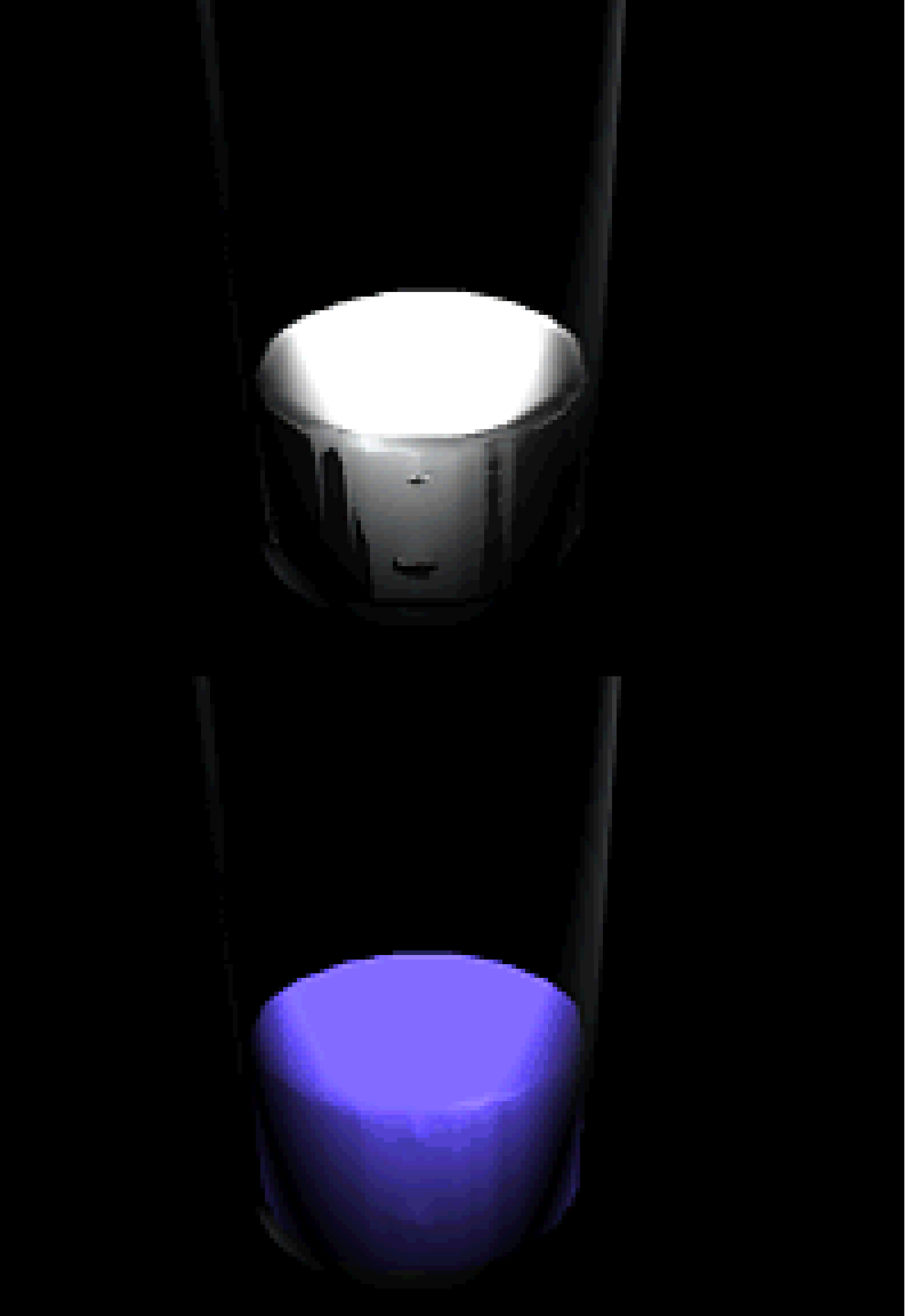

VISCOSITY

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DEFINITION

- A fluid’s ability to flow is called viscosity.
- Viscosity arises from the mutual COHESIVE FORCES between molecules of a liquid; the stronger the forces are, the more viscous the liquid.

STRESS

- is defined as **F / A :** force per unit area
- If the force applied is
  - perpendicular to the area it is called tensile stress (or pressure).
  - tangent (parallel), it is called shear stress.

Tangential stress

The force due to one plane within a streamline, on another adjacent plane, will be a viscous force.

It will act tangentially against the streamline.

In this diagram, the upper streamline is travelling faster than the lower one and their relative velocity is Δv.

The force on the upper streamline is in the opposite direction to the relative velocity so is a retarding force. The force on the lower streamline accelerates the streamline.

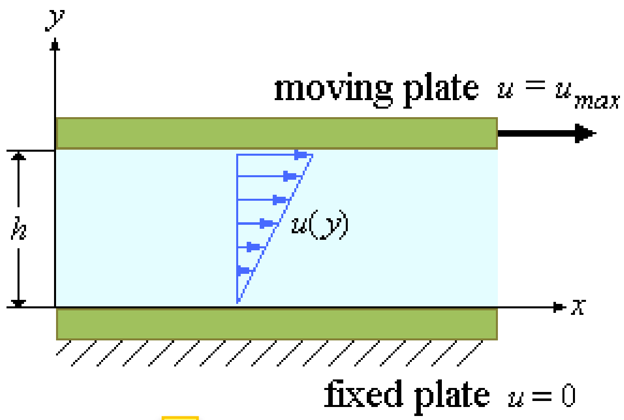
The tangential stress is sometimes called the shear or shearing stress and is defined by:

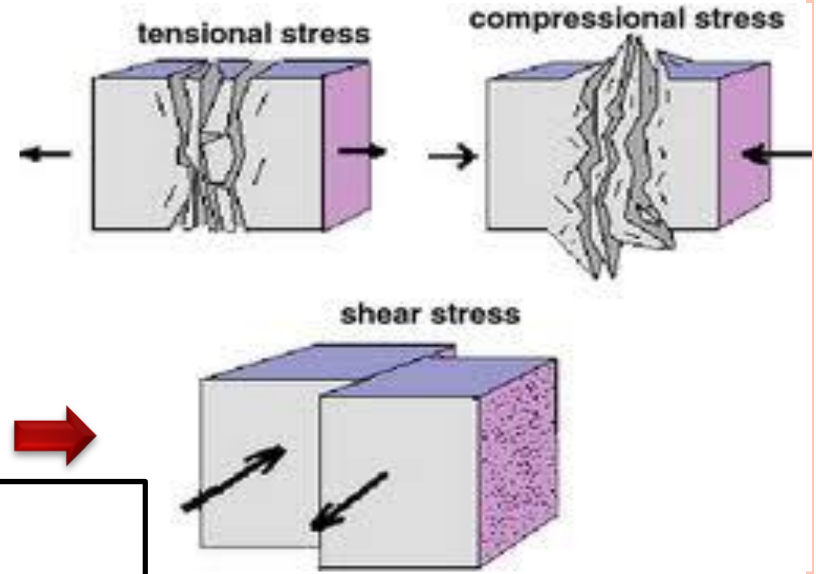
Tangential stress = force / area

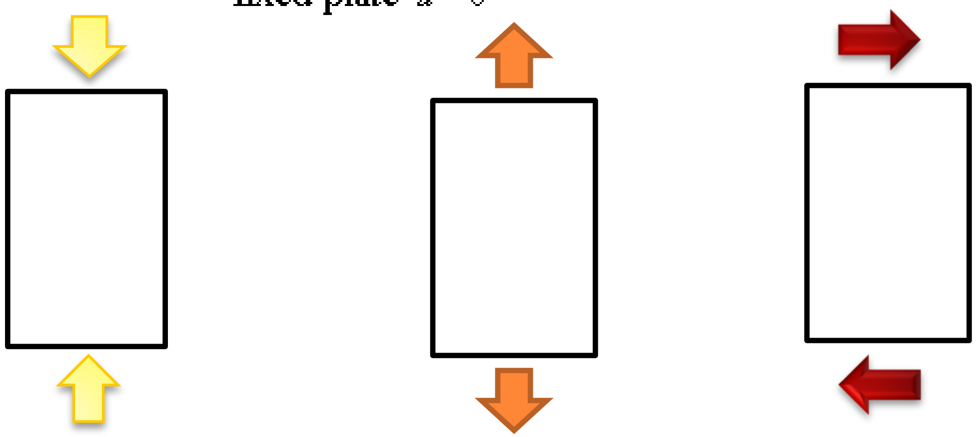
$$\boldsymbol{\tau = F / A}$$

The units of tangential stress are Pa (Pascal)

Shear stress in a moving fluid





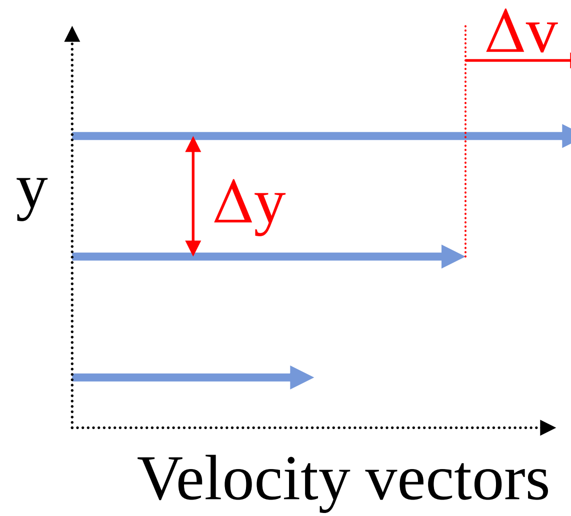


compressional stress

tensile stress

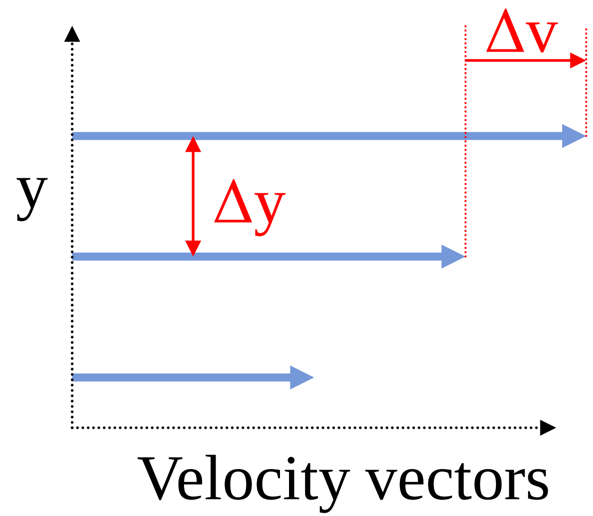
shear stress

Velocity gradient



This tangential stress creates varying velocities, perpendicular to the direction of streamline flow, as represented by the vectors in this diagram.

The rate of change of velocity across the streamlines is called the *velocity gradient*.



Velocity gradient = Δv/Δy = Change in velocity/Change in y

What are the units of velocity gradient?

From the definition, they will be ms^{-1} / m.

The units of velocity gradient are s^{-1}.

The greater the viscosity of the liquid, the smaller the velocity gradient.

If you spin a cup of water, the water at the edge will turn immediately but as there is low viscosity and a high velocity gradient, the water further into the cup might not spin at all at first.

Coefficient of viscosity

Viscosity is related to the tangential stress and the velocity gradient.

We can define the coefficient of viscosity as follows:

$$\textbf{Coefficient of viscosity, } \eta = \frac{\textbf{tangential stress}}{\textbf{velocity gradient}}$$

Where $\eta = (F/A) / (\Delta v/\Delta y)$.

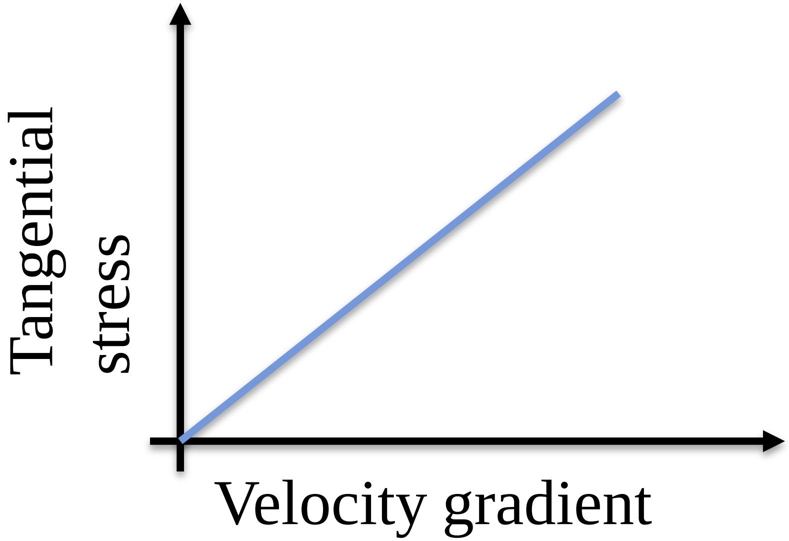
What are its units?

Pa s (pascal seconds)

This is also Nsm^{-2}

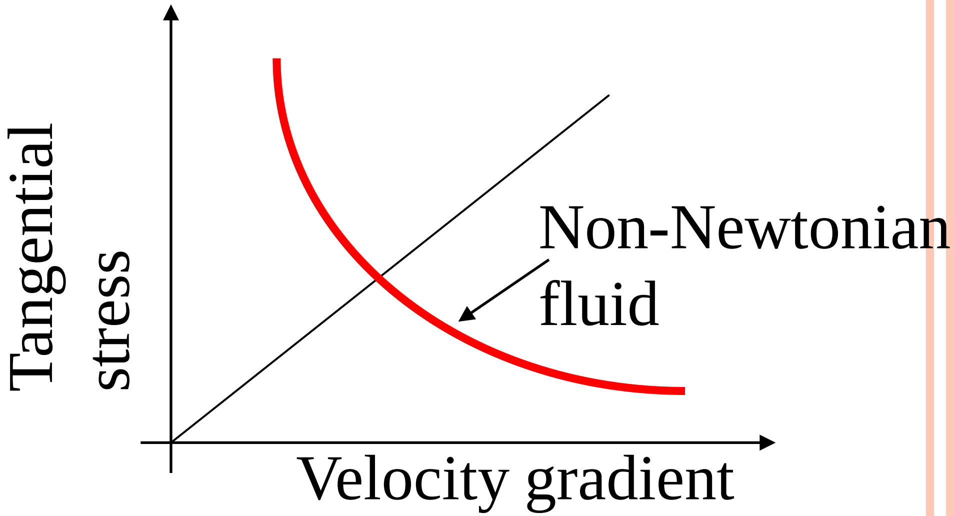
If we plot tangential stress against velocity gradient, we should obtain a straight line. The gradient (slope) of the line will be the coefficient of viscosity.

Note that the coefficient of velocity is extremely temperature dependent.



Fluids which obey this relationship are called Newtonian fluids.

Fluids that do not are called non-Newtonian fluids. The graph will be non-linear for non-Newtonian fluids.



Non-drip paint is a non-Newtonian fluid. With small tangential stresses, there is a large velocity gradient but this soon falls as the tangential stress increases.

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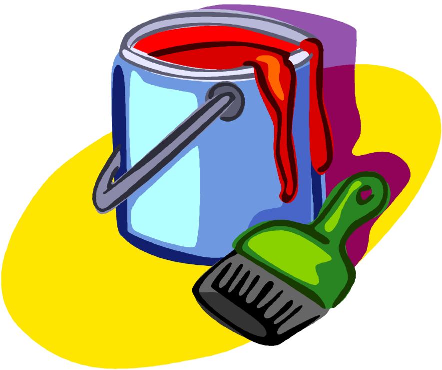


When on the brush and not being worked, it is very viscous – has a high viscosity.

This means that it will not drip off the brush easily.

As soon as it is worked, the viscosity falls and it becomes spreadable.

- Flow through a blood vessel is determined by two factors:

  (1) the pressure difference between the two ends of the vessel, which is the force that pushes the blood through the vessel

  (2) the impediment to blood flow through the vessel, that is, vascular resistance.

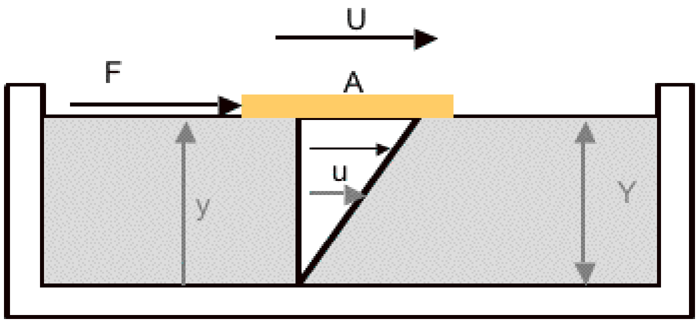
$$Q = \Delta P / R$$

- Resistance on the other hand is : $R = 8 \eta l / \pi r^4$
- So, for the flow rate

  $Q = \pi \Delta P r^4 / 8 \eta l$ which is the Hagen-Poiseuille Law

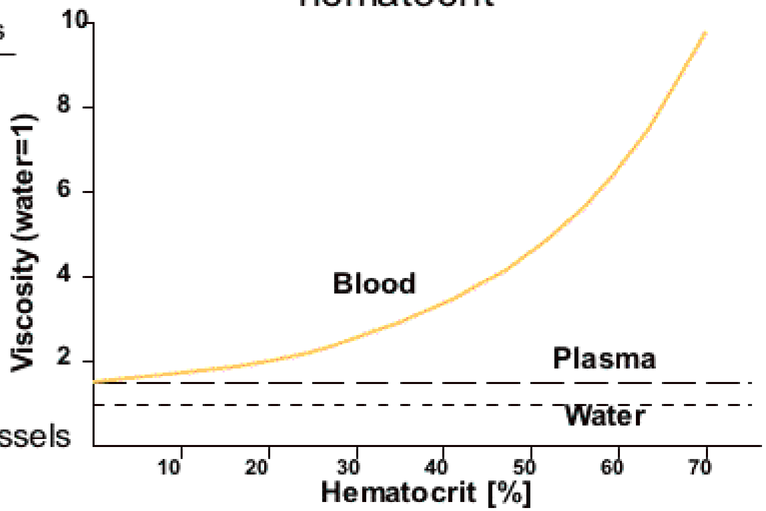
- The viscosity of blood varies (relative viscosity) depending on:
  (1) the quantity of suspended cells
  (2) the content of the plasma
  (3) the dimensions (radius vs. length) of the conducting tube.

Viscosity



$$\eta = \frac{\tau}{\frac{du}{dy}} = \frac{F/A}{U/Y} = \frac{\text{Shear stress}}{\text{Shear rate}}$$

- Viscosity increased by
  - vessel diameter
  - reduced flow velocity
  - constrictions in vessels
  - hematocrit



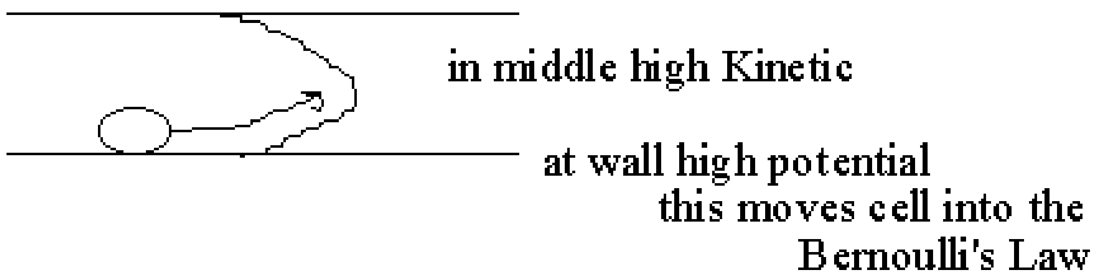
No good formula for viscosity in small vessels

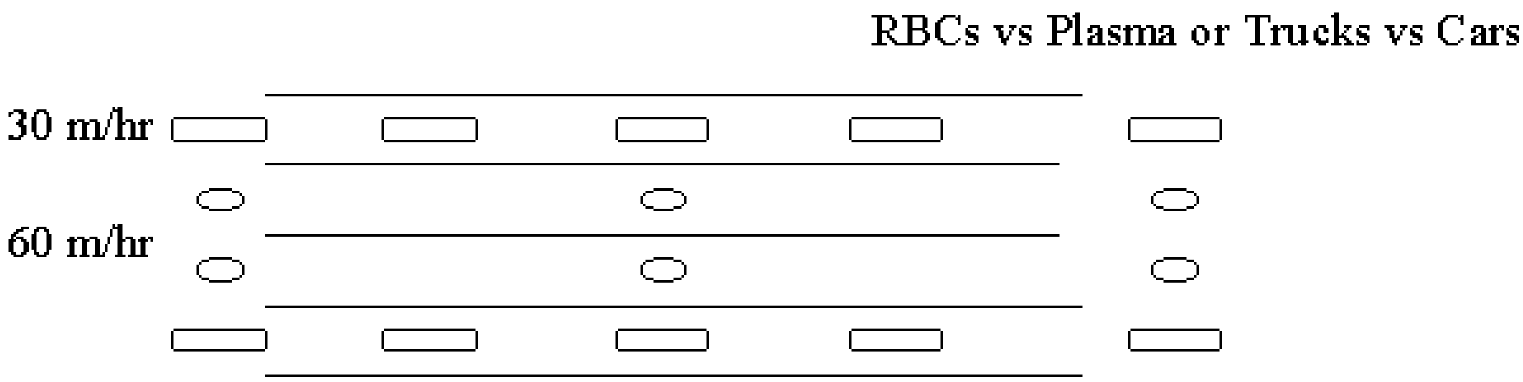
www.cvrti.utah.edu/~macleod/bioen/ be3202/notes/l12-hemo/ - 11k

- Which of the following statements below is/are correct of viscosity?

I. increased by hematocrit count
II. increased by reduced flow velocity
III. increased by constriction in vessels

- only plasma along wall
- more plasma than red blood cells in small vessels - thus reduced hematocrit
- Whenever the velocity of flow is low, VISCOSITY INCREASES





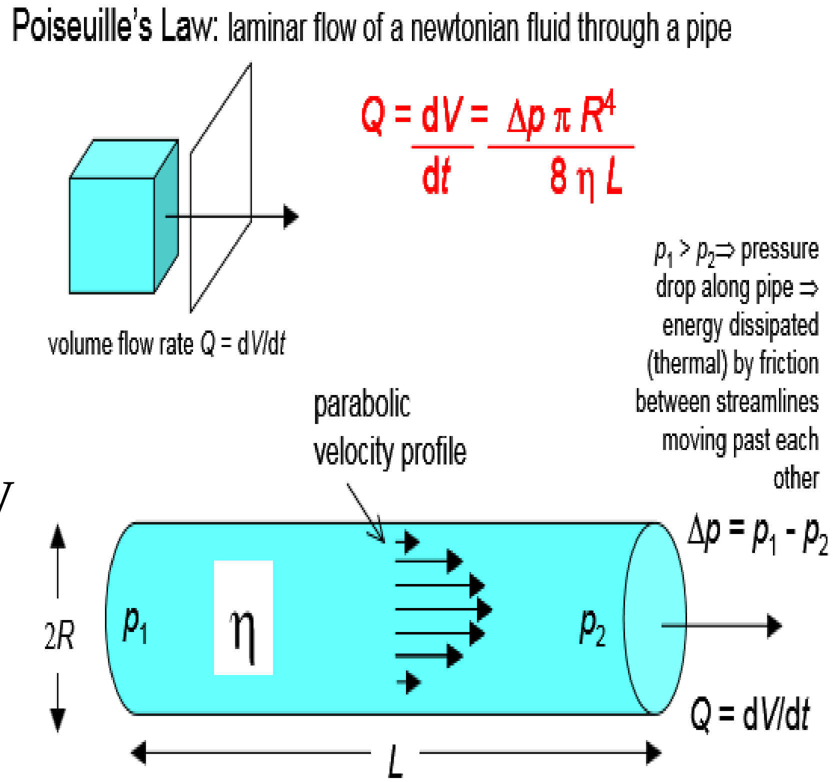
Flow of a fluid through a pipe
Poiseuille' s Law

In trying to find out what factors control how fast fluids can flow through pipes, the following factors are easy to isolate:

- The pressure difference between the ends of the pipe. The bigger the pressure difference, the faster will be the flow.
- The length of the pipe. More liquid will flow through a shorter than a longer pipe in the same time
- The radius of the pipe. More liquid will flow through a wide than a narrow pipe in the same time.

The volume flow rate
Q = dV/dt
of a fluid of viscosity η,
through a pipe of radius R,
and length L, when driven by
a pressure difference ΔP is given by

$$dV/dt = Q = \Delta P \pi R^4 / (8 \eta L)$$



This is known as Poiseuille' s Law.

• Poiseuille's law only applies to newtonian fluids.

• Non-newtonian liquids do not obey Poiseuille's law because their viscosities are velocity dependent.

The assumption of streamlined (laminar) flow is built in to Poiseuille's law.

If turbulence occurs than you must be very careful about using Poiseuille's law to calculate flow rates.

If turbulence does occur in the flow then the volume flow rate is dramatically reduce.

Alternative view of Poiseuille's Law Consider an electrical circuit in which a potential V between the ends of a resistance R results in a current I. Then the flow is determined by the ratio of potential to resistance.

flow (current) = potential / resistance

$I = V / R$

Poiseuille's Law can be arranged in this form

$Q = \Delta p / (8 \eta L / \pi R^4)$

flow $\Rightarrow$ Q potential $\Rightarrow$ Δp resistance $\Rightarrow$ $(8 \eta L / \pi R^4)$

resistance $\propto L$ resistance $\propto \eta$ resistance $\propto (1 / R^4)$

- Viscosity of blood at normal temperature is 3 x that of water.
- Water 20°C 0.01 poise (1 centipoise)

  1.005×10^{-3} Pa.s
- Blood 20°C 0.03 poise

- Blood 37°C 3.015×10^{-3} Pa.s
- Blood plasma 37°C 1.81×10^{-3} Pa.

GASSES

In a gas, molecules can drift under thermal motion as well as travel in the direction of bulk fluid flow.

What happens to the layers of gas in streamline flow if a molecule passes from a slower layer to a faster one?

- A faster molecule entering a slower layer will, on average, speed the slower layer up
- A slower molecule entering a faster layer will, on average, slow the faster layer down