perspective of MLAM that focuses on the quality of the application, while the second is the user perspective that focuses on the usability of the application.

3.1 Quality Aspects of Mobile Learning Applications

When an institution considers adopting mobile learning, therefore it is of critical importance to consider its quality (Arsovski & Arsovski, 2012) the quality aspect of mobile learning application is important because its gives the user satisfaction but the quality is achieved when the system is well designed based on a standard model (Melchor & Julián, 2008). However, Arsovski and Arsovski (2012) stated that evaluating the quality of mobile learning applications should be carried out in view of two ways which are listed below:

3.1.1 Technical perspectives

Mobile learning applications are regarded utterly as software applications that are designed typically to run on smart phones (Pocatilu & Vetrici, 2009) for this reasons mobile learning applications could be evaluated using software quality characteristics (Pocatilu & Boja, 2009). However, international standard organization ISO 9126 created a model to evaluate the quality of software products adopting accessibility, functionality, usability, efficiency, maintainability, portability and quality in use as its criteria. Despite the fact that there are many software quality models for evaluating quality in mobile applications McCall (1977), Boehm (1978), FURPS, Dromey, ISO-9126, ISO-25010, Bertoa, Gecuamo Model, Alvaro Model, Rawashdeh Model (Miguel, Mauricio and Rodríguez, 2014; Alvaro and de Almeida & Meira, 2007; Morisio, Stamelos and Tsoukias, 2002) but the ISO/IEC 9126 standard is mostly adopted for this purpose and mostly used from the technical perspective viewpoint (Chatterjee, Ghosh &
Bandyopadhyay, 2009). The following Figure 3.1 depicts the quality criteria and sub criteria of ISO 9126.

![ISO 9126 software quality criteria](image)

**Figure 3.1: ISO 9126 software quality criteria**

### 3.1.2 Non-technical perspectives

Mobile learning applications are designed to be used by end users which are mostly academicians. These MLAM’s are specifically designed using the software architecture model, thus these MLAM do not differ from the native applications in terms of design, what differentiate them is the content. Therefore, these non-technical aspects can be used to evaluate the user satisfaction of other applications just as the ISO 9126 quality model which can be used to evaluate the quality of any software. Therefore, the satisfaction of these academicians should be put into a heavy consideration when determining the overall quality of MLAM. This is because MLAM quality from the technical point of view alone does not necessarily gives the user satisfaction which is important, so the quality from the user point of view must be determined to ensure an overall effective evaluation process (Boja & Batagan, 2009).

Consequently, according to Arsovski and Arsovski (2012) one of the most populous model based on user satisfaction quality evaluation is the service quality model (SERVQUAL) but it cannot be adopted for the evaluation of MLAM’s because the model belongs to electronic services and has numerous properties or criteria.

Barnes and Vidgen (2002) proposed a different model which is WebQual, but the model was designed primarily for e-commerce which proposed it criteria as trust, empathy, usability, design and information. But Yoo and Donthu proposed a different model which proposed it criteria as ease of use, processing speed security and aesthetic and named the model as SITEQUAL. However, after a thorough literature review it was difficult to find a paper with a
model that was based utterly on user satisfaction purposely developed for MLAM. For this reason the study conceptualizes a model which is based on user satisfaction with the considerations of Kurilova and Vinogradova (2015) learning activity quality model and Boja et al. (2011) quality evaluation model.

In accordance to the proposed framework each criteria was selected based on a constructive factor, the ‘loading time’ criterion was selected because it surpasses other criteria in terms of importance (Boja & Batagan, 2009). ‘Ease of use’ appears in many studies which shows its importance amongst quality criteria, for this reason the criterion was included in the framework (Arsovski & Arsovski, 2012; Kurilova & Vinogradova, 2015). ‘Quality of content’ was selected because a quality MLAM is severely dependent on its content, the criterion was selected with regards to this factor (Arsovski & Arsovski, 2012). Before an application is termed qualitative, the application has to be in conformance with the user goal, the ‘conformance with learning goal’ was selected for this factor. The following Figure 3.2 shows the proposed user satisfaction quality model criteria for MLAM.

![Figure 3.2: MLAM quality criteria model based on user satisfaction](image)

### 3.2 Selection and Evaluation Criteria for MLAM

The MCDM approach is dependent on criteria and alternatives to achieve its goal, for this reason this study proposed its criteria based on the proposed framework which constitutes the technical and non-technical aspects (user satisfaction) to achieve its aim. The criteria consists of eleven criterion in which was used for selecting from the conflicting five alternatives. The following Table 3.1 shows the proposed criteria for this study.
Table 3.1: Proposed MLAM criteria

<table>
<thead>
<tr>
<th>Label</th>
<th>Proposed Criteria</th>
<th>Corresponding Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Functionality</td>
<td>$C_1$</td>
</tr>
<tr>
<td></td>
<td>Reliability</td>
<td>$C_2$</td>
</tr>
<tr>
<td></td>
<td>Usability</td>
<td>$C_3$</td>
</tr>
<tr>
<td></td>
<td>Efficiency</td>
<td>$C_4$</td>
</tr>
<tr>
<td></td>
<td>Maintainability</td>
<td>$C_5$</td>
</tr>
<tr>
<td></td>
<td>Portability</td>
<td>$C_6$</td>
</tr>
<tr>
<td></td>
<td>Quality in use</td>
<td>$C_7$</td>
</tr>
<tr>
<td></td>
<td>Ease of use</td>
<td>$C_8$</td>
</tr>
<tr>
<td></td>
<td>Loading time</td>
<td>$C_9$</td>
</tr>
<tr>
<td></td>
<td>Quality of content</td>
<td>$C_{10}$</td>
</tr>
<tr>
<td></td>
<td>Conformance with learning goal</td>
<td>$C_{11}$</td>
</tr>
</tbody>
</table>

Technical Perspective (ISO 9126)

Non-Technical Perspective (user satisfaction) (Kurilova & Vinogradova, 2015; Boja et al., 2011; Arsovski & Arsovski, 2012)

3.3 Samples of Mobile Learning Applications for Mathematics

MLAM comes in different forms depending on its design, functionalities, purpose, limitations and target audience. These applications are distributed digitally via the official Google Play store, which are either available freely or at cost. The Google Play store host millions of android apps of different categories such social, games, education, security etc. These apps are also rated by users based on their experience. However, for this study five MLAM’s for adults were selected as sampled applications for evaluation based on their respective Google App Store user rating of at least 4.0 out of 5. The Figure 3.3 below shows their respective Google App Store ratings. The following Table 3.2 shows the respective user ratings of each selected sampled application. Hence, the study targeted the android applications only for its sample because it’s open source and it’s the most populous mobile application store.
Table 3.2: MLAM Google App Store ratings

<table>
<thead>
<tr>
<th>MLAM</th>
<th>Ratings (0-5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathway</td>
<td>4.1</td>
</tr>
<tr>
<td>Malmaths</td>
<td>4.6</td>
</tr>
<tr>
<td>Cymaths</td>
<td>4.6</td>
</tr>
<tr>
<td>Mathematics</td>
<td>4.1</td>
</tr>
<tr>
<td>Mathspapa</td>
<td>4.7</td>
</tr>
</tbody>
</table>
3.3.1 Mathway

The Mathway is a mobile learning application used for learning mathematics with millions of users. It’s solve different mathematical problems ranging from complex calculus to basic algebra equations with instant result. The application provides an anytime anywhere learning and also home works. Its’ like a tutor on the go in your palm.

![Mathway worksheet](image)

*Figure 3.3: Mathway worksheet*
3.3.2 Malmaths

The Malmath is a mobile learning application used for learning mathematics with lots of users. It can also be used to solve mathematical problem with step by step descriptions and displays some graphical view when needed. It can be used to solve some range of mathematical fields such as integrals, derivatives, limits, trigonometry, logarithms, equations, algebra. It helps students to comprehend the understanding procedure and other people who have issues on their homework. It is useful for secondary school and students, instructors and guardians.

![Malmath worksheet](image)

**Figure 3.4:** Malmath worksheet
3.3.3 Cymaths

Cymath is a mobile learning application built mainly for learning mathematics and solving some mathematical equations. The application has millions of users worldwide. The application uses an algorithm which solves mathematical problems on the go ranging from calculus and algebra. The supported topics include equation solving, product rule, exponents, quotient rule, chain rule complex numbers, quadratic equations, trigonometry, partial fraction, factoring, logarithms, polynomial division, and u-substitution.

![Cymath worksheet](image)

**Figure 3.5:** Cymath worksheet
3.3.4 Mathematics

Mathematic application is a powerful mobile learning application used to learn and teach mathematics anytime, anywhere at the user finger tips. It ease learning mathematics by providing most of the formulas ranging from basic to complex within some few seconds. It cover the algebra field of mathematics, matrices, determinants and vectors. It also comes with some features which enables a student make conversion of units, from one unit to another. The whole features combines to make learning mathematics much easier than in the classroom.

Figure 3.6: Mathematics worksheet
3.3.5 Mathspapa

Mathspapa is a mobile application designed purposely to be used for learning mathematics and solve some mathematical problems, it has lots of users worldwide. The application has an inbuilt algorithm for solving mathematical equations step by step, such as systems of two equations, quadratic equations, graphical equations, linear equations with different methods. This application also support working in an offline mode. The application help student learn mathematic in a step by step way.

![MathPapa](image)

**Figure 3.7:** Mathspapa worksheet
This chapter explains in details the research methodology of the whole study, by which the research model, the instrumentation used in the research, techniques adopted in data analysis, research procedure and lastly research schedules.

4.1 Quality and User Satisfaction Model

The aim of this thesis is to select and evaluate different MLAM’s using MCDM methods. Many models have been proposed related to this thesis based on the literatures review, but the findings shows that there are few tangible quality model designed for evaluating mobile learning applications which were based on the technical point of view and user satisfaction point of view combined.

Therefore the study adopted the technical perspective and user satisfaction perspective as the main factor in determining the evaluation of the mobile learning application. This implies that the model was designed around the technical (ISO 9126) and non-technical (user satisfaction) combined because a technically faultless mobile learning application does not warrant a quality assessment by the potential user. Hence, this does not nullify the importance of the technical aspects of the mobile learning applications during evaluation.

As a result, the study identifies the following model for evaluating mobile learning applications after making numerous literature reviews on the subject matter. The following Figure 4.1 depicted the proposed model for evaluating mobile learning applications for mathematics.
Figure 4.1: Quality and user satisfaction model for evaluating MLAM
The above Figure 4.1 demonstrates the used values of the weight factors for both points of view. The best effect on the quality is based on the technical (ISO 9126) and non-technical (user satisfaction) assessments given by a decision maker.

4.2 Fuzzy Set

Fuzzy are set of numbers whose quantity is not specific, it can also be addressed as an extension of the popular Boolean logic whose sets is not just 0 and 1 but a connection of different values where each and every value is assigned a weight. It can also be stated as a set of values ranging from one interval to another. Fuzzy gives more dynamism to expression. So far there are many types of fuzzy numbers such as triangular, trapezoidal, octagonal, pyramid, pentagonal, diamond and hexagonal fuzzy numbers (Pathinathan & Ponnivalavan, 2015) thus they are used based on certain situation at place. However, Klir and Yuan (1995) stated that triangular fuzzy number and trapezoidal fuzzy numbers are the most widely used. For this reason this study adopts the triangular fuzzy number.

4.2.1 Triangular Fuzzy Number (TFN)

This type of fuzzy number consists of the set of three real numbers ranging from minimum, most expected and maximum weights. The Figure 4.2 below depicts the triangular fuzzy number with its three values, \(a_1\), \(a_2\) and \(a_3\).

![Figure 4.2: Triangular Fuzzy Number](image)
4.3 The FAHP Method

Saaty (1980) developed a new decision making model called AHP in which presumably became one of the most popular decision making methodology. This method divides the problem into smaller chunks of group in such a hierarchical form. However, the method encounters problems because it utilizes the exact values provided by decision maker to express it intake of the alternatives by pair wise comparison. Then, Chang (1996) modified this approach by proposing a triangular fuzzy numbers through utilizing pair wise comparison scale of FAHP. In addition, by utilizing the extent analysis technique for synthetic extent estimation for the pair wise comparisons.

Hence, during decision making processes the FAHP method is utilized in determining the weight of each criterion. However, the pairwise comparisons in the decision matrix are fuzzy numbers.

The following steps were adopted in selecting the most appropriate MLAM. Based on Chang (1996) technique, each object is selected and each goal respectively undergoes an extent analysis.

- **Step 1**: The value of fuzzy synthetic extent with respect to the $i^{th}$ object is determined as

$$S_i = \sum_{j=1}^{m} M_{gi}^j \oplus \left[ \sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j \right]^{-1}$$  \hspace{1cm} (1)

To derive $\sum_{j=1}^{m} M_{gi}^j$, the fuzzy addition operation of $m$ extent analysis values for the certain matrix is performed such as

$$\sum_{j=1}^{m} M_{gi}^j = \left( \sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right)$$  \hspace{1cm} (2)

And to acquire $[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j]$, by performing the fuzzy addition operation of $M_{gi}^j (j = 1, 2, \ldots)$ such that

$$\left[ \sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j \right]^{-1} = \left( \sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right)$$  \hspace{1cm} (3)
And \( \left[ \sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi} \right]^{-1} \) can be calculated by the inverse of Eq. (3), as follows:

\[
\left[ \sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi} \right]^{-1} = \left( \frac{1}{\sum_{i=1}^{n} u_i} \right) \left( \frac{1}{\sum_{i=1}^{n} u_i} \right) \left( \frac{1}{\sum_{i=1}^{n} u_i} \right)
\]

\( (4) \)

- **Step 2:** as \( M_1 = (l_1, m_1, u_1) \), and \( M_2 = (l_2, m_2, u_2) \) are two triangular fuzzy numbers, the degree of possibility of \( M_2 \geq M_1 \) is defined as

\[
V = (M_2 \geq M_1) = \operatorname{SUP}_{y \geq x} \left[ \min(\mu_{M_1}(x), \mu_{M_2}(y)) \right]
\]

\( (5) \)

And can be equivalently expressed as follows:

\[
V(M_1 \geq M_2) = hgt(M_1 \cap M_2) = \mu_{M_1}(d) = \begin{cases} 1 & \text{if } m_1 \geq m_2 \\ 0 & \text{if } l_2 \geq u_1 \\ \frac{(l_2 - u_1)}{(m_1 - u_1) - (m_2 - l_2)} & \text{Otherwise} \end{cases}
\]

\( (6) \)

Where \( d \), as shown in Figure 4.3, is the ordinate of the highest intersection point \( D \) between \( \mu(M_1) \) and \( \mu(M_2) \). To compare \( M_1 = (l_1, m_1, u_1) \), and \( M_2 = (l_2, m_2, u_2) \), we need both the values of \( V(M_1 \geq M_2) \) and \( V(M_2 \geq M_1) \).

**Figure 4.3:** Intersection between \( M_1 \) and \( M_2 \)

- **Step 3:** The degree possibility for a convex fuzzy number to be greater than \( k \) convex fuzzy \( M_i \) \((i = 1, 2, \ldots, k)\) numbers can be defined by

\[
V(M \geq M_1, M_2, \ldots, M_k) = \bigvee \left[ M \geq M_1 \text{ and } M \geq M_2 \text{ and } M \geq M_k \right]
\]
Assume that
\[ d'(A_i)=\min V(S_i \geq S_k) \text{ for } k=1,2,\ldots,n : k \neq i \] Then the weight vector is given by
\[ W^+ = (d'(A_1), d'(A_2), \ldots, d'(A_n))^T \] (8)

Where \( A_i \) (i=1,2,\ldots,n) are n elements.

**Step 4:** Via normalization, the normalized weight vectors are
\[ w = (d(A_1), d(A_2), \ldots, d(A_n))^T \] (9)

Where \( w \) is a non-fuzzy number.

### 4.4 The TOPSIS Method

Hwang & Yoon (1981) developed the TOPSIS technique, the method requires just an insignificant number of contributions from the user and its yield is straightforward. The main subjective parameters are the weights related with criteria. The essential thought of TOPSIS strategy is that the best solution is the one which has the least distance to the perfect arrangement and the uttermost separation from the counter perfect arrangement (Ishizaka & Nemery, 2013).

Hence, according to his method the alternative that is nearest to the ideal solution is the best, vice versa. According to Hwang and Yoon (1981) the following shows how the method is calculated.

**Step 1:** Evaluate the normalized decision matrix with value \( r_{ij} \):

\[ r_{ij} = x_{ij} \sqrt{\sum_{i=1}^{m} x_{ij}^2} i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n. \] (10)

**Step 2:** Evaluate the weighted normalized decision matrix with value \( v_{ij} \):

\[ v_{ij} = r_{ij} \times w_j \text{ i=1, 2,\ldots, m and j = 1, 2,\ldots, n.} \] (11)
Where, \( w_j \) is the assigned weight of each \( j^{th} \) criterion and \( \sum_{j=1}^{n} w_j = 1 \).

- **Step 3:** Get the ideal (\( A^* \)) and negative ideal (\( A^- \)) solutions.

\[
A^* = \left\{ \left( \max_i v_{ij} \mid j \in C_b \right), \left( \min_i v_{ij} \mid j \in C_c \right) \right\} = \{v_j^* \mid j = 1, 2, ..., m\} \tag{12}
\]

\[
A^- = \left\{ \left( \min_i v_{ij} \mid j \in C_b \right), \left( \max_i v_{ij} \mid j \in C_c \right) \right\} = \{v_j^- \mid j = 1, 2, ..., m\} \tag{13}
\]

- **Step 4:** Evaluate the separation measures using the \( m \)-dimensional Euclidean distance of each alternative from positive ideal solution and negative ideal solution:

\[
S_i^* = \sqrt{\sum_{j=1}^{m} (v_{ij} - v_j^*)^2}, j = 1, 2, ..., m \tag{14}
\]

\[
S_i^- = \sqrt{\sum_{j=1}^{m} (v_{ij} - v_j^-)^2}, j = 1, 2, ..., m \tag{15}
\]

- **Step 5:** Evaluate the relative closeness to ideal solution of the alternative \( A_i \) with respect to \( A^* \) is defined below:

\[
RC_i^* = \frac{S_i^-}{S_i^* + S_i^-}, i = 1, 2, ..., m \tag{16}
\]

- **Step 6:** Finally, rank in order of it preferences.

### 4.5 The Selection and Evaluation Method

The present study integrates two MCDM techniques FAHP and TOPSIS due to the nature of its aim to select and evaluate MLAM’s. The FAHP method is the most widely used MCDM technique but it comes with some disadvantages that makes the values differ based on the form of hierarchy structure and also maintaining consistency is problematic amongst responses (Chalúpková, E., & Franek, 2014). As a result, the study omitted the final part of the FAHP method which is the alternatives and integrated the TOPSIS method to produce the final ranking. Furthermore, the verdict to combine two MCDM techniques was also in order to improve the evaluation process and because the two methods are usually combined in complex
decision issues (Yang et al., 2007; Chen & Tzeng, 2004). Accordingly, this approach makes a study of the proposed model’s practical behavior based on the assumption of initial priorities and values from the decision maker. Subsequently, the MLAM’s were ranked based on the proposed combined eleven criteria. Hence, for this study five MLAM’s were sampled and the author is the decision maker.

Figure 4.4: Hierarchy structure for decision making problem for MLAM’s
4.6 Research Procedure

The study was conducted in accordance to the following steps in a chronological manner:

1. A review of various literature was conducted based on the related research subject area, to investigate why, how previous studies were conducted, and the findings of previous literature and to find out the study lapses. This provided a road map of how the study was conducted.

2. A research proposal was accepted by the department to embark on the study.

3. A conceptualize framework was developed for assessing the quality of MLAM in accordance to the result of the literature review and some MLAM were sampled for the study.

4. Later on, a research model based on technical and user satisfaction was developed with the criteria for evaluating the sampled MLAM.

5. The selection and evaluation of the five sampled MLAM was conducted using FAHP and TOPSIS methods respectively.

6. Subsequently, discussion of the result and recommendations was done.

4.7 Research Schedules

The completion of the study took 16 weeks as depicted in the Table 4.1. Figure 4.5 shows the Gantt chart for the study.

Table 4.1: Research schedules

<table>
<thead>
<tr>
<th>Work done</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thesis proposal and seeking approval</td>
<td>4 Weeks</td>
</tr>
<tr>
<td>Writing thesis</td>
<td>6 Weeks</td>
</tr>
<tr>
<td>Sample collection, selection and evaluation</td>
<td>1 Weeks</td>
</tr>
<tr>
<td>Final thesis draft</td>
<td>1 Weeks</td>
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<tr>
<td>Reading, discussion and correction based on the feedback of supervisor</td>
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<td><strong>Total</strong></td>
<td><strong>16 weeks</strong></td>
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</table>
Figure 4.5: Study Gantt chart
This chapter shows and discussed the selection and evaluation process in selecting the best MLAM out of five sampled conflicting alternatives which are Mathsway, Malmaths, Cymaths, Mathematics and Mathspapa using FAHP and TOPSIS method.

5.1 Case Study: Evaluation of MLAM

The following illustrates the selection and evaluation of MLAM based on an actual experiment. The carried out experiment was implemented on five MLAM’s using a triangular fuzzy numbers (TFN). To start off the FAHP evaluation process, a pairwise matrix for comparison was created using a corresponding linguistics scale. The Table 5.1 defines the proposed linguistics scale with its corresponding TFN and TFN\(^{-1}\).

- **FAHP Method**

  **Table 5.1**: The linguistic scale with its corresponding TFN (Brajković et al., 2015).

<table>
<thead>
<tr>
<th>Linguistic Scale</th>
<th>Description</th>
<th>TFN</th>
<th>TFN(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same significance</td>
<td>When two criteria contribution to a goal is same.</td>
<td>(1, 1, 1)</td>
<td>(1, 1, 1)</td>
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<tr>
<td></td>
<td>When one criterion slightly preside the other criterion’s contribution to a goal.</td>
<td>(0.33, 0.5, 1)</td>
<td>(1, 2, 3)</td>
</tr>
<tr>
<td>Average significance</td>
<td>When one criterion clearly preside the other criterion’s contribution to a goal.</td>
<td>(0.75, 1, 1.25)</td>
<td>(0.8, 1, 1.33)</td>
</tr>
<tr>
<td>Significant</td>
<td>When one criterion clearly preside the other criterion’s contribution to a goal.</td>
<td>(0.75, 1, 1.25)</td>
<td>(0.8, 1, 1.33)</td>
</tr>
<tr>
<td>Very significant</td>
<td>When one criterion clearly dominate the other criterion’s contribution to a goal.</td>
<td>(1, 2, 3)</td>
<td>(0.33, 0.5, 1)</td>
</tr>
<tr>
<td>Very strong significant</td>
<td>When one criterion clearly dominate the other criterion’s contribution to a goal.</td>
<td>(1.33, 2, 4)</td>
<td>(0.25, 0.5, 0.75)</td>
</tr>
</tbody>
</table>
The below Table 5.2 is the developed fuzzy evaluation matrix for making a pairwise comparison of each criterion. The matrix was developed by the author with a Mathematical and Computer Science background. Furthermore, the matrix was cross checked by the supervisor with a Mathematical background as well.

**Table 5.2: The developed fuzzy evaluation matrix**

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<th></th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
<th>C₆</th>
<th>C₇</th>
<th>C₈</th>
<th>C₉</th>
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</tr>
</tbody>
</table>

39
A hierarchy structure was developed in the form of a fuzzy evaluation matrix. The above Table 5.2 depicts the evaluation matrix according to the adopted method FAHP.

Table 5.3: Summation of rows and columns

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<tr>
<th>Criteria</th>
<th>Sum of Rows</th>
<th>Sum of Columns</th>
</tr>
</thead>
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<td>C_1</td>
<td>(9.16, 14, 19.5)</td>
<td>(7.92, 11, 15.91)</td>
</tr>
<tr>
<td>C_2</td>
<td>(8.91, 12, 16)</td>
<td>(9.06, 12, 15.99)</td>
</tr>
<tr>
<td>C_3</td>
<td>(9.08, 13.5, 18.25)</td>
<td>(7.99, 10.5, 14.65)</td>
</tr>
<tr>
<td>C_4</td>
<td>(7.82, 11, 15.5)</td>
<td>(9.26, 14, 19.66)</td>
</tr>
<tr>
<td>C_5</td>
<td>(8.56, 12, 16.49)</td>
<td>(8.51, 12, 16.41)</td>
</tr>
<tr>
<td>C_6</td>
<td>(9.48, 11.5, 14.24)</td>
<td>(9.38, 11.5, 14.08)</td>
</tr>
<tr>
<td>C_7</td>
<td>(8.34, 12.5, 17.91)</td>
<td>(8.29, 12.5, 17.83)</td>
</tr>
<tr>
<td>C_8</td>
<td>(9.88, 14.5, 19.58)</td>
<td>(7.87, 10, 13.58)</td>
</tr>
<tr>
<td>C_9</td>
<td>(7.94, 10.5, 14.57)</td>
<td>(9.13, 13.5, 18.33)</td>
</tr>
<tr>
<td>C_10</td>
<td>(9.06, 12, 15.99)</td>
<td>(8.91, 12, 15.75)</td>
</tr>
<tr>
<td>C_11</td>
<td>(7.25, 10.5, 15.66)</td>
<td>(9.16, 15, 21.5)</td>
</tr>
</tbody>
</table>

Sum of rows or columns: (95.48, 134, 183.69)

In the wake of forming the fuzzy pairwise comparison matrix, FAHP determines the weight of each criterion. However, in respect to FAHP methodology the synthesis extent values should be determined. The synthesis extent values can be determined according to Equation 1. The following shows the calculations of all the synthesis extent values.

\[
SC_1 = (9.16, 14, 19.5) \otimes (95.48, 134, 183.69)^{-1} = (0.0499, 0.1045, 0.2042)
\]

\[
SC_2 = (8.91, 12, 16) \otimes (95.48, 134, 183.69)^{-1} = (0.0485, 0.0896, 0.1676)
\]

\[
SC_3 = (9.08, 13.5, 18.25) \otimes (95.48, 134, 183.69)^{-1} = (0.0493, 0.1007, 0.1911)
\]

\[
SC_4 = (7.82, 11, 15.5) \otimes (95.48, 134, 183.69)^{-1} = (0.0426, 0.0821, 0.1623)
\]

\[
SC_5 = (8.56, 12, 16.49) \otimes (95.48, 134, 183.69)^{-1} = (0.0466, 0.0896, 0.1727)
\]
$SC_6 = (9.48, 11.5, 14.24) \otimes (95.48, 134, 183.69)^{-1} = (0.0516, 0.0859, 0.1491)$

$SC_7 = (8.34, 12.5, 17.91) \otimes (95.48, 134, 183.69)^{-1} = (0.0454, 0.0933, 0.1876)$

$SC_8 = (9.88, 14.5, 19.58) \otimes (95.48, 134, 183.69)^{-1} = (0.0537, 0.1082, 0.2051)$

$SC_9 = (7.94, 10.5, 14.57) \otimes (95.48, 134, 183.69)^{-1} = (0.0432, 0.0784, 0.1526)$

$SC_{10} = (9.06, 12, 15.99) \otimes (95.48, 134, 183.69)^{-1} = (0.0493, 0.0896, 0.1675)$

$SC_{11} = (7.25, 10.5, 15.66) \otimes (95.48, 134, 183.69)^{-1} = (0.0395, 0.0784, 0.1640)$

<table>
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<tr>
<th>Criteria</th>
<th>$SC_i$</th>
</tr>
</thead>
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</tr>
<tr>
<td>C2</td>
<td>(0.0485, 0.0896, 0.1676)</td>
</tr>
<tr>
<td>C3</td>
<td>(0.0493, 0.1007, 0.1911)</td>
</tr>
<tr>
<td>C4</td>
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<tr>
<td>C5</td>
<td>(0.0466, 0.0896, 0.1727)</td>
</tr>
<tr>
<td>C6</td>
<td>(0.0516, 0.0859, 0.1491)</td>
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<td>C7</td>
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<tr>
<td>C11</td>
<td>(0.0395, 0.0784, 0.1640)</td>
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</tbody>
</table>

Table 5.4: The result of the synthesis extent values of each criteria

The above synthesis extent values were used to make comparison using Equation 6 to obtain the following results.

$V (SC_1 \geq SC_2) = 1$, $V (SC_1 \geq SC_3) = 1$, $V (SC_1 \geq SC_4) = 1$, $V (SC_1 \geq SC_5) = 1$, $V (SC_1 \geq SC_6) = 1$, $V (SC_1 \geq SC_7) = 1$, $V (SC_1 \geq SC_8) = 0.9760$, $V (SC_1 \geq SC_9) = 1$, $V (SC_1 \geq SC_{10}) = 1$, $V (SC_1 \geq SC_{11}) = 1$
\[ V(\text{SC}_2 \geq \text{SC}_1) = 0.8876, \quad V(\text{SC}_2 \geq \text{SC}_3) = 0.9142, \quad V(\text{SC}_2 \geq \text{SC}_4) = 1, \quad V(\text{SC}_2 \geq \text{SC}_5) = 1, \quad V(\text{SC}_2 \geq \text{SC}_6) = 1, \quad V(\text{SC}_2 \geq \text{SC}_7) = 0.9706, \quad V(\text{SC}_2 \geq \text{SC}_8) = 0.8596, \quad V(\text{SC}_2 \geq \text{SC}_9) = 1, \quad V(\text{SC}_2 \geq \text{SC}_{10}) = 1, \quad V(\text{SC}_2 \geq \text{SC}_{11}) = 1 \]

\[ V(\text{SC}_3 \geq \text{SC}_1) = 0.9738, \quad V(\text{SC}_3 \geq \text{SC}_2) = 1, \quad V(\text{SC}_3 \geq \text{SC}_4) = 1, \quad V(\text{SC}_3 \geq \text{SC}_5) = 1, \quad V(\text{SC}_3 \geq \text{SC}_6) = 1, \quad V(\text{SC}_3 \geq \text{SC}_7) = 1, \quad V(\text{SC}_3 \geq \text{SC}_8) = 0.9482, \quad V(\text{SC}_3 \geq \text{SC}_9) = 1, \quad V(\text{SC}_3 \geq \text{SC}_{10}) = 1, \quad V(\text{SC}_3 \geq \text{SC}_{11}) = 1 \]

\[ V(\text{SC}_4 \geq \text{SC}_1) = 0.8338, \quad V(\text{SC}_4 \geq \text{SC}_2) = 0.9382, \quad V(\text{SC}_4 \geq \text{SC}_3) = 0.8587, \quad V(\text{SC}_4 \geq \text{SC}_5) = 0.9391, \quad V(\text{SC}_4 \geq \text{SC}_6) = 0.9668, \quad V(\text{SC}_4 \geq \text{SC}_7) = 0.9126, \quad V(\text{SC}_4 \geq \text{SC}_8) = 0.8062, \quad V(\text{SC}_4 \geq \text{SC}_9) = 1, \quad V(\text{SC}_4 \geq \text{SC}_{10}) = 0.9378, \quad V(\text{SC}_4 \geq \text{SC}_{11}) = 1 \]

\[ V(\text{SC}_5 \geq \text{SC}_1) = 0.8918, \quad V(\text{SC}_5 \geq \text{SC}_2) = 1, \quad V(\text{SC}_5 \geq \text{SC}_3) = 0.9175, \quad V(\text{SC}_5 \geq \text{SC}_4) = 1, \quad V(\text{SC}_5 \geq \text{SC}_6) = 1, \quad V(\text{SC}_5 \geq \text{SC}_7) = 0.9718, \quad V(\text{SC}_5 \geq \text{SC}_8) = 0.8648, \quad V(\text{SC}_5 \geq \text{SC}_9) = 1, \quad V(\text{SC}_5 \geq \text{SC}_{10}) = 1, \quad V(\text{SC}_5 \geq \text{SC}_{11}) = 1 \]

\[ V(\text{SC}_6 \geq \text{SC}_1) = 0.8421, \quad V(\text{SC}_6 \geq \text{SC}_2) = 0.9645, \quad V(\text{SC}_6 \geq \text{SC}_3) = 0.8709, \quad V(\text{SC}_6 \geq \text{SC}_4) = 1, \quad V(\text{SC}_6 \geq \text{SC}_5) = 0.9652, \quad V(\text{SC}_6 \geq \text{SC}_7) = 0.9334, \quad V(\text{SC}_6 \geq \text{SC}_8) = 0.8105, \quad V(\text{SC}_6 \geq \text{SC}_9) = 1, \quad V(\text{SC}_6 \geq \text{SC}_{10}) = 0.9643, \quad V(\text{SC}_6 \geq \text{SC}_{11}) = 1 \]

\[ V(\text{SC}_7 \geq \text{SC}_1) = 0.9248, \quad V(\text{SC}_7 \geq \text{SC}_2) = 1, \quad V(\text{SC}_7 \geq \text{SC}_3) = 0.9492, \quad V(\text{SC}_7 \geq \text{SC}_4) = 1, \quad V(\text{SC}_7 \geq \text{SC}_5) = 1, \quad V(\text{SC}_7 \geq \text{SC}_6) = 1, \quad V(\text{SC}_7 \geq \text{SC}_8) = 0.8999, \quad V(\text{SC}_7 \geq \text{SC}_9) = 1, \quad V(\text{SC}_7 \geq \text{SC}_{10}) = 1, \quad V(\text{SC}_7 \geq \text{SC}_{11}) = 1 \]

\[ V(\text{SC}_8 \geq \text{SC}_1) = 1, \quad V(\text{SC}_8 \geq \text{SC}_2) = 1, \quad V(\text{SC}_8 \geq \text{SC}_3) = 1, \quad V(\text{SC}_8 \geq \text{SC}_4) = 1, \quad V(\text{SC}_8 \geq \text{SC}_5) = 1, \quad V(\text{SC}_8 \geq \text{SC}_6) = 1, \quad V(\text{SC}_8 \geq \text{SC}_7) = 1, \quad V(\text{SC}_8 \geq \text{SC}_9) = 1, \quad V(\text{SC}_8 \geq \text{SC}_{10}) = 1, \quad V(\text{SC}_8 \geq \text{SC}_{11}) = 1 \]

\[ V(\text{SC}_9 \geq \text{SC}_1) = 0.7974, \quad V(\text{SC}_9 \geq \text{SC}_2) = 0.9029, \quad V(\text{SC}_9 \geq \text{SC}_3) = 0.8225, \quad V(\text{SC}_9 \geq \text{SC}_4) = 0.9675, \quad V(\text{SC}_9 \geq \text{SC}_5) = 0.9044, \quad V(\text{SC}_9 \geq \text{SC}_6) = 0.9309, \quad V(\text{SC}_9 \geq \text{SC}_7) = 0.8780, \quad V(\text{SC}_9 \geq \text{SC}_8) = 0.7685, \quad V(\text{SC}_9 \geq \text{SC}_{10}) = 0.9022, \quad V(\text{SC}_9 \geq \text{SC}_{11}) = 1 \]

\[ V(\text{SC}_{10} \geq \text{SC}_1) = 0.8743, \quad V(\text{SC}_{10} \geq \text{SC}_2) = 1, \quad V(\text{SC}_{10} \geq \text{SC}_3) = 0.9142, \quad V(\text{SC}_{10} \geq \text{SC}_4) = 1, \quad V(\text{SC}_{10} \geq \text{SC}_5) = 1, \quad V(\text{SC}_{10} \geq \text{SC}_6) = 1, \quad V(\text{SC}_{10} \geq \text{SC}_7) = 0.9706, \quad V(\text{SC}_{10} \geq \text{SC}_8) = 0.8595, \quad V(\text{SC}_{10} \geq \text{SC}_9) = 1, \quad V(\text{SC}_{10} \geq \text{SC}_{11}) = 1 \]
\[ V (SC_{11} \geq SC_1) = 0.8138, \ V (SC_{11} \geq SC_2) = 0.9116, \ V (SC_{11} \geq SC_3) = 0.8372, \ V (SC_{11} \geq SC_4) = 0.9704, \ V (SC_{11} \geq SC_5) = 0.9129, \ V (SC_{11} \geq SC_6) = 0.9374, \ V (SC_{11} \geq SC_7) = 0.8883, \ V (SC_{11} \geq SC_8) = 0.7873, \ V (SC_{11} \geq SC_9) = 1, \ V (SC_{11} \geq SC_{10}) = 0.9110 \]

Then, with the use of Equation 8 each weight is calculated.

\[ d'(C_1) = \min (1, 1, 1, 1, 1, 0.9760, 1, 1, 1) = 0.9760 \]
\[ d'(C_2) = \min (0.8876, 0.9142, 1, 1, 1, 0.9706, 0.8596, 1, 1, 1) = 0.8596 \]
\[ d'(C_3) = \min (0.9738, 1, 1, 1, 1, 0.9482, 1, 1, 1) = 0.9482 \]
\[ d'(C_4) = \min (0.8338, 0.9382, 0.8587, 0.9391, 0.9668, 0.9126, 0.8062, 1, 0.9378, 1) = 0.8062 \]
\[ d'(C_5) = \min (0.8918, 1, 0.9175, 1, 0.9718, 0.8648, 1, 1, 1) = 0.8648 \]
\[ d'(C_6) = \min (0.8421, 0.9645, 0.8709, 1, 0.9652, 0.9334, 0.8105, 1, 0.9643, 1) = 0.8105 \]
\[ d'(C_7) = \min (0.9248, 1, 0.9492, 1, 1, 1, 0.8999, 1, 1, 1) = 0.8999 \]
\[ d'(C_8) = \min (1, 1, 1, 1, 1, 1, 1, 1, 1) = 1 \]
\[ d'(C_9) = \min (0.7974, 0.9029, 0.8225, 0.9675, 0.9044, 0.9309, 0.8780, 0.7685, 0.9022, 1) = 0.7685 \]
\[ d'(C_{10}) = \min (0.8743, 1, 0.9142, 1, 1, 1, 0.9706, 0.8595, 1, 1) = 0.8595 \]
\[ d'(C_{11}) = \min (0.8138, 0.9116, 0.8372, 0.9704, 0.9129, 0.9374, 0.8883, 0.7873, 1, 0.9110) = 0.7873 \]

Then, the resulting priority weight forms the vector below.

\[ W' = (0.9760, 0.8596, 0.9482, 0.8062, 0.8648, 0.8105, 0.8999, 1, 0.7685, 0.8595, 0.7873) \]

Then, using Equation 9 the priority weights are normalized to form the following vector with regards to the main goal.

\[ W = (0.1019, 0.0897, 0.0990, 0.0842, 0.0903, 0.0846, 0.0939, 0.1044, 0.0802, 0.0897, 0.0822) \]
• **TOPSIS method**

After obtaining the weights through some series of FAHP method, then the TOPSIS method was used to calculate the ranking of the alternatives. However, according to TOPSIS method the first step is to form a decision matrix. This matrix is formed based on the proposed eleven criteria with regards to the MLAM alternatives which is used for the ranking in Table 5.5 below. The decision matrix in Table 5.5 was rated by the decision maker. However, the criteria where rated with respect to the five MLAM’s accordingly and the rating scores of all the criteria were based on a scale of 0-10 except criterion $C_9$ (Loading time) which was measured in seconds.

**Table 5.5: Decision matrix**

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
<th>$C_7$</th>
<th>$C_8$</th>
<th>$C_9$</th>
<th>$C_{10}$</th>
<th>$C_{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathway</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>8</td>
<td>7</td>
<td>8</td>
<td>6</td>
<td>8</td>
<td>15</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Malmaths</td>
<td>9</td>
<td>6</td>
<td>5</td>
<td>7</td>
<td>6</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>12</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>Cymaths</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>8</td>
<td>6</td>
<td>7</td>
<td>9</td>
<td>8</td>
<td>10</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Mathematics</td>
<td>7</td>
<td>8</td>
<td>7</td>
<td>9</td>
<td>7</td>
<td>7</td>
<td>6</td>
<td>8</td>
<td>19</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Mathspapa</td>
<td>9</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>8</td>
<td>9</td>
<td>5</td>
<td>8</td>
<td>15</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

The decision matrix is then normalized using Equation 10 as shown in Table 5.6 below.

**Table 5.6: Normalized decision matrix**

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
<th>$C_7$</th>
<th>$C_8$</th>
<th>$C_9$</th>
<th>$C_{10}$</th>
<th>$C_{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathway</td>
<td>0.43</td>
<td>0.45</td>
<td>0.44</td>
<td>0.47</td>
<td>0.46</td>
<td>0.46</td>
<td>0.39</td>
<td>0.45</td>
<td>0.46</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>Malmaths</td>
<td>0.49</td>
<td>0.38</td>
<td>0.37</td>
<td>0.41</td>
<td>0.39</td>
<td>0.46</td>
<td>0.51</td>
<td>0.45</td>
<td>0.37</td>
<td>0.50</td>
<td>0.45</td>
</tr>
<tr>
<td>Cymaths</td>
<td>0.43</td>
<td>0.45</td>
<td>0.44</td>
<td>0.47</td>
<td>0.39</td>
<td>0.40</td>
<td>0.58</td>
<td>0.45</td>
<td>0.31</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>Mathematics</td>
<td>0.38</td>
<td>0.51</td>
<td>0.52</td>
<td>0.52</td>
<td>0.46</td>
<td>0.40</td>
<td>0.39</td>
<td>0.45</td>
<td>0.59</td>
<td>0.39</td>
<td>0.45</td>
</tr>
<tr>
<td>Mathspapa</td>
<td>0.49</td>
<td>0.45</td>
<td>0.45</td>
<td>0.35</td>
<td>0.52</td>
<td>0.51</td>
<td>0.32</td>
<td>0.45</td>
<td>0.46</td>
<td>0.45</td>
<td>0.45</td>
</tr>
</tbody>
</table>
Each normalized weight is then evaluated using Equation 11 by multiplying each weight with its corresponding decision matrix. The following Table 5.7 shows the result.

### Table 5.7: Weighted normalized decision matrix

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathway</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.05</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Malmaths</td>
<td>0.05</td>
<td>0.03</td>
<td>0.04</td>
<td>0.03</td>
<td>0.04</td>
<td>0.05</td>
<td>0.05</td>
<td>0.03</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>Cymaths</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.03</td>
<td>0.05</td>
<td>0.05</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>Mathematics</td>
<td>0.04</td>
<td>0.05</td>
<td>0.05</td>
<td>0.04</td>
<td>0.04</td>
<td>0.03</td>
<td>0.04</td>
<td>0.05</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>Mathspapa</td>
<td>0.05</td>
<td>0.04</td>
<td>0.04</td>
<td>0.03</td>
<td>0.05</td>
<td>0.04</td>
<td>0.03</td>
<td>0.05</td>
<td>0.04</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Then by obtaining the minimum and maximum value of each criterion the ideal negative and positive solutions are determined using the Equation 12 and Equation 13.

\[ A^+ = (0.0498, 0.0457, 0.0514, 0.0442, 0.0472, 0.0435, 0.0543, 0.0467, 0.0469, 0.0450, 0.0368) \]

\[ A^- = (0.0387, 0.0342, 0.0367, 0.0295, 0.0354, 0.0338, 0.0302, 0.0467, 0.0247, 0.0350, 0.0368) \]

Using Equation 14 and Equation 15, the length of each of the five alternative from negative ideal solution and positive ideal solution is determined with regards to each criterion.

\[ S_1^+ = \sqrt{(0.0443 - 0.0498)^2 + (0.0400 - 0.0457)^2 + (0.0440 - 0.0514)^2 + (0.0393 - 0.0442)^2 + (0.0413 - 0.0472)^2 + (0.0386 - 0.0435)^2 + (0.0362 - 0.0543)^2 + (0.0467 - 0.0467)^2 + (0.0370 - 0.0469)^2 + (0.0400 - 0.0450)^2 + (0.0368 - 0.0368)^2} \]

\[ S_1^- = \sqrt{(0.0443 - 0.0387)^2 + (0.0400 - 0.0342)^2 + (0.0440 - 0.0367)^2 + (0.0393 - 0.0295)^2 + (0.0413 - 0.0354)^2 + (0.0386 - 0.0338)^2 + (0.0362 - 0.0302)^2 + (0.0467 - 0.0467)^2 + (0.0370 - 0.0247)^2 + (0.0400 - 0.0350)^2 + (0.0368 - 0.0368)^2} \]

\[ S_2^+ = \sqrt{(0.0498 - 0.0498)^2 + (0.0342 - 0.0457)^2 + (0.0367 - 0.0514)^2 + (0.0344 - 0.0442)^2 + (0.0354 - 0.0472)^2 + (0.0386 - 0.0435)^2 + (0.0483 - 0.0543)^2 + (0.0467 - 0.0467)^2 + (0.0296 - 0.0469)^2 + (0.0450 - 0.0450)^2 + (0.0368 - 0.0368)^2} \]
\[ S_2^- = \sqrt{(0.0498 - 0.0387)^2 + (0.0342 - 0.0342)^2 + (0.0367 - 0.0367)^2 + (0.0344 - 0.0295)^2 + (0.0354 - 0.0354)^2 + (0.0386 - 0.0338)^2 + (0.0483 - 0.0302)^2 + (0.0467 - 0.0467)^2 + (0.0296 - 0.0247)^2 + (0.0450 - 0.0350)^2 + (0.0368 - 0.0368)^2} \]

\[ S_3^+ = \sqrt{(0.0443 - 0.0498)^2 + (0.0400 - 0.0457)^2 + (0.0440 - 0.0514)^2 + (0.0393 - 0.0442)^2 + (0.0354 - 0.0472)^2 + (0.0338 - 0.0435)^2 + (0.0543 - 0.0543)^2 + (0.0467 - 0.0467)^2 + (0.0247 - 0.0469)^2 + (0.0400 - 0.0450)^2 + (0.0368 - 0.0368)^2} \]

\[ S_3^- = \sqrt{(0.0443 - 0.0387)^2 + (0.0400 - 0.0342)^2 + (0.0440 - 0.0367)^2 + (0.0393 - 0.0295)^2 + (0.0354 - 0.0354)^2 + (0.0338 - 0.0338)^2 + (0.0543 - 0.0302)^2 + (0.0467 - 0.0467)^2 + (0.0247 - 0.0247)^2 + (0.0400 - 0.0350)^2 + (0.0368 - 0.0368)^2} \]

\[ S_4^+ = \sqrt{(0.0387 - 0.0498)^2 + (0.0457 - 0.0457)^2 + (0.0514 - 0.0514)^2 + (0.0442 - 0.0442)^2 + (0.0413 - 0.0472)^2 + (0.0338 - 0.0435)^2 + (0.0362 - 0.0543)^2 + (0.0467 - 0.0467)^2 + (0.0469 - 0.0469)^2 + (0.0350 - 0.0450)^2 + (0.0368 - 0.0368)^2} \]

\[ S_4^- = \sqrt{(0.0387 - 0.0387)^2 + (0.0457 - 0.0342)^2 + (0.0514 - 0.0367)^2 + (0.0442 - 0.0295)^2 + (0.0413 - 0.0354)^2 + (0.0338 - 0.0338)^2 + (0.0362 - 0.0302)^2 + (0.0467 - 0.0467)^2 + (0.0469 - 0.0247)^2 + (0.0350 - 0.0350)^2 + (0.0368 - 0.0368)^2} \]

\[ S_5^+ = \sqrt{(0.0498 - 0.0498)^2 + (0.0400 - 0.0457)^2 + (0.0440 - 0.0514)^2 + (0.0295 - 0.0442)^2 + (0.0472 - 0.0472)^2 + (0.0435 - 0.0435)^2 + (0.0302 - 0.0543)^2 + (0.0467 - 0.0467)^2 + (0.0370 - 0.0469)^2 + (0.0400 - 0.0450)^2 + (0.0368 - 0.0368)^2} \]

\[ S_5^- = \sqrt{(0.0498 - 0.0387)^2 + (0.0400 - 0.0342)^2 + (0.0440 - 0.0367)^2 + (0.0295 - 0.0295)^2 + (0.0472 - 0.0354)^2 + (0.0435 - 0.0338)^2 + (0.0302 - 0.0302)^2 + (0.0467 - 0.0467)^2 + (0.0370 - 0.0247)^2 + (0.0400 - 0.0350)^2 + (0.0368 - 0.0368)^2} \]
By using Equation 16 the relative closeness to the ideal solution of each alternative was calculated. Below Table 5.8 shows the result of the final ranking of the alternatives.

<table>
<thead>
<tr>
<th>Application</th>
<th>$S_i^*$</th>
<th>$S_i^-$</th>
<th>$R_{Ci}$</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathway</td>
<td>0.02551</td>
<td>0.02201</td>
<td>0.463</td>
<td>3</td>
</tr>
<tr>
<td>Malmaths</td>
<td>0.03071</td>
<td>0.02494</td>
<td>0.448</td>
<td>4</td>
</tr>
<tr>
<td>Cymaths</td>
<td>0.02988</td>
<td>0.02864</td>
<td>0.489</td>
<td>2</td>
</tr>
<tr>
<td>Mathematics</td>
<td>0.02607</td>
<td>0.03359</td>
<td>0.563</td>
<td>1</td>
</tr>
<tr>
<td>Mathspapa</td>
<td>0.03174</td>
<td>0.02489</td>
<td>0.440</td>
<td>5</td>
</tr>
</tbody>
</table>

5.2 Results

Hence, with regards to the obtained result in Table 5.8 the most preferred choice amongst the MLAM’s is the ‘Mathematics’ application. The ‘Cymaths’ application followed suit with regards to preference. The third application is ‘Mathway’ based on the attained result. ‘Malmaths’ application is the fourth choice. The last preferred application is ‘Mathspapa’. The obtained result differed from the Google Play store user ratings because the user ratings was done based on the users judgment of the applications while the result of this study was influenced by the values of the decision maker preferences with dissimilar criteria. In addition, the final ranking of the MLAM’s may be different in another study based on the values of the decision maker preferences and the framework.

Hence, there is no doubt that the study decision making had yielded an optimal result because two appealing MCDM techniques were adopted in order to get an optimal result, the FAHP and TOPSIS are one of the most used MCDM techniques, and the study balanced the model by proposing a non-technical aspect instead of sticking to just the technical aspect of the ISO 9126 quality model. Sticking with these aforementioned measures will surely yield a good decision making.
5.3 Discussion

The study adopted a proposed framework which comprises of technical and non-technical aspects of software quality criteria which is different from many frameworks such as McCall (1977), Boehm (1978), FURPS, Dromey, ISO-9126, ISO-25010, Bertoa, Gecuamo Model, Alvaro Model, Rawashdeh Model (Miguel, Mauricio and Rodríguez, 2014; Alvaro and de Almeida & Meira, 2007; Morisio, Stamelos and Tsoukias, 2002) because these frameworks only focused on the technical aspects of the software which does not guarantee a user satisfaction of the application. The proposed framework was used to evaluate the MLAM’s using FAHP and TOPSIS methods respectively. However, Brajković et al. (2015), Ballı and Korukoğlu (2009), Işık, and Büyüközkan (2007) and Volarić et al. (2014) all conducted a research with a similar methodology which combined FAHP and TOPSIS methods to enhance result, thus with different empirical study and results reported.

This study combined technical aspect quality framework the ISO 9126 and proposed a non-technical aspect framework, the user satisfaction to improve the result. In addition the study adopted the triangular fuzzy number which is an extension of real numbers that provides more dynamism during evaluation and lets the incorporation of ambiguity on limits. And also combining FAHP and TOPSIS to enhance the accuracy of the projected result.
CHAPTER 6
CONCLUSION AND RECOMMENDATIONS

The study winds up with a conclusion and recommendation for future works.

6.1 Conclusion

There is no doubt that the manual selection of MLAM had resulted many individuals in making selection prematurely. The study had shown how to combine both fuzzy AHP and TOPSIS to weight and rank MLAM alternatives respectively in order to make a more reliable selection within a stipulated period of time. Prior to the evaluation process, a framework which consist of technical and non-technical aspect of MLAM was developed for assessing the quality of MLAM through an empirical study. The empirical study shows that application “Mathematics” was ranked first while application “Mathspapa” was ranked last with regards solely to this study. For this study, the researcher acted as the decision maker. Hence, the final ranking of the MLAM may be different in another study based on the decision maker’s preferences and adopted criteria.

6.2 Recommendations

For future related study, other MCDM methods such as AHP, ANP, FAHP, SMART, TOPSIS, ELECTRE and PROMETHEE can be adopted to advance the selection and evaluation process of MLAM. In addition, more MLAM’s can be sampled for the empirical study and there should be more professionals who should act as the decision makers in future works, also increasing the number of decision makers/experts could yield to a better result.
REFERENCES


Cumming, T. M. (2014). Does Mobile Technology Have a Place in Differentiated Instruction?. In *Mobile Pedagogy and Perspectives on Teaching and Learning, 2*(3), 132-149.


