INTEGRAL SLIDING MODE CONTROL FOR THREE DEGREE OF FREEDOM ROBOT MANIPULATOR

A THESIS SUBMITTED TO THE GRADUATE SCHOOL OF APPLIED SCIENCES OF NEAR EAST UNIVERSITY

By
ANTENEH TADESSE YITBARK

In Partial Fulfillment of the Requirements for the Degree of Master of Sciences in Mechatronics Engineering

NICOSIA, 2019

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Approval of Director of Graduate School of Applied Sciences

Prof.Dr.Nadire CAVUS

We certify this thesis is satisfactory for the award of the degree of Master of Sciences in Mechatronics Engineering

Examine committee in charge:

Assist. Prof. Dr. Parvaneh ESMAİLİ	Department of Electrical and Electronic Engineering, NEU (supervisor)
Prof. Dr. Bülent BİLGEHAN	Department of Electrical and Electronic Engineering, NEU
Assist. Prof. Dr. Ali SERENER	Department of Electrical and Electronic Engineering, NEU

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original of this work.

Name, Last name: Anteneh Tadesse Yitbark

Signature:

Date:

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To my Beloved Mother Yabundeje Melkamu…

ABSTRACT

Industrial robot manipulators are multi-body system having nonlinear, coupled dynamics and parameter fluctuations, the nonlinear and coupled dynamics includes: - gravitational forces which depend on position, carioles and centrifugal force. Reaction forces in joints due to the acceleration of other links, friction force, etc. In this paper an ISMC with fuzzy supervisory is presented to control and eliminate the trajectory tracking error of a three degree of freedom robot manipulating arm and also the modeled robot manipulator's stability is analyzed using the direct method of Lyapunov stability theorem and the supervisory adaptive fuzzy logic inference system was serving as a tuning mechanism for the gain parameter of the sliding surface also it helps to converge and maintain the sliding surface to zero. The kinematics and dynamics of the robot manipulator are designed and briefly discussed before the suggested controller is implemented. The effectiveness of the presented control method has been evaluated in the simulation environment using MATLAB Simulink software. we have also compared the result of the presented controller which is the integral sliding mode and integral sliding mode with fuzzy supervisory with 20% of matched uncertainty imposed to the system and without uncertainty. in the system simulation, the presented control provides 0.0001435 rad of root mean square error (RMS) without uncertainty and 0.002188 rad of RMS when 20% of disturbance is applied on it. The suggested control model was also implemented on PUMA 560 robot six DOF manipulator while the first 3 joints are fixed and the result of the model is compared with another model which was implemented on similar well-known PUMA 560 robot manipulator.

Keywords**:** *kinematics; dynamics; fuzzy logic; Lyapunov stability theorem; trajectory tracking; ISMC; root mean square*

ÖZET

Endüstriyel robot manipülatörleri, doğrusal olmayan, birleştirilmiş dinamikler ve parametre dalgalanmalarına sahip çok gövdeli bir sistemdir, doğrusal olmayan ve birleştirilmiş dinamikler şunları içerir: - pozisyona, karyolalara ve merkezkaç kuvvetine bağlı yerçekimi kuvvetleri. Diğer bağlantıların, sürtünme kuvvetinin, vb. Hızlandırılmasından dolayı eklemlerdeki reaksiyon kuvvetleri. Bu yazıda, 3-DOF robot manipülatör bir kolun yörünge izleme hatasını kontrol etmek ve ortadan kaldırmak için bulanık bir denetime sahip bir ISMC ve ayrıca modellenmiş robot manipülatörünün dengesi Lyapunov stabilite teoreminin direkt metodu kullanılarak analiz edilir ve denetleyici adaptif bulanık mantık çıkarım sistemi, kayma yüzeyinin kazanç parametresi için bir ayarlama mekanizması görevi görmekte olup, kayma yüzeyinin sıfıra yakınlaşmasına ve korunmasına yardımcı olmaktadır. Manipülatörün kinematik ve dinamikleri, önerilen kontrolör uygulanmadan önce tasarlanmış ve kısaca tartışılmıştır. Sunulan kontrol yönteminin etkinliği simülasyon ortamında MATLAB Simulink yazılımı kullanılarak değerlendirilmiştir. Ayrılmaz kayma modu ve bütünleşik kayma modu olan sunulan denetleyicinin, bulanık denetleyici ile sisteme uygulanan ve belirsizliğin% 20'sini karşılayan belirsizlik ile karşılaştırdık. Sistem simülasyonunda sunulan kontrol, belirsizliği olan 0.0001435 radyasyon kök ortalama kare hatası (RMS) ve üzerine% 20 bozulma uygulandığında 0.002188 radyon kök ortalama kare hatası (RMS) sağlar. Önerilen kontrol modeli aynı zamanda PUMA 560 robot altı DOF manipülatöründe uygulanmış, ilk 3 eklem sabitlenmiş ve modelin sonucu benzer PUMA 560 robot manipülatöründe uygulanan başka bir model ile karşılaştırılmıştır.

Anahtar Kelimeler: kinematik; dinamiği; Bulanık mantık; Lyapunov kararlılık teoremi; yörünge izleme; ISMC; Kök kare ortalama

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LIST OF ABBREVIATIONS

DOF:	Degree of Freedom
SMC:	Sliding Mode control
ISMC:	Integral Sliding Mode Control
RMS:	Root Mean Square
RMSE:	Root Mean Square Error
ANFIS:	Adaptive Neuro Fuzzy Inference System
FFNN:	Feed Forward Neural Network
ANNISMC:	Adaptive Neural Network Integral Sliding Mode Control
ISSOSM:	Integral Suboptional Second Order Sliding Mode
PUMA:	Programmable Universal Machine for Assembly
COMAU:	COnsorzio MAcchine Utensili
MST:	Mean Square Tracking
SSOSM:	Suboptional Second Order Sliding Mode
MPC:	Model Predictive Control
ASMC:	Adaptive Sliding Mode Control

CHAPTER 1
INTRODUCTION

1.1. Background

Robotics, as the name suggests, is the science and study of robots, the word comes from the Czech word "ROBOTA" meaning (forced labor) which was coined by Karel Capek in 1921. Robots as by what comes first to mind, are man-made computerized machines which perform tasks on human commands or by themselves and seeks to make work easier for human beings.

A robot is an integrated system made up of sensors, manipulators, control systems, power supplies, and software, which work co-dependently in order to perform a task. Robots come in all different shapes and sizes but frequently classed by their degree of freedom, each direction of movement on the robot is considered as an axis of movement, each single movement axis is considered as one degree of freedom, the most common movement of a robot manipulator which is yaw, pitch and roll helps to locate the tools in a work area and are called position axes.

Every robot has a controller, which continuously reads from sensors like motor encoder, force sensors, vision sensor and depth sensor which updates the actuator commands so as to achieve the desired robot behavior.

Nowadays, there are many ways of controlling the trajectory tracking of a robot manipulating arm, the control methodology of the robot arm based on their controller is mainly divided as a model-based controller and non-model-based controller. these are adaptive control, variable structure control and computed torque under the model-based. For this research, we choose integral sliding mode controller which has a robust controlling approach and insensitive for disturbance and uncertainty in the nonlinear system and help to develop a precisely controlled way for the dynamic movement of the robot arm. Variable structure control is a way of controlling dynamics of a model-based linear and nonlinear system where the control law is changing accordingly with some pre-defined rules in the control process and have different type based on the controlling method used among them sliding mode control is widely used in controlling of a dynamic system with uncertainty and disturbance but it has its own drawbacks which will be discussed below.

1.1.1.Sliding mode control

It is the main and widely used type of variable structure control methodology for both linear and nonlinear system, which is capable of changing the dynamics of the system by applying a high frequency switching control law, even if the system have good response for nonlinear and discontinuous system it has major drawback of the chattering effect to the system many researchers have been proposing and applying different type of control method for nonlinear multi input multi output system and others, (Vadim Utkin, 1992) the stability of the system in sliding mode control will be granted if the matching condition is verified when there is uncertainty and external disturbance on the system but the system is sensitive for the disturbance in reaching phase, there is no guarantee for robustness in reaching phase, several researchers suggest imposing reachability time will eliminate the sensitivity of the system for uncertainty and disturbance in reaching phase or by choosing a non-linear sliding surface which starts from the initial point, nowadays due to its speed, nonlinearity and switching nature sliding mode is widely applied to more controlling systems even if it has a chattering problem which is harmful to actuators leading to low accuracy in trajectory tracking, cause wear and heat the mechanical parts.

Different researcher have been proposing many techniques to reduce or eliminate the chattering problem of a system mainly in to two branches which are estimated uncertainty method (uses system uncertainty estimator to compensate the uncertainty) and boundary layer saturation method (replacing the discontinuous method with linear (saturation) method with small switching surface), many researchers use estimated uncertainty method option using artificial intelligence, fuzzy logic and neural network but when we try to eliminate the nonlinear unstable system’s chattering effect it causes a trajectory tracking error for our system nevertheless to overcome this error we suggested a chattering free, insensitive to dynamic uncertainties and disturbance and robust ways of controller which is integral sliding mode control(ISMC) for trajectory tracking control of 3 degree of freedom robot manipulator.

1.1.2. Integral sliding mode control

It is an improved version of SMC for designing an enhanced accuracy, robustness, undisturbed and chattering free dynamic control mechanism for a nonlinear system like robot manipulation. In this method, the guaranteed stability of the system is the result of imposing the sliding mode throughout the system but the overshoot, undershot and response time may be poor if the initial error is very large(Zheng, et al., 2015). In classical sliding mode control (SMC), since the SMC does not have information about the stability of the system during reaching phase of the system so, the system robustness and stability which are exposed to matched and unmatched uncertainty and external disturbances(gravitational forces, Coriolis and centrifugal forces) can be achieved only after the occurrence of the sliding mode we do not have a guarantee for the system stability during the reaching phase but the ISMC can overcome this problem and achieve the desired accuracy and robustness for a nonlinear system control.

In this research paper, to overcome the windup issue tuning the gain parameter of the sliding manifold of the nonlinear control system using an adaptive fuzzy logic inference system will give a promising result (Lu, 2015).

1.1.3. Adaptive fuzzy logic gain tune

Fuzzy logic is a way of computing based on the degree of truth instead of crisp or Boolean value. In this research, we will implement the well-known adaptive fuzzy inference system to automatically tune the gain of the nonlinear control system parameter which does not have prior information about the parameters of the system. Combination of robust control with adaptive fuzzy logic inference system for controlling a trajectory tracking of a robot manipulator has been suggested by the different researcher so far.

For a system which have a changing dynamic parameter are need to be trained online, the adaptive fuzzy will try to tune the gain parameter without any information about the dynamic system behavior, by combining the ISMC with adaptive fuzzy logic gain scheduler, we can overcome the problem of instability and the increment of rise time and get good trajectory tracking controller

For this research, we will implement an adaptive fuzzy inference system to tune the gain parameter of the integral sliding surface to remain the surface to zero s=0 and the gain parameter of the integral and derivative component of $\dot{s}$, K_p and K_i to obtain the desired output.

1.2. Statement of the Problem

Robot manipulators are a multi-body system having nonlinear, coupled dynamics and parameter fluctuations. The nonlinear and coupled dynamics represent example: - gravitational forces which depend on position, carioles and centrifugal force. Reaction forces in joints due to the acceleration of other links, friction force, etc... In trajectory tracking application, such complex systems are mostly controlled using PID (proportional integral derivative) controller and sliding mode control.

Applying the pre-tuned integral sliding mode using a fuzzy logic will be implemented to overcome those trajectory tracking problems and get the required accuracy, effectiveness and robust dynamic trajectory tracking control system for our 3 degree of freedom robot manipulator. For example, in robotics systems use for assembling operations with heavy workpieces carried by manipulator's gripper, a robot with high speed operating, etc. are exposed for dynamic uncertainties and external disturbance applying the classical sliding mode controller or PID controller will not provide the stable system.

Different researchers have presented to avoid the shortcoming of sliding mode control using a nonlinear controller and different approaches have been suggested for designing the nonlinear control,

1. The ISMC.
2. The adaptive fuzzy logic gain tune for ISMC.

1.3. Objectives

1.3.1. General Objective

Developing a trajectory tracking using an Adaptive fuzzy integral sliding mode controller for the 3 DOF robot manipulator.

1.3.2. Specific Objective

- To develop dynamics and kinematics of a 3-DOF robot manipulator
- To develop an ISMC for the robot manipulating arm
- To develop a tuning parameter for ISMC using adaptive fuzzy logic

- To compare the AFISMC with ISMC
- To simulate and demonstrate an AFISMC of a 3-DOF robot manipulating arm

1.4 Methodology

The suggested thesis first concentrated at the study of the literature about controller design, theoretical and structural backgrounds about robot manipulator and controller. Then after we get a good understanding about robot manipulator and controller from the reviewed literature, we have compared the performance of the various controller and then we have selected our technique which can remove the shortcoming of the PID and sliding mode controller.

Since an appropriate dynamic model equation has very important in designing the robot controller. To develop the model, we have divided this thesis into two main parts. First, we developed the robot manipulator equation of motion. This stars from the equation of position and orientation description, forward and inverse kinematics, dynamic analysis and forces, kinematic and potential energy by using Lagrange equation and second, we will design the overall nonlinear system model of the manipulator dynamics. After the dynamic models have been developed, based on the ISMC theory the three degrees of freedom robot manipulator controllers are designed.

CHAPTER 2
LITERATURE REVIEW

In a research made (Esmaili & Haron, 2017) the researcher suggested an adaptive neural-based ISMC method. The suggested method was designed for synchronization of multiple robot manipulators which do not have direct communication between them. However, the communication carries outs through an object as a communication point and use both direct and un-direct implicit communication ways throughout the system. In which it has a huge advantage to overcome the failures occurrences in case of un-direct communication. In the suggested system kinematics and dynamics constraint of the system is considered instead of direct force control for synchronization of multiple robot manipulators with improved integral sliding mode controller.

The researchers briefly explain and drive a synchronization robust and accurate nonlinear control method by the combination of adaptive NN with ISMC method for a nonlinear system. In order to eliminate the tracking and the synchronization error by considering different kind of uncertainty that may happen to the system which is exposed to external disturbance. For implementation purpose the researchers used a well-known two 6-degree of freedom PUMA 560 robot manipulators are chosen and the parametric gain for K_P and K_I is set to be 2 and 0.0001.

In this paper (Esmaili & Haron, 2017) the researchers try to consider four different cases which are: a system without uncertainty, with lumped uncertainty, with grasping uncertainty and with both lumped and grasping uncertainties. Based on these cases the paper tries to analyze, test and compare the performance of the suggested system with Adaptive Neuro-Fuzzy Inference Systems (ANFIS) method.

The paper suggests a feed-forward neural network architecture (FFNN) that have an input layer, hidden layer, and output layer. The neural model has 6 input nodes representing each parameter, and 1 hidden layer which have 10 nodes, and 6 output nodes representing the output parameters. The designed neural network architecture is trained using 71 sample data and validated using 71 validation set samples. After training the designed neural network the researchers obtained a mean squared error of $1.9641e^{-11}$.

The researchers compared the ANFIS and the suggested ANNISMC methods based on the four uncertainty cases. These two methods are compared based on three different measurement factors which are Position Tracking Error (PE), Coupled Error (CE), and Synchronization Errors (SE). The execution result shows us that they have a significant difference. The researchers compared the result of those two systems by their values differences of the results.

For the first case, the value difference between the two methods using the CE, PE, and SE factor errors are 0.000293346, 4.22121e^{-4}, and 0.2951946e^{-4} are respectively showing the suggested system is better. And for the second case, the value difference of the CE, PE, and SE values are 0.2951946e^{-4},0.2769245 e^{-4}, and 0.2769245 e^{-4} also, indicating that the ANNISMC method is better. Furthermore, in the third case, the value of the difference between the two methods for the CE, PE, and Se are 0.2769245 e^{-4},0.2769245e^{-4}, and 0.008291 clamming that the suggested method is better than the ANFIS method. Finally, the results for the fourth case shows that the suggested model is better in which the value of the difference for the CE, PE, and SE errors factors are 0.313392 e^{-4}, 0.3075458 e^{-4}, and 0.2718546 e^{-4}. According to the paper in total, the suggested ANNISMC method has a 3% percent of improvement on the ANFIS method.

In a research made(Ferrara & Incremona, 2015) the researchers suggested an algorithm based integral suboptimal second-order sliding mode control system (ISSOSM). The suggested algorithm is a nonlinear control mode that provides a mechanism for reducing the reaching phase of a system which is controlled by a sliding control method. In which the reaching phases generally exists in any kinds of system that are controlled by a sliding mechanism. The suggested algorithm increases the efficiency of the suggested system by extending the sliding mode to the time intervals in which the default sliding mechanism doesn't include. These reasons make the suggested algorithm suitable and applicable for real industrial robots to improve and optimize the efficiency and accuracy of the controlling model of the robots. Also, the suggested algorithm provides a second-order sliding mode in which the control mode of the system has a relative degree of one which results in a positive chattering alleviation effect.

Furthermore, the researchers carried out an experimental test of the suggested algorithm on COMAU SMART3-S2 industrial robot. Additionally, the researchers compared their suggested controller algorithm model with other standard and suboptimal algorithms. Their comparison results show the suggested algorithm model is satisfactory than the others compared. The researchers compared their suggested model with two different robot controlling models. The first controlling model which is compared with their model is proportional derivate (PD) controller and the second model is the suboptimal second-order sliding mode (SSOSM). Those three controllers' models are compared based on their mean square tracking (MST) error and the suggested model (ISSOSM) is better than the others according to their comparison results. The results of the root mean square (RMS) error of the PD, SSOSM, and ISSOSM are 0.016, 0.0056, and 0.0037 respectively. Their RMS result shows the suggested model which is ISSOSM has the smallest tracking error making it the best among the three models.

Also, in research (Incremona, et al., 2017) the researchers introduced a model that can solve the problem of motion controlling for robot manipulators. The suggested model is based on hierarchical multiloop control scheme and the paper presents the design and implementation of the introduced model. The main elements of the suggested model are the multi-input and multi-output based on an inverse dynamic controlling mechanism and an integrated sliding control (ISC) model with a model predictive control (MPC). When some uncertainties occur as a result of unrecognized and unmodeled dynamics the ISM is responsible for adjusting those uncertainties. Usually, the ISM will become responsible if those uncertainties are not handled by the inverse dynamic method. The MPC is attached with the external loop which gives a guarantee for the evolution of the control system in order to make it as efficient as possible, also through the accomplishments of the inputs and outputs constraints at the same time.

As the researchers presented in the paper the primary reason for the ISM to be used is its robustness while dealing with multiple and wide class of uncertainties. Additionally, the second main reason is the ability of the ISM to apply and enforce a sliding mode of the control system from the beginning of the time start. The ability to enforce the sliding control from the initial time start will give the system the ability to reduce and solve the optimization problems of the MPC. The researchers have simulated the testing and examination of the presented model using COMAU Smart3-S2. In which this model is an industrial robot manipulator. The researchers compared their suggested system MPC/ISM with the standalone MPC methods according to the results of the root mean square error (RMS) and the RMS value of the models. The robotic system they were examining and testing their suggested strategies were a 6-joint based robot but for the sake of their strategies and simplicity they locked the three joints and carried out their experiment only on the other three joints. However, they have explained that their suggested approach can be applied for 6-joint robotic models as well and even can be generalized for general use.

Finally, the root mean square error (RMSE) results obtained by the researchers for the three unlocked joints using the standalone MPC algorithm model are 0.2105, 0.3749, and 1.0458 for the 1st, 2nd, and 3rd joints respectively. And the RMS results of the suggested system (MPC/ISM) are 0.0070, 0.0102, and 0.0152 for the 1st, 2nd, and 3rd joints respectively. Thus, we can see that the mean square error of the suggested system for each joint was smaller than the standalone models showing the suggested model is better. And the result of the RMS value using the standalone MPC model for joint 1, joint 2 and joint 3 are 8.0303, 13.4839, and 41.2172 respectively. Additionally, the RMS value of the suggested system for joint 1, joint 2, and joint 3 is 8.06, 13.4217, and 34.8530 respectively. Therefore, in general, we can conclude that according to the experimental results of RMS and RMS error the suggested system is better than the standalone MPC robotic model.

Additionally, an adaptive controller based integral sliding model and time delay estimation (TDE) approaches are suggested in research (Lee, et al., 2017) for robot manipulators. The researchers suggested an estimator for uncertainties that occurs in robot dynamics, such uncertainness might be variations of parameters and disturbances according to the researcher's explanation. The sliding integral is responsible for eliminating and reducing the reaching phase and noisy actions of the conventional sliding models. Additionally, the researchers used adaptation gain dynamics to obtain a better and higher degree of accuracy. According to the research's explanation, the suggested adaptive control model is chattering free, robust and highly accurate. An experimental examination is carried on the suggested system to verify its accuracy and efficiency. The extermination is conducted on a programmable universal machine for industrial robot manipulators.

According to their paper (Lee et al., 2017), fuzzy logic and artificial neural network approaches are better and efficient for handling nonlinearities and uncertainties. However, these models require an intense number of parameters and complex rules so, they become nearly impossible or difficult to be applied. The controller suggested by the researchers is not model dependent as a result of the TDE being model-independent. As the researches indicated the suggested system can be considered as an improved form of the original TDC model. The researches designed an adaptive rule in order to activate the ASMC term also an additional algorithm was implemented to remove the problem of singularity. Thus, in the suggested system the singularity and the reaching phase are removed and also the robustness of the model is improved thanks to the integral sliding surface.

The suggested algorithm was written and implemented using c++ in Linux operating system environment. For the purpose of verifying the effectiveness and accuracy of the suggested system (TDE based AISMC), the researches performed experimentation on three joint robots of the Samsung Faraman-AT2 using AC servo which is used to serve as a power transmission motor. The researchers compared the efficiency of the suggested algorithm through the RMS values of the simulation with a previously made model. Also, the researchers compared their model with a different constant value of *k* in which *k* is a constant adaptive gain value. The researches evaluate the efficiency of the suggested system by comparing it with three different *k* constant values of adaptive gain which can be grouped as small, proper and large.

The comparison is performed for each joint of the robot. The results of the RMS value of the first joint for the suggested system, smaller constant k value, proper constant k value, large constant k value, and previous design are 7.97, 14.86, 9.70, 30.53, and 25.81 respectively. And the RMS values for the second joint for the suggested system, smaller *k*, proper *k*, larger *k*, and the previously done model are 12.61, 32.62, 16.45, 20.06, and 65.74 respectively. Additionally, the RMS results of the third joint are12.47, 35.19, 17.35, 20.82, 73.83 for the models of the suggested system, smaller k constant value, proper *k* constant value, larger *k* constant value, and previously done model respectively. The comparison of the RMS value of each joint show that the suggested model is better than the previously done model and models having a constant adaptive gain value.

Furthermore, in research (Zhang et al., 2018) the researchers suggested an integral sliding control tracking mechanism for robot manipulators. In which the suggested model provides a solution for the tracking problems of continuous sliding control for robot manipulators with the existence of uncertainties and disturbances from the external environments. The researchers first suggest a control mode which is based on a non-chattering integral terminal sliding. For which the sliding control is integrated with an observer. In order to prove the global finite time of the tracking mode of the robot controller system, the researchers used the Lyapunov stability theory. The paper presents an integral terminal sliding mode control (ITSMC) for the main goal of tracking robot manipulators in finite time under the existences of parametric uncertainties and disturbances that can occur from the external environment.

Additionally, the paper suggests a tracking mode that has a chattering free SMC so that it can guarantee the errors of the siding surface and tracking process is initially suggested. The trajectories of the suggested model don’t meet the boundaries of the layer like the existing SMC models rather it meets the sliding surface within a finite time. Also, the tracking activity of the suggested model ITSMC is assured for the finite-time convergence. The researchers performed an extensive examination of the suggested controller system through the comparisons of two different controllers. In which these two are a continuous terminal sliding mode controls and a fast terminal sliding mode control.

According to their explanation, the suggested controller gains an improved controller tracking performance in the presence of uncertainties and external disturbances. Also, the suggested controller is better than the CTSMC and FTSMC because of its chattering free and has a great and fast steady stage accuracy under uncertainties and disturbances that might happen from the external environment. The researchers used a two-DOF robot for executing the simulation operation in order to demonstrate the suggested ITSMC tracking accuracy and performance in which the robot to be used for simulation has only two joints.

The researchers used Integrated Absolute Error (IAE) and energy of control input (ECI) for evaluating the accuracy and performance of the tracking process and energy required for control. The experimentation results for the IAE and ECI of the suggested system (ITSMC) are 451.30 and 589.35 for joint 1 and 244.48 and 851.56 for joint 2 respectively. And the values of IAE and ECI for CTSMC are 626.37 and 588.58 for joint 1 and 335.86 and 954.89

for joint 2 respectively. Finlay the results of IAE and ECI for the third FTSMC mode are 642.68 and 504.63 for joint 1 and 355.92 and 840.03 for joint 2 respectively. According to the result of their examination, we can conclude that the suggested system ITSMC is better than CTSMC and FTSMC models.

Additionally, in another paper (Enrique, et al., 2010) the researchers suggest an integral control for robot manipulators which basis a nested sliding mode control. The researchers suggest a solution to the problem of tracking robot manipulators with the help and application of ISM and NSM. The suggested controller has the robustness of the nested sliding mode and the ability to reduce the gains of the sliding functions from the ISM. Since the suggested controller is modeled with the combination of ISM using the sliding control mode as a basis it improves the robustness of the model and guarantees the tracking process.

The researches performed a simulation operation to prove the designed algorithm performs as expected. They used a two-link planar robot manipulator having perturbations under the existences of the uncertainties, external disturbances, and variations of parameters. In which the simulation robot has two joints. According to their results and conclusion, the response for joint 1 and joint 2 are appreciable and satisfactory. The suggested controller model has rejected the external disturbances, uncertainties, and also the parameter variations. Also, the researches stated that the result of the simulation for the tracking error was approximately zero.

In a paper (Dumlu, 2018) the researchers suggested a six degree of freedom trajectory tracking control for robotic manipulator based on a suggested FOAISM. The suggested model achieves many different advantages such as a finite time convergence, without chattering, improved tracking precious, and improved robustness using an adaptive integral sliding mode and fractional order control mechanisms. The adaptive method has been used to evaluate and handle the uncertainties and the unknown dynamics of a robot control system without the use of upper bounds. Additionally, for the purpose of proving the efficiency of the suggested system for real-time experimentation of the model using an industrial robot that has six joints.

Similarly, to the papers (Lee et al., 2017) reviewed before this paper presents that there are many other good approaches, however, among the many different robust control methods the SMC is a better and robust control approach for a system which is nonlinear and the involves unmodeled dynamics, uncertainties and external disturbances also the paper presents the use of fraction order calculus (FO) with the adaptive ISMC to create a new approach that uses a fractional-order adaptive ISMC that is examined on an industrial robot manipulating arm of six-DOF. Thus, the suggested system comprises a fractional-order (FO) with adaptive feedback for the purpose of improving the efficiency and performance of a robotic manipulating arm.

The main aim or reason for integrating fraction order (FO) calculus concept with adaptive integral sling mode controller (AISMC) as the paper presents is to design a fast-finite time convergence, accurate and chattering (high-frequency oscillation) free control, and a better robust model for industrial robot manipulators. The control model is designed so that it can deal and compensate with the unknown and un-modeled dynamics without the need of prior knowledge about the nonlinear model. Also, the aim of the control is tracking each joint of the robot based on their angles using the given trajectory reference. In order to track the joints based on the given trajectory reference, the tracking error should be minimized in any cases.

The researches carried experimentation to show the efficiency and effectiveness of the suggested FO-AISMC system and compared their experimentation results with the classic ISMC. The experimentation is performed on an industrial robot manipulator called Denso VP-6242G which has six rotational joints and each joint are controlled by servo motors. The results of the joints for classic ISMC and FO-AISMC are compared based on their root mean square error (RMSE) values. The RMSE experimentation result of the classic ISMC model for joint 1, 2, 3, 5, and 6 are 0.0088, 0.0047, 0.0058, 0.0062, and 0.0047 respectively. And the RMSE experimentation results of the suggested FO-AISMC model for joint 1, 2, 3, 5, and 6 are 0.0041, 0.0024, 0.0039, 0.0043, and 0.0031 respectively. Therefore, based on their experimentation result we can conclude that the suggested fractional order (FO-AISMC) system is better than the classic ISMC model.

Also, in another paper (Heydaryan & Majd, 2017) suggests an integral sliding mode using bilateral teleoperation method that can be applied to the n-DOF. The researchers used a second ordered sliding mode to predict or estimate the unknown input forces of the model from the environment which are acting on the robot manipulators and for estimating the actual state of the system. The ISM which is used by the researches is used provides a better and efficient practical implementation of the suggested system. The bilateral teleoperation constitutes the power and ability to operate remote side manipulator by using a master manipulator from another environment for controlling the save manipulator located in another environment. This operation is performed by sending a signal commands from the master manipulators to the salve. And the command receiver side which is the salve manipulator will simulate the master-slave based on the command received. However, the operation of this command transmission faces a greater problem of time delaying in the operation of command transmission.

Additionally, the researchers suggest (Heydaryan & Majd, 2017) a positioning force bilateral teleoperation design based on two different operations for designing the master and slave manipulator controllers. The first approach of the suggested system is a positioning force bilateral system it uses conventional sliding mode and an impedance controller for the slave controller and the master manipulator side respectively. This first approach also constitutes a constant time delay that is used to improve the transparency of the suggested model. Additionally, for the purpose of estimating the forces, positions, and velocity that are acting on both slave and master manipulators of the suggested system they implemented an observer of second-order sliding mode.

Furthermore, in the second approach, the researchers used a new ISMC for the slave manipulator side in order to remove the chattering as easily as possible. Also, the suggested designed algorithm is much safer and easier for implementing it on a mechanical system. Unlike other models the suggested algorithm doesn't cause the mechanical actuators to wear and tear therefore, it is much suitable for mechanical systems.

The suggested system’s efficiency and accuracy are evaluated through simulation in which the master and slave manipulators are taken as 2-DOF manipulators and also parameters for the simulation operation of the manipulators are specified as a mass of 1-kilogram and a link length of 1-meter. The simulation experiment is carried by considering an external human forces and environmental forces acting on the master manipulator and both the external forces are estimated well by the suggested model. Also, a simulation examination is carried on to check the performance of the mode on position tracking in which the results show there is no contact between the slave and the environment. Moreover, when a simulation is done by applying the conventional sliding mode approach to the slave manipulator the delayed trajectory of the master manipulator’s first degree of freedom is tracked showing that the tracking process worked very well and sufficiently. Furthermore, when the suggested integral siding control mode is applied to the tracking of the joint angels it is also, satisfactory and efficient. In addition to these simulation results when the conventional siding mode is compared with the suggested integral siding mode it shows a large performance variance in the teleoperation process.

Basically, the first paper on the integral sliding system was suggested (V. Utkin & Jingxin Shi, 2002) in 1996 by V. Utkin and J. Shi as an improved model of the conventional sliding mode approach. In which the suggested integral sliding model has a similar order of motion equation with original mode but it has a reduced number of inputs dimension of the control. Therefore, this reduced number of input dimensions improves and assures the robustness of the suggested control mode during the entire system process starting from the beginning of the response time instance.

Also, the researchers suggested an alternative way when the control matrix can't be used for generating a sliding mode control through the discussion of the associated decoupling problem. This alternative way presented by the researches is by using a sliding mode transformation for generating the SMC. For the purpose of removing the chattering problem, the researches removed the discontinuous control activity from the control and integrate it into the internal dynamic process in order to generate the SMC. This technique of removing the chattering problem while delivering a higher degree of robustness gives a better advantage than the conventional chattering removing approach.

Usually, many of the papers which are reviewed before and will be reviewed after are the extension of the concept of this first paper. The robustness of the ISMC is the main reason and intension for other researchers to use and improve this suggested algorithm. The switching part of the suggested integral sliding mode is designed with two different parts. In which the first part involves the conventional sliding mode by using a combination of the linear states of the model. And the second part is designed as the integral terms which are presented by the suggested system. Additionally, the researchers suggest an integral feedback design of the performances of the integral sliding mode system.

In research (Rsetam, et al., 2017) the researchers suggested an optimal second-order ISM control for the purpose of reducing the challenges that occur while dealing with a robot manipulator having a flexible joint. The suggested system (OSOISMC) is intended to improve and obtain a better robustness performance of robot manipulator having a single-link flexible joint and the researchers primarily designed a linear quadratic regulator of an integral sliding mode that can serve as a nominal and switching control so that the control guarantees the robustness of the entire system. For the purpose of decreasing the chattering problem of the suggested the system the researches come up with a second order ISM that has a nonsingular terminal sliding surface also, the second-order ISMC gives the fine convergence. Simulation experimentation is carried out to prove the performance of the suggested system.

The suggested algorithm is compared with a linear quadratic regulator (LQR) and the classic or conventional integral sliding mode control (ISMC) based on the results of the tracking performances and chattering results. The LQR control obtains the performance of the system without the presence of the uncertainties and smaller control efforts. Then the uncertainties and the disturbance of the system are tackled by the ISMC form the initial start of the process. This integral sling mode controller which will result in improving the robustness and performance of the suggested system.

The experimentation is performed by maintaining some control parameters for each of the comparing algorithms in order to make a fair comparison and prove the performance and efficiency of the suggested system. The external disturbance was rejected by the suggested system by improving the response tracking without the effect of the chattering. But the LQR controller was unable to reject the disturbances like the suggested system because it was not designed with ability to reject disturbances. Therefore, the suggested system was much efficient than the LQR model based on rejecting the disturbances. However, the ISMC was designed to reject the disturbance but it results was not as efficient as the suggested system because the chattering problem was not handled in the ISMC system and it has a greater effect on it.

Also, in a paper (Ablay, 2014) the researchers suggested algorithms of two different control approaches which are sliding surface mode based on a coefficient ratio and an ISMC are presented for multivariable systems. The problem of the design of the sliding surface is reduced by using a constant closed-loop system. For the purpose of efficient and robust tracking, a sliding surface based on a coefficient ratio design is used for the suggested system. The researchers implemented the suggested algorithms on a strike aircraft and flexible robot manipulators in order to verify the performance and efficiency of the suggested algorithms and also, the simulation results are presented in the paper.

In order to design the sliding surface of the suggested system, it uses a coefficient ratio-based algorithm that follows four respective steps. The first step of the algorithm is finding τ from a given t where τ=t/3 and the second step is to compute a transformation matrix T which will be used to convert the system into a regular form. The third step of the suggested algorithm is by using the coefficient ratio to design and find the desired spectrum of the system using the characteristic polynomial and finally, the last step is generating the sliding surface using the robust Eigen structure approach.

Additionally, the researches presented another approach for designing the sliding surface and the algorithm is presented in the paper having four consecutive processes. The second suggested algorithm first finds the τ from a given settling time t in where τ= t/3 just like the previous algorithm and secondly it computes the transformation matrix for transforming the reduced-order initial variable into a controllable form. Then in the third stage based on the coefficients, the characters polynomial is calculated for the desired spectrum and at last the sliding surface is obtained by computing the original initial system with the matrix. The coefficient ratio-based method constitutes the advantages of the linear quadratic regulator (LQR) and Eigen structure assignment-based approaches. The numerical simulation result of the suggested systems on a flexible robot manipulator and aircraft are presented on the paper. The examination results show that on both simulation systems the coefficient ratio based and feedback designs provide with similar and better performances of the suggested algorithms.

Furthermore, in research (Nadda & Swarup, 2018) the researchers suggested a nonlinear ISMC for solving robot manipulators position obtaining the problem. The paper presents and proves the efficiency of the suggested system by taking under consideration of the external disturbances, load variations, and other external factors. The performance of the suggested nonlinear ISMC system is evaluated by comparing its performance results with the performance of the linear proportional integral derivative control approaches (PID). The paper presents the performance results by considering a robot manipulator having two degree of freedom (2-DOF) and an integral sliding mode (ISM) control is suggested to support the position using the desired or required value and also the paper proves and guarantees the support of the suggested model is presented.

The researchers removed the discontinues function and replaced it with a saturation function instead of the sign function in order to eliminate the chattering that occurs in the system due to the presence of the sign function. According to the simulation results which are presented in the paper the position tracking of the suggested algorithm is much better than t the PID control model showing the suggested algorithm is efficient and have a better performance than the proportional integral derivative mode control approach. Additionally, the suggested algorithm uses a smaller time for the purpose of stabilizing the positions of the controllers however, the PID control model takes a larger time for the same purpose. This shows the suggested algorithm is robust and faster than the PID controller model.

On the paper (Kumar, Kumar, & Gaidhane, 2018) the researcher tries to compare the performance of integer order fuzzy PID, fuzzy fractional-order PID and conventional PID for trajectory tracking controller of a five-degree of freedom redundant robotic manipulator through Matlab simulation. The redundant 5 DOF robot manipulator is a highly nonlinear multi-input multi-output manipulator whose performance is poorly affected by the matched uncertainty and external disturbance.

For tuning the gain parameter of the proposed controller the researcher used a technique called artificial bee colony optimization technique or ABC, the performance of the proposed system tested using a Matlab Simulink with integral absolute time error (ITAE) and compared each controllers value as 0.2220, 0.8487 and 1.5505 for fuzzy $PI^{\lambda}D^{\mu}$ than fuzzy PID and PID respectively and among them the best controller selected which have small ITAE which is fuzzy $PI^{\lambda}D^{\mu}$ for trajectory tracking application.

In research paper(H., Bendary, & Elserafi, 2016) the researcher suggested a fractional order fuzzy PID controller for the trajectory tracking purpose of a multi input multi output highly nonlinear robot manipulator and implemented the proposed controller on the first three joints of PUMA 560 robot manipulator while the rest three are fixed. The research also designs different type of controller namely PID, FOPID and FUZZY-PID to compare and analyze the result of each different controller with fractional order fuzzy PID.

The kinematics and dynamics model of the robot manipulator is also designed while the matched uncertainty is considered and imposed as input to check the robustness of the controller with matched uncertainty. On the proposed controller the fuzzy logic is served as a supervisory tuning for the gain parameter of the controller while the error and the error derivative were serves as input for the fuzzy logic. These four different controllers provide different RMS performance on the same model as 0.04676 rad, 0.001799 rad, 0.02278 rad and 0.0008419 rad for PID, FOPID, Fuzzy-PID and FO-Fuzzy-PID respectively.

As we studied other related works there is a trajectory tracking problem still needs to be fixed in order to make our system to perform in the highly nonlinear environment and matched uncertainty, so we will propose an adaptive fuzzy integral sliding mode controller that will overcome the trajectory tracking problem.

CHAPTER 3
PROPOSED METHODOLOGY

Different robot has different type of connection of links through their joint, for a robot manipulator to have a rotational movement the links should be joined with revolute joint whereas for sliding movement we need to attach the links in a prismatic way, so that each link can achieve the required movement and desired positions using the forward kinematics and inverse kinematics. The independent input movement that will be achieved by the relative movement of the links in a manipulator is described using degree of freedom, each consecutive links in a joint have its own degree of freedom but differ based on the connection of each independent links, for PUMA560 robot there are 6 revolute joints connected to perform the required trajectory, the kinematics and dynamics of the PUMA560 robot have been studied to apply different type of controller in order to choose the best controller performance, for this research purpose we fixed the first 3 links in order to study the rest 3 links of the robot manipulator.

3.1. Kinematics and Dynamics of Robot Manipulator

For describing, analyzing and designing a robot manipulator a brief knowledge about the kinematics and dynamics of the manipulator plays a key role, the representation and explanation of how the motion, position, and acceleration of the robot manipulator and the cause of the motion will be briefly discussed below.

3.1.1. Kinematics

Kinematics is the analysis of the motion, positions, and acceleration of a rigid body without considering the force or torque associated which those motions. As we know motion is the crucial feature for robotic manipulators while designing the robot's movement and analysis the system robot kinematics is the fundamental aspect.

To represent the kinematics of a mechanism or a rigid body we use two different possible ways which are Cartesian and Quaternion space. For this research purpose, we use Cartesian space representation of the robot manipulator in space.

3.1.1.1. Forward kinematics of robot manipulator

computing the angle parameter of the given mechanism will determine and describes the position and orientation of the end effector of the robot manipulator. For forward kinematics, the first step is to find the homogenous transformation matrix of the manipulator from the first link to the end effector link

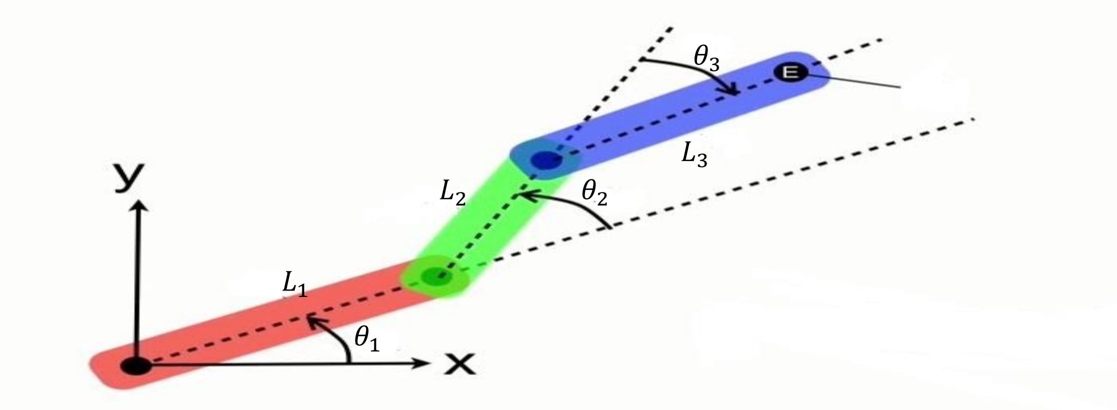

Figure 3.1: Frame representation of 3 DOF robot manipulator

The Denavit-Hatrenberg (DH) convention will be used to find the homogenous transformation matrix. And the value of each link is fully described below

Table 3.1: DH parameter of a 3 DOF planner robot manipulator

Link	a_i	d_i	θ_i	α_i
1	L_1	0	θ_1	0
2	L_2	0	θ_2	0
3	L_3	0	θ_3	0

Using the general transformation matrix to find the transformation matrix we can derive the homogenous transformation matrix for each link and using matrix multiplication we can get the forward kinematics of the end effector.

$$ {}_{i-1}^{\;\;i}T = \begin{bmatrix} \cos(\theta_i) & \cos(\alpha_i) * -\sin(\theta_i) & \sin(\alpha_i)\ \sin(\theta_i) & \mathrm{a}_i \cos(\theta_i) \\ \sin(\theta_i) & \cos(\alpha_i) * \cos(\theta_i) & \sin(\alpha_i) * -\cos(\theta_i) & \mathrm{a}_i \sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.8) $$

$$ {}_0^3T = {}_0^1T\,{}_1^2T\,{}_2^3T \tag{3.9} $$

$$ {}_0^1T = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & L_1(\cos\theta_1) \\ \sin\theta_1 & \cos\theta_1 & 0 & L_1\,(\sin\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_1^2T = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & L_2\,(\cos\theta_2) \\ \sin\theta_2 & \cos\theta_2 & 0 & L_2\,(\sin\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_2^3T = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & L_3\,(\cos\theta_3) \\ \sin\theta_3 & \cos\theta_3 & 0 & L_3\,(\sin\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The total homogenous transformation matrices are the matrix multiplication of the individual rotational matrixes and translation matrixes

$$ {}_0^3T = \begin{bmatrix} \cos(\theta_1+\theta_2+\theta_3) & -\sin(\theta_1+\theta_2+\theta_3) & 0 & L_1\cos(\theta_1)+L_2\cos(\theta_1+\theta_2)+L_3\cos(\theta_1+\theta_2+\theta_3) \\ \sin(\theta_1+\theta_2+\theta_3) & \cos(\theta_1+\theta_2+\theta_3) & 0 & L_1\sin(\theta_1)+L_2\sin(\theta_1+\theta_2)+L_3\sin(\theta_1+\theta_2+\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.10} $$

The homogenous transformation matrix defined in (3.10) is the one that defines the forward kinematics of the 3-DOF robotic arm shown in Figure 3.3. From this matrix, the end effectors position. and orientation is a non-linear function of joint variables P (x, y) = f (θ). Having derived the forward kinematics or direct kinematics of Figure 3.3, it's now possible to define the end-effector's position and orientation from the individual joint angles ($\theta_1, \theta_2 and\ \theta_3$).

$$ \mathrm{X} = L_1\cos(\theta_1) + L_2\cos(\theta_1+\theta_2) + L_3\cos(\theta_1+\theta_2+\theta_3) \tag{3.11} $$

$$ \mathrm{Y} = L_1\sin(\theta_1) + L_2\sin(\theta_1+\theta_2) + L_3\sin(\theta_1+\theta_2+\theta_3) \tag{3.12} $$

To determine the velocity of the robot's end-effector the Jacobean matrix can be applied. The end effector's velocity in the base frame has two components, the first component is the linear velocity(V_n^0) and the second component is the angular velocity (ω_n^0) for n number of joints.

$$\begin{bmatrix} V_n^0 \\ \omega_n^0 \end{bmatrix} = \begin{bmatrix} \dot{X}_n^0 \\ \dot{Y}_n^0 \\ \dot{Z}_n^0 \\ {\omega_x}_n^0 \\ {\omega_y}_n^0 \\ {\omega_z}_n^0 \end{bmatrix} = \mathrm{J}\dot{q} = \mathrm{J} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \tag{3.13}$$

Where $\begin{bmatrix} V_n^0 \\ \omega_n^0 \end{bmatrix}$ is end effector's velocity

V_n^0 is the linear velocity of the end effector from frame 0 to frame n in the base frame

ω_n^0 is end effector's rotational velocity from frame 0 to frame n in the base frame

$\dot{q}$ is the time derivative of each joint parameters

J is a Jacobean matrix

Jacobian matrix is a matrix that transforms the joint velocity $\dot{q}$ into the end effectors velocity. Rank deficiency of Jacobean represents singularity. This means that the joint angular velocities become infinite when the determinant of the Jacobian matrix component becomes zero. This is when the angular position of the third elbow joint θ_3 is either 0^0 or180^0 which leads to the loss of solution number of the inverse kinematics.

$$\mathrm{J} = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \theta_3} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3} \\ \frac{\partial z}{\partial \theta_1} & \frac{\partial z}{\partial \theta_2} & \frac{\partial z}{\partial \theta_3} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \tag{3.14}$$

$$a_{11} = L_2 \sin(\theta_1 + \theta_2) - L_1 \sin(\theta_1) - L_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

$$a_{12} = -L_2 \sin(\theta_1 + \theta_2) - L_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

$$a_{13} = -L_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

$a_{21} = L_2\cos(\theta_1+\theta_2) + L_1\cos(\theta_1) + L_3\cos(\theta_1+\theta_2+\theta_3)$

$a_{22} = L_2\cos(\theta_1+\theta_2) + L_3\cos(\theta_1+\theta_2+\theta_3)$

$a_{23} = L_3\cos(\theta_1+\theta_2+\theta_3)$

$$\begin{pmatrix}\dot{X}\\ \dot{Y}\end{pmatrix} = J * \dot{\theta} \qquad (3.15)$$

3.1.1.2. Inverse kinematics

Inverse kinematics is a function that tells us the joint angle we need to attain a particular robot end-effector position. The inverse kinematics tries to answer what should the joint angle be set in order for the robot end-effector to get to the particular pose? Which is the key for arm type robot? There are several approaches to Figure out the inverse kinematics of a robotic manipulator, for our robot arm manipulator we will use the geometry method as described below.

Figure 3.2: Kinematic representation of a manipulator

From the transformation matrix, we found the forward kinematics of the robot arm manipulator from equation 3.11 and equation 3.12 respectively as follow

$X = L_1\cos(\theta_1) + L_2\cos(\theta_1+\theta_2) + L_3\cos(\theta_1+\theta_2+\theta_3)$

$Y = L_1\sin(\theta_1) + L_2\sin(\theta_1+\theta_2) + L_3\sin(\theta_1+\theta_2+\theta_3)$

$$\phi_e = \theta_1+\theta_2+\theta_3 \qquad (3.16)$$

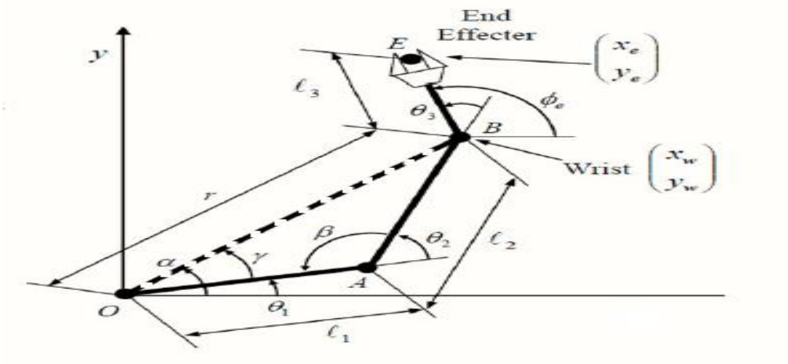

Figure 3.3: Inverse kinematics of a 3 DOF planner robot manipulator

$$x_w = x_e - l_3 \cos \phi_e \tag{3.17}$$

$$y_w = y_e - l_3 \sin \phi_e \tag{3.18}$$

$$l_1^2 + l_2^2 - 2l_1 l_2 \cos \beta \tag{3.19}$$

$$\beta = \cos^{-1}\left(\frac{l_1^2 + l_2^2 - r^2}{2l_1 l_2}\right) \tag{3.20}$$

$$\gamma = \cos^{-1}\left(\frac{r^2 + l_1^2 - l_2^2}{2rr_1}\right) \tag{3.21}$$

Where

$$r^2 = x_w^2 + y_w^2$$

$$\theta_2 = \pi - \beta$$

Similarly, $r^2 + l_1^2 - 2rl_1 \cos \gamma = l_2^2$

$$\alpha = \tan^{-1}\left(\frac{y_w}{x_w}\right)$$

$$\theta_1 = \alpha - \gamma$$

Then,

$$\theta_3 = \phi_e - \theta_1 - \theta_2 \tag{3.22}$$

3.1.2. Dynamic analysis and forces

In the previous section, we studied the kinematics position and differential motion of the robot. Dynamics of the robot is related to the accelerations, loads, masses and inertias. In order to be able to accelerate a mass, we need to exert a force on it. Similarly, to cause an angular acceleration in a rotation body a torque must be exerted on it.

Actuators with a sufficient amount of torque and force are required in order to get an accelerated robot link which will move them to proper and required velocity and acceleration if the link doesn't get the necessary amount of torque and force it will not move and preserve the required positional accuracy.

The dynamic calculation that controls the movement of the link will tell us the about strongness of the actuator, to do so a relationship of inertia(I), mass(m), force(F), angular acceleration and torque is necessary. Using the information achieved from the above relation and considering the external loads we can design the proper robot dynamics.

Using the Lagrangian method, we can derive the dynamic equation of motion for 3 DOF robot manipulating arm of the below Figure3.4.

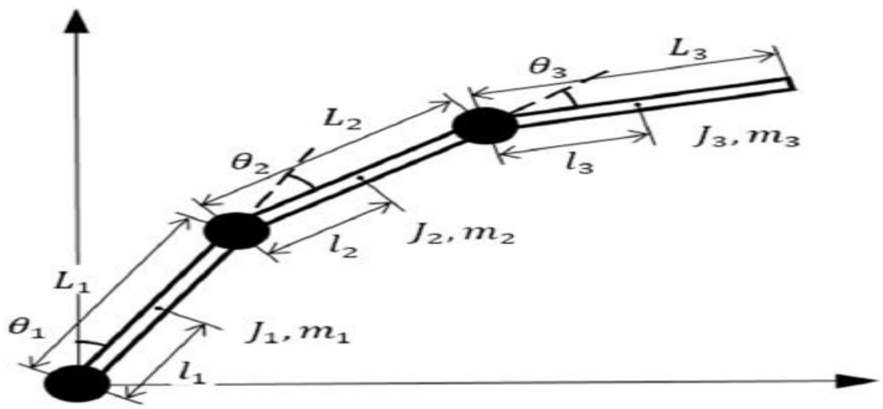

Figure 3.4: Dynamics representation of 3 DOF planner robot manipulator

For link 1, 2 and 3 the kinetic and potential energies are illustrated as follow(Al-Shabi et al., 2017)

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}_i} - \frac{\partial \mathcal{L}}{\partial \theta_i} = \tau_i \tag{3.23}$$

$$\mathcal{L} = k_1 + k_2 + k_3 + p_1 + p_2 + p_3 \tag{3.24}$$

$$k_i = \frac{1}{2} m_i ||v_i||^2 + \frac{1}{2} I_i \left(\sum_i \dot{\theta}_i\right)^2 \tag{3.25}$$

$$p_i = m_i g l_{i,y} \tag{3.26}$$

Where: $||v_i||$ the magnitude of the velocity

$l_{i,y}$ the position of the centroid that is dependent on joint coordinate

As we know the Lagrangian equation is a sum of the kinetic and potential energy, so to drive the components of lagrangian we need to compute the derivative of each components and use the kinetic and potential energy formula.

$$x_1 = L_1 cos\theta_1$$

$$y_1 = L_1 sin\theta_1$$

$$x_2 = L_1 cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$

$$y_2 = L_1 sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$$

$$x_3 = L_1 cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) + L_3 \cos(\theta_1 + \theta_2 + \theta_3)$$

$$y_3 = L_1 sin\theta_1 + L_2 \sin(\theta_1 + \theta_2) + L_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

$$\dot{x}_1 = -L_1 \dot{\theta}_1 sin\theta_1 \tag{3.27}$$

$$\dot{y}_1 = L_1 \dot{\theta}_1 cos\theta_1 \tag{3.28}$$

$$\dot{x}_2 = -L_1 \dot{\theta}_1 sin\theta_1 - L_2(\dot{\theta}_1 + \dot{\theta}_2)\sin(\theta_1 + \theta_2) \tag{3.29}$$

$$\dot{y}_2 = L_1 \dot{\theta}_1 cos\theta_1 + L_2(\dot{\theta}_1 + \dot{\theta}_2)\cos(\theta_1 + \theta_2) \tag{3.30}$$

$$\dot{x}_3 = -L_1 \dot{\theta}_1 sin\theta_1 - L_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)\sin(\theta_1 + \theta_2) - L_3(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)\sin(\theta_1 + \theta_2 + \theta_3) \tag{3.31}$$

$$\dot{y}_3 = L_1 \dot{\theta}_1 cos\theta_1 + L_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)\cos(\theta_1 + \theta_2) + L_3(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)\cos(\theta_1 + \theta_2 + \theta_3) \tag{3.32}$$

After calculating the torque using the derivation of the Lagrangian equation we can use it to drive the final dynamic equation of three degree of freedom robot arm manipulator which includes asymmetric and positive definite inertia matrix ($M(\theta)$, Coriolis and centrifugal matrix ($c(\theta,\dot{\theta})$) and the gravitational matrix ($g(\theta)$).

$$\tau = M(\theta)\ddot{\theta} + c(\theta,\dot{\theta}) + g(\theta) \tag{3.33}$$

$$M(\theta) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{3.34}$$

$$c(\theta,\dot{\theta}) = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{3.35}$$

$$g(\theta) = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} \tag{3.36}$$

Table 3.2: Physical parameter of the 3 DOF planner robot manipulator

Parameter	**Notation**	**Value**
Link length 1	L_1	0.2m
Link length 2	L_2	0.2m
Link length 3	L_3	0.2m
Mass of link 1	m_1	1kg
Mass of link 2	m_2	1kg
Mass of link 3	m_3	1kg
Link 1 center of mass	l_{c1}	0.1m
Link 2 center of mass	l_{c2}	0.1m
Link 3 center of mass	l_{c3}	0.1m
Inertia of link 1	I_1	0.5kg*m^2
Inertia link 2	I_2	0.5kg*m^2
Inertia link 3	I_3	0.5kg*m^2
Gravitational acceleration	g	9.8m/s^2

3.2. The Proposed Adaptive Fuzzy Integral Sliding Mode (AFISMC)

Control of a nonlinear system is classified based on the model as model-based and non-model based, for non-model-based system detailed mathematical knowledge is not necessary whereas for the model-based system the detailed analysis of dynamics of the system is required to obtain the desired control mechanism.

The variable structure control system is a robust way of controlling uncertainties and external disturbance where the switching function control determines which structural control law will be applied by changing them. Sliding mode control is one of the methods in the variable structure control system for robust control of a system which is insensitive for uncertainty and disturbance for a nonlinear system.

Since sliding mode control is affected by external disturbance during the reaching phase, it is difficult to control some uncertainties even the matched one. To overcome this problem an efficient and precise way of controlling method has been suggested which is integral sliding mode controller. (Edwards et al., 2016).

3.2.1. Integral sliding mode control

For a nonlinear system with disturbance and parametric uncertainties, a sliding mode controller is an efficient and accurate way to handle the system dynamics, the compensated dynamic using sliding mode control system becomes insensitive for the matched and unmatched disturbance but as a drawback, the controller will develop a chattering problem for the system.

For a linear system, the traditional way of controlling which is proportional integral derivative (PID) and proportional derivative (PD) can handle but when the nonlinear dynamics overcome the linear dynamics those technique fails to compensate the dynamics.

The total torque will be written as

$$\tau = [\tau_1 \quad \tau_1 \quad \tau_3]^T = M(\theta)\ddot{\theta} + c\big(\theta, \dot{\theta}\big) + g(\theta) \qquad (3.37)$$

Where $\theta, \dot{\theta}, \ddot{\theta}$ represents the vectors of joint position, velocity and acceleration respectively, $M(\theta) \in R^{nxn}$ is inertia matrix, $c(\theta, \dot{\theta}) \in R^{nx1}$ is centripetal Coriolis and $g(\theta) \ \in R^{nx1}$ is gravitational matrix

For any n number of degree of freedom robot the generalized dynamic equation is given by the equation below

$$\tau = M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta)$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} \tag{3.38}$$

From the above equation, we can get each torque individually

$$\tau_1 = [((a_{11} * \ddot{\theta}_1) + (a_{12} * \ddot{\theta}_2) + (a_{13} * \ddot{\theta}_3)) + b_1 + g_1] \tag{3.39}$$

$$\tau_2 = [((a_{21} * \ddot{\theta}_1) + (a_{22} * \ddot{\theta}_2) + (a_{23} * \ddot{\theta}_3)) + b_2 + g_2] \tag{3.40}$$

$$\tau_3 = [((a_{31} * \ddot{\theta}_1) + (a_{32} * \ddot{\theta}_2) + (a_{33} * \ddot{\theta}_3)) + b_3 + g_3] \tag{3.41}$$

$$\ddot{\theta} = M^{-1}(\theta)\big(\tau - c(\theta, \dot{\theta}) - g(\theta)\big) \tag{3.42}$$

When a sliding mode have the equation of motion same order as the original system is called an ISMC (integral sliding mode controller). The logic behind the ISMC methodology is to find an expression which satisfies $x(t) \equiv x_0(t)$ at all-time $t > 0$, the equivalent control(u_{1eq}) of the control part, should obey the expression

$$B(x)u_{1eq} = -h(x, t) \tag{3.43}$$

The first step for designing the ISMC for a dynamic model whose state-space equation is expressed as

$$\dot{x} = f(x) + B(x)u + h(x, t) \tag{3.44}$$

Where $x \in R^n$ a representation of a state vector, $u \in R^m$ is a representation of control input of the system and $h(x, t)$ is a representation of perturbation vector caused by the dynamic system of uncertainty and disturbance here the perturbation vector is assumed to ensure the matched condition.

The control law for the dynamic system of the state space equation is described as following

$$u = u_0 + u_1 \tag{3.45}$$

In which u_0 represents the performance of the nominal control and u_1 represents the discontinuous control, which is responsible to reject the uncertainty and disturbance of a

perturbation vector for the dynamic manipulation of the robot arm to ensure the sliding motion(Esmaili & Haron, 2017).

The sliding manifold of the dynamic system is represented as

$$s = s_0(x) + z, \qquad s, s_0(x), z \in R \tag{3.46}$$

$$z = s_0(x) - s$$

$$\dot{z} = \frac{\partial s_0(x)}{\partial x}$$

$$\dot{z} = -\frac{\partial s_0}{\partial x}\dot{x} = \frac{\partial s_0}{\partial x}(f(x) + B(x)u_0 x), \quad Z(0) = -s_0\big(x(0)\big) \tag{3.47}$$

The above equation 3.19 contains both a linear combination in sliding mode $s_0(x)$ of the system state and expression for integral design Z. To fulfill the initial condition of $Z(0)$ the condition $s(0) = 0$ must be satisfied, and it is shown that the discontinuity control rejects the uncertainty and disturbance parameters(Vadim Utkin, Guldner, & Shi, 2017).

When we come to the real practical application the high-frequency vibration will lead the system to chattering problem when applying the discontinuous control u_1 to the dynamic model of a robotic manipulator, in order to overcome this chattering problem and reduce the effect on our dynamic system the equation will be modified to

$$s = s_0(x) + z,$$

$$\dot{z} = -\frac{\partial s_0}{\partial x}\dot{x} = \frac{\partial s_0}{\partial x}\{f(x) + B(x)u - B(x)u_1\} \tag{3.48}$$

$$z(0) = -s_0\big(x(0)\big)$$

$$u = u_0 + u_{1eq}$$

By solving the time derivative of s and compute to zero ($\dot{s} = 0$) for control part(u_1) it can be checked that u_{1eq} of u_1 cancels the uncertainty and disturbances of the dynamic robot manipulator(Vadim Utkin, 1992; Vadim Utkin et al., 2017).

An equational expression that describes the motion caused due to the applied force on robot manipulator is called dynamic equation of the robot arm. The dynamic equation of the manipulator helps to relate, define and express displacement, velocity, and acceleration of the manipulator when force is applied to it.

$$M(\theta)\ddot{\theta} + c\big(\theta,\dot{\theta}\big) + g(\theta) = \tau$$

To make the design easy we change the variables as

$$M(\theta)\ddot{\theta} + N\big(\theta,\dot{\theta}\big) = \tau \tag{3.49}$$

where $N\big(\theta,\dot{\theta}\big) = c\big(\theta,\dot{\theta}\big) + g(\theta)$, this expression doesn't contain the acceleration term $\ddot{\theta}$, let's express the dynamics model as a combination of with ideal part and the uncertainty and disturbances term (perturbation term)

$$M_0(\theta)\ddot{\theta} + N_0\big(\theta,\dot{\theta}\big) = \tau + H(\theta,\dot{\theta},\ddot{\theta}) \tag{3.50}$$

Where

$$M_0(\theta) = M(\theta) - \Delta M$$

$$N_0\big(\theta,\dot{\theta}\big) = N\big(\theta,\dot{\theta}\big) - \Delta N,$$

$$H\big(\theta,\dot{\theta},\ddot{\theta}\big) = -(\Delta M\ddot{\theta} + \Delta N)$$

ΔM And ΔN are unknown matrices of the dynamic manipulator's motion vectors of $M(\theta)\ and\ N\big(\theta,\dot{\theta}\big)$ and the perturbation vector, $H\big(\theta,\dot{\theta},\ddot{\theta}\big)$ satisfies the matching condition.

Following the control law of the dynamic model of the robot manipulator given on equation 3.45 and the computed torque control method stated in (Dodds, 2015).

The system can be expressed as follow

$$\tau = \tau_0 + \tau_1$$

$$\ddot{\theta} = \ddot{\theta}_d + K_d\dot{e} + K_pe$$

$$\tau_0 = M_0(\theta)\left[\ddot{\theta}_d + K_d\dot{e} + K_pe\right] + N_0\left(\theta, \dot{\theta}\right) \tag{3.51}$$

The trajectory tracking error $e = \theta_a - \theta_d$ and $\dot{e} = \dot{\theta}_a - \dot{\theta}_d$, where θ_d is the ideal or desired position and θ_a is the practical position, the nominal value of the inertia matric and Coriolis and gravitational force are $M_0(\theta)\ and\ N_0\left(\theta, \dot{\theta}\right)\ respectively,\ K_p\ and\ K_i$ are (nxn) diagonal matrices which ensure the stability of the error system, $K_p = diag\left(K_p, 1, \ldots\ldots K_p\ , n\right) > 0\ and\ K_i = diag(K_i, 1, \ldots\ldots K_i\ , n) > 0$.

The sliding manifold expressed in equation 3.46 can be rewritten using a definite positive gain and identity (nxn) matrix as follow

$$s = s_0(x) + z$$

$$s_0 = [C \quad I]\begin{bmatrix} e \\ \dot{e} \end{bmatrix}$$

$$\dot{z} = -[C \quad I]\begin{bmatrix} \dot{e} \\ M_0^{-1}N_0 + M_0^{-1}\tau_0 - \ddot{\theta}_d \end{bmatrix} \tag{3.52}$$

$$z(0) = -ce(0) + \dot{e}(0)$$

Substituting τ_0 in equation 3.25 vector s can be more simplified as

$$s = \dot{e} + K_pe + K_i\int_0^t e(\xi)d\xi - \dot{e}(0) - K_pe(0) \tag{3.53}$$

From the above, we can see that the condition $s(0) = 0$ is satisfied and resulting in the occurrence of sliding mode starting from $t = 0$. Equation 3.53 represents the natural occurrence of the basic design equation of ISM. The time derivative of $s(t)$ is

$$\frac{ds}{dt} = \dot{s} = \dot{s}_0 + \dot{z} = \zeta_1 + \zeta_2\tau_0 + M^{-1}\tau_1 \tag{3.54}$$

Where $\zeta_1 = (M_0^{-1}N_0 - M^{-1}N) and\ \zeta_2 = (M^{-1} - M_0^{-1})$ these parameters represent the mismatch between the nominal and real parameters $M_0, N_0(\theta, \dot{\theta})\ and\ M(\theta), N(\theta, \dot{\theta})$ respectively, $\zeta_1 and\ \zeta_2\tau_0$ are bounded.

To reject overall uncertainty and disturbance torque $H(\theta, \dot{\theta}, \ddot{\theta})$ of the system, we design a discontinuous control τ_1 with a positive constant gain $\varGamma_0$ as follow.

$$\tau_1 = -\Gamma_0 sign(s) \tag{3.55}$$

To stabilize the sliding mode, we will consider a Lyapunov function as

$$V = {}^1/_{2s^Ts} > 0\ for\ s \neq 0$$

$$\dot{\mathrm{V}} = \mathrm{s}^\mathrm{T}\dot{\mathrm{s}} = \mathrm{s}^\mathrm{T}(\zeta_1 + \zeta_2\tau_0) - \mathrm{s}^\mathrm{T}\mathrm{M}^{-1}\Gamma_0\mathrm{sat(s)} \tag{3.56}$$

The term $\mathrm{s}^\mathrm{T}\mathrm{M}^{-1}\Gamma_0\mathrm{sat(s)}$ will be positive because of the term M^{-1} and $\varGamma_0$ are positive for, $s \neq 0$ on the above we clearly show the occurrence of sliding mode by bounding the system to manifolds $s(t) = 0$, the equivalent torque can be found by taking the time derivative of s to zero $\dot{s} = 0$.

$$\tau_{1eq} = -M(\zeta_1 + \zeta_2\tau_0) \tag{3.57}$$

Equating equation 3.49 and equation 3.57 we will get a simplified form of the equation

$$M_0(\theta)\ddot{\theta} + N_0(\theta, \dot{\theta}) = \tau_0 \tag{3.58}$$

The general integral sliding mode controller equation for a robot manipulator is stated below,

$$s = \dot{e} + K_p e + K_i \int_0^t e(\xi)d\xi - \dot{e}(0) - K_p e(0) \tag{3.59}$$

$$\tau_1 = -\varGamma_0 sat(s)$$

$$\tau = \tau_0 + \tau_1$$

Implementation of the integral sliding mode on 3 DOF robot manipulator

$$\tau = \tau_0 + \tau_1$$

$$\tau_0 = \tau_{eq} = [M^{-1}\ (B + G) + \dot{S}]M$$

Where

$$\tau_0 = \begin{bmatrix} \tau_{eq1} \\ \tau_{eq2} \\ \tau_{eq3} \end{bmatrix}, \acute{\Gamma} = \begin{bmatrix} \acute{\Gamma}_1 \\ \acute{\Gamma}_2 \\ \acute{\Gamma}_3 \end{bmatrix}, \left({S}/{\emptyset}\right) = \begin{bmatrix} {S}/{\emptyset_1} \\ {S}/{\emptyset_2} \\ {S}/{\emptyset_3} \end{bmatrix}$$

$$sat\left({S}/{\emptyset}\right) = \begin{cases} 1 & \left({S}/{\emptyset} > 1\right) \\ -1 & \left({S}/{\emptyset} < 1\right) \\ {S}/{\emptyset} & -1 < {S}/{\emptyset} > 1 \end{cases}$$

$$\dot{S} = \begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \\ \dot{S}_3 \end{bmatrix}, \tau_1 = \tau_{sat} = \acute{\Gamma} * sat\left({S}/{\emptyset}\right)$$

$$\tau_{sat} = \begin{bmatrix} \tau_{sat1} \\ \tau_{sat2} \\ \tau_{sat3} \end{bmatrix}$$

$$\tau = \left[M^{-1}\,(N + G) + \dot{S}\right]M + \acute{\Gamma} * sat\left({S}/{\emptyset}\right) \quad (3.60)$$

3.2.2. The proposed fuzzy control-based adaptation mechanism

Fuzzy logic is a way of computing based on the degree of truth instead of crisp or Boolean value. In this research, we will implement the well-known adaptive fuzzy inference system to automatically tune the gain of the nonlinear control system which does not have prior information about the parameters of the system. Combination of robust control with adaptive fuzzy logic inference system for controlling a trajectory tracking of a robot manipulator has been suggested by the different researcher so far.

For a system which has a changing dynamic parameter are need to be trained online, the adaptive fuzzy will try to tune the gain parameter without any information about the dynamic system behavior, by combining the integral sliding mode controller with adaptive fuzzy logic gain scheduler, we can get accurate trajectory tracking controller

For this research, we will implement an adaptive fuzzy inference system to tune the gain of the integral sliding surface K_s to remain the surface to zero s=0 and the gain of the integral and derivative parameters of the $\dot{s}$, K_p and K_i to obtain the desired output.

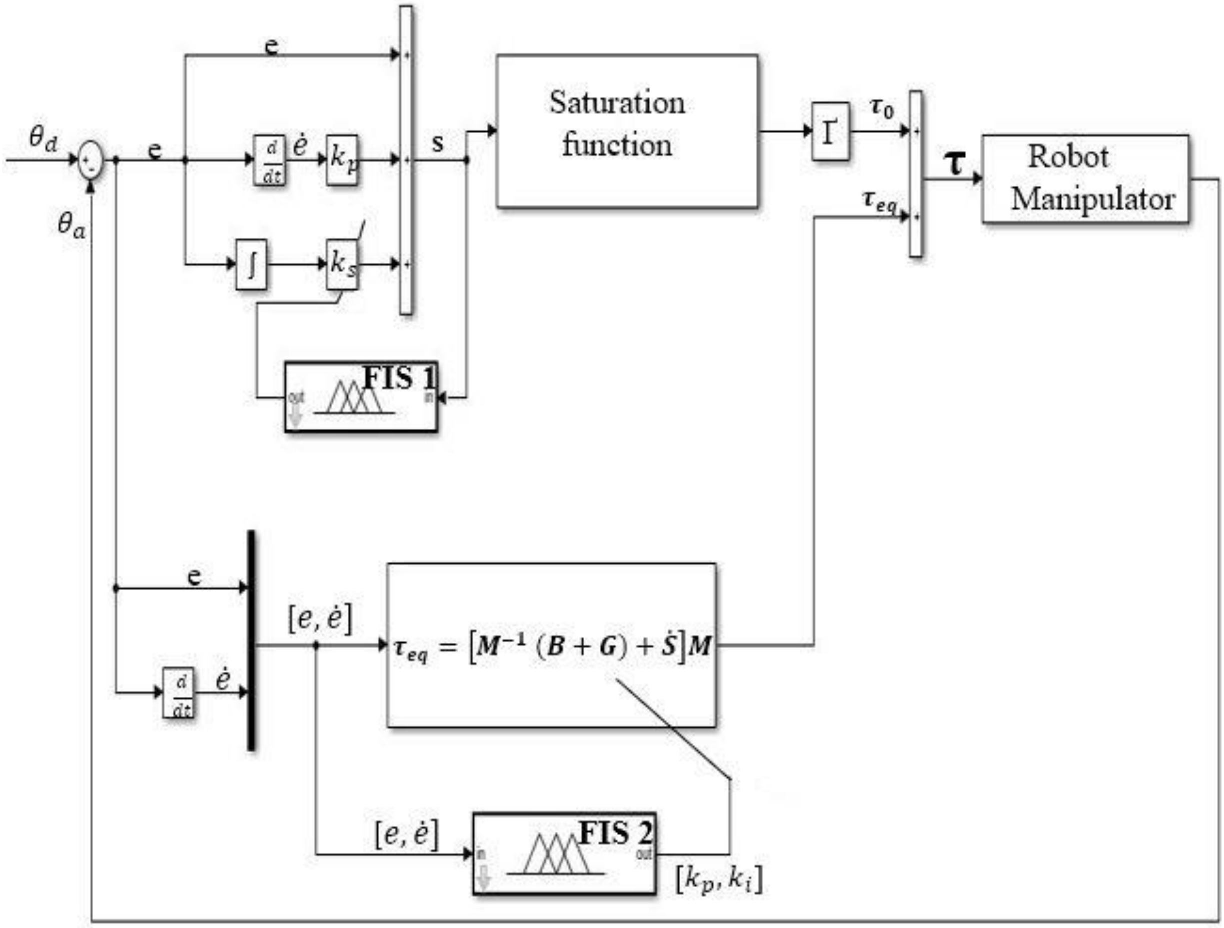

Figure 3.5: Block diagram of an adaptive fuzzy gain scheduling sliding mode controller

The above Figure shows that the general structure of the controller integrated with the robot manipulator where the adaptive fuzzy logic is individually applied to the sliding surface s to tune gain parameter K_s every time so that the sliding surface sustain on the s=0 and the second adaptive fuzzy logic is responsible to tune the gain parameter of the equivalent torque controller part. After properly tuned the total torque f the controller is feed back to the robot manipulator kinematics to perform the required trajectory.

3.2.2.1. Fuzzy logic to tune sliding surface

The sliding surface of integral sliding mode controller should be equal to zero in order to have a desirable reduced order dynamic when constrained to it. In ideal system the sliding surface is designed to satisfy the condition s=0 but when there is uncertainty the surfaces will have some difficulties to achieve the required, so the fuzzy logic will improve the sliding surface performance by approximate the sliding manifold to zero. The fuzzy logic we use the output of the sliding manifold as input and take the gain parameter of the integral part as output then using the described fuzzy rule the system gain will be tuned.

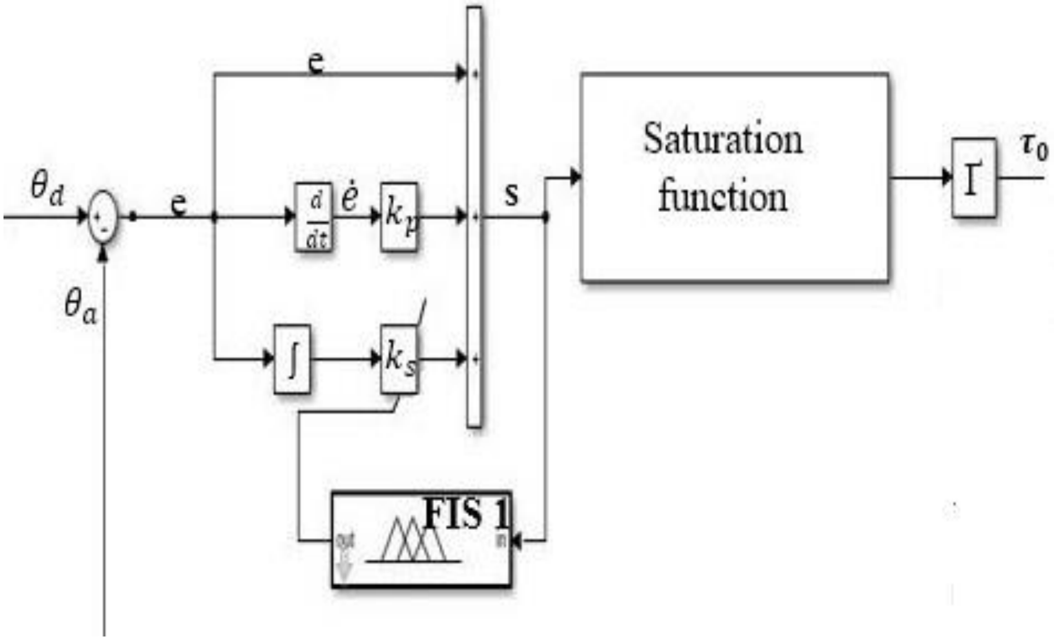

Figure 3.6: Block diagram of the adaptive fuzzy inference tuner of the gain

To design the fuzzy inference for sliding surface, we will follow some steps:

Step1: the controller will have one input and one output and seven linguistic membership function each, the input is the sliding surface s and the output is gain of an integral part K_s.

Step2: the linguistic membership function for our fuzzy inference system are NB (Negative Big), NM(Negative Medium), NS(Negative Small), Z(zero), PS(positive Small), PM(Positive Medium) and PB(Positive Big) with the range of -0.01 to 0.01 and quantized as -0.01, -0.006667, -0.003333, 1.735e-18, 0.003333, 0.006667 and 0.01 also for the gain K_i the linguistic membership function are LL(Large Left), ML(Medium Left), SL(Small Left), Z(Zero), SR(Small Right), MR(Medium Right) and LR(Large Right) with the range of -0.01 to 0.01 and quantized as -0.01, -0.006667, -0.003333, 1.735e-18, 0.003333, 0.006667 and 0.01.

Step3: the shape of the membership function is selected as a triangular membership function

Step4: the main goal of this fuzzy inference is to make the sliding surface equals to zero for that purpose we will state some rules and generalized as follow

Table 3.3: Control rule of the fuzzy inference of sliding surface

S	NB	MM	NS	Z	PS	PM	PB
K_s	LL	LM	LS	Z	SR	MR	LR

The general rule of the fuzzy logic controller tries to sustain the sliding surface to zero, which helps the control system to maintain stability and robustness.

If (S is NB) then (K_s is LL)

If (S is MM) then (K_s is LM)

If (S is NS) then (K_s is LS)

If (S is Z) then (K_s is Z)

If (S is PS) then (K_s is SR)

If (S is PM) then (K_s is MR)

If (S is PB) then (K_s is LR)

Step5: Defuzzification: for this step, we use the center of gravity.

3.2.2.2. Fuzzy logic to tune equivalent control law gains

The second adaptive fuzzy inference system to tune the gain of $\dot{s}$ will use the data of the error and derivative of the error as input and gives moderate value for the gain parameter k_p and k_i.

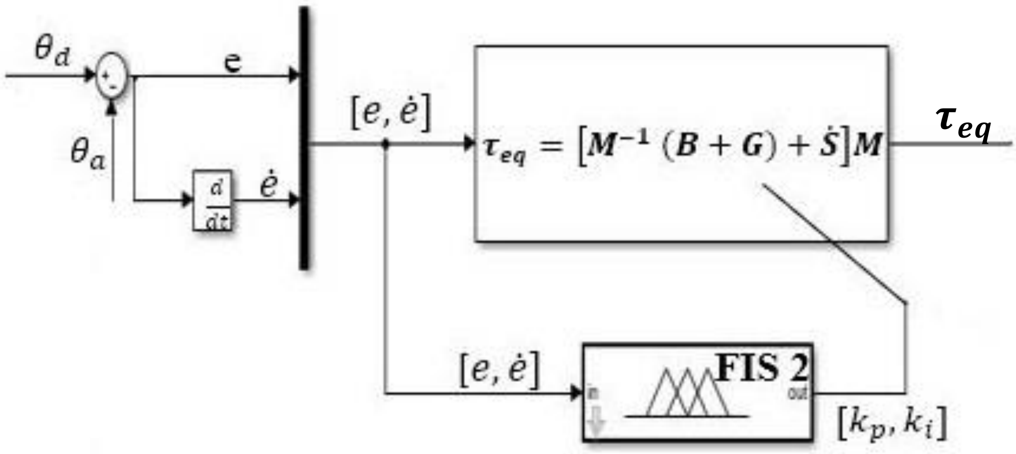

Figure 3.7: Block diagram of the adaptive fuzzy inference tuner of the gain

Step1: the controller will have two input and two output and seven linguistic membership function each, the input are the error and derivative of the error($\dot{e}$ and e) and the outputs are the gains K_p and K_i.

Step2: the linguistic membership function for our fuzzy inference system are NB (Negative Big), N(Negative), Z(zero), P(Positive), and PB(Positive Big) with the range of -0.01 to 0.01 and quantized as -0.01, -0.006667, -0.003333, 1.735e-18, 0.003333, 0.006667 and 0.01 also for the gain K_i the linguistic membership function are LL(Large Left), ML(Medium Left), SL(Small Left), Z(Zero), SR(Small Right), MR(Medium Right) and LR(Large Right) with the range of -0.01 to 0.01 and quantized as -0.01, -0.006667, -0.003333, 1.735e-18, 0.003333, 0.006667 and 0.01.

Step3: the shape of the membership function is selected as a triangular membership function

Step4: the operating rule for the membership function is stated in the below Table

Table 3.4: Fuzzy rule for K_p variable

		e				
		NB	**N**	**Z**	**P**	**PB**
	NB	Z	Z	Z	PS	PM
	N	Z	PS	PS	PS	PM
$\dot{e}$	Z	Z	PS	PM	P	PB
	P	PM	P	P	P	PB
	PB	PM	P	PB	PB	PB

Table 3.5: Fuzzy rule for K_i variable

		e				
		NB	**N**	**Z**	**P**	**PB**
	NB	PB	Z	PM	PS	Z
	N	PB	P	P	PM	Z
$\dot{e}$	Z	P	P	PM	PS	Z
	P	PM	P	PS	PS	Z
	PB	PM	PS	Z	Z	Z

The system will have 25 rules that vary in the given range of value

If (e is NB) and ($\dot{e}$ is NB) then (k_p is Z) and (k_i is PB)

If (e is Z) and ($\dot{e}$ is NB) then (k_p is Z) and (k_i is Z)

If (e is Z) and ($\dot{e}$ is NB) then (k_p is Z) and (k_i is PB)

If (e is P) and ($\dot{e}$ is NB) then (k_p is PS) and (k_i is PS)

If (e is PB) and ($\dot{e}$ is NB) then (k_p is PM) and (k_i is Z)

If (e is NB) and ($\dot{e}$ is N) then (k_p is Z) and (k_i is PB)

If (e is N) and ($\dot{e}$ is N) then (k_p is PS) and (k_i is P)

If (e is Z) and ($\dot{e}$ is N) then (k_p is PS) and (k_i is P)

If (e is P) and ($\dot{e}$ is Z) then (k_p is PS) and (k_i is PM)

If (e is PB) and ($\dot{e}$ is Z) then (k_p is PM) and (k_i is Z)

If (e is NB) and ($\dot{e}$ is Z) then (k_p is Z) and (k_i is P)

If (e is N) and ($\dot{e}$ is Z) then (k_p is PS) and (k_i is P)

If (e is Z) and ($\dot{e}$ is Z) then (k_p is PM) and (k_i is PM)

If (e is P) and ($\dot{e}$ is Z) then (k_p is P) and (k_i is PS)

If (e is PB) and ($\dot{e}$ is Z) then (k_p is PB) and (k_i is Z)

If (e is NB) and ($\dot{e}$ is P) then (k_p is PM) and (k_i is PM)

If (e is N) and ($\dot{e}$ is P) then (k_p is P) and (k_i is P)

If (e is Z) and ($\dot{e}$ is P) then (k_p is P) and (k_i is PS)

If (e is P) and ($\dot{e}$ is P) then (k_p is P) and (k_i is PS)

If (e is PB) and ($\dot{e}$ is P) then (k_p is PB) and (k_i is Z)

If (e is NB) and ($\dot{e}$ is PB) then (k_p is PM) and (k_i is PM)

If (e is N) and ($\dot{e}$ is PB) then (k_p is P) and (k_i is PS)

If (e is Z) and ($\dot{e}$ is PB) then (k_p is PB) and (k_i is Z)

If (e is P) and ($\dot{e}$ is PB) then (k_p is PB) and (k_i is Z)

If (e is PB) and ($\dot{e}$ is PB) then (k_p is PB) and (k_i is Z)

Step 5: Defuzzification: for this step we use the center of gravity method and the block diagram

The proposed system will use the fuzzy logic controller to adaptively tune the gain parameter of the controller automatically to give a good performance and robust system.

CHAPTER 4
SIMULATION DESIGN AND RESULT DISCUSSION

The simulation test of the ISMC with adaptive fuzzy logic to tune the gain parameter of the controller for 3 DOF planner robot manipulator is carried out using the Simulink feature of Matlab 2019a. In this chapter, we will build the control system for both integral sliding mode and AFISMC with and without uncertainty for three degree of freedom robot manipulator whose mathematical modeling has been studied in the previous chapter briefly.

The robustness of the designed controller is tested based on the power of disturbance elimination and insensitive to parameter variation in the adaptive fuzzy logic integral sliding mode and integral sliding mode controller. The value of disturbance applied to the parameters of the system is predefined as 20% of the input signal. Based on the result ADISMC is more robust than ISMC and SMC.

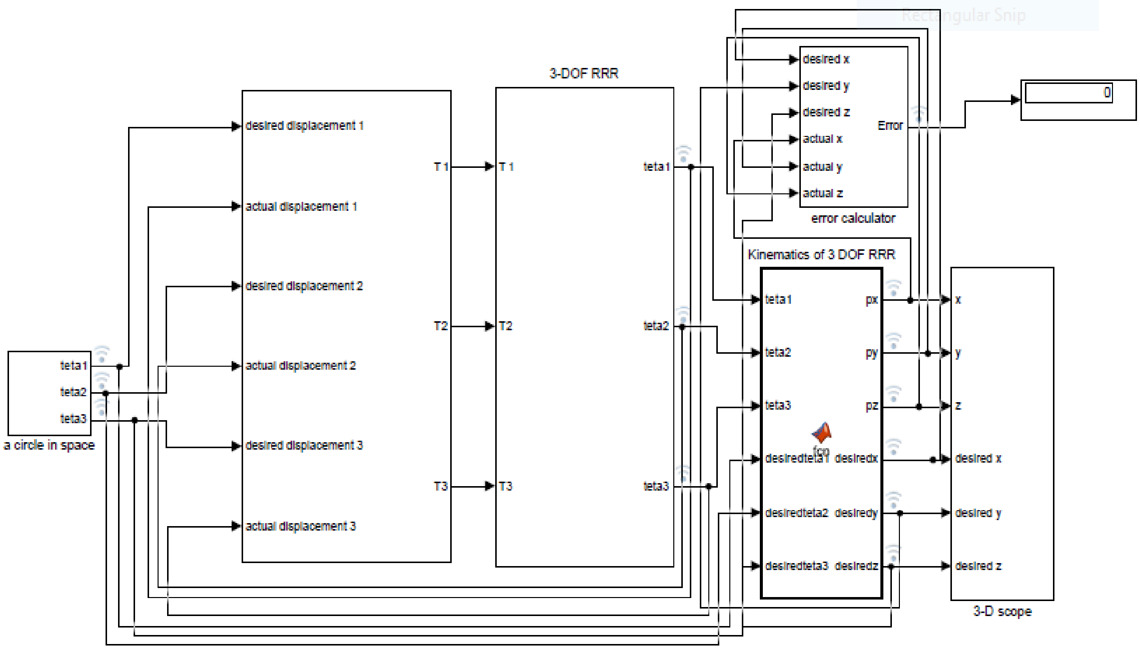

Figure 4.1: Overall adaptive fuzzy integral sliding mode control

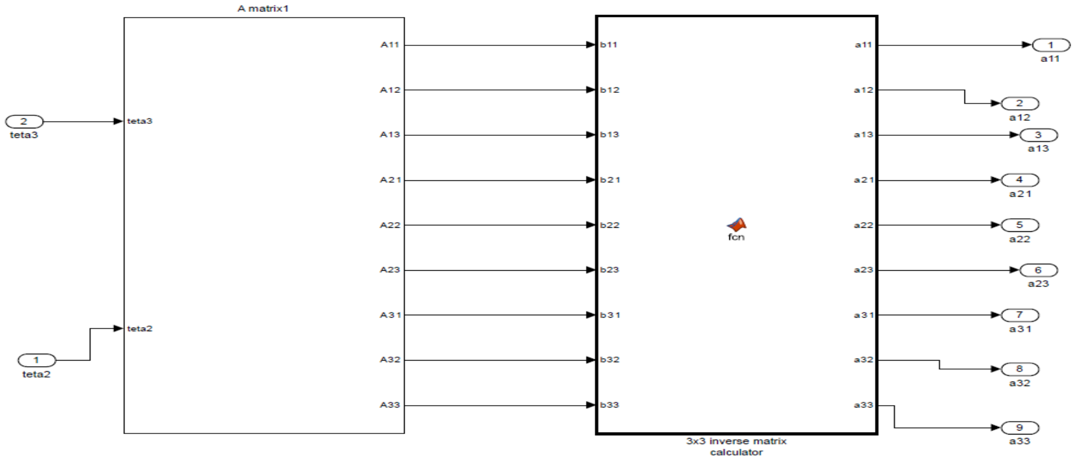

Figure 4.2: Inverse kinematics calculator

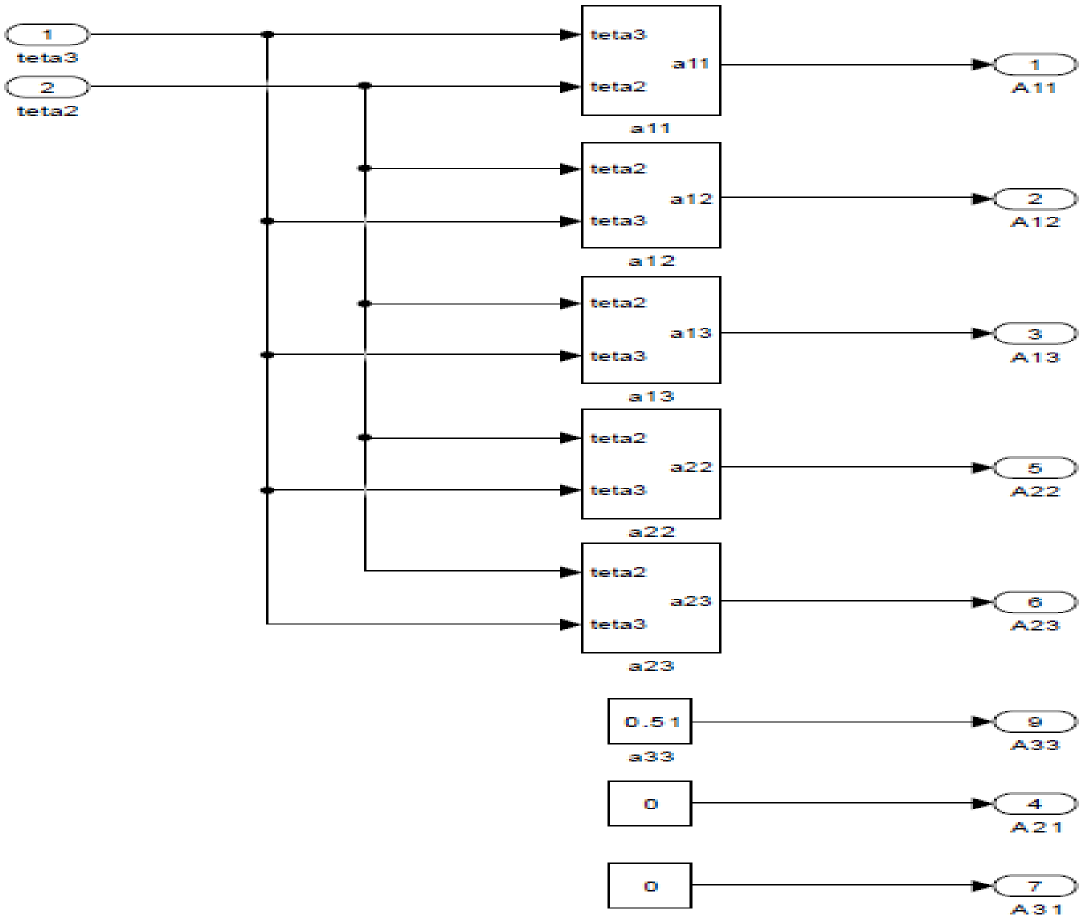

Figure 4.3: Inertia matrix calculator

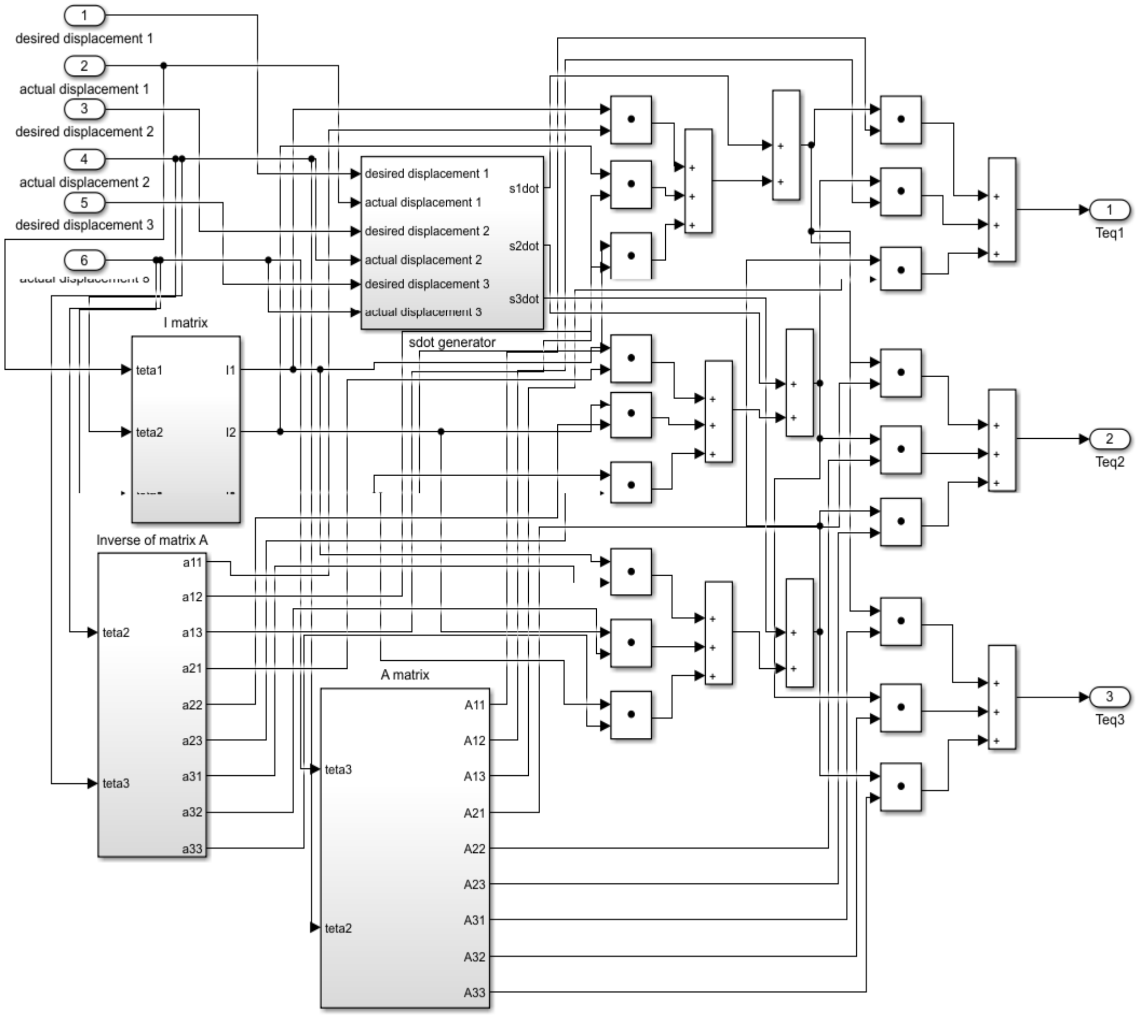

Figure 4.4: Equivalent control law generator

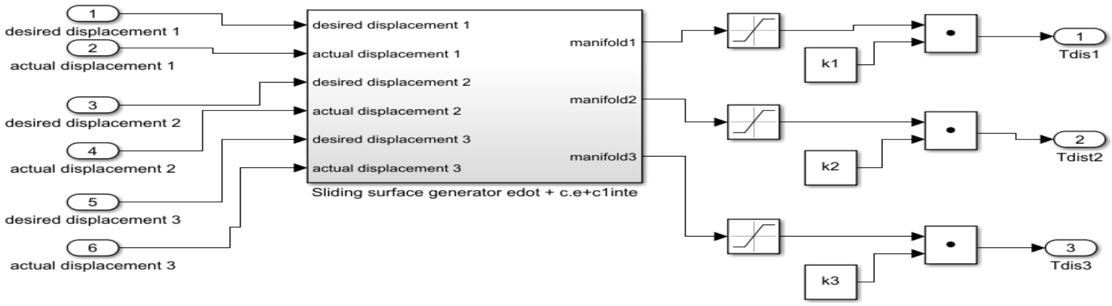

Figure 4.5: Discontinues control law generator

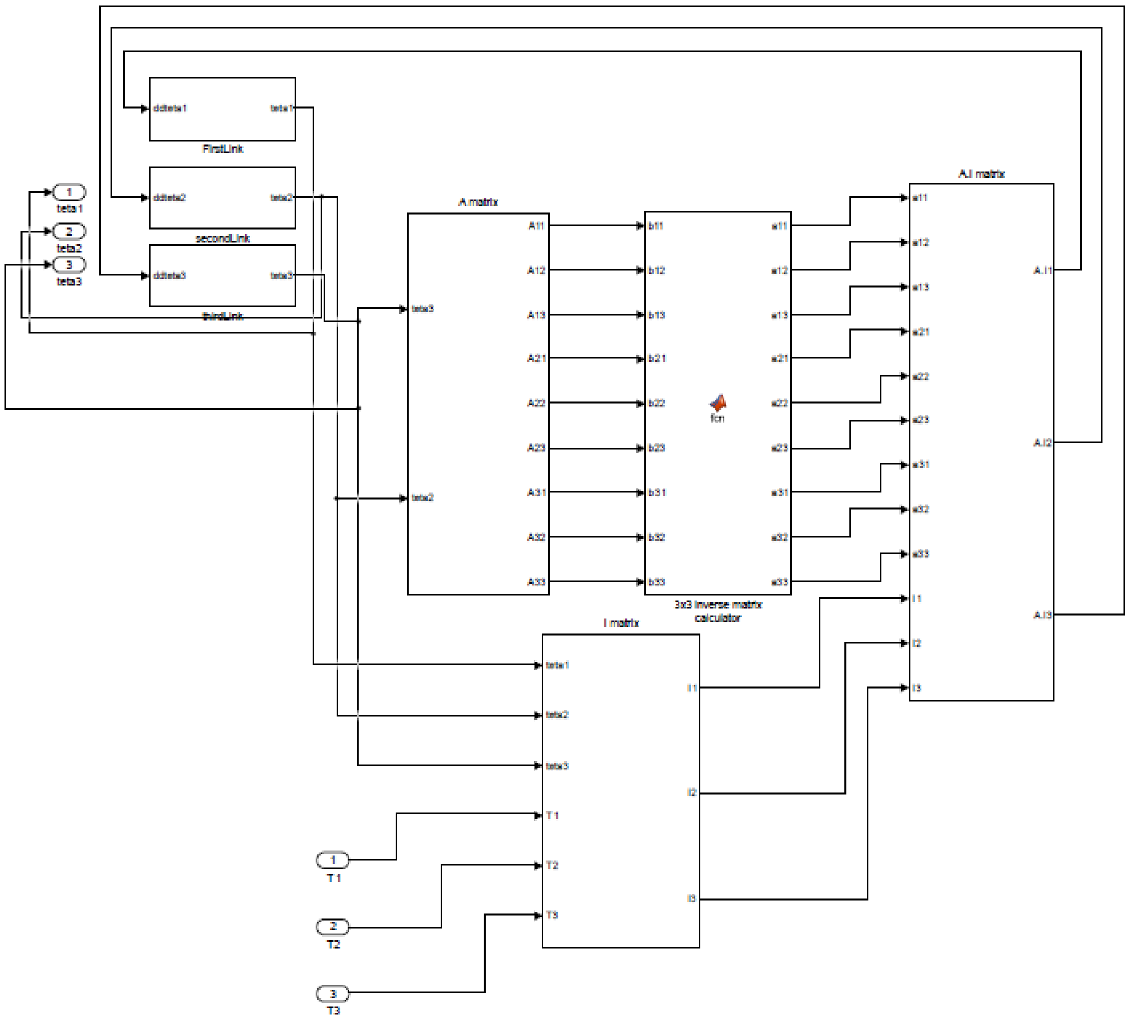

Figure 4.6: Dynamic equation of the 3 DOF planner manipulator

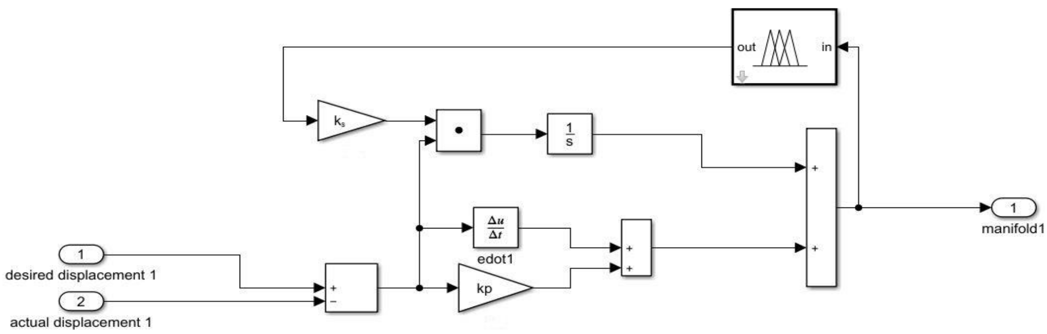

Figure 4.7: Sliding surface of the AFISMC

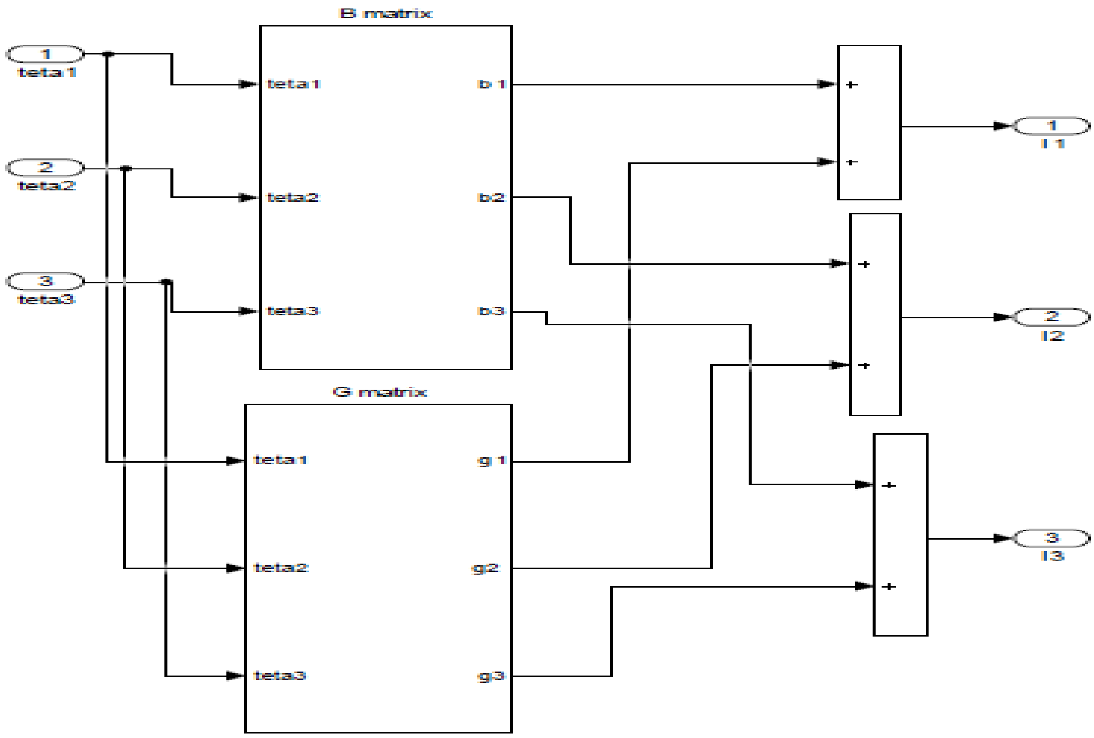

Figure 4.8: I matrix

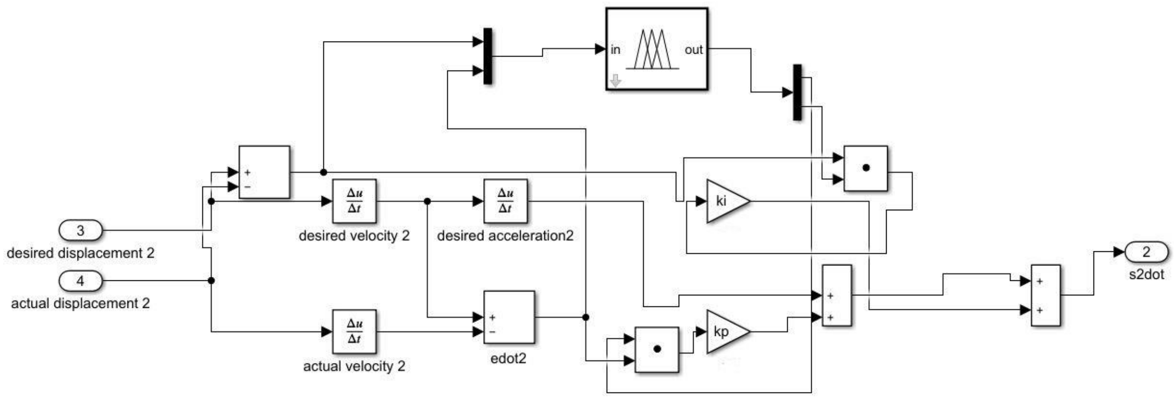

Figure 4.9: Sdot generator for AFISMC

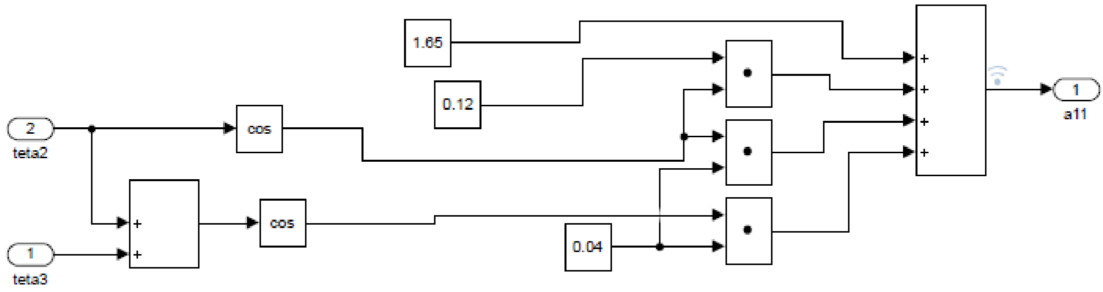

Figure 4.10: A11 without uncertainty

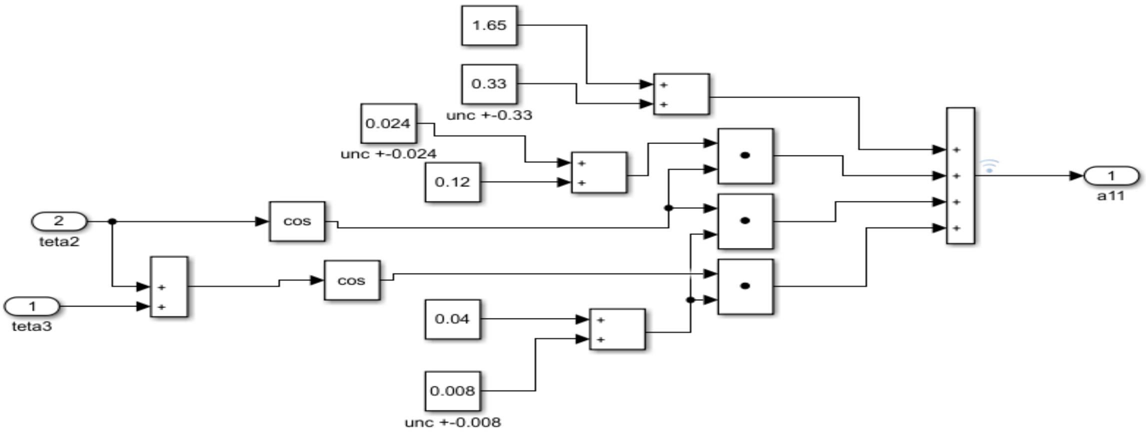

Figure 4.11: Inertia matrix a11 with 20% uncertainty added

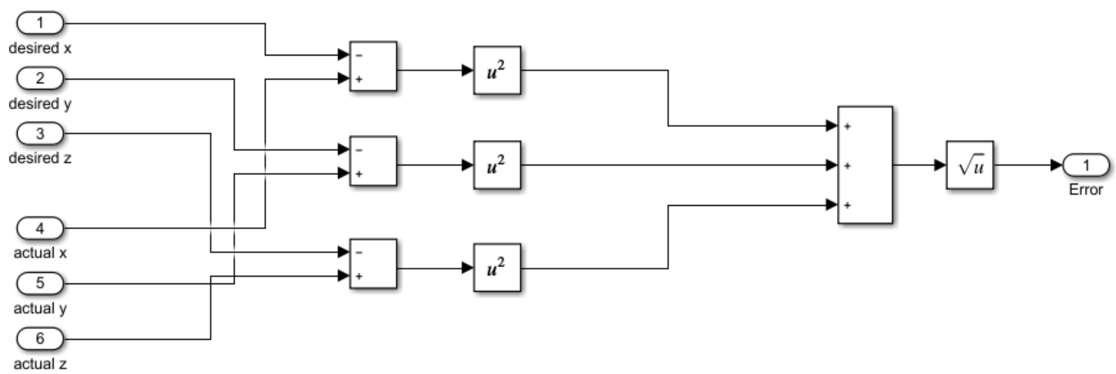

Figure 4.12: RMS of the desired controller

4.1. Performance of Integral Sliding Mode Controller

The adaptive fuzzy integral sliding mode controller (AFISMC) response without uncertainty and external disturbance for the three degree of freedom robot manipulator for desired and actual displacement with ramp response as input is shown in Figure 4.13 and 4.14 respectively and with sine wave as input is shown in Figure 4.15 and 4.16 respectively. Figure 4.17, 4.18 and 4.19 shows the performance of the adaptive integral sliding mode controller for trajectory tracking of link one, two and three with equivalent step response signal is applied. After testing the system performance without applying any external disturbance and uncertainty we tried to impose 20% of the input signal as uncertainty to the system and analyze the efficiency of the control system. Figure 4.20, 4.21, 4.22 and 4.23 shows the desired and actual displacement with ramp response and sine response as input respectively.

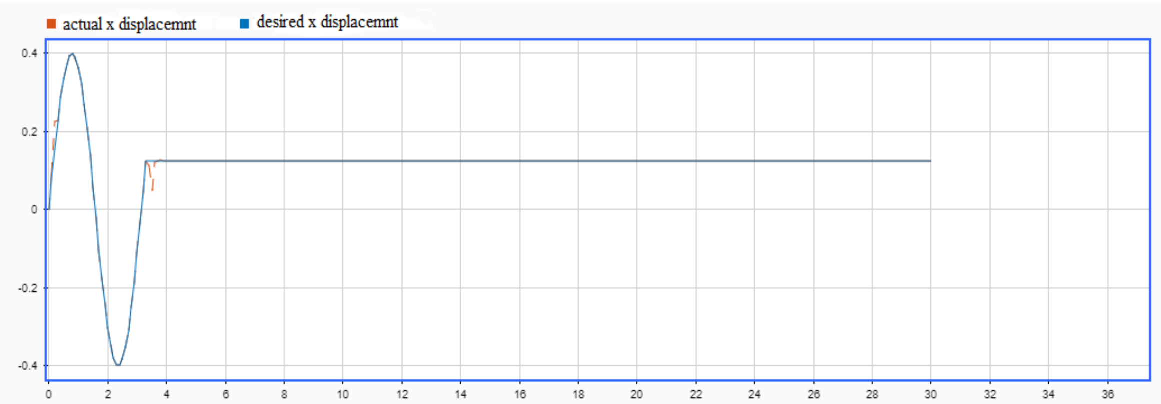

Figure 4.13: θ_d and θ_a of the manipulator with ramp response

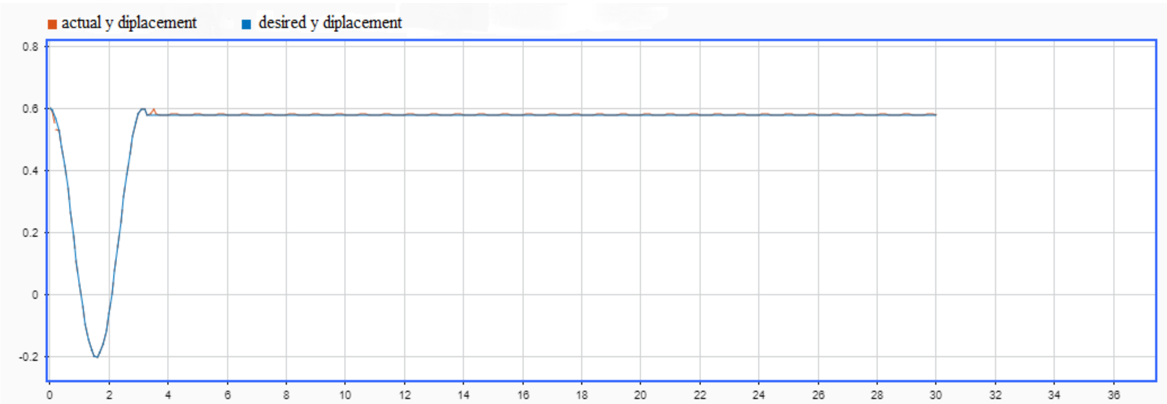

Figure 4.14: θ_d and θ_a of the manipulator with ramp response

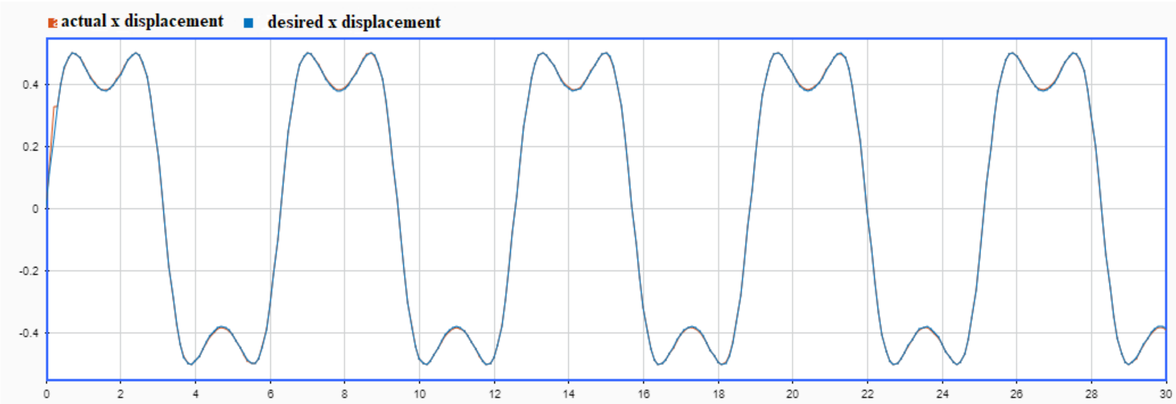

Figure 4.15: θ_d and θ_a of the manipulator with sine wave

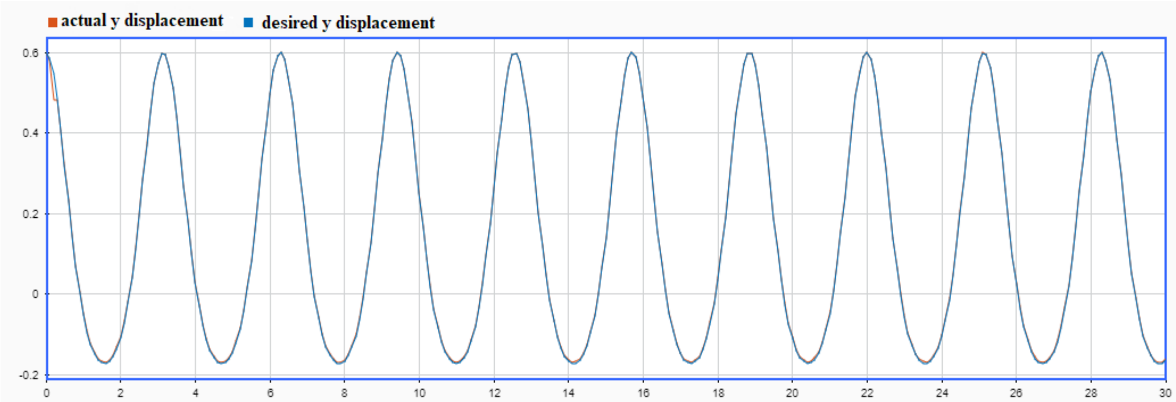

Figure 4.16: θ_d and θ_a of the manipulator with sine wave

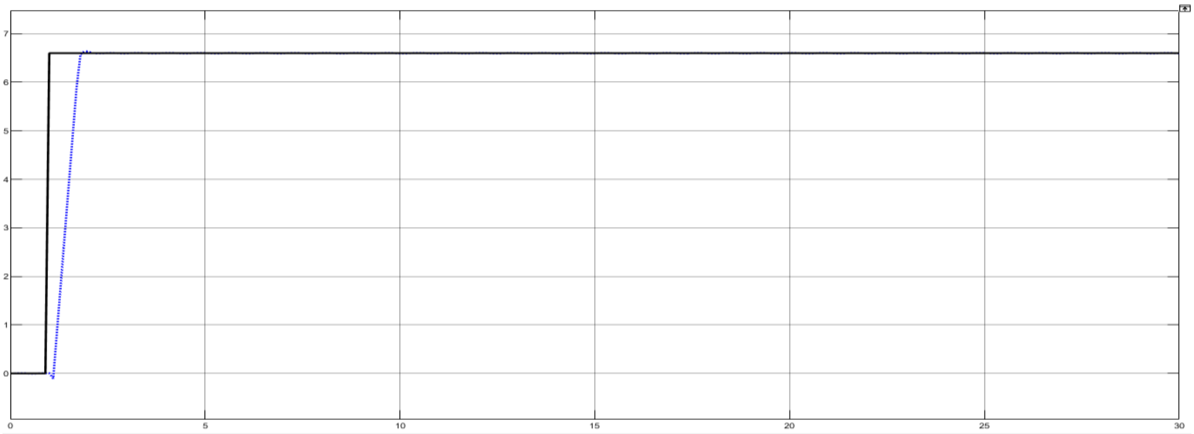

Figure 4.17: First link trajectory without disturbance

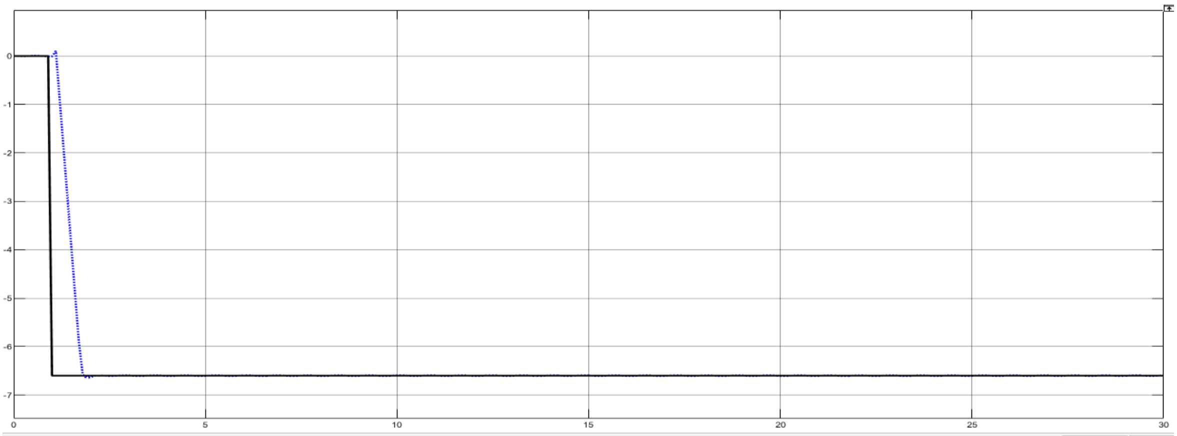

Figure 4.18: Second link trajectory without disturbance

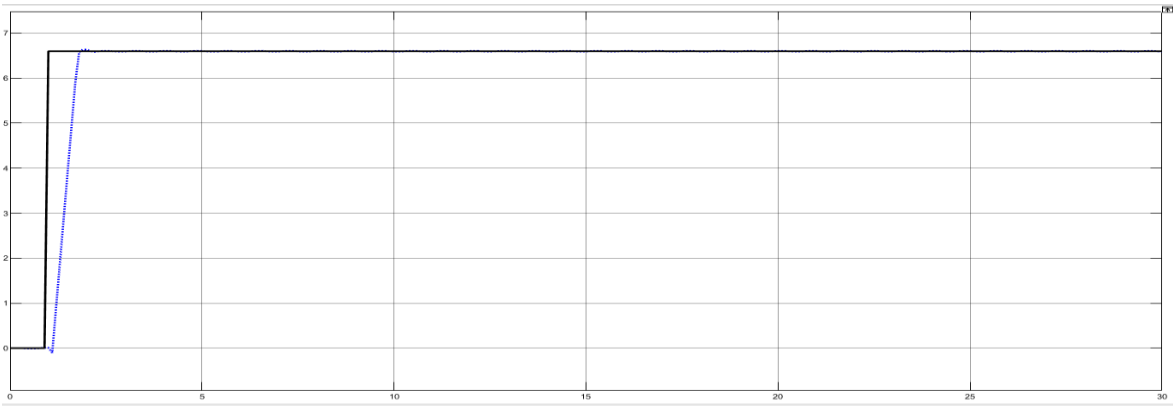

Figure 4.19: Third link trajectory without disturbance

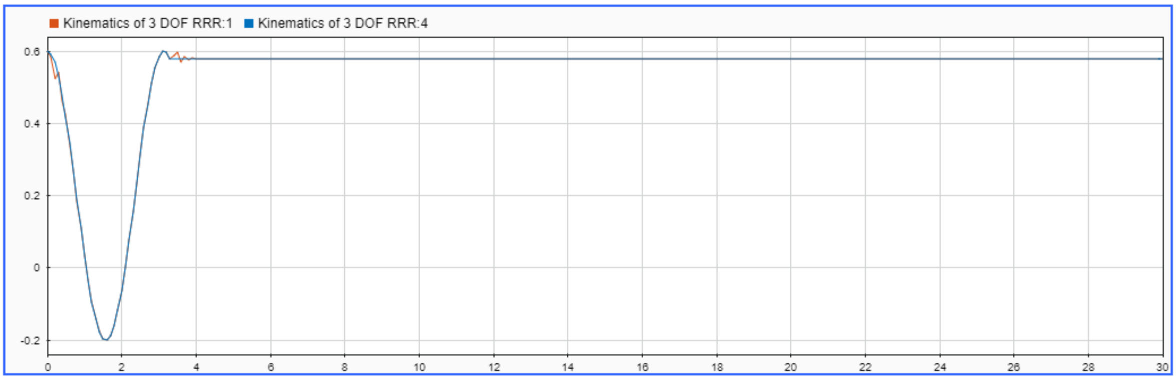

Figure 4.20: θ_d and θ_a of the manipulator with ramp response and 20% uncertainty

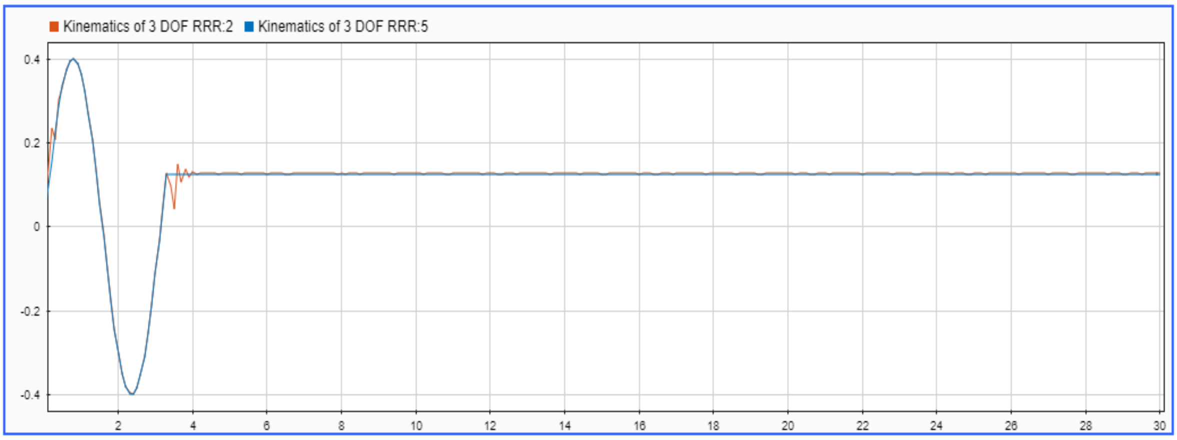

Figure 4.21: θ_d and θ_a of the manipulator with ramp response and 20% uncertainty

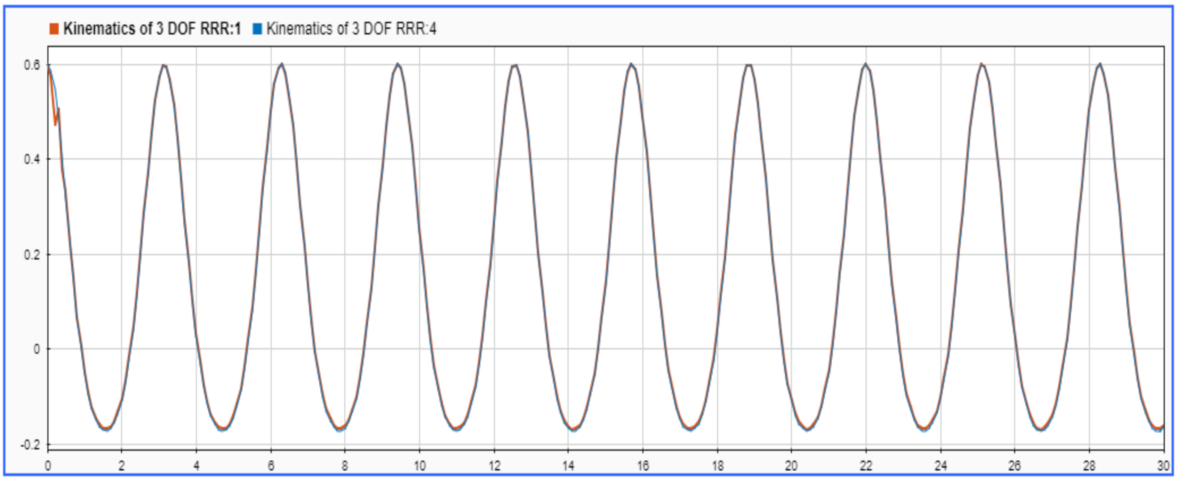

Figure 4.22: θ_d and θ_a of the manipulator with sine response and 20% uncertainty

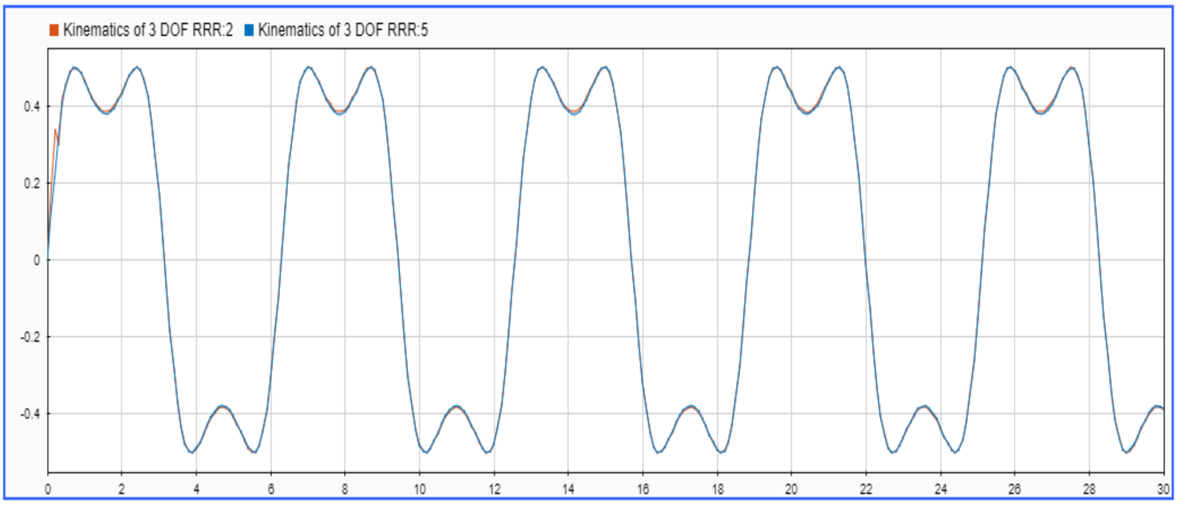

Figure 4.23: θ_d and θ_a of the manipulator with sine response and 20% uncertainty

The comparison between the adaptive fuzzy integral sliding mode and integral sliding mode for step input response is shown below and the AFISMC tries to eliminate the undershoot of the system and have better settling and rise time than the integral slinging mode controller for 3 DOF robot manipulator.

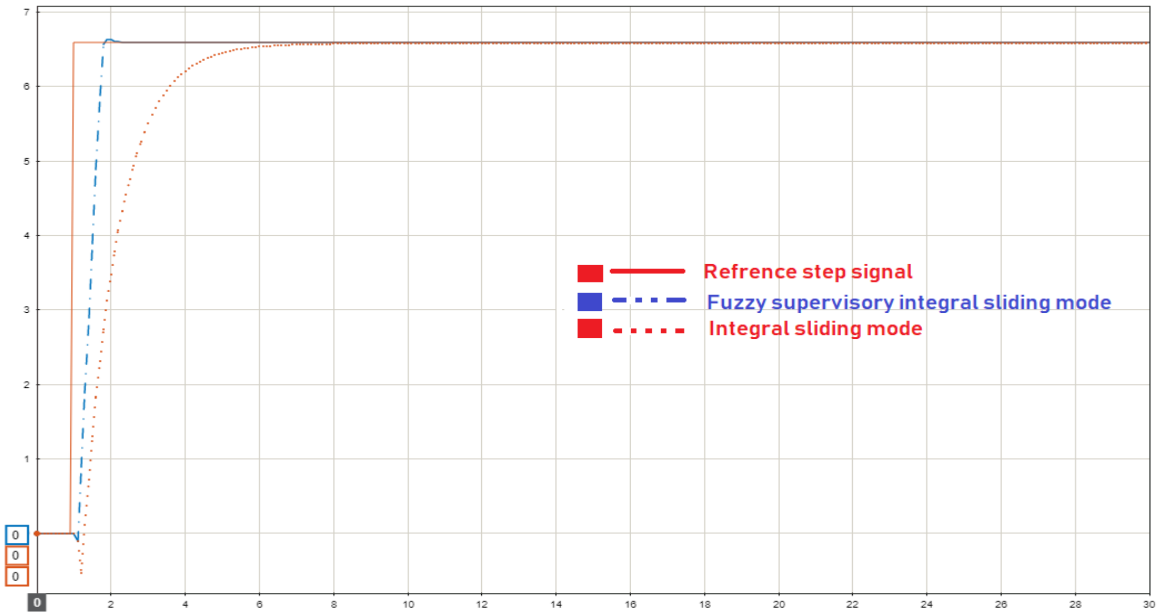

Figure 4.24: Comparison between FISMC and ISMC

In Figure 4.24 the trajectory tracking control system with a tuned gain by fuzzy logic algorithm clearly shows that the settling, rise time and the undershoot of the modeled system is lower than the conventional integral sliding mode control.

The trajectory tracking response of each controller for the same reference trajectory is shown in the Figure 4.25, Figure 4.26, Figure 4.27 and Figure 4.28, the result clearly shows that the adaptive fuzzy integral sliding mode controller has a better trajectory tracking compared to others which clearly overlaps on the reference trajectory. Whereas the adaptive fuzzy integral sliding mode controller with 20% matched uncertainty added on it lacks to follow the trajectory at the beginning of the trajectory but achieve the reference trajectory as soon as it begins. The integral sliding mode shows almost the same result as the adaptive fuzzy integral sliding mode with uncertainty.

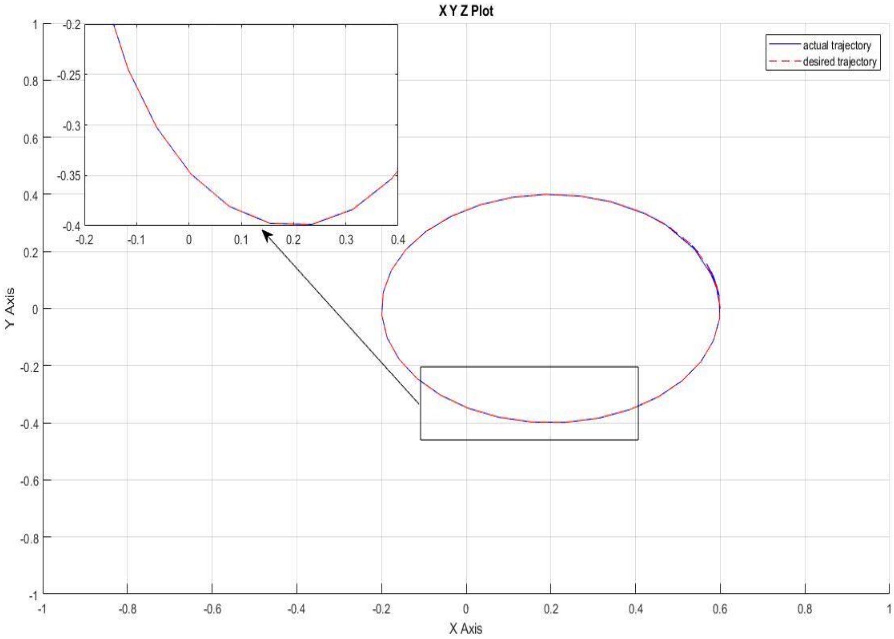

Figure 4.25: AFISM trajectory tracking

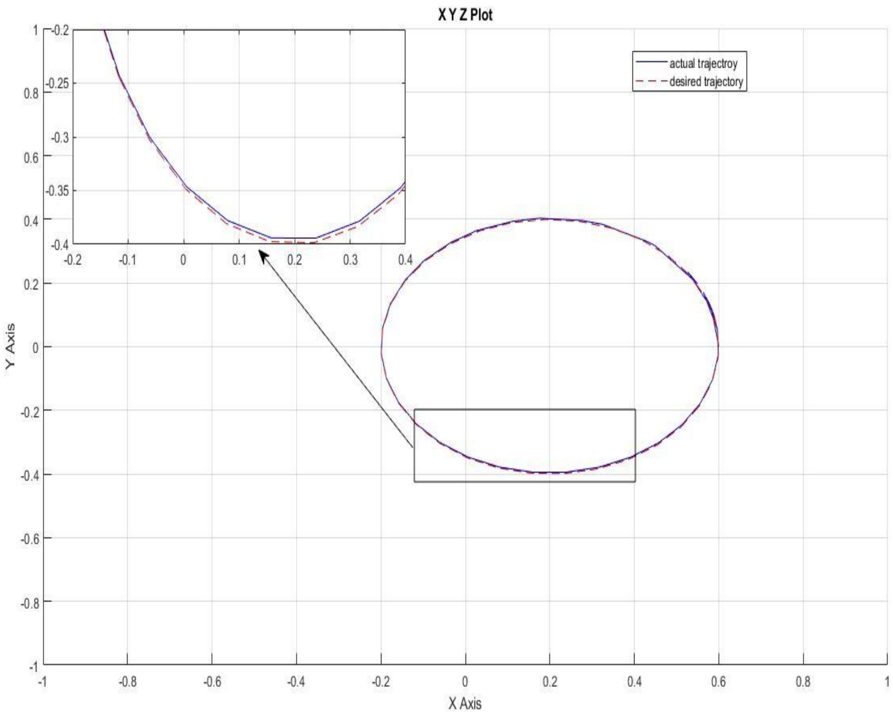

Figure 4.26: AFISM with 20% uncertainty trajectory tracking

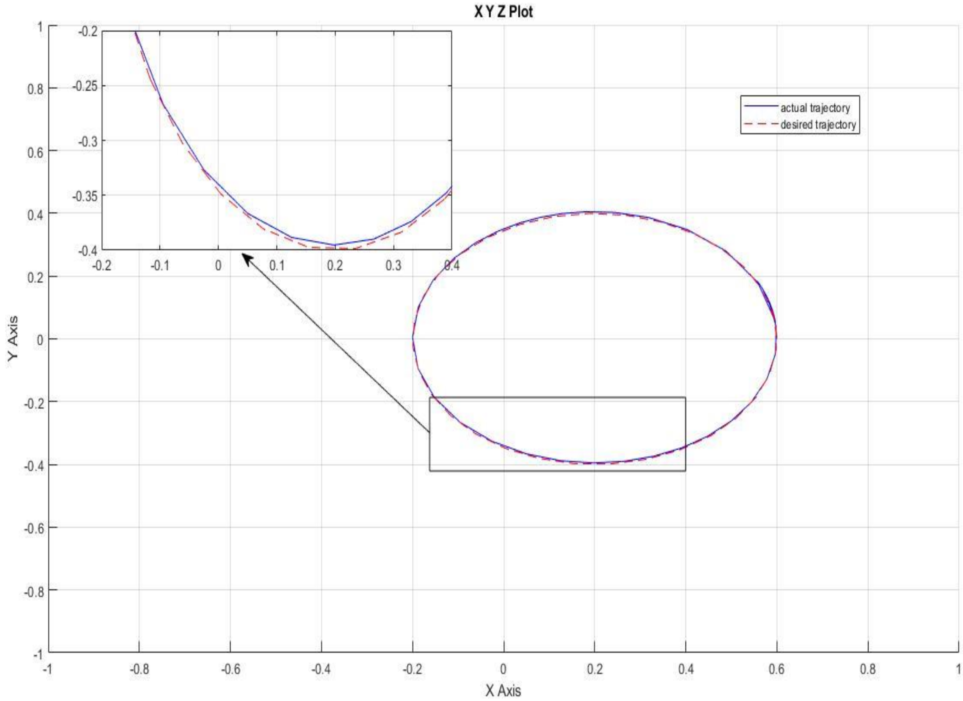

Figure 4.27: ISM trajectory tracking

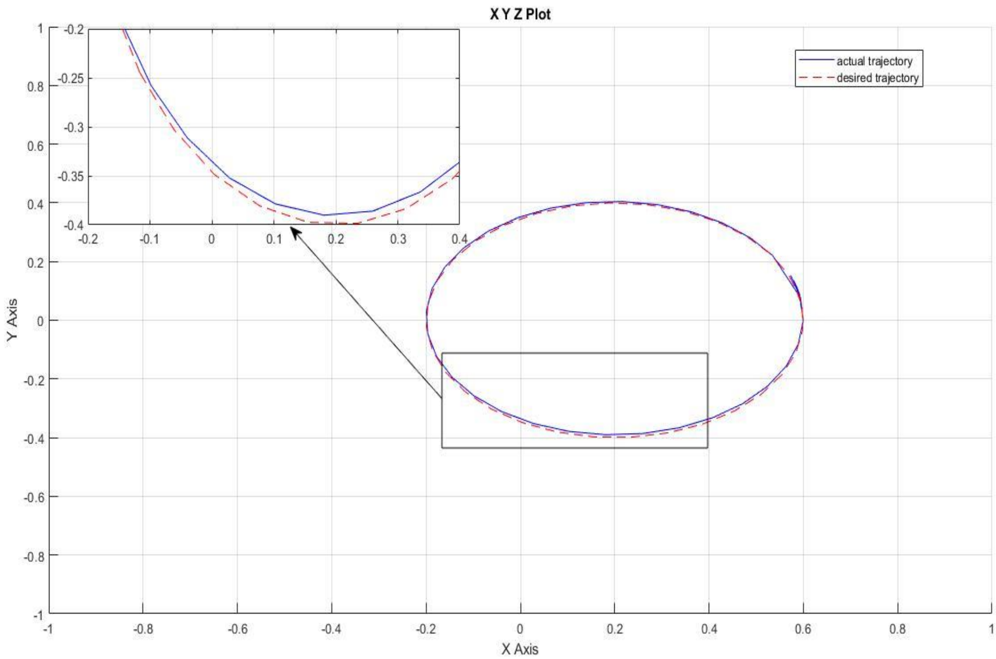

Figure 4.28: ISMC with 20% uncertainty trajectory tracking

As shown in the Table below the adaptive tuned gain of the integral sliding mode controller have a better performance than the reset.

Table 4.1: RMS result of the trajectory tracking

Controller	RMS (rad)
AFISMC	0.0001435
AFISMC +20% uncertainty	0.002188
ISMC	0.002306
ISMC +20% uncertainty	0.004908

On paper (H. et al., 2016) the researchers suggest a fraction order fuzzy PID for trajectory tracking of robot arm and tested on PUMA 560 robot arm for the first 3 joints and the result was discussed as follow and compare their result using different methodology among them the fractional order fuzzy PID give a better result. As the paper presents the fuzzy logic was served as a supervisory to automatically tune the gain parameter of the PID to get a better result. The RMS value of the trajectory error of the different method was listed as 0.04676 rad, 0.001799 rad, 0.02278 rad and 0.0008419 rad for PID, FOPID, Fuzzy-PID and FO-Fuzzy-PID respectively.

We implement the integral sliding mode controller with tuned gain using fuzzy logic and get a better result. Table 4.2 shows the root mean square error (RMS) of the trajectory tracking of our suggested control method on PUMA 560 robot dynamics.

$$\text{RMS}=\frac{\sqrt{(x_{act}-x_{des})^2+(y_{act}-y_{des})^2+(z_{act}-z_{des})^2}}{3} \qquad (4.1)$$

Table 4.2: Suggested system RMS trajectory error on Puma 560

Controller type	RMS (rad)
AFISMC	0.0001517

$$ITAE = \int t|e|\text{dt} \qquad (4.2)$$

On paper(Kumar et al., 2018) the researcher applied Fuzzy $PI^{\lambda}D^{\mu}$, Fuzzy PID and PID controllers for trajectory tracking with integral time absolute error (ITAE) of 0.2220, 0.8487 and 1.5508 or the robot manipulator, while our proposed system have a good performance with ITAE of 0.03315.

CHAPTER 5
CONCLUSION

In trajectory tracking of a robot, manipulators robustness plays a key feature, obtaining robustness in a system with matched uncertainty and external disturbance is a difficult and main target, in this paper we suggested a modified ISMC to obtain the robustness for the trajectory tracking of a 3-DOF robot manipulator.

In this study ISMC with fuzzy supervisory to tune the gain parameter of the 3-DOF robot manipulator has been implemented to achieve the required trajectory tracking of the robot arm with minimum trajectory error and highly robust system. the final result we achieved from the simulation has been compared with ISM and AFISM while applied 20% of inputs matched uncertainty to the system and tries to analyze the results. after comparing our result, it is clear that the automatic fuzzy gain tuning mechanism provides a better result to our trajectory tracking system model compared to the traditional ISMC.

We implement the suggested system on the well-known PUMA 560 six degree of freedom robot manipulator while the three joint are fixed and compare the achieved result with other researcher work who design a robust trajectory tracking robot manipulator on PUMA 560 robot manipulator and get a better performance on the trajectory tracking and less root mean square error (RMS) relative to them.

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APPENDIX 1: DYNAMIC EQUATION OF THE ROBOT MANIPULATOR

$$a_{11} = m_1 l_{c1}^2 + m_2(L_1^2 + l_{c2}^2 + 2L_1 l_{c2} cos\theta_2)$$
$$+ m_3(L_1^2 + L_2^2 + l_{c3}^2 + 2L_1 L_2 cos\theta_2 + 2L_1 l_{c3}\cos(\theta_2 + \theta_3)$$
$$+ 2L_2 l_{c3}\cos(\theta_3)) + I_1 + I_2 + I_3$$

$$a_{12} = m_2(l_{c2}^2 + L_1 l_{c2}\cos(\theta_3)) + m_3(L_2^2 + l_{c3}^2 + L_1 L_2 cos\theta_2$$
$$+ L_1 l_{c3}\cos(\theta_2 + \theta_3) + 2L_2 l_{c3} cos\theta_3) + I_2 + I_3$$

$$a_{13} = m_3(l_{c3}^2 + L_1 l_{c3}\cos(\theta_2 + \theta_3) + L_2 l_{c3} cos\theta_3) + I_3$$

$$a_{22} = m_2 l_{c2}^2 + m_3(L_2^2 + l_{c3}^2 + 2L_2 l_{c3}\cos(\theta_2 + \theta_3)) + I_2 + I_3$$

$$a_{23} = m_3(l_{c3}^2 + L_2 l_{c3} cos\theta_3) + I_3$$

$$a_{33} = m_3 l_{c3}^2 + I_3$$

$$b_1 = -m_2 L_1 l_{c2}\left(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2\right)\sin\theta_2$$
$$- m_3\left[L_1 L_2\left(2\dot{\theta}_1 + \dot{\theta}_2\right)\sin\theta_2 + L_1\, l_{c3}\left(2\dot{\theta}_1 + \dot{\theta}_2\right.\right.$$
$$\left.\left. + \dot{\theta}_3\right)\sin(\theta_2 + \theta_3) + L_1 l_{c3}\left(2\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3\right)\sin\theta_3\right]$$

$$b_2 = -m_2\left[L_1 l_{c2}\left(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2\right)\sin\theta_2 + L_2 l_{c2}\dot{\theta}_1\dot{\theta}_2\sin\theta_2\right]$$
$$- m_3\left[L_1 L_2\left(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2\right) + L_1 l_{c3}\left(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_1\dot{\theta}_3\right)\sin(\theta_2 + \theta_3)\right.$$
$$+ L_2 l_{c3}\left(\dot{\theta}_1^2 + \dot{2}\theta_1\dot{\theta}_2 + \dot{\theta}_1\dot{\theta}_3 + \dot{\theta}_2^2 + \dot{\theta}_2\dot{\theta}_3\right)\sin\theta_3$$
$$+ L_1 L_2\dot{\theta}_1\dot{\theta}_2\sin\theta_2 + L_1 l_{c3}\dot{\theta}_1\dot{\theta}_2\sin(\theta_2 + \theta_3)$$
$$\left. + L_2 l_{c3}\left(2\dot{\theta}_1 + \dot{2}\theta_2 + \dot{\theta}_3\right)\sin\theta_3\right]$$

$$b_3 = -m_3[L_1 l_{c3}\left(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_1\dot{\theta}_3\right)\sin(\theta_2 + \theta_3) + L_2 l_{c3}\left(\dot{\theta}_1^2 + \dot{2}\theta_1\dot{\theta}_2 + \dot{\theta}_1\dot{\theta}_3\right.$$
$$\left. + \dot{\theta}_2^2 + \dot{\theta}_2\dot{\theta}_3\right)\sin\theta_3 + L_1 l_{c3}\dot{\theta}_1\dot{\theta}_3\sin(\theta_2 + \theta_3)$$
$$+ L_2 l_{c3}\left(\dot{\theta}_1\dot{\theta}_3 + \dot{\theta}_2\dot{\theta}_3\right)\sin\theta_3]$$

$$g_1 = g[\cos\theta_1(m_1 l_{c1} + m_2 L_1 + m_3 L_1) + \cos(\theta_1 + \theta_2)\,(m_2 l_{c2} + m_3 L_2)$$
$$+ \cos(\theta_1 + \theta_2 + \theta_3)\,(m_3 l_{c3})]$$

$$g_2 = g[(m_2 l_{c2} + m_3 L_2)\cos(\theta_1 + \theta_2) + m_3 l_{c3}\cos(\theta_1 + \theta_2 + \theta_3)]$$

$$g_3 = g(m_3 l_{c3}\cos(\theta_1 + \theta_2 + \theta_3))$$

After inserting the value of each parameter to the dynamic's equation

$$a_{11} = m_1 l_{c1}^2 + m_2(L_1^2 + l_{c2}^2 + 2L_1 l_{c2} cos\theta_2) + m_3(L_1^2 + L_2^2 + l_{c3}^2 + 2L_1L_2cos\theta_2 + 2L_1 l_{c3}\cos(\theta_2+\theta_3) + 2L_2 l_{c3}\cos(\theta_3)) + I_1 + I_2 + I_3$$

$$a_{11} = 1.65 + 0.12cos\theta_2 + 0.04\cos(\theta_2+\theta_3) + 0.04\cos(\theta_3)$$

$$a_{12} = m_2(l_{c2}^2 + L_1 l_{c2}\cos(\theta_3)) + m_3(L_2^2 + l_{c3}^2 + L_1L_2cos\theta_2 + L_1 l_{c3}\cos(\theta_2+\theta_3) + 2L_2 l_{c3}cos\theta_3) + I_2 + I_3$$

$$a_{12} = 1.06 + 0.06cos\theta_3 + 0.04cos\theta_2 + 0.02\cos(\theta_2+\theta_3)$$

$$a_{13} = 0.51 + 0.02\cos(\theta_2+\theta_3) + 0.02cos\theta_3$$

$$a_{21} = 0$$

$$a_{22} = 1.06 + 0.04\cos(\theta_2+\theta_3)$$

$$a_{23} = 0.51 + 0.02\cos(\theta_2+\theta_3)$$

$$a_{31} = 0$$

$$a_{32} = 0$$

$$a_{33} = 0.51$$

$$b_1 = -m_2 L_1 l_{c2}(2\dot{\theta}_1 + \dot{\theta}_2)\sin\theta_2\dot{\theta}_2 - m_3\left[L_1L_2(2\dot{\theta}_1+\dot{\theta}_2)\sin\theta_2 + L_1\, l_{c3}(2\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)\sin(\theta_2+\theta_3) + L_1 l_{c3}(2\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)\sin\theta_3\right]$$

$$b_1 = -0.02(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2)\sin\theta_2 - \left[0.04(2\dot{\theta}_1+\dot{\theta}_2)\sin\theta_2 + 0.02\,(2\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)\sin(\theta_2+\theta_3) + 0.02(2\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)\sin\theta_3\right]$$

$$b_2 = -\left[0.02(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)\sin\theta_2 + 0.02\dot{\theta}_1\dot{\theta}_2\sin\theta_2\right] - \left[0.04(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2) + 0.02(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_1\dot{\theta}_3)\sin(\theta_2+\theta_3) + 0.02(\dot{\theta}_1^2 + 2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_1\dot{\theta}_3 + \dot{\theta}_2^2 + \dot{\theta}_2\dot{\theta}_3)\sin\theta_3 + 0.04\dot{\theta}_1\sin\theta_2\,\dot{\theta}_2 + 0.02\dot{\theta}_1\dot{\theta}_2\sin(\theta_2+\theta_3) + 0.02(2\dot{\theta}_1 + 2\dot{\theta}_2 + \dot{\theta}_3)\sin\theta_3\right]$$

$$b_3 = -[0.02(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_1\dot{\theta}_3)\sin(\theta_2 + \theta_3)$$
$$+ 0.02(\dot{\theta}_1^2 + 2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_1\dot{\theta}_3 + \dot{\theta}_2^2 + \dot{\theta}_2\dot{\theta}_3)\sin\theta_3$$
$$+ 0.02\dot{\theta}_1\dot{\theta}_3\sin(\theta_2 + \theta_3) + 0.02(\dot{\theta}_1\dot{\theta}_3 + \dot{\theta}_2\dot{\theta}_3)\sin\theta_3]$$

$$g_1 = 4.905 cos\theta_1 + 2.943\cos(\theta_1 + \theta_2) + 0.981\cos(\theta_1 + \theta_2 + \theta_3)$$

$$g_2 = 2.943\cos(\theta_1 + \theta_2) + 0.981\cos(\theta_1 + \theta_2 + \theta_3)$$

$$g_3 = 0.981\cos(\theta_1 + \theta_2 + \theta_3)$$