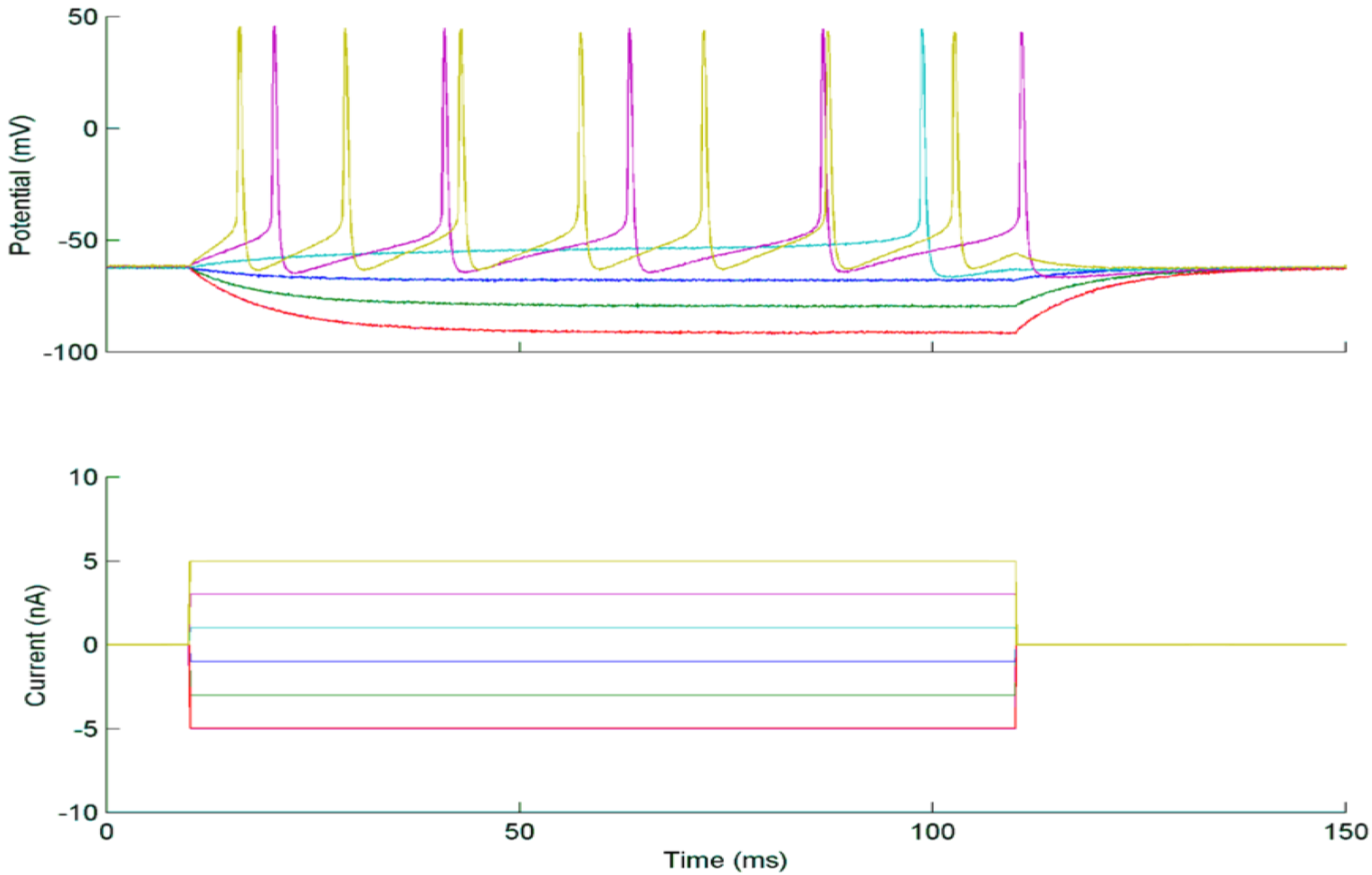
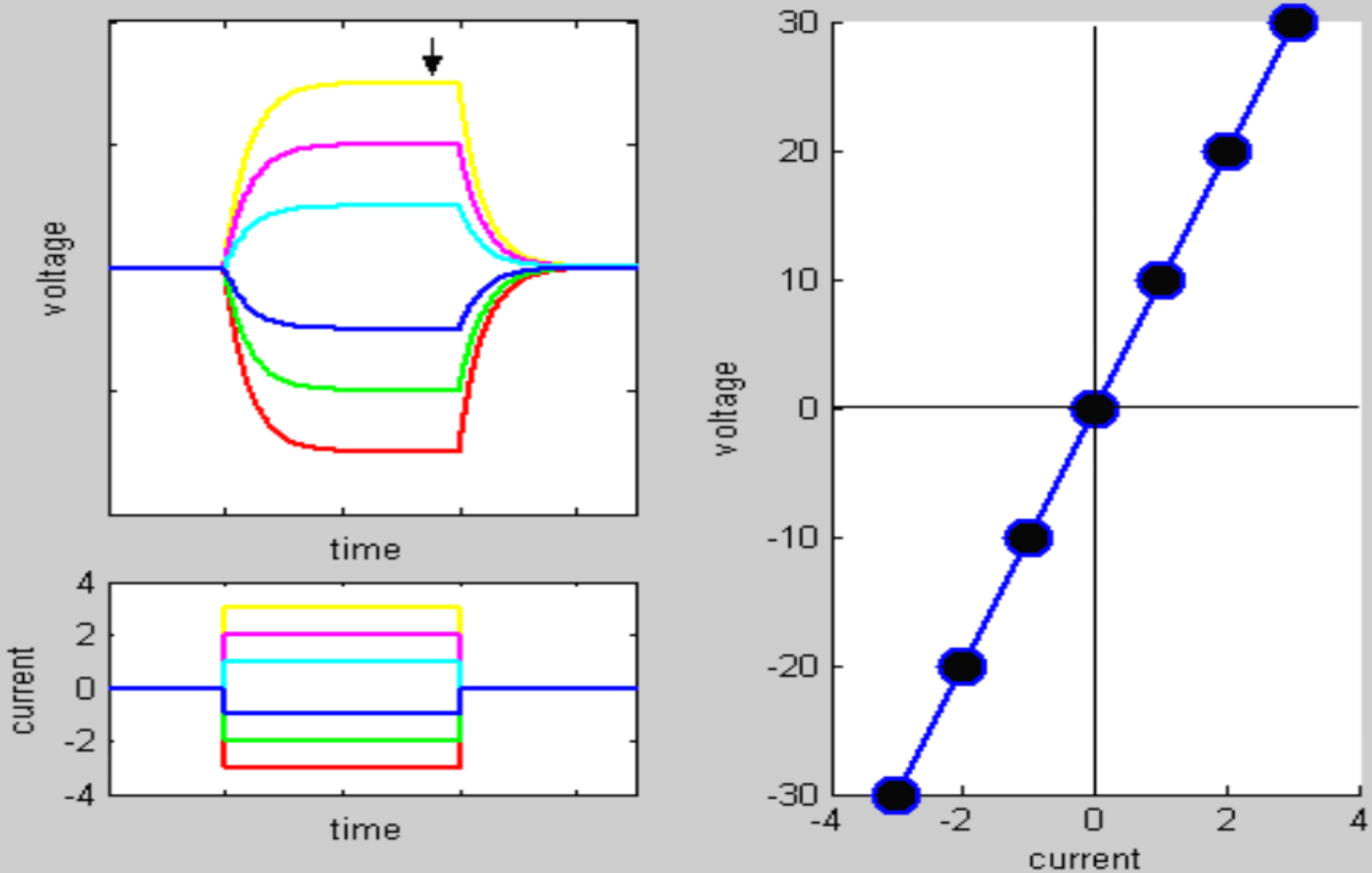
Stucture Function Relationship in Nerve Cells

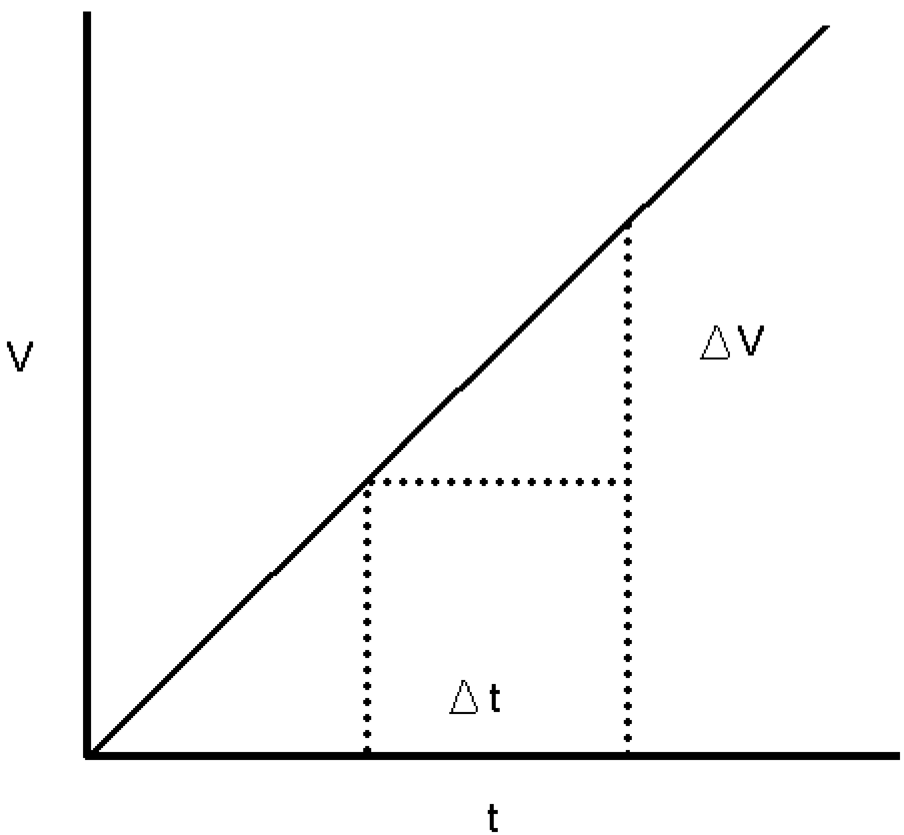
Dr. Aslı AYKAÇ



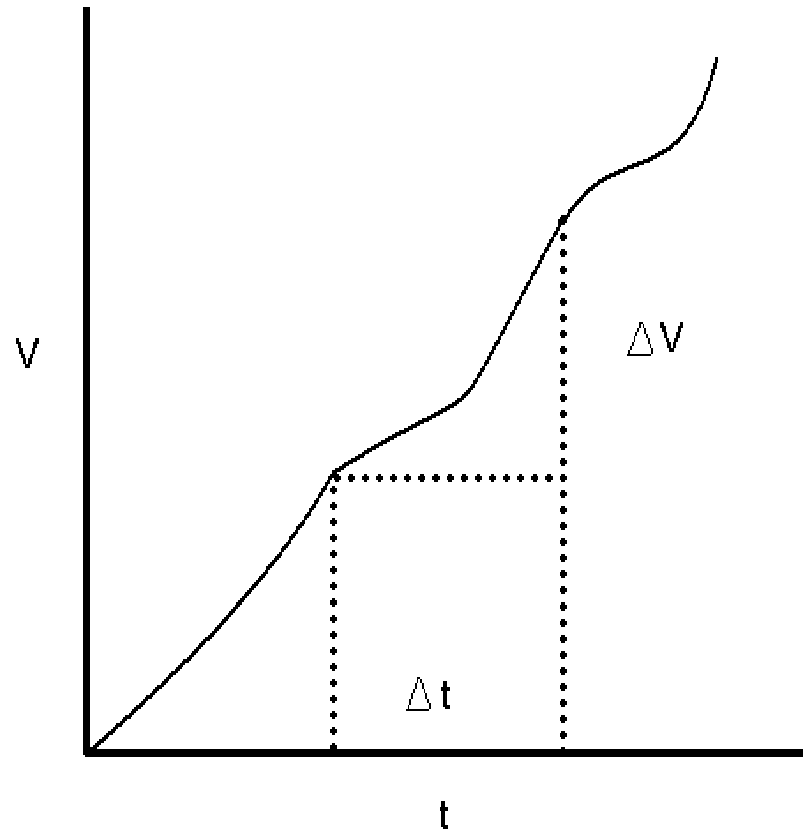
50
0
-50
-100
Potential (mV)
10
5
0
-5
-10
Current (nA)
0
50
100
150
Time (ms)



voltage
time
4
2
0
-2
-4
current
time
30
20
10
0
-10
-20
-30
-4
-2
0
2
4
voltage
current



a=ΔV/Δt



$$Lim = \Delta V / \Delta t = \frac{dV}{dt}$$

$$\Delta t \Rightarrow 0$$

$$F = ma$$

the equation could be arranged as first order differential equation

$$F = m\frac{dv}{dt}$$

or

the equation could be arranged as first order differential equation

$$F = ma$$

$$F = m\frac{dv}{dt}$$

or

since

$$v = \frac{dx}{dt}$$

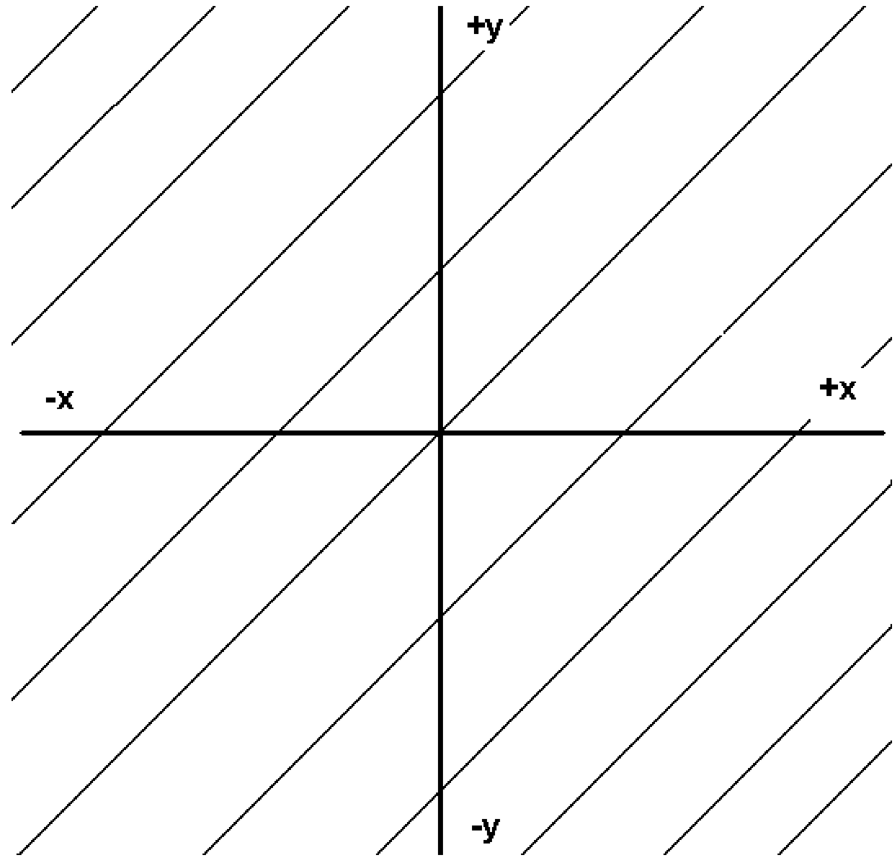
$$F = m\frac{d^2x}{dt^2} = m\frac{d}{dt}\left(\frac{dx}{dt}\right)$$

as a second order differential equation important

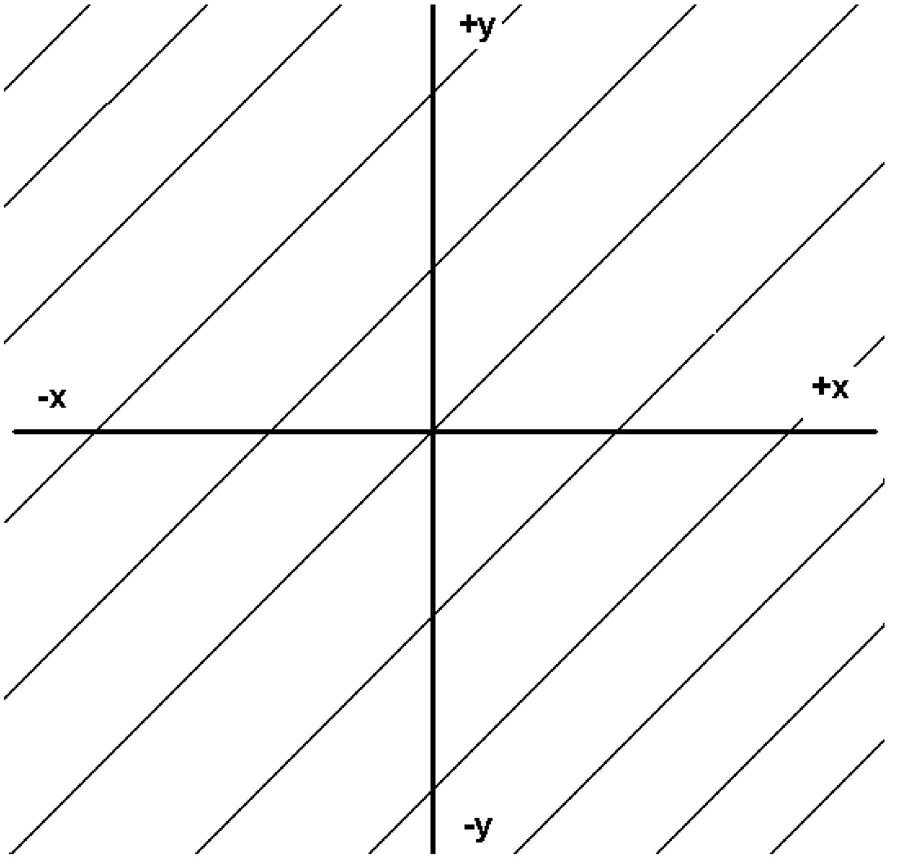
$$\frac{d^2x}{dt^2} \neq \left(\frac{dx}{dt}\right)^2$$

$$\frac{dy}{dx} = 1 \quad \Rightarrow \quad y = x + A$$

is the general solution



$$\frac{dy}{dx} = 1 \quad \Rightarrow \quad y = x + A$$



is the general solution

İf additional conditions are given

$$x = 0 \quad için \quad y = 0$$

the condition is satisfied only

$$A = 0$$

thus

$$y = x$$

The solution is valid under some specific conditions the solution is a particular solution.

$$I = \frac{V}{R}$$

$$I = \frac{V}{R}$$

$$Q = CV$$

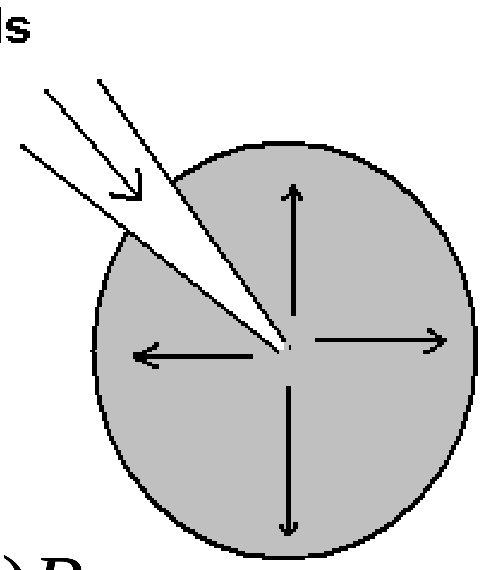
$$I = \frac{V}{R}$$

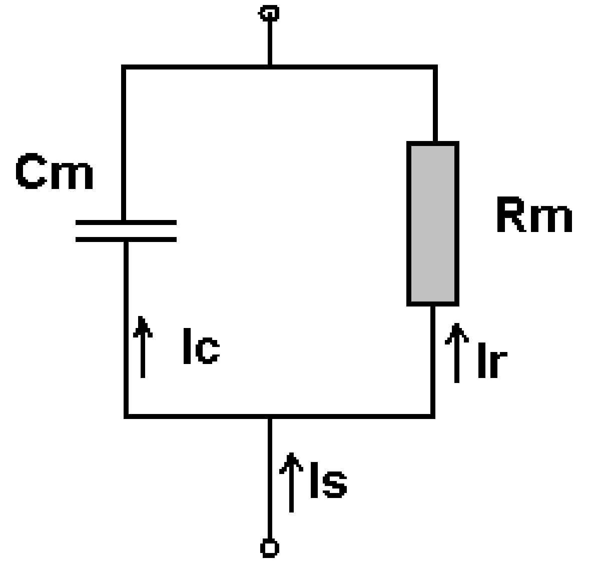
$$Q = CV$$

as a definition rate of change in the amount of charged particle per unit time is the current

$$I_C = \frac{dQ}{dt} = C\frac{dv}{dt}$$

$$I_s = I_c + I_r$$





$$E_r = E_c = E_m$$

$$E = I_r R_m = (I_s - I_c) R_m$$

$$I_c = C \frac{dE}{dt}$$

$$\frac{dE}{dt} R_m C_m + E = I_s R_m$$

$$E = I_s R_m (1 - e^{-t / RmCm})$$

$$E = I_s R_m (1 - e^{-t/RmCm})$$

$$t \Rightarrow \infty \qquad E_\infty = IsRm$$

$$\Delta E = E_\infty - E_o \quad \propto \quad IsRm$$

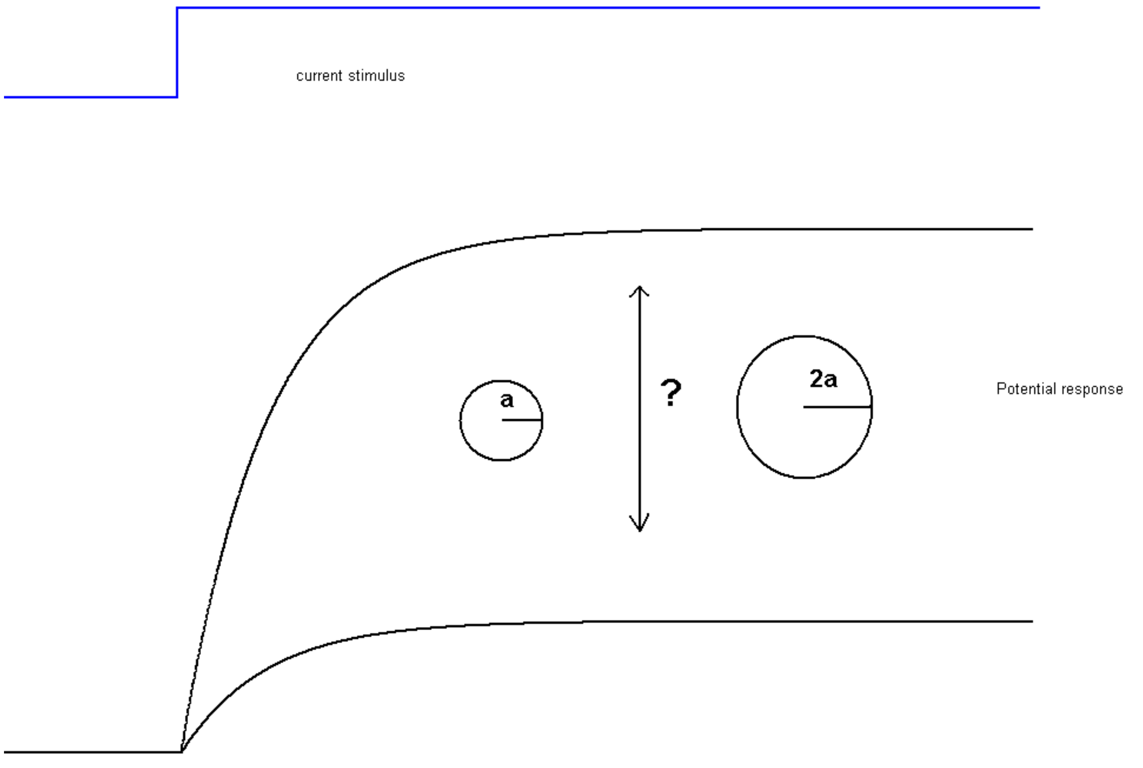
$$I_m = \frac{Is}{4\pi a^2}$$

$$Rinput = R_N = \frac{E_\infty}{I_S} = \frac{Rm}{4\pi a^2}$$

$$R_N \propto \frac{1}{a^2}$$

$$E = I_s R_m (1 - e^{-t/RmCm})$$

$$\Delta E \quad \propto \quad R_N$$



$$E = I_s R_m (1 - e^{-t/RmCm})$$

-the maximal amplitude of the passive membrane potential is defined by the input resistance of the cell.

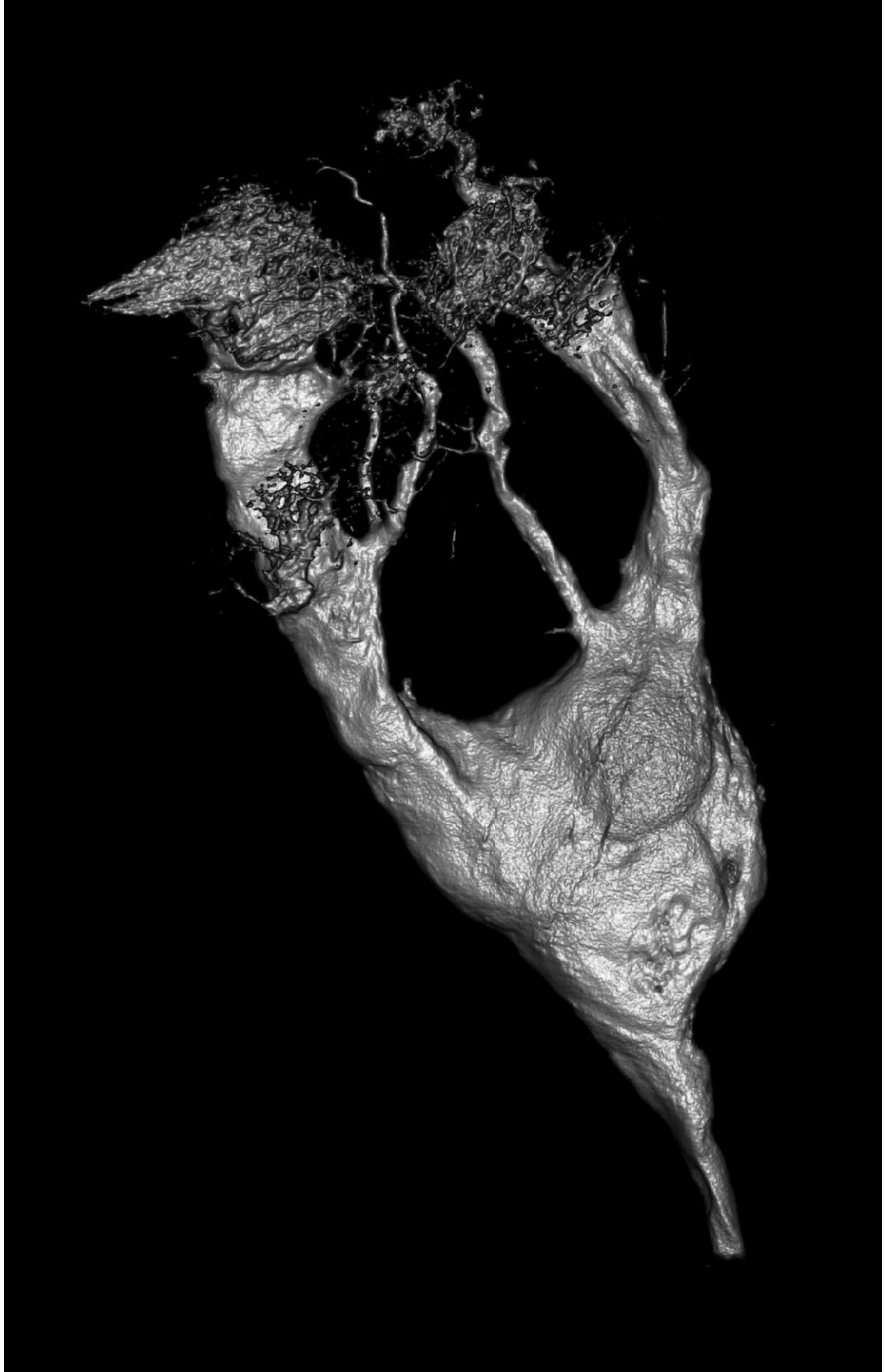
$$t = \infty \quad \Rightarrow \quad E = IsRm$$

-Membrane capacitance Cm prolongs the time course of the electrical signals

$(\tau_m = R_m C_m)$.

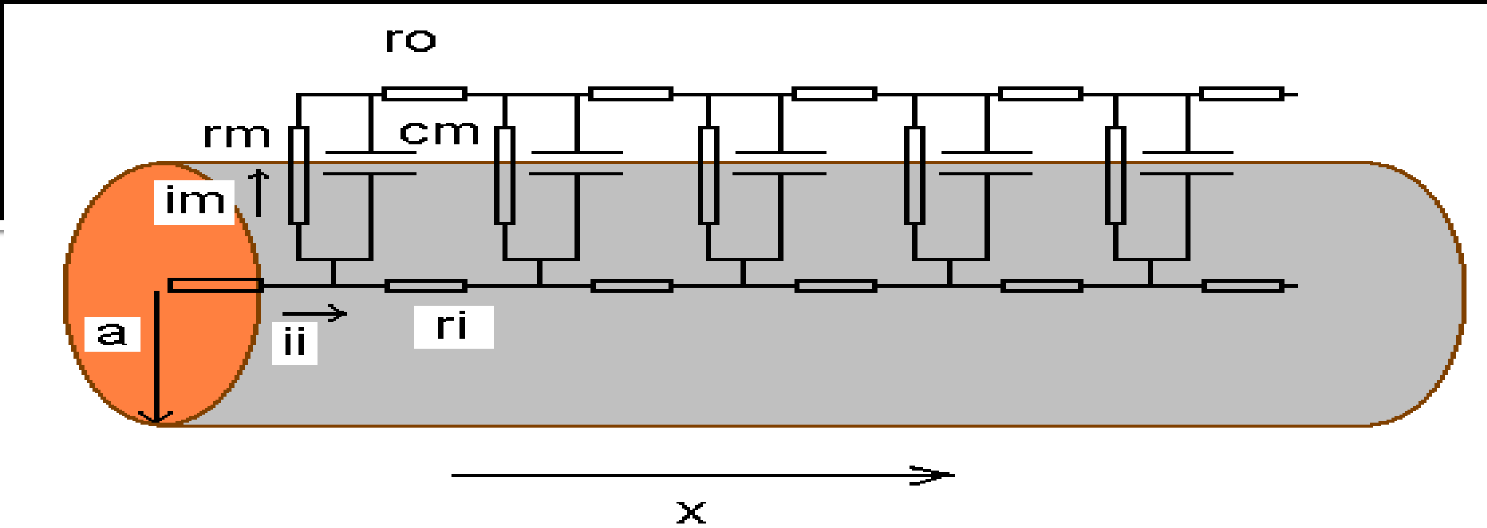
-Membrane capacitance is proportional to the surface area while the input
-resistance is inversely proportional.

$$Rinput = R_N = \frac{E_\infty}{I_S} = \frac{Rm}{4\pi a^2}$$



Assumptions in driving the cable equation

- Axon is tubular in shape
- The whole membrane is homegenous.
- Physical properties are constant and are not dependent on voltage
- Axonal currents are unidirectional (radial currents are ignored)
- Extracellular solution is very conductive, its resistivity is ignored



- *Vm (x ve t)* changes as a function of time and distance
- Voltage change is in the form of a reduction
- Rate of change is related to $r_i i_i$
- *Axoplasmic current* i_i will get smaller by distance since it flows over the membrane

$$\frac{dVm(x,t)}{dx} = -r_i i_i$$

$$\frac{di}{dx} = -i_m$$

$$\frac{d^2Vm}{dx^2} = -r_i \frac{di}{dx} \quad = i_m r_i$$

$$i_m = i_c + i_r \quad = \quad Cm\frac{dVm}{dt} + \frac{Vm}{rm}$$

$$\frac{1}{r_i}\frac{d^2Vm}{dx^2} = Cm\frac{dVm}{dt} + \frac{Vm}{rm}$$

with refer to the resistivity of a 1 cm^2 membrane, and $1 cm^3$ axoplasm

Ri	*specific intracellular resistivity*	*(Ω-cm)*
Rm	*specific membrane resistance*	*(Ω-cm^2)*
Cm	*specific membrane capacitance*	*(F/cm^2)*

for an axon in any shape

ri	*intracellular resistivity*	*(Ω/cm)*
rm	*membrane resistance*	*(Ω-cm)*
cm	*membrane capacitance*	*(F/cm)*

Considering the tubular shape of the axon

$$Ri = \pi a^2 ri \qquad Rm = 2\pi a\, rm \qquad Cm = cm / 2\pi a$$

$$\frac{1}{r_i}\frac{d^2Vm}{dx^2} = Cm\frac{dVm}{dt} + \frac{Vm}{rm}$$

$$\lambda = \sqrt{\frac{r_m}{r_i}} = \sqrt{\frac{aRm}{2Ri}}$$

$$\lambda^2\frac{d^2Vm}{dx^2} = t_m\frac{dVm}{dt} + Vm$$

$$X = x/\lambda$$

$$T = t/t_m$$

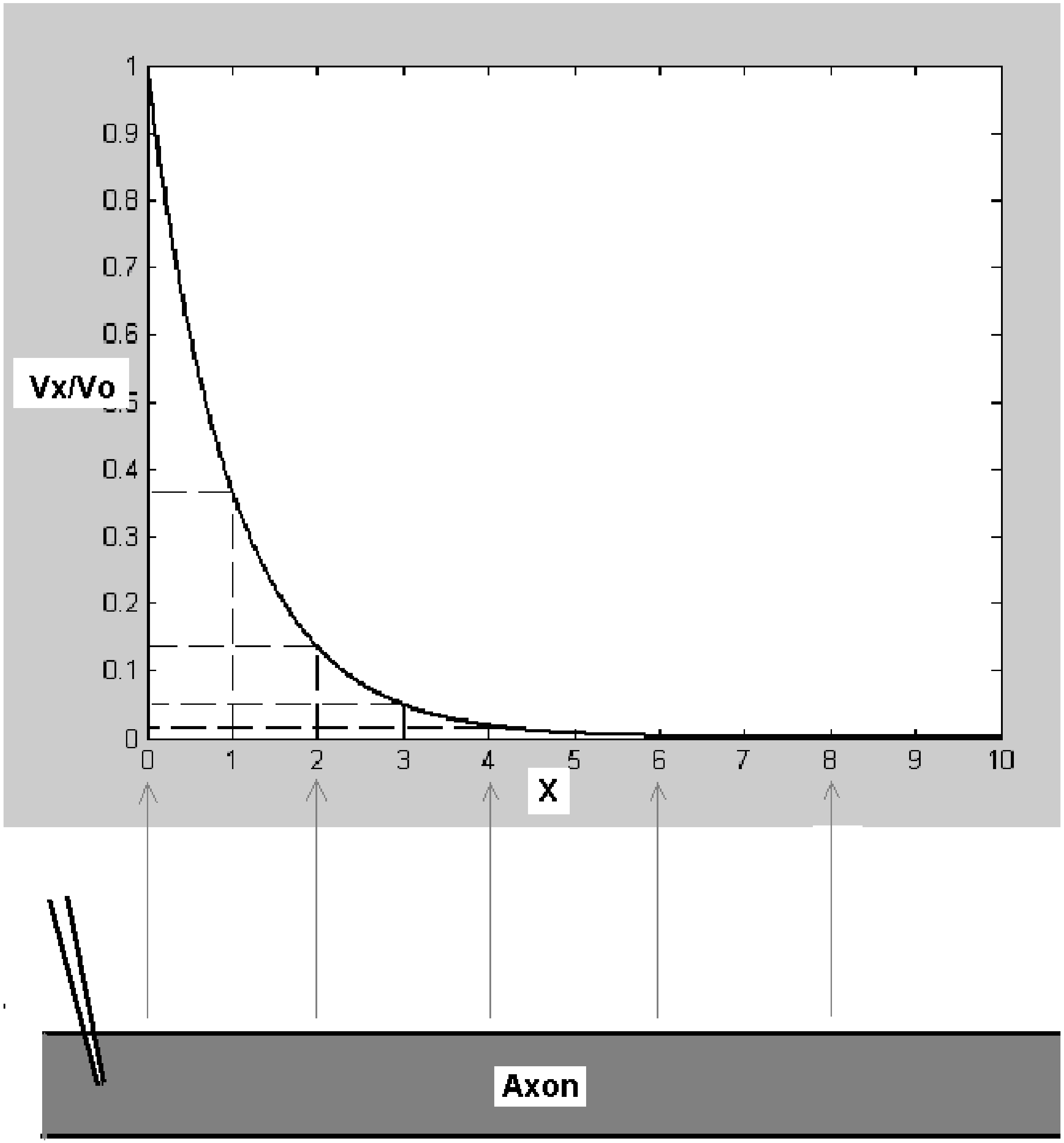
$$\frac{d^2Vm}{dX^2} - \frac{dVm}{dT} - Vm = 0$$

$$\frac{d^2Vm}{dX^2} - \frac{dVm}{dT} - Vm = 0$$

$$T \rightarrow \infty \qquad Vm(\infty, X) = \frac{riIo\lambda}{2} e^{-X}$$

$$Vm(\infty, x) = \frac{riIo\lambda}{2} e^{-x/\lambda}$$

$$\frac{V_X}{Vo} = e^{-X}$$

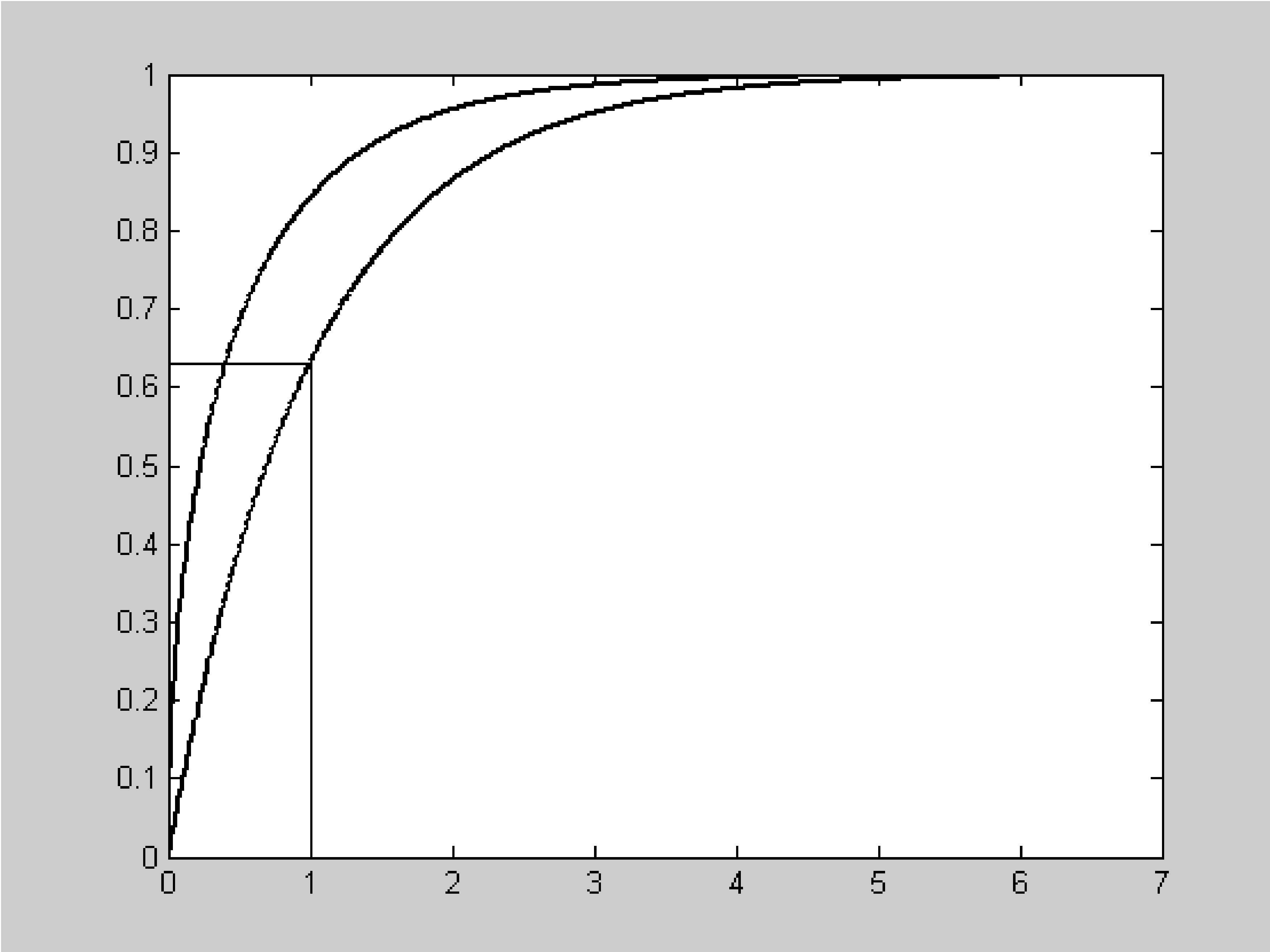


$$\lambda = \sqrt{\frac{r_m}{r_i}} = \sqrt{\frac{aRm}{2Ri}} \qquad \lambda \propto \sqrt{a}$$

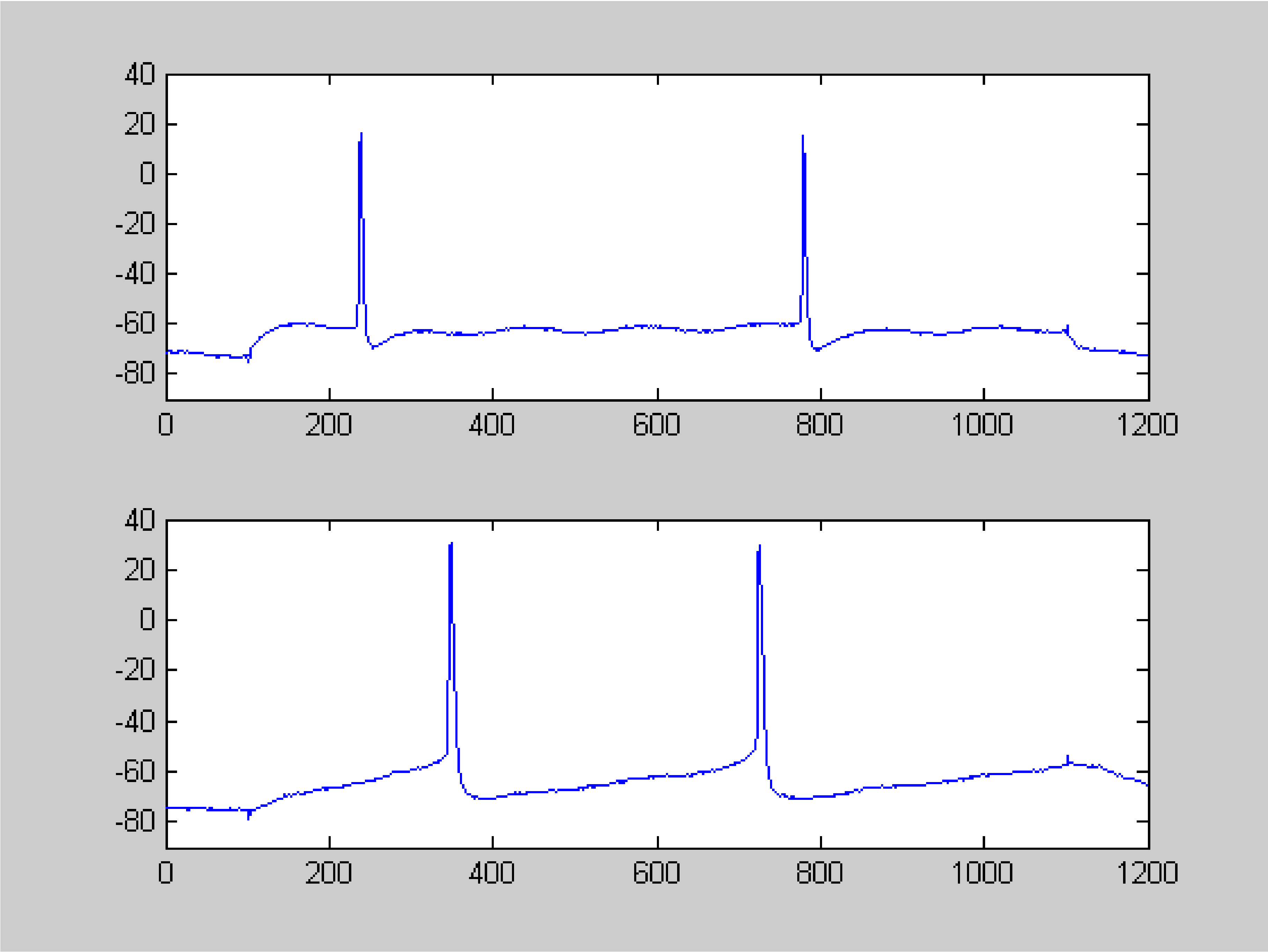
$$X = 0 \quad ve \quad T$$

$$Vm(T,0) = \frac{riIo\lambda}{2} erf(\sqrt{T})$$

$$erf(\sqrt{T})$$



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
7


40
20
0
-20
-40
-60
-80
0
200
400
600
800
1000
1200
40
20
0
-20
-40
-60
-80
0
200
400
600
800
1000
1200

$$\frac{Vm(T,X)}{Vm(\infty,X)} = \frac{1}{2}$$

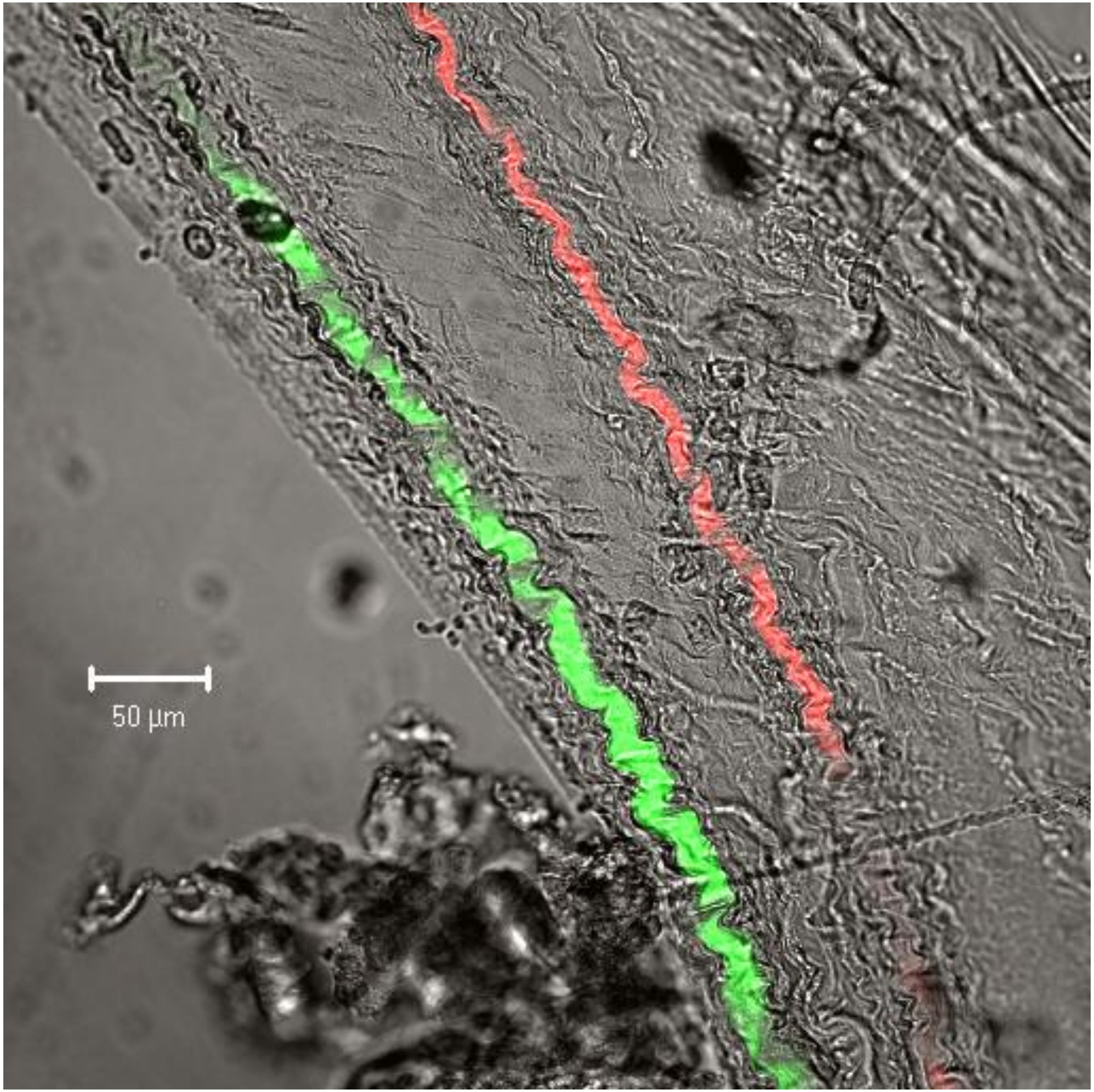
$$X = 2T - 0.5 \quad veya \quad x = \frac{2\lambda}{t_m} t - 0.5\lambda$$

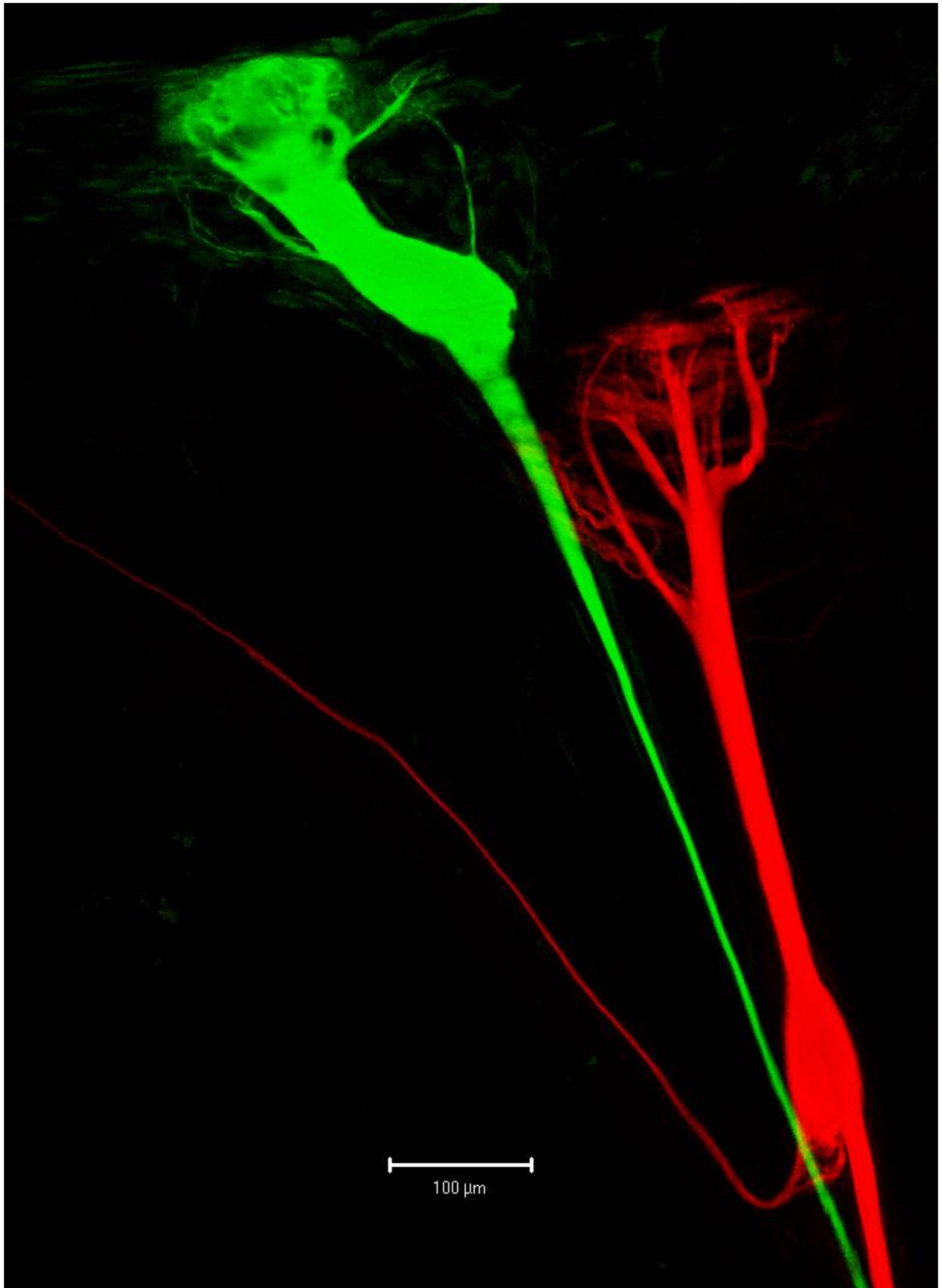
$$\theta = \frac{dx}{dt}$$

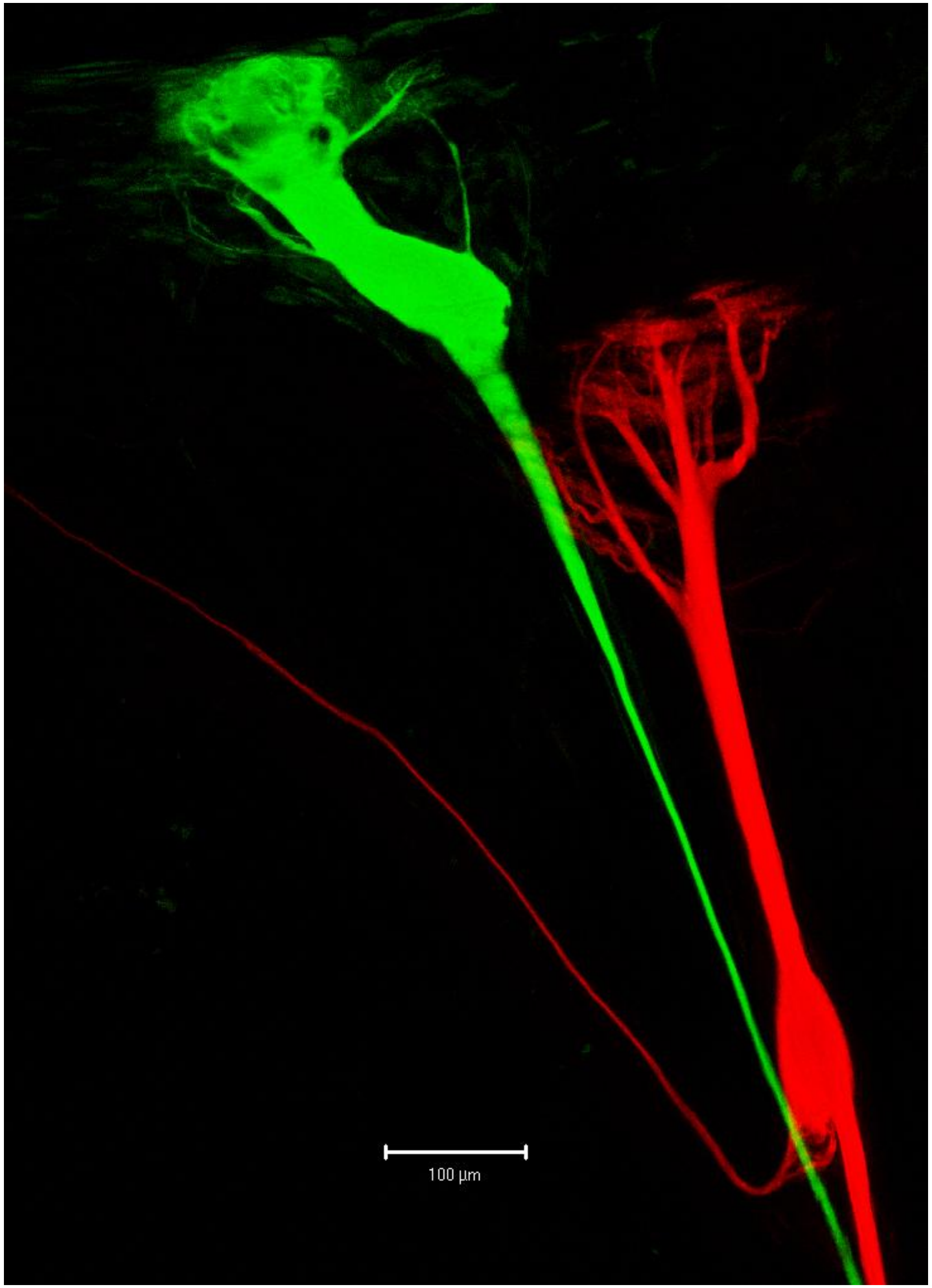
$$\theta = \frac{dx}{dt} = \frac{2\lambda}{t_m} = \left(\frac{2a}{RmRiCm^2}\right)^{1/2}$$

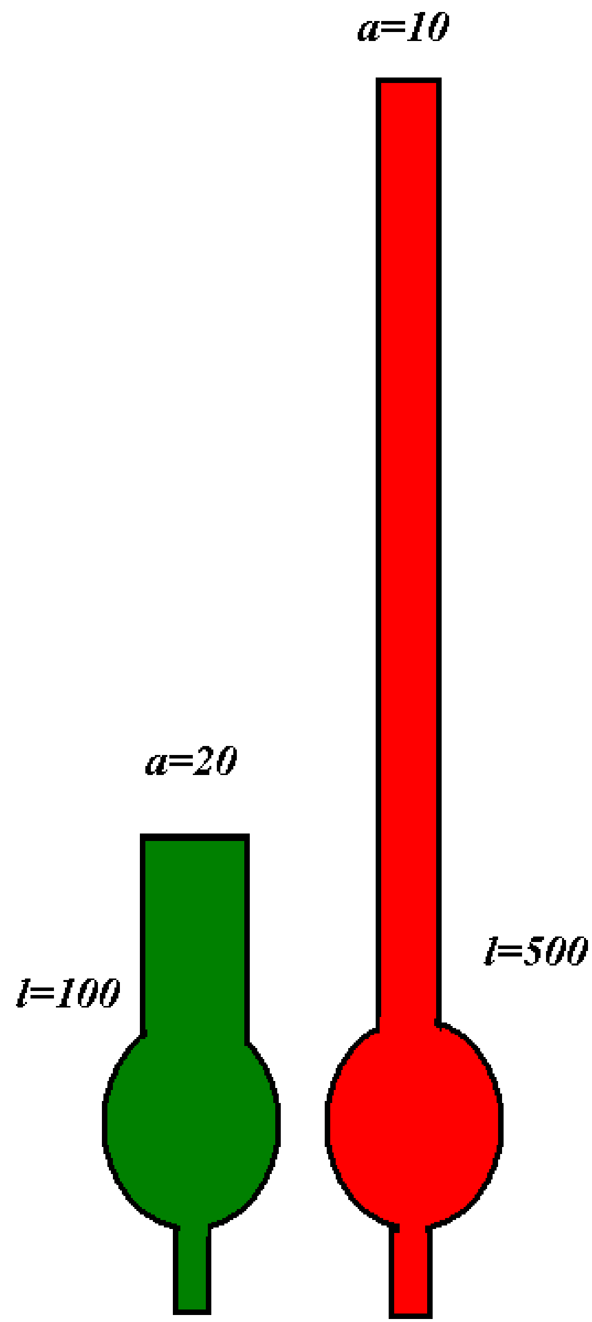
$$\theta \propto \sqrt{a}$$

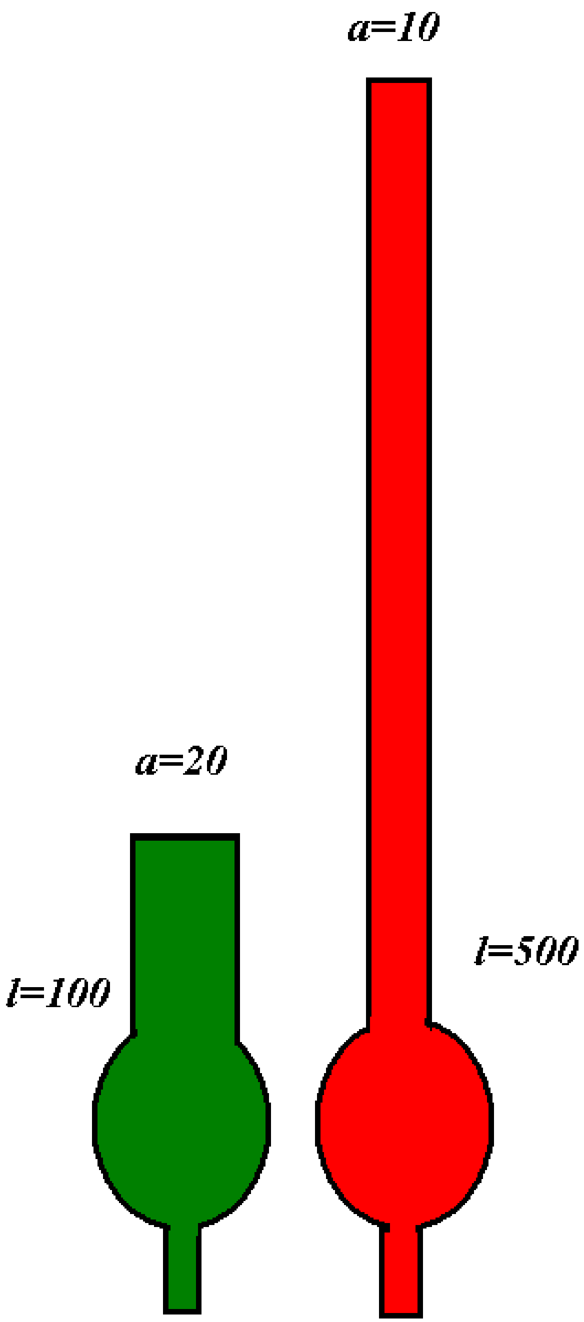
- [redacted] *amplitude of the pasive potential response*
- *Length constant λ defines the amplitude of the propagated electrical signal.*
- *Length constant becomes longer as the diameter of the axon increases*
- *As in the spherical cell membrane capacitance prolongs the time course of the passive signals ($\tau m = RmCm$).*
- *Rate of passive spread is faster in axons with large diameter.*


50 µm


100 µm


100 µm


a=10
a=20
l=100
l=500



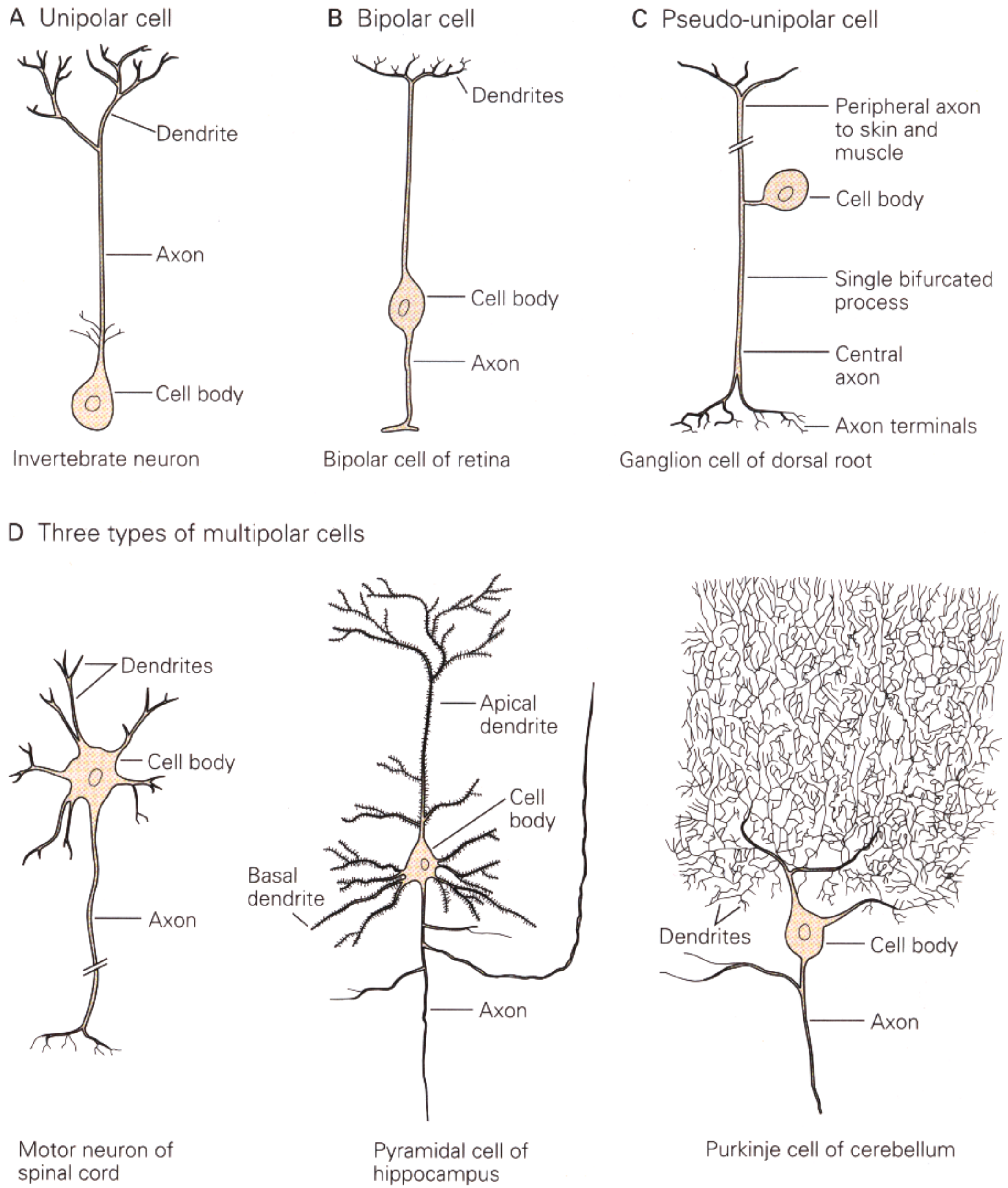
λgreen =2,24 mm
λred =1,58 mm

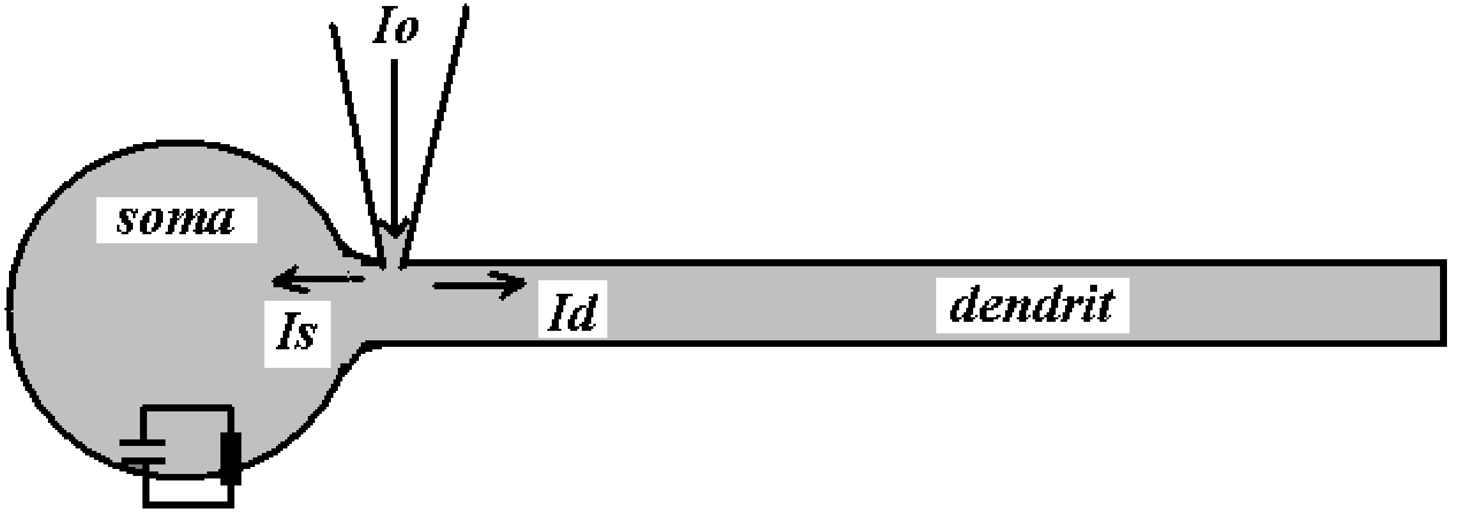
lgreen=100 µm
L=l/λ=0,1/2.24=0,045
lred=500 µm
L=0,5/1.58=0,32

İf both dendrites are depolirized to the potential level of Vo, as can be calculated by the the equation

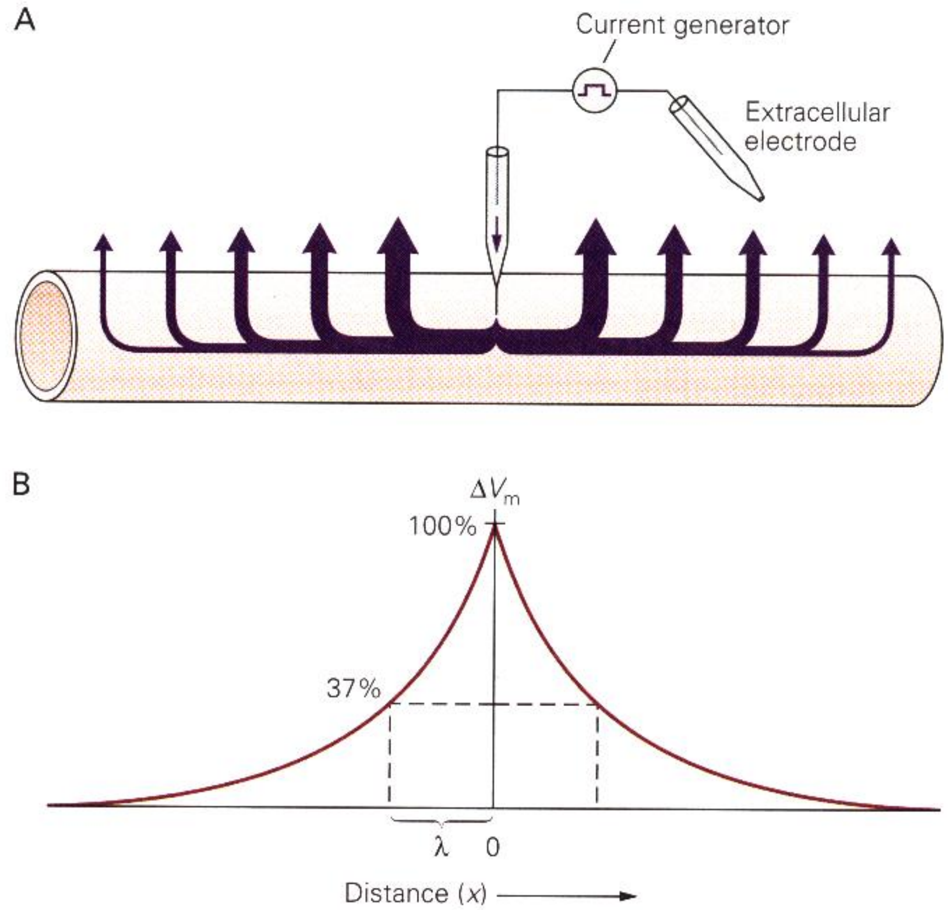
$$\frac{V_X}{Vo} = e^{-x/\lambda}$$

96 % of Vo will propagate to the green soma. However only 72% of Vo will arrive to the red soma

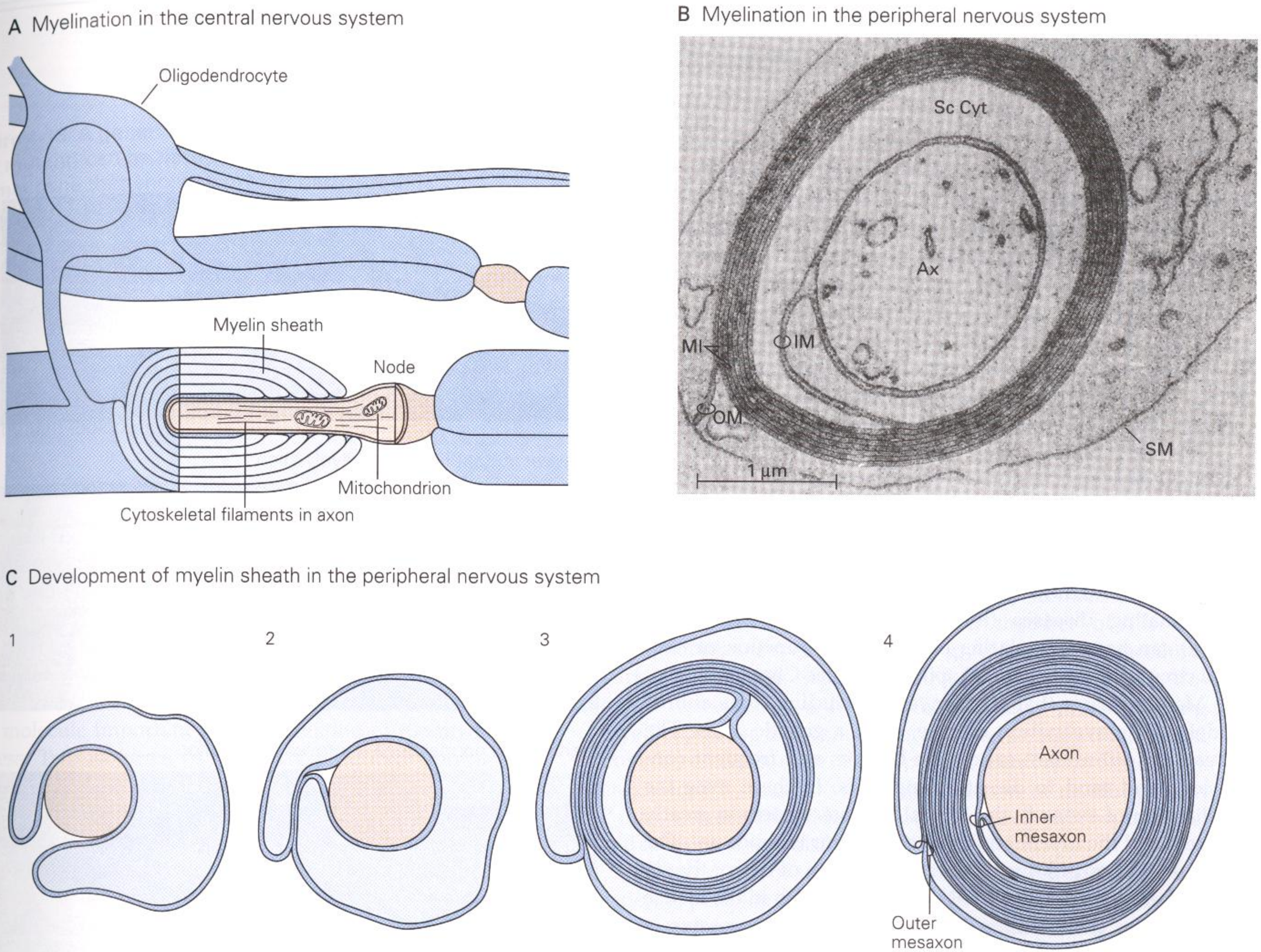

A Unipolar cell
Dendrite
Axon
Cell body
Invertebrate neuron
B Bipolar cell
Dendrites
Cell body
Axon
Bipolar cell of retina
C Pseudo-unipolar cell
Peripheral axon to skin and muscle
Cell body
Single bifurcated process
Central axon
Axon terminals
Ganglion cell of dorsal root
D Three types of multipolar cells
Dendrites
Cell body
Axon
Motor neuron of spinal cord
Apical dendrite
Cell body
Basal dendrite
Axon
Pyramidal cell of hippocampus
Dendrites
Cell body
Axon
Purkinje cell of cerebellum

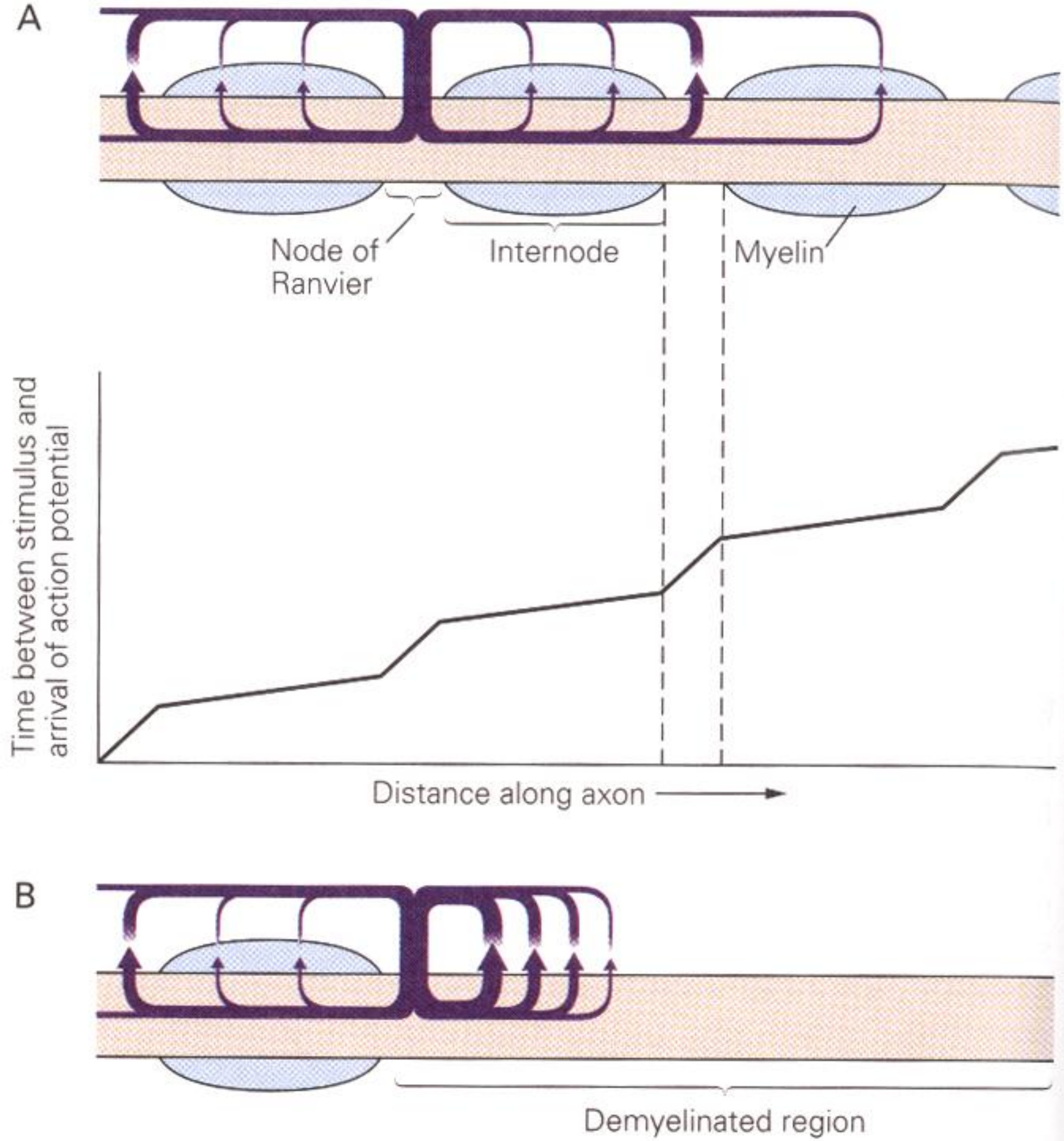


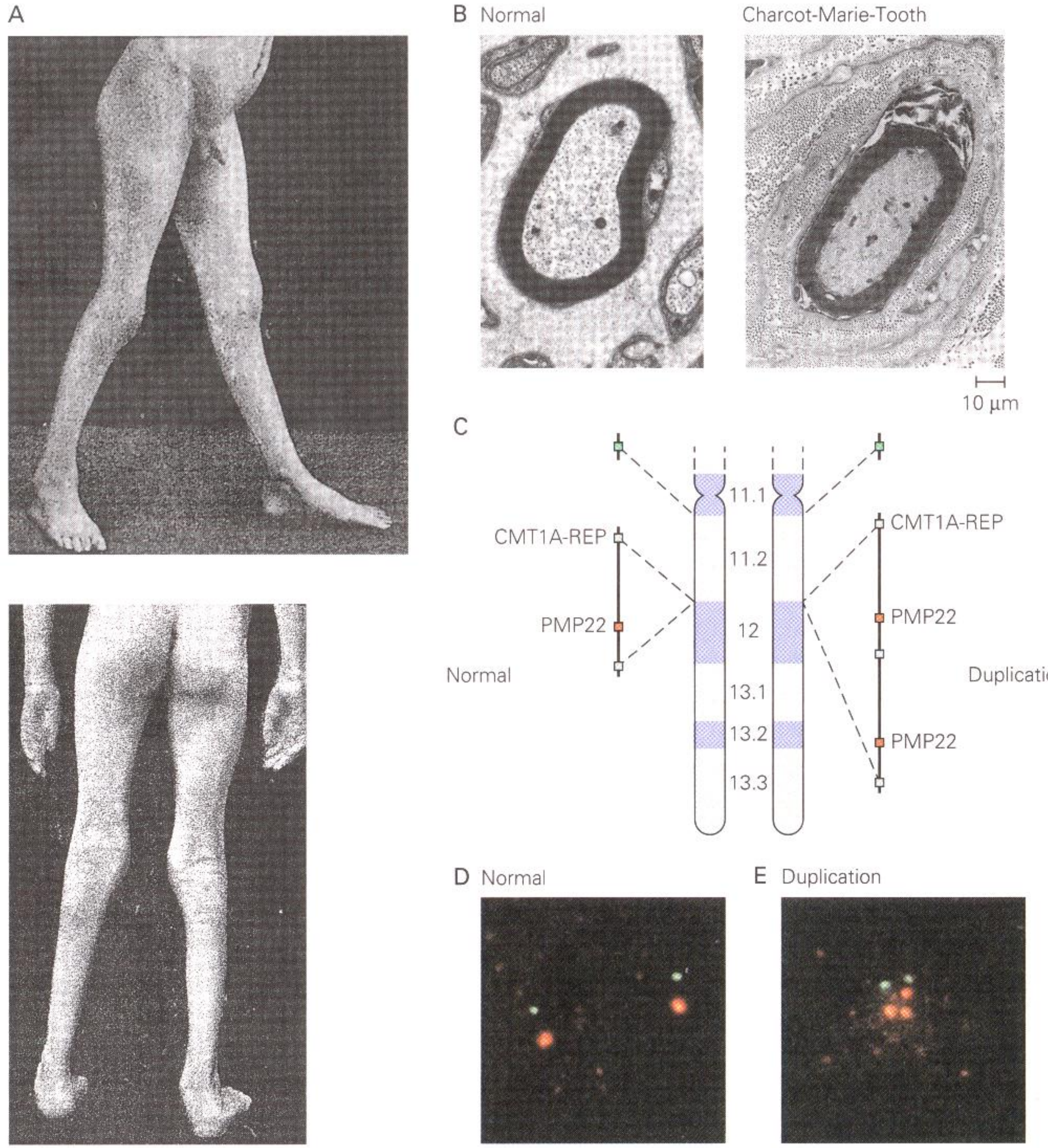
$$\rho = \frac{G_s}{G_d} \approx \frac{I_s}{I_d}$$



A
Current generator
Extracellular
electrode
B
ΔVm
100%
37%
λ
0
Distance (x)


A Myelination in the central nervous system
Oligodendrocyte
Myelin sheath
Node
Mitochondrion
Cytoskeletal filaments in axon
B Myelination in the peripheral nervous system
Sc Cyt
Ax
MI
IM
OM
SM
1 μm
C Development of myelin sheath in the peripheral nervous system
1
2
3
4
Axon
Inner mesaxon
Outer mesaxon


A
Node of Ranvier
Internode
Myelin
Time between stimulus and arrival of action potential
Distance along axon
B
Demyelinated region


A
B Normal
Charcot-Marie-Tooth
10 μm
C
11.1
11.2
12
13.1
13.2
13.3
CMT1A-REP
PMP22
Normal
CMT1A-REP
PMP22
PMP22
Duplicatio
D Normal
E Duplication