# DIRECTION OF ARRIVAL ESTIMATION OF WIDEBAND RF SOURCES

**A THESIS SUBMITTED TO THE GRADUATE SCHOOL OF APPLIED SCIENCE OF NEAR EAST UNIVERSITY**

**By**

**AMR ABDELNASER ABDELHAK ABDELBARI**

**In Partial Fulfillment of the Requirements for the Degree of Master of Science in Electrical and Electronic Engineering**

**NICOSIA, 2018**

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**Approval of Director of Graduate School of Applied Sciences**

**Prof.Dr.Nadire CAVUS**

**We certify this thesis is satisfactory for the award of the degree of Master of Science in Electrical and Electronic Engineering**

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Eternally, my deep gratefulness goes to my parents for their unconditional support, end-less love and encouragement.

To my parents...

## ABSTRACT

DOA estimation methods seem to be very useful in the reconstruction of the original received signals and help on the estimation of its location which is highly applicable in radars, sonars, seismic exploration, and military surveillance. DOA estimation methods try to figure out the parameters hidden in the sensors data using different mathematical techniques and physical properties of the geometry of the array of antennas and the impinging signals themselves.

In this thesis, the DOA estimation methodology of wideband signals is studied and its computational cost and execution time are investigated. Further, recent well-known wideband algorithms are also investigated and simulated. Finally, The simulation results show the main factors that affect the computational costs of DOA methods and how to control it to reduce the complexity of the algorithms while maintaining a high resolution.



***Keyword:*** Array Signal Processing; Direction-Of-Arrival; DOA; IMUSIC; CSS; WAVES; TOPS

# ÖZET

DOA kestirim yöntemleri, asıl alınan sinyallerin yeniden yapılandırılmasında çok faydalı görünmektedir ve radarlarda, sonarlarda, sismik keşiflerde ve askeri gözetimde yüksek oranda geçerli olan yeri tahmin etmede yardımcı olmaktadır. DOA kestirim yöntemleri, farklı matematiksel teknikleri ve anten dizisinin geometrisinin fiziksel özelliklerini ve çarpışma sinyallerini kullanarak sensörler verilerinde saklı parametreleri belirlemeye çalışır.

Bu tezde, geniş bantlı sinyallerin DOA kestirim metodolojisi çalışılmış, hesaplama maliyeti ve yürütme süresi incelenmiştir. Ayrıca, son zamanlarda bilinen geniş bantlı algoritmalar da incelenmekte ve simüle edilmektedir. Son olarak, simülasyon sonuçları DOA yöntemlerinin hesaplama maliyetlerini etkileyen ana faktörleri ve yüksek çözünürlüğü korurken algoritmaların karmaşıklığını azaltmak için nasıl kontrol edileceğini göstermektedir.

***Anahtar Kelimeler:*** Dizi Sinyal İşleme; Varış Yönü; DOA; IMUSIC; CSS; WAVES; TOPS

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| | |
|---|---|
| **A** | Matrix of Steering Vectors |
| **ADC** | Analog to Digital Converter |
| $\mathbf{C}^n$ | Complex Number Space |
| **CSS** | Coherent Signal Subspace |
| **DOA** | Direction of Arrival |
| **DSP** | Digital Signal Processing |
| $\mathbf{E}\{\cdot\}$ | The Statistical Expectation |
| $\mathbf{E}_s$ | The Signal Subspace Eigenvectors |
| $\mathbf{E}_n$ | The Noise Subspace Eigenvectors |
| **ESPRIT** | Estimation of Signal Parameters via Rotational Invariance Technique |
| **EVD** | Eigen Value Decomposition |
| $\mathbf{F}(\theta)$ | The Spatial Spectrum |
| **FFT** | Fast Fourier Transform |
| **I** | The Identity Matrix |
| **IMUSIC** | Incoherent Multiple Signal Parameter Estimation |
| **MATLAB** | Simulation Program |
| **MP** | Matrix Pencil |
| **MS** | Mobile Station |
| **MUSIC** | Multiple Signal Parameter Estimation |
| $\mathbf{O}(\cdot)$ | Big-O notation |

| | |
|---|---|
| $\mathbf{P}_{ss}$ | Signal Correlation Matrix |
| **range**$\{\cdot\}$ | Range of Matrix |
| **RMSE** | Root Mean Square Estimate |
| **RFI** | Radio frequency Interference |
| **SLFM** | Source Line Fitting Method |
| **SNR** | Signal to Noise Ratio |
| **SST** | Signal Subspace Transformation |
| **SVD** | Singular Value Decomposition |
| **TOFES** | Test of Orthogonality of Frequency Subspaces |
| **TOPS** | Test of Orthogonality of Projected Subspaces |
| **ULA** | Uniform Linear Array |
| $\mathbf{V}_{xx}$ | The Covariance Matrix |
| **WAVES** | Weighted Average of Signal Subspace |
| **WSF** | Weighted subspace Fitting |
| $\lambda$ | Wavelength |
| $\Phi$ | Spatial Frequency |
| $\theta$ | Angle of Arrival |
| $\sigma^2$ | Noise Variance |
| $\phi$ | Search Angle |

# CHAPTER 1

# INTRODUCTION

Radio frequency (RF) signals are the electromagnetic waves that carry the information through the wireless medium on many wireless communication systems between its two side, the transmitter, and the receiver. The RF spectrum has a wide range of frequencies which divided into sub-bands for different applications e.g. mobile communication, satellite, emergency, police and military use. The main purpose of the communication system is to transfer the maximum amount of information with minimum errors (Monzingo, Haupt, & Miller, 2011).

Recently, modern communication systems have increased rapidly on its capacity and on the number of served users. Some cell phone infrastructures have reached its capacity of more than 100 $\%$ of the number of served users. More intelligent solutions are required for the communication system to be capable of handling this growing demands. One of the most interesting technology today is using multiple antennas for transmitting and receiving. The antenna is the device that transfers the electromagnetic signal from the transmitter into the medium e.g. air, then the signal propagates through the medium and finally another antenna transfer the signal from the air into the receiver. Using one antenna is already deployed in many communication systems and it has reached its limitations on size and overall gain. On the other hand, using multiple antennas increases the capability of design to meet the requirements and improve the characteristics of transmission e.g. higher gain and more directivity (Balanis, 2007).

In this thesis, an array of antennas or radioactive sensor elements on the receiving side are investigated which have significant benefits in both communication and radar systems. The array processing field studies array of sensor elements and its Recent developments on the computational processors and digital signal process make it possible to carry many processes on the data received. Array processing gained a lot of interests and have been deployed on many applications e.g. radars, mobile communication, acoustics, aerospace, and satellite communication. On top of the array process topics, arises the direction of arrival (DOA) estimation and its application in many fields (Monzingo et al., 2011).

DOA estimation methods try to extract the true angles of the exact number of the received noisy signals which impinging on the array of sensors. This estimation is very useful in recovering the signal of interest and suppressing the noise and other interfering signals. Also estimating the signal sources angles helps on the estimation of its location which is highly applicable in radars, sonars, seismic exploration, and military surveillance (Krim & Viberg, 1996). The idea of DOA estimation is to come up with the arrived signals true angles using the received data collected from the sensors which have the power of the signals mixed with noise (Balanis, 2007).

This thesis is arranged as follows: Chapter 1 introduces the array signal processing; the DOA estimation, and its applications; the difference between the narrowband and wideband signals; the computational cost and the complexity of each algorithm and how to calculate it as it an important factor on rating the efficiency of the algorithms. Chapter 2 shows the signal model for both narrowband and wideband signals and explains the idea of orthogonality of subspaces and finally reviews the well-known DOA estimation methods and the latest improvements on it. Chapter 3 is the core of this thesis which shows different methods of DOA estimation of the wideband signal. Chapter 4 previews the simulation environment and the analysis metrics used on it and the assumptions that have been taken within the simulation. Chapter 5 shows the results of the simulation and discuss the computational cost of each wideband methods and how to improve it. Finally, chapter 6 concludes the thesis and view the future work that can be done to improve and develop a better DOA estimation method.

## 1.1 Array Signal Processing

Signal processing is about forming the signals into the most suitable mold for transmission through different kind of medium e.g. free space, air, water, metals, and optic materials without losing its assets. It's also about receiving signals from the surrounding medium and getting several types of information from it e.g. the information that carried by the signal, the direction of arrival. Array signal process is the field that focuses on manipulation of signals induced or impinging by an array of radiative sensors (Johnson & Dudgeon, 1993; Godara, 1997). The Array of sensors are used to measure the propagating wavefield and transferring the field energy to electrical energy. The wavefield is assumed to have information about the signal sources and so its sampled into a data set to be processed to extract as much as possible information form it. Multiple snapshots are taken to get a high resolution of the signals, though the signal processing using different kind of algorithms can determine the number of sources, locate them and reconstruct the signals itself (Johnson & Dudgeon, 1993).

Array signal processing is a detection and estimation problem. The detection of multiple signals using an array of sensor elements has been an attractive research topic for decades due to its ability to overcome the limitation on diversity, beam width and beamforming of single antenna (Krim & Viberg, 1996; Er, Cantoni, & Lee, 1990). The estimation problem is in the core of the array signal process and in the signal process in general which tries to figure out the values of some parameters which is estimated to be the most possible values closed to the true values e.g. angles of arrivals, number of signals (Kay, 1993).

## 1.2 Motivations

### 1.2.1 DOA estimation problem

Several plane waves (Narrowband or wideband) impinging on an array of sensor elements from different locations which we will assume they are emission points on the far-field space. The Array of sensors are arranged in a linear uniform way called uniform linear array (ULA) as in Figure 1.1. The sensor elements are equally spaced and the important parameter in array processing is the manfield of the array of sensors. The estimation methods try to use the characteristics parameters of both the incident signals and the geometry of the array to estimate these parameters. One of the important parameters is the angle of arrival of the signal and its components which shown in Figure 1.1 as $\theta_i$. Each indecent signal on each element will be shifted in time by the array geometry related parameters which we can easily relate it to the angle of arrival (Satish & L. Kashyap, 2018). This will be discussed in details in chapter 2.

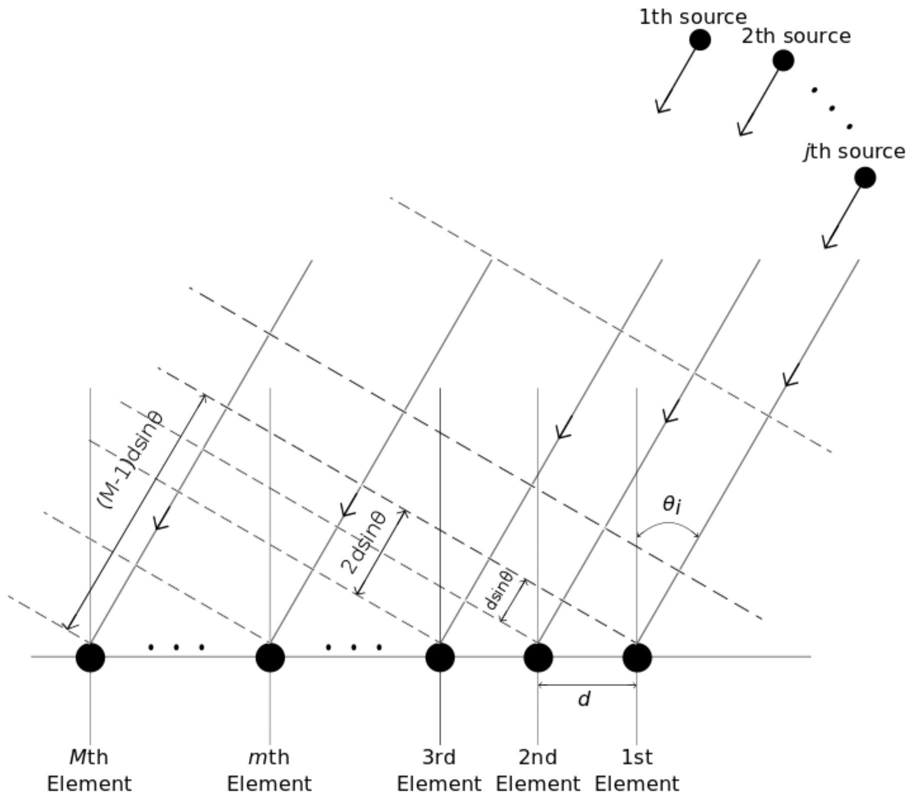


**Figure 1.1:** The direction of arrival DOA problem

DOA estimation has been investigated for decades and many researches explored the nature of exploiting the sampled data so as to estimate the true angles of the incident signals, however, in this thesis we are more concerned about the estimation of direction of arrival of wideband signals due its difficulty to figure out its components and also we focus on the computational cost of the recent methods due to its importance in the estimation in the real application and how to improve it without losing the resolution of the DOA method.

### 1.2.2 Application of DOA estimation

In this section, we will get a deep sight about the importance of the DOA estimation by reviewing some applications in real life. These applications have different phenomenology and related parameters to estimate and extract from the signals impinging on the array of sensors. This section gives a hint about every field and the considerable position of DOA estimation within it.

*Radar*

The elementary job of the radar system is to detect and locate objects depending on a simple electromagnetic system. A signal emitted at a pre-specified frequency which hit the objects and scattered into many directions. One of this reflected rays come back in the direction of the radar to be detected by a receiving antenna which delivers it to the receiver. The most important parameter extracted from it is the range between the radar and the object locations. Other parameters include the presence of the target object, the exact location (the direction of arrival and the range), relative velocity and other target related characteristics (Skolnik, 2001).

Radar using one antenna in the traditional radar systems can detect multiple targets with large angular spacing between them. Once the sources are too close or the noise is spatially colored, the radar cannot detect and process the received signals in an optimum way. On the other hand, using array signal processing provides more flexibility and higher capacity to overcome the limitations of the conventional radar system (Haykin, Litva, & Shepherd, 1993).

Phased array radar is one of the most recent developments which attracts many researches because of its ability to control the beam position which gives more angle of freedom to direct the beam over the whole possible search area (Pell, 1988). Recent advances in the technology of communication systems and digital signal processing (DSP) make it possible to implement a digital phased array radar and provide more advanced architectures for the beamformer and the processing of the sampled data of the received signals. In the digital phased array, the weights of the array may be altered to the signal related parameters to overcome the interference. (Talisa, O'Haver, Comberiate, Sharp, & Somerlock, 2016).

*Sonar*

Acoustics waves propagate better on water than on air with higher speed and less attenuation over long distances. Sonar technology is similar to radar technology which used in detection and location of objects and determination of objects characteristics. The different between sonar and radar is that sonar is used underwater and use acoustic waves instead of electromagnetic waves which used in radar systems (Lurton, 2002).

Array signal processing is applied to the sonar systems and provide many benefits where the resolution of the sonar array can be improved to estimate the DOA of the targets. Several array geometries are designed to meet the tough underwater environment characteristics (Zhang, Gao, Chen, & Fu, 2013). Another improvement is the integration of DSP into the array sonar technology which introduces various methods and techniques which has a high impact on estimating a very precise value of the angle of arrival of the present targets (Li, 2012).

*Seismology*

Earthquakes usually accompanied with radiation of waves called seismic waves which propagate through the earth. Studying this waves can improve our knowledge about the tectonics of the earth and the underlying physics of earthquakes which help on expecting and avoiding the destructive influences on populated areas. Seismology also involves the improvements on the instruments and the physics of estimating and detecting the seismic waves which provide information about the inner layers of the earth and the beginning geographical locations and the center of the earthquakes. Many researches have been made through the last century and nowadays many seismic stations are located all over the affected areas which help on discovering the earthquakes early and warming the surrounded cities and populated areas (Shearer, 2009; Udías & Vallina, 1999).

DOA estimation also used to detect and locate the earth reservoirs e.g. oil and natural gas by receiving elastic waves caused naturally or by explosions and extract the related parameters of the underlying layers and the concentration of several reservoirs (Miron, Le Bihan, & Mars, 2005). Different aspects are affecting DOA estimation using seismic array including the geometry of the sensors itself and the nature of the seismic waves which may results on estimation errors (Maranò, Fäh, & Lu, 2014). The earthquakes are often in a sequence of events which demand a highly sensitive and quick detection so several seismic stations are used and the cooperation of this sites to get closely analogous waveforms requires more effective estimation techniques. As the different results of regional sites are assumed to be insignificant where a small difference between neighbors sites led to ambiguity on the detection and estimation of the earthquakes DOAs (Gibbons, Ringdal, & Kværna, 2008).

The different properties of each field where the DOA estimation presents led to several specified and deep studies to fit these characteristics and come up with an improved and may be unique solutions for each different application.

*Astronomy*

Astronomy includes the studying of objects beyond the planet Earth and the investigation of underlying physics of this objects and the interaction between them to gain a better understanding about our universe and the nature of elementary physics and other galaxies and stars origin which contribute to our knowledge of the history of the humanity and the whole universe (Fraknoi, Morrison, & Wolff, 2016). This investigation is done by the observations of the aerospace electromagnetic spectrum from the terrestrial stations and adjacent areas e.g. satellites and space shuttle missions (Vogt, 2001).

A telescope is an instrument that receives the incoming signals and extracts the information from it. The telescope is the elementary resource for almost all the knowledge of astrophysics. Several frequency bands e.g. radio and x-ray are analyzed using a different kind of foci systems. The impinging signals are suffering from low energy and different kind of noise though there is a huge research interest on developing better instruments that are more sensitive and very low noisy techniques and algorithms to extract stellar objects related properties e.g. the rate of impinging, the direction of arrival and the polarization (Bradt, 2004).

*Mobile Communication*

The remote connection between users at different locations including voice conversation, text and multimedia massaging and Internet data exchange using mobile equipment are called mobile communication. Mobile communication systems have been subjected to many significant developments from analog to digital and from serving a few users with speech conversation and text messaging only to crowded areas with more services e.g. Internet and video gaming and still attracting researchers interests. Several kinds of cellular systems are introduced depending on the coverage areas and the bit rate e.g. ground mobile networks, mobile satellite. The fundamental job of any mobile communication system is to keep the connection with mobile stations (MSs) both active and inactive ones and provide the services upon demand and satisfying the minimum requirements for different type of services (Stüber, 2000; Arokiamary, 2009).

The highly growing demands for mobile communications services e.g. more capacity, higher data rate, and low latency increase the needs for new developments. Array signal processing is used by mobile communication systems which help on locating and tracking of the MSs which signals strength vary in power and signal to noise ratio (SNR). Adaptive beamformer techniques depend on array processing are used to strengthen the downlink signal power and maximize the SNR in the direction of the served users while canceling the surrounding interference and jamming by nulling the beam in the direction of their sources (Godara, 1997; A. Al-Nuaimi, Shubair, & O. Al-Midfa, 2005). The DOA estimation also used in the satellite communication systems to determine the attitude of the communication satellites to adjust the beamforming toward the desired direction and so maximizing the signal power and keeping a good data link between the satellite and the ground stations. Different kind of sensors is used to determine the direction of arrival of the signal that contains the attitude information (Yang, He, Jin, Xiong, & Xu, 2014).

*Military Surveillance*

Locating and tracking both moving and standing multiple targets in real time are the most uses of DOA estimation techniques in military reconnaissance and surveillance. Radar and sonar systems are among these systems which have been previously mentioned in this section. The radio frequency interferences (RFI) in such systems are caused inadvertently from surrounding communication systems and intentionally induced by military jammer (Xiaohong, Xue, & Liu, 2014). Not just estimate the DOA of the targets repeatedly but also identify and associate the DOA information with their sources which led to various data association techniques (Cai, Shi, & Zhu, 2017). The surveillance and reconnaissance system includes various information gathering and processing subsystems e.g. wireless sensors network and manned surveillance system which help on the final evaluation of the situation and decision making. These systems help also on the coverage of larger area using unmanned systems e.g. drones, motion detectors (Astapov, Preden, Ehala, & Riid, 2014).

All the above applications illustrate the importance of the DOA estimation on real life and provide a clear motivation for our work on the array processing field.

## 1.3 Narrowband and Wideband Signals

The band of frequencies that is between the lowest and highest frequencies determined at 10 dB below the highest radiation power of the radio frequency signal is called bandwidth (Sabath, Mokole, & Samaddar, 2005). These frequency bands are a fundamental parameter in the communication system and it's affecting the design and operation of almost all the communication devices, algorithms, and systems due to the limitation of handling very few bands in the same efficient and economic system. In this section, the simple difference between narrowband and wideband RF signals are previewed which affect the estimation of DOA information as we will see later (Yoon, Kaplan, & McClellan, 2006).

### 1.3.1 Narrowband signals

The very small portion of frequencies that is proportional to the center frequency of the signal ($\leq 0.01$) is called narrowband signal which is generated by merging the information

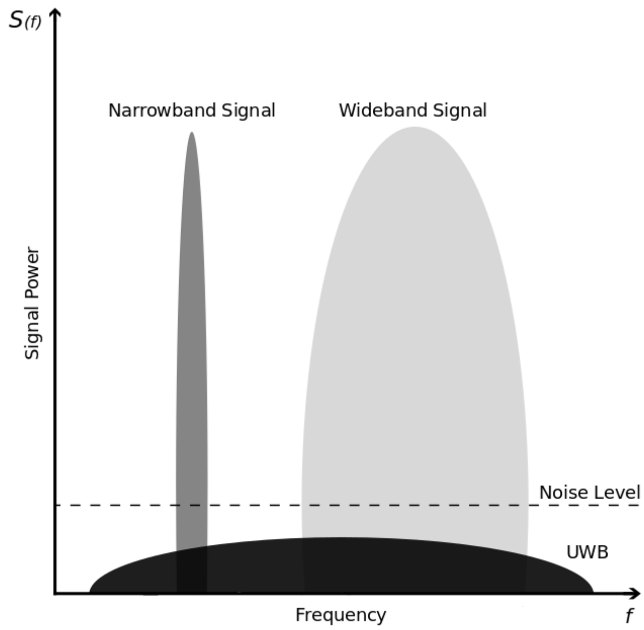


**Figure 1.2:** The narrowband and wideband and ultra-wideband signals

into one sinusoidal carrier using different modulation techniques and is small enough to be treated as one frequency on the detection side. Due to the difficulty of generating a true sinusoidal wave, the bandwidth of the signal is spreading lightly over the two sides of the central frequency. In general, narrowband signals occupy small portions of the frequency spectrum, affected by less noise and need less transmission power which make it very effective for many applications where the available spectrum is limited and the served users is too many (Barras, Ellinger, & Jäckel, 2002; Sabath et al., 2005).

### 1.3.2 Wideband signals

Figure 1.2 clarifies the difference between narrowband and wideband signals which mainly in bandwidth. Any signal spreads over a large portion of the frequency spectrum that is proportional to its center frequency ($0.01 \leq ... \leq 0.25$) is called wideband signal. The wider bandwidth means higher data rate which is also effective for many applications. For example, the radar system uses wideband signals to achieve high resolution and better detection of targets (Barras et al., 2002; Yoon, Kaplan, & H. McClellan, 2006).

In the next chapter, the difference between narrowband and wideband in the estimation of

DOA will be investigated and a signal model for each type will be reviewed.

## 1.4 Computational Costs and The Complexity of An Algorithm (O)

Computational complexity is simply about the determination of needed resources for a given algorithm from the available resources which are limited in many applications. It's a key metric of how efficient an algorithm does where computational processing is the main consumer of time, storage and capacity for a specific algorithm. It's also a sign of negative results on an efficiency metric if the computational costs of an algorithm are too high to be considered in real applications (Gács & Lovász, 2000).

The calculation of complexity is often an approximation of the exact number of operations per algorithm due to the complex mathematical expressions of the algorithms. Though higher order terms of the expressions are only considered for the running time of the algorithm (Sipser, 2012). This done when the inputs are very large which is the case in DOA estimation methods.

For example, the function $f(n) = 9n^4 + 2n^3 + 10n$ includes three terms where the highest order term is $9n^4$ which computational cost can be approximated as $n^4$. The multiplication and addition are of the same order of complexity $O(1)$. The notation $O(1)$ is called asymptotic notation or big-O notation which used to illustrate the complexity of a function or an algorithm (Sipser, 2012). Another example illustrates the complexity of the matrix. If we have a matrix $A$ of size [$m \times n$] multiplied by matrix $B$ of size [$n \times k$] then the complexity is $O(mk(2n-1))$ which is approximately $O(2mnk)$.

In this thesis, the complexity of well known wideband DOA estimation methods is investigated to show that the reduction of complexity will be a great improvement which would be vital for many applications.

# CHAPTER 2

# LITERATURE REVIEW

In this chapter, the basics of DOA estimation for both narrowband and wideband signals will be reviewed then the most well-known estimation methods will be previewed which applied to narrowband signals directly. Also, we will review the main idea behind these estimation methods which is the orthogonality of signal and noise subspaces.

## 2.1 The Signal Model

In Figure 1.1, an array of $M$ radioactive elements are linearly arranged. This array geometry is called uniform linear array (ULA) which we will assume through this dissertation. The array elements are equally spaced by a distance $d$ between every two successive elements that is not larger than half the wavelength $\lambda$ of the highest frequency of the impinging signals. A $D$ signals are emitted far away from the array that each wavefront impinges on all elements from the same direction of emission with the same angle of arrival $\theta$.

As the first element of the array is assumed to be the reference for the array, each wave impinges on the following elements will travel a longer distance than the wave impinge on the first element with an additional distance equal to

$$(m-1) \cdot d \cdot \sin(\theta_k), \tag{2.1}$$

where $m = 1, 2, 3 \ldots M$ is the index of array elements. The signal received by $m$th array element without noise is

$$x_m(t) = s_k(t) \cdot \exp(\frac{-j2\pi(m-1)d\sin(\theta_k)}{\lambda}), \tag{2.2}$$

$$x_m(t) = a_m(\theta_k) \cdot s_k(t), \tag{2.3}$$

where $s_k(t)$ is the $k$th received wavefront. The exponential term multiplied by the wavefront is the steering vector $a(\theta_k)$ which has the DOA information for the $k$th signal.

$$\Phi_k = \frac{-2\pi(m-1)d\sin(\theta_k)}{\lambda}, \tag{2.4}$$

called the spatial frequency that is related to the source of the $k$th received wavefront impinging on the array with $\theta_k$ angle. (Chen, Gokeda, & Yu, 2010).

The signal model in (Schmidt, 1986) assumes $D$ signals with noise which could be merged to the signal on the air or on the receiving equipment. Though the signal model is

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} = \begin{bmatrix} a(\theta_1) & a(\theta_2) & \dots & a(\theta_D) \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_D \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_M \end{bmatrix} \tag{2.5}$$

or

$$\mathbf{X} = \mathbf{AS} + \mathbf{N}. \tag{2.6}$$

The steering matrix $\mathbf{A}$ depends on the array elements locations related to the reference element coordinates and the angles of arrival of the received signals. Through the next sections, we will review how to extract this DOA information.

### 2.1.1 Narrowband signals

The steering matrix $\mathbf{A}$ for narrowband signal is

$$\mathbf{A} = \begin{bmatrix} a(\theta_1) & a(\theta_2) & \dots & a(\theta_D) \end{bmatrix}^T \tag{2.7}$$

$$= \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{-j\varPhi_1} & e^{-j\varPhi_2} & \dots & e^{-j\varPhi_D} \\ e^{-j2\varPhi_1} & e^{-j2\varPhi_2} & \dots & e^{-j2\varPhi_D} \\ \vdots & \vdots & \vdots & \vdots \\ e^{-j(M-1)\varPhi_1} & e^{(-j(M-1)\varPhi_2} & \dots & e^{-j(M-1)\varPhi_D} \end{bmatrix} \tag{2.8}$$

This matrix is a vandermond matrix where $\theta_i \neq \theta_j, i \neq j$ which is a kind of centro-symmetric matrix (Strang, 2009).

To get a high resolution, $N$ snapshots are taken over a wide angle of search to estimate an accurate value for the angles of arrival. The received signals are merged with noise that we assume it is uncorrelated while the wavefronts received by each array element are correlated because of arriving from the direction of the same sources. To extract the DOA information, we calculate the covariance matrix $[M \times M]$ of the data as follows

$$\mathbf{V}_{xx} = E\{\mathbf{X}(t)\mathbf{X}^H(t)\} = \mathbf{A}\mathbf{P}_{ss}\mathbf{A}^H + \sigma^2\mathbf{I}, \tag{2.9}$$

where $E\{.\}$ is the statistical expectation, $\mathbf{P}_{ss}$ is the signal correlation matrix, $\sigma^2$ is the noises variance and $\mathbf{I}$ is the identity matrix (Chen et al., 2010). Due to difficulty in obtaining the real correlation matrix, the statistical expectation is estimated as following

$$\hat{\mathbf{V}}_{xx} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{X}(t_i)\mathbf{X}^H(t_i). \tag{2.10}$$

$\mathbf{V}_{xx}$ is a Hermitian matrix that all its eigenvalues are real and all its eigenvectors are orthogonal when they are corresponding to different eigenvalues (Strang, 2009).

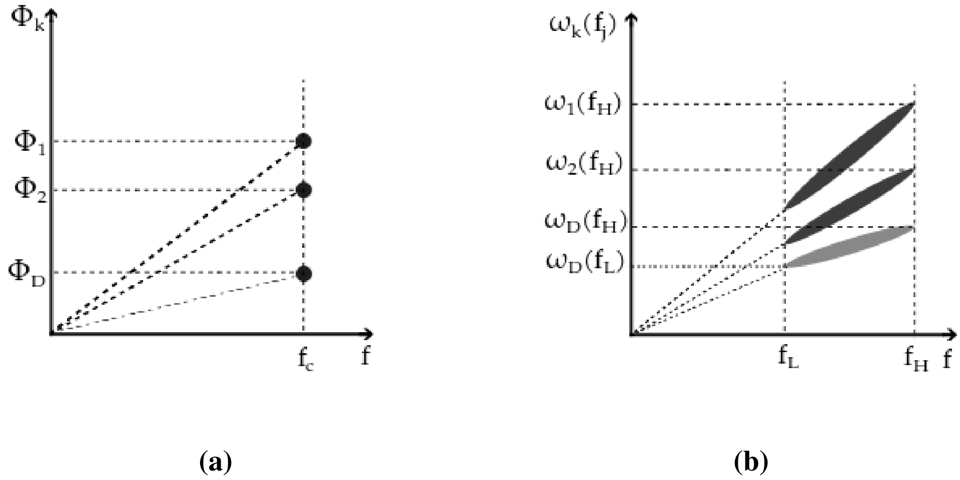


**Figure 2.1:** Visual illustration of the relation between the spatial and physical frequencies and the main difference between (a) narrowband signals and (b) wideband signals

### 2.1.2 Wideband signals

In wideband signals, the $\mathbf{A}$ steering matrix depends on frequency and not only the angle of arrival as in narrowband signals where the shift in time is considered as phase shift which is approximated as constant over the bandwidth. Figure 2.1 illustrates this dependency and show that narrowband signal is treated as one frequency while the wideband signal is divided to a number of narrowband signals by sampling it using Fourier Transform into $L$ frequency bins so the DOA estimation methods for narrowband can be applied (Yoon, Kaplan, & H. McClellan, 2006).

Though the signal model for a $D$ wideband incident wavefronts on a $M$ array elements is

$$\mathbf{X}(f_j) = \mathbf{A}(f_j, \theta)\mathbf{S}(f_j) + \mathbf{N}(f_j) = \sum_{j=1}^{L} \mathbf{S}_k(f_j)e^{-j2\pi f_j u_m \sin\theta_k} + \mathbf{N}_m(f_j) \tag{2.11}$$

where $u_m = \dfrac{d_m}{v}$ and $d_m = (m-1)d$ is the linearly distance between any array element and the reference element, $v$ is the wave propagation speed, $j = 1, 2, 3 \cdots L$ frequency bins, $k = 1, 2, 3 \cdots D$ number of incident wavefronts and $m = 1, 2, 3 \cdots M$ number of array elements. All this data are captured for $n = 1, 2, 3 \cdots N$ number of snapshots. The steering vector for frequency bin $f_j$ of any incident signal at angle $\theta_k$ will take the following form

$$\mathbf{A}_m(f_j,\theta_k) = \begin{bmatrix}\mathbf{a}_1(f_j,\theta_k) & \mathbf{a}_2(f_j,\theta_k) & \dots & \mathbf{a}_M(f_j,\theta_k)\end{bmatrix}^T \tag{2.12}$$

$$= \begin{bmatrix}1 & e^{-j2\pi f_j u_1 \sin\theta_k} & e^{-j2\pi f_j u_2 \sin\theta_k} & \dots & e^{-j2\pi f_j u_M \sin\theta_k}\end{bmatrix}^T \tag{2.13}$$

Equation 2.11 is the fundamental vector matrix for further processing to extract the DOA information with high resolution at a reasonable time.

## 2.2 The Signal and Noise Subspaces

Any matrix consists of vector columns that span a space of any combination of this columns. Hermitian matrix is a type of complex matrix that consists of complex vectors that is a subspace of the complex numbers space $\mathbf{C}^n$. Orthogonality of two vectors is that the dot product of them must equal zero which led to that each subspace of different vectors is orthogonal to another subspace of vectors. These subspaces are called orthogonal to each other (Strang, 2009).

The main idea behind the orthogonality is that the covariance matrix $[M \times P]$ declared in Equation 2.10 is a Hermitian matrix where all its eigenvectors are orthogonal. If the $D$ incident wavefronts is less than the $M$ array elements such that ($k \leq M; k \leq P$), then the matrix of rank $k$ and if $k = M$ then its full rank. The rank of the matrix is its dimension. This matrix can be spanned by any unitary matrix that includes any subset of columns of the covariance matrix. The covariance matrix eigen decomposition is

$$\mathbf{V}_{xx} w_r = e_r w_r = \sigma^2 w_r \tag{2.14}$$

and its eigenvalues can be sort as following

$$e_1 \geq e_2 \geq \cdots \geq e_D \geq e_{D+1} \geq \cdots \geq e_M \geq 0 \tag{2.15}$$

which can be distinguished to two sets; the highest eigenvalues corresponding to the signals and smallest eigenvalues are corresponding to noises. A signal and noise subspaces spanning the corresponding eigenvectors of the two eigenvalues sets can be arranged respectively as following

$$\mathbf{E}_s = \begin{bmatrix} W_1 & W_2 & \dots & W_D \end{bmatrix} \tag{2.16}$$

$$\mathbf{E}_n = \begin{bmatrix} W_{D+1} & W_{D+2} & \dots & W_M \end{bmatrix} \tag{2.17}$$

The number of incident signals is critical in determining these subspaces. Signal subspace is the column subspace and the noise subspace is the null row space of the covariance matrix (Paulraj et al., 1993). After determining the signal subspace, the DOA information can be extracted from it (Chen et al., 2010).

For more information about the matrix subspaces, refer to this good reference (Strang, 2009) in linear algebra written by Prof. Strang Gilbert.

### 2.3 DOA Estimation Methods

Several approaches tried to introduce high-resolution DOA estimation methods. Here we reviewed the most well-known methods which gain huge interests in the last decades both in real life application and research developments due to its potential characteristics (Paulraj et al., 1993).

#### 2.3.1 MUSIC

Based on the idea of signal and noise subspaces; the Multiple SIgnal Parameter Estimation (MUSIC) method claims that when the number of impinging wavefronts on the array is lower than the number of its elements ($D \leq M$), then the $\mathbf{AP}_{ss}\mathbf{A}^H$ is singular and

$$|\mathbf{AP}_{ss}\mathbf{A}^H| = |\mathbf{V}_{xx} - e\mathbf{I}| = 0, \tag{2.18}$$

where the $D$ highest eigenvalues of the covariance matrix should be corresponding to the incident signals and the rest ($M - d$) eigenvalues is equivalent to zero where $e_{min}\mathbf{I} = \sigma^2\mathbf{I}$ but due to the shortage of getting more snapshots of the sampled data in practical, its values variance can be in cluster. Its limit goes to zero when the number of data samples goes to infinity (Schmidt, 1986).

Let the corresponding eigenvectors $w_r$ of the covariance matrix including the noise eigenvalues only which are the minimum as following

$$\mathbf{V}_{xx} w_r = e_{r,min} w_r, \tag{2.19}$$

where $r = D+1, D+2, \cdots, M$. Substitute Equation 2.9 into 2.19, then the equation will be

$$(\mathbf{A}\mathbf{P}_{ss}\mathbf{A}^H + e_{min}\mathbf{I}) w_r = e_{r,min} w_r, \tag{2.20}$$

and since $\mathbf{P}_{ss}$ is non-singular and $\mathbf{A}$ is full rank, then

$$\mathbf{A}\mathbf{P}_{ss}\mathbf{A}^H w_r = 0 \tag{2.21}$$

or

$$\mathbf{A}^H w_r = 0 \tag{2.22}$$

which indicate that the eigenvectors of noise subspace are orthogonal to the columns of steering matrix that are in the signal subspace. This means that nulling the corresponding noise subspace in the steering matrix will declare the DOA of incident signals but due to the lack in snapshots, some noise will appear. The power spectrum for the MUSIC algorithm is

$$\mathbf{F}(\theta) = \frac{1}{\mathbf{A}^H(\theta)\mathbf{E}_n\mathbf{E}_n^H\mathbf{A}(\theta)} \tag{2.23}$$

where $\mathbf{E}_n$ is a matrix of noise eigenvectors declared in Equation 2.17. When the denominator in Equation 2.23 goes to zero for the true angles of the signals, the power spectrum will have peaks indicating this angles.

### 2.3.2 Root-MUSIC

MUSIC algorithm gives peaks on the true angles of arrival and to find them a human observation or some sort of searching algorithm is needed which is not too practical. Therefore, to extract the peaks root-MUSIC was introduced and proposed a final polynomial form as following

$$\mathbf{Q}(\theta) = \sum_{u=-N+1}^{N+1} x_u z^{-u} \tag{2.24}$$

and peaks of the MUSIC algorithm are the roots of this polynomial.

Let's assume that

$$\mathbf{G} = \mathbf{E}_n \mathbf{E}_n^H \tag{2.25}$$

then the MUSIC spectrum will be

$$\mathbf{F}(\theta)^{-1} = \sum_{v=1}^{N} \sum_{u=1}^{N} z^u \mathbf{G}_{vu} z^{-v} \tag{2.26}$$

$$= \sum_{v=1}^{N} \sum_{u=1}^{N} z^{u-v} \mathbf{G}_{vu} \tag{2.27}$$

where the $z$ is the steering vectors. By letting $h = u - v$, the Equation 2.27 can be simplified to

$$\mathbf{F}(\theta)^{-1} = \sum_{h=-N+1}^{N-1} z^h \mathbf{G}_h \tag{2.28}$$

where

$$\mathbf{G}_h = \sum_{h=u-v} \mathbf{G}_{vu} \tag{2.29}$$

which is the summation of all $\mathbf{G}$ diagonal elements. The angles of arrival are corresponding to the $M$ roots close to the unit circle (Barabell, 1983). After determining this roots, the angle of arrival will be

$$\theta = \sin^{-1}(\frac{\lambda}{2\pi d})arg(z_1) \tag{2.30}$$

### 2.3.3 ESPRIT

ESPRIT stands for Estimation of Signal Parameters via Rotational Invariance Technique which is a well known DOA estimation method depending on subdividing the steering array to two well-displaced arrays that exploit the DOA information by taking the eigen decomposition of the estimated data covariance matrix mentioned in Equation 2.10 and this two sub-arrays (Paulraj, Roy, & Kailath, 1986).

The two sub-arrays will have a signal model given by

$$\mathbf{X}_1 = \mathbf{AS} + \mathbf{N}_1 \tag{2.31}$$

$$\mathbf{X}_2 = \mathbf{A\Gamma S} + \mathbf{N}_2, \tag{2.32}$$

where $\mathbf{N}_1$ and $\mathbf{N}_2$ are the additive noise vectors added to the two arrays respectively, and $\mathbf{\Gamma}$ is the rotation $[M \times M]$ matrix expresses the displacement between the two arrays and defined by

$$\mathbf{A} = \begin{bmatrix} z_1 & 0 & \dots & 0 \\ 0 & z_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & z_M \end{bmatrix}, \tag{2.33}$$

where $z_m = e^{j\lambda d \cos\theta_m}$ and $d$ is the distance between the two arrays. The cross covariance matrix that relates the two arrays is

$$\mathbf{V}_{x_1 x_2} = E\{\mathbf{X}_1(t)\mathbf{X}_2^H(t)\} = \mathbf{A\Gamma}^H\mathbf{P}_{ss}\mathbf{A}^H, \tag{2.34}$$

and according to eigen decomposition of covariance matrix stated in Equation 2.20, we can say that

$$\mathbf{V}_{xx} - e_{min}\mathbf{I} = \mathbf{A}\mathbf{P}_{ss}\mathbf{A}^H = \mathbf{Q}_{xx}, \tag{2.35}$$

which implies

$$\mathbf{Q}_{xx} - \beta\mathbf{V}_{x_1x_2} = \mathbf{A}\mathbf{P}_{ss}\mathbf{A}^H - \beta\mathbf{A}\mathbf{\Gamma}^H\mathbf{P}_{ss}\mathbf{A}^H = \mathbf{A}\mathbf{P}_{ss}(\mathbf{I} - \beta\mathbf{\Gamma}^H)\mathbf{A}^H \tag{2.36}$$

where the eigenvalues of the subspace spanned by both $\mathbf{Q}_{xx}$ and $\mathbf{V}_{x_1x_2}$ and equal to the diagonal elements of $\mathbf{\Gamma}$. Other eigenvalues in the null space of this pair are all equal to zeros.

Though, the ESPRIT method tries to get the eigen decomposition of the covariance matrix, then calculate the matrices in Equation 2.34 and 2.35 and get the $d$ eigenvalues of the matrix pair $\{\mathbf{A}\mathbf{\Gamma}^H\mathbf{P}_{ss}\mathbf{A}^H, \mathbf{A}\mathbf{P}_{ss}\mathbf{A}^H\}$ that within the unit circle which construct the $\Gamma$ subspace (Roy, Paulraj, & Kailath, 1986).

### 2.3.4 Latest improvements

In recent decades, DOA estimation got a huge interest in the field of research and developments to explore further areas and developing the methods to be more efficient and robust. This areas of interest include different geometry arrays, wideband signals, and correlated signals. Some of this improvements are discussed in this references (Yoon, Kaplan, & H. McClellan, 2006; Chen et al., 2010; Krim & Viberg, 1996; Paulraj et al., 1993; Amin & Zhang, 2009). In this thesis, we are more interesting about the DOA estimation of wideband signals and its methods.

CHAPTER 3

# WIDEBAND DOA ESTIMATION METHODOLOGY

In this chapter, we will discuss the various methods of wideband DOA estimation that can be categorized into two main ways (Incoherent and coherent) of processing the received signal data that was expressed in Equation 2.10. The basic concept of wideband DOA estimation illustrated in Figure 3.1 is to use Fast Fourier Transform (FFT) or a filtering technique on the received data to subdivide the wideband into a number of narrowbands. Then the data correlation matrix for each sub-band is estimated and then several techniques are proposed to apply as we will see later. The final result of those methods is to get a matrix that we can apply one of the narrowband DOA estimation methods on it. Often MUSIC algorithm is applied (Demmel, 2009).

## 3.1 Incoherent Methods

The incoherent methods apply narrowband estimation methods directly on the correlation matrices of each sub-band and then stratify some techniques to merge the produced estimates in a way that results in a high resolution and accurate evaluation of the true angles of the sources (el Ouargui, Frikel, & Said, 2018).

### 3.1.1 IMUSIC

In incoherent MUSIC; after the correlation matrix for each frequency bin is estimated, the eigen decomposition is applied for each correlation matrix as in the narrowband signals to

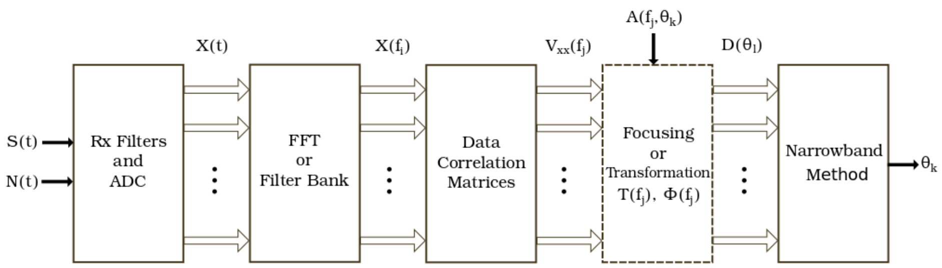


**Figure 3.1:** The basic concept of the DOA estimation of a wideband signals

get the signal and noise subspaces. Then the average is taken, for eigenvectors of the noise subspaces, for all frequency bins to estimate the DOA. IMUSIC spectrum is calculated by either one of the following

$$\mathbf{F}_1(\theta) = \frac{\mathbf{a}^H(f_j,\theta)\mathbf{a}(f_j,\theta)}{\frac{1}{L}\sum_{j=1}^{L}\frac{1}{M-D}\sum_{m=D+1}^{M}\mathbf{a}^H(f_j,\theta)\mathbf{E}_n(f_j)\mathbf{E}_n^H(f_j)\mathbf{a}(f_j,\theta)}, \tag{3.1}$$

$$\mathbf{F}_2(\theta) = \frac{\mathbf{a}^H(f_j,\theta)\mathbf{a}(f_j,\theta)}{\prod_{j=1}^{L}\frac{1}{M-D}\sum_{m=D+1}^{M}[\mathbf{a}^H(f_j,\theta)\mathbf{E}_n(f_j)\mathbf{E}_n^H(f_j)\mathbf{a}(f_j,\theta)]^{\frac{1}{L}}}. \tag{3.2}$$

where the $j = 1, 2, \cdots L$ is the number of frequency bins and $m = D+1, D+2, \cdots M$ is the number of eigenvectors corresponding to the noise subspace. The peaks over the produced spectrum are corresponding to the true angles of the sources. This method called Incoherent MUSIC due to the use of same narrowband MUSIC method for all frequency bins at once (Wax, Shan, & Kailath, 1984).

This method is usually effective in high SNR conditions and also when the signals are well separated from each other while suffering from errors and produce side peaks at wrong angles when the SNR is low which is the case in many situations. Also, the level of noise is assumed to be flat over the frequency range which is not the usual condition (Yoon, Kaplan, & H. McClellan, 2006). This drawbacks in incoherent methods led to the proposition of the coherent methods which overcome this issues.

### 3.2 Coherent Methods

The coherent methods try to apply some focusing and transformation strategies and therefore, achieving a single universal covariance matrix that corresponding to the wideband signal which based on the fact that noise subspace vectors are orthogonal to the signal corresponding vectors in the steering matrix. Thus profiting from the assembling of non uniformly distributed DOA information in different frequencies over the wideband spectrum of the target signals and also reducing the complexity of calculating the eigen decomposition for each frequency bin (Yoon, Kaplan, & McClellan, 2006; Wang & Kaveh, 1985).

### 3.2.1 CSS

Coherent Signal Subspace (CSS) method was the first to propose the coherent scheme which basically depends on the pre-estimation of the DOA using one of the narrowband methods and then applying a focusing matrix thus, transforms the correlation matrices for all frequencies into a reference focusing frequency $f_0$ and it is assumed that a transformation matrix $\mathbf{T}(f_j)$ transforms the steering matrix into a new steering matrix at $f_0$ given by

$$\mathbf{T}(f_j)\mathbf{A}(f_j) = \mathbf{A}(f_0) \tag{3.3}$$

Let the correlation matrix of the $j$th frequency calculated by

$$\mathbf{V}_{xx}(f_j) = \mathbf{A}(f_j,\theta)\mathbf{P}_{ss}(f_j)\mathbf{A}^H(f_j,\theta) + \sigma^2(f_j)\mathbf{I}, \tag{3.4}$$

where $\mathbf{P}_{ss}$ is the signal correlation matrix, $\sigma^2$ is the noises variance and $\mathbf{I}$ is the identity matrix. The general correlation matrix for all frequency bins which called universal spatial correlation matrix (USCM) is calculated as follows

$$\hat{\mathbf{V}}_{f0}(f_j) = \sum_{j=1}^{L} \mathbf{T}(f_j)\mathbf{V}_{xx}(f_j)\mathbf{T}^H(f_j) \tag{3.5}$$

where the $\mathbf{T}(f_j)$ is the transformation matrix from $f_j$ to $f_0$ which is a diagonal and unitary matrix given by

$$\mathbf{T}(f_j) = \begin{bmatrix} \frac{a_1(f_0,\theta_0)}{a_1(f_k,\theta_0)} & 0 & \dots & 0 \\ 0 & \frac{a_2(f_0,\theta_0)}{a_2(f_k,\theta_0)} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \frac{a_M(f_0,\theta_0)}{a_M(f_k,\theta_0)} \end{bmatrix}. \tag{3.6}$$

where the steering vector $a_m(f_0,\theta_0)$ is $m$th element at the estimated $\theta_0$ angle using Capon or Periodogram method on the preliminary step (Wang & Kaveh, 1985). This is the simplest way to estimate the transformation matrix. Another higher resolution and more accurate method is the signal subspace transformation (SST) matrix proposed in (Doron & Weiss, 1992). From equations 3.5 and 3.6 and using the assumption in 3.3, the general correlation matrix can be calculated as following

$$\hat{\mathbf{V}}_{f0}(f_j) = \sum_{j=1}^{L} [\mathbf{T}(f_j)\mathbf{A}(f_j,\theta)\mathbf{P}_{ss}(f_j)\mathbf{T}^H(f_j)\mathbf{A}^H(f_j,\theta) + \sigma^2(f_j)\mathbf{T}(f_j)\mathbf{T}^H(f_j)] \quad (3.7)$$

$$= \sum_{j=1}^{L} [\mathbf{A}(f_0,\theta)\mathbf{P}_{ss}(f_j)\mathbf{A}^H(f_0,\theta) + \sigma^2(f_j)\mathbf{T}(f_j)\mathbf{T}^H(f_j)] \quad (3.8)$$

$$= \mathbf{A}(f_0,\theta_0)\sum_{j=1}^{L} [\mathbf{P}_{ss}(f_j)]\mathbf{A}^H(f_0,\theta_0) + B\sum_{j=1}^{L} \sigma^2(f_j)\mathbf{I} \quad (3.9)$$

it's can be further simplified to

$$\hat{\mathbf{V}}_{f0}(f_j) = \mathbf{A}(f_0,\theta_0)\mathbf{P}_{ss}(f_0)\mathbf{A}^H(f_0,\theta_0) + \sigma^2(f_0)\mathbf{I} \quad (3.10)$$

where

$$\mathbf{P}_{ss}(f_0) = \sum_{j=1}^{L} \mathbf{P}_{ss}(f_j) \quad (3.11)$$

and

$$\sigma^2(f_0) = B\sum_{j=1}^{L} \sigma^2(f_j) \quad (3.12)$$

Equation 3.10 is approximately the same as in narrowband method and MUSIC can be used to estimate the DOA information. It can be easily shown that the matrix in 3.10 has a smallest $M - d$ eigenvalues corresponding to the noise subspace and therefore MUSIC algorithm can be applied (Wang & Kaveh, 1985).

CSS estimation method is depending in the first place on the initial estimation of the focusing angles and how it close to the true DOA angles which may dominate the results of the algorithm and led to the wrong estimation in case of poor evaluation of the focusing angles (Pal & Vaidyanathan, 2009). Also, CSS is an iterative method and each iterate is evaluated in only one direction, thus, if there are multiple well-separated sources, then the iterations will increase. Also, the successive iterates help in improving the following estimation. Despite the multiple iterations of CSS method, it takes less computational costs compared with the IMUSIC because it is applying the eigen decomposition once on the general correlation matrix while IMUSIC applies the eigen decomposition to all frequency bins correlation matrices. CSS method acts well in the low SNR due to the taking the average estimation of the coherent frequencies. It is also more robust to the noise than IMUSIC which act better in high SNR (Wang & Kaveh, 1985; Yoon, Kaplan, & McClellan, 2006).

### 3.2.2 WAVES

Weighted Average of Signal Subspace (WAVES) method is a modified algorithm that based on Weighted subspace fitting (WSF) method (Viberg, Ottersten, & Kailath, 1991). WSF method exploits the DOA parameters using the signal subspaces for frequency bin as following

$$\hat{\theta}_k = \arg \min_{k=1,2,\cdots,D} \{\sum_{j=1}^{L} |\mathbf{A}(f_j,\theta)\mathbf{Y}(f_j) - \mathbf{E}_s(f_j)\mathbf{G}(f_j)|_F^2\} \tag{3.13}$$

where $\mathbf{G}(f_j)$ is a diagonal matrix acting as a weighting matrix which diagonal elements calculated by

$$\mathbf{G}(f_j)_{kk} = \frac{e_k(f_j) - \sigma^2}{(e_k(f_j)\sigma^2)^{\frac{1}{2}}} \tag{3.14}$$

where $e_k(f_j)$ is the largest eigenvalue corresponding to the $k$th signal at the $j$th frequency and

$$\mathbf{Y}(f_j) = \mathbf{G}(f_j)\mathbf{E}_s(f_j)\mathbf{A}^{\dagger}(f_j,\theta_j) \tag{3.15}$$

where $\mathbf{E}_s(f_j)$ is the signal subspace at the $j$th frequency and $\mathbf{A}^{\dagger}(f_j,\theta_j)$ is the pseudo-inverse matrix of the steering matrix calculated by

$$\mathbf{A}^{\dagger}(f_j,\theta) = (\mathbf{A}^T(f_j,\theta)\mathbf{A}(f_j,\theta))^{-1}\mathbf{A}^T(f_j,\theta) \tag{3.16}$$

WAVES method uses the idea of transformation matrix in Equation 3.6 that used by CSS method to transform the weighted signal subspaces into a universal one and applies this transformation matrix to the WSF (di Claudio & Parisi, 2001). Though Equation 3.13 will be

$$\hat{\theta}_k = \arg \min_{k=1,2,\cdots,D} \{\sum_{j=1}^{L} |\mathbf{A}(f_0,\theta)\mathbf{Y}(f_j) - \mathbf{T}(f_j)\mathbf{E}_s(f_j)\mathbf{G}(f_j)|_F^2\} \tag{3.17}$$

WAVES method suggests a new matrix $\hat{\mathbf{Z}}(f_j)$ with rank $D$ to get the signal subspace as following

$$\hat{\mathbf{Z}}(f_j) = \begin{bmatrix}\mathbf{T}(f_1)\mathbf{E}_s(f_1)\mathbf{G}(f_1) & \mathbf{T}(f_2)\mathbf{E}_s(f_2)\mathbf{G}(f_2) & \cdots & \mathbf{T}(f_L)\mathbf{E}_s(f_L)\mathbf{G}(f_L)\end{bmatrix} \tag{3.18}$$

but due to noise, its rank will be a full rank and after applying the SVD on $\hat{\mathbf{Z}}(f_j)$ matrix as following

$$SVD\{\hat{\mathbf{Z}}(f_j)\} = \begin{bmatrix}\mathbf{E}_s & \mathbf{E}_n\end{bmatrix} \begin{bmatrix} e_s & 0 \\ 0 & e_n \end{bmatrix} \begin{bmatrix} W_s \\ W_n \end{bmatrix} \tag{3.19}$$

the universal signal and noise subspaces can be obtained. The left singular vectors $\mathbf{E}_n$ corresponding to the $M - D$ universal noise subspace which can be used in MUSIC algorithm to get the DOA angles (di Claudio & Parisi, 2001).

WAVES method performs better than CSS method due to the use of the signal subspaces instead of the correlation matrices itself but suffering from complexity which is much more than CSS method due to the calculation of eigen decomposition for all frequency bins of the signal. Also, WAVES method depends on the initial estimation of focusing angles to get the transformation matrix which affects the accuracy of the whole algorithm DOA estimation (di Claudio & Parisi, 2001; Yoon, 2004).

### 3.2.3 TOPS

In (Yoon, Kaplan, & McClellan, 2006; Yoon, 2004), a new DOA method called Test of Orthogonality of Projected Subspaces (TOPS) is proposed to overcome the main drawback of CSS and WAVES methods which is the dependency on the initial focusing estimation. TOPS method depends on the transformation of the steering matrix at $j$th frequency to another focusing frequency called reference frequency. Unlike the CSS and WAVES methods, in which the transformation matrix is applied once, the transformation applied to all frequencies in TOPS method. The transformation matrix is diagonal and given by Equation 3.6. Applying this matrix $\mathbf{\Psi}$ at the $l$th frequency and $l$th angle to the array steering matrix $\mathbf{a}_m(f_j, \theta_j)$ at $j$th frequency and $j$th angle gives

$$\mathbf{\Psi}_{m,m}(f_l, \theta_l)\mathbf{a}_m(f_j, \theta_j) = e^{-j2\pi f_l v_m \sin(\theta_l)} e^{-j2\pi f_j v_m \sin(\theta_j)} \tag{3.20}$$

$$= e^{-j2\pi v_m (f_l + f_j)(\frac{f_l \sin(\theta_l)}{f_l + f_j} + \frac{f_j \sin(\theta_j)}{f_l + f_j})} \tag{3.21}$$

$$= e^{-j2\pi v_m f_h \sin(\theta_h)} \tag{3.22}$$

$$= \mathbf{a}_m(f_h, \theta_h) \tag{3.23}$$

where $f_h = f_l + f_j$ and

$$\sin(\theta_h) = \frac{f_l \sin(\theta_l)}{f_l + f_j} + \frac{f_j \sin(\theta_j)}{f_l + f_j} \tag{3.24}$$

this transforms the steering matrix from frequency $j$ and angle $\theta_j$ to frequency $h$ and angle $h$ which is the aim of the TOPS to superimpose all frequencies into one frequency bin using the idea of transformation. Note that, $\sin(\theta_h) = \sin(\theta_j)$ when $\theta_j = \theta_l$.

Then the eigen decomposition of all frequencies bins correlation matrices is calculated to obtain the signal subspaces $\mathbf{E}_s(f_j)$. This $\mathbf{E}_s(f_j)$ is the signal subspace that spans the same range spanned by the steering matrix. Though it can be assumed that

$$\mathbf{E}_s(f_j) = \mathbf{A}_m(f_j, \theta)\mathbf{C}_j \tag{3.25}$$

where $\mathbf{C}_j$ is a full rank matrix. Now, applying the transformation matrix to Equation 3.25 will give

$$\boldsymbol{\Psi}_{m,m}(\Delta f, \vartheta)\mathbf{E}_s(f_j) = \boldsymbol{\Psi}_{m,m}(\Delta f, \vartheta)\mathbf{A}_m(f_j, \theta)\mathbf{C}_j \tag{3.26}$$

$$= \mathbf{A}_m(f_h, \theta)\mathbf{C}_j \tag{3.27}$$

where $\Delta f = f_i - f_j$ and

$$\sin(\theta_h) = \frac{\Delta f \sin(\vartheta)}{f_i} + \frac{f_j \sin(\theta_j)}{f_i}. \tag{3.28}$$

Here the transformation is using this matrix $\boldsymbol{\Psi}_{m,m}(\Delta f, \vartheta)$ to transform from the $j$th frequency to the $r$th frequency. The transformation led to the elementary idea underlying TOPS method which is the range space spanned by the transformed signal subspace is the same as of the original steering matrix $\mathbf{A}_m(f_j, \theta)$ as follows

$$range\{\boldsymbol{\Psi}_{m,m}(\Delta f, \vartheta)\mathbf{E}_s(f_j)\} = range\{\mathbf{A}_m(f_j, \theta)\} \tag{3.29}$$

.

Though, TOPS method constructs the following matrix

$$\mathbf{Q}(\vartheta) = \Big[\mathbf{Y}^H(f_1)\mathbf{E}_n(f_1) \quad \mathbf{Y}^H(f_2)\mathbf{E}_n(f_2) \quad \cdots \quad \mathbf{Y}^H(f_L)\mathbf{E}_n(f_L)\Big] \tag{3.30}$$

which used to estimated the true angles by applying the SVD and taking the least singular values and where

$$\mathbf{Y}(f_j) = \mathbf{P}(f_j)\boldsymbol{\Psi}_{m,m}(\Delta f_j, \vartheta)\mathbf{E}_s(f_0) \tag{3.31}$$

where $\Delta f_j = f_j - f_0$ and $\mathbf{P}(f_j)$ is the projection matrix used to reduce the errors occur on the calculation of $\mathbf{Q}$ matrix and calculated by

$$\mathbf{P}(f_j) = \mathbf{I} - (\mathbf{a}_m^H(f_j, \vartheta)\mathbf{a}_m(f_j, \vartheta))^{-1}\mathbf{a}_m(f_j, \vartheta)\mathbf{a}_m^H(f_j, \vartheta). \tag{3.32}$$

which is a projection in the null space of $\mathbf{a}_m(f_j, \vartheta)$. Finally the TOPS spectrum will be

$$F(\theta) = \arg\min(\frac{1}{l_{min}(\theta)}) \tag{3.33}$$

where the $l_{min}(\theta)$ is the minimum signaler values of the $Q$ matrix and the true angles occur when $Q$ matrix losses its rank (Yoon, Kaplan, & McClellan, 2006).

The reference frequency $f_0$ is a key factor in TOPS method and selected for the highest difference between the least eigenvalue corresponding to signals and the highest eigenvalues corresponding to noises, therefore, achieving the highest resolution by selecting lowest interference with noise.

TOPS method compared to CSS and WAVES methods outperforms their performance in high SNR conditions and overcomes their drawbacks of initial estimation and the wrong estimation when on noise exists. TOPS method is very complex due to the calculation of eigen decomposition for all frequency bins and the estimation of the transformation matrix for all hypothesized search angles which add more computational costs (Yoon, 2004).

### 3.2.4 Other significant methods and improvements

A lot of research still have interests in wideband DOA estimation subject due to its wide range of applications and although because there are a lot of points still not satisfied properly e.g. a low computational cost, higher robust to noise and very low SNR condition.

In (Wei & Jun, 2009), a new method called Source Line Fitting Method (SLFM) proposed which exploit the time delay information to estimate the DOA information. It is assuming that the distance between array elements to be larger than half the wavelength, and therefore, it does not use the phase information in the steering vectors. SLFM applies a ADC to the sensors data and uses a line fitting method on the output. The plotted result shows lines that refer to each wideband signals and SLFM uses its slop to estimate the true angle of each source. SLFM uses a developed algorithm for the line fitting to estimate the DOA angles which based on Matrix Pencil (MP) algorithm (Abed-Meraim & Hua, 1997). SLFM method does not use a filter of FFT to subdivide the wideband signal to multiple narrowbands and also it has the advantage of lower computational costs than other coherent methods.

Another significant method called Test of Orthogonality of Frequency Subspaces (TOFES) proposed based on TOPS method. TOFES basically modifies the Equation of $Q$ matrix in 3.30 to have the ordinary MUSIC denominator as the elements of the matrix calculated using SVD instead of EVD for each frequency bin as following

$$\mathbf{Q}(\theta) = \left[\mathbf{A}^H(f_1,\theta)\mathbf{E}_n(f_1)\mathbf{E}_n(f_1)^H\mathbf{A}(f_1,\theta) \quad \cdots \quad \mathbf{A}^H(f_L,\theta)\mathbf{E}_n(f_L)\mathbf{E}_n(f_L)^H\mathbf{A}(f_L,\theta)\right] \tag{3.34}$$

then the SVD is applied as in 3.33 and the DOAs can be obtained by searching for the peaks. The results show that TOFES outperforms other coherent methods in high and moderate SNR condition. The main advantage of TOFES is that it does not need any initial focusing estimation and also the transformation not used so it stands amidst the incoherent and coherent methods (Yu, Liu, Huang, Zhou, & Xu, 2007).

Other significant DOA methods include the use of a non-uniform array of sensors e.g. L-shaped and sparse arrays. Some of this developments are mentioned in this references (Shen et al., 2014; Ioushua, Yair, Cohen, & Eldar, 2017; El Ouargui, Safi, & Frikel, 2018).

# CHAPTER 4

# SIMULATION

On this thesis, we build up a simulation model to demonstrate the DOA problem and implement the wideband DOA methods on it. Many published papers and dissertations have studied the resolution, accuracy, and efficiency of this algorithms (Amin & Zhang, 2009; Yoon, 2004). Though, we focus on the investigation of various aspects that directly affect the computational costs of running these algorithms. In this chapter, the simulation environment, parameters and assumptions are presented.

## 4.1 Simulation Environment

Simply, simulation is a way of testing the performance of various techniques and algorithms in real-like environments under certain conditions and circumstances before deploying the most effective and realizable among these algorithms in a real-life application. Monte Carlo Simulation is the most well-known kind of imitation that uses random choices and decisions in the simulation stages to provide a full examination of the maximum possible number of various conditions that might happen in the real-life applications (Raychaudhuri, 2008). we use MATLAB program version (R2014a) to create a whole simulation environment for several wideband DOA estimation cases. The aim is to test the distinct aspects affecting the computational costs of DOA estimation techniques (Traat, n.d.).

## 4.2 Simulation Parameters and Assumptions

An array of sensors in a (ULA) geometry are assumed. The number of array elements $M = 10$ which equally spaced by a distance $d = 0.5$ with a value normalized to half the wavelength of the indecent waves. This normalization help on testing several kinds of signals with different frequencies without changing the element location. The first array element is the elementary element and all calculations of delay shift caused by the array of sensors are related to its location.

A number of wideband signals $D = 3$ are emitted from far-field sources that the wavefronts

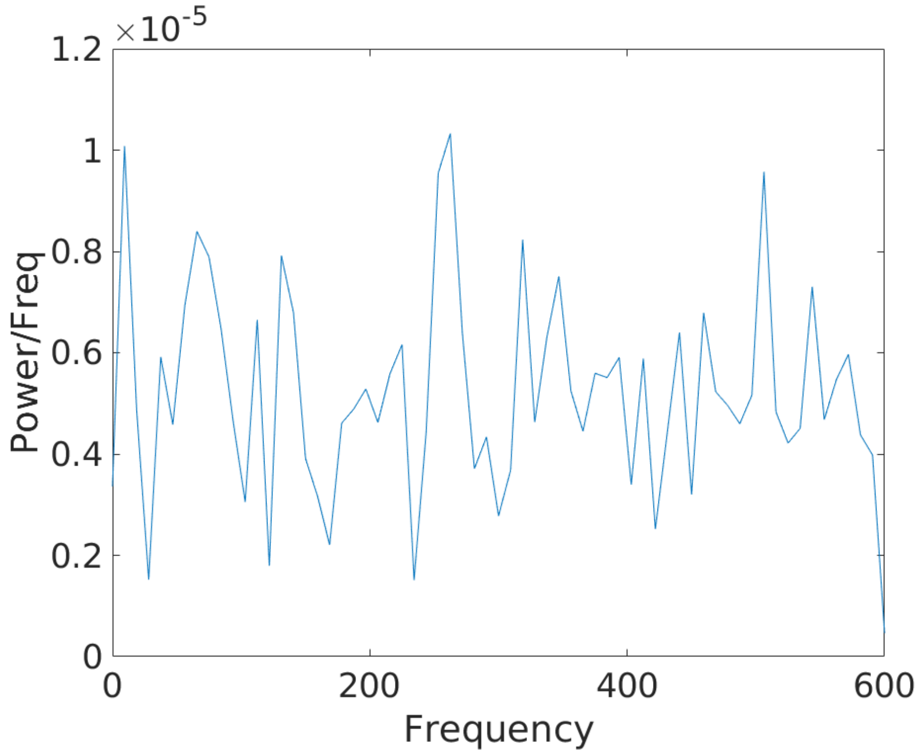


**Figure 4.1:** Power spectrum of the generated signals

impinging on the array elements are in parallel. This assumption removes any additional calculation related to the near-field radioactive sources. The signals bandwidth are within the same range of frequencies where $fl = 250$, $fh = 450$, $f0 = 350$ and $Fs = 1200$; the lowest frequency, the highest frequency, the center frequency, and the Nyquist sampling frequency respectively. The bandwidth is $bw = fh - fl = 450 - 250 = 200$. These values are all used in a normalized way to the center frequency in the calculation of all algorithms, though it's easy to change it in the simulation script to different frequency bands and get the results. The power of the signals is 10 times the power of the noise. The signal demonstrated as a wideband signal which spreads over $L$ frequencies, expressed by

$$s_i(t) = \sum_{j=1}^{L} g_i(t) \exp(j2\pi f_j t) \ + n_i(t) \tag{4.1}$$

where $g_i(t)$ is a Gaussian random variable represents the magnitude of the signal and $n_i(t)$ is a random variable represents the Additive White Gaussian noise. A one-dimensional case is assumed where the sources are located at 20°, 40°and 65°.

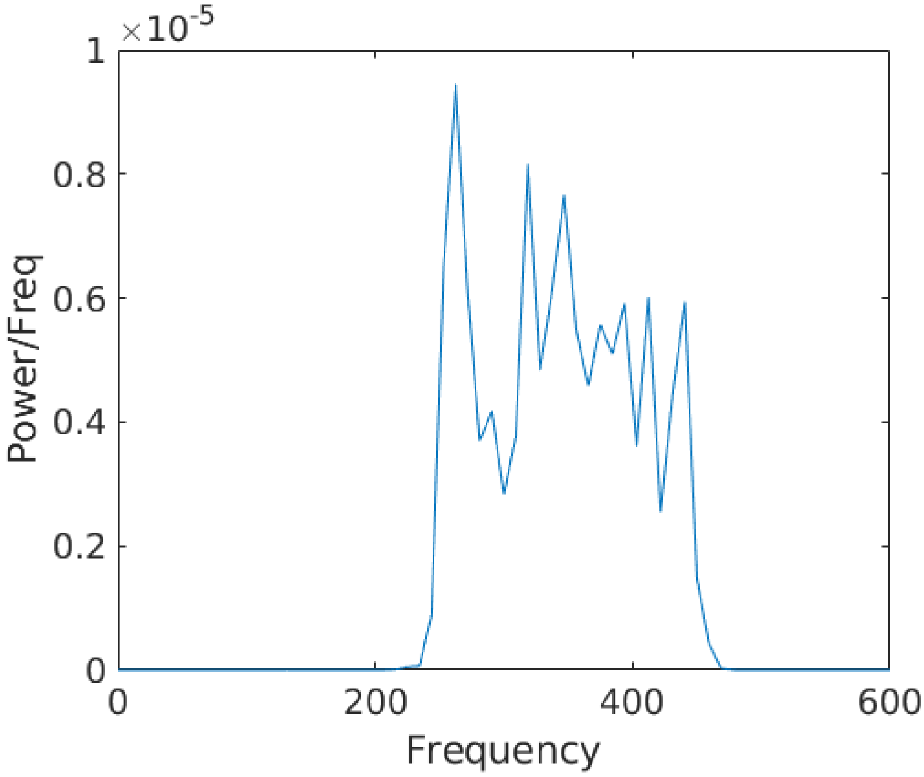


**Figure 4.2:** Power spectrum of the generated signals after Butterworth filter

The sensors' data are generated for 20 snapshots and the Fast Fourier Transform (FFT) is applied at a $rate = 128$. The random generation of the signal spread it on a wider frequency spectrum than the selected bandwidth as shown in figure 4.1. Though a Butterworth filter is used to sharply determine the incoming signal frequency band only as shown in figure 4.2.

The selection of reference frequency for coherent DOA methods are assumed for simplicity of the simulation to be the signal center frequency $f_c$. However, it should be noticed that the selection of reference frequency takes an important role in the determination of the resolution of the DOA methods. In this thesis, the main focus is on the complexity of the algorithms not the accuracy of it. Though such an assumption is made.

### 4.3 Performance Metrics

A four well-known performance and analysis metrics are used in this simulation; spatial spectrum, Root Mean Square Estimate (RMSE), mean time elapsed and the complexity of each algorithm.

**Table 4.1:** The symbols of complexity factors

| **Factor** | **Symbol** |
|---|---|
| Number of Search angles | $\phi_k$ |
| Number of Sensors | M |
| Number of Signals | D |
| Number of freq. bins | L |
| Number of snapshots | N |
| Number of FFT taps | T |

*Spatial Spectrum*

Spatial spectrum shows the estimated angles as peaks among a wide search angles $-90 < \phi_k < 90$. The peaks appear where the power spectrum in MUSIC algorithm loses its rank. In some methods, the spatial spectrum uses the normalized inverse of the smallest singular values e.g. TOPS to show the estimated angles as peaks.

*RMSE*

RMSEs of estimated DOA show the accuracy of an algorithm and how closely the estimated angles to the true angles of the signals sources. RMSE calculated by

$$RMSE = \sqrt{\frac{1}{200}\sum_{r=1}^{200}(\hat{\theta}_r - \theta)^2}, \tag{4.2}$$

where $\hat{\theta}_r$ is the estimated DOA angle at the $r$th simulation round. For more than one DOA sources, an average is taken over the number of indecent signals (Baig & Malik, 2013).

*Mean Time Elapsed*

The time elapsed by each algorithm to estimate the DOA is averaged by the number of simulation rounds so as to get a comparative overview of different algorithms on a time consuming metric. The simulation rounds are calculated for each SNR value which varies from -5 dB to 30 dB.

**Table 4.2:** The complexity of common mathematical multiplications and stages used within DOA algorithms

| **Stage** | **Complexity** |
|---|---|
| FFT Subband extraction | $T \cdot \log(T)$ |
| Correlation matrix | $L \cdot N \cdot O(M^2)$ |
| EVD | $O(M^3)$ |
| SVD | $O(M^3)$ |
| Matrix multiplication | $O(mnp)$ |

*Complexity of an Algorithm*

The computational cost of each algorithm is divided into multiple additive stages from the FFT process to the finding of peaks. Tables 4.1 and 4.2 summarizes the essential factors that play a role in the calculation of complexity and the common stages used in several DOA methods, respectively (Baig & Malik, 2013; Golub, Van Loan, & of Mathematical Sciences, 2013; Talagala, Zhang, & Abhayapala, 2013). In this thesis, the complexity of an algorithm is varied against variation in the number of Sensors, the number of signals and number of frequency bins.

# CHAPTER 5

# RESULTS AND DISCUSSION

The simulation results of wideband DOA estimation methods with the parameters specified and under the assumptions mentioned in chapter 4 are presented in this chapter. Then a discussion about that results and various observations and notes are presented.

## 5.1 Results

Four cases are being investigated through this simulation. The first case is the default case where M=10, D=3, and L=22; M is the number of sensors, D is the number of incident signals and L is the number of frequency bins, respectively. The second case with a change in the number of sensors where M=8, D=3, and L=22. The third case with a change in the number of incident signals where M=10, D=5, and L=22. Finally, the fourth case with a change in the number of frequency bins where M=10, D=3, and L=12.

### 5.1.1 Spatial spectrum

For each case, two spatial spectrums are presented. For low SNR condition (10 dB) and high SNR condition (25 dB). Figures 5.1 and 5.2 show the spatial spectrum for the first case. All the wideband DOA methods estimate the true angles with minimum errors. For low SNR, TOPS method shows a little bias in the third source but in high SNR, it estimated all the DOA accurately.

Figures 5.3 and 5.4 show the spatial spectrum for the second case where the number of sensors is degraded to 8 elements instead of 10 in the first case. It's showing that all methods estimate the DOA correctly while IMUSIC method stead has side fluctuations around -60 degree.

Figures 5.5 and 5.6 shows the spatial spectrum for the third case where the number of impinging signals is increased to 5 while the number of sensors still 10 elements. All the DOA methods have got the true angles correctly except for TOPS method which encounters many peaks between the estimated DOAs. In high SNR, TOPS acts better than low SNR.

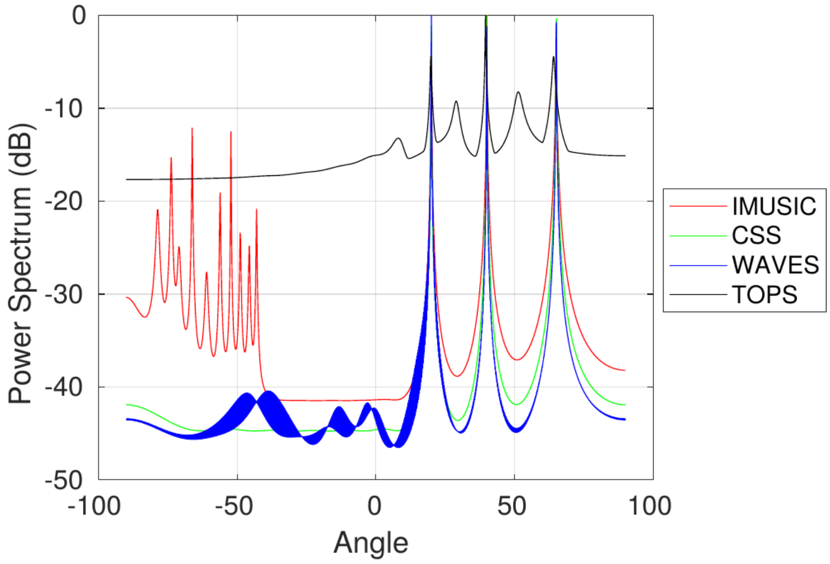


**Figure 5.1:** Spatial Spectrum at SNR = 10 dB (M=10, D=3, L=22)

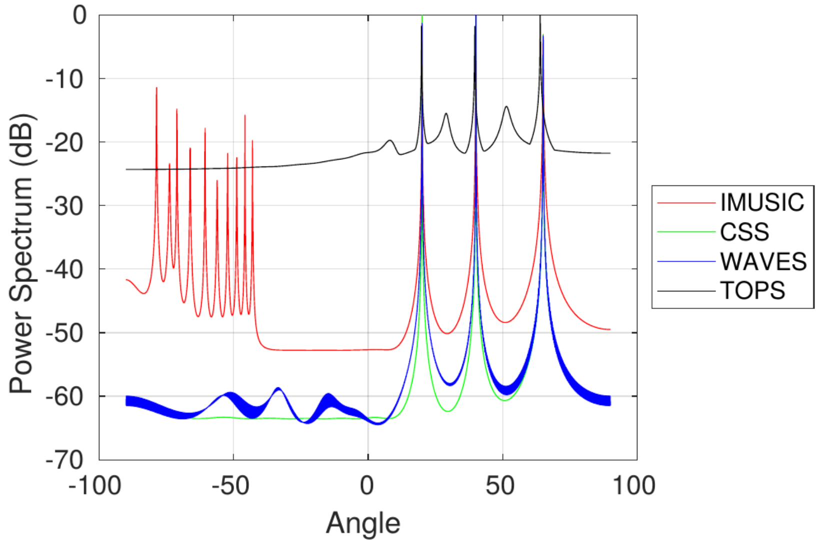


**Figure 5.2:** Spatial Spectrum at SNR = 25 dB (M=10, D=3, L=22)

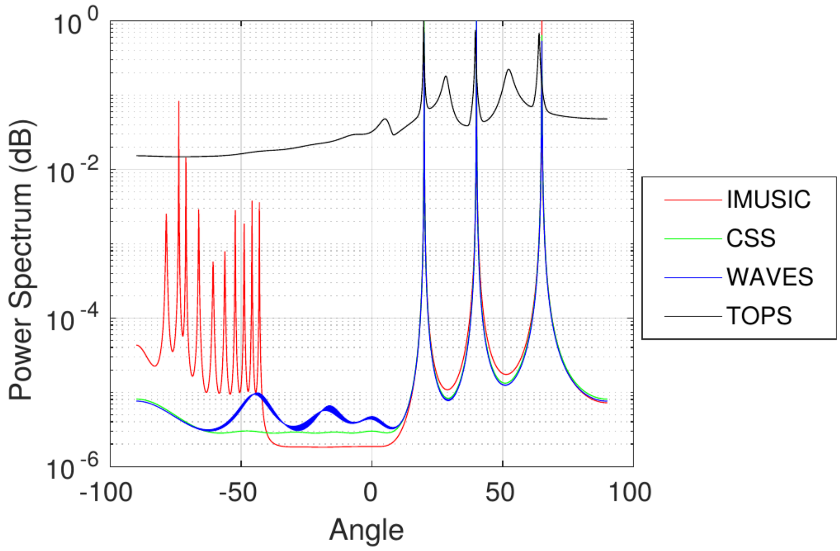


**Figure 5.3:** Spatial Spectrum at SNR = 10 dB (M=8, D=3, L=22)

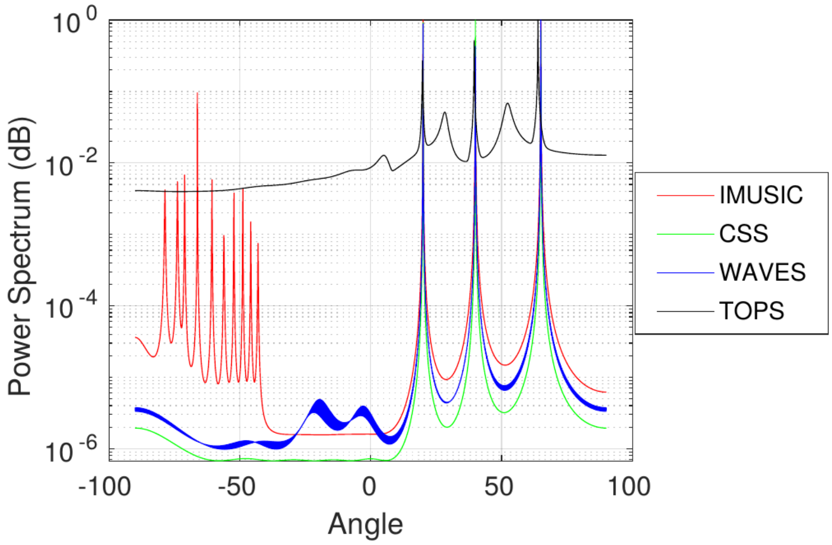


**Figure 5.4:** Spatial Spectrum at SNR = 25 dB (M=8, D=3, L=22)

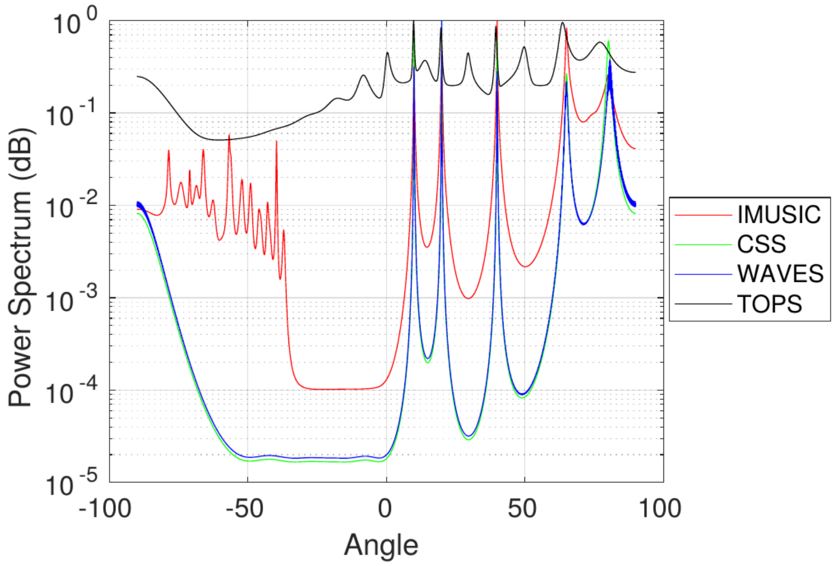


**Figure 5.5:** Spatial Spectrum at SNR = 10 dB (M=10, D=5, L=22)

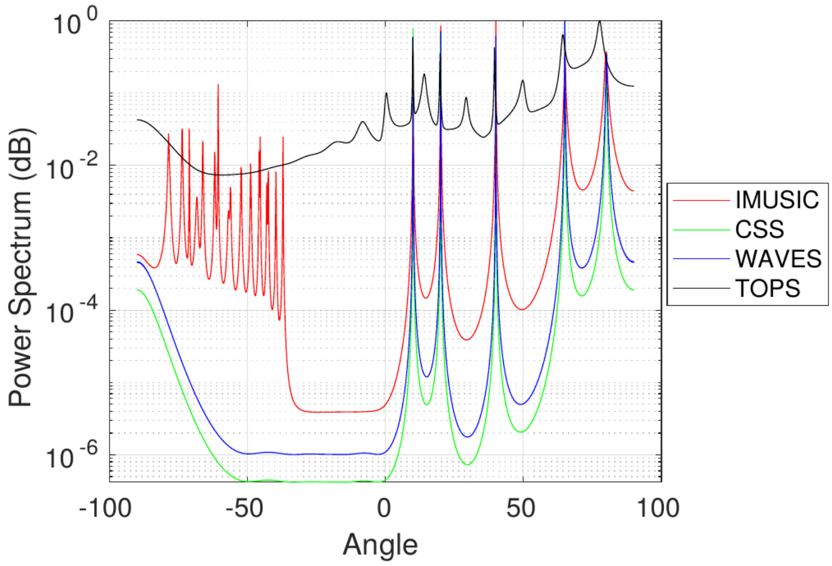


**Figure 5.6:** Spatial Spectrum at SNR = 25 dB (M=10, D=5, L=22)

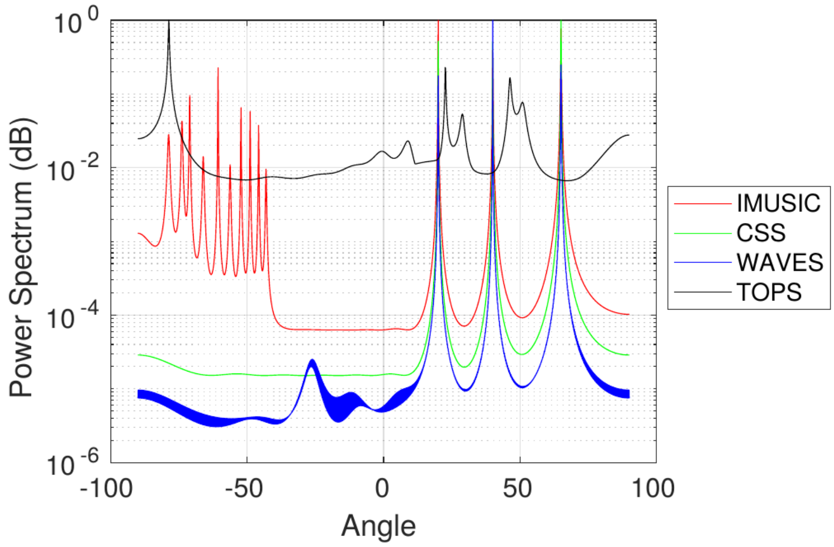


**Figure 5.7:** Spatial Spectrum at SNR = 10 dB (M=10, D=3, L=12)

Figures 5.7 and 5.8 show the spatial spectrum for the fourth case where the number of frequency bins is degraded to 12 bins. At low SNR, the fluctuations of IMUSIC method are at high dB which are comparable with the true estimations. TOPS method failed to estimate the true DOAs and show high fluctuations at wrong DOAs. At high SNR, the power of the estimation is better for WAVES and CSS. IMUSIC fluctuations degraded but not vanished. TOPS still affected by the low number of frequency bins.

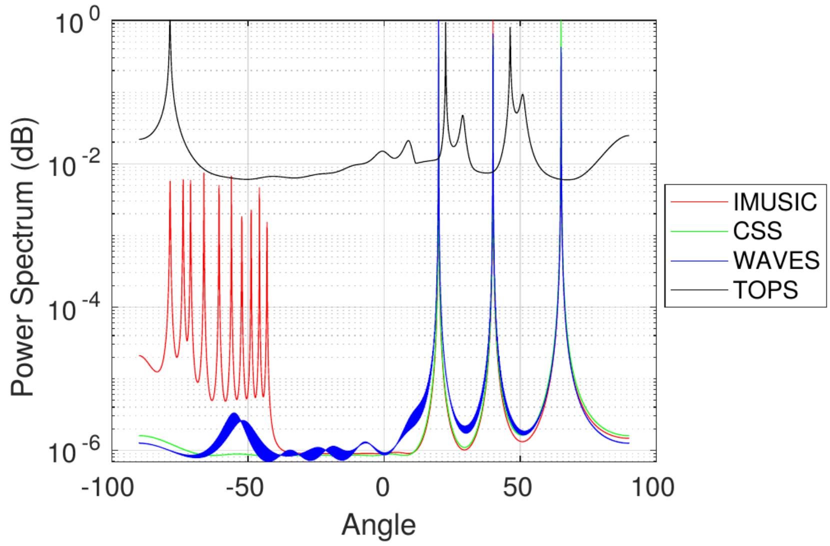


**Figure 5.8:** Spatial Spectrum at SNR = 25 dB (M=10, D=3, L=12)

### 5.1.2 RMSEs

RMSEs figures show the degree of errors in DOA estimation in the four cases. In Figure 5.9, the CSS shows better results than the rest of methods. IMUSIC was the worst especially in high SNR where coherent methods act better than it due to the exploitation of all frequency bins spatial information. All methods act better in high SNR than low SNR. Figure 5.10 shows a worth case for CSS method than in the first case due to degradation in sensors number which also affects all methods both in low and high SNR. Figure 5.11 shows a decrease in the performance of WAVES method which is outperformed by TOPS and CSS methods performance. IMUSIC acts better than WAVES in low SNR but after 20 dB WAVES achieves better resolution. Finally, Figure 5.12 shows that TOPS method is highly affected by the number of frequency bins while other methods performance is not affected too much from the first case.

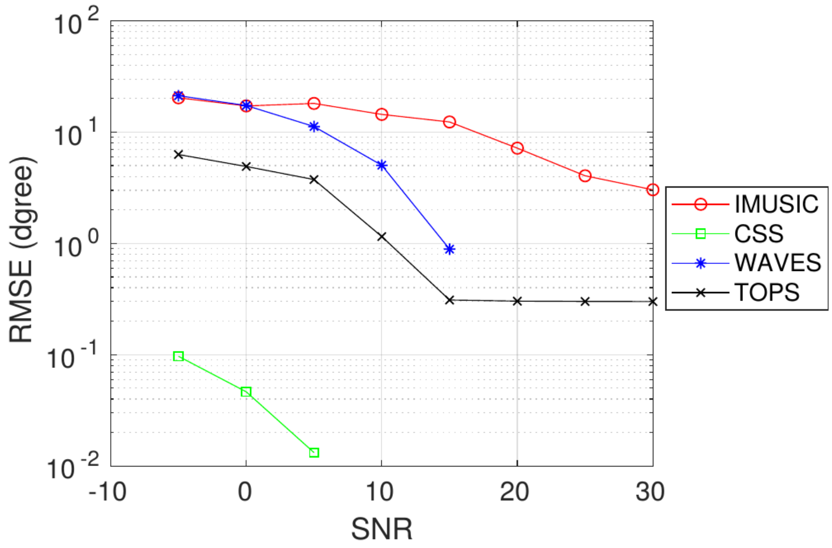


**Figure 5.9:** RMSE where (M=10, D=3, L=22)

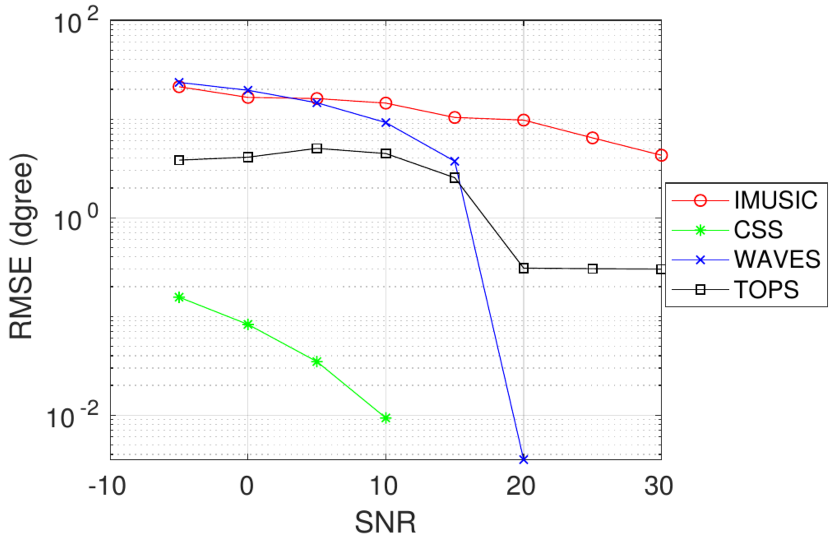


**Figure 5.10:** RMSE where (M=8, D=3, L=22)

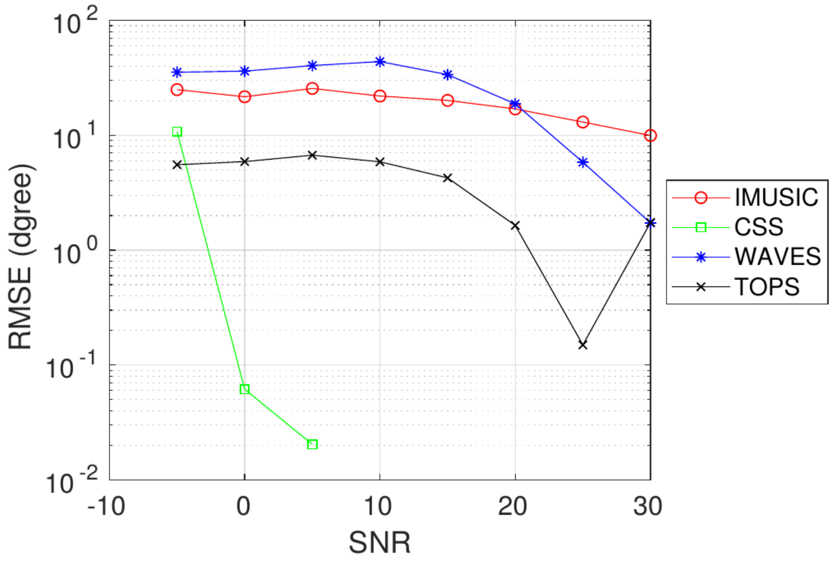


**Figure 5.11:** RMSE where (M=10, D=5, L=22)

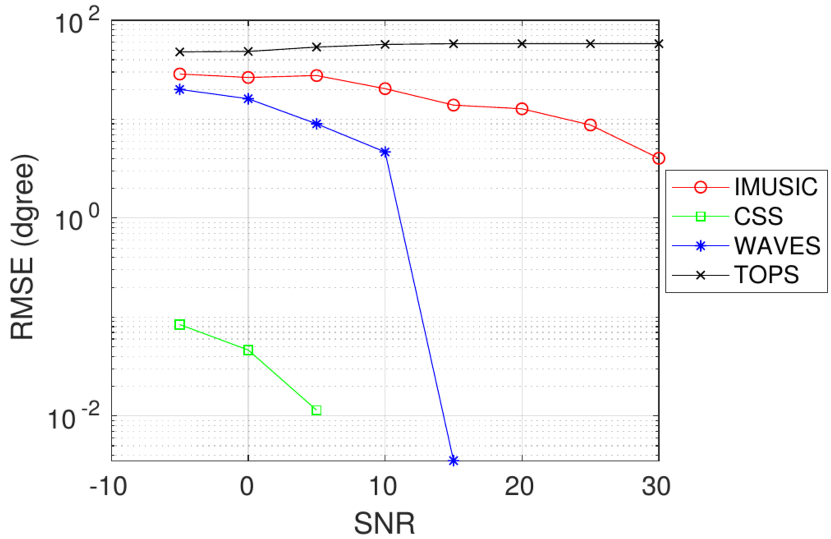


**Figure 5.12:** RMSE where (M=10, D=3, L=12)

### 5.1.3 Computational costs

In this section, the complexity of each DOA method is calculated and varied against three main factors to see how much it is being affected by each factor. These factors are the number of sensors (M), the number of signals (D) and the number of frequency bins (L). The calculations are made according to the Tables 4.2 and 4.1 and are approximated and rounded, though the most effective factors on each stage of each algorithm are only evaluated.

*IMUSIC*

IMUSIC algorithm takes the sensors data and applies the FFT to convert it to an L narrow-bands. Then the correlation matrix for each subband is calculated. IMUSIC take the average of all correlation matrices. The EVD is applied to get the noise subspace which used in MUSIC algorithm to find the peaks. Table 5.1 illustrates the complexity of IMUSIC algorithm stages expressed in Big-O notation.

*CSS*

The stages of CSS algorithm differ from the stages of IMUSIC that it does not take the average of all correlation matrices. Instead, an initial estimation of DOA is performed using a low-resolution method e.g. Capon to estimate the DOAs in every correlation matrix related to each subband. Then a focusing method is used e.g. RSS method. Finally, the EVD is applied to get the noise subspace which used in MUSIC algorithm to find the peaks. Table 5.2 illustrates the complexity of each stage of CSS algorithm and shows more stages and so more complexity than IMUSIC method.

**Table 5.1:** The complexity of IMUSIC method mathematical multiplications and stages

| **Stage** | **Complexity** |
|---|---|
| FFT Subband extraction | $O(T \cdot \log(T))$ |
| Correlation matrix | $L \cdot N \cdot O(M^2)$ |
| EVD | $O(M^3)$ |
| Narrowband MUSIC | $O(M^2 \cdot (M - D) + \phi_k \cdot (M^2 + M))$ |

**Table 5.2:** The complexity of CSS method mathematical multiplications and stages

| Stage | Complexity |
|---|---|
| FFT Subband extraction | $O(T \cdot \log(T))$ |
| Correlation matrix | $L \cdot N \cdot O(M^2)$ |
| Initial estimation (Capon) | $O(M^2 \cdot (M - D) + \phi_k \cdot (M^2 + M))$ |
| RSS | $L \cdot O(M^2 \cdot D + D^2 \cdot M)$ |
| $D_l$ | $L \cdot O(2M^3)$ |
| EVD | $O(M^3)$ |
| Narrowband MUSIC | $O(M^2 \cdot (M - D) + \phi_k \cdot (M^2 + M))$ |

**Table 5.3:** The complexity of WAVES method mathematical multiplications and stages

| Stage | Complexity |
|---|---|
| FFT Subband extraction | $O(T \cdot \log(T))$ |
| Correlation matrix | $L \cdot N \cdot O(M^2)$ |
| Initial estimation (Capon) | $O(M^2 \cdot (M - D) + \phi_k \cdot (M^2 + M))$ |
| EVD | $L \cdot O(M^3)$ |
| RSS | $L \cdot O(M^2 \cdot D + D^2 \cdot M)$ |
| $D_l$ | $L \cdot O(M^2 \cdot D + D^2 \cdot M)$ |
| SVD | $O(M^3)$ |
| Narrowband MUSIC | $O(M^2 \cdot (M - D) + \phi_k \cdot (M^2 + M))$ |

*WAVES*

Table 5.3 shows that WAVES algorithm uses the same stages as CSS method and adds the EVD stage related to the transformation matrix. Then the universal matrix $D_l$ calculated. All these calculations are done for every frequency bins which add more computational costs to the overall cost of the algorithm. Finally, the SVD applied to $D_l$ and the MUSIC algorithm uses the minimum singular values instead of the eigenvectors of noise subspace.

*TOPS*

TOPS algorithm calculates the EVD for each correlation matrix and then the $D_l$ matrix is calculated for each subband and finally, the minimum singular values are used to find the estimated DOAs. Table 5.4 illustrates computational costs of each stage.

### 5.1.4 The factors affecting computational costs

Figures 5.13, 5.14 and 5.15 show the computational costs of DOA estimation methods against sensors, signals and frequency bins, respectively. The figures show that CSS and

**Table 5.4:** The complexity of TOPS method mathematical multiplications and stages

| Stage | Complexity |
|---|---|
| FFT Subband extraction | $O(T \cdot \log(T))$ |
| Correlation matrix | $L \cdot N \cdot O(M^2)$ |
| EVD | $L \cdot O(M^3)$ |
| $D_l$ | $L \cdot O(M^2 \cdot D + M^2 \cdot D + D \cdot M(M-D))$ |
| SVD | $\phi_k \cdot O(M^3)$ |

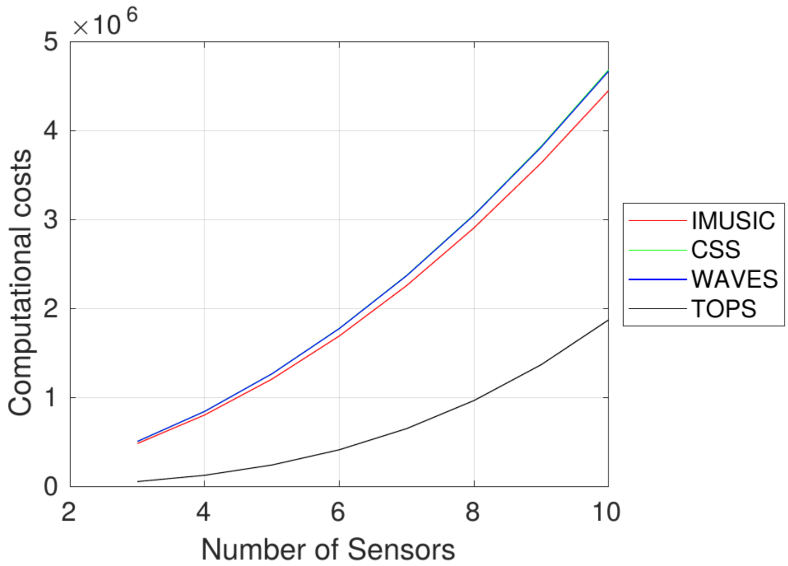


**Figure 5.13:** Computational Costs when varying number of sensors

WAVES methods suffering from high complexity than other methods. TOPS method achieves the lowest computational cost among DOA methods and only in Figure 5.15, when the number of frequency bins is low, the TOPS method achieves high costs than other methods.

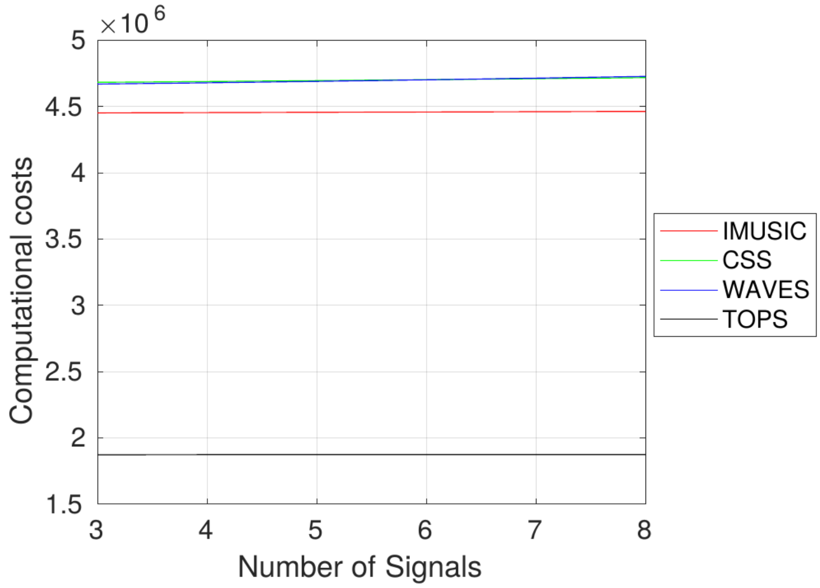


**Figure 5.14:** Computational Costs when varying number of signals

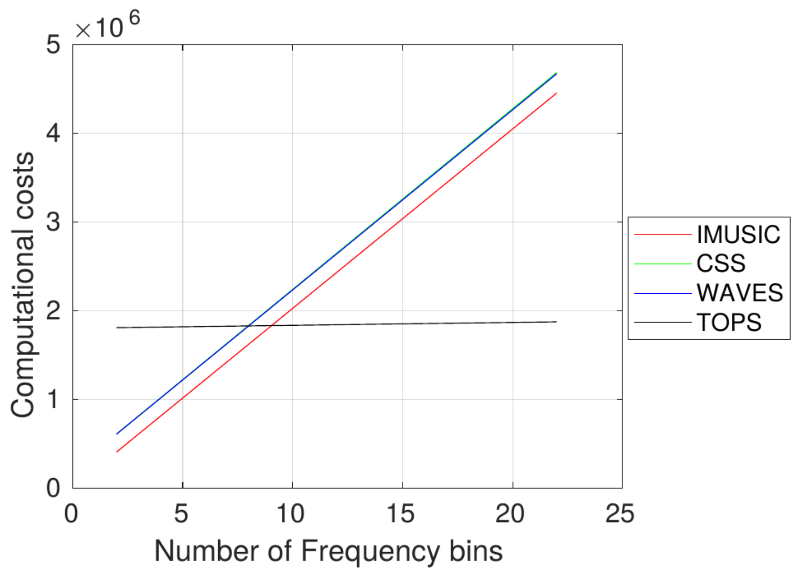


**Figure 5.15:** Computational Costs when varying number of frequency bins

### 5.1.5 Mean time elapsed

For the first case where (M = 10, D = 3 and L = 22), Figure 5.16 shows the mean time elapsed by each algorithm and shows that CSS and WAVES are much higher in execution time than TOPS and IMUSIC. The mean execution time over all simulation rounds for each algorithm is for IMUSIC = 3.552917e-02, for CSS = 6.118259e-01, for WAVES = 5.862703e-01 and for TOPS = 1.848878e-01. For the second case Where (M = 8, D = 3 and L = 22), Figure 5.17 shows that the gap between both WAVES and CSS methods and both TOPS and IMUSIC are less in time. Also the WAVES took much time to execute than other methods. The mean execution time over all simulation rounds for each algorithm is for for IMUSIC = 3.184238e-02, for CSS = 3.986155e-01, for WAVES = 4.810482e-01 and for TOPS = 1.418370e-01. Figure 5.18 illustrate the execution time for the third case where the change is in the number of signals and shows same results as for the first case. Mean elapsed time for IMUSIC = 6.350877e-01, for CSS = 1.116885e+01, for WAVES = 1.034425e+01 and for TOPS = 3.019424e+00. Figure 5.19 shows a slightly different execution times for each simulation round for both WAVES and CSS but still lower than other cases. Here the fourth case where the number of frequency bins are reduced to 12 bins. The Mean elapsed time for IMUSIC = 1.830228e-02, for CSS = 3.149867e-01, for WAVES = 3.030944e-01 and for TOPS = 8.821636e-02.

The execution time analysis of all algorithms shows that the initial estimations are taking the most amount of the execution time. Following the creation of the correlation matrix, the multiplications of focusing matrices and the construction of the main universal matrices $D_l$ are taking much time of the execution time of each algorithm. CSS and WAVES suffering from high execution time because of the need for all three time-consuming stages mentioned. TOPS algorithm on the other side taking much less time to execute because its only need to calculate the correlation matrices and the universal matrix only. IMUSIC algorithm achieving the fastest time due to less time-consuming stages involved.

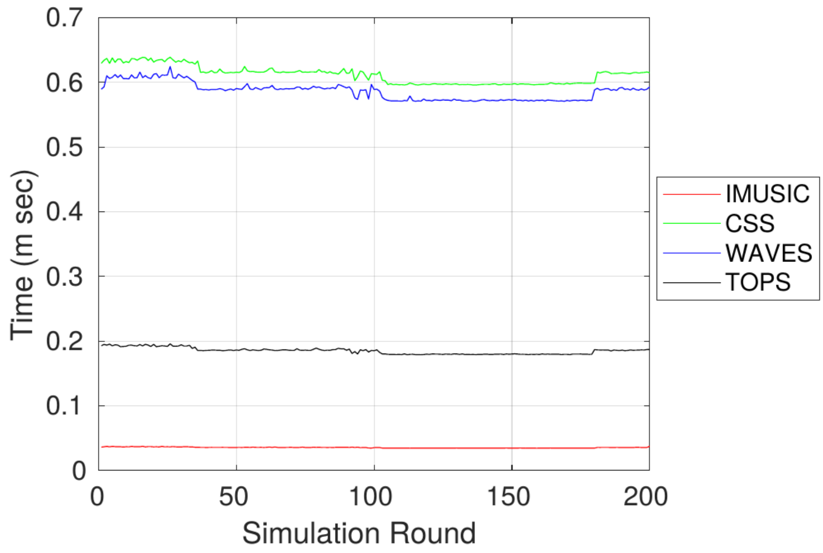


**Figure 5.16:** Mean time elapsed where (M=10, D=3, L=22)

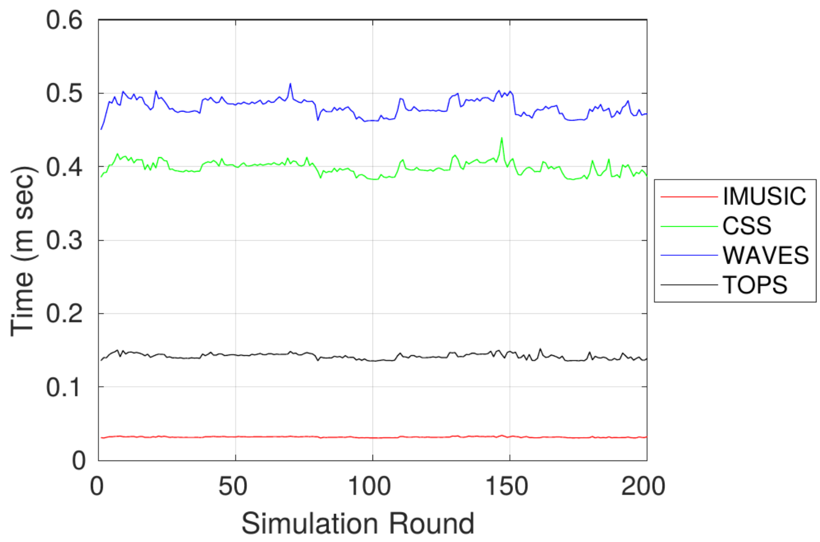


**Figure 5.17:** Mean time elapsed where (M=8, D=3, L=22)

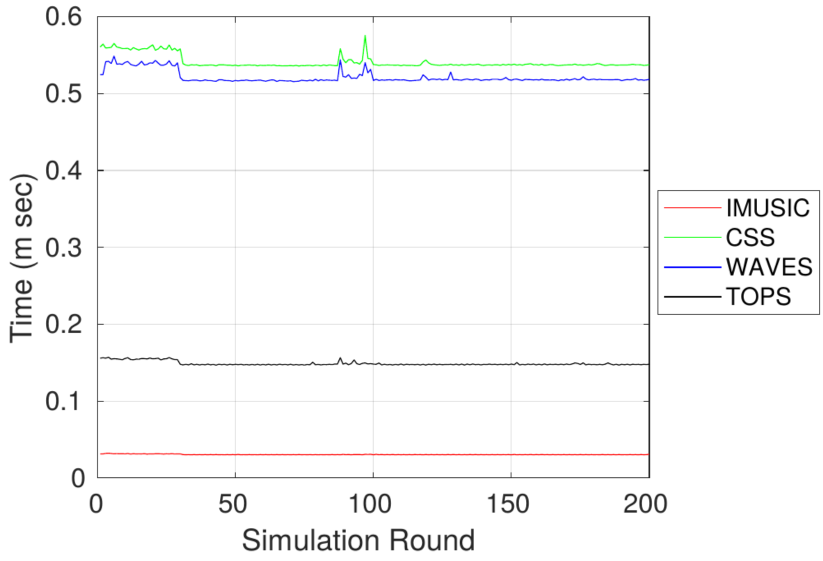


**Figure 5.18:** Mean time elapsed where (M=10, D=5, L=22)

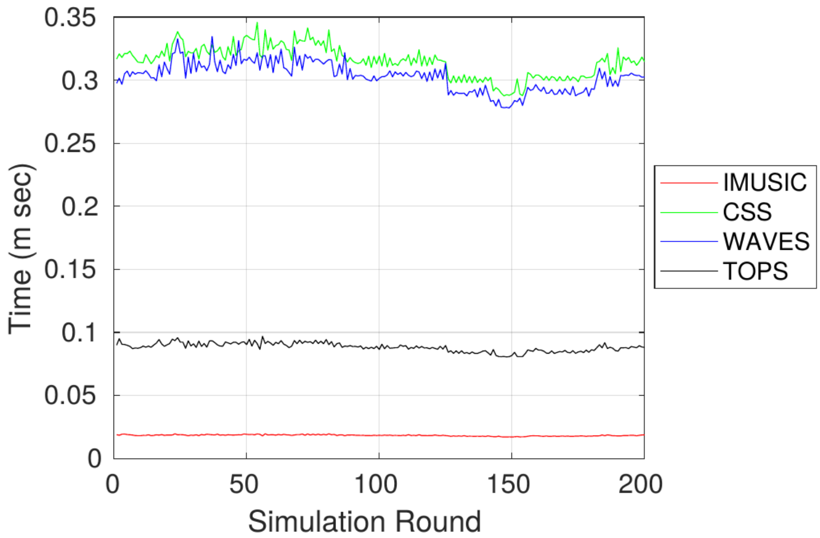


**Figure 5.19:** Mean time elapsed where (M=10, D=3, L=12)

## 5.2 Discussion

The main questions on this thesis were: what are the elementary aspects that affecting the computational costs of different wideband DOA methods? and how much does that affect the execution time of each algorithm? and finally how to improve these algorithms to be less complex?

The main observation is that each algorithm has some advantages over other methods but also suffers from different aspects than others which may lead the improvements on it to less computational technique but may lose its advantages. Though recent research focuses on the improvements of the method itself to gain better resolution and higher accuracy while maintaining less complexity (Talagala et al., 2013; Hayashi & Ohtsuki, 2016). Another observation of the most complex algorithms CSS and WAVES is that their estimations almost depend on the accuracy of the initial estimation which also the main time-consuming stage. Though any errors on it mean both wrong DOA estimation and a consumption of a huge time (0.6-0.7 m sec for every single estimation) which is the worst case might happen. Also, some advantages might be achieved by reducing the time consumed and maintain the same resolution at least like in the fourth case for CSS, WAVES and IMUSIC algorithms where the number of frequency is reduced while still estimating the true DOAs with much less complexity and execution time. TOPS algorithm resolution highly depends on the selection of the reference frequency while IMUSIC algorithm suffers from the combining of all frequency bins including poor ones which led to many fluctuations (Yoon, Kaplan, & McClellan, 2006). Poor frequency bins mean that ones with fewer signals and DOA related information that can contribute to the estimation of the true DOAs or on the reconstruction of the original signals.

### 5.2.1 Improvements to be made

The main advantage observed of the initial estimation stage is to predict which frequency bins has most of the DOA information and so the most valuable bins to use in the algorithm. Though, the development of a good procedure for the selection of reference frequency bin could help on the reduction of complexity of DOA algorithm and achieving a higher resolution. The advantage achieved by improving the usage of a few sensors and less number of frequency bins could through away the problem of the wrong estimation. Such an improvement could be made using different techniques in the same system to achieve the ultimate goal of correct estimation of the true DOAs in much less execution time.

# CHAPTER 6

# CONCLUSION

## 6.1 Conclusion

DOA estimation methods try to extract the true angles of the exact number of the received noisy signals which impinging on the array of sensors. This estimation seems very useful in the reconstruction of the original signal and helps on the estimation of its location which is highly applicable in radars, sonars, seismic exploration, and military surveillance. In general DOA problem is an estimation problem which tries to figure out the parameters hidden in the sensors data using different mathematical techniques and physical properties of the geometry of the array of antenna and the impinging signals themselves. Matrix multiplication and manipulation are almost the core of the exploitation of the DOA information e.g. matrix sub-spaces orthogonality, eigen compensation, and singular decomposition. Due to the natural differences between narrowband and wideband signals, the DOA estimation of both signals is different and so the methods estimating the DOA in a narrowband are not sufficient for wideband DOA estimation. Wideband DOA methods are classified into two main categories; incoherent e.g. IMUSIC and coherent methods e.g CSS, WAVES and TOPS. Incoherent methods took the average of all frequency bins while coherent methods exploit the underneath characteristics of frequency bins, though to make use of DOA related parameters in a weighted way and gain better resolution.

One of the most important aspects of this methods is the complexity of an algorithm and execution time consumed to estimate the true DOAs. Though several wideband DOA estimation methods proposed in the recent research that use different techniques to achieve a high-resolution estimation while maintaining low computational costs. The finding in this thesis shows that there are some dominant factors controlling the complexity of each algorithm. Exploring this factors within the well known DOA methods while maintaining a high resolution is investigated through the results and discussion.

## 6.2 Future Work

Further studying of different DOA methods to gain a better understanding of its main parameters underlying these methods. Also investigating the use of different techniques for the selection of good frequency bins so as to maintain a low profile initial estimation stage and improve the quality of estimation in the incoherent methods and coherent methods that do not use initial stage.

In the words of applications, an implementation of a specific system using different algorithms to achieve maximum benefits of DOA estimation in those specific applications without any improvement in the DOA algorithms itself. Also, the investigation of characteristics of the nature of the application signals and relate it to the implementation and development of the DOA algorithm would help in a better algorithm.

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