Hemodynamics II

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Laplace's Law

- Relates the pressure difference across a closed elastic membrane on liquid film to the tension in the membrane or film.

(Thick Wall) $P_{aorta} \approx 100$ mm Hg $\quad v_{aorta} = 33$ cm / sec

(Thin Wall) $P_{capillary} \approx 15\text{-}35$ mm Hg $\quad v_{cap} = 1000$ x less

- **Laplace :** The wall tension required to withstand a given fluid pressure is proportional to the vessel radius

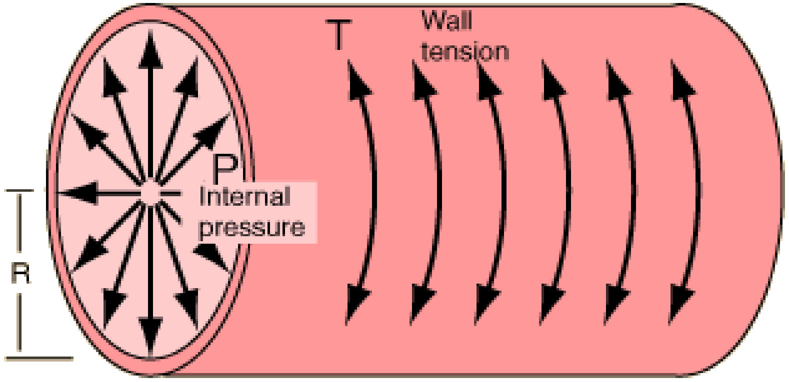
Tension

For a cylindrical membrane: **T = P . R**

R = radius of curvature

T = tension in the membrane

P = fluid pressure acting outward against the membrane



Cylindrical Vessel
T = PR

The Components of Vessel Wall Tension

- Elastic tension results from stretching of the vessel wall, resistance to stretch of elastic fibers
- Active tension is due to the contraction of smooth muscle fibers in the vessel wall
- Interfacial tension plasma does not completely "wet" the surface of the blood vessel wall. Therefore, surface tension is also a factor

While blood P between the aorta and the capillaries varies ONLY by a factor of 4, vessel radius changes by a factor of 10^4

Wall Tension in Aorta and Capillary

- **AORTA :** $P_{aorta} \approx 100$ mm Hg $r = 0.9$ cm $d \approx 1.8 - 2.0$

  $$T = (100 \text{ mm Hg}) \; \frac{1333 \text{ dynes / cm}^2}{\text{mm Hg}} \; (0.9 \text{ cm})$$

  $$= 1.2 \times 10^5 \text{ dynes / cm}$$

- **CAPILLARY:** $P_{cap} \approx 30$ mm Hg $r \sim 4\ \mu$m

  $$T = (30 \text{ mm Hg}) \; \frac{1333 \text{ dynes / cm}^2}{\text{mm Hg}} \; (4 \times 10^{-4} \text{ cm})$$

  $$= 16 \text{ dynes / cm}$$

  A factor of 7500 smaller than the wall tension in the aorta

Wall Tension in Blood Vessels

- Larger blood vessels have to produce a higher wall tension to support the same pressure difference.
- Laplace’s law explains why the walls of an enlarged heart or enlarged blood vessel must provide an even greater tension to withstand the fluid pressure.
- Expension of arterial walls is limited by **COLLAGEN** fibers.

Pressure Drop: Experimental Results in Dogs

Expected pressure drops

Aorta	4 mm Hg
Large arteries	5 mm Hg
Branch arteries to termination points at arterioles	15 mm Hg
Arterioles	39 mm Hg
Capillaries	26 mm Hg
Venous system including venules	7 mm Hg

Pressure Drop across the Arterioles is Larger!

Suprising!

- Pressure drop proportional to L / r^4

  Capillary 8 μm $\varnothing$ L = 0.1 mm

  Arterioles 20 μm $\varnothing$ L = 2 mm

$\Re$ (cap.) $\approx$ x 20 $\Re$ (arterioles) ; but $\Delta P = \Im \times \Re$

- The large pressure loss in arterioles is caused by their small individual radius and relatively fast velocity of flow (relative to capillaries).
- Capillary: Relatively low total resistance (parallel arrangement) and slow flow.
  - Arterioles 1.5 cm/s
  - Capillaries 0.04 cm/s

Control of Volume Flow Rate: Example

A person in a resting state with systolic pressure $P \approx 120$ mm Hg.
Sudden demand for vigorous exercise (dog chasing him !).
Volume flow rate should increase to five times the resting value.
If no vasodilation, how much would his blood pressure have to increase ?
If no pressure increase, what percentage dilation would be required to handle the demand?

Five fold $\uparrow$ in $\mathfrak{I}$ $\therefore$ 5 x increase in P

120 mm Hg x 5 = 600 mm Hg Physiologically unreasonable

$\mathfrak{I} = K\,(P_1 - P_2)\,r^4$

To get $5 \times \mathfrak{I}$ $\quad r' = ?$

$(r')^4 = 5\, r^4$

$(c\, r)^4 = 5\, r^4$

$c^4 r^4 = 5\, r^4$

$c^4 = 5 \Rightarrow c = 5^{1/4} = 1.5 \quad ; \quad r' = 1.5\, r$

Therefore a factor of 1.5 or a 50% increase in the internal radius of a blood vessel would give a 5 fold increase in volume flow rate.

Flow Resistance: Example

The aorta of an average adult human r = 1.3 cm = 1.3×10^{-2} m

What are the (A) resistance and (B) pressure drop over 20 cm = 0.2 m distance, assuming a flow rate of 100 cm^3 /sec. ?

$\eta = 2.084 \times 10^{-3}$ Pa.s.

(A) $R = \dfrac{8\,\eta\, L}{\pi\, r^4} = \dfrac{8}{\pi}\,\dfrac{(2.084 \times 10^{-3}\ \text{Pa.s.})\ (0.2\ \text{m})}{(1.3 \times 10^{-2}\ \text{m})^4}$

$= 3.72 \times 10^{4}\ \text{Pa.s.m}^{-3}$

$= 3.72 \times 10^{4}\ \text{Pa.s.m}^{-3}\ \dfrac{(1\ \text{torr s. cm}^{-3})}{1.33 \times 10^{8}\ \text{Pa.s.m}^{3}}$

$= 2.79 \times 10^{-4}\ \text{torr.s.cm}^{-3}$

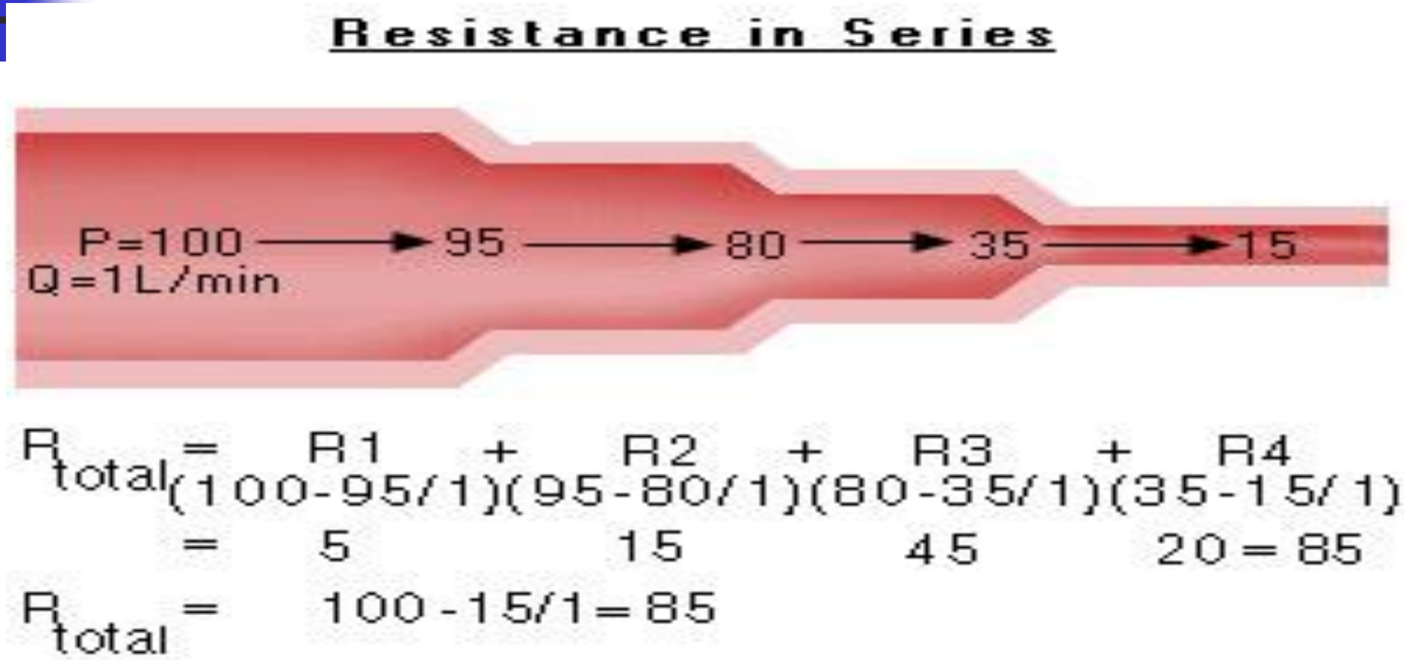
$\Delta P = R \times \Im$

$= (2.79 \times 10\text{-}4\ \text{torr s.cm}^{-3})\ (100\ \text{cm}^3/\text{s})$

$= 0.0279$ torr (1 torr = 1 mm Hg)

- Very small compared to the total pressure drop in the system which is ~ 100 torr.
- **Most of the flow resistance and pressure drops occur in the smaller arteries and vascular beds of the body.**

Series and Parallel Resistances



When a given volume of blood flows through several flow resistances in turn, they are said to be in series

Resistance in Series

$$\Delta \mathbf{P} = \Delta \mathbf{P}_1 + \Delta \mathbf{P}_2 + \ldots\ldots\ldots + \Delta \mathbf{P}_n$$

$$= \Im\,(\Re_{f1} + \Re_{f2} + \ldots\ldots\ldots + \Re_{fn})$$

The N resistances are equivalent to a single resistance, $\Re_s$ chosen so that

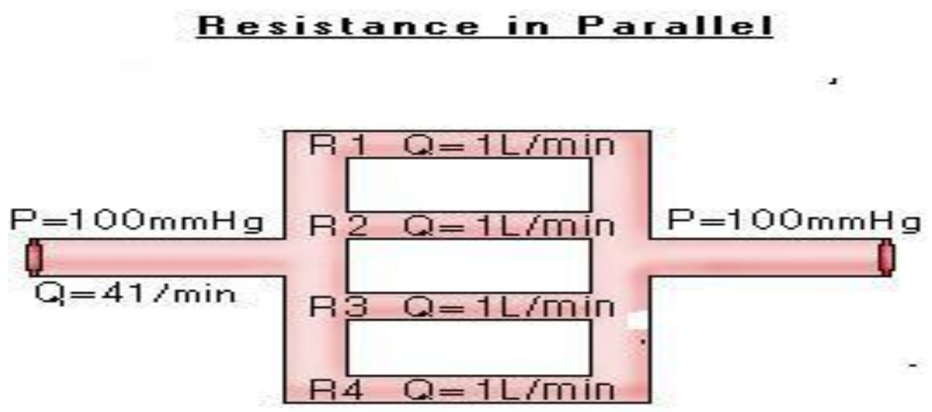
$$\Delta \mathbf{P} = \Im\,\Re_s \quad \text{or} \quad \Delta \mathbf{P} = \mathbf{Q}\,\Re_s$$

$$\Re_s = \Re_{f1} + \Re_{f2} + \ldots\ldots\ldots + \Re_{fn} \text{ (series)}$$

Note flow in $\Im$ or $\mathbf{Q}$

Resistance in Parallel

- If a number of resistances are in parallel, then the fluid splits up, Q_1 through $\Re_{f1}$, Q_2 through $\Re_{f2}$ and so on.



Resistance in Parallel

- The pressure drop ΔP across each resistance is the same, so applying
  $\Delta P = Q\,\Re_f$ to each resistance gives

$$Q_1 = \frac{\Delta P}{\Re_{f1}} \quad Q_2 = \frac{\Delta P}{\Re_{f2}} \quad Q_n = \frac{\Delta P}{\Re_{fn}}$$

- Thus

$$Q = Q_1 + Q_2 + \ldots\ldots\ldots\ldots\ldots\ldots + Q_n$$

$$\Delta P\left(\frac{1}{\Re_{f1}} + \frac{1}{\Re_{f2}} + \ldots\ldots\ldots\ldots\ldots\ldots + \frac{1}{\Re_{fn}}\right)$$

Resistance in Parallel

If we were this system of parallel resistances by a single resistance, $\mathfrak{R}_p$, we would have

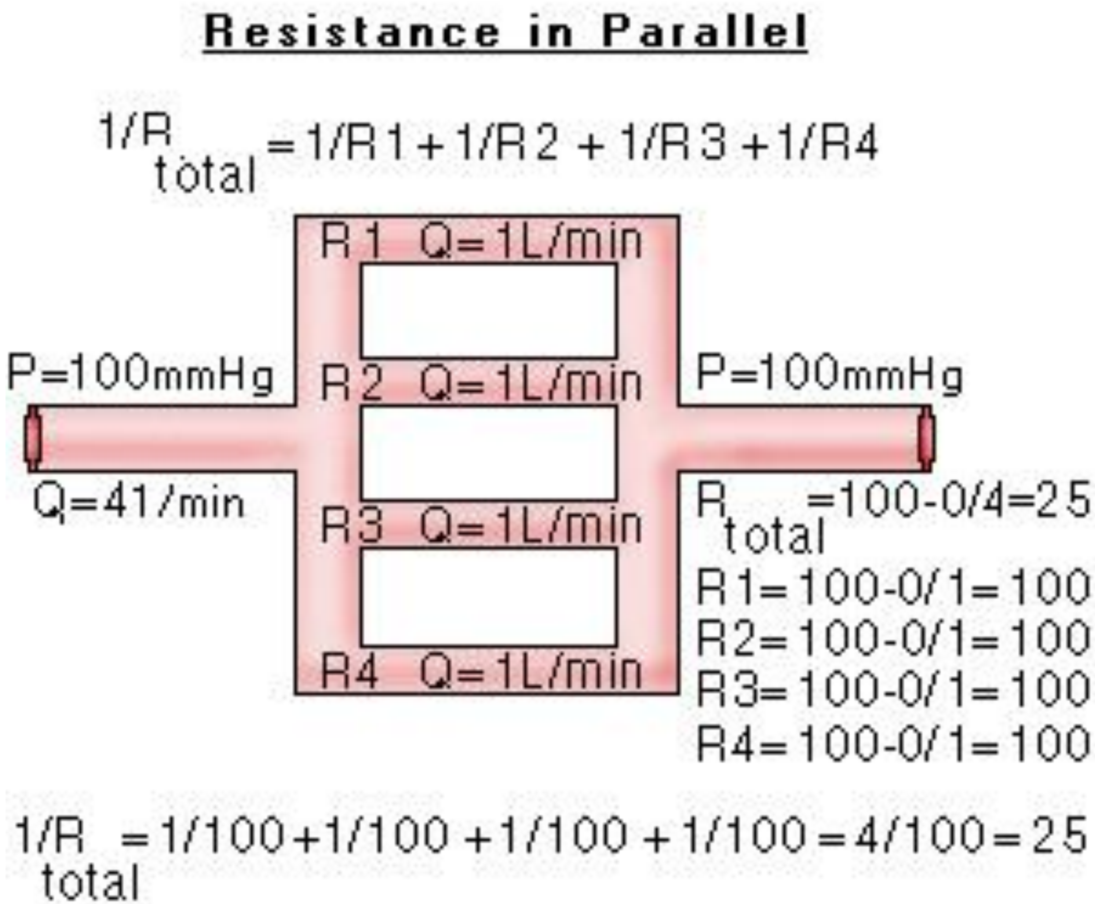
$$Q = \frac{\Delta P}{\mathfrak{R}_p}$$

So, for N resistances in parallel, the equivalent resitance is

$$\frac{1}{\mathfrak{R}_p} = \frac{1}{\mathfrak{R}_{f1}} + \frac{1}{\mathfrak{R}_{f2}} + \dots \frac{1}{\mathfrak{R}_{fn}}$$

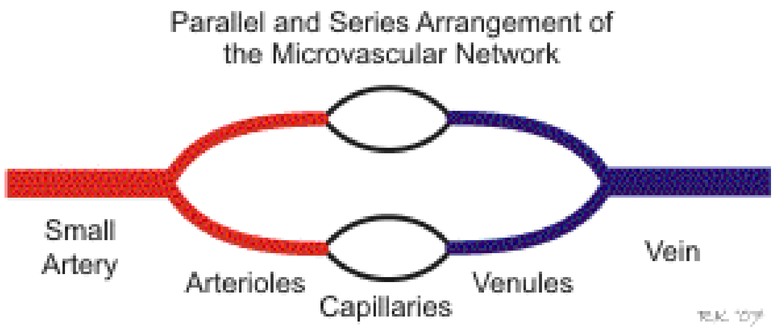
The relationship for series and parallel resistances are exactly the same for electrical resistances in circuits.

Resistance in Parallel



Resistance of Vascular Beds

http://www.cvphysiology.com/Hemodynamics/H005.htm



For an in-series resistance network the total resistance is equal to the sum of the small artery (RA), arterioles (Ra), capillaries (Rc), venules (Rv), and vein (RV) resistances.

$$\mathbf{R_T = R_A + R_a + R_c + R_v + R_V}$$

Assume, RA = 20, Ra = 50, Rc = 20, Rv = 8, RV = 1

Therefore, RT = 20 + 50 +20 +6 + 4 = 100

Example: the resistance of capillaries

A. Net resistance of one capillary

$$\frac{8\,\eta\,L}{\pi\, r^4} = \frac{8}{\pi}\,\frac{(2.084 \times 10^{-3}\ \text{Pa.s.})\ (10^{-3}\ \text{m})}{(4.0 \times 10^{-6}\ \text{m})^4}$$

$$= 2.073 \times 10^{16}\ \text{Pa.s.m}^{-3}$$

$$= 3.72 \times 10^{4}\ \text{Pa.s.m}^{-3} \quad \frac{(1\ \text{torr s.}\ \text{cm}^{-3}\ )}{1.33 \times 10^{8}\ \text{Pa.s.m}^{3}}$$

$$= 1.56 \times 10^{8}\ \text{torr.s.cm}^{-3}$$