COMPARING DIFFERENT MULTI-CRITERIA DECISION MAKING METHODS TO EVALUATE MOBILE APPLICATIONS FOR MATHEMATICS LEARNING

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To my family….

ABSTRACT

The abundance of mobile applications for learning particularly for mathematics (MLAM) which a fundamental field in teaching and learning in the application stores like Google Play store or Apple have led users with confusion on the selection of right application. Multi-criteria decision making methods come in handy in solving these issues. The aim of this study is; firstly to compare 3 different multi-criteria methods such as hybrid (FAHP and ELECTRE I) and 2 stand-alone methods FAHP and ELECTRE I) to evaluate and rank MLAMs, Secondly, to develop and propose an executable stand-alone computer application for evaluating 5 top rated MLAMs by using C# in Visual Studio with the outcomes of 1 hybrid and 2 stand-alone methods.

Criteria was decided from an adopted framework to evaluate both technical and pedagogical requirements of MLAMs. Results showed that, in terms of ease of use for ranking limited number of alternatives with few criteria hybrid method is the most efficient than stand-alone fuzzy AHP and lastly ELECTRE I . In the case of more number of alternatives and criteria, ELECTRE I seems more efficient. So far, in this area, no studies located to compare different multi criteria methods. Moreover, an executable program to compare the outcomes of multiple methods provides useful support for decision makers.

Keywords: ELECTRE; FAHP; mobile learning application for mathematics; pedagogical requirements; technical requirements

ÖZET

Özellikle matematik için öğrenme için mobil uygulamaların bolluğu (MLAM) hangi öğretim ve Google Play Store veya Apple gibi uygulama mağazalarında öğrenme temel bir alan doğru uygulamanın seçiminde karışıklık ile kullanıcıları açmıştır. Çok kriterli karar verme yöntemleri bu sorunları çözmede kullanışlıdır. Bu çalışmanın amacı; İlk olarak Hybrid (FAHP ve ELECTRE I) ve 2 tek başına Yöntem FAHP ve ELECTRE I) değerlendirmek ve rank MLAMs, ikinci, geliştirmek ve 5 üst değerlendirmek için çalıştırılabilir bir stand-alone bilgisayar uygulaması önermek için 3 farklı çok kriterli yöntemleri karşılaştırmak 1 karma ve 2 tek başına yöntemleri ile Visual Studio 'da C# kullanarak MLAMs derecelendirilmiş.

Kriterler, MLAMs teknik ve pedagojik gerekliliklerini değerlendirmek için kabul edilen bir çerçeveden karar verildi. Sonuçlar, birkaç kriterli hibrit metodu ile sınırlı sayıda alternatiflerin derecelendirmesi için kullanım kolaylığı açısından, tek başına FAHP ve son derece ELECTRE en etkili olduğunu gösterdi. Alternatifler ve kriterler daha fazla sayıda durumda, ELECTRE ben daha verimli görünüyor. Şimdiye kadar, bu alanda, hiçbir çalışma farklı çok kriterler yöntemleri karşılaştırmak için yer. Ayrıca, birden fazla yöntemin sonuçlar karşılaştırmak için çalıştırılabilir bir program karar vericiler için yararlı destek sağlar.

Anahtar Kelimeler: ELECTRE; FAHP; matematik için mobil öğrenme uygulaması; pedagojik gereklilikler; Teknik gereklilikler

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LIST OF ABBREVIATIONS

AHP:	Analytic Hierarchy Process.
ANP:	Analytic Network Process.
DEMATEL:	Decision Making Trial and Evaluation Laboratory.
ELECTRE:	Elimination and Choice Expressing Reality.
FAHP:	Fuzzy Analytic Hierarchy Process.
TFN:	Triangular Fuzzy Number.
TOPSIS:	Technique for Order of Preference by Similarity to Ideal Solution.
SMART:	Specific Measurable Achievable Responsible Time-related.
ISO 9126:	International Organization for Standardization.
MLE:	Mobile Learning Environment.
MCDM:	Multi-Criteria Decision Making
MLAM:	Mobile Learning Application for Mathematics.
MADM:	Multi-Attribute Decision Making
PROMETHEE:	Preference Ranking Organization Method for Enrichment of Evaluations.
PC:	Personal Computer.

CHAPTER 1

INTRODUCTION

1.1 Background

Mathematics subject is essential in the field of computer science that majority of students have difficulties in learning according to literature research. Mathematics has a very important function in the development of the human mind. It inspires creativity; implanting logical and systematic thinking. Mathematics is a natural field of study for learning and having a good knowledge to appreciate the fundamentals of many computer science concepts because major computer science uses mathematics and logic in declarative mode as well as in calculation mode.

As important as the subject Mathematics is there should be sufficient learning ways for it and in the context of this study several mobile learning applications for learning Mathematics exits and all fall under mobile learning applications for mathematics. However, there are no sufficient evaluation for these applications, only few researchers have carried out a form of evaluation or "testing" of MLAMs.

Skillen (2015) detailed that there are more than 4,000 MLAMs to choose from which had pined for courses for bunches alternatives for making choice on what versatile realizing condition to receive. This scenario has led many into making a premature selection of MLAMs because making an efficient selection from over 4,000 applications seems tedious and time- consuming, thus making right selection is crucial to enhancing its continuous usage and developments. Hence, to address this problem a model was adopted to provide a roadmap for making a reliable ranking of MLAM different but conflicting options is inevitable.

Making a manual selection among several MLAM is tiresome, time-consuming and has led many individuals in making an inept selection, therefore, there is a need to select and evaluate MLAM in an independent way. For this purpose, utilizing MCDM approaches in the selection and evaluation of MLAM provides less time consuming and an optimum result.

To evaluate MLAMs applying a hybrid multi-criteria decision-making method exists in literature, which was by Basaran and Haruna (2017). They proposed a model defining both user satisfaction and quality aspects used in evaluating MLAMs by utilizing the Fuzzy AHP and TOPSIS techniques together. There are several MCDM methods developed and utilized, MCDM is a mechanism used for selecting the best option out of multiple alternatives in most cases conflicting decision criteria (Pomerol & Barba, 2012). There are numerous MCDM techniques such as ANP, FAHP, SMART, ELECTRE, PROMETHEE, TOPSIS etc.

Using FAHP, ELECTRE I in this study is the initial attempt because no other study has used three methods in a way this study did; a comparison of results of these three methods was derived in this study and a hybrid method and stand-alone methods respectively were utilized. FAHP was used because it is widely used while ELECTRE I is preferred for the ranking the MLAMs as its purpose. In order to enhance the process of decision making normally different methods are merged together.

The popular AHP approach is well known and as of late the most utilized approach for making decisions, in the process of making decisions out of different choices, which their rankings are based on numerous attributes that are expressed utilizing distinctive scales. The AHP approach gives flexibility in the process of making decision and assists decision makers to assign priority, which helps in making good decisions by taking qualitative and quantitative aspects into considering (Forman & Gass 2001). However, there is an extension to the AHP method, which provides more flexibility, and dynamism in weighting criteria reducing ambiguity and it is called Fuzzy Analytic Hierarchy Process (FAHP).

FAHP is an expansion, also an upgrade of AHP technique that utilizes fuzzy logic, numbers, and sets. It utilizes fuzzy numbers rather than assigning particular numbers to determine criteria weighting, these fuzzy sets provided more flexibility in assigning weights to the criteria. In addition, the method utilized fuzzy numbers to decide the advantages of single criteria to its other (Shaout & Yousif, 2014; Sun, Lin & Tzeng, 2009). Specifically, the MCDM complex problems were much simpler with FAHP strategy with managing and understanding qualitative and quantitative data during utilization. AHP is first utilized to compute criteria weights then the FAHP fo the utilization of fuzzy numbers are used to combine the presence of fuzzy values because of subjective evaluation, then, ELECTRE I is used for alternatives ranking.

The ELECTRE I technique for picking the best activity from a given arrangement of activities was contrived in 1965. The ELECTRE is for 'Disposal ET Choix Traduisant la REalite (Elimination and Choice Expressing the Reality). ELECTRE is an outstanding MCDM strategy that has a background marked by fruitful genuine applications. It has been connected in past to different sorts of basic leadership circumstances. ELECTRE I requires a contribution of criteria assessments for the options, called choice network, inclination data, communicated as weights, limits, and different parameters (Sevkli, 2010).

During the conduction of this study there was only at the time one software quality model with a purpose of evaluating MLAM's (Başaran and Haruna, 2017). However, many models have been designed for evaluating software qualities generally such as ISO-9126, 25010, FURPS. According to a software quality model reviewed by Miguel et al. (2014), it was put forward that ISO-9126 model was optimal and the most used for the evaluation of software qualities, for the reasons stated above and that MLAM's are regarded utterly as software applications that are designed typically to run on smartphones (Pocatilu & Vetrici, 2009) MLAM's could be evaluated using the ISO 9126 software quality characteristics (Pocatilu & Boja, 2009).

As mentioned above there are numerous frameworks for mobile learning exist but the most common characteristic of most is towards the technological requirements, no study has been towards the pedagogical aspects, this study aims to adopt a framework from two viewpoints; technical aspect and the pedagogical aspect simply because looking into this study mobile applications for Mathematics will be evaluated with criteria selected from each aspect. Pedagogical requirements are important due to its relation to educational references and development and this study looks to evaluate Mathematics learning applications. To carry out this evaluation for MLAMs with MCDM methods criteria are important because they are the metrics for evaluation as well as alternatives, which are the sampled MLAMs for this study.

What makes this study different from other similar is this study first implemented an hybrid method in FAHP and ELECTRE I for the weights determination and ranking of alternatives for MLAM's, then implemented a stand-alone in FAHP for the weights determination and ranking of alternatives for MLAM's and finally a stand-alone ELECTRE I for the weights determination and ranking of alternatives for MLAM's also.

In addition an executable stand-alone computer application is developed for evaluating 5 top rated MLAMs by using C# in Visual Studio with the outcomes of 1 hybrid and 2 stand-alone methods, this application has never been developed, a few study have developed stand-alone software for a type of MCDM but not a hybrid or even a comparison.

A sub-characteristic of a requirement in models is used in this study to evaluate MLAM's towards the technical requirements in addition to the pedagogical requirements, with reason to be called Multi-Criteria decision-making methods. The appropriate aspect for mobile learning should be open and satisfactory to the learning condition, which leads to a first viewpoint for evaluation, which is a technical oriented perspective. It is important not to base this study alone on the technical quality aspects alone when conducting an evaluation, Shee and Wang, (2008) highlighted this importance and this study will be including another perspective known as the human-computer interaction.

Since mobile devices are getting progressively present, decision-makers have coordinated this into the instructive situations. Keegan (2002) foretold, "Mobile learning is a harbinger of the future of learning". Developed applications for education have a wide range, from K– 12 to advanced education and corporate learning settings, from formal and casual figuring out how to classroom learning, separate learning, and field studies. Regardless of the numerous sorts of services mobile learning offers, there is a lack of pedagogical considerations and technical considerations (Traxler, 2007).

Another viewpoint for the evaluation is from a pedagogical perspective. This brings up that mobile learning grows new aptitudes and ways to deal with ensure the educational viability which is the effectiveness of pedagogy. It is viewed as that the setting of a mobile condition ought to be taken into consideration during evaluation. Criteria for this perspective includes an approved arrangement of focal properties of m-learning: coordinated effort, validness, and personalization (Kearney, et al., 2012). Alsumait and Al-Osaimi (2010) buttressed that for a learning application to perform optimally in the market, it must be analyzed not only in terms of its usability also the educational benefits together with its interactive ability. This is important because e-learning and m-learning applications should satisfy some HCI factors like effectiveness, satisfaction of interfaces. It should equally satisfy the pedagogy of learning and educational domains. The aforementioned criteria were used for achieving the study goal through decision-making mechanisms, which are grouped into two as single criterion and multiple criteria decision making. Since the study was based on numerous criteria, the most appropriate method is the MCDM.

MCDM is a mechanism used for selecting the best option out of multiple alternatives in most cases conflicting decision criteria (Pomerol & Barba, 2012). There are numerous MCDM techniques such as ANP, FAHP, SMART, ELECTRE, PROMETHEE, TOPSIS etc. In order to enhance the process of decision making normally different methods are merged together. According to Saaty and Vargas (2012), they stated that a well-known exceptional MCDM method is FAHP, which can be used alongside TOPSIS in mostly complex decision-making.

Therefore, the aim of this study is; firstly, employ and compare 3 different multi-criteria methods such as hybrid (FAHP and ELECTRE I) and 2 stand-alone methods FAHP and ELECTRE I) to evaluate and rank MLAMs, Secondly, to develop and propose an executable stand-alone computer application for evaluating 5 top rated MLAMs by using C# in Visual Studio with the outcomes of 1 hybrid and 2 stand-alone methods. Conclusively, this study adopted the Economides (2008) pedagogical requirements, and technical requirements in relation to the ISO 9126 model. The resulting framework was used to evaluate the selected five sampled MLAM's using both FAHP and ELECTRE I methods for weights determination and alternatives ranking respectively.

1.2 The Problem

At the time of conducting this study concerning the evaluation of MLAM, using MCDM methods there was only one framework developed for evaluating MLAM (Başaran and Haruna 2017). In addition, making a manual selection among several MLAM is tiresome, time-consuming and has led many individuals in making an inept selection, therefore, there is a need to select and evaluate MLAM in an independent way. For this purpose, utilizing MCDM approaches in the selection and evaluation of MLAM provides less time consuming and an optimum result.

Mathematics is an essential subject in the field of computer science that majority of students have difficulties in learning according to literature research. Mathematics has a very important function in the development of the human mind. It inspires creativity; implanting logical and systematic thinking.

1.3 Aim of the Study

Firstly to compare 3 different multi-criteria methods such as hybrid (FAHP and ELECTRE I) and 2 stand-alone methods FAHP and ELECTRE I) to evaluate and rank MLAMs, Secondly, to develop and propose an executable stand-alone computer application for evaluating 5 top rated MLAMs by using C# in Visual Studio with the outcomes of 1 hybrid and 2 stand-alone methods. In addition, the study also identifies an optimal mobile learning application for mathematics among given ones with respect to the adopted framework criteria through applying MCDM techniques (FAHP & ELECTRE I).

The qualities of five highly rated mobile applications for mathematics are compared. The criteria employed in the evaluation is explained in the framework adopted. It is believed to be the second mobile learning applications for mathematics evaluation that utilizes MCDM techniques.

1.4 Research Objectives

Adopting both technical and pedagogical requirements of mobile learning application for mathematics. Also, evaluating MLAM with respect to the adopted framework through applying MCDM techniques (FAHP and ELECTRE I) for a comparison and stand-alone.

1.5 Significance of the Study

As long as the quality evaluation based on technical requirements are wide, other aspects such as the pedagogical requirements should not be far, including them makes this study distinct from former studies. This study can serve MLAM users in decision making during the selections of MLAM in such an autonomous timesaving manner and on the other hand optimizing the decision results during the selection process. Thus, the developed framework will also help in making an efficient quality assessment of MLAM. This study can serve the future researchers as their reference in making a related research in this subject area, which can provide them with ideas in order to make their overall work a lot easier.

1.6 The Limitations of the Study

Whilst this study will accomplish its aim, there are restrictions attached to it due logistics and time.

- The study deals only on MLAMs for adult users.

- The total number of MLAMs for selection and evaluation are five for the purpose of this study
- This study only uses three MCDM methods though a comparison between a combination of both and stand-alone are utilized.

1.7 Overview of the Study

The thesis comprises of 6 chapters in all:

The First chapter brings an introduction that explains in details the definition of the problem, what the study aims at, and importance of the study.

The Second chapter dwells on the literature review on mobile devices, mobile applications, and m-learning applications, features of m-learning, m-learning for mathematics, limitations of MLAM, MCDM, AHP, FAHP and ELECTRE I.

The Third chapter introduces the conceptual framework.

The Fourth chapter explains the methodology used in selecting, evaluation and writing thesis.

The Fifth chapter is the empirical study and discussed the result.

The Sixth chapter concludes the study with future recommendations.

CHAPTER 2

LITERATURE REVIEW

This is the literature review on evaluation by different researchers, where different studies previously published in this subject area of the research was analyzed, examined their findings and study their missing gaps.

2.1 Related Research

In adopting a framework for evaluating the quality of MLAM using the approach mentioned in the first chapter, a subset of different previous kind literatures below were selected for review because they fall within the subject area of this study, they are; Smart mobile phones, Mobile learning applications, Features of mobile learning, Mobile learning application for mathematics, FAHP and ELECTRE and Limitations of mobile learning application for mathematics. However, the literature review focused on the information introduced in articles, journals and other related reports, with the expectations that the findings were accurate and reliable.

2.2 Smart Mobile Devices

In 2012, the typical use of mobile devices raised by 81% more than 2011 and the normal download of applications raised to 342 MB for every month and per mobile devices, contrasted with 189 MB in 2011 (Cisco Visual Networking Index, 2014) at a global level. Gartner (2014) reports at 2011 yearly 19 % rate of increment in offers of mobile devices. In 2013, about 102 billion of applications was downloaded, versus 64 billion out of 2012. In 2018, it is normal that this number will increment to 254 billion downloads (Gartner, 2014). These measurements demonstrate that mobile devices have possessed the day by day lives of shoppers: at home, at work, and in the public.

Boja and Batagan (2011) investigated the characteristics of mobile devices used for activities of learning, reported the rapid growth of mobile devices had opened ways for easy accessibility, collaboration, portability and motivated users.

Mobile devices are way less expensive than a personal computer and are used by many because they are more affordable and in the case of mobile phones, is almost a necessity to have. Therefore, most users will go for the choice.

Han, Kim, Yun, Hong, and Kim (2004) investigated mobile devices attributes that are essential to user satisfaction, they identified size, color, shape and material at its design attributes. They stated that user satisfaction would help in identifying some essential features of mobile device during design.

Ziefle and Bay (2006) reiterated the importance of considering some critical technological ability of mobile devices when designing a mobile application. They recommended that mobile applications developer should concentrate more on the device sub-characteristics in other to attain a user satisfaction.
After the presentation of mobile devices in the 1980s, they have turned out to be largely utilized, they are transforming into the new PCs as its users now operate serval mobile devices on the expansion rather than work areas to get to data because of its mobility Lobo (2011). Cell phones are not utilized for correspondence and collaboration; they can be close to home and compact Prensky (2012). Because of the imperative dispersion of portable advancements, most students today as of now have their own particular mobile devices and they have a conceivable preferred standpoint for applying innovation in the instructive field, as they are less. (Traxler & Leach, 2006). Wang et al., (2005) accounted that mobile devices can be used to pass on online courses to understudies in this age. While Prensky (2012) solicited, for what reason not exploit from these gadgets for instructive purposes? He underscored that understudies will have the capacity to learn "anything, if developers designed it right." In this age, mobile devices are presently extremely overwhelming. Utilizing them has expanded over the utilization of consistent calls or messages, enhanced capacities of cell phones can run applications, in this manner, advanced the utilization and now it has the ability to adapting anywhere (Cota et.al, 2014).

2.3 Mobile Learning Applications

Sha, Looi, Chen, Seow and Wong (2012) investigated learning during a mobile learning application through measure and recognizing self-regulation in these technologies. They acknowledged that a lot of analysis is required to research the skin elements and within variables (e.g., earlier info, inspiration) basic students targeted versatile taking in, the current concentrate on paper and precisely investigates however, the speculations and approaches of self-regulated learning (SRL) will facilitate to interrupt down and comprehend the procedures of moveable learning. They determined info gathered from two rudimentary science categories in Singapore that showed that the informative SRL model of versatile learning planned in their study lighted up the connections between three elements of moveable taking in: students' self-reports of mental procedures, samples of web learning conducted within the mobile learning surroundings (MLE), and learning accomplishment. There also documented that factual investigations created three principle discoveries. They explicit, understudy inspiration for this case will represent whether or not and to what degree the scholars will effectively participate in versatile learning exercises metacognitive, motivationally, and behaviorally. Secondly, the impact of students' self-reported inspiration on their learning accomplishment is interceded by their activity engagement in a very pre-outlined action within the MLE. Thirdly, students' impression of parental self-reliance support is not simply connected with their inspiration in class adapting, to boot connected with their real practices in self-moving their learning.

Hamdan et al. (2012) stated that, the benefits are first, learning material can be carried anywhere thus m-learning managed to reduce the use of lecture halls and tutorial rooms. Other than that, m-learning has limited the presence of teachers in the classroom to teach, overcome the limited time of teaching and content of teaching and the small number of students can be overcome. This is additionally bolstered by Corbeil and Valdes-Corbeil (2007) which versatility enables figuring out how to stretch out past the conventional classroom. The m-learning helped separate learning as clients of convenient gadgets can break the tie of a home PC. Third, mobile learning is advancing deep-rooted learning in which anybody can utilize versatile learning applications and not just subject to the understudies and educators.

Concerning assessment of versatile learning condition Vavoula and Sharples (2009) archived that assessment ought to be deferred unless a simple and naturally simple to utilize condition is produced and also Vavoula and Sharples (2009) detailed that portable learning is controlled by its condition more than the classroom direction, repeating the significance of learning condition and its many-sided quality. Standard measures for studying student's satisfaction with data frameworks and for assessing student satisfaction concerning classroom educating are not sensible for portable based learning framework (Shee and Wang, 2008). Detailed by Kurilovas and Vinogradova (2015) that cutting edge versatile learning uses tablets in perspective of basic reasoning, personalization, joint exertion, and flipped class are more versatile than regular ones, and they have more potential for feedbacks. M-learning is a brand of education delivery called distance learning, it is much something beyond utilizing a mobile device to get data and interface with clients, and it is about the mobility of the student. A few m-learning applications have been produced to make adapting simple however before when these applications are been made accessible to the users a few tests ought to be performed to guarantee it is dependable and it meets the particular criteria or necessities.

In general, mobile learning is an educational method that makes it possible to reach educational content anywhere without depending on a specific location, benefit from services dynamically and communicate with others. It immediately fulfills the needs of users as an individual and increases their productivity and performance efficiency (Keskin, 2010). In other words, mobile is a learning which is possible anytime and anywhere by using mobile devices like smartphones, tablets, PDA. We can simply define mobile learning as performing education through mobile devices and systems. Willacy and Calder (2017) defined mobile learning applications as mobile applications that make it possible for users to exercise learning in a changeable position. It establishes an anytime, anywhere learning environment (Martin, et al., 2013; Peng, et al., 2009; Uzunboylu, Cavus & Ercag, 2009). This technique provides greater flexibility to the learner, which as a result had foster high adoption by many individuals and learning institutions.

2.4 Mobile Learning Applications for Mathematics (MLAMs)

Lately, specialists composed an m-learning application that backs showing Mathematics and its distinctive classes, for example, statistics, algebra, geometry, and others (Drigas and Pappas, 2015). In any case, mobile applications for arithmetic enable its users to assess scientific capacities, giving graphical capacities and give a few sorts of portable adding machines. There are applications created to deal with estimation endeavors and instructive applications for practicing on numerical and scientific aptitudes. Innovations that offer help to numerical science in cell phones have in like manner been growing over the span of the latest decade (Bjerede, Atkins & Dede, 2010). The mobile instructive contraptions for arithmetic can help understudies in basic reasoning or critical thinking, redesign energy about numerical thoughts, give capably portrayals of considerations and stimulate general meta-intellectual limits (Pierce, Stacey and Barkatsas, 2007). The consistent utilization of versatile innovations over the traverse of arithmetic would empower the difference in portable learning applications and furthermore help understudies to upgrade their learning targets. In any case, Skillen (2015) detailed that there are more than 4, 000 portable applications for taking in arithmetic to choose from which had pined for courses for bunches alternatives for making choice on what versatile realizing condition to receive. This scenario has led many into making a premature selection of MLAMs because making an efficient selection from over 4,000 applications seems tedious and time- consuming, thus making right selection is crucial to enhancing its continuous usage and developments. Hence, to address this problem a software quality model to provide a roadmap for making a reliable selection of mobile learning application for mathematics from different but conflicting options is inevitable.

While in its beginning periods, investigate on the utilization of portable innovation with a specific spotlight on Mathematics instruction is quickly developing as a region of premium. This intrigue is creating as cell phones and applications turn out to be even more effectively open and well known among understudies. Distinguish the examination of the viability of versatile applications similar to another developing territory of research. To-date the vast majority of the exploration on applications has been directed by huge media organizations with an emphasis on advertising and the trial of gadgets. It is a dependable fact that applications for hand-held gadgets. A large number of this attention is on Mathematics. A current inquiry uncovered in excess of 4000 applications for Mathematics training. Nonetheless, there is constrained research in Mathematics instruction delineating the utilization of Apps and going with teaching methods. With the expansion of gadgets into classrooms and going with Apps, instructors and teachers need to think about their viability in supporting adapting, especially in connection to Mathematics. In its position explanation, the U.S. National Council of Teachers of Mathematics (NCTM) assert that "Technology is an essential tool for learning Mathematics in the 21st century, and all schools must ensure that all their students have access to technology".

Like other researchers, Skillen (2015) identified that "Despite the rapid expansion of the use of apps in the Educational domain, there is a lack of empirical studies as to their effectiveness in supporting learning, particularly in relation to Mathematics". This absence of available research likewise reaches out to the employments of Apps by instructor instructors in pre-service education training programs.

There is prove that instructors in New South Wales schools are not sure utilizing cell phones (e.g., tablets, cell phone) in showing science (Handal, Campbell, Cavanagh, Petocz, and Kelly, 2012). Absence of expert improvement and assets appear to be main considerations in blocking the usage of versatile innovation in schools. What's more, questions have been raised about the nature of some instructive programming programs, which appear to be created from a business viewpoint with minimal academic thought.

Different models for sorting arithmetic instruction programming as per assignment introduction have been distributed. Among them, Handal and Herrington (2003) distributed a scientific classification sorting PC based learning assets into six gatherings to be specific, drills, instructional exercises, amusements, reproductions, hypermedia devices and open learning situations. Set on a subjective engagement continuum, penetrate and rehearse assets were arranged at the least end of the range since they are generally repetition learning based. In any case, those apparatuses with an introduction towards recreations and open instruments spoke to a change towards more constructivist-arranged teaching pedagogies. So also, Kurz, Middleton, and Yanik (2005) distributed a scientific classification for school science programming in light of a device reason origination. It included four fundamental bunches, specifically, audit and practice, general, particular, condition and correspondence. These scientific classifications were intended to sort out instructive innovation, which did exclude portable learning applications. The previous add new points of view to the learning background, for example, touch screen affordances (Watlington, 2001) featuring the requirement for rethinking current frameworks.

In their classifications, Handal and Herrington (2003) and Kurz, Middleton, and Yanik (2005) recommend various criteria in the regions of educational affordances and operational affordances that rise as basic highlights of media extravagance in computerized undertakings. Such criteria speak to the ability of the application to encourage expanding levels of critical thinking and intellectual open-endedness. These assignment wealth highlights would include: (a) levels of intelligence between the student and the application, (b) ability to encourage profound learning, (c) student's control, engagement and criticism, (d) supporting instructional models, (e) acknowledgment of various learning styles and (f) open doors for coordinated effort (Handal and Herrington, 2003).

Roberts, Spencer, Vänskä, and Eskelinen (2015) inspected the difficulties of Nokia portable learning for arithmetic in South Africa by looking into the estimating impacts of these situations. Their investigation gave a record of the examination of the ponder take-up and usage of the Nokia Mobile Mathematics profited by 3,957 Grade 10 students. They measured the effect of the organization on the school arithmetic satisfaction of 1,950 of these students in excess of one scholastic session. Their investigation revealed that 21% of the Grade 10 science students purposefully and reliably made usage of this adaptable learning resource outside of instructive time, with little commitment from their instructors. They found that over the get-together of 1,950 students, there was a typical diminishing in arithmetic accomplishment from Grade 9 to Grade 10 of 15 rate centers.

Botzer and Yerushalmy (2007) explored the learning experiences inside a versatile getting the hang of setting, likewise investigated how socio-social and orchestrated learning edges are reflected in these experiences. The paper demonstrated a pilot logical examination wanted to take a gander at how socio-social and masterminded learning edges are reflected in learning experiences inside a novel versatile learning condition, Math4Mobile, a portable application for learning science. The relevant examination focused on four understudies in a number juggling systems course who were possessed with a numerical undertaking in perspective of the cell phone applications. They found that usage of the versatile condition redesigned the individuals' engagement in the showing of real circumstances and added to the planned exertion between individuals.

The MLAMs chosen could be used to improve arithmetic skills, for graphs' representation, geometrical objects calculations, algebra problem solving and mathematical programming. MLAMs chosen has to motivate the students and make mathematics as a subject more enjoyable and interactive than the ordinary teaching practices.

2.5 FAHP and ELECTRE I

Basaran and Haruna (2017) proposed a model defining both quality and user satisfaction used to evaluate MLAMs by utilizing the Fuzzy AHP and TOPSIS approaches. 11 criteria used were based on the technical and non-technical aspects specific to the mathematics applications. The technical aspects were culled from the ISO 9126 model of while then on-technical aspects were culled from the user satisfaction model used for the evaluation. The weight of each criterion derived was determined through using the Fuzzy AHP approach while the alternatives: MLAMs are ranked applying TOPSIS.

Kaya and Kahraman (2011) proposed a methodology that was carried out on a hybrid approach; fuzzy AHP–ELECTRE approach .The criteria weights were computed with the FAHP method, eight criteria were used in this study. In addition, fuzzy ELECTRE I was utilized to the alternatives. The study ended with an aggregate matrix to rank the alternatives finally.

Charilas et,al (2009) applied Fuzzy AHP and ELECTRE to cover the issue of a network selection where the network alternatives were ranked, utilized fuzzy numbers since the importance of criteria cannot be exactly defined to integrate subjective judgement in decision-making.

Samaras et. al (2014) assessed risk levels in three particular dams using AHP and ELECTRE 1 approach, these dams are the alternatives in this study as five criteria were specific to assessing these dams. Results showed that the first dam is the most exposed to risk, the third is the least exposed after utilizing AHP, and using ELECTRE I approach showed that the third dam is the least exposed to risk and the second dam the most. It was stated that AHP was used because it gives a hierarchical segmentation of a decision while the ELECTRE I method was used because this approach supports decision for the final selection of an alternative from all options.

Guo (2010) proposed an M-commerce partner selection method that uses a hybrid MCDM approach, AHP and ELECTRE 1.Used a set of 13 criteria and 5 m-commerce candidate partners where AHP determined the weight of the 13 criteria and ELECTRE I ranks the candidate partners.

CHAPTER 3

CONCEPTUAL FRAMEWORK

Here, what is discussed is a framework that was adopted for selecting and MLAMs. The adopted framework is based on two categories. First, it is the technical requirements, second, the pedagogical requirements of the application and this framework is not specific to only mobile learning applications for mathematics.

As it was described in the Introduction, there are many types of handheld devices and they have several requirements that should be suited for mobile learning, this study proceeds to develop an evaluation framework. This framework will contain criteria for evaluating various mobile learning for mathematics devices.

Only a couple of studies have been directed concentrating on quality measurements, including those proposed by the ISO 9126 measurements, to assess the nature of versatile applications. Some have concentrated on stretching out the ISO 9126 measurements to specific kinds of versatile applications (M-Learning, M-Commerce, MWeb, and so on. Parsons and Ryu (2006), while others have proposed new measurements for all classifications of portable applications to assess quality attributes, such as Usability Framework, Gafni (2008), one study in particular relates with a great deal to this study and it was carried out in 2017; Integrating FAHP and TOPSIS for technical and non-technical aspects evaluation of mobile learning applications for mathematics Basaran and Haruna (2017) it uses MCDM methods in evaluating MLAMs too.

For the framework, the selection of criteria is based on the technical and pedagogical requirements adapted from Economides (2008). The study presented and broke down four requirements' dimensions: economical, socio-cultural, technical and pedagogical. It also goes at length in discussing sub-characteristics of the four dimensions listed above and their relations to mobile learning applications. The technical and pedagogical requirements were chosen due to their importance to MLAMs in the context of this study and to avoid ambiguity during the evaluation.

This investigation embraces a structure to help evaluators; instructors manage the constraints of cell phones, and achieve an abnormal state of value in the product that works in portable learning situations. The structure is made out of two viewpoints, which incorporate two prerequisites, and is intended to recognize criteria that might be impacted by the impediments of MLAM and gives an arrangement of criteria that can be contemplated while assessing these applications.

3.1 Technical Requirements

Mobility refers "the ability to move or be moved freely and easily", its contain advantages that defend mobile learning.

Technical requirements because quality requirements must be considered. The MLAMs ought to fulfill the student's needs while upgrading the learning quality. MLAM run on smartphones and devices and are evaluated based on software quality characteristics. (Pocatilu & Boja 2009). Quality fuses the traits of the MLAs that bear on their ability to satisfy the understudy's needs. Awakened by the ISO/IEC 9126 standard for programming quality which we in like manner widen. By setting up and caring for rules, associations would more have the capacity to satisfactorily reengineer their learning redid to pass on great quality online education.

Therefore, for the Technical requirements, this study adopts some of the requirements put forward by Economides (2008) which was prompted. Eight requirements were considered in the study mentioned. For the purpose of this study, three of the requirements will be adopted to be combined with the pedagogical requirements to put forward the framework, the three selected were chosen due to their importance and relation to MLAM's requirements. They are; User interface, Reliability & Maintainability, and Efficiency & Performance. Each of the requirements have sub-characteristics that were adopted based off the judgment of the author.

User Interface: The requirements selected are explained below:
Usability, it ought to be seen effectively, learn, recollect and utilize its capacities. It ought to be basic and advantageous to work (e.g. least number of clicks to discover and show data, least

number of scrolls to show data). It should neither occupy nor intellectually over-burden the student. Alternatively, maybe, it ought to pull in the student's consideration and core interest.

Navigation & Orientation, it ought to be simple, basic and instinctive to explore. They ought to be exact and reliable. There ought to be elective methods for route (e.g. alternate routes) with an appropriate number of levels. It should offer numerous route and introduction offices (e.g. sitemap, record, next, past, home, exit, fix, re-try, history trail, forecast, spare, print).

Reliability and Maintainability: The requirements selected are explained below:

Error Free, the MLA task ought to be right and exact without any blunders (flaws). It ought to effectively detect, evaluate, and adjust as indicated by the mindfulness parameters. Its activity ought to be steady and comparative states ought to be dealt with comparably. For instance, students at a similar execution level ought to be taken evaluations at a similar trouble level.

Easiness of Installation, it ought to be easy and quick to setup on any appropriate device.

Easiness of Upgrade, it ought to be easy and quick to change and upgrade it.

• **Efficiency and Performance**: The requirements selected are explained below:
Responsiveness, its reaction to a change (student's activity, condition's change, and so on.) ought to be quick and suitable. For instance, when the framework recognizes that the student enters a particular territory a message is transmitted to him instantly. There ought to be no deferrals in reacting to the student's solicitations and correspondences. The student ought not to see any postponements. The deferral of preparing, putting away, and imparting information ought to be littler than the limit for proficient application sending. For instance, the deferral of transmitting voice ought to be littler than an edge. Likewise, the preparing unit ought to be used proficiently.

Energy Consumption, the MLA ought not to utilize a great deal of energy. Energy protections procedures might be connected to save energy when no movement is occurred. It is imperative to have long energy self-sufficiency in mobile learning.

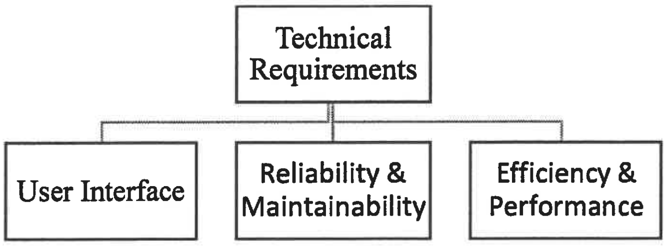

Figure 3.1: MLAM quality criteria model based on technical aspects

3.2 Pedagogical Requirements

Pedagogical requirements are important because incorporating criteria in the training angle MLAs that help such necessities may upgrade the learning and fulfillment of the understudies. They may expand the student's engagement, inspiration, learning, capability, and capacities. This is important and was selected as it consist of the activities of educating or basically the activities that impart knowledge as the alternatives in this study are for learning Mathematics. The Pedagogical requirements for this study are:

Content Quality, it ought to be legitimate, reliable, and exact. It ought to be founded on as of now adequate hypotheses that will get by for quite a while. It ought to be valuable and proper for the indented instructive targets, ages, and level of students. It should unbiasedly show an assortment of "perspectives" without separating concerning age, sexual orientation, race, religious, political thoughts and so on. It ought to give experiential open doors in view of logical cases. It ought to likewise propel the student.

Content Presentation, it ought to be founded on an assortment of media (e.g. content, picture, picture, charts, graphs, sound, video, and submersion) of high caliber (e.g. determination, number of hues, invigorate rate, sound constancy, mono or stereo sound). There ought to be the correct blend of media objects at the proper positions in the substance.

Content Organization, it ought to be basic, measured and adaptable. There ought to be an assortment of elective substance's associations for various students (e.g. successive versus arbitrary student) and circumstances. It ought to be anything but difficult to explore giving numerous route devices (e.g. Table, Map, Directories). Its structure ought to be natural, legitimate and proper for the students (e.g. age) and the instructive exercises. There ought to be few route levels (i.e. small tree depth) all together the student not to be lost. The sequencing among the modules ought to be appropriate and critical subjects ought to have need.

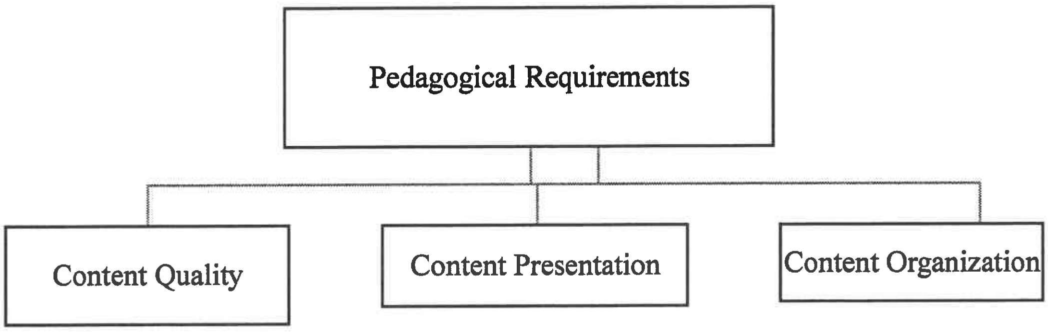

Figure 3.2: MLAM quality criteria model based pedagogical aspects

3.3 Selection and Evaluation Criteria for MLAM

The MCDM approach is dependent on criteria and alternatives to achieve its goal, for this reason, this study adopted its criteria based on the framework proposed by Economides (2008) which constitutes the technical and pedagogical requirements to achieve its aim. The criteria consist of ten criterion in total in which was used for selecting from the conflicting five alternatives. The following Table below shows the proposed criteria for this study

Table 3.1: Adopted MLAM Criteria

Label	Proposed Criteria	Corresponding Values
Technical Requirements (Economides, 2008)	Usability	C_1
	Navigation and Orientation	C_2
	Error Free	C_3
	Easiness of installation	C_4
	Easiness of upgrade	C_5
Pedagogical Requirements (Economides, 2008)	Responsiveness	C_6
	Energy Consumption	C_7
	Content Quality	C_8
	Content Presentation	C_9
	Content Organization	C_{10}

3.4 Using Fuzzy Sets

Fuzzy was introduced in this study to express the phonetic terms in the basic leadership process to determine uncertainty, unclearness of human judgment. Additionally, it is valuable just when information are fresh, managing an extremely lopsided size of judgment; vulnerability of human judgment does not assess common dialect; if positioning is somewhat loose; and the subjective judgment of observation, assessment, change, and choice in view of inclination of chiefs has incredible impact on the AHP comes about. To conquer such dubiousness and subjectivity of human judgment process, these set hypothesis have been presented.
Decision makers express their conclusions as far as linguistic scales. These scales are changed over into fuzzy numbers with the assistance of various enrollment capacities. At that point, it turns out to be anything but difficult to take care of MCDM issues. Along these lines, fuzzy set theory has turned into an accommodating device for systematizing human exercises with vulnerability-based data.

3.5 Using Fuzzy AHP

FAHP is a powerful method used to prioritize criteria and alternatives based on a pair-wise comparison of both. Used also due to its popularity as a method for software prioritization. Used also because from the literature it has been shown that it is a highly dependable since the great level of pair-wise comparison makes the process immune to comparison errors. Values assigned in AHP's pair-wise comparison are based on experience, intuition and real data.

3.6 Using ELECTRE I

This method was used due to its unique ranking ability only unlike the other kinds of ELECTRE that are used for selection only. It utilizes a satisfactory or selected set of alternatives, and works with concordance records. They measure the force of the contentions supporting the statement that activity (an) outranks activity (b). There is likewise a conflict list, that is, the amount or power of contradicted contentions inside the criteria under examination, which challenges the declaration that (an) outranks (b). This MCDM approach has a special capacity to point out the exact motives of a decision-maker suggest an appropriate result through its ranking.

CHAPTER 4

METHODOLOGY

This chapter explains in details what research methodology was used and explains the steps for the MCDM approaches selected for this study for this study.

4.1 Adopted Framework

The aim of this study is; firstly, employ and compare 3 different multi-criteria methods such as hybrid (FAHP and ELECTRE I) and 2 stand-alone methods FAHP and ELECTRE I) to evaluate and rank MLAMs, Secondly, to develop and propose an executable stand-alone computer application for evaluating 5 top rated MLAMs by using C# in Visual Studio with the outcomes of 1 hybrid and 2 stand-alone methods. The criteria were characterized in view of the technical and pedagogical prerequisites of the alternatives.

This study adopted criteria proposed by Economides (2008) in relation to MLAM, also criteria that the author is able to judge based on the level of expertise; technical requirements and pedagogical requirements were selected the former with sub criteria making it logical and right to use MCDM for the evaluation of the applications mentioned in the previous chapter. This does not nullify the importance of other criteria that was not adopted for the framework.

As a result, the study identifies with the following model for evaluating mobile learning applications for mathematics after several literature reviews of this subject. The figure below depicts the adopted model for evaluating mobile learning applications for mathematics.

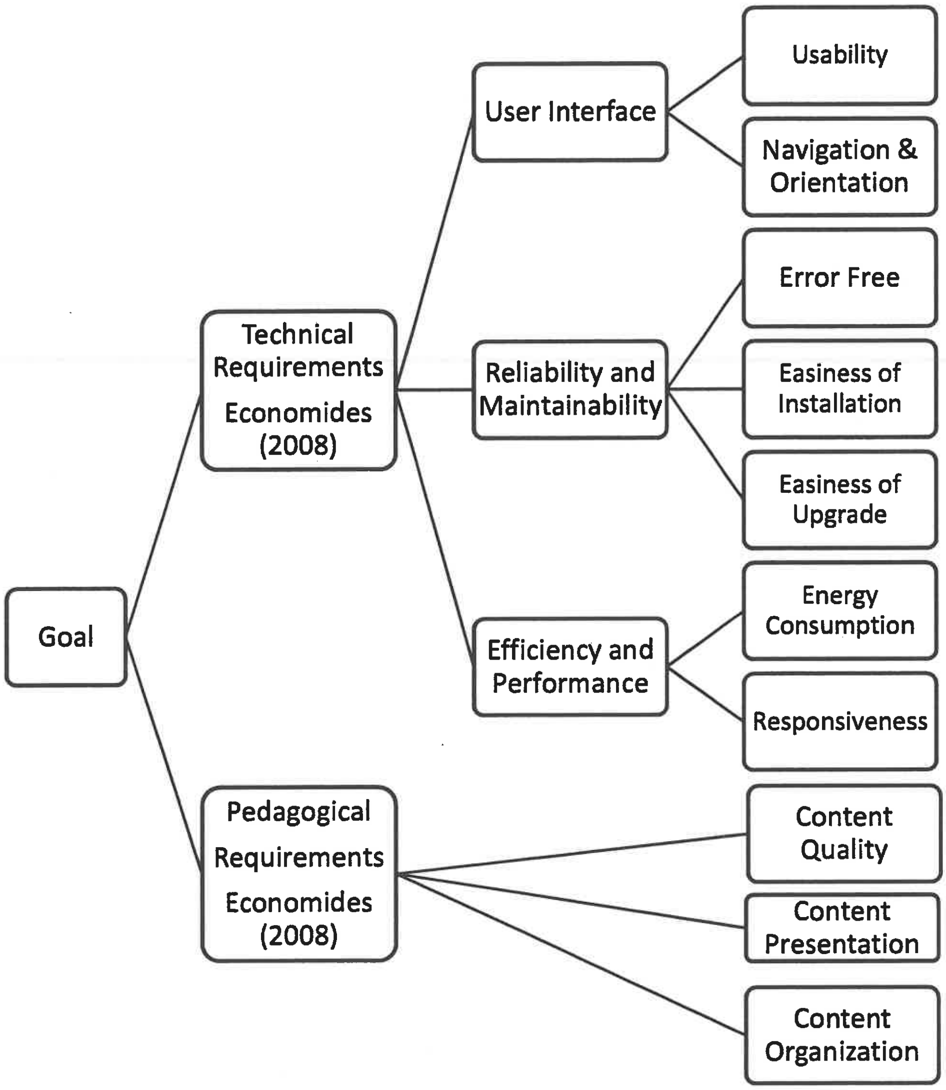

Figure 4.1: Framework for evaluation

4.2 Samples of Mobile Learning Applications for Mathematics

There are several MLAM's and they come in different forms depending on features like design, functionalities, purpose, limitations and target audience. This study targeted the Android applications only for its sample because it's open source and it is the most populous mobile application store. These applications are distributed digitally via the official Google Play store as this study focuses on the Android OS platform, which is either available freely or at cost. The

Google Play store host millions of Android apps of different categories such social, games, education, security etc. Users based on their experience also rate these apps. However, for this study five MLAMs for adults is selected as sampled applications for evaluation based on their respective Google App Store user rating of at least 4.0 out of 5. The Table 3.2 below shows their respective Google App Store ratings. Google play store ratings is selected because it displays the current store rating as well as version. Users can rate an application with stars from 1.0 to 5.0, 5.0 being the most elevated conceivable rating, and these appraisals are collected by Google Play per application to figure a general normal rating for every application. As mobile applications continue to grow rapidly and gain popularity, different platforms have been developed to create and allow users to download these applications. According to Appbrain (2017), 3.4 million applications in October 2017 are available for download and this application store gives users the ability to express opinions through reviews and ratings. A survey conducted in 2015 shows that 69% of the end users consider the application rating as a significant deciding factor when downloading an application. Subsequently, 77% of the end users will seldom download an application that has a rating that is below 3 stars (Martin, 2017). Hence, the success of an application is closely tied to the ratings and reviews that it receives.

Table 4.1: MLAMs Google Play Store ratings

MLAM	Rating (0-5)
Mathematics	4.1
Cymaths	4.5
MalMaths	4.6
MathsPapa	4.7
Maths42	4.7

4.2.1 Mathematics

The Mathematics application is a powerful mobile learning application used to learn and teach mathematics anytime, anywhere at the users fingertips. It eases learning mathematics by providing most of the formulas ranging from basic to complex within some few seconds. It covers the algebra field of mathematics, matrices, determinants and, vectors. It also comes with some features, which enables a student make the conversion of units, from one unit to another. The whole features combine to make learning mathematics much easier than in the classroom.

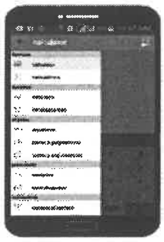

Figure 3.3: Mathematics Sample

4.2.2 Cymaths

The Cymaths is a mobile learning application built mainly for learning mathematics and solving some mathematical equations. The application has millions of users worldwide. The application uses an algorithm, which solves mathematical problems on the go ranging from calculus and algebra. The supported topics include equation solving, product rule, exponents, quotient rule, chain rule complex numbers, quadratic equations, trigonometry, partial fraction, factoring, logarithms, polynomial division, and u-substitution.

Figure 3.4: Cymaths Sample

4.2.3 MalMath

The MalMath is a mobile learning application used for learning mathematics with lots of users. It can also be used to solve a mathematical problem with systematic descriptions and displays some graphical view when needed. It can be used to solve some range of mathematical fields.

Figure 3.5: MalMath Sample

4.2.4 MathPapa

The MathPapa is a mobile application developed purposely to be used for learning mathematics and solve some mathematical problems; it has many users worldwide. The application has an inbuilt algorithm for solving mathematical equations systematically, such as systems of two equations, quadratic equations, graphical equations, linear equations with different methods. This application also supports working in an offline mode. The application help student learns mathematically in a systematic way.

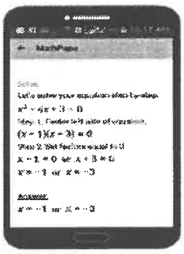

Figure 3.6: MathPapa Sample

4.2.5 Math42

Math42 is a mobile application developed purposely to help with intelligent approaches to the solution, systematic solutions of their problem, an Assessment Center. It solves different mathematical problems ranging from roots, derivation, sum rule, product rule, and quotient rule to integration, partial fraction, and many others.

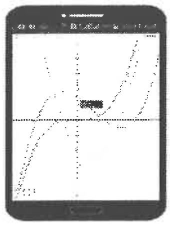

Figure 3.7: Math42 Sample

4.3 Triangular Fuzzy Number (TFN) - Fuzzy Set

Fuzzy is set of numbers whose quantity is not specific, it can also be addressed as an extension of the popular Boolean logic whose sets is not just 0 and 1 but a connection of different values where each and every value is assigned a weight. It can also be stated as a set of values ranging from one interval to another. Fuzzy gives more dynamism to expression. So far, many types of fuzzy numbers exist such as trapezoidal, octagonal, pyramid, triangular, pentagonal, diamond and hexagonal fuzzy numbers Pathinathan and Ponnivalavan (2015) thus they are used based on the certain situation at a place. However, Klir and Yuan (1995) noted that triangular fuzzy and trapezoidal fuzzy numbers are the majorly used. For this reason, this study adopts the triangular fuzzy number. This type of fuzzy number consists of the set of three real numbers ranging from minimum, most expected and maximum weights. The Figure below depicts the triangular fuzzy number with its three values, $a_{1,}a_2$ and a_3 .

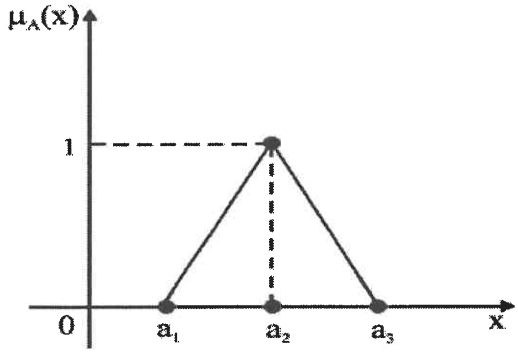

Figure 4.2: Triangular Fuzzy Number

4.4 Hybrid (FAHP & ELECTRE I) Method

To begin the hybrid method; FAHP starts to weight the criteria and ELECTRE I is used for the rankings. Saaty (1980) developed a new decision-making model called AHP in which presumably became one of the most popular decision-making methodologies. This method divides the problem into smaller chunks of a group in such a hierarchical form. However, the method encounters problems because it utilizes the exact values provided by the decision maker to express its intake of the alternatives by a pair-wise comparison. Then, Chang (1996) modified this approach by proposing TFN through utilizing comparisons in FAHP. Hence, during decision-making process the FAHP technique is utilized in calculating the weight of each criterion. However, the pairwise comparisons within the decision matrix are fuzzy numbers. The following steps were adopted in selecting the most appropriate MLAMs.

Step 1: The value of fuzzy artificial extent with regard to the i^{th} item is calculated as

$$S_i = \sum_{j=1}^{m} M_{gi}^{j} \oplus \left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} \tag{4.1}$$

To derive $\sum_{j=1}^{m} M_{gi}^{j}$, the fuzzy addition operation of m volume evaluation values for the certain matrix is done using

$$\sum_{j=1}^{m} M_{gi}^{j} = \left(\sum_{j=1}^{m} l_j , \sum_{j=1}^{m} m_j , \sum_{j=1}^{m} u_j\right) \tag{4.2}$$

And to acquire $\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]$, by performing the fuzzy addition operation of $M_{gi}^{j}(j = 1, 2, \ldots,)$ such that

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} = \left(\sum_{j=1}^{m} l_j , \sum_{j=1}^{m} m_j , \sum_{j=1}^{m} u_j\right) \tag{4.3}$$

In addition, $\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1}$ can be determined by the inverse of Eq. (3), as follows:

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} u_i}\right) \tag{4.4}$$

Step 2: as $M_1 = (l_1, m_1, u_1)$, and $M_2 = (l_2, m_2, u_2)$ are triangular fuzzy numbers, the degree of opportunity of $M_2 \geq M_1$ is defined as

$$V = (M_2 \geq M_1) = \underset{y \geq x}{\mathrm{SUP}}[\min(\mu_{M_1}(x), \mu_{M_2}(y))] \quad (4.5)$$

Also can be conveyed:

$$V(M_1 \geq M_2) = hgt(M_1 \cap M_2) = \mu_{M_1}(d) = \begin{cases} 1 & if\ m_1 \geq m_2 \\ 0 & if\ l_2 \geq u_1 \\ & \dfrac{(l_2 - u_1)}{(m_1 - u_1) - (m_2 - l_2)} \end{cases} \quad (4.6)$$

In which d, as shown inside the parent underneath, is the ordinate of the best intersection factor D between $\mu(M_1)$ and $\mu(M_2)$. To compare $M_1 = (l_1, m_1, u_1)$, and $M_2 = (l_2, m_2, u_2)$, we want both the values of $V (M_1 \geq M_2)$ and $V (M_2 \geq M_1)$.

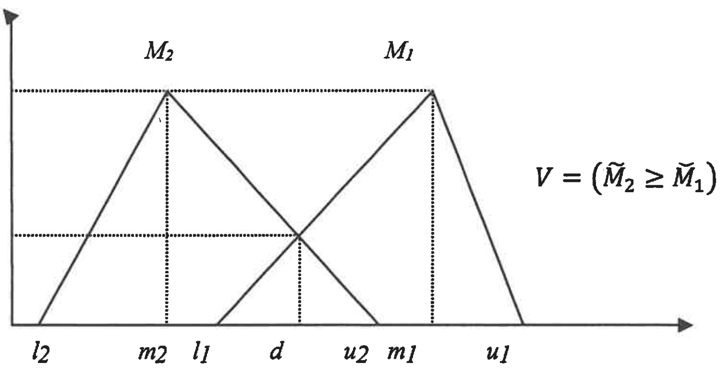

Figure 4.3: Intersection between M_1 and M_2

Step 3: The level of possibility for a convex fuzzy variety to be more than *k* convex M_i (i= 1, 2... k) numbers may be described by way of

V ($M \geq M_1, M_2 \ldots M_k$) $= v\ [M \geq M_1$ *and* $M \geq M_2$ *and* $M \geq M_k] = \min v\ M \geq M_1$, *i*=1, 2... *k* (4.7)
expect that

d' (A_i)=min*V* ($S_i \geq S_k$) *for* *k*=1,2,...,*n* ;*k*≠*i* Then the weight vector is;

$$W' = (d'(A_1), d'(A'_2 \ldots d'(A_n))^T \quad (4.8)$$

Where A_i (i=1, 2... n) are n elements.

Step 4: after normalization, the weight vectors are;

$$w = (d\,(A_1),\, d\,(A_2) \ldots d\,(A_n))^T \tag{4.9}$$

Where w is a non- fuzzy number

Benayoun Roy et al. (1966) formulated ELECTRE firstly, Firstly presented ELECTRE I upper-score have become unique almost about the choice systems they contained irrespective of whether or not they're making use of weight statistics or no longer and the results uncovered and were named as ELECTRE II, III, IV and ELECTRE TRI. ELECTRE I is chosen to solve the problem of application selection and evaluation for decision makers because it could improve the decision-making process by making it more realistic by firstly considering both quantitative and qualitative criteria, secondly taking criteria importance into the decision-making process, and fourthly validating concordance and discordance simultaneously.

The fundamental guideline of the ELECTRE technique is to decide the predominance relations by directing combined correlations between the options independently for every rule. The steps of the ELECTRE method are listen then mathematically explained; Preparation of Decision Matrix, Calculation of the standardized choice grid, Calculate the weighted standardized choice network, determine the concordance and discordance units, Calculate the concordance, Calculate the discordance, Make calculations of gain.

Step 5: Decision Matrix In decision matrix columns will give you the criteria (n) and rows will state the alternatives (m). This will be the standard matrix for determining the grounds of the process.

$$A_{ij} = \begin{bmatrix} r_{11} & r_{12} \cdots & r_{1n} \\ r_{21} & r_{22} \cdots & r_{2n} \\ \\ \cdots & \cdots \cdots \cdots & \cdots \\ r_{m1} & r_{m2} \cdots & r_{mn} \end{bmatrix} \tag{4.10}$$

Step 6: Calculating the normalized decision matrix. Decision matrix is be normalized with the below formula and will give us normalized decision matrix.

$$x_{ij} = \frac{r_{ij}}{\sqrt{\sum_{i=1}^{n} r_{ij}^2}} \qquad i = 1,2 \ldots \ldots \ldots m \;\; j = 1,2 \ldots \ldots . n \tag{4.11}$$

And the following equation for the criteria;

$$x_{ij} = \frac{\frac{1}{r_{ij}}}{\sqrt{\sum_{i=1}^{n}\left(\frac{1}{r_{ij}}\right)^2}} \qquad i = 1,2 \ldots\ldots\ldots m \quad j = 1,2 \ldots\ldots.n \tag{4.12}$$

Based on the calculated normalized decision matrix X is showed as;

$$X_{ij} = \begin{bmatrix} x_{11} & x_{12} \cdots & x_{1n} \\ x_{21} & x_{22} \cdots & x_{2n} \\ \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ x_{m1} & x_{m2} \cdots & x_{mn} \end{bmatrix} \tag{4.13}$$

Step 7: Calculate weighted normalized choice matrix, weight determined by AHP process above is multiplied by the normalized matrix and weighted normalized matrix is now available.

$$v_{ij} = w_j \,.\, x_{ij} \tag{4.14}$$

It is proven that matrix W is a diagonal matrix which values of its major diameter is W and rest values are 0.

$$W = \begin{bmatrix} w_1 & \cdots & 0 \\ \cdots & w_2 & \cdots \\ \\ 0 & \cdots & w_n \\ \ldots & \ldots\ldots & \ldots. \end{bmatrix} \tag{4.15}$$

Step 8: Decide the concordance and discordance sets. The net weighted normalized matrix information is as compared for each pair and consequences are evaluated as beneath: If the alternative is higher than or identical to other element of pair it's far considered beneath concordance set and is described by way of C.

$$C\,(p,q) = \{j,\ v_{pj} \geq v_{qj}\} \tag{4.16}$$

If alternative is worse than the other element of the pair for relevant criteria it's far considered beneath discordance set and described with the aid of D.

$$D\,(p,q) = \{j,\ v_{pj} < v_{qj}\} \tag{4.17}$$

Step 9: Concordance matrix is the matrix generated by adding the values of weights of Concordance set elements.

$$C_{pq} \;=\; \sum_{j^*} w_{j^*} \qquad (4.18)$$

Step 10: Discordance matrix is prepared by dividing discordance set members values to the total value of the whole set.

$$Dpq = \frac{\Sigma_{j+}|v_{pj}+-v_{qj}+|}{\Sigma_j|v_{pj}-v_{qj}|} \qquad (4.19)$$

Step 11: Outranking relations between alternatives are determined. A_p outranks A_q when $Cpq \geq C$ and $D\,pq \leq$ D where C and D are the averages of C_{pq} and D_{pq} respectively.

Step 12: To determine the global matrix, calculate the intersection of the matrix F and G. The elements of this matrix are described:

$$h_{k/} = f_{pq} \cdot g_{pq} \qquad (4.20)$$

4.5 FAHP Method

Step 1: The value of fuzzy artificial extent with regard to the i^{th} item is calculated as

$$S_i = \sum_{j=1}^{m} M_{gi}^{j} \oplus \left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1}$$

To derive $\sum_{j=1}^{m} M_{gi}^{j}$, the fuzzy addition operation of m volume evaluation values for the certain matrix is done using

$$\sum_{j=1}^{m} M_{gi}^{j} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right)$$

And to acquire $\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]$, by performing the fuzzy addition operation of $M_{gi}^{j}(j = 1, 2, \dots,)$ such that

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right)$$

In addition, $\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1}$ can be determined by the inverse of Eq. (3), as follows:

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} u_i}\right)$$

Step 2: as $M_1 = (l_1, m_1, u_1)$, and $M_2 = (l_2, m_2, u_2)$ are triangular fuzzy numbers, the degree of opportunity of $M_2 \geq M_1$ is defined as

$$V = (M_2 \geq M_1) = \sup_{y \geq x}\left[\min(\mu_{M_1}(x), \mu_{M_2}(y))\right]$$

Also can be conveyed:

$$V(M_1 \geq M_2) = hgt(M_1 \cap M_2) = \mu_{M_1}(d) = \begin{cases} 1 & if\ m_1 \geq m_2 \\ 0 & if\ l_2 \geq u_1 \\ \dfrac{(l_2 - u_1)}{(m_1 - u_1) - (m_2 - l_2)} & \end{cases}$$

In which d, as shown inside the parent underneath, is the ordinate of the best intersection factor D between $\mu(M_1)$ and $\mu(M_2)$. To compare $M_1 = (l_1, m_1, u_1)$, and $M_2 = (l_2, m_2, u_2)$, we want both the values of $V\ (M_1 \geq M_2)$ and $V\ (M_2 \geq M_1)$.

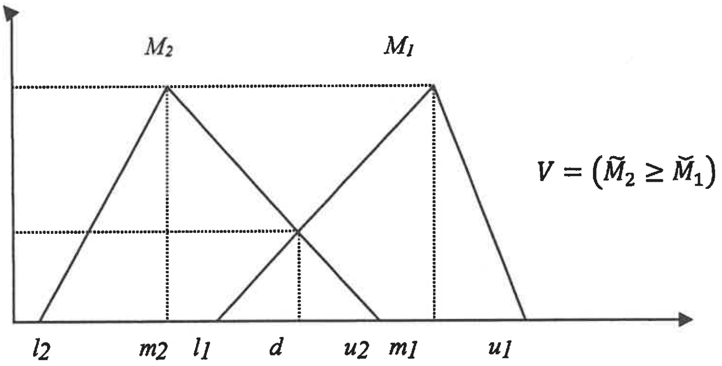

Figure 4.4: Intersection between M_1 and M_2

Step 3: The level of possibility for a convex fuzzy variety to be more than *k* convex M_i (i= 1, 2... k) numbers may be described by way of

$V\ (M \geq M_1, M_2 \ldots M_k) = v\ [M \geq M_1\ and\ M \geq M_2\ and\ M \geq M_k]$

$= \min v\ M \geq M_1,\ i = 1, 2 \ldots k$

Expect that

d' (A_i)=min$V$ ($S_i \geq S_k$) for k=1,2,…,n ;$k \neq i$ Then the weight vector is;

$W' = (d'(A_1), d'(A'_2 \ldots d'(A_n))^T$

Where A_i (i=1, 2… n) are n elements.

Step 4: after normalization, the weight vectors are;

$w = (d(A_1), d(A_2) \ldots d(A_n))^T$

Where w is a non- fuzzy number

Step 5: Combine the entire top values and the lower values one at a time, after which divide them by the overall sum of α price:

$(x_{ij}) = (a_{ij}.b_{ij}.c_{ij})$

$l_{ij} = \min\{a_{ijk}\}$, $m_{ij} = \frac{1}{K}\sum_{k=1}^{K} b_{ijk'}$ $u_{ij} = \max\{d_{ijk}\}$

4.6 ELECTRE I

The ELECTRE I approach follows several steps and those steps are describes as beneath. Think that our problem has a alternatives $E1, E2, E3, \ldots, Ea$ and b decision criteria $F1, F2, F$three,…,Fb . Then, every alternative is evaluated with appreciate to those b criteria.

Step 1: firstly, K wide variety of choice makers are shaped within the choice-making method $D1, D2$, three… DK. DMs evaluate the weight of every criterion with a linguistic variable. Then, those linguistic variables are transformed into fuzzy numbers (l, m, u). For k=1, 2… K and j=1, 2… b and the aggregated fuzzy importance weights can be decided follows;

$$\alpha_j^l = \min\{\gamma_{jk}\}\alpha_j^m = \frac{1}{K}\sum_{k=1}^{K}\gamma_{jk}\ \alpha_j^u = \max_k\{\gamma_{jk}\}$$

After the calculation of weights, the aggregated fuzzy significance weights are normalized as;

$\tilde{w} = (w_j^l, w_j^{\mathrm{m}}, w_j^u)$

Where,

$$w_j^l = \frac{1/\alpha_j^l}{\Sigma_{j=1}^n 1/\alpha_j^l} w_j^m = \frac{1/\alpha_j^m}{\Sigma_{j=1}^n 1/\alpha_j^m} w_j^u = \frac{1/\alpha_j^u}{\Sigma_{j=1}^n 1/\alpha_j^u}$$

Finally, the matrix of normalized aggregated fuzzy importance weight matrix is constructed as

$$\overline{W} = [\widetilde{w}_1, \widetilde{w}_2, \dots, \widetilde{w}_b]$$

Step 2: A selection matrix denoted with the aid of $X= (xij)$ axb is fashioned with respect to every criterion

$$x = \begin{vmatrix} x_{11} & x_{1b} \\ \vdots & \vdots \\ x_{a1} & x_{ab} \end{vmatrix}$$

Step 3: The decision matrix $R= (rij)$ axb is normalized by way of the use of calculating rij, which represents the normalized value of criteria.

$$r_{ij} = \frac{x_{ij}}{\sqrt{\Sigma_{i--1}^{a} x_{ij}^2}}$$

$$R = \begin{vmatrix} r_{11} & r_{1n} \\ \vdots & \vdots \\ r_{m1} & r_{mn} \end{vmatrix}$$

Step 4: Seeing that each criterion has a exclusive weight, the weighted normalized decision matrix is formed by way of multiplying the importance weights of criteria and the values inside the normalized fuzzy selection matrix. . $V = (v_{ij})_{axb}$ for i=1,2,...,a and j=1,2,...,b where $v_{ij} = r_{ij}\ x\ \widetilde{w}_j$

$$V^l = \begin{vmatrix} v_{11}^l & v_{1b}^l \\ \vdots & \vdots \\ v_{a1}^l & v_{ab}^l \end{vmatrix} \quad V^m = \begin{vmatrix} v_{11}^m & v_{1b}^m \\ \vdots & \vdots \\ v_{a1}^m & v_{ab}^m \end{vmatrix} V^u = \begin{vmatrix} v_{11}^u & v_{1b}^u \\ \vdots & \vdots \\ v_{a1}^u & v_{ab}^u \end{vmatrix}$$

Step 5: On this step, the concordance indices and sets are calculated with the use of the weighted normalized fuzzy choice matrix and pairwise assessment most of the options, respectively. If p and q are two options, the concordance index Cpq represents the pairwise contrast between p and $_q$ ($A_p \rightarrow A_q$). C_{pq} is the gathering of attributes where Ap is higher than or equal to Aq.

$$C_{pq}^{l} = \sum_{j^+}^{-} w_j^l \; C_{pq}^{m} = \sum_{j^+}^{-} w_j^m \; C_{pq}^{u} = \sum_{j^+}^{-} w_j^u$$

Where j^+ are attributes contained inside the concordance set Cpq.

Step 6: The discordance indices mean the variances in judgement between alternatives p and q ($Ap \longrightarrow Aq$). Dpq Represents that Ap is worse than or equal to Aq. The discordance indices are calculated as;

$$D_{pq}^{l} = \frac{\Sigma_{j^+} |v_{pj^+}^{l} - v_{qj^+}^{l}|}{\Sigma_j |v_{pj}^{l} - v_{qj}^{l}|} \; D_{pq}^{m} = \frac{\Sigma_{j^+} |v_{pj^+}^{m} - v_{qj^+}^{m}|}{\Sigma_j |v_{pj}^{m} - v_{qj}^{m}|} \; D_{pq}^{u} = \frac{\Sigma_{j^+} |v_{pj^+}^{u} - v_{qj^+}^{u}|}{\Sigma_j |v_{pj}^{u} - v_{qj}^{u}|}$$

Where j^+ are attributes contained inside the concordance set D_{pq}.

Step 7: The very last concordance and discordance indices are computed as follows:

$$C_{pq}^{*} = \sqrt{\prod_{z=1}^{Z} C_{pq}^{Z}}$$

$$D_{pq}^{*} = \sqrt{\prod_{z=1}^{Z} D_{pq}^{Z}}$$

Where $Z = 3$

Step 8: In the end, net concordance and discordance indices are calculated to decide high-quality alternative. Alternative with the minimal net concordance index and most internet discordance index is the satisfactory alternative among all alternatives.

$$\tilde{C}_i = \sum_{i=1}^{a} C_{pq} - \sum_{i=1}^{a} C_{qp} \; \overline{D} = \sum_{i=1}^{a} D_{pq} - \sum_{i=1}^{a} D_{qp}$$

4.7 Ranking Method

This study integrates two MCDM techniques FAHP and ELECTRE I due to the nature of its aim to select and evaluate MLAMs. The FAHP method is the most widely used MCDM technique but it comes with some disadvantages that make the values differ based on the form of hierarchy structure and also maintaining consistency is problematic amongst responses (Chalúpková, E., & Franek, 2014). As a result, the study omitted the final part of the FAHP method, which is the alternatives and integrated the ELECTRE method to produce the final ranking. Furthermore, the verdict to combine two MCDM techniques was also to improve the evaluation and because the two methods are usually combined in complex, decision issues (Yang et al., 2007; Chen & Tzeng, 2004). Accordingly, this approach makes a study of the proposed model's practical behavior based on the assumption of initial priorities and values from the decision maker. Subsequently, the MLAMs were ranked based on the proposed combined eleven criteria. Hence, for this study, five MLAMs were sampled and the author is the decision maker.

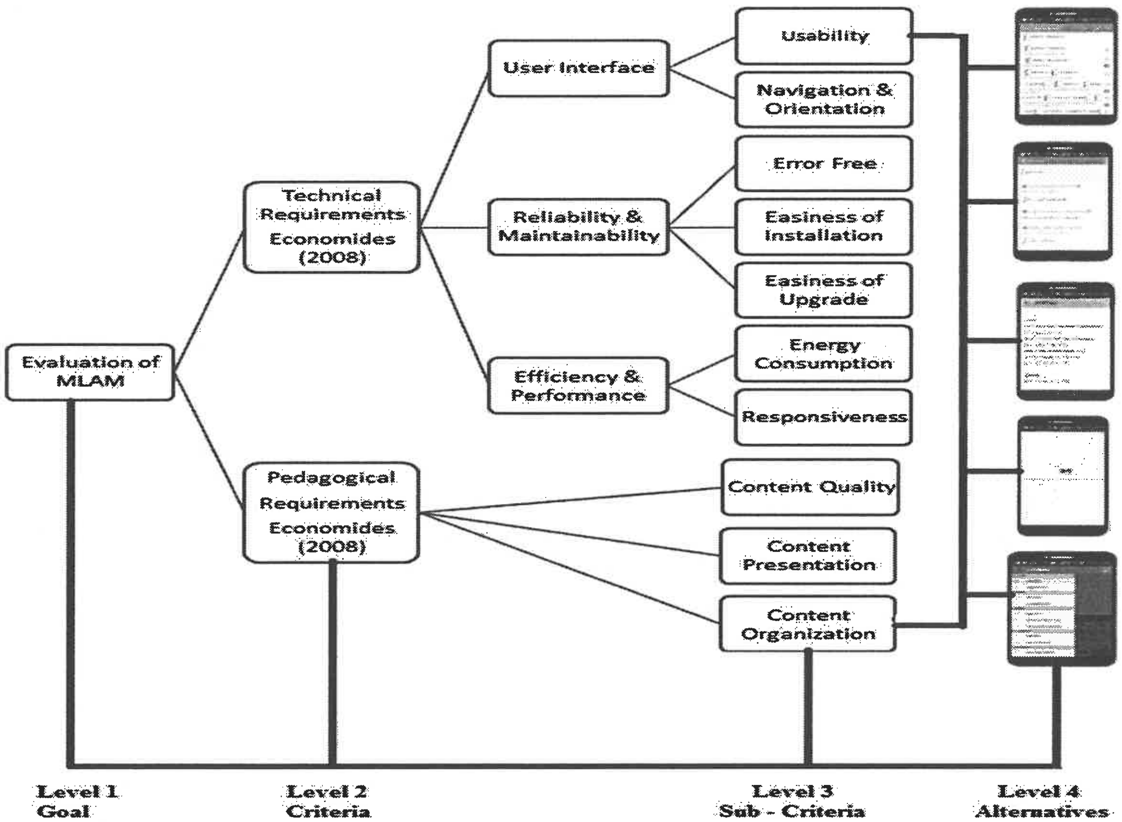

Figure 4.5: Structure for decision-making challenges for MLAMs

The figure above shows the entities involved for the evaluation, from the alternatives to the criteria.

4.8 Research Procedure

The study was conducted in accordance to the following steps in a chronological manner:

1. Literature review was conducted based on the related research subject area, to investigate why, how previous studies were conducted, and the findings of previous literature and to find out the study lapses. This provided a guide of how the study was conducted.
2. A conceptualize framework was adopted for assessing the quality of MLAM in accordance with the result of the literature review and some MLAM were sampled for the study.
3. Later on, a model based on technical and pedagogical was adopted with the ten criteria for evaluating the selected samples MLAM and five alternatives were selected.
4. The ranking of the five sampled MLAM were conducted using FAHP and ELECTRE I methods.
5. Developing a software application for the

4.9 Research Schedule

This study was conducted from September 2017 to March 2018. Figure 4.6 describes each task and the duration it takes during the research, while and Figure 4.7 is the Gantt chart for the thesis.

Table 4.2: Research schedule of the study

Task	Duration
Identify research area	7days
Related research	13 days
Writing thesis proposal	5 days
Proposal Review	4 days
Thesis approval	2 days
Writing Thesis	30 days
Thesis Review	14 days
Temporary Thesis Draft	5 days
Analysis of SmartRank	10 days
Design	9 days
Implementation	14 days
Reading, discussion and correction based on feedback of Supervisor	14 days
Last Revision of Thesis	10 days

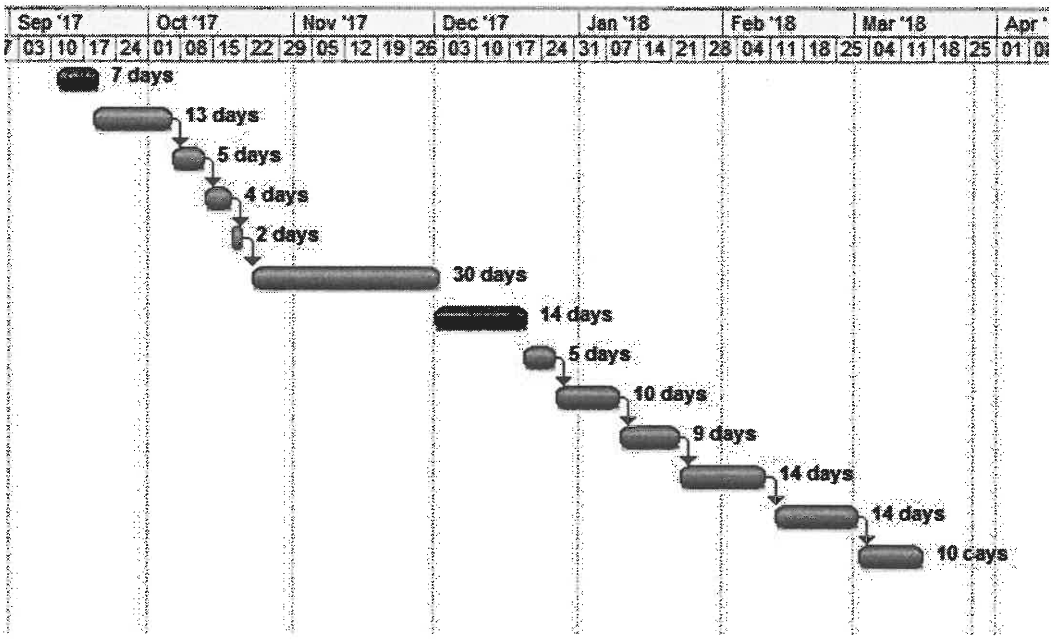

Figure 4.6: Gantt chart

CHAPTER 5

RESULTS AND DISCUSSION

The first part of this chapter is related to the results of the steps followed in hybrid (FAHP& ELECTRE I), FAHP and ELECTRE I separately. The second part involves the proposed stand-alone application of these three procedures.

5.1 Results of Hybrid (FAHP &ELECTRE I) in Ranking MLAMs

The following illustrates the selection and evaluation of MLAM based on an actual experiment. The conducted experiment was implemented on five MLAMs using triangular fuzzy numbers (TFN). To start FAHP method, a pairwise matrix for correlation was made utilizing a relating fuzzy scale. The Table beneath characterizes the proposed fuzzy scale with its relating TFN and TFN^{-1}. This is **Step 1** from the equation.

Table 5.1: Linguistic scale with its corresponding TFN (Brajković et al., 2015)

Linguistic Scale	Description	TFN	TFN^{-1}
Same significance	At the point when two criteria commitment to an objective is same.	(1, 1, 1)	(1, 1, 1)
Average significance	When one criterion slightly presides the other criterion's contribution to a goal.	(0.33, 0.5, 1)	(1, 2, 3)
Significant	When one criterion clearly presides the other criterion's contribution to a goal.	(0.75, 1, 1.25)	(0.8, 1, 1.33)
Very significant	When one criterion presides the other criterion's contribution to a goal	(1, 2, 3)	(0.33, 0.5, 1)
Very strong significant	When one criterion dominates the other criterion's contribution to a goal	(1.33, 2, 4)	(0.25, 0.5, 0.75)

Table 5.2: Developed Pair-wise Fuzzy Decision Matrix

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	
C_1	(1, 1, 1)	(0.75, 1, 1.25)	(1, 1, 1)	(1, 2, 3)	(1, 2, 3)	(0.75, 1, 1.25)	(1, 1, 1)	(0.33, 0.5, 1)	(0.33, 0.5, 1)	(0.33, 0.5, 1)	C_1
C_2	(0.8, 1, 1.33)	(1, 1, 1)	(1, 1, 1)	(0.75, 1, 1.25)	(0.75, 1, 1.25)	(1, 2, 3)	(0.33, 0.5, 1)	(0.33, 0.5, 1)	(1, 1, 1)	(1, 1, 1)	C_2
C_3	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	(1, 2, 3)	(1, 2, 3)	(0.75, 1, 1.25)	(0.33, 0.5, 1)	(1, 1, 1)	(0.75, 1, 1.25)	(0.75, 1, 1.25)	C_3
C_4	(0.33, 0.5, 1)	(0.8, 1, 1.33)	(0.33, 0.5, 1)	(1, 1, 1)	(1, 1, 1)	(0.75, 1, 1)	(0.33, 0.5, 1)	(0.33, 0.5, 1)	(0.33, 0.5, 1)	(0.33, 0.5, 1)	C_4
C_5	(0.33, 0.5, 1)	(0.8, 1, 1.33)	(0.33, 0.5, 1)	(1, 1, 1)	(1, 1, 1)	(0.33, 0.5, 1)	(0.33, 0.5, 1)	(0.33, 0.5, 1)	(0.33, 0.5, 1)	(0.33, 0.5, 1)	C_5
C_6	(0.8, 1, 1.33)	(0.8, 1, 1.33)	(0.8, 1, 1.33)	(1, 2, 3)	(1, 2, 3)	(1, 1, 1)	(0.33, 0.5, 1)	(0.33, 0.5, 1)	(0.33, 0.5, 1)	(0.33, 0.5, 1)	C_6
C_7	(1, 1, 1)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	C_7
C_8	(1, 2, 3)	(1, 2, 3)	(1, 1, 1)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(1, 1, 1)	(1, 1, 1)	(0.75, 1, 1.25)	(0.75, 1, 1.25)	C_8
C_9	(1, 2, 3)	(1, 1, 1)	(0.8, 1, 1.33)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(1, 1, 1)	(0.8, 1, 1.33)	(1, 1, 1)	(1, 1, 1.)	C_9
C_{10}	(1, 2, 3)	(1, 1, 1)	(0.8, 1, 1.33)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(1, 1, 1)	(0.8, 1, 1.33)	(1, 1, 1)	(1, 1, 1)	C_{10}
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	

The Table 5.2 above is the pair-wise fuzzy matrix of the criteria selected, the matrix was cross-checked by the supervisor with a Mathematical background as well for correct judgment. A hierarchy structure is then developed in a form of a fuzzy evaluation matrix.

Table 5.3: Addition of rows and columns

Criteria	Rows Sum	Columns Sum
C_1	(7.49, 10.5, 14.5)	(8.26, 12, 16.66)
C_2	(7.96, 10, 12.83)	(9.15, 12, 15.24)
C_3	(8.58, 11.5, 14.75)	(8.06, 10, 12.99)
C_4	(5.53, 7, 10.33)	(9.75, 17, 24.25)
C_5	(5.11, 6.5, 10.33)	(9.75, 17, 24.25)
C_6	(6.72, 10, 14.99)	(8.58, 14.5, 20.5)
C_7	(10, 15, 20)	(6.65, 7.5, 10)
C_8	(9.5, 15, 20.5)	(6.25, 7.5, 10.66)
C_9	(9.6, 14, 18.66)	(6.82, 8, 10.5)
C_{10}	(9.6, 14. 18.66)	(6.82, 8, 10.5)
	Sum of rows or columns	(80.09, 113.5, 155.55)

After forming the fuzzy pairwise comparison matrix, FAHP determines weight of each criterion. However, in respect to FAHP method, the synthesis extent values should be determined. The following shows the calculations of all the synthesis extent values which is the **Step 2**.

SC_1= (7.49, 10.5, 14.5) $\otimes$ (80.09, 113.5, 155.55)$^{-1}$ = (0.0482, 0.0925, 0.1810)

SC_2= (7.96, 10, 12.83)$\otimes$ (80.09, 113.5, 155.55)$^{-1}$ = (0.0512, 0.0881, 0.1602)

SC_3= (8.58, 11.5, 14.75)$\otimes$ (80.09, 113.5, 155.55)$^{-1}$ = (0.0552, 0.1013, 0.1842)

SC_4= (5.53, 7, 10.33)$\otimes$ (80.09, 113.5, 155.55)$^{-1}$ = (0.0356, 0.0617, 0.1290)

SC_5= (5.11, 6.5, 10.33) $\otimes$ (80.09, 113.5, 155.55)$^{-1}$ = (0.0329, 0.0573, 0.1290)

SC_6= (6.72, 10, 14.99) $\otimes$ (80.09, 113.5, 155.55)$^{-1}$ = (0.0432, 0.0881, 0.1872)

SC_7= (10, 15, 20) ⊗ $(80.09, 113.5, 155.55)^{-1}$ = (0.0643, 0.1322, 0.2497)

SC_8= (9.5, 15, 20.5) ⊗ $(80.09, 113.5, 155.55)^{-1}$ = (0.0611, 0.1322, 0.2560)

SC_9= (9.6, 14, 18.66) ⊗ $(80.09, 113.5, 155.55)^{-1}$ = (0.0617, 0.1233, 0.2330)

SC_{10}= (9.6, 14, 18.66) ⊗ $(80.09, 113.5, 155.55)^{-1}$ = (0.0617, 0.1233, 0.2330)

Using **Step 3** from the equation of the Hybrid method, to find the highest intersection value.

Table 5.4: The result of the synthesis extent values of each criterion

Criteria	SC_i
C_1	(0.0482, 0.0925, 0.1810)
C_2	(0.0512, 0.0881, 0.1602)
C_3	(0.0552, 0.1013, 0.1842)
C_4	(0.0356, 0.0617, 0.1290)
C_5	(0.0329, 0.0573, 0.1290)
C_6	(0.0432, 0.0881, 0.1872)
C_7	(0.0643, 0.1322, 0.2497)
C_8	(0.0611, 0.1322, 0.2560)
C_9	(0.0617, 0.1233, 0.2330)
C_{10}	(0.0617, 0.1233, 0.2330)

$V(SC_1 \geq SC_2) = 1$, $V(SC_1 \geq SC_3) = 0.9346$, $V(SC_1 \geq SC_4) = 1$, $V(SC_1 \geq SC_5) = 1$, $V(SC_1 \geq SC_6) = 1$, $V(SC_1 \geq SC_7) = 0.7461$, $V(SC_1 \geq SC_8) = 0.7512$, $V(SC_1 \geq SC_9) = 0.7948$, $V(SC_1 \geq SC_{10}) = 0.7948$,

$d'(C_1) = \min(1, 0.9346, 1, 1, 1, 0.7461, 0.7512, 0.7948, 0.7948) = 0.7461$

$V(SC_2 \geq SC_1) = 0.9621$, $V(SC_2 \geq SC_3) = 0.8883$, $V(SC_2 \geq SC_4) = 1$, $V(SC_2 \geq SC_5) = 1$, $V(SC_2 \geq SC_6) = 1$, $V(SC_2 \geq SC_7) = 0.685$, $V(SC_2 \geq SC_8) = 0.6920$, $V(SC_2 \geq SC_9) = 0.7378$, $V(SC_2 \geq SC_{10}) = 0.7378$,

$d'(C_2) = \min(0.9621, 0.8883, 1, 1, 1, 0.685, 0.6920, 0.7378, 0.7378) = 0.685$

$V(SC_3 \geq SC_1) = 1$, $V(SC_3 \geq SC_2) = 1$, $V(SC_3 \geq SC_4) = 1$, $V(SC_3 \geq SC_5) = 1$, $V(SC_3 \geq SC_6) = 1$, $V(SC_3 \geq SC_7) = 0.7951$, $V(SC_3 \geq SC_8) = 0.7994$, $V(SC_3 \geq SC_9) = 0.8477$, $V(SC_3 \geq SC_{10} = 0.8477$,

$d'(C_3) = \min(1, 1, 1, 1, 1, 0.7951, 0.7994, 0.8477, 0.8477) = 0.7951$

V ($SC_4 \geq SC_1$) = 0.7240, V ($SC_4 \geq SC_2$) = 0.7466, V ($SC_4 \geq SC_3$) = 0.6507, V ($SC_4 \geq SC_5$) = 1, V ($SC_4 \geq SC_6$) = 0.7647, V ($SC_4 \geq SC_7$) = 0.4785, V ($SC_4 \geq SC_8$) = 0.4906, V ($SC_4 \geq SC_9$) = 0.5221, V ($SC_4 \geq SC_{10}$) = 0.5221,

$d'(C_4)$ = min (0.7240, 0.7466, 0.6507, 1, 0.7647, 0.4785, 0.4906, 0.5221, 0.5221) = 0.4785

V ($SC_5 \geq SC_1$) = 0.6965, V ($SC_5 \geq SC_2$) = 0.7163, V ($SC_5 \geq SC_3$) = 0.6264, V ($SC_5 \geq SC_4$) = 0.9550, V ($SC_5 \geq SC_6$) = 0.7358, V ($SC_5 \geq SC_7$) = 0.4634, V ($SC_5 \geq SC_8$) = 0.4754, V ($SC_5 \geq SC_9$) = 0.5048, V ($SC_5 \geq SC_{10}$) = 0.5048,

$d'(C_5)$ = min (0.6965, 0.7163, 0.6264, 0.9550, 0.7358, 0.4634, 0.4754, 0.5048, 0.5048) = 0.4634

V ($SC_6 \geq SC_1$) = 0.9693, V ($SC_6 \geq SC_2$) = 1, V ($SC_6 \geq SC_3$) = 0.9090, V ($SC_6 \geq SC_4$) = 1, V ($SC_6 \geq SC_5$) = 1, V ($SC_6 \geq SC_7$) = 0.7359, V ($SC_6 \geq SC_8$) = 0.7408, V ($SC_6 \geq SC_9$) = 0.7810, V ($SC_6 \geq SC_{10}$) = 0.7810,

$d'(C_6)$ = min (0.9693, 1, 0.9090, 1, 1, 0.7359, 0.7408, 0.7809, 0.7809) = 0.7359

V ($SC_7 \geq SC_1$) = 1, V ($SC_7 \geq SC_2$) = 1, V ($SC_7 \geq SC_3$) = 1, V ($SC_7 \geq SC_4$) = 1, V ($SC_7 \geq SC_5$) = 1, V ($SC_7 \geq SC_6$) = 1, V ($SC_7 \geq SC_8$) = 1, V ($SC_7 \geq SC_9$) = 1, V ($SC_7 \geq SC_{10}$) = 1

$d'(C_7)$ = min (1, 1, 1, 1, 1, 1, 1, 1, 1) = 1

V ($SC_8 \geq SC_1$) = 1, V ($SC_8 \geq SC_2$) = 1, V ($SC_8 \geq SC_3$) = 1, V ($SC_8 \geq SC_4$) = 1, V ($SC_8 \geq SC_5$) = 1, V ($SC_8 \geq SC_6$) = 1, V ($SC_8 \geq SC_7$) = 1, V ($SC_8 \geq SC_9$) = 1, V ($SC_8 \geq SC_{10}$) = 1

$d'(C_8)$ = min (1, 1, 1, 1, 1, 1, 1, 1, 1) = 1

V ($SC_9 \geq SC_1$) = 1, V ($SC_9 \geq SC_2$) = 1, V ($SC_9 \geq SC_3$) = 1, V ($SC_9 \geq SC_4$) = 1, V ($SC_9 \geq SC_5$) = 1, V ($SC_9 \geq SC_6$) = 1, V ($SC_9 \geq SC_7$) = 0.9498, V ($SC_9 \geq SC_8$) = 0.9507, V ($SC_9 \geq SC_{10}$) = 1,

$d'(C_9)$ = min (1, 1, 1, 1, 1, 1, 0.9498, 0.9507, 1) = 0.9498

V ($SC_{10} \geq SC_1$) = 1, V ($SC_{10} \geq SC_2$) = 1, V ($SC_{10} \geq SC_3$) = 1, V ($SC_{10} \geq SC_4$) = 1, V ($SC_{10} \geq SC_5$) = 1, V ($SC_{10} \geq SC_6$) = 1, V ($SC_{10} \geq SC_7$) = 0.9498, V ($SC_{10} \geq SC_8$) = 0.9507, V ($SC_{10} \geq SC_9$) = 1,

$d'(C_{10})$ = min (1, 1, 1, 1, 1, 1, 0.9498, 0.9507, 1) = 0.9498

Next, the resulting priority weight forms the vector below are derived from the minimum value from each criteria set above. Using Step 3 is how the minimum values are derived from the 10 values if each criteria.

W' = (0.7461, 0.685, 0.7951, 0.4785, 0.4634, 0.7359, 1, 1, 0.9498, 0.9498). Sum of W' = 7.8036

Then, **Step 4** the priority weights are normalized to get the global weights.

W = (0.0956, 0.0877, 0.1018, 0.0613, 0.0593, 0.0943, 0.1281, 0.1281, 0.1217, 0.1217)

Here, the ELECTRE I method is utilized to determine the best Mathematics applications as alternatives. The consensus evaluation values for five alternatives are given in Table below. The weights were derived at through some series of steps brought by the FAHP method, and then the ELECTRE I for ranking of alternatives. The suitability rating for the alternatives on each criterion will be the value of the numbers one to five, namely: 1,2,3,4 and 5 with matching values; very bad, bad, enough, good and very good. The Table below shows first the matrix for the alternatives compared to each criteria, which is **Step 5.**

Table 5.5: The Criteria vs Alternative decision matrix

Criteria/ Alternatives	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
Mathematics	4	3	4	5	5	3	3	5	2	5
Cymaths	2	4	4	5	5	3	3	4	2	3
MalMaths	4	4	4	5	5	3	3	4	2	3
MathsPapa	5	5	4	5	5	4	4	4	3	4
Maths42	5	5	4	5	5	4	4	4	3	4

The values in Table **5.5** were derived by the judging each alternative per the criteria within a range from 1 to 5, it is also important to state that the judgment was made according to the current expertise of the decision maker.

Table 5.6: Normalizing the decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
Mathematics	16	9	16	25	25	9	9	25	4	9
Cymaths	16	16	16	25	25	9	9	16	4	9
MalMaths	16	16	16	25	25	9	9	16	4	9
MathPapa	25	25	16	25	25	25	16	16	9	16
Maths42	25	25	16	25	25	25	16	16	9	16
SUM	98	91	80	125	125	77	59	89	30	59
SQRT	9.89	9.53	8.94	11.18	11.18	8.77	7.68	9.43	5.47	7.68

The values in the Table 5.6 above is derived by taking the square root of each value, then a sum of each column is added and then a final square root of each sum is done. Then using **Step 6** for calculating the normalized matrix.

Table 5.7: Results from normalizing the decision matrix.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
Mathematics	0.404	0.314	0.447	0.447	0.447	0.341	0.390	0.529	0.365	0.390
Cymaths	0.404	0.419	0.447	0.447	0.447	0.341	0.390	0.423	0.365	0.390
MalMaths	0.404	0.419	0.447	0.447	0.447	0.341	0.390	0.423	0.365	0.520
MathPapa	0.505	0.524	0.447	0.447	0.447	0.455	0.520	0.423	0.547	0.520
Maths42	0.505	0.524	0.447	0.447	0.447	0.455	0.520	0.423	0.547	0.520

The values in the Table 5.7 above is as a result of dividing each value in Table 5.6 by the square root values in Table 5.6 above

Table 5.8: Weighted Normalized Decision Matrix

Weights (*W*)	0.095	0.087	0.101	0.061	0.059	0.094	0.128	0.128	0.121	0.121
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
Mathematics	0.386	0.027	0.045	0.027	0.026	0.032	0.050	0.068	0.044	0.048
Cymaths	0.386	0.036	0.045	0.027	0.026	0.032	0.050	0.054	0.044	0.048
MalMaths	0.386	0.036	0.045	0.027	0.026	0.032	0.050	0.054	0.044	0.063
MathsPapa	0.048	0.046	0.045	0.027	0.026	0.043	0.067	0.054	0.067	0.063
Math42	0.048	0.046	0.045	0.027	0.026	0.043	0.067	0.054	0.067	0.063

The values in the Table 5.8 above were derived simply from the multiplication of each value in Table 5.7 by the weights of the criteria *W* derived using **Step 7** from the equations of the hybrid method.

Table 5.9: Concordance Matrix

	Mathematics	Cymaths	MalMaths	MathPapa	Math42
Mathematics	0	0.9123	0.9123	0.3515	0.3515
Cymaths	0.8721	0	1	0.3515	0.3515
MalMath	0.8721	1	0	0.3515	0.3515
MathPapa	0.8721	1	1	0	1
Math42	0.8721	1	1	1	0
Sum of Colums	3.488	3.912	3.912	2.054	2.054

Step 8 is used for the concordance sets as well as using **Step 9** for the concordance domination matrix.

Table 5.10: Matrix of Concordance Domination

	Mathematics	Cymaths	MalMaths	MathPapa	Maths42
Mathematics	0	1	1	0	0
Cymaths	1	0	1	0	0
MalMaths	1	1	0	0	0
MathPapa	1	1	1	0	1
Maths42	1	1	1	1	0

Step 8 again is used for the discordance sets as well as using **Step 10** for the discordance domination matrix.

Table 5.11: Discordance Matrix

	Mathematics	Cymaths	MalMaths	MathsPapa	Math42
Mathematics	0	0.5	0.5	1	1
Cymaths	0.5	0	0	0.875	0.875
MalMaths	0.5	0	0	0.875	0.875
MathsPapa	0.5	0	0	0	0
Maths42	0.5	0	0	0	0
Sum of Colums	2	0.5	0.5	2.75	2.75

Table 5.12: Matrix of Discordance Domination

	Mathematics	Cymaths	MalMath	MathPapa	Math42
Mathematics	1	0	0	0	0
Cymaths	0	1	1	0	0
MalMath	0	1	1	0	0
MathPapa	0	1	1	1	1
Maths42	0	1	1	1	1

Using the final **Step 11 & 12** to calculate the aggregate dominance matrix. The Table 5.13 called the Aggregate Dominance Matrix is a combination of the matrices of concordance domination and discordance domination using the AND operator.

Table 5.13: Aggregate Dominance Matrix.

	Mathematics	Cymaths	MalMaths	MathsPapa	Math42
Mathematics	0	0	0	0	1
Cymaths	0	0	0	1	1
MalMaths	0	0	0	1	0
MathsPapa	1	0	0	1	1
Maths42	1	0	1	1	0

Finally, we can eliminate the less favorable alternative and rank them as done in the table above. Using the ELECTRE I method, the best alternative will be carried out scanning the columns of the Aggregate Dominance Matrix and each column with the least amount of 1 is the best. According to the ranking, the best Mathematics learning application will then be Cymaths followed by MalMath, Mathematics, Math42 and MathsPapa.

5.2 Results of FAHP in Ranking MLAMs

There are five applications to be selected from and they are; Mathematics, Cymaths, MalMath, MathPapa, and Math42. From the previous calculations, the author used the FAHP method to determine the weights for the ranking with ELECTRE I. However, this study also conducts a standalone with FAHP method. Therefore, a continuation of the Step 4 in the equation for FAHP where the weights of the criteria were derived will lead to the ranking/selection here. From using **Step 4** above, the priority weights are normalized to form the following vector about the main goal. Here, the considered applications ought to contain credit to all criteria. Due to this, Liberatore et al., (1992) put out a rating scale that is five-point and was utilized to decide the match pair-wise comparison judgment matrix (PCJM) as appeared in Table 5.14 beneath. It is accepted that the distinction in relative significance between two nearby scales regarding a specific scale is consistent at two times, and gets the comparing PCJM for the rating scales. The framework is then converted into the biggest eigenvalue issue and the subsequent need weights of extraordinary, great, normal, reasonable, and poor are found as 0.513, 0.261, 0.129, 0.063, and 0.034, individually which is Step 5 from the FAHP method equations. The previous steps leading to the calculation of the normalized weight suing Step 4 has been calculated in the results of the hybrid method above, the pair-wise comparison for a 5 rating scale to each alternative is as a result of using Step 5 of the FAHP method.

Table 5.14: Pair-wise Comparison judgment matrix for a five rating scale

Rating Scale	O	G	A	F	P	Relative weights
Outstanding (O)	1	3	5	7	9	0.513
Good (G)	1/3	1	3	5	7	0.261
Average (A)	1/5	1/3	1	3	5	0.129
Fair (F)	1/7	1/5	1/3	1	3	0.063
Poor (P)	1/9	1/7	1/5	1/3	1	0.034

Table 5.15: Applying of FAHP to A_1 (Mathematics)

Criteria	Weights	Mathematics		
		Rating	Score	Result
C_1	0.0956	G	0.261	0.0250
C_2	0.0877	G	0.261	0.0229
C_3	0.1018	G	0.261	0.0266
C_4	0.0613	O	0.513	0.0314
C_5	0.0593	O	0.513	0.0304
C_6	0.0943	A	0.129	0.0122
C_7	0.1281	A	0.129	0.0165
C_8	0.1281	G	0.261	0.0334
C_9	0.1217	G	0.261	0.0318
C_{10}	0.1217	G	0.261	0.0318
Total scores				0.262

Table 5.16: Applying of FAHP to A_2 (Cymaths)

Criteria	Weights	Cymaths		
		Rating	Score	Result
C_1	0.0956	A	0.129	0.0123
C_2	0.0877	A	0.129	0.0113
C_3	0.1018	O	0.513	0.0522
C_4	0.0613	O	0.513	0.0314
C_5	0.0593	O	0.513	0.0304
C_6	0.0943	F	0.063	0.0059
C_7	0.1281	F	0.063	0.0081
C_8	0.1281	A	0.129	0.0165
C_9	0.1217	G	0.261	0.0318
C_{10}	0.1217	A	0.129	0.0157
Total scores				0.2156

Table 5.17: Applying FAHP to $\mathbf{A_3}$ (MalMath)

Criteria	Weights	MalMath		
		Rating	Score	Result
C_1	0.0956	G	0.261	0.0250
C_2	0.0877	O	0.513	0.0450
C_3	0.1018	A	0.129	0.0131
C_4	0.0613	O	0.513	0.0314
C_5	0.0593	O	0.513	0.0304
C_6	0.0943	A	0.129	0.0122
C_7	0.1281	A	0.129	0.0165
C_8	0.1281	G	0.261	0.0334
C_9	0.1217	G	0.261	0.0318
C_{10}	0.1217	A	0.129	0.0157
Total scores				0.2545

Table 5.18: Applying of FAHP to $\mathbf{A_4}$ (MathPapa)

Criteria	Weights	MathPapa		
		Rating	Score	Result
C_1	0.0956	G	0.261	0.0250
C_2	0.0877	G	0.261	0.0229
C_3	0.1018	G	0.261	0.0266
C_4	0.0613	O	0.513	0.0314
C_5	0.0593	O	0.513	0.0304
C_6	0.0943	G	0.261	0.0246
C_7	0.1281	G	0.261	0.0334
C_8	0.1281	A	0.129	0.0165
C_9	0.1217	G	0.261	0.0318
C_{10}	0.1217	G	0.261	0.0318
Total scores				0.2744

Table 5.19: Applying of FAHP to A_5 (Math42)

Criteria	Weights	Math42		
		Rating	Score	Result
C_1	0.0956	G	0.261	0.0250
C_2	0.0877	O	0.513	0.0450
C_3	0.1018	G	0.261	0.0266
C_4	0.0613	O	0.513	0.0314
C_5	0.0593	O	0.513	0.0304
C_6	0.0943	G	0.261	0.0246
C_7	0.1281	O	0.513	0.0657
C_8	0.1281	O	0.513	0.0657
C_9	0.1217	O	0.513	0.0624
C_{10}	0.1217	O	0.513	0.0624
Total scores				0.4392

Table 5.20: FAHP Stand-alone results

	Final rating	Ranking
Malmaths	0.1812	3
Mathematics	0.1491	5
Cymaths	0.1760	4
MathsPapa	0.1890	2
Math42	0.3038	1

According to the result from the Table 5.20 above, Math42 is ranked the best after using the FAHP stand-alone. The final rating is derived by adding up the total scores of each Alternative to give a grand total and then diving each total score by the grand total.

5.3 Results of ELECTRE I in Ranking MLAMs

Weighting Criteria: The relative significance of the criteria should be effected. Not the majority of the criteria will be similarly critical. To set up the weights to be connected to the criteria the decision-makers were solicited to rank the relative significance from the criteria and to give a value score in the range of 0 and 100 in the table below, this system is known as the SMART strategy technique Chou and Chang (2008).

In this study, significance weights of criteria were surveyed utilizing fuzzy fundamental method considering the potential fuzzy subjective judgment in the midst of the evaluation process. Fuzzy scale weights address ten linguistic elements, which were used to depict the importance of weights of every criteria; Same significance, Average significance, Significant, Very significant, Very strong significant, with their respective inverses.

The Table 5.21 below is the **Step 1** which evaluates each criterion with a linguistic value.

Table 5.21: Pair-wise comparison matrix for criteria in terms of Fuzzy Linguistic variables

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
C_1	-	S	SS	VS	VS	S	SS	AS	AS	AS
C_2	S’	-	SS	S	S	VS	AS	AS	SS	SS
C_3	SS’	SS’	-	S	S	S	AS	SS	S	S
C_4	VS’	S’	S’	-	SS	S	AS	AS	AS	AS
C_5	VS’	S’	VS’	SS	-	SS	AS	AS	AS	AS
C_6	S’	VS’	S’	S’	SS	-	SS	AS	AS	AS
C_7	SS’	AS’	AS’	AS’	AS’	SS	-	SS	SS	SS
C_8	AS’	AS’	SS’	AS’	AS’	AS’	SS	-	S	S
C_9	AS’	SS’	S’	AS’	AS’	AS’	SS	S’	-	SS
C_{10}	AS’	SS’	S’	AS’	AS’	AS’	SS	S’	SS	-

Using **Step 2**, a matrix is developed with respect to each criterion again.

Table 5.22: Pairwise comparison matrix for criteria in terms of Fuzzy scale

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
C_1	-	0.825	0.880	0.770	0.770	0.825	0.650	0.770	0.770	0.825
C_2	0.170	-	0.825	0.825	0.825	0.770	0.530	0.770	0.530	0.530
C_3	0.050	0.170	-	0.825	0.825	0.825	0.650	0.770	0.770	0.770
C_4	0.290	0.170	0.170	-	0.530	0.290	0.170	0.290	0.290	0.290
C_5	0.290	0.170	0.170	0.530	-	0.290	0.170	0.290	0.290	0.290
C_6	0.170	0.290	0.170	0.770	0.770	-	0.170	0.290	0.290	0.290
C_7	0.410	0.530	0.410	0.825	0.825	0.825	-	0.410	0.410	0.410
C_8	0.290	0.290	0.290	0.770	0.770	0.770	0.650	-	0.770	0.770
C_9	0.290	0.530	0.290	0.770	0.770	0.770	0.650	0.290	-	0.530
C_{10}	0.170	0.530	0.290	0.770	0.770	0.770	0.650	0.290	0.530	-

Sum for Criteria rows divided by 10: C1= 7.085/10 = 0.7085, C2= 5.775/10 = 0.5775, C3= 5.655/10 = 0.5655, C4= 2.49/10 = 0.249, C5= 2.49/10 = 0.249, C6= 3.21/10 = 0.321, C7= 5.055/10 = 0.5055, C8= 5.37/10 = 0.537, C9= 4.89/10 = 0.489, C10= 4.77/10 = 0.477

Sum of Priority weights = 4.679

Calculate the normalized weights using **Step 3&4** by dividing each priority weight by the sum above.

Normalized weights: C1= 0.1514, C2= 0.1234, C3= 0.1208, C4= 0.0532, C5= 0.0532, C6= 0.0686, C7= 0.1080, C8= 0.1147, C9= 0.1045, C10= 0.1019

Table 5.23: Performance rating matrix of five alternatives associated with ten criteria

Cri./Alt.	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
Mathematics	50	40	40	100	100	50	40	40	40	50
Cymaths	60	60	50	100	100	50	50	40	30	50
MalMaths	50	70	50	100	100	50	50	50	60	50
MathsPapa	80	80	60	100	100	40	70	70	70	50
Maths42	70	80	70	100	100	50	70	80	80	80

Using **Step 5** for the concordance matrix.

Table 5.24: Concordance Matrix

Concordance Matrix		Mathematics	Cymaths	MalMaths	MathsPapa
Mathematics	0	0.5104	0.4406	0.2848	0.18
Cymaths	0.8925	0	0.6761	0.2848	0.18
MalMaths	1	0.8442	0	0.2848	0.18
MathsPapa	0.9295	0.9295	0.9295	0	0.4747
Maths42	1	1	1	0.8442	0

Using **Step 7** for the concordance domination matrix

Table 5.25: Concordance Domination Matrix

	Mathematics	Cymaths	MalMaths	MathsPapa	Math42
Mathematics	0	0	0	0	0
Cymaths	1	0	1	0	0
MalMaths	1	1	0	0	0
MathsPapa	1	1	1	0	0
Maths42	1	1	1	1	0

Using **Step 6** for the discordance matrix and **Step 7** for the discordance domination matrix

Table5.26: Discordance Matrix

Discordance Matrix	Mathematics	Cymaths	MalMaths	MathsPapa	Math42
Mathematics	0	0.3394	0.5091	0.6788	0.8
Cymaths	0.2	0	0.6	0.8	1
MalMaths	0	0.1806	0	0.5419	0.6
MathsPapa	0.2333	0.2333	0.2333	0	0.6
Maths42	0	0	0	0.1806	0

Table 5.27: Discordance Domination Matrix

	Mathematics	Cymaths	MalMaths	MathsPapa	Math42
Mathematics	1	1	0	0	0
Cymaths	1	1	0	0	0
MalMaths	1	1	1	0	0
MathsPapa	1	1	1	1	0
Maths42	1	1	1	1	1

Finally, a multiplication of the concordance and discordance matrix domination is used to calculate the aggregate matrix using **Step 8.**

Table 5.28: Aggregate/Global Matrix

	Mathematics	Cymaths	MalMath	MathPapa	Math42
Mathematics	0	0	0	0	0
Cymaths	1	0	0	0	0
MalMath	1	1	0	0	0
MathPapa	1	1	1	0	1
Maths42	1	1	1	1	1

From the Aggregate Dominance Matrix table above, Using ELECTRE I as a stand-alone the best alternative is determined by scanning through the columns and selecting the column has the least number of one should be the best alternative. Therefore, the best alternative is Maths42.

Table 5.29: Final ranking of the alternatives

Method	Final results
Hybrid	Cymaths > Malmaths > Mathematics > Maths42 > MathPapa
FAHP	Math42 > MathPapa > MalMath > Mathematics > Cymaths
ELECTRE I	Cymaths > Malmaths > Mathematics > Maths42 > MathPapa

5.4 The proposed computer software for three methods.

The use of different criteria and different decision-maker's judgment will result in different rankings. The two methods employed for this study yielded an optimum result and can be utilized in the future. In terms of a higher number of alternatives and criteria ELECTRE I seems more efficient due to an understandable method steps that is not only shorter compared to the other methods but more error free and less time consuming. This study not only carried out an evaluation but an application which was named SmartRank that also carried out the evaluation of the three methods was develop, below are snapchats of the steps involved when using the application; The Figure 5.1 below is the first page that is shown when SmartRank is run and it shows the name of the application as well as two buttons "Start Evaluation" which leads to Step 1 of the evaluation and "Instructions" which show how to use the application and what this application is about.

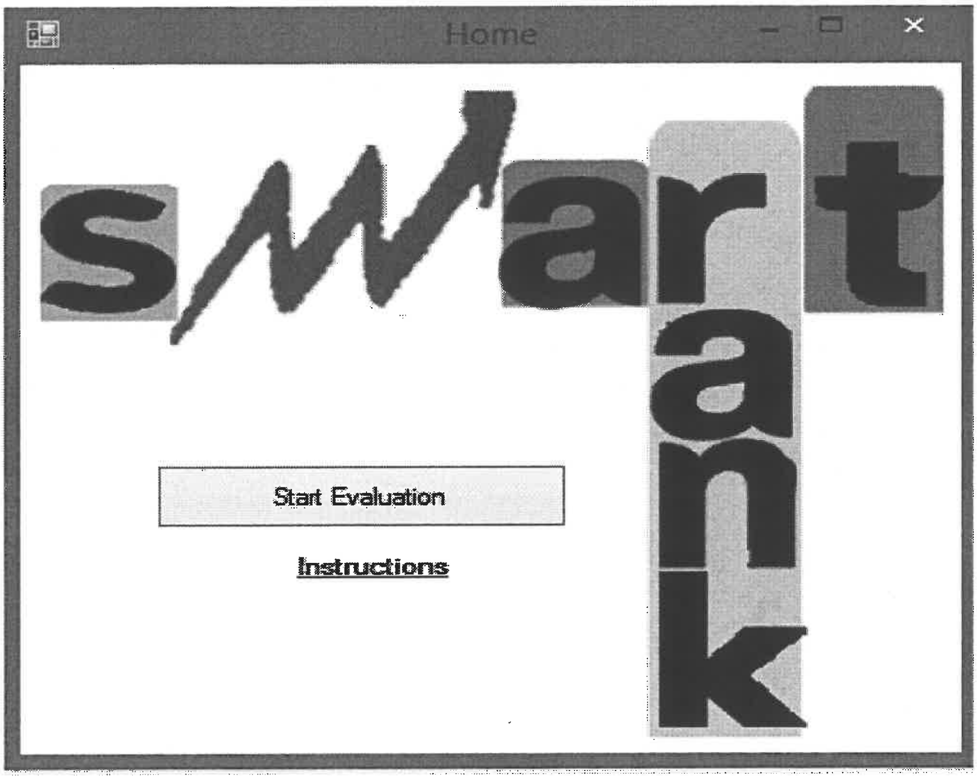

Figure 5.1: Homepage

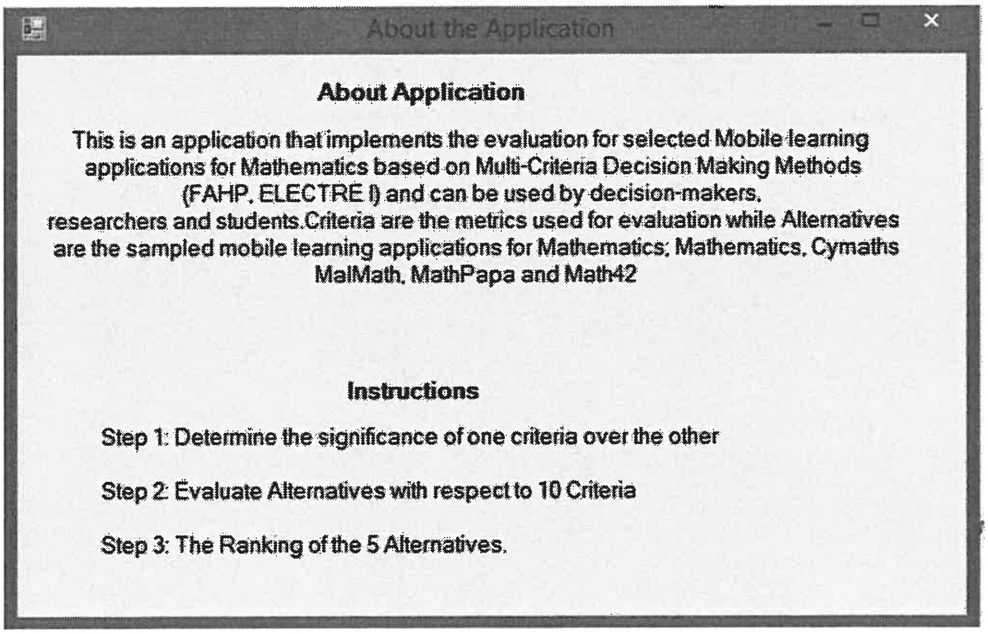

Figure 5.2: Instructions Page

The Figure 5.2 is the Instructions page from the "Instructions" button from the Homepage, this talk about what SmartRank does and the steps on how it is being used.

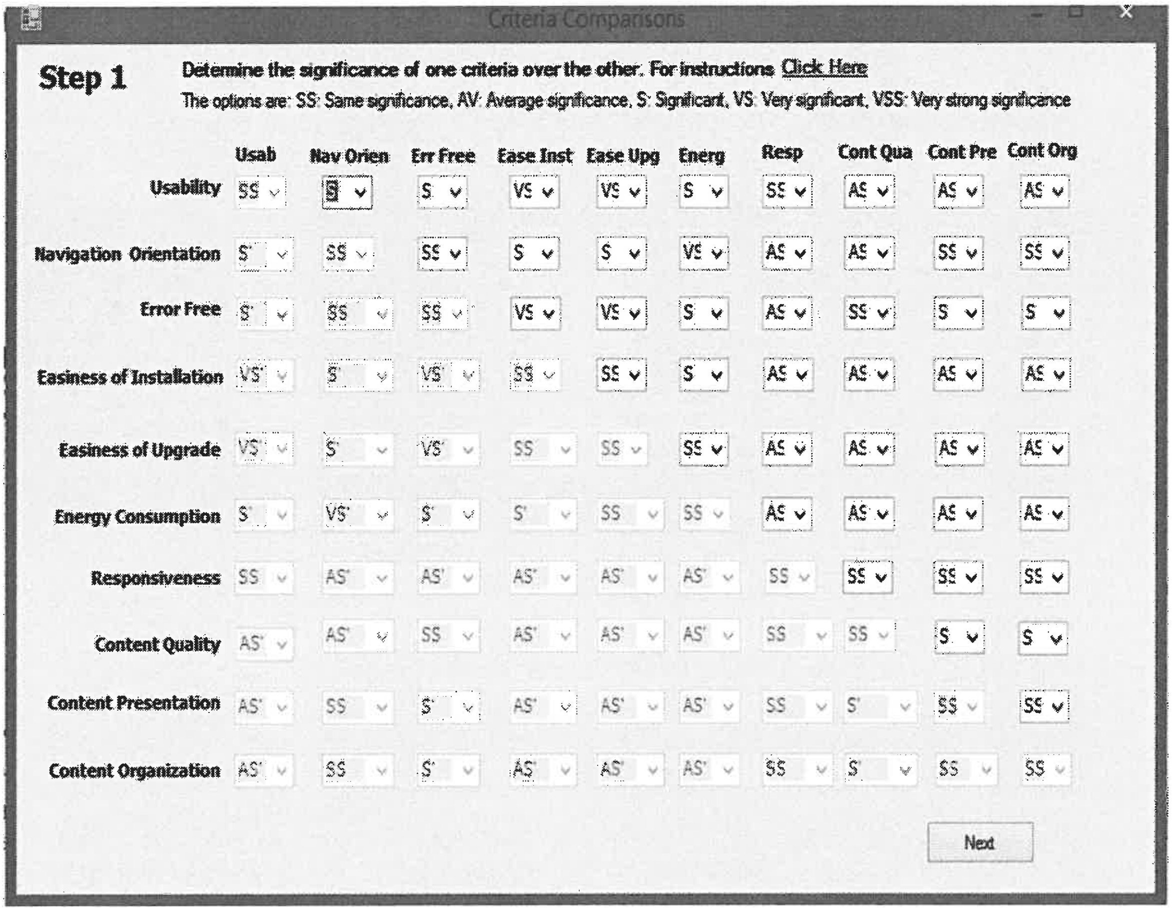

Figure 5.3: Criteria Significance Page

The Figure 5.3 is where the significance of one criteria over the other criteria is carried out which is called a pair-wise comparison. Also shows the meaning of the acronyms and has a "Click Here" button.

The Figure 5.4 below is the resulting page that shows up after the "Click Here" button from Figure 5.3 is clicked, here the selected Criteria are defined and the sampled Alternatives are mentioned.

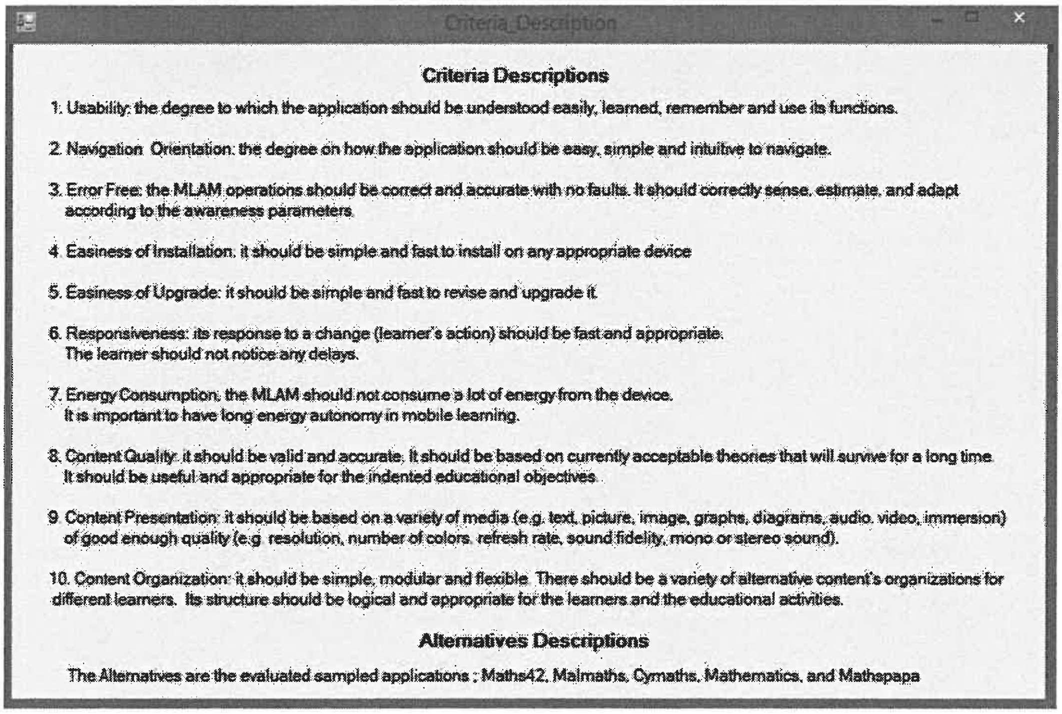

Figure 5.4: Criteria & Alternatives Description Page

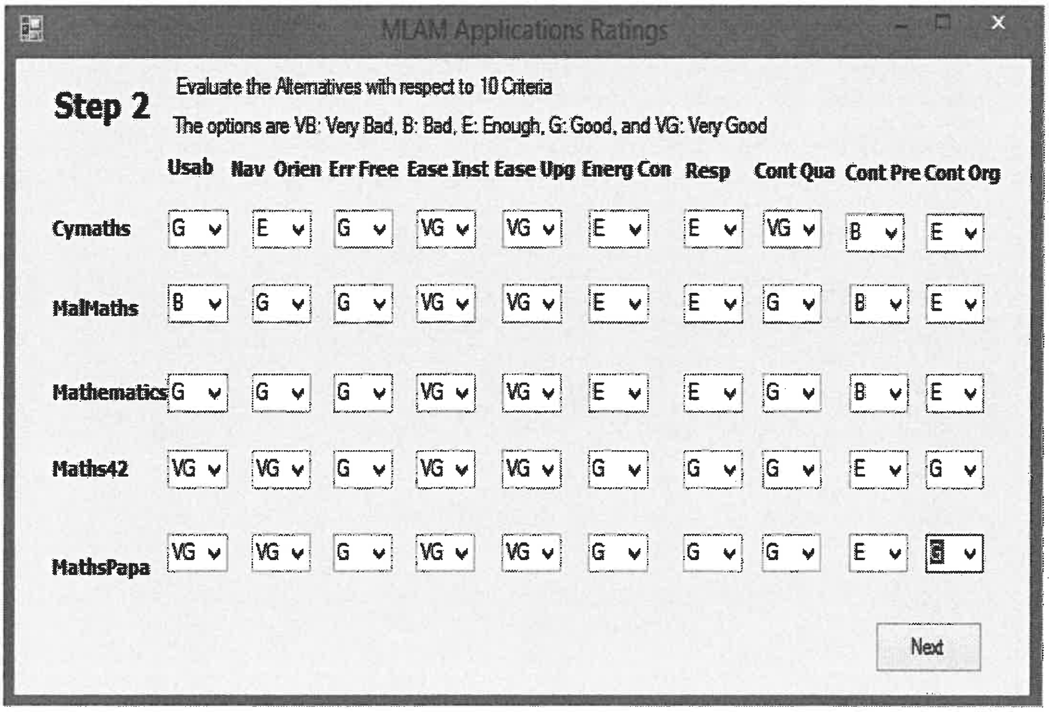

Figure 5.5: Alternatives Significance Page

The Figure 5.5 above is where the significance of the five Alternatives are calculated versus the selected Criteria. The acronyms of the values for each options of rating are also shown to the user of SmartRank here.

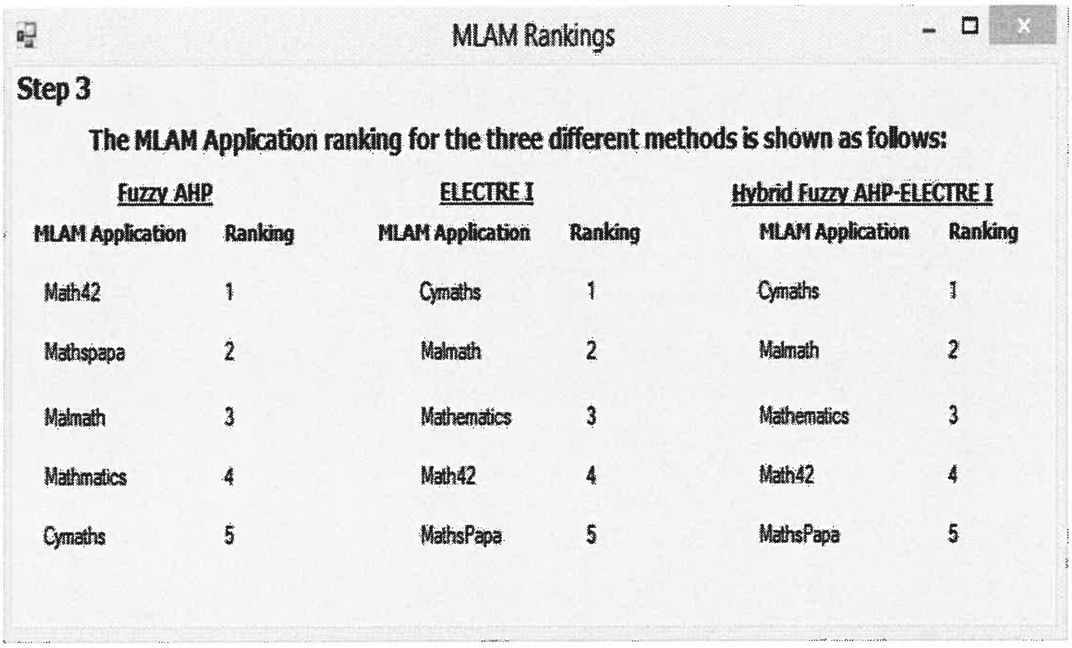

Figure 5.6: Rankings Page

The final page shown in Figure 5.6 above shows the rankings of the Alternatives for each of the three methods used in the study.

5.5 Discussion

With this study part of a framework was adopted and two categories in relation to mobile learning applications for mathematics were specifically adopted; Technical requirements and Pedagogical requirements. Requirements here are also known as Criteria, which make up a framework. The framework originated from Economides (2008) and this was used because the study specifically applies to MLAMs. It is important to state that the selected criteria are not standards set for mobile learning applications for mathematics; they were based off the judgment and level of expertise of the decision-maker. The adopted framework was then used for the evaluation of the MLAMs using two methods; FAHP and ELECTRE I respectively. This study as earlier stated combined the technical criteria and the pedagogical criteria to propose a framework for the evaluation and to improve the result. The triangular fuzzy number was adopted which is simply an extension of real numbers which provides dynamism during evaluation for accuracy, this was utilized with FAHP and ELECTRE I. The choice of the number of criteria depends on the decision-maker, as it can be subjective, studies selected criteria based on the alternatives, which evaluations will be carried on, therefore, the number of criteria cannot be definitive, and it is subjective. The alternatives are similar when it comes to the choice because of their importance in the evaluation, the number is based off what the evaluation is tending to ad what is likely to be achieved.

In previous studies, it was discovered that AHP does not endure ranking inconsistencies as to why Fuzzy was integrated, the manner by which choices are ranked by AHP uses less data than the ELECTRE techniques. In this study, the alternatives rankings based of the 10 criteria by utilizing the FAHP and the ELECTRE I separately.

Basaran and Haruna (2017) proposed a model defining both technical and non-technical aspects used to evaluate MLAMs by utilizing the Fuzzy AHP together with the TOPSIS methods. 11 criteria used defined quality and user satisfaction aspects specific MLAMs. The technical aspects (quality) were culled from the ISO 9126 model of while then non-technical aspects (user-satisfaction) were culled from the user satisfaction model used for the evaluation. The weight of each criterion derived was determined through using the Fuzzy AHP approach while the alternatives: MLAMs ranking was applied with the TOPSIS method. The alternatives chosen were based on top five user ratings on the Google play Store. This is the only study that has evaluated MLAMs with MCDM methods from the literature, in comparison with this study three methods were used to evaluate MLAMS and a comparison between methods were done.
Other studies differ from this study in a light that mostly one MCDM method is used or two but a comparison between methods is not common among decision-makers, three methods used in this study and a comparison is done which is a first. In addition, several MCDM methods have been developed into software for easy calculations, still they all have been stand-alone, and here the three methods were integrated to give results, which is a first too.

However, Ozdemir, Y. S. (2017), Ka, B. (2011) and Kaya, T., & Kahraman, C. (2011), all performed a research with a similar technique which mixed FAHP and ELECTRE methods to enhance result, for that reason with extraordinary empirical take a look at and effects reported, this study is a first to use the FAHP, ELECTRE I combined as an hybrid method and separately as stand-alone methods for an evaluation.

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

The study ends with a concluding remark and further future recommendation.

6.1 Conclusion

In terms of ease of use for ranking a limited number of alternatives with few criteria the hybrid method is the most efficient than stand-alone fuzzy AHP and lastly ELECTRE I due to the complexity of the steps when FAHP and ELECTRE 1 are implemented, the calculations are not complex compared to the stand-alone.

In terms of a higher number of alternatives and criteria ELECTRE I seems more efficient due to a fairly understandable method steps that is not only shorter compared to the other methods but more error free and less time consuming.

So far, in this area, no studies have been located to compare three different multi-criteria methods, this study is a first that implements a comparison between three methods; a hybrid; FAHP and ELECTRE I, two stand-alone methods; FAHP, ELECTRE I.

In addition, an executable program to compare not only one method but also the results of different three methods provides useful insight for decision makers. Most studies focused on either the development of an application for the evaluation or a manual evaluation and calculation is implemented but this study implemented both the manual evaluation with the three MCDM methods and developed a software for these methods, which has not been done.

In recent years, many researchers and developers expected and proposed a rapid increase in the number of users of MLAMs and its services. In order to increase its quality, testing and evaluations have been made by various researchers in several ways. One growing method is the Multi-criteria decision-making methods, which now plays a critical role in real-life problems even though they may not be accurate all the time. As an evaluation of MLAMs requires concurrent thought of a few comparative and clashing criteria, MCDM methods are very useful in handling this.

In this study, the criteria weights were generated using the Fuzzy AHP method, which is highly rated as one of the most reliable weights assigning methods in MCDM literature. Next ELECTRE I is used to evaluate and rank the mobile learning applications for mathematics. Before the evaluation process, a framework, which consists of technical and pedagogical criteria of MLAM, was developed for assessing the quality of MLAM through an empirical study. "Math42" was ranked as the first application while "Mathematics" was ranked last based only in this study. The decision maker for this study is the researcher with supervision. In addition, as stated earlier in this study different criteria will yield different ranking based in the level of expertise and judgment by the decision maker.

6.2 Recommendations

For future research, the MCDM methods used can be applied to several other industries, not only these methods but also other MCDM methods as they can be used to real life problems. Larger number of criteria can also be used without limitations but the decision maker should have capable knowledge of the methods as they can be tedious which lead to a final recommendation that a general software be developed that houses all the current MCDM methods and can be upgraded incase a new method is developed, this will greatly assist decision makers and researchers especially not to limit the number of alternatives and criteria for whatever purpose.

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APPENDIX: SIMILARITY REPORT

AUTHOR	TITLE	SIMILARITY	GRADE	RESPONSE	FILE	PAPER ID	DATE
Oluwatobi John Adura...	CHAPTER1	4%	-	-		965027572	17-May-2018
Oluwatobi John Adura...	CHAPTER2	12%	-	-		965027575	17-May-2018
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