NEAR EAST UNIVERSITY
GRADUATE SCHOOL OF SOCIAL SCIENCES
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MASTER'S PROGRAMME

MASTER'S THESIS

CONDITIONAL VOLATILITY OF TURKISH REAL ESTATE INVESTMENT TRUSTS

FIRASS ALI

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Thesis Defense

CONDITIONAL VOLATILITY OF TURKISH REAL ESTATE INVESTMENT TRUSTS

We certify the thesis is satisfactory for the award of degree of
Master of Banking and Finance

Prepared by
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Abstract:

In this paper, we estimate the conditional volatility in the excess returns of the real estate investment trust index (XGMYO) and Borsa Istanbul 100 index (XU100) in the Istanbul Stock exchange. We apply three models which are GARCH, EGARCH and GARCH-GJR to their daily excess return. A comparison was conducted to examine which of the following models is superior at forecasting future excess return in REITs. While GARCH model fails to account for coefficient restrictions, asymmetry and leverage effect, EGARCH and GARCH-GJR succeed to encompass those limitations. Our empirical outcomes find that EGARCH is the most efficient model to estimate the conditional beta in the Turkish REIT sector.

Key Words: GARCH, EGARCH, GJR GARCH, Conditional Beta, REIT

ÖZ:

Bu araştırma, koşullu oynaklığın İstanbul Menkul Kıymetler Borsası'ndaki Gayrimenkul Yatırımları Endeksi (XGMYO) ve Borsa İstanbul 100 Endeksi'ndeki (XU100) getirilerin olması gerekenin üzerinde olması durumundaki değerlerini hesaplamaktadır. Günlük elde edilen fazla getiriler için esas olarak üç model uygulanmıştır: GARCH, EGARCH VE GARCH-GJR. Belirtilen modellerin arasında hangisinin GYO endeksindeki gelecek fazlalık getirileri öngörme açısından daha üstün olduğunu bulmak için karşılaştırmalı araştırma yürütülmüştür. Bu doğrultuda, GARCH modelinin katsayı sınırlamalarını, asimetri ve kaldıraç gücünü hesaba katmadığını, EGARCH ve GARCH-GJR modellerinin ise belirtilen kısıtlamaları kapsadıkları saptanmıştır. Elde edilen ampirik sonuçlar, EGARCH modelinin Türkiye GYO sektöründeki en etkili model olduğunu göstermektedir.

Anahtar Kelimeler: GARCH, EGARCH, GJR GARCH, Beta şartlı, GYO

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Chapter 1: Introduction:

1.1. Introduction:

Beta stability has always been a shaded area of study. While in the capital asset pricing model (CAPM) beta is assumed to be constant over time, researchers found that beta experience a stochastic behavior due to micro and macroeconomic factors, where it moves randomly through time (Fabozzi and Francis, 1978). One of the first steps toward modeling the time varying behavior of beta was done by Engle(1982) when he introduced the autoregressive conditionally heteroskedastic model(ARCH) that allows the conditional variance to change through time as a function of past errors, yet leaving the conditional variance constant. This model makes the conditional variance prediction error at any time *t* a function of time where the variables are exogenous and lagged endogenous, and beta is a vector of unknown parameters. This model evolved to a more generalized form by Bollerslev (1986), to the GARCH model (Generalized autoregressive conditional heteroskedastic), that allows more lag structure and a longer memory of volatility. Yet GARCH model have three major drawbacks. First, a negative correlation between current returns and future returns volatility was found by Black (1976), indicating that volatility tend to increase when receiving bad news and yields lower return than expected, whereas volatility tend to decrease when receiving good news and yields less return than expected. Second, the model imposes parameter restrictions that can be violated by estimated coefficient. And finally the last drawback is the difficulty in interpretation of the persistence of the shocks to conditional variance (Nelson, 1991). Numerous models were evolved to account for those drawbacks, and two of them will be handled in this paper. The first model is the EGARCH (Exponential GARCH) developed by Nelson (1991), and the second model is the GARCH-GJR model developed by Glosten, Jagannathan and Runkle (1993). Both of the models successfully account for these drawbacks where they take into consideration the leverage effect, asymmetry and coefficient restrictions.

On the other hand, real estate investment trust is a recent trend invading the financial market. REITs were created to securitize the real estate in every developed/developing country by allowing REITs to invest and finance real estate projects, lands and buildings. They gained reputation among investors due to their high return, inflation hedging and tax shelter advantages. Frequent studies aimed to study the REITs behavior, and their relation to the overall stock market due to their high gain potential. Therefore we aim in this article to study the relationship between the REIT index return in turkey known as "XGMYO" and the overall index return of the market known as "XU100" of the Istanbul stock exchange by modeling the stochastic behavior of excess returns. In addition, Turkish REITs returns experienced high volatility throughout the years. Therefore when modeled correctly, investing in REITs becomes very profitable.

1.2. Aims of study:

In this paper we aim to model the conditional volatility of the real estate investment trust industry in Turkey. We apply three models that are proven to be efficient in most published articles. The three models are GARCH, Exponential GARCH and GJR-GARCH. We also use two different distributions for each model which are student t and generalized normal distribution. We aim to find the optimal model between these three that can efficiently describe and forecast the Turkish REITs industry.

1.3. Importance of this study:

Turkish REITs offers investors with high profitable opportunities as well as efficient hedging strategies. Due to their historical performance where if you bought all the REIT stocks in the index, or simply an exchange traded fund that imitate the index's performance in July 15, 2003 at 9,660.8 Turkish liras. You'd have a 340.65% capital gain on your investment where in May 18, 2017 it reached 42,570.65 Turkish Liras, alongside the return from dividends.

Therefore if we find a model that can efficiently forecast the REITs stock prices, it will help us create an optimal portfolio of long/short positions that can yield positive returns.

On the other hand, conditional volatility of REITs sector is a neglected area of study in the Turkish economy. Few articles exists that aim to model their performance therefore this paper is one of few other efforts to study the stochastic behavior of REITs.

In section 2 we discuss the literature behind the models and the methodology. In section 3 we discuss the Turkish REITs industry, its performance and legal framework. In section 4 we provide the data and their relative analysis. In section 5 we analyze our results. And finally in section 6 we conclude our findings.

Chapter 2: REITs in Turkey:

The real estate investment trust is a capital market instrument that represents real estate projects, which serve as a bridge between corporate capital financing and the real estate sector. REITs serve as a mean for financing residential and commercial projects, and an investment opportunity for investors in the capital market. They are regulated by the capital market board (CMB), yet Turkish ones have several advantages over other countries. First Turkish REITs are tax exempted, i.e. they don't pay corporation or income taxes. Investors are expected to pay taxes only on dividends. On the other hand, another advantage is that REITs doesn't have to pay dividends on a regular basis, rather they can reinvest their earnings in new or existing projects. And finally REITs managers are not restricted to specific types of product investments or a geographic location; rather they are restricted to not invest more than 49% of their asset in foreign real estate. Therefore Turkish REITs are an attractive investment for local and foreign investors, and when forecasted properly offers great return opportunity.

Turkish REIT index is found under the name of "BIST Gayrimenkul Yatırım Ortaklıkları" and the ticker "XGMYO". This index consists of 27 Turkish real estate investment trust companies. These companies vary in their market capitalization from 51.70 million Turkish liras for Marti GYO, to 11.40 billion Turkish liras for Emlak Konut GYO.

We conduct our study on Turkish REITs from July 15, 2003 until May 18, 2017 as shown in figure 1:

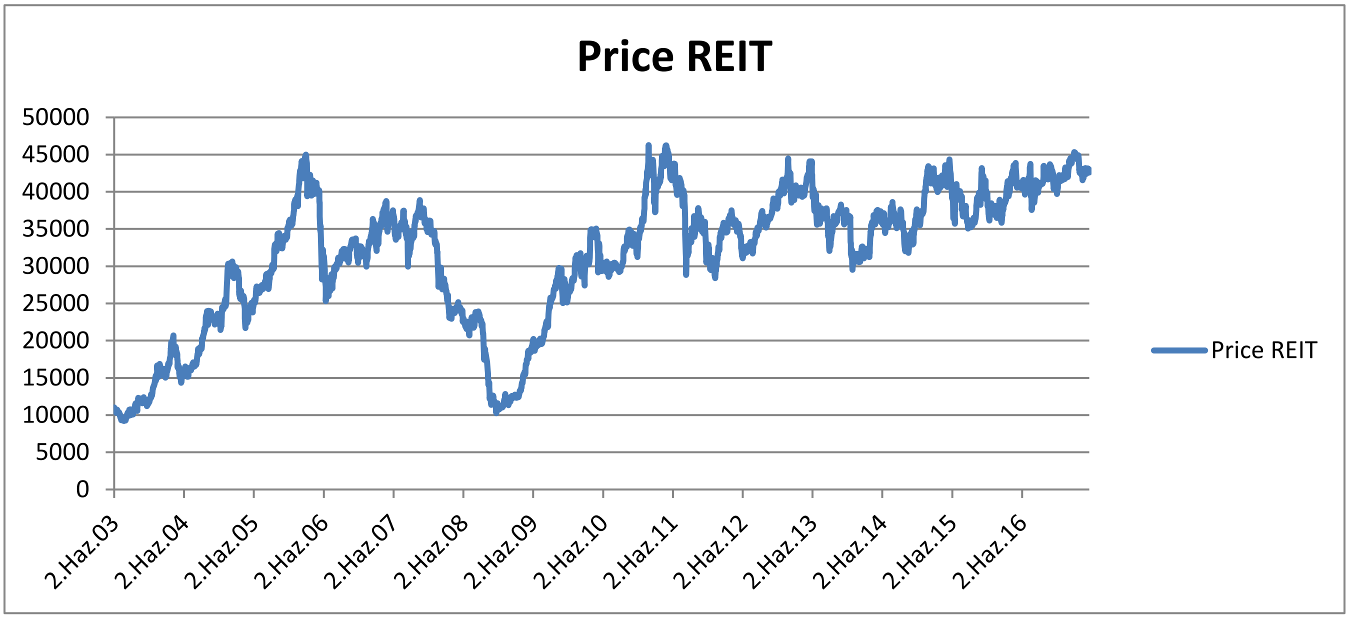

Figure 1

In order to assess the performance of REITs, we divide the time span over three major sections. The first section is from July 15, 2003 until December 31, 2008, where REITs prices increased drastically from 9,660.8 to 34,722.55 Turkish Liras scoring 259.41% increase in price. The second section starts from Jan2, 2008 until Nov 20, 2008. In this time period the REITs had its worst performance since its inception. Due to the global financial crisis of 2008 that was caused by the oversupply of subprime mortgage debt and the creation of collateralized debt obligations (CDOs) that supported toxic debt. The crisis of 2008 known as the worst financial crisis since the great depression of 1930 hit the Turkish market as well. Where XU100 hit the lowest in November 20, 2008 and reached 21,228.27 Turkish Liras from 54,708.42 Turkish Liras at the beginning of the year as shown in figure 2.

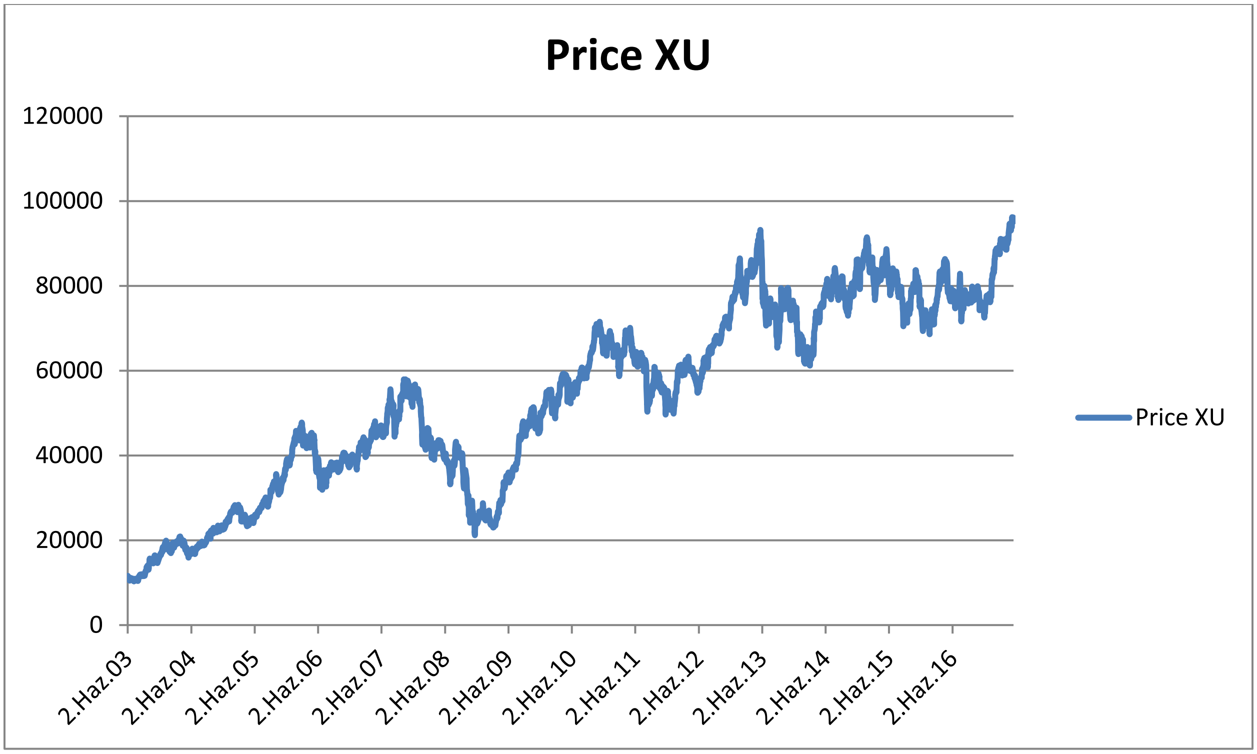

Figure 2

In the same day REITs reached 10,269.12 Turkish Liras scoring a loss of -70.14%. The correction began as of November 21, 2008 and prices started increasing at a slow pace till the end of 2008. Our third section starts from November 21, 2008 until May 18, 2017 where REITs recovered and scored new highs. Their performance recorded 314.55% since the crisis compared to a 348.21% increase in the price of XU100.

Chapter 3: Literature Revue:

3.1. Theoretical Part:

According to the capital asset pricing model developed by Sharpe (1964), every security bares systematic and unsystematic risk, while unsystematic risk can be diversified; systematic risk denoted beta cannot due to its correlation with other asset returns in the same market or portfolio. In CAPM, unconditional beta is assumed to be constant through time, i.e. all investors have the same expectations of the variance, mean and covariance of returns. It can be calculated using the following formula:

$$\beta = \frac{cov(R_M, R_i)}{Var(R_M)}$$

Where R_i denotes the return of the REIT sector that is the return of XGYMO, and R_M denotes the return of the stock market XU100. We use ordinary least square method to estimate beta (OLS) assuming that the error terms are identically and independently distributed (IDD).

Yet if the covariance between the market's return and that of the stock market is not constant, then our Beta itself isn't constant. We know from Fabozzi and Francis (1978) that the beta coefficient moves randomly through time. Beta depends on the successive price changes of an asset. In addition it depends on the effect of good news and bad news on the price of that same asset. On the other hand, if the volatility of an asset's price at time *t-1* affects its price at time *t*, then we need to account for the volatility effect on the price changes. Therefore it's a must to build a model that can estimate the conditional beta while taking into consideration the volatility effect of each price at time *t* with its preceding one.

Engle (1982) developed the first autoregressive conditional heteroskedasticity model (ARCH) that allows volatility to evolve over time by specifying the conditional variance as a function of past squared errors. The model aims to model the conditional volatility and is given by:

$$\varepsilon_t = \sigma_t z_t$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$$

Where ε_t denotes the error term, z_t is a random variable following IID with mean 0 and variance equal to 1. σ_t is the standard deviation, and $\omega > 0$, $\alpha > 0$ and $i > 0$.

In order to validate his model, Engle (1983) estimated the variance of inflation in the United Kingdom. He conducted his study on quarterly data that ranged over the course of 19 years from 1958 until 1977. He also used quarterly manual wage rates as his independent variable. His estimation found that the model is in good fit, and his estimation errors were less than 1%. His ARCH model allowed a conventional regression specification for the mean function, and a stochastically efficient change of variance.

He then conducted another study using his ARCH model on the inflation rate in the United States. His main finding was that the variance of inflation in the late forties and fifties were higher than the variance in the sixties that is in its turn higher than the variance in the seventies. He then tested the same model in an effort to estimate the same inflation in the United States a year later. He found that uncertainty of inflation tends to change over time (Engle, 1983).

ARCH successfully models the conditional beta and takes into account the ARCH effect on the variance and price. Yet it has several drawbacks that make the model weak and unsuitable for most variables. First the model assumes symmetry in shocks. This means that negative and positive shocks have the same effect on volatility, while in actual terms negative and positive shocks have different magnitude. The second weakness is that the model assumes that volatility continues for a short period. And finally the third weakness is that it's restrictive which creates a serious problem for high order ARCH models.

3.1.1. GARCH Model:

ARCH inspired many other researchers to create a model that follows the ARCH steps but solve for its drawbacks. One of the most pioneering and well known models is the GARCH model. This model developed by Bollerslev (1986) aims to model the successive price changes through a moving average of their past conditional variances, and their dependence on the past behavior of the squared residuals. The squared residuals indicate that if errors at time t-1 are large in absolute value, then they will probably be large at time t. This creates a clustering manner of volatility. It differentiates from the ARCH model by three main points. First GARCH allows more flexible lag structure by adding more lags to conditional variances. Second it provides a longer memory of returns whereas ARCH is categorized as a short memory model (Elyasiani, 1998). And third it permits a parsimonious description. This model introduced the GARCH effect, and it is caused by business cycle, margin requirements, information patterns, dividend yield, and money supply that cause volatility clustering (Bollerslev et al, 1992). The model is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \, \sigma_{t-j}^2$$

Where σ_t^2 denotes the conditional variance, ω is the intercept, α and β are the coefficients, ε_{t-i}^2 is the residual squared lagged, and σ_{t-j}^2 the GARCH variable lagged.

After explaining his GARCH model, Bollerslev gave an empirical example where he modeled inflation in the United States. He used quarterly inflation data from 1948 until 1983 and used the implicit price deflator for GNP as his independent variable (Bollerslev, 1986). He found that GARCH model not only provide a better fit than ARCH model, but also exhibits a more efficient lag structure.

This model received positive criticism and was widely adopted by most practitioners. The GARCH (1.1) didn't just sufficiently fit most economic time series data (Bollerslev, 1987); it was also the foundation of different GARCH models that evolved and has been used to model the conditional beta of different stock markets throughout the years. Two of the most common models that were created were the exponential general autoregressive conditional heteroskedastic model (EGARCH), and the general autoregressive conditional heteroskedastic with threshold model (GARCH GJR, or GARCH (p.q) with threshold).

3.1.2. EGARCH Model:

According to Nelson (1991), the GARCH model suffers from several limitations. Therefore Exponential GARCH model was developed to account for those limitations accordingly. The first constraint that GARCH model suffers from is the negative correlation observed by Black (1976) between the returns of a stock and the returns of volatility. This indicates that bad news result in a greater volatility and good news result in a lower volatility. Yet the GARCH model only takes into consideration the magnitude, and ignores the sign of returns. Therefore EGARCH was developed to include the oscillatory behavior ignored by GARCH. The second limitation is the non-negativity restriction imposed on the parameters α and β in the ARCH equation. When restricted to non-negativity, the σ_t^2 remains non negative with probability 1 at any time *t*. The third limitation also observed by Poterba and Summers (1986) is the issue of persistence of shocks to the conditional variance. Whether the shocks are transitory or persistent, definite or indefinite, what will their effect be on volatility?

Therefore the EGARCH model came to improve the ARCH model by first lagging z_t, second taking the Ln (z_t) for linearity, and third making g (z_t) a function of sign of z_t as well as magnitude. The EGARCH model variance equation is given as follows:

$$\text{Ln } (\sigma_t^2) = \omega + \sum_{i=1}^{q} \alpha_i \frac{|\varepsilon_{t-i}| + \delta_i \, \varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{p} \beta_j \, \sigma_{t-j}^2$$

Where ε_{t-1}> 0 when there's good news and ε_{t-i} = $(1 + \delta_i)$ $|\varepsilon_{t-i}|$. On the other hand when ε_{t-1}> 0 following bad news, then ε_{t-i} = $(1 - \delta_i)$ $|\varepsilon_{t-i}|$.

This model not only captures the size and sign effects, but also the leverage effect. Where leverage effect is the negative correlation between volatility returns and stock returns. This is due to a higher Debt/Equity ratio in the CAPM model, where the value of equity decreases to account for a higher risk as a result to an increase in volatility.

In order to test his model, Engle (1991) estimated the conditional variance of the excess returns for the value-weighted market index from the Center for Research in Security Prices tapes. He used daily data ranging from July 1962 until December 1987. He finds four important results.

First it exist a negative correlation between conditional variance and the estimated risk premium. Second, there's a high significance in the asymmetry between changes in volatility and returns. Third, shocks are persistent. Fourth, the distribution of shock returns exhibit fat and thick tails. Fifth, trading days contribute more to volatility than non-trading days (Nelson, 1991).

3.1.3 GARCH-GJR:

This model was developed by Glosten, Runkle and Jagannathan(1993) to account for the drawbacks of the GARCH-M model. They found that the negative and positive shocks have different impacts on the conditional variance. Therefore to account for those asymmetries, described as a seasonal variation, they added a dummy variable S_{t-1} to the original model that takes a value of 0 when innovations ε_{t-1} are positive, and a value of 1 when ε_{t-1} are negative. Therefore when the coefficient of S_{t-1} is negative and significant, then the positive shocks have smaller effect that the negative ones. In addition to seasonal pattern, this model also considered the leverage effect when α is the impulse of positive shocks, and $(\alpha + \delta)$ is the impulse of negative shocks. The GJR-Model is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \delta S_{t-i}^{-} \varepsilon_{t-i}^2 \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

In this model, $\hat{\alpha} + \hat{\delta}$ shows the asymmetry in the impact of good news, whereas $\hat{\alpha}$ shows the asymmetry in the impact of a bad news on our conditional volatility.

Therefore in order to test their model, they conducted a study on the relation between monthly risk return on the Center for Research in Security Prices value-weighted index of New York stock exchange equities and the risk free rate of the Treasury bills form Ibbotson & Associates. They came to conclude five findings. First there's a negative statistical significant relation between conditional variance and conditional mean. Second, risk free rate contains information about future volatility. Third seasonal volatility is statistically significant during the moths of January and October. Fourth, the excess return's conditional volatility isn't exceedingly persistent. And finally positive residuals cause a decrease in variance, while negative residuals causes an increase in variance.

3.2.1. Empirical Part:

These three models are widely used nowadays and are proven to efficiently estimate the conditional variance, studying the relationship between two variables and forecasting the conditional volatility. Several studies used at least one of these models such as Hansen (2005) who conducted a study comparing 330 ARCH-type models using two sets of data. The first data consists of dollar spot exchange rate, and the second data consists of IBM stock returns. He found that GARCH isn't outperformed by more sophisticated models, yet it fails to account for leverage effect for IBM return data.

Lee, Chen and Rui(2017) on the other hand conducted a study on the daily return of the Chinese stock market using GARCH and EGARCH model; they found strong evidence of time varying volatility and a long memory of returns yet they didn’t find any relationship between expected risk and expected return.

Brooks, Faff and McKenzie (1998) used a multivariate GARH model to estimate the conditional volatility for 24 industry groups in the Australian stock exchange using monthly data. They compared these results with two other models, the Kalman Filter approach and the Schwert and Seguin approach. They found that both GARCH and Kalman Filter were both efficient in improving out-of-sample and in-sample forecasts for the robustness test.

Gokbulut and Pekkaya (2014) estimated the volatility in the Turkish Stock market using GARCH models family. They used daily index data, as well as interest rate, and foreign exchange market data from 2002 until 2014. They found that CGARCH and TGARCH have superiority at forecasting the volatility in the Turkish stock market index due to their outperformance in the robustness test.

Franses and Van Dijk (1996) estimated the volatility of several European stock market indices. They used GARCH, GJR-GARCH and non-linear Quadratic GARCH on weekly return. They found that QGARCH is the best model at forecasting while GJR-GARCH isn’t recommended for forecasting.

Contrary to Franses and Van Dijk, Brailford and Faff (1996) conducted a study to compare the forecasting capabilities of different forecasting models on the Australian stock market. They used the Statex-Actuaries Accumulation Index as their dependent variable and the data ranged from 1974 until 1993. Their forecasting models were GARCH, GJR-GARCH, historical mean, exponential smoothing, simple regression, moving average, random walk and exponentially weighted average models. Their results were that the ARCH class models and the simple regression have the highest accuracy in forecasting volatility. Their decision was based on four criteria that are the mean absolute error, root mean squared errors, mean error and mean absolute percentage error. Moreover out of all the ARCH models, GJR-GARCH was the best at forecasting the Australian stock market returns.

Dutta (2014) estimated the conditional voliatlity in the U.S. and Japan daily exchange rate from 2000 until 2012. He used three GARCH family models that are GARCH (1.1), EGARCH and GJR-GARCH following a GED distribution. He found that positive shocks to the exchange rate are more redundant than negative ones and that there exists size effect of news due to asymmetries in volatility.

In this article we handle real estate investment trusts; therefore looking at similar studies we find several that aim to model their behavior. Peterson and Hsieh (1997) tried studying the relation between EREITs and the stock market. They conducted Fama and French's (1993) five factor model on EREIT returns and found that risk premium on REITs are similar to that of a market portfolio of stocks. And that the risk premium of mortgage REITs is significantly related to two bond market factors and three stock market factors in returns. Chan, Hendershott and Sanders (1990) also used a multi factor capital asset pricing model. They found that EREIT are less sensitive to the factors specified in the model than stock returns. But they do have significance in explaining EREIT return. The five macroeconomic factors were expected and unexpected inflation, industrial pollution, and risk and term structure of interest rate as specified by Chenn, Roll and Ross (1986).

Devaney (2001) on the other hand used a 4 factor arbitrage pricing theory model to invest the relation of EREITs with interest rates. He implemented a GARCH-M model in the mean to test for changes in risk premium through time. He found that interest rates and their relative conditional variance has an inverse relation with EREITs, and that mortgage EREITs are more related to interest rates than equity ones.

Stevenson (2002) on the other hand used the univariate models GARCH and EGARCH to analyze the volatility of the U.S. REIT sector to equity and fixed income sectors. He found a relation between Equity REITs and small cap stocks, and a relationship between equity REITs and other REIT sectors.

Yuan, Sun and Zhang conducted a study using four GARCH models on the daily price of REITs in the Unites States. They use GARCH, EGARCH, GARCH-GRJ and APARCH and compare between them using value at risk estimations. They find that GARCH-GJR is the best model at estimating REITS volatility in the U.S.

Moreover, Winniford (2003) conducted a study on the seasonal volatility of the EREIT sector using GARCH and P-GARCH model. He used the Wilshire REIT index and the National Association of Real Estate Investment Trusts EREIT index data. His study covered the period from February 1972 until December 2002. He found that EREITs are more seasonally volatile than the stock market and highly sensitive to news. Plus he found that the months of April, June, September, October and December exhibit the highest seasonal volatility patterns in the overall return.

Loo (2016) conducted a study on the Asian REIT market. He studied their volatility behavior using ARCH family models. His results suggested that EGARCH model was the best one from the ARCH family at forecasting volatility in Asian REIT market.

In addition, Cotter and Stevenson (2006) examined the REITs volatility using the VAR-GARCH model between REITs and US equity sector. He found a weak relation between the equity sector and REITS by using monthly returns. Rather he suggests that daily returns are more efficient than monthly ones. These studies gave us a reason to further investigate the GARCH models family and their application on the REITs sector.

On the other hand few studies aimed to model the volatility in the Turkish REIT industry. Aksoy and Ulusoy used a GARCH (1.1) and EGARCH to study the Turkish REITs where they search for daily, weekly and monthly variations in index returns. They found that calendar anomalies exist in the REITs index and BIST index on weekly and monthly variations.

3.2.2. Articles Summary:

The articles used are summarized in the following table:

Author	Method	Variables	Results
Hansen(2005)	330 ARCH	• Dollar spot exchange rate • IBM stock returns	No sophisticated model outperform GARCH
Lee, Chen and Rui	• GARCH • EGARCH	Chinese stock market	• Long memory in volatility • No relation between expected return and expected risk
Brooks, Faff and McKenzie (1998)	GARCH	24 Industry groups in Australian Stock Exchange	Passes the robustness test for in-sample and out-of-sample forecasting
Peterson and Hsieh(1997)	Fama Five Factor model	EREITs	Risk premium on REITs are similar to risk premium of a market portfolio of stocks
Sanders(1990)	Multi factor capital price model	EREITs	Significance in explaining EREIT return by the five macroeconomic factors picked
Devaney(2001)	4 factor APT model	ERITs with interest rates	Interest rate and EREITs have inverse relation in the conditional variance

Stevenson(2002)	GARCH EGARCH	U.S. REIT	• There exist a relation between equity REITs and small cap stocks. And one between EREITs and REIT stocks.
Winniford(2003)	GARCH and P-GARCH	EREITs	There exist a seasonal volatility in the EREITs return
Cotter and Stevenson(2006)	VAR-GARCH	REIT U.S.	Weak relation between equity sector and REITs on the monthly return basis.
Aksoy and Gulsoy(2015)	GARCH, EGARCH	Turkish REITs	Existence of calendar anomalies in the Turkish REITs.
Yuan,Sun and Zhang	GARCH,EGARC, GARCH-GRJ and APARCH	U.S.REITs	GARCH-GRJ is the best model at VAR estimation
Gokbulut and Pekkaya(2014)	GARCH family	Turkish stock market	CGARCH and TGARCH have the best forecasting ability
Franses and Van Dijk(1996)	GARCH,GJR-GARCH,QGARCH	European stock market indices	QGARCH is the best model at forecasting
Brailford and Faff (1996)	GARCH, GJR-GARCH	Statex-Actuaries Accumulation Index	GJR-GARCH is the best at forecasting
Dutta (2014)	GARCH,EGARCH,GJR-GARCH	US-Japan daily exchange rate	• Positive shocks are more redundant than negative ones. • Asymmetry exists in the exchange rate's volatility.
Loo(2016)	GARCH family	Asian reit market	EGARCH is the best at forecasting

Table 1

Chapter 4: Methodology:

In this paper we're studying the excess return of the real estate investment trust index as our dependent variable, while taking the Borsa Istanbul index as our independent variable. We use the daily closing price of Turkish REITs index "XGMYO" and of the Borsa Istanbul index "XU100". The return is calculated as the logarithm of the percentage change in daily closing price as follows:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \times 100$$

The excess return is calculated using the same method followed by Aksoy and Ulusoy(2015) in their EGARCH application on Turkish REITs. Where excess return is calculated using mean adjusted return approach:

$$AR_t = R_t - \bar{R}$$

Where AR_t is the abnormal return at time *t,* R_t is the daily return for REITs, and $\bar{R}$ is the daily average return of REITs between *t* = -30 (Jun 3,2003) until *t* = -11 (Jun 30,2003) , and Jun15,2003 is our event date at *t* = 0. The statistical significance of our abnormal returns is calculated through the standardized abnormal return explained by Brown and Warner (1985) where:

$$SAR_t = \frac{AR_t}{SD(AR)_t}$$

And

$$SD(AR)_t = \sqrt{\frac{1}{T_0}\sum_{t=1}^{T_0} AR_t^2}$$

The Abnormal returns for XU100 is calculated the same way as that of REITs.

On the other hand the three models used are the following:

- GARCH (1.1):

$$\sigma_t^2 = \omega + \alpha\, \varepsilon_{t-1}^2 + \beta\, \sigma_{t-1}^2$$

- EGARCH(1.1):

$$\text{Ln}\ (\sigma_t^2) == \omega + \sum_{i=1}^{q} \alpha_i\ \frac{|\varepsilon_{t-i}| + \delta_i\, \varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{p} \beta_j\, \sigma_{t-j}^2$$

- GARCH-GJR (with threshold):

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i\, \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \delta S_{t-i}^{-} \varepsilon_{t-i}^2 \sum_{j=1}^{p} \beta_j\, \sigma_{t-j}^2$$

We run these three models using two different distributions, student t and generalized normal distribution. The logic behind using these two distributions is discussed later in this paper. After estimating these six models we test each mode for serial correlation using the Correlogram of standardized square residuals, normality of the distribution using the Jarque-Bera test and for ARCH effect using the ARCH LM Test. If the model successfully passes these three tests then the model is eligible for application

We then compare their values of AIC (Akaike info criteria) and SIC (Schwartz info criteria) the LogL (Log Likelihood). The lowest the values for AIC and SIC, the better the model. While the highest the value for LogL the better. And finally we forecast each model independently by first dividing our sample on two years interval. Therefore we forecast seven samples of two years period for each model and for each distribution.

We finally compare the root mean squared error (RMSE) and mean absolute error (MAE) of each sample forecasted first by the rest of the years. Then we compare the values between the different distributions to decide which distribution is the better fitting our data. And finally we compare between the different models to pick the best one with lowest errors at forecasting the conditional volatility of REITs.

Chapter 5: Analysis:

5.1.1 Summary statistics:

Analyzing our REITs daily data we first plot the residuals graph. We notice the high fluctuation in the residuals through time. Where a high volatility is followed by a high one and a low volatility is followed by a low one. This indicates that we can apply a GARCH model to this data.

We then plot the summary statistics table for our 3464 daily closing price of REITs. We notice that the value of our mean and standard deviation are positive, indicating that positive returns are more dominant than the negative ones in the REITs sector. In addition the value of skewness (-0.479594) is far from our standard deviation indicating that our data is negatively skewed. And our kurtosis is 6.317923 indicating that our data is also leptokurtic as shown in the following table.

	Observatio ns	Mean	Median	Std. Dev.	Skewness	Kurtosis
Daily AR REIT	3464	0.44724	0.49226	1.77826	-0.4796	6.31792

Table 2

On the other hand, XU100 experience similar attributes. First the mean and standard deviation are positive. Second it’s negatively skewed with a value of -0.1633. Third is leptokurtic with a kurtosis of 6.72361 as shown in the following table:

	Observatio ns	Mean	Median	Std. Dev.	Skewness	Kurtosis
Daily AR XU100	3464	0.37261	0.41308	1.72653	-0.1633	6.72361

Table 3

We then run our estimation with AR REIT as a dependent variable and AR UX100 as an independent one through ordinary least square method (OLS) as shown in table 2. We get:

$$AR\_REIT_t = 0.150958 + 0.795166 \ * AR\_XU_t + \varepsilon_t$$

We find that our R-squared is equal 59.60% which means that 59.60% of our dependent variable is explained by our independent one.

5.1.2. Test for normality:

Then we test for normlity using the Jarque-Bera test. We find that our Jarque-Bera value of 1721.703 is at P-value of (0.00) for the daily returns in table 4. Indicating that we should reject our null hypothesis of normal distrubtion. We also found our Jarque-Bera value for our residuals in our estimated model which is 2016.621 at (0.00) P-value. Which also indicates that our squared returns aren't normally distrubted.

	Observations	Jarque-Bera	P-Value
Daily AR REIT	3464	1721.703	0.000
Daily AR XU100	3464	2016.621	0.000
Residuals	3464	2016.621	0.000

Table 4

5.1.3. Test for stationary:

We test the stationary of our data at level using the Augment Dickey-Fuller test. We find that our t-statistic for REITs return is (-55.66963) and it's significant at 1, 5 and 10% where t-critical is (-3.432051), (-2.86211) and (-2.567153). Therefore we accept the null hypothesis that our data is and has no unit root.

In addition we find our data for the return of XU100 is also stationary where our t-statistic are (-57.33808) and it's significant at 1,5 and 10% where our t-critical are (-3.432051), (-2.86211) and (-2.567153).

		T-Statistic	Prob.*
AR REIT	ADF test statistic	-55.66963	0.0001
AR XU100	ADF test statistic	-57.33808	0.0001
	Test critical values:		
	1% level	-3.432051	
	5% level	-2.862177	
	10% level	-2.567153	

Table 5

5.1.4. Test for autocorrelation and serial correlation:

To check for autocorrelation we use the Ljung-Box Q test on the squared residuals. We reject the null hypothesis of autocorrelation since our P-values are (0.00) and significant at all lags as summarized in the following table:

	Autocorrelation	Prob
Q(2)	0.104	0.000
Q(10)	0.046	0.000
Q(20)	0.036	0.000
Q(30)	0.026	0.000

Table 6

We then use the Breusch-Godfrey LM test to c heck for serial correlation. We also reject the null hypothesis of existence of serial correlation since our Prob. Chi-Square for 2 lags is 0.1547 which is statistically significant in table 7.

F-statistic	Obs*R-squared	Prob.Chi Square(2)
1.86746	3.735205	0.1545

Table 7

5.1.5. ARCH LM Test:

We use the ARCH LM test to check for our Arch effect in our model. We find heteroskedasticity in our model since P-value is (0.00) and we reject our null hypothesis of homoscedasticity Therefore our model suffers from ARCH effects and can be used to estimate GARCH models.

F-statistic	Obs*R-squared	Prob.Chi Square(2)
130.0105	125.376	0.000

Table 8

5.1.6. Distribution Hypothesis:

According to our previous tests we found that our model experiences a non-normal distribution. Our return has heavy fat tails and a leptokurtic distribution. Therefore in our study we run the different GARCH models through a generalized normal distribution (GED) and a student t distribution. We later compare between them

5.2. Models Estimation:

5.2.1. GARCH (1.1) Estimation:

We run the GARCH model and report our findings from Eviews in the following table 9:[1]

GARCH(1.1)	Student t	GED
$\hat{\alpha}$	0.138752	0.132239
$\hat{\beta}$	0.798698	0.792393
$\hat{\omega}$	0.092740	0.099364
Log Likelihood	-5010.711	-5024.850
Akaike info criteria	2.896484	2.904648
Schwartz info criteria	2.907137	2.904648
Jarque-Bera	1317.390 (0.000000)	1277.437 (0.000000)

The first thing to pinpoint in this table is the sum of $\hat{\alpha}$ and $\hat{\beta}$, where if $\hat{\alpha} + \hat{\beta} < 1$, it means that our results are stationary, while a value larger than 1 indicates that there's a unit root. In both student t and GED distribution our $\hat{\alpha} + \hat{\beta}$ is less than 1 (0.93745 and 0.924632 respectively), therefore our model is stationary and it does experience volatility shocks. On the other hand all our coefficients $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\omega}$ are significant at all levels 1, 5 and 10%. We then test both models for serial correlation using the correlogram of standardized squared residuals, we find that Q (30) test rejects the null hypothesis of serial correlation since P values are more than 5% at all lags. We then run the Jarque-bera test where we reject the normality of distribution since our data is negatively skewed, leptokurtic and our Jarque-Bera p-value is 0.00. And finally we run the ARCH LM test; we find absence of ARCH effects in both where prob chi square is 0.2508

[1] Tests results are in tables 1C-2C-3C-4C-5C-6C

for student t, and 0.2055 for GED. These tests indicate that GARCH model successfully solves for ARCH effect and effectively model the volatility.

In addition it's important to compare the three main criteria AIC (Akaike info criteria) and SIC (Schwartz info criteria) that are all smaller for student t than GED and the LogL (Log Likelihood) is larger in student t than GED. This indicates a better model following the student t in GARCH (1.1).

5.2.2. EGARCH (1.1) Estimation:

We run the EGARCH (1.1) model and report our findings from Eviews in in the following table 10:[2]

EGARCH(1.1)	Student t	GED
$\hat{\alpha}$	0.265186	0.252702
$\hat{\beta}$	0.931929	0.927172
$\hat{\omega}$	-0.184449	-0.127929
$\hat{\delta}$	0.000411	-0.006530
Log Likelihood	-5009.290	-5024.083
Akaike info criteria	2.896242	2.904782
Schwartz info criteria	2.908670	2.917210
Jarque-Bera	1349.436 (0.000000)	1286.455 (0.000000)

We first notice the asymmetry in our model where good news affects conditional volatility by $1 + \hat{\delta} = 1.000411$ and bad news affect it by $\left|-1 + \hat{\delta}\right| = 0.999589$ for t distribution. Whereas good news affects our conditional volatility by $1 + \hat{\delta} = 0.99347$ and bad news affect it by $\left|-1 + \hat{\delta}\right| = 1.00653$ for GED distribution.

[2] Tests results are in tables 7C-8C-9C-10C-11C-12C

We deduct that for the t student distribution the impact of good news is larger in magnitude than the impact of bad news. However in the GED distribution, bad news has a larger impact than that of good news.

We then notice that our coefficients are all significant at 1, 5 and 10% in both distributions except $\widehat{\delta}$. We then test for serial correlation where we find presence of serial correlation at first lag only. In addition we find presence of ARCH effect using ARCH LM test in both distributions with a prob chi squared of 0.0457 for t, and 0.0267 for GED Both of our estimations reject the normality of distributions using Jarque-Bera test.

We then find similar results to GARCH (1.1), where AIK and SIC and Logl are also in favor of student t, since they give us better values than GED.

5.2.3. GARCH-GJR Estimation:

We run the GARCH-GJR model and report our findings from Eviews in the following table 11:[3]

GARCH-GJR(1.1)	Student t	GED
$\widehat{\alpha}$	0.138279	0.127590
$\hat{\beta}$	0.798586	0.790595
$\widehat{\omega}$	0.092782	0.100273
$\widehat{\delta}$	0.001168	0.012274
Log Likelihood	-5010.710	-5024.722
Akaike info criteria	2.897061	2.905152
Schwartz info criteria	2.909489	2.917580
Jarque-Bera	1316.287 (0.000000)	1265.501 (0.000000)

[3] Tests results are in tables 13C-14C-15C-16C-17C-18C

We first calculate the asymmetry in our model. Where the impact of good news on conditional volatility is found by $\hat{\alpha} + \hat{\delta}$ = 0.936865 and bad news impact by $\hat{\alpha}$ = 0.138279 for student t. whereas the impact of good news is $\hat{\alpha} + \hat{\hat{\delta}}$ = 0.918185 and bad news impact is $\hat{\alpha}$ = 0.127590 for GED. This indicates that good news in GARCH-GJR affect volatility more than bad news.

On the other hand, running the serial correlation test we find no serial correlation in our Q (30) test. In addition to the absence of ARCH effect with a prob chi squared of 0.2520 and 0.2173 respectively. And finally the results of Jarque-Bera test reject the normality in both estimations.

Moreover it's important to notice that our results for GARCH-GJR follows EGARCH and GARCH when comparing the models using AIC, SIC and Logl where student t yields better values than GED.

5.3. Model Comparison:

Comparing so far between the models and their distributions we find the following table 12:

	GARCH		EGARCH		GARCH-GJR	
	Student t	GED	Student t	GED	Student t	GED
AIC	2.89648	2.90465	2.89624	2.904782	2.897061	2.905152
SIC	2.90714	2.9153	2.90867	2.91721	2.909489	2.91758
LogL	-5010.7	-5024.9	-5009.3	-5024.083	-5010.71	-5024.722

Looking at our results so far we find that first the Turkish REITs conditional volatility is more efficient when using student t distribution. Since first AIC and SIC are lower for student t in GARCH, EGARCH and GARCH-GJR than GED. AIC and SIC are the negative log likelihood penalized for a number of parameters. It's a measure of a model's fitness where the lower the value the better the model. In addition student t also gives us the higher values for LogL where the higher the values the better fit the model is.

On the other hand, we find very close competition in GARCH models between GARCH-GJR, EGARCH and GARCH in the t student distribution therefore in order to pick the best model, we forecast each model over two years span. The reason why we picked two years is to avoid any overlapping problem and any sample effect. Due to the bulkiness of the forecasting data results, we only mention 2015-2017 time-lapse. We then compare our root mean squared error and mean absolute error values for our models. We summarize our findings in table 13:

	Student t	GED	Student t	GED	Student t	GED
RMSE	0.8333	0.83416	0.832454	0.833027	0.833308	0.834258
MAE	0.587297	0.58785	0.586907	0.587414	0.587304	0.587947

Comparing the root mean squared error (RMSE) and mean absolute error (MAE) we first find the same result using SIC, AIC and LogL. Which is that student t provides better value for RMSE and MAE. Therefore we can come to a conclusion that Turkish REITs market experiences a student t distribution. Second we find that EGARCH following the t student distribution have the lowest values of 0.832454(RMSE) and 0.586907 (MAE). We compare the forecasted values over the scale of two years to the actual ones; we find that EGARCH following the student t yields the closest values to actual.

The results we found agrees with Aksoy's and Ulusoy's(2015) findings that EGARCH is the best model at forecasting conditional volatility in the Turkish real estate investment trust stock market. Even though the model suffers from serial correlation and ARCH effect, its forecasting ability of our variable surpasses both GARCH and GARCH-GJR that don't suffer from any serial correlation or ARCH effect.

Chapter 6: Conclusion:

In this paper we estimate the conditional volatility of Turkish REITs return and we study its relationship with the overall market index. We therefore used three GARCH models that empirically are the best at estimating volatility which are GARCH, EGARCH and GARCH-GJR. We compare between these models over three steps. The first is through choosing which distribution better fits the Turkish REITs industry. We found that the student t gives us a higher description of the distribution of fat tails and skewed leptokurtic data. The second step was comparing the three models using the Akaike info criteria, Schwartz info criteria and Log Likelihood criteria. Yet we find that their values are very close and indecisive. The third step was through estimating and forecasting each model. We find that EGARCH models hold the lowest value of root mean squared errors and mean absolute error values. Therefore it was our best model at estimating the conditional variance.

Yet the GARCH model had few drawbacks that are important to pinpoint. First the EGARCH model was suffering from serial correlation at the first few lags. Second the model failed at the ARCH LM test where we find presence of ARCH effect. GARCH and GARCH-GRJ on the other hand doesn't suffer from these drawbacks but still their forecasting ability is weaker than EGARCH.

In addition our results agrees to Aksoy's and Ulusoy's(2015) study on the Turkish real estate investment trust, where they found that EGARCH was efficient at modeling the conditional volatility of Turkish REITs and accounting for the calendar anomalies in weekly and daily data.

Potential future studies regarding Turkish REITs would be through using the rest of the GARCH family models in the Turkish market. Or through using the Kalman-Filter approach and Schwert-Seguin approach to forecast the conditional volatility. A comparison between these two approaches and the GARCH family is a great starting point since several studies favorite the Kalman-Filter approach over the GARCH family derivations.

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Appendix A:

Table 1A: Summary Statistics:

	Observations	Mean	Median	Std. Dev.	Skewness	Kurtosis	Jarque-Bera
Daily AR REIT	3464	0.447241	0.492262	1.778255	-0.479594	6.317923	1721.703 (0.000000)
Daily AR XU100	3464	0.372605	0.413082	1.726530	-0.163311	6.723612	2016.621 (0.000000)

Table 2A: OLS estimation:

Dependent Variable: AR_REITS
Method: Least Squares
Date: 05/29/17 Time: 10:53
Sample: 7/15/2003 5/18/2017
Included observations: 3464

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.150958	0.019648	7.683027	0.0000
AR_XU	0.795166	0.011126	71.47146	0.0000

R-squared	0.596041	Mean dependent var	0.447241
Adjusted R-squared	0.595924	S.D. dependent var	1.778255
S.E. of regression	1.130383	Akaike info criterion	3.083567
Sum squared resid	4423.622	Schwarz criterion	3.087118
Log likelihood	-5338.737	Hannan-Quinn criter.	3.084835
F-statistic	5108.170	Durbin-Watson stat	1.939411
Prob(F-statistic)	0.000000		

Table 3A: GARCH (1.1): Student t:

Dependent Variable: AR_REITS
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)
Date: 05/29/17 Time: 11:10
Sample: 7/15/2003 5/18/2017
Included observations: 3464
Convergence achieved after 34 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.148714	0.015774	9.427603	0.0000
AR_XU	0.760742	0.008919	85.29621	0.0000
		Variance Equation		
C	0.092740	0.018525	5.006165	0.0000
RESID(-1)^2	0.138752	0.019277	7.197802	0.0000
GARCH(-1)	0.798698	0.025621	31.17356	0.0000
T-DIST. DOF	5.007633	0.449249	11.14666	0.0000

R-squared	0.594852	Mean dependent var	0.447241
Adjusted R-squared	0.594735	S.D. dependent var	1.778255
S.E. of regression	1.132045	Akaike info criterion	2.896484
Sum squared resid	4436.641	Schwarz criterion	2.907137
Log likelihood	-5010.711	Hannan-Quinn criter.	2.900288
Durbin-Watson stat	1.933169		

Table 4A: GARCH (1.1): GED:

Dependent Variable: AR_REITS
Method: ML ARCH - Generalized error distribution (GED) (BFGS / Marquardt steps)
Date: 05/29/17 Time: 11:16
Sample: 7/15/2003 5/18/2017
Included observations: 3464
Convergence achieved after 32 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.145083	0.015190	9.551045	0.0000
AR_XU	0.758303	0.008733	86.82918	0.0000
		Variance Equation		
C	0.099364	0.020041	4.958055	0.0000
RESID(-1)^2	0.132239	0.018558	7.125641	0.0000
GARCH(-1)	0.792393	0.028313	27.98648	0.0000
GED PARAMETER	1.254271	0.035132	35.70189	0.0000

R-squared	0.594638	Mean dependent var	0.447241
Adjusted R-squared	0.594521	S.D. dependent var	1.778255
S.E. of regression	1.132343	Akaike info criterion	2.904648
Sum squared resid	4438.982	Schwarz criterion	2.915300
Log likelihood	-5024.850	Hannan-Quinn criter.	2.908452
Durbin-Watson stat	1.932518		

Table 5A: EGARCH (1.1): Student t:

Dependent Variable: AR_REITS
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)
Date: 05/29/17 Time: 11:36
Sample: 7/15/2003 5/18/2017
Included observations: 3464
Convergence achieved after 39 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.148092	0.015580	9.505435	0.0000
AR_XU	0.763264	0.008941	85.36409	0.0000
Variance Equation				
C(3)	-0.184449	0.019304	-9.555212	0.0000
C(4)	0.265186	0.028298	9.371208	0.0000
C(5)	0.000411	0.015883	0.025905	0.9793
C(6)	0.931929	0.013558	68.73680	0.0000
T-DIST. DOF	5.011457	0.448880	11.16437	0.0000

R-squared	0.595012	Mean dependent var	0.447241
Adjusted R-squared	0.594895	S.D. dependent var	1.778255
S.E. of regression	1.131820	Akaike info criterion	2.896242
Sum squared resid	4434.882	Schwarz criterion	2.908670
Log likelihood	-5009.290	Hannan-Quinn criter.	2.900679
Durbin-Watson stat	1.933612		

Table 6A: EGARCH (1.1): GED:

Dependent Variable: AR_REITS
Method: ML ARCH - Generalized error distribution (GED) (BFGS / Marquardt steps)
Date: 05/29/17 Time: 11:38
Sample: 7/15/2003 5/18/2017
Included observations: 3464
Convergence achieved after 46 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.142397	0.015095	9.433477	0.0000
AR_XU	0.761681	0.008795	86.60809	0.0000
Variance Equation				
C(3)	-0.178980	0.019260	-9.292752	0.0000
C(4)	0.252702	0.027549	9.172668	0.0000
C(5)	-0.006530	0.015207	-0.429423	0.6676
C(6)	0.927172	0.014631	63.37231	0.0000
GED PARAMETER	1.256364	0.035264	35.62737	0.0000

R-squared	0.594844	Mean dependent var	0.447241
Adjusted R-squared	0.594727	S.D. dependent var	1.778255
S.E. of regression	1.132056	Akaike info criterion	2.904782
Sum squared resid	4436.729	Schwarz criterion	2.917210
Log likelihood	-5024.083	Hannan-Quinn criter.	2.909220
Durbin-Watson stat	1.933003		

Table 7A: GARCH-GJR (1.1): Student t:

Dependent Variable: AR_REITS
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)
Date: 05/29/17 Time: 12:20
Sample: 7/15/2003 5/18/2017
Included observations: 3464
Convergence achieved after 43 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0) + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.148607	0.015877	9.360082	0.0000
AR_XU	0.760722	0.008920	85.27953	0.0000
Variance Equation				
C	0.092782	0.018544	5.003339	0.0000
RESID(-1)^2	0.138279	0.023251	5.947235	0.0000
RESID(-1)^2*(RESID(-1)<0)	0.001168	0.026077	0.044799	0.9643
GARCH(-1)	0.798586	0.025639	31.14781	0.0000
T-DIST. DOF	5.008364	0.450448	11.11864	0.0000

R-squared	0.594849	Mean dependent var	0.447241
Adjusted R-squared	0.594732	S.D. dependent var	1.778255
S.E. of regression	1.132048	Akaike info criterion	2.897061
Sum squared resid	4436.668	Schwarz criterion	2.909489
Log likelihood	-5010.710	Hannan-Quinn criter.	2.901499
Durbin-Watson stat	1.933160		

Table 8A: GARCH-GJR (1.1): GED:

Dependent Variable: AR_REITS
Method: ML ARCH - Generalized error distribution (GED) (BFGS / Marquardt steps)
Date: 05/29/17 Time: 12:21
Sample: 7/15/2003 5/18/2017
Included observations: 3464
Convergence achieved after 36 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0) + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.144004	0.015309	9.406624	0.0000
AR_XU	0.758045	0.008742	86.71201	0.0000
Variance Equation				
C	0.100273	0.020219	4.959230	0.0000
RESID(-1)^2	0.127590	0.021667	5.888595	0.0000
RESID(-1)^2*(RESID(-1)<0)	0.012274	0.025163	0.487765	0.6257
GARCH(-1)	0.790595	0.028595	27.64781	0.0000
GED PARAMETER	1.254832	0.035290	35.55794	0.0000

R-squared	0.594605	Mean dependent var	0.447241
Adjusted R-squared	0.594488	S.D. dependent var	1.778255
S.E. of regression	1.132390	Akaike info criterion	2.905152
Sum squared resid	4439.344	Schwarz criterion	2.917580
Log likelihood	-5024.722	Hannan-Quinn criter.	2.909589
Durbin-Watson stat	1.932403		

Appendix B:

Table 1B: Summary Statistics: Residuals:

	Observations	Mean	Median	Std. Dev.	Skewness	Kurtosis	Jarque-Bera
Residuals	3464	2.17E-16	-0.006351	1.097344	0.011645	5.666228	1026.111 (0.000000)

Table 2B: ADF for Excess Return on REITs:

Null Hypothesis: AR_REITS has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=29)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-55.66963	0.0001
Test critical values:	1% level	-3.432051	
	5% level	-2.862177	
	10% level	-2.567153	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(AR_REITS)
Method: Least Squares
Date: 05/29/17 Time: 10:56
Sample (adjusted): 7/16/2003 5/18/2017
Included observations: 3463 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR_REITS(-1)	-0.944684	0.016969	-55.66963	0.0000
C	0.421875	0.031116	13.55834	0.0000

R-squared	0.472417	Mean dependent var	-0.000920
Adjusted R-squared	0.472265	S.D. dependent var	2.444316
S.E. of regression	1.775682	Akaike info criterion	3.986824
Sum squared resid	10912.70	Schwarz criterion	3.990376
Log likelihood	-6901.186	Hannan-Quinn criter.	3.988092
F-statistic	3099.108	Durbin-Watson stat	2.000735
Prob(F-statistic)	0.000000		

Table 3B: ADF for Excess Return on XU100:

Null Hypothesis: AR_XU has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=29)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-57.33808	0.0001
Test critical values:	1% level	-3.432051	
	5% level	-2.862177	
	10% level	-2.567153	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(AR_XU)
Method: Least Squares
Date: 05/29/17 Time: 10:57
Sample (adjusted): 7/16/2003 5/18/2017
Included observations: 3463 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR_XU(-1)	-0.974069	0.016988	-57.33808	0.0000
C	0.362261	0.030006	12.07299	0.0000

R-squared	0.487157	Mean dependent var	-0.000870
Adjusted R-squared	0.487009	S.D. dependent var	2.409810
S.E. of regression	1.725988	Akaike info criterion	3.930053
Sum squared resid	10310.43	Schwarz criterion	3.933605
Log likelihood	-6802.887	Hannan-Quinn criter.	3.931321
F-statistic	3287.655	Durbin-Watson stat	1.999151
Prob(F-statistic)	0.000000		

Table 4B: Ljung Box Q:

Date: 05/29/17 Time: 11:00
Sample: 7/15/2003 5/18/2017
Included observations: 3464

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.190	0.190	125.51	0.000
		2	0.104	0.070	163.06	0.000
		3	0.075	0.045	182.75	0.000
		4	0.100	0.075	217.33	0.000
		5	0.132	0.098	277.83	0.000
		6	0.079	0.027	299.58	0.000
		7	0.077	0.038	320.34	0.000
		8	0.042	0.000	326.34	0.000
		9	0.055	0.022	336.93	0.000
		10	0.046	0.010	344.42	0.000
		11	0.070	0.040	361.42	0.000
		12	0.079	0.044	383.26	0.000
		13	0.033	-0.007	386.98	0.000
		14	0.029	0.001	389.98	0.000
		15	0.040	0.016	395.42	0.000
		16	0.048	0.017	403.60	0.000
		17	0.025	-0.008	405.84	0.000
		18	0.011	-0.012	406.23	0.000
		19	0.014	-0.002	406.92	0.000
		20	0.036	0.022	411.52	0.000
		21	0.024	0.000	413.47	0.000
		22	0.016	-0.000	414.39	0.000
		23	0.041	0.029	420.30	0.000
		24	0.031	0.010	423.56	0.000
		25	0.023	0.003	425.37	0.000
		26	0.036	0.021	429.95	0.000
		27	0.024	0.001	431.96	0.000
		28	0.001	-0.022	431.96	0.000
		29	0.010	0.001	432.32	0.000
		30	0.026	0.015	434.63	0.000
		31	0.009	-0.011	434.88	0.000
		32	0.031	0.021	438.30	0.000
		33	0.015	0.003	439.12	0.000
		34	-0.003	-0.016	439.14	0.000
		35	0.004	-0.006	439.20	0.000
		36	0.009	0.001	439.48	0.000

Table 5B: Breusch-Godfrey LM Test (2 Lags):

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	1.867460	Prob. F(2,3460)	0.1547
Obs*R-squared	3.735205	Prob. Chi-Square(2)	0.1545

Table 6B: ARCH LM Test:

Heteroskedasticity Test: ARCH

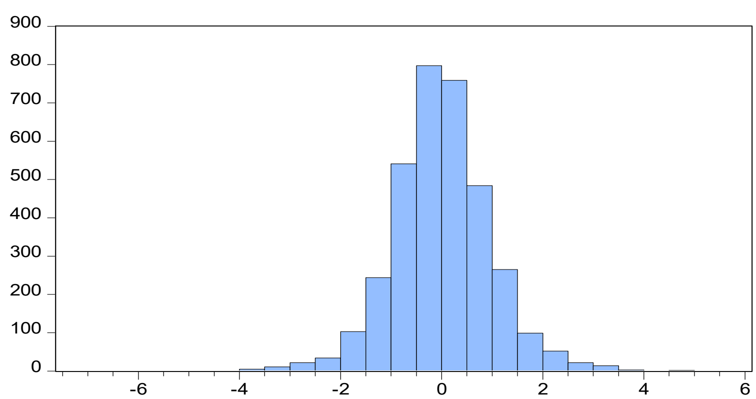
F-statistic	130.0105	Prob. F(1,3461)	0.0000
Obs*R-squared	125.3760	Prob. Chi-Square(1)	0.0000

Appendix C:

Table 1C: Correlogram of Standardized Residuals Squared: GARCH Student t:

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.020	0.020	1.3206	0.250
		2	-0.019	-0.019	2.5818	0.275
		3	-0.032	-0.031	6.1175	0.106
		4	-0.022	-0.021	7.7335	0.102
		5	0.012	0.012	8.2580	0.143
		6	-0.003	-0.005	8.2926	0.217
		7	-0.013	-0.014	8.8663	0.262
		8	-0.013	-0.012	9.4368	0.307
		9	-0.022	-0.022	11.072	0.271
		10	-0.024	-0.024	12.993	0.224
		11	0.010	0.009	13.332	0.272
		12	0.012	0.009	13.796	0.314
		13	-0.018	-0.020	14.891	0.314
		14	-0.012	-0.011	15.374	0.353
		15	0.012	0.013	15.881	0.390
		16	0.036	0.034	20.462	0.200
		17	-0.010	-0.014	20.814	0.235
		18	-0.029	-0.027	23.749	0.163
		19	-0.021	-0.018	25.311	0.151
		20	0.004	0.004	25.364	0.188
		21	-0.013	-0.017	25.996	0.207
		22	0.003	0.002	26.038	0.250
		23	0.006	0.005	26.154	0.294
		24	0.035	0.036	30.513	0.168
		25	-0.012	-0.013	31.045	0.188
		26	0.026	0.028	33.319	0.153
		27	0.003	0.000	33.347	0.186
		28	-0.016	-0.018	34.294	0.191
		29	-0.001	0.001	34.298	0.229
		30	0.007	0.009	34.459	0.263

Table 2C: Histogram Normality Test: Jarque-Bera: GARCH Student t:



Series: Standardized Residuals
Sample 7/15/2003 5/18/2017
Observations 3464

Mean	0.009544
Median	-0.016106
Maximum	5.718137
Minimum	-7.104156
Std. Dev.	0.993750
Skewness	0.009375
Kurtosis	6.021102
Jarque-Bera	1317.390
Probability	0.000000

Table 3C: ARCH LM Test: ARCH Student t:

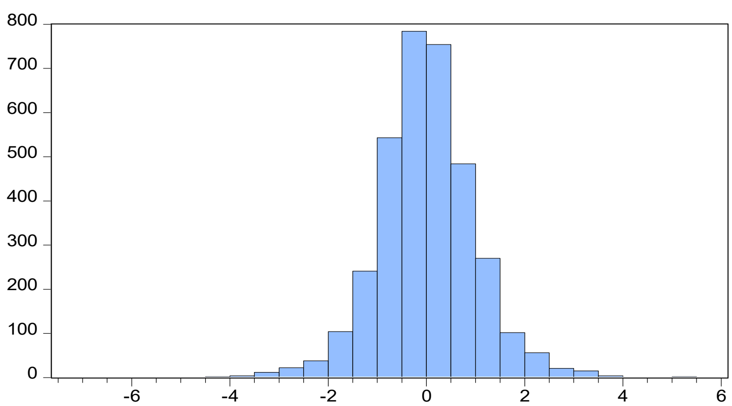
Heteroskedasticity Test: ARCH

F-statistic	1.318837	Prob. F(1,3461)	0.2509
Obs*R-squared	1.319096	Prob. Chi-Square(1)	0.2508

Table 4C: Correlogram of Standardized Residuals Squared: GARCH GED:

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.022	0.022	1.6044	0.205
		2	-0.018	-0.018	2.6910	0.260
		3	-0.031	-0.030	6.0365	0.110
		4	-0.020	-0.019	7.4672	0.113
		5	0.015	0.014	8.2174	0.145
		6	-0.001	-0.004	8.2235	0.222
		7	-0.011	-0.012	8.6706	0.277
		8	-0.012	-0.011	9.1647	0.329
		9	-0.020	-0.020	10.561	0.307
		10	-0.022	-0.023	12.266	0.268
		11	0.012	0.011	12.760	0.309
		12	0.014	0.011	13.401	0.341
		13	-0.017	-0.019	14.366	0.349
		14	-0.011	-0.010	14.770	0.394
		15	0.013	0.015	15.376	0.425
		16	0.038	0.035	20.348	0.205
		17	-0.009	-0.013	20.628	0.243
		18	-0.028	-0.027	23.441	0.174
		19	-0.020	-0.017	24.852	0.165
		20	0.005	0.005	24.943	0.204
		21	-0.012	-0.016	25.473	0.227
		22	0.004	0.003	25.533	0.272
		23	0.007	0.006	25.702	0.315
		24	0.035	0.036	30.002	0.185
		25	-0.011	-0.012	30.458	0.208
		26	0.026	0.029	32.849	0.167
		27	0.004	0.001	32.896	0.201
		28	-0.016	-0.017	33.796	0.208
		29	-0.001	0.002	33.796	0.247
		30	0.007	0.010	33.964	0.282

Table 5C: Histogram Normality Test: Jarque-Bera: GARCH GED:



Series: Standardized Residuals
Sample 7/15/2003 5/18/2017
Observations 3464

Mean	0.014261
Median	-0.011046
Maximum	5.792994
Minimum	-7.133574
Std. Dev.	1.007466
Skewness	0.017821
Kurtosis	5.974783
Jarque-Bera	1277.437
Probability	0.000000

Table 6C: ARCH LM Test: GARCH GED:

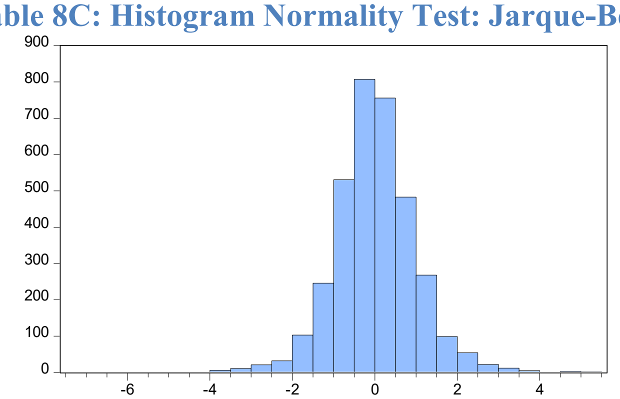
Heteroskedasticity Test: ARCH

F-statistic	1.602423	Prob. F(1,3461)	0.2056
Obs*R-squared	1.602607	Prob. Chi-Square(1)	0.2055

Table 7C: Correlogram of Standardized Residuals Squared: EGARCH Student t:

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.034	0.034	3.9985	0.046
		2	-0.015	-0.017	4.8247	0.090
		3	-0.030	-0.029	7.9554	0.047
		4	-0.019	-0.017	9.2278	0.056
		5	0.019	0.019	10.485	0.063
		6	0.001	-0.002	10.488	0.106
		7	-0.012	-0.013	10.990	0.139
		8	-0.011	-0.009	11.381	0.181
		9	-0.023	-0.022	13.255	0.151
		10	-0.021	-0.021	14.771	0.141
		11	0.009	0.009	15.067	0.179
		12	0.012	0.010	15.597	0.210
		13	-0.019	-0.021	16.842	0.207
		14	-0.012	-0.010	17.321	0.239
		15	0.011	0.013	17.745	0.276
		16	0.033	0.030	21.534	0.159
		17	-0.011	-0.015	21.940	0.187
		18	-0.028	-0.026	24.583	0.137
		19	-0.022	-0.019	26.299	0.122
		20	0.003	0.003	26.329	0.155
		21	-0.011	-0.014	26.720	0.180
		22	0.003	0.002	26.754	0.221
		23	0.008	0.008	27.000	0.256
		24	0.032	0.032	30.510	0.168
		25	-0.014	-0.015	31.165	0.184
		26	0.019	0.021	32.402	0.180
		27	0.001	-0.002	32.408	0.217
		28	-0.018	-0.020	33.494	0.218
		29	-0.004	-0.002	33.538	0.257
		30	0.008	0.010	33.745	0.291

Table 8C: Histogram Normality Test: Jarque-Bera: EGARCH Student t:



Series: Standardized Residuals
Sample 7/15/2003 5/18/2017
Observations 3464

Mean	0.009582
Median	-0.018098
Maximum	5.333285
Minimum	-7.259509
Std. Dev.	0.994321
Skewness	-0.014275
Kurtosis	6.057553
Jarque-Bera	1349.436
Probability	0.000000

Table 9C: ARCH LM Test: EGARCH Student t:

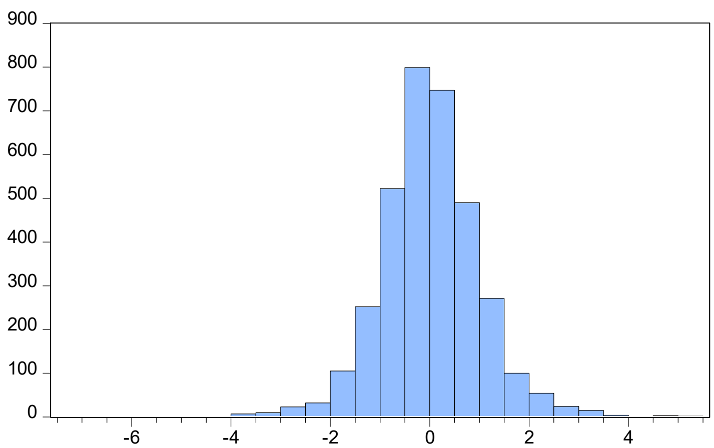
Heteroskedasticity Test: ARCH

F-statistic	3.996178	Prob. F(1,3461)	0.0457
Obs*R-squared	3.993876	Prob. Chi-Square(1)	0.0457

Table 10C: Correlogram of Standardized Residuals Squared: EGARCH GED:

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.038	0.038	4.9148	0.027
		2	-0.013	-0.014	5.4948	0.064
		3	-0.029	-0.028	8.3719	0.039
		4	-0.017	-0.015	9.4187	0.051
		5	0.022	0.022	11.033	0.051
		6	0.003	-0.000	11.057	0.087
		7	-0.010	-0.011	11.422	0.121
		8	-0.010	-0.008	11.752	0.163
		9	-0.022	-0.020	13.373	0.146
		10	-0.020	-0.020	14.767	0.141
		11	0.010	0.011	15.149	0.176
		12	0.014	0.012	15.833	0.199
		13	-0.018	-0.020	16.962	0.201
		14	-0.011	-0.008	17.361	0.237
		15	0.012	0.014	17.850	0.271
		16	0.034	0.031	21.865	0.148
		17	-0.010	-0.014	22.187	0.178
		18	-0.027	-0.025	24.683	0.134
		19	-0.021	-0.017	26.185	0.125
		20	0.004	0.005	26.253	0.158
		21	-0.010	-0.013	26.582	0.185
		22	0.003	0.003	26.623	0.226
		23	0.009	0.009	26.937	0.259
		24	0.033	0.033	30.709	0.162
		25	-0.013	-0.014	31.276	0.180
		26	0.020	0.022	32.626	0.173
		27	0.002	-0.001	32.643	0.209
		28	-0.017	-0.019	33.689	0.211
		29	-0.003	-0.001	33.722	0.250
		30	0.008	0.010	33.921	0.284

Table 11C: Histogram Normality Test: Jarque-Bera: EGARCH GED:



Series: Standardized Residuals
Sample 7/15/2003 5/18/2017
Observations 3464

Mean	0.016097
Median	-0.011934
Maximum	5.375499
Minimum	-7.272162
Std. Dev.	1.007547
Skewness	-0.002743
Kurtosis	5.985474
Jarque-Bera	1286.455
Probability	0.000000

Table 12C: ARCH LM Test: EGARCH GED:

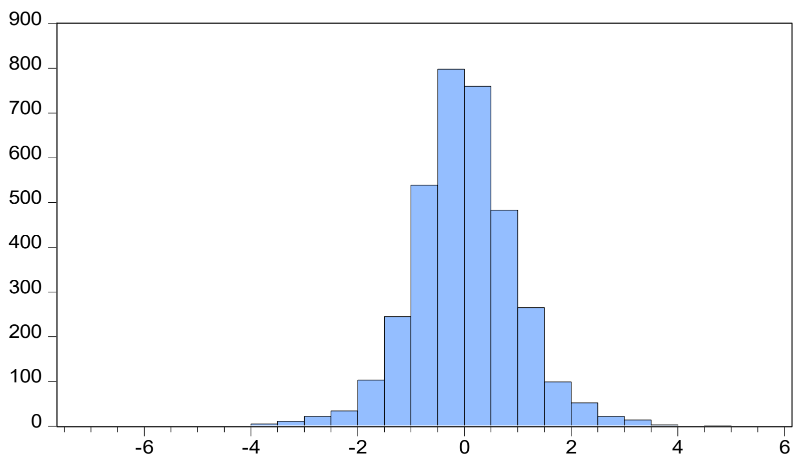
Heteroskedasticity Test: ARCH

F-statistic	4.913280	Prob. F(1,3461)	0.0267
Obs*R-squared	4.909150	Prob. Chi-Square(1)	0.0267

Table 13C: Correlogram of Standardized Residuals Squared: GARCH-GJR Student t:

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.019	0.019	1.3139	0.252
		2	-0.019	-0.019	2.5757	0.276
		3	-0.032	-0.031	6.1198	0.106
		4	-0.022	-0.021	7.7384	0.102
		5	0.012	0.012	8.2594	0.143
		6	-0.003	-0.005	8.2948	0.217
		7	-0.013	-0.014	8.8669	0.262
		8	-0.013	-0.012	9.4347	0.307
		9	-0.022	-0.022	11.067	0.271
		10	-0.024	-0.024	12.993	0.224
		11	0.010	0.009	13.329	0.272
		12	0.012	0.009	13.792	0.314
		13	-0.018	-0.020	14.883	0.315
		14	-0.012	-0.011	15.362	0.354
		15	0.012	0.013	15.873	0.391
		16	0.036	0.034	20.472	0.200
		17	-0.010	-0.014	20.821	0.234
		18	-0.029	-0.027	23.752	0.163
		19	-0.021	-0.018	25.306	0.151
		20	0.004	0.004	25.360	0.188
		21	-0.013	-0.017	25.989	0.207
		22	0.003	0.002	26.031	0.250
		23	0.006	0.005	26.146	0.294
		24	0.036	0.036	30.546	0.167
		25	-0.012	-0.013	31.076	0.187
		26	0.026	0.028	33.353	0.152
		27	0.003	0.000	33.381	0.185
		28	-0.016	-0.018	34.328	0.190
		29	-0.001	0.001	34.331	0.227
		30	0.007	0.009	34.493	0.262

Table 14C: Histogram Normality Test: Jarque-Bera: GARCH-GJR Student t:



Series: Standardized Residuals
Sample 7/15/2003 5/18/2017
Observations 3464

Mean	0.009671
Median	-0.015981
Maximum	5.715079
Minimum	-7.105185
Std. Dev.	0.993745
Skewness	0.009503
Kurtosis	6.019836
Jarque-Bera	1316.287
Probability	0.000000

Table 15C: ARCH LM Test: GARCH-GJR Student t:

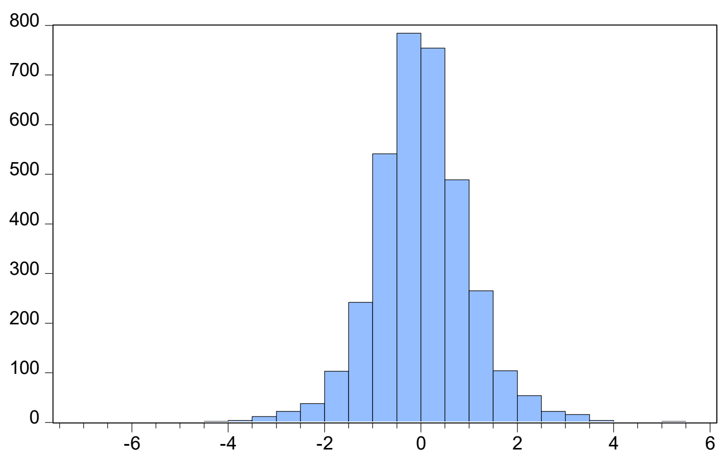
Heteroskedasticity Test: ARCH

F-statistic	1.312124	Prob. F(1,3461)	0.2521
Obs*R-squared	1.312385	Prob. Chi-Square(1)	0.2520

Table 16C: Correlogram of Standardized Residuals Squared: GARCH-GJR GED:

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.021	0.021	1.5236	0.217
		2	-0.018	-0.018	2.6124	0.271
		3	-0.031	-0.031	6.0384	0.110
		4	-0.020	-0.019	7.4865	0.112
		5	0.014	0.014	8.2008	0.146
		6	-0.002	-0.004	8.2102	0.223
		7	-0.011	-0.012	8.6360	0.280
		8	-0.012	-0.011	9.0965	0.334
		9	-0.020	-0.019	10.453	0.315
		10	-0.022	-0.023	12.196	0.272
		11	0.012	0.011	12.659	0.316
		12	0.014	0.011	13.298	0.348
		13	-0.016	-0.018	14.211	0.359
		14	-0.010	-0.009	14.574	0.408
		15	0.014	0.015	15.243	0.434
		16	0.039	0.036	20.434	0.201
		17	-0.008	-0.012	20.684	0.241
		18	-0.028	-0.026	23.448	0.174
		19	-0.019	-0.016	24.768	0.168
		20	0.006	0.006	24.885	0.206
		21	-0.012	-0.015	25.381	0.231
		22	0.004	0.003	25.444	0.276
		23	0.007	0.006	25.604	0.320
		24	0.037	0.038	30.349	0.173
		25	-0.011	-0.011	30.773	0.197
		26	0.026	0.029	33.205	0.156
		27	0.004	0.002	33.256	0.189
		28	-0.016	-0.017	34.149	0.196
		29	-0.000	0.002	34.150	0.234
		30	0.007	0.010	34.319	0.268

Table 17C: Histogram Normality Test: Jarque-Bera: GARCH-GJR GED:



Series: Standardized Residuals
Sample 7/15/2003 5/18/2017
Observations 3464

Mean	0.015604
Median	-0.009938
Maximum	5.761163
Minimum	-7.143041
Std. Dev.	1.007362
Skewness	0.019622
Kurtosis	5.960804
Jarque-Bera	1265.501
Probability	0.000000

Table 18C: ARCH LM Test: GARCH-GJR GED:

Heteroskedasticity Test: ARCH

F-statistic	1.521698	Prob. F(1,3461)	0.2174
Obs*R-squared	1.521908	Prob. Chi-Square(1)	0.2173