

Chapter 9: Phase Diagrams

ISSUES TO ADDRESS...

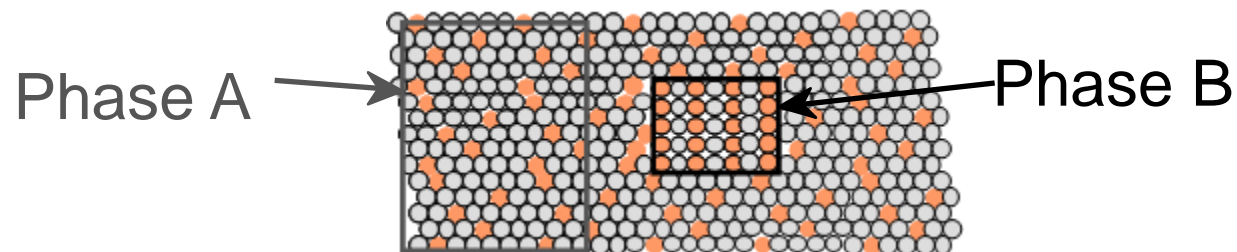
- When we combine two elements...
 what is the resulting equilibrium state?
- In particular, if we specify...
 - the composition (e.g., wt% Cu - wt% Ni), and
 - the temperature (T)

then...

How many phases form?

What is the composition of each phase?

What is the amount of each phase?



- Nickel atom
- Copper atom



Phase Equilibria: Solubility Limit

- **Solution** – solid, liquid, or gas solutions, single phase
- **Mixture** – more than one phase

Adapted from Fig. 9.1,
Callister & Rethwisch 8e.

- **Solubility Limit:**

Maximum concentration for which only a single phase solution exists.

Question: What is the solubility limit for sugar in water at 20°C?

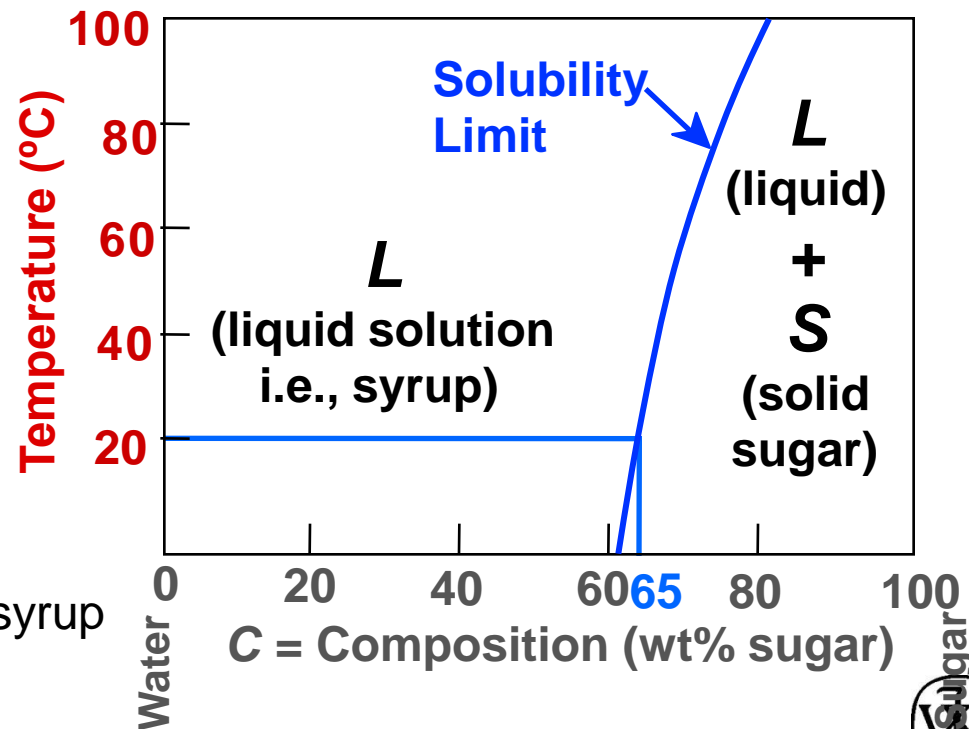
Answer: 65 wt% sugar.

At 20°C, if $C < 65$ wt% sugar: syrup

At 20°C, if $C > 65$ wt% sugar:

syrup + sugar

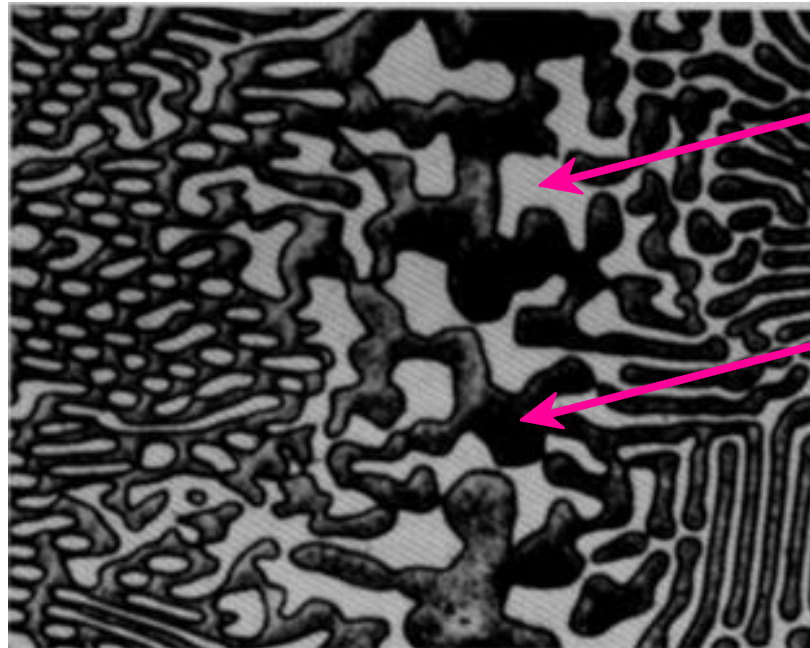
Sugar/Water Phase Diagram



Components and Phases

- **Components:**
The elements or compounds which are present in the alloy (e.g., Al and Cu)
- **Phases:**
The physically and chemically distinct material regions that form (e.g., α and β).

Aluminum-
Copper
Alloy



β (lighter
phase)

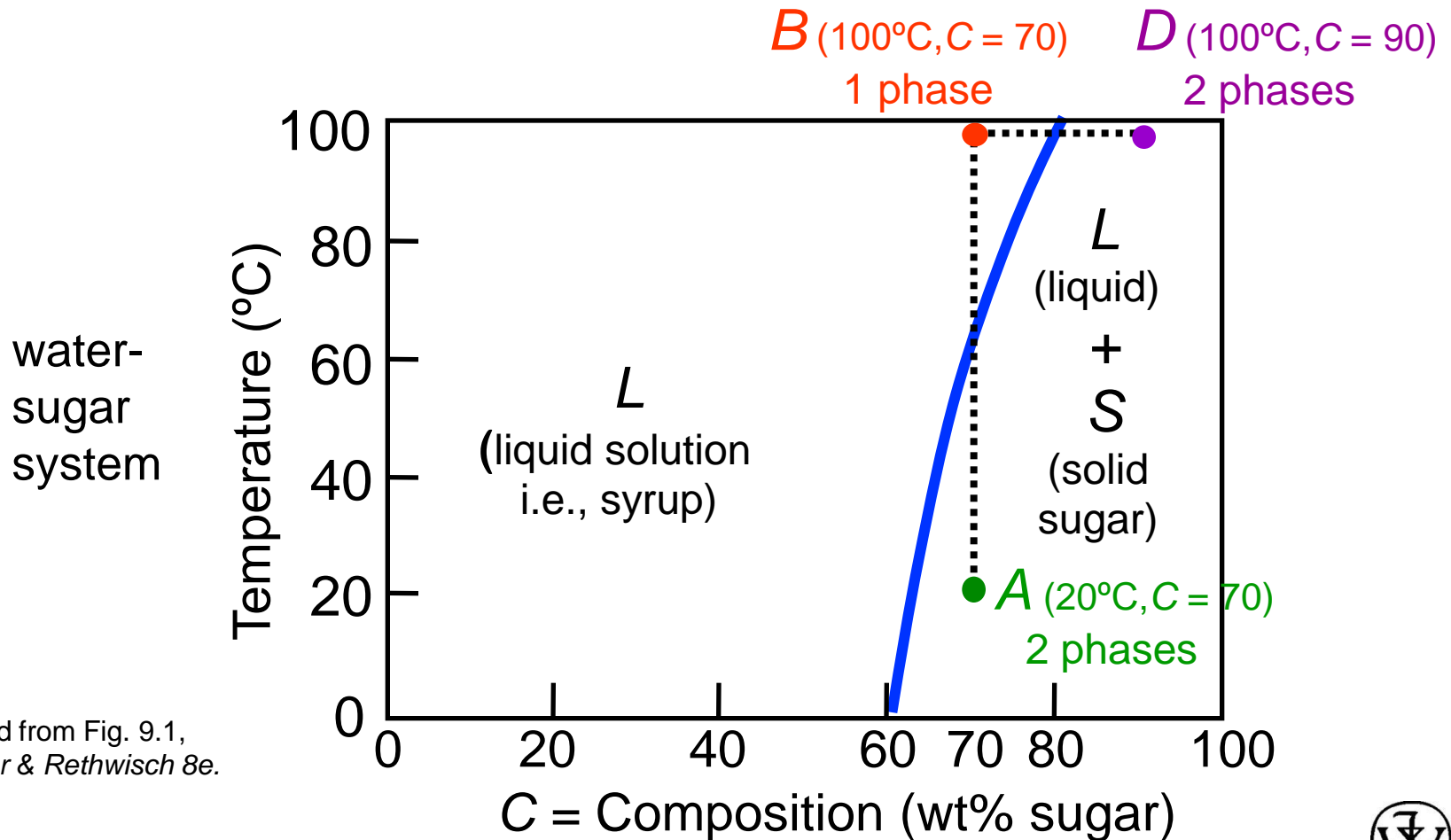
α (darker
phase)

Adapted from chapter-opening photograph, Chapter 9, *Callister, Materials Science & Engineering: An Introduction, 3e.*



Effect of Temperature & Composition

- Altering T can change # of phases: path A to B .
- Altering C can change # of phases: path B to D .



Adapted from Fig. 9.1,
Callister & Rethwisch 8e.



Criteria for Solid Solubility

Simple system (e.g., Ni-Cu solution)

	Crystal Structure	electroneg	r (nm)
Ni	FCC	1.9	0.1246
Cu	FCC	1.8	0.1278

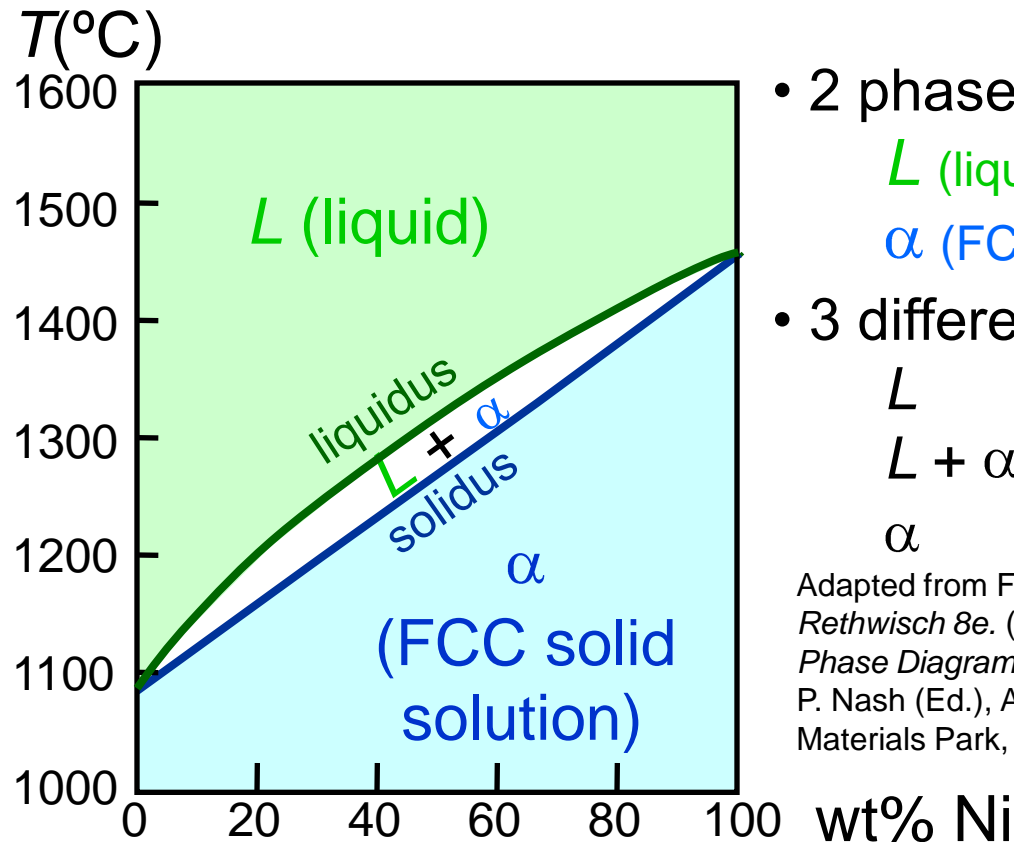
- Both have the same crystal structure (FCC) and have similar electronegativities and atomic radii ([W. Hume – Rothery rules](#)) suggesting high mutual solubility.
- Ni and Cu are totally soluble in one another for all proportions.



Phase Diagrams

- Indicate phases as a function of T , C , and P .
- For this course:
 - binary systems: just 2 components.
 - independent variables: T and C ($P = 1$ atm is almost always used).

Phase
Diagram
for Cu-Ni
system



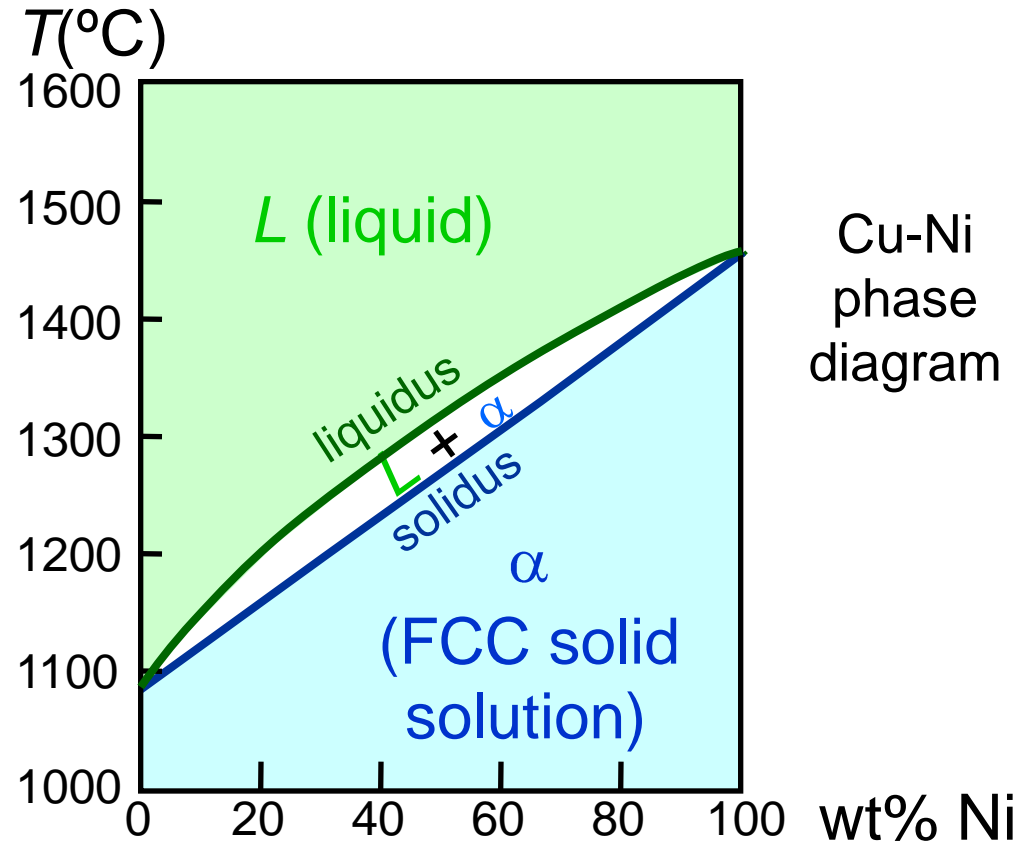
- 2 phases:
 - L (liquid)
 - α (FCC solid solution)
- 3 different phase fields:
 - L
 - $L + \alpha$
 - α

Adapted from Fig. 9.3(a), Callister & Rethwisch 8e. (Fig. 9.3(a) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH (1991).



Isomorphous Binary Phase Diagram

- Phase diagram:
Cu-Ni system.
- System is:
 - binary
i.e., 2 components:
Cu and Ni.
 - isomorphous
i.e., complete solubility of one component in another; α phase field extends from 0 to 100 wt% Ni.



Adapted from Fig. 9.3(a), *Callister & Rethwisch 8e*. (Fig. 9.3(a) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH (1991).



Phase Diagrams:

Determination of phase(s) present

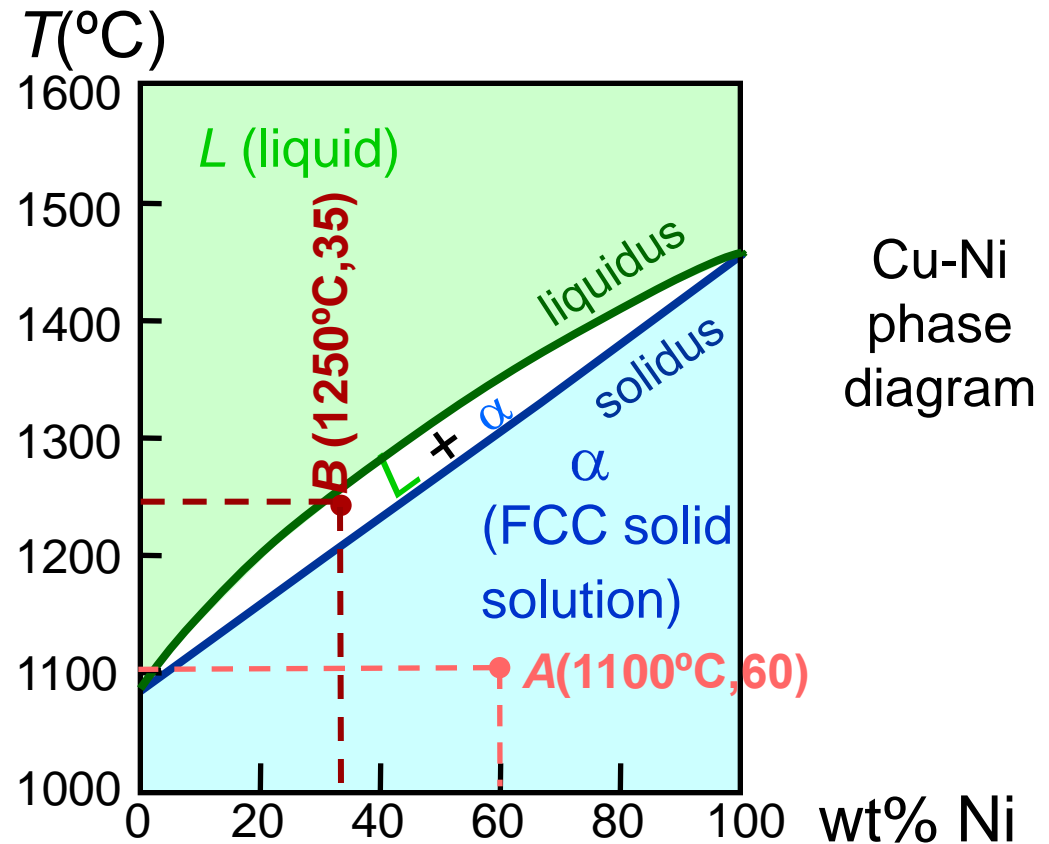
- Rule 1: If we know T and C_0 , then we know:
 - which phase(s) is (are) present.

- Examples:

$A(1100^{\circ}\text{C}, 60 \text{ wt\% Ni})$:
1 phase: α

$B(1250^{\circ}\text{C}, 35 \text{ wt\% Ni})$:
2 phases: $L + \alpha$

Adapted from Fig. 9.3(a), *Callister & Rethwisch 8e*. (Fig. 9.3(a) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH (1991).



Phase Diagrams:

Determination of phase compositions

- Rule 2: If we know T and C_0 , then we can determine:
 - the composition of each phase.

- Examples:

Consider $C_0 = 35$ wt% Ni

At $T_A = 1320^\circ\text{C}$:

Only Liquid (L) present

$C_L = C_0$ (= 35 wt% Ni)

At $T_D = 1190^\circ\text{C}$:

Only Solid (α) present

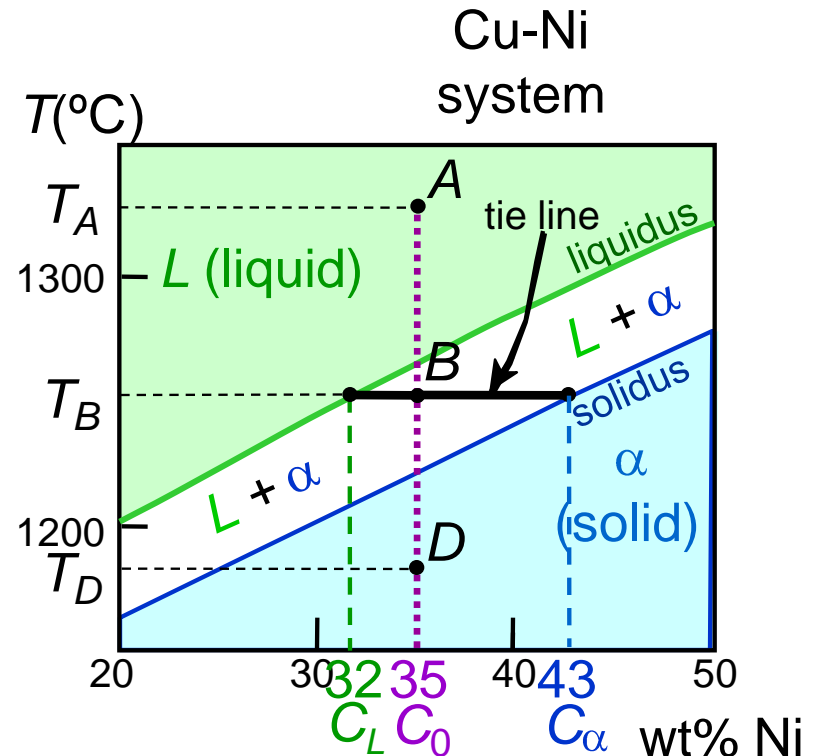
$C_\alpha = C_0$ (= 35 wt% Ni)

At $T_B = 1250^\circ\text{C}$:

Both α and L present

$C_L = C_{\text{liquidus}}$ (= 32 wt% Ni)

$C_\alpha = C_{\text{solidus}}$ (= 43 wt% Ni)



Adapted from Fig. 9.3(a), Callister & Rethwisch 8e. (Fig. 9.3(a) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH (1991).



Phase Diagrams:

Determination of phase weight fractions

- Rule 3: If we know T and C_0 , then can determine:
 - the weight fraction of each phase.

- Examples:

Consider $C_0 = 35 \text{ wt\% Ni}$

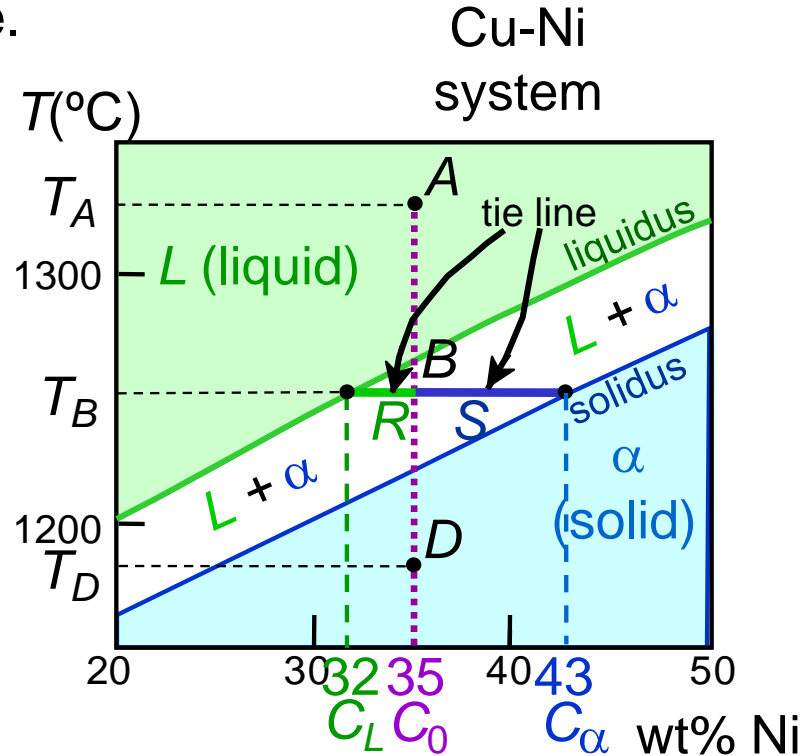
At T_A : Only Liquid (L) present

$$W_L = 1.00, W_\alpha = 0$$

At T_D : Only Solid (α) present

$$W_L = 0, W_\alpha = 1.00$$

At T_B : Both α and L present



$$W_L = \frac{S}{R+S} = \frac{43 - 35}{43 - 32} = 0.73$$

$$W_\alpha = \frac{R}{R+S} = 0.27$$

$$W_L = \frac{S}{R+S} = \frac{C_\alpha - C_0}{C_\alpha - C_L} \quad W_\alpha = \frac{R}{R+S} = \frac{C_0 - C_L}{C_\alpha - C_L}$$

Lever Rule

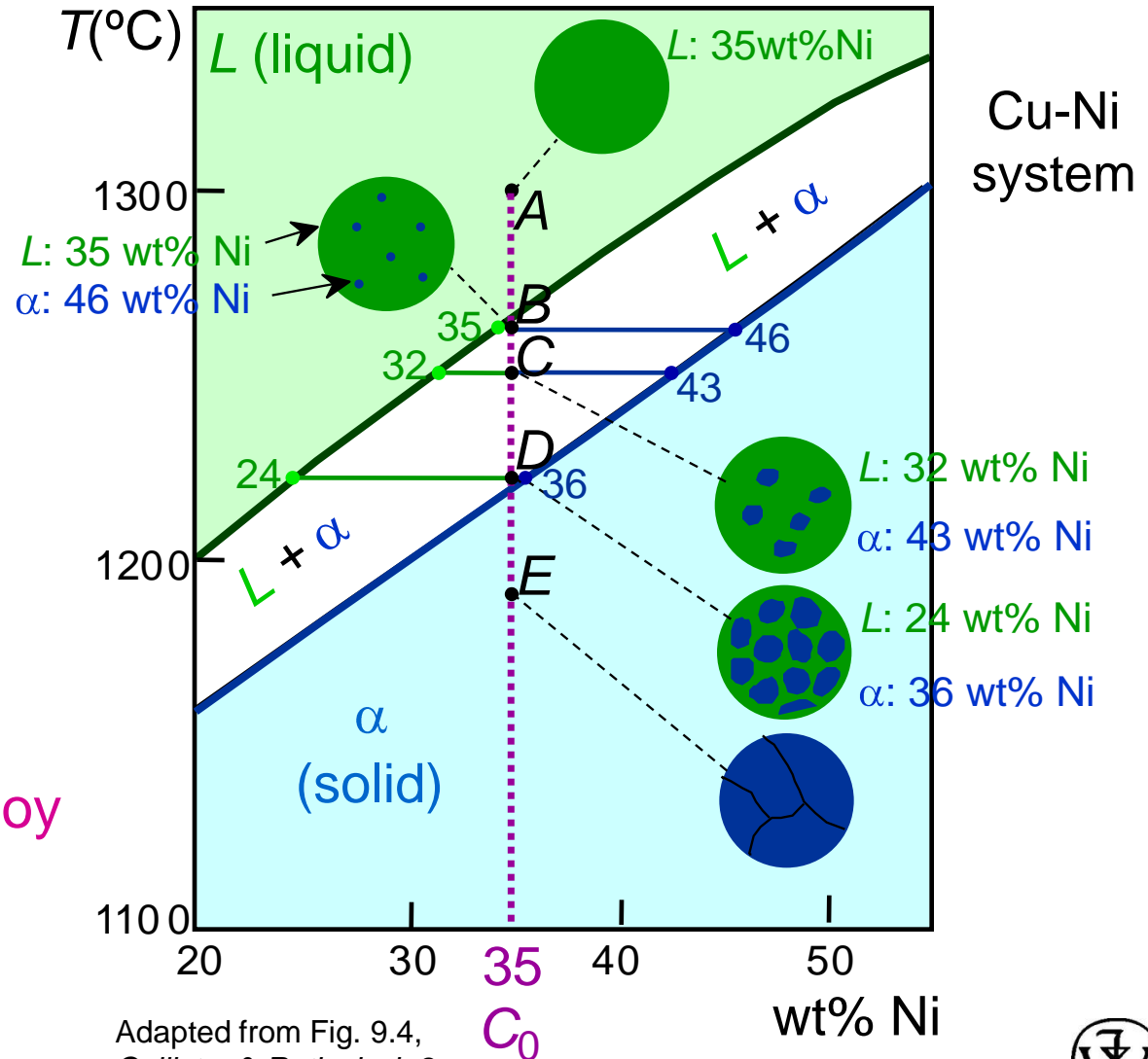


Development of Microstructure

Ex: Cooling of a Cu-Ni Alloy

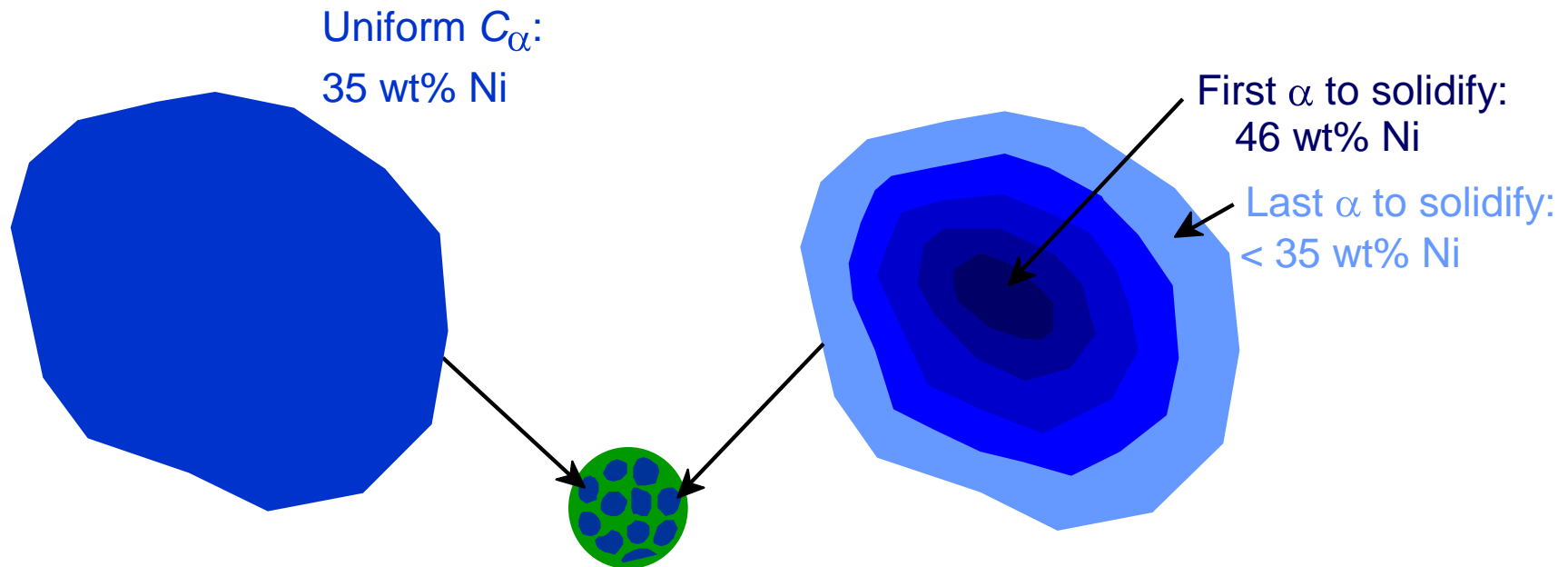
- Phase diagram: Cu-Ni system.

- Consider microstructural changes that accompany the cooling of a $C_0 = 35 \text{ wt\% Ni}$ alloy



Cored vs Equilibrium Structures

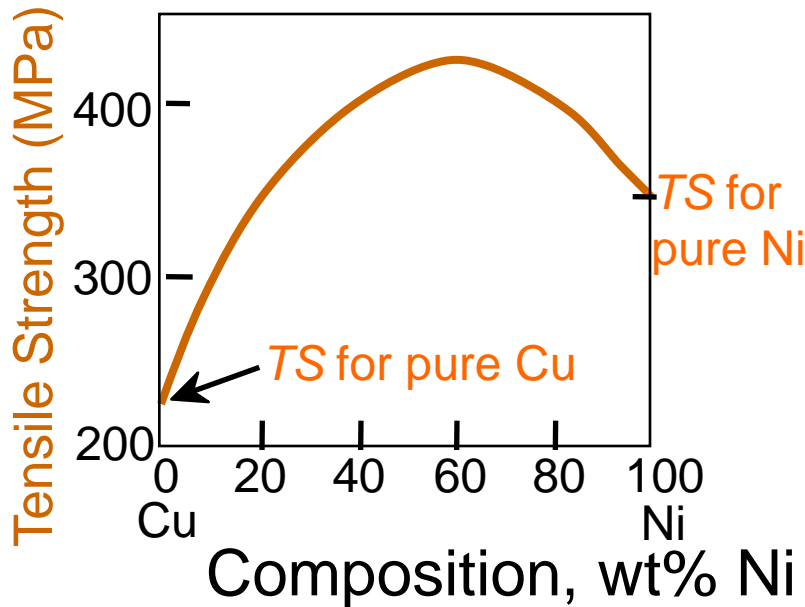
- C_{α} changes as we solidify.
- Cu-Ni case: First α to solidify has $C_{\alpha} = 46$ wt% Ni.
Last α to solidify has $C_{\alpha} = 35$ wt% Ni.
- Slow rate of cooling:
Equilibrium structure
- Fast rate of cooling:
Cored structure



Mechanical Properties: Cu-Ni System

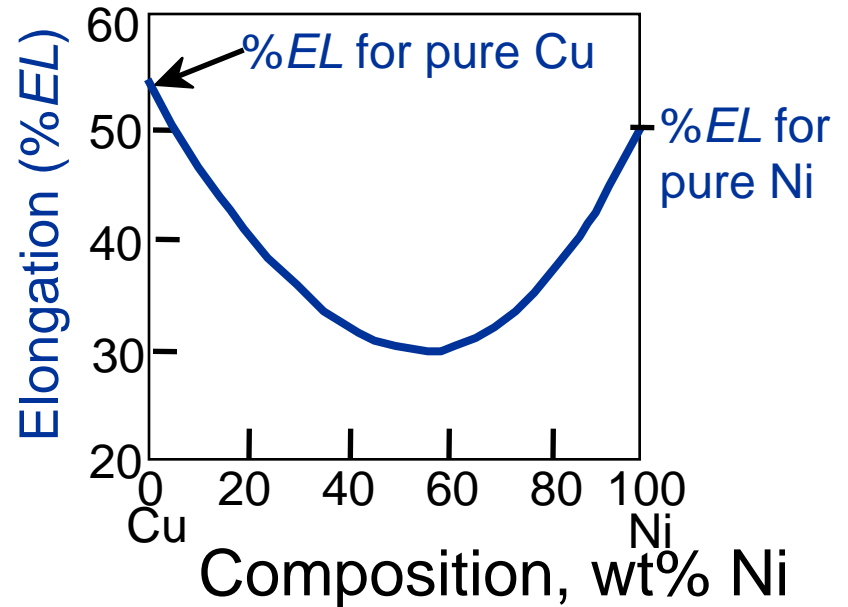
- Effect of solid solution strengthening on:

-- Tensile strength (*TS*)



Adapted from Fig. 9.6(a),
Callister & Rethwisch 8e.

-- Ductility (%*EL*)



Adapted from Fig. 9.6(b),
Callister & Rethwisch 8e.



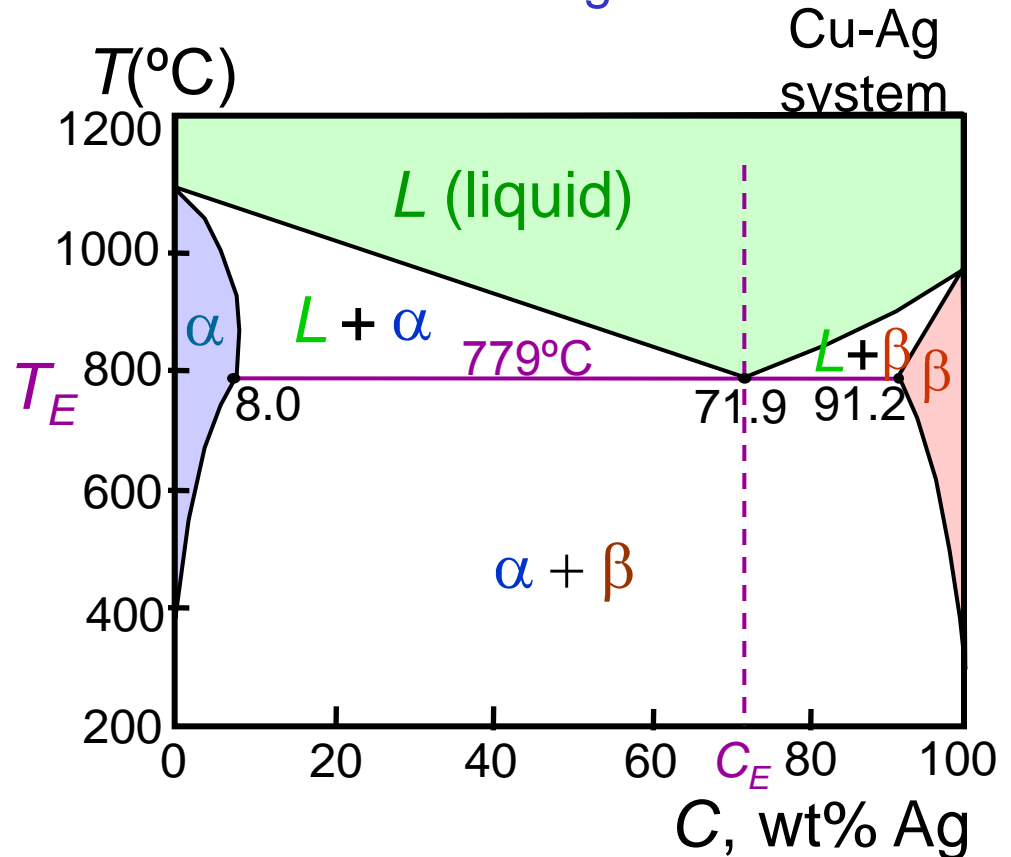
Binary-Eutectic Systems

2 components

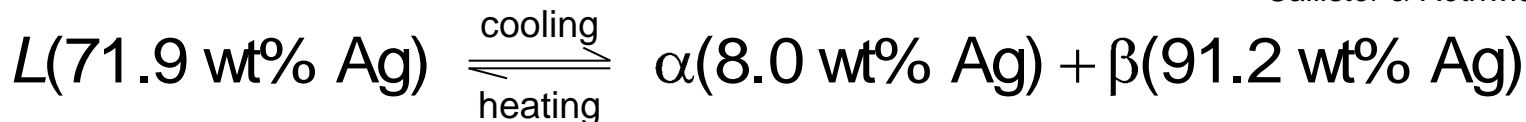
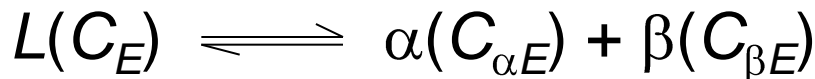
has a special composition with a min. melting T .

Ex.: Cu-Ag system

- 3 single phase regions (L , α , β)
- Limited solubility:
 - α : mostly Cu
 - β : mostly Ag
- T_E : No liquid below T_E
- C_E : Composition at temperature T_E



Eutectic reaction



Adapted from Fig. 9.7,
Callister & Rethwisch 8e.



EX 1: Pb-Sn Eutectic System

- For a 40 wt% Sn-60 wt% Pb alloy at 150°C, determine:
 - the phases present

Answer: $\alpha + \beta$

- the phase compositions

Answer: $C_\alpha = 11$ wt% Sn
 $C_\beta = 99$ wt% Sn

- the relative amount of each phase

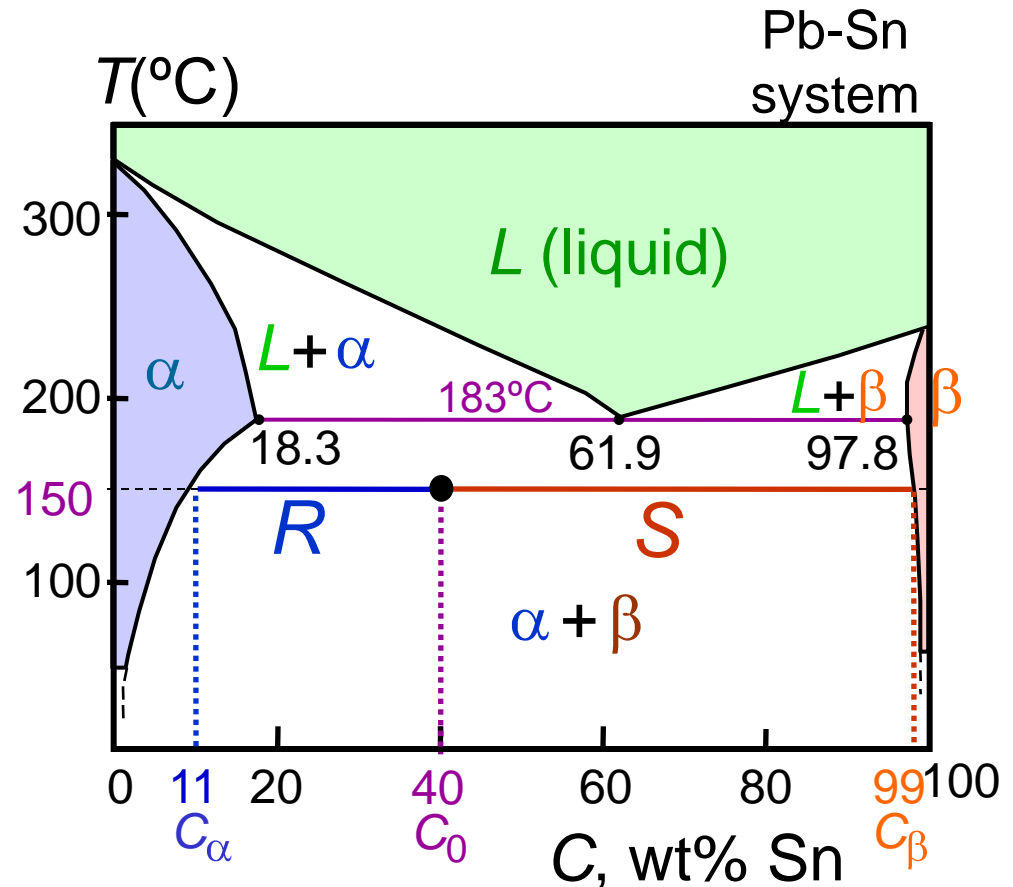
Answer:

$$W_\alpha = \frac{S}{R+S} = \frac{C_\beta - C_0}{C_\beta - C_\alpha}$$

$$= \frac{99 - 40}{99 - 11} = \frac{59}{88} = 0.67$$

$$W_\beta = \frac{R}{R+S} = \frac{C_0 - C_\alpha}{C_\beta - C_\alpha}$$

$$= \frac{40 - 11}{99 - 11} = \frac{29}{88} = 0.33$$



Adapted from Fig. 9.8,
 Callister & Rethwisch 8e.



EX 2: Pb-Sn Eutectic System

- For a 40 wt% Sn-60 wt% Pb alloy at 220°C, determine:
 - the phases present:

Answer: $\alpha + L$

- the phase compositions

Answer: $C_\alpha = 17$ wt% Sn
 $C_L = 46$ wt% Sn

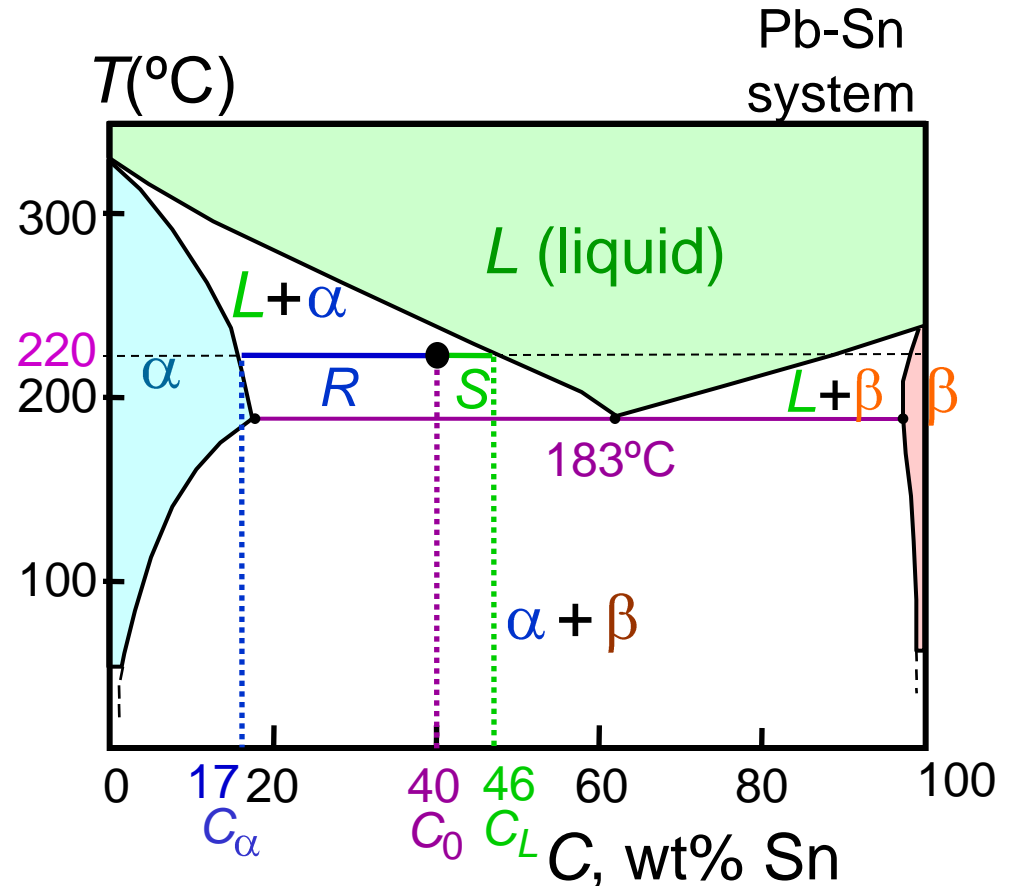
- the relative amount of each phase

Answer:

$$W_\alpha = \frac{C_L - C_0}{C_L - C_\alpha} = \frac{46 - 40}{46 - 17}$$

$$= \frac{6}{29} = 0.21$$

$$W_L = \frac{C_0 - C_\alpha}{C_L - C_\alpha} = \frac{23}{29} = 0.79$$

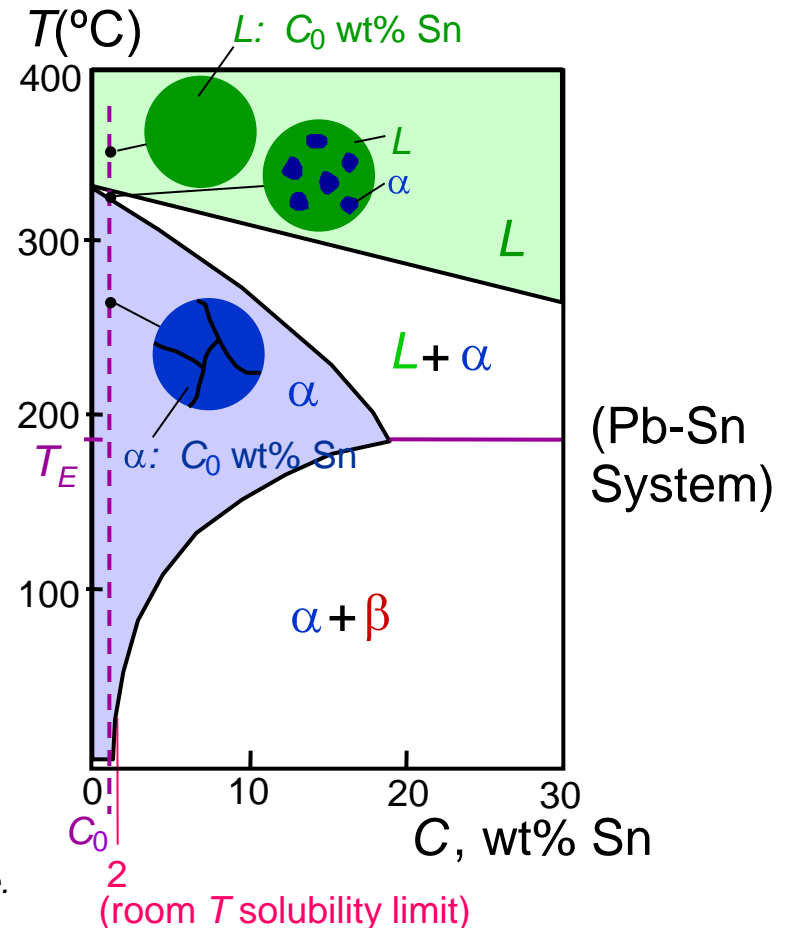


Adapted from Fig. 9.8,
 Callister & Rethwisch 8e.



Microstructural Developments in Eutectic Systems I

- For alloys for which $C_0 < 2 \text{ wt\% Sn}$
- Result: at room temperature -- polycrystalline with grains of α phase having composition C_0

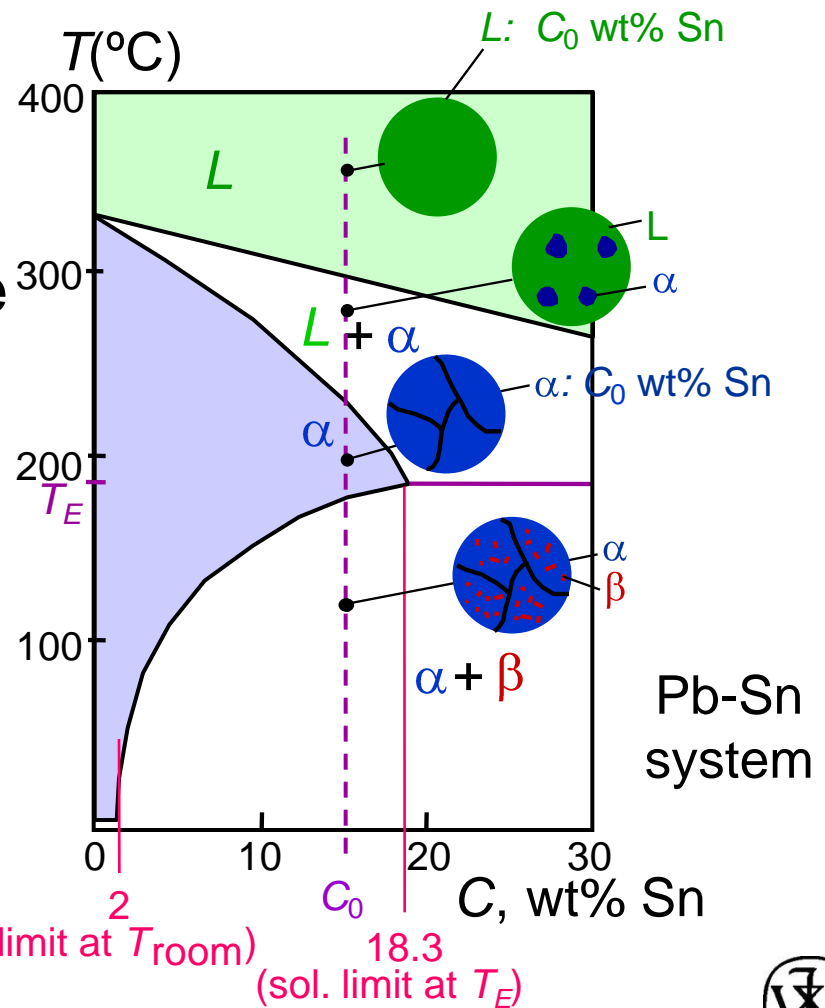


Adapted from Fig. 9.11,
Callister & Rethwisch 8e.



Microstructural Developments in Eutectic Systems II

- For alloys for which $2 \text{ wt\% Sn} < C_0 < 18.3 \text{ wt\% Sn}$
- Result: at temperatures in $\alpha + \beta$ range -- polycrystalline with α grains and small β -phase particles



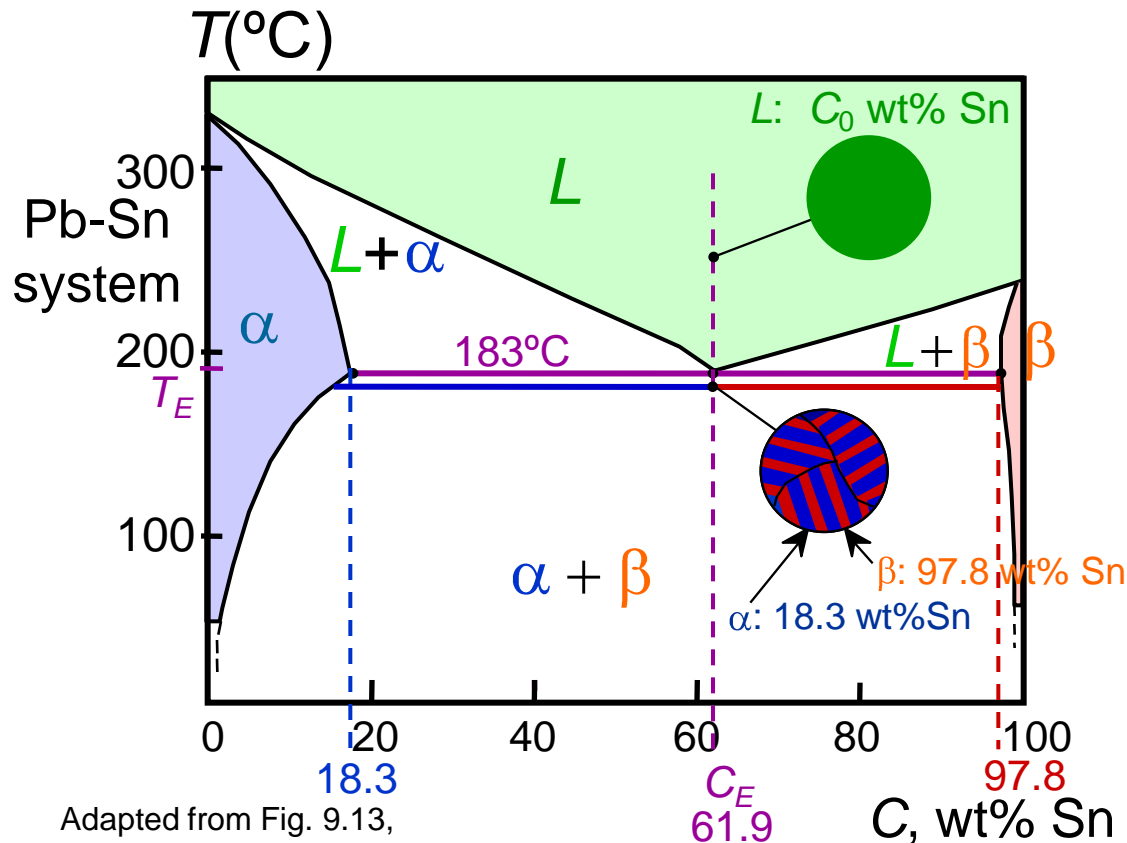
Adapted from Fig. 9.12,
Callister & Rethwisch 8e.

(sol. limit at T_{room})
 C_0 18.3
(sol. limit at T_E)



Microstructural Developments in Eutectic Systems III

- For alloy of composition $C_0 = C_E$
- Result: Eutectic microstructure (lamellar structure)
-- alternating layers (lamellae) of α and β phases.



Adapted from Fig. 9.13,
Callister & Rethwisch 8e.

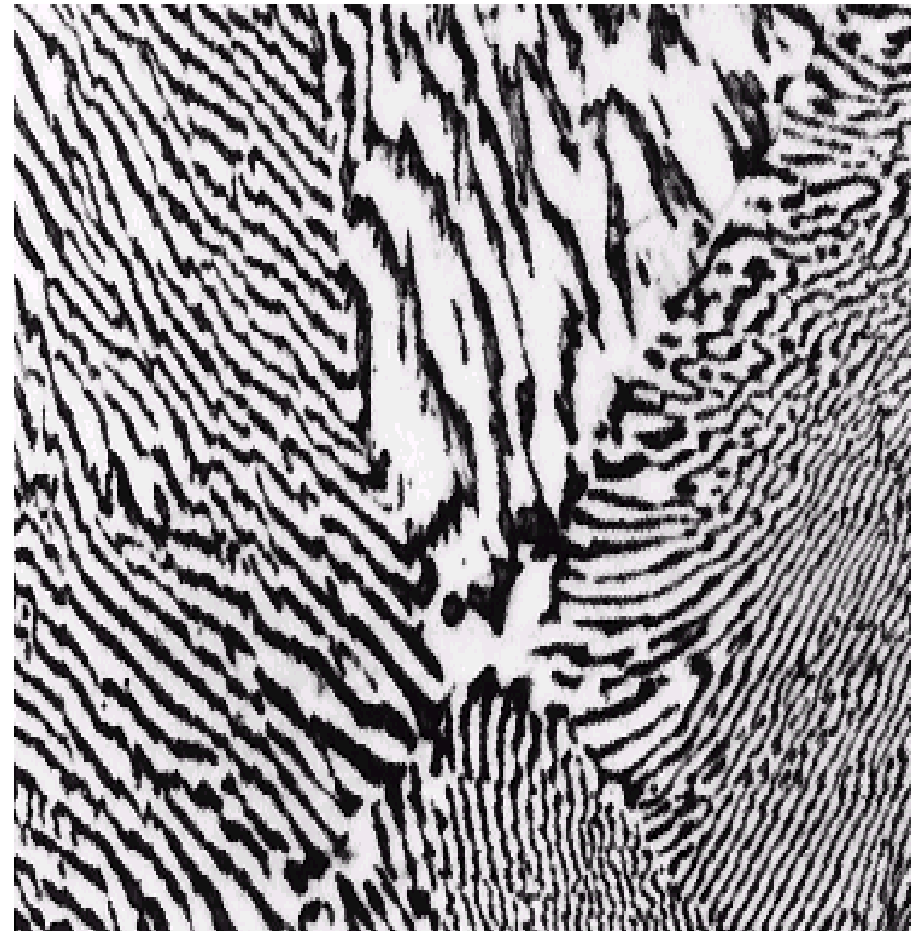
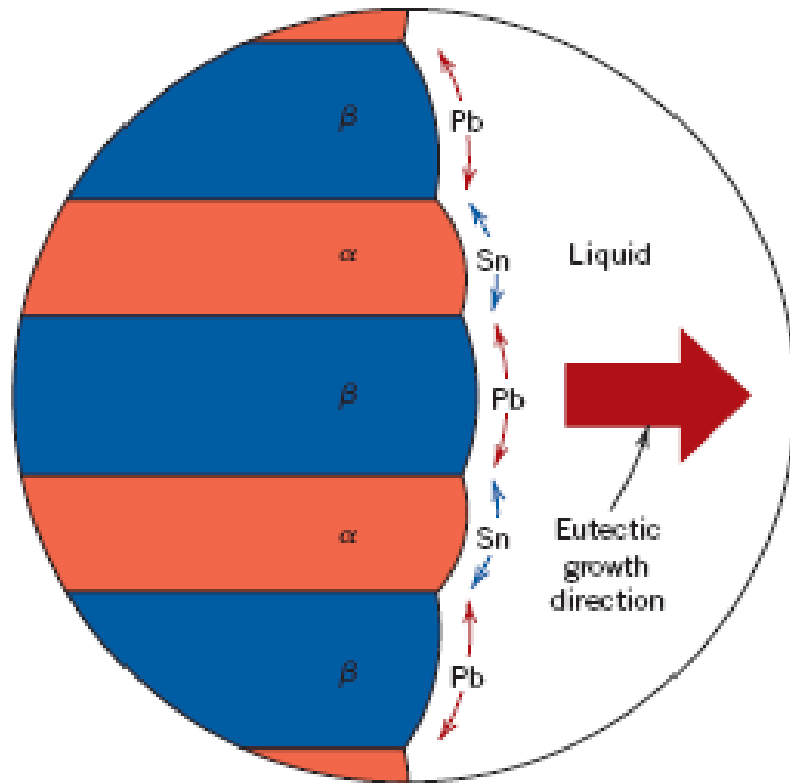
Micrograph of Pb-Sn
eutectic
microstructure



160 μm

Adapted from Fig. 9.14,
Callister & Rethwisch 8e.

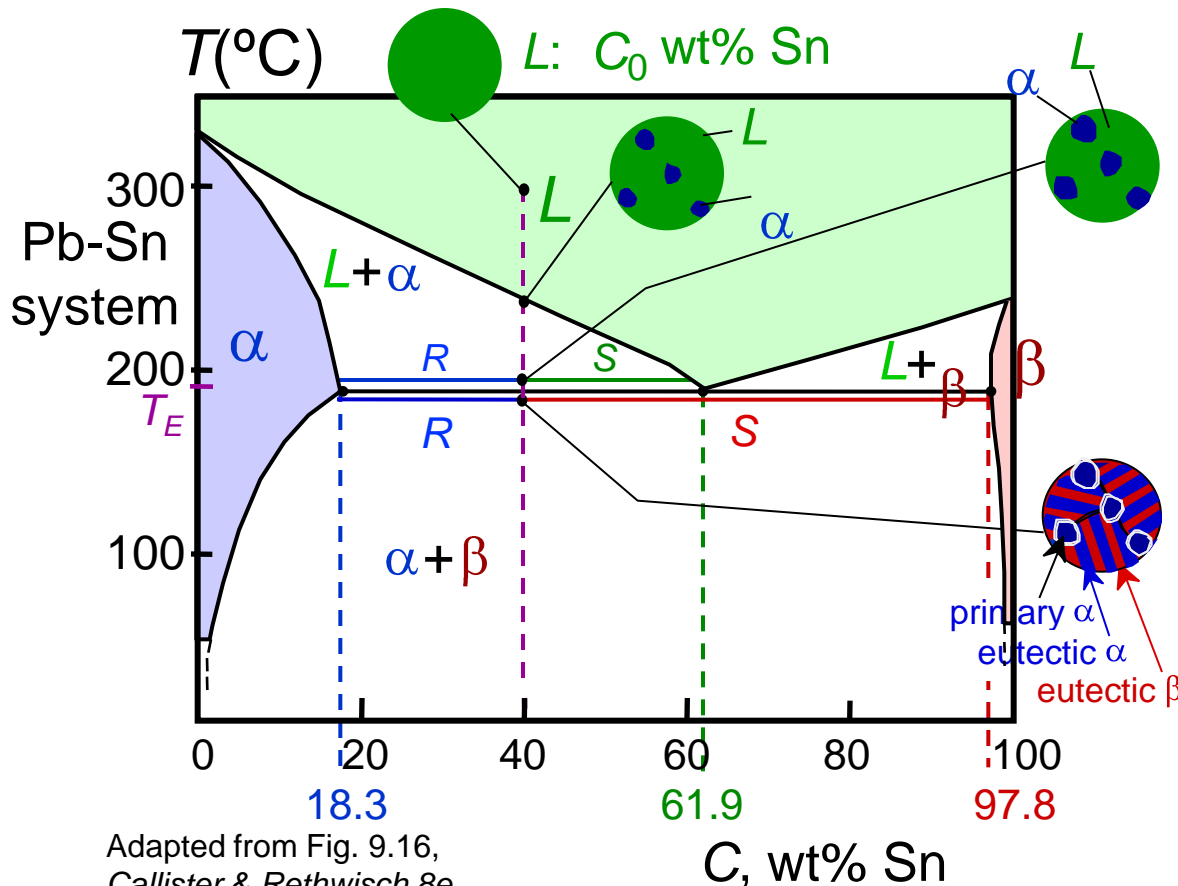
Lamellar Eutectic Structure



Adapted from Figs. 9.14 & 9.15, *Callister & Rethwisch 8e*.

Microstructural Developments in Eutectic Systems IV

- For alloys for which $18.3 \text{ wt\% Sn} < C_0 < 61.9 \text{ wt\% Sn}$
- Result: α phase particles and a eutectic microconstituent



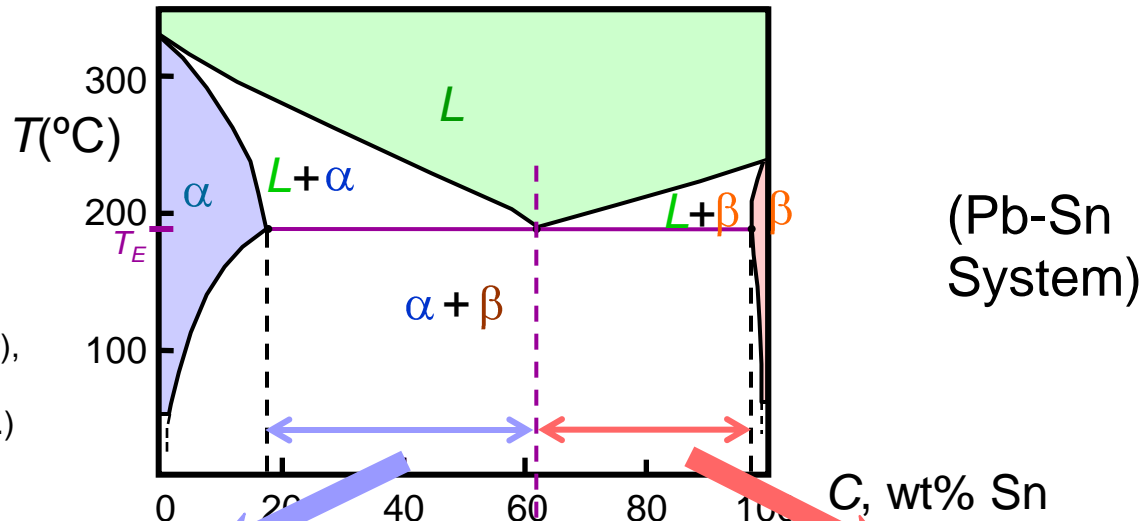
Adapted from Fig. 9.16,
Callister & Rethwisch 8e.

- Just above T_E :
 $C_{\alpha} = 18.3 \text{ wt\% Sn}$
 $C_L = 61.9 \text{ wt\% Sn}$
 $W_{\alpha} = \frac{S}{R+S} = 0.50$
 $W_L = (1 - W_{\alpha}) = 0.50$
- Just below T_E :
 $C_{\alpha} = 18.3 \text{ wt\% Sn}$
 $C_{\beta} = 97.8 \text{ wt\% Sn}$
 $W_{\alpha} = \frac{S}{R+S} = 0.73$
 $W_{\beta} = 0.27$

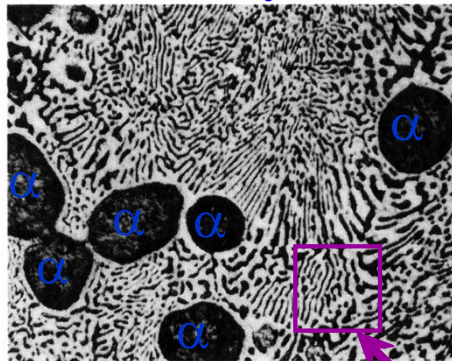


Hypoeutectic & Hypereutectic

Adapted from Fig. 9.8,
Callister & Rethwisch 8e.
 (Fig. 10.8 adapted from
Binary Phase Diagrams,
 2nd ed., Vol. 3, T.B.
 Massalski (Editor-in-Chief),
 ASM International,
 Materials Park, OH, 1990.)



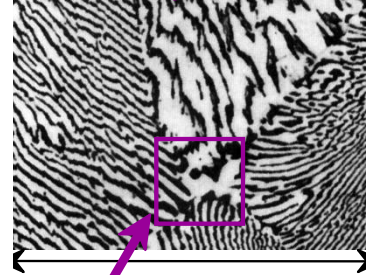
hypoeutectic: $C_0 = 50 \text{ wt\% Sn}$



Adapted from
 Fig. 9.17, *Callister & Rethwisch 8e.*

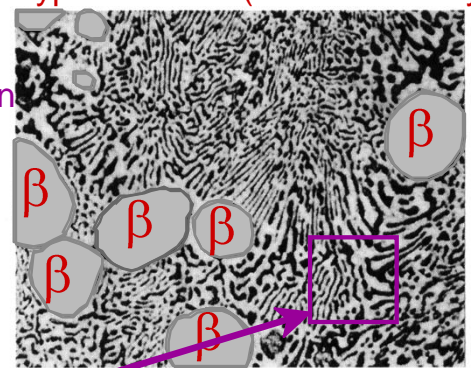
eutectic
 61.9

eutectic: $C_0 = 61.9 \text{ wt\% Sn}$



eutectic micro-constituent
 Adapted from Fig. 9.14,
Callister & Rethwisch 8e.

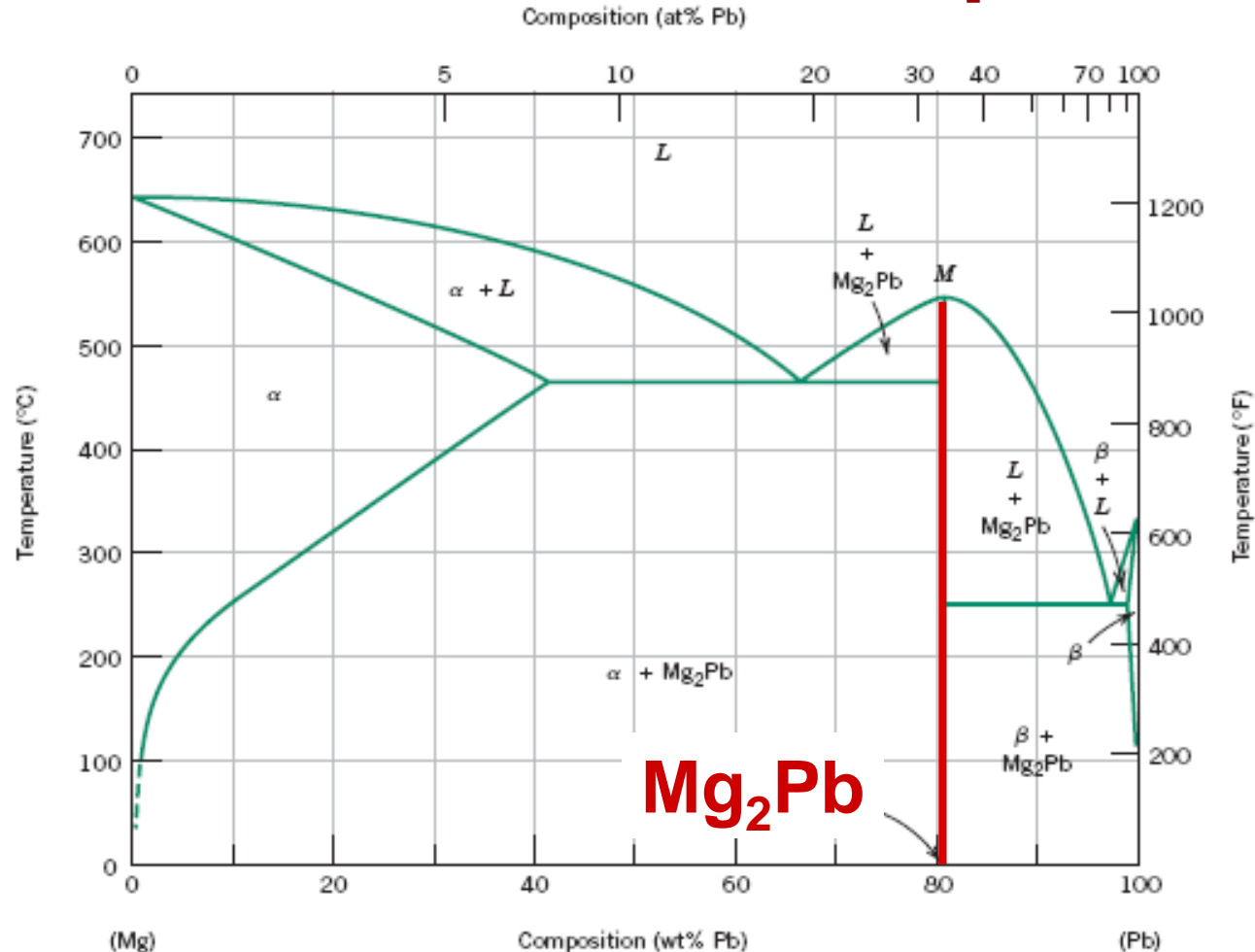
hypereutectic: (illustration only)



Adapted from Fig. 9.17,
Callister & Rethwisch 8e.
 (Illustration only)



Intermetallic Compounds



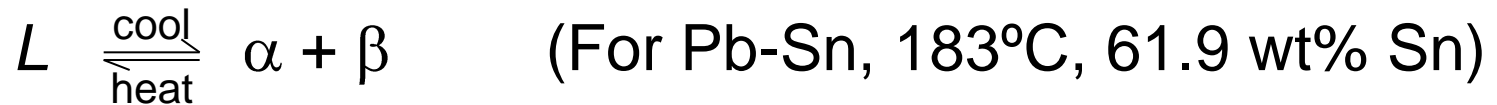
Adapted from
Fig. 9.20, Callister &
Rethwisch 8e.

Note: intermetallic compound exists as a line on the diagram - not an area - because of stoichiometry (i.e. composition of a compound is a fixed value).

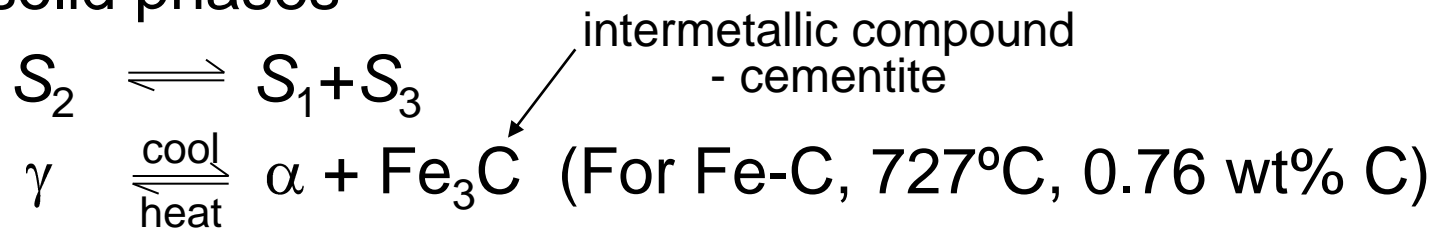


Eutectic, Eutectoid, & Peritectic

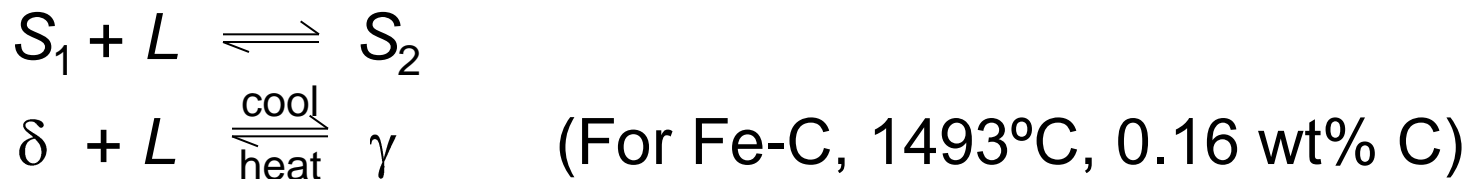
- **Eutectic** - liquid transforms to two solid phases



- **Eutectoid** – one solid phase transforms to two other solid phases



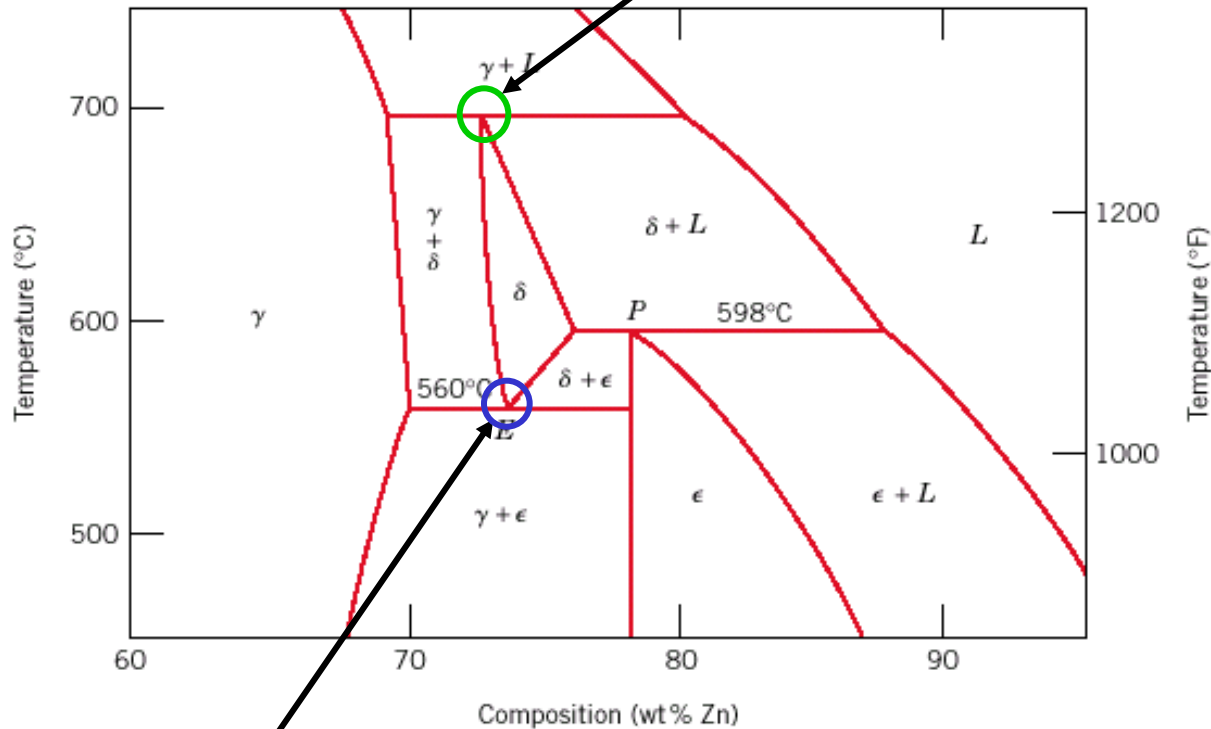
- **Peritectic** - liquid and one solid phase transform to a second solid phase



Eutectoid & Peritectic

Cu-Zn Phase diagram

Peritectic transformation $\gamma + L \rightleftharpoons \delta$



Adapted from Fig. 9.21,
Callister & Rethwisch 8e.

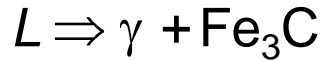
Eutectoid transformation $\delta \rightleftharpoons \gamma + \epsilon$



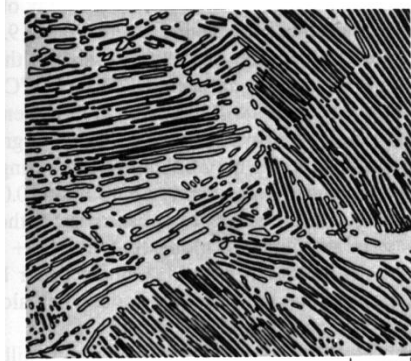
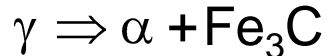
Iron-Carbon (Fe-C) Phase Diagram

- 2 important points

- Eutectic (A):

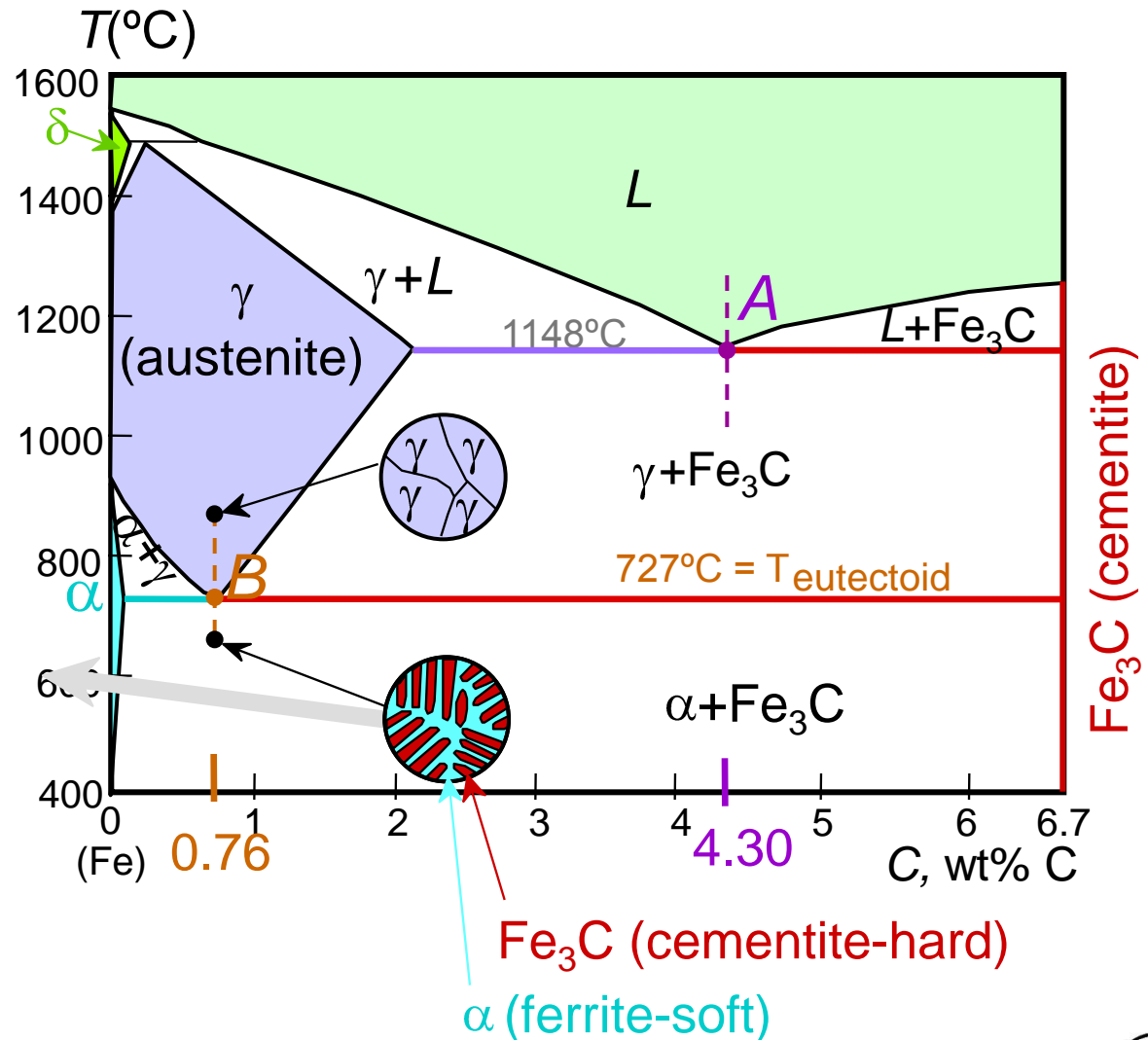


- Eutectoid (B):

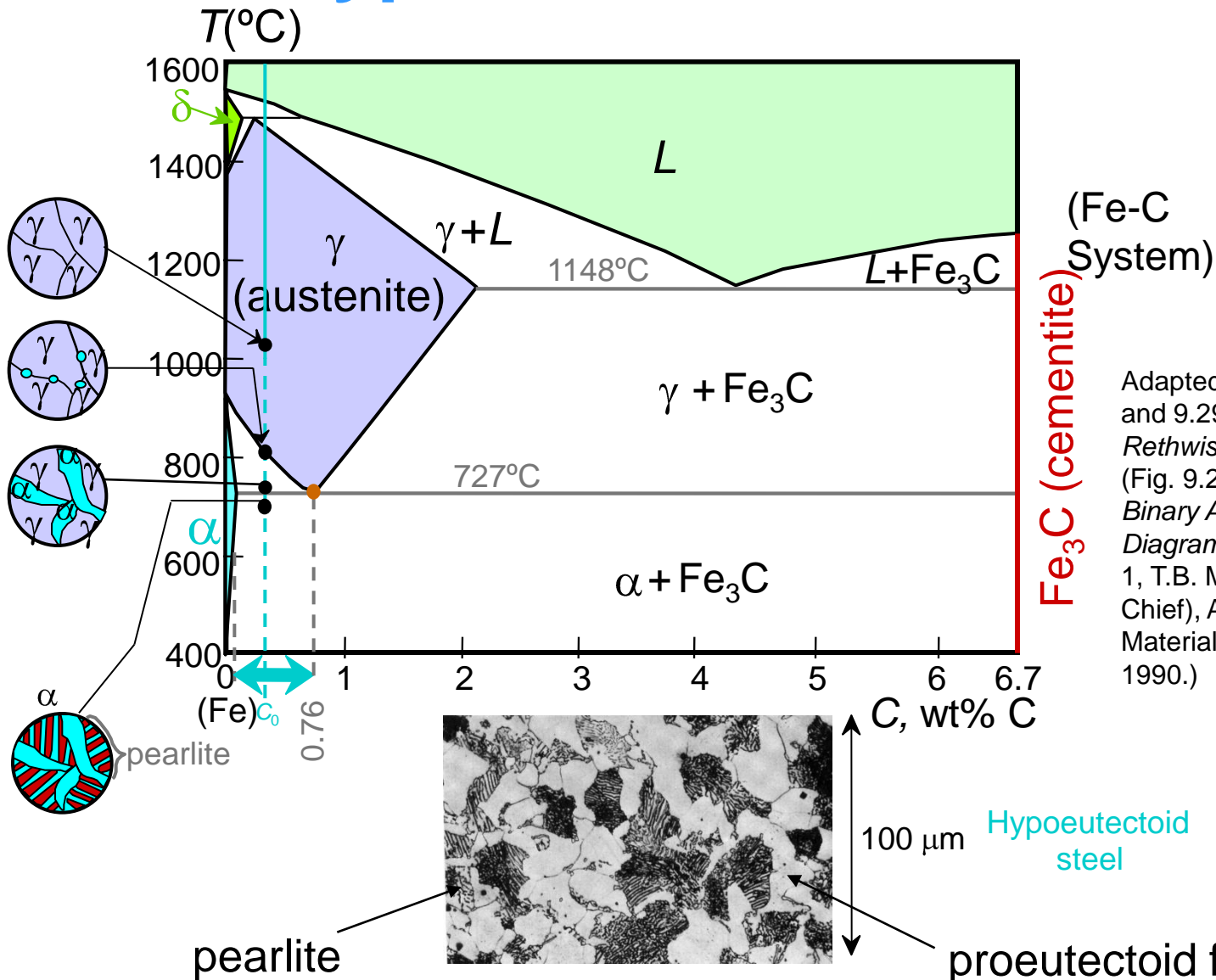


120 μm

Result: Pearlite = alternating layers of α and Fe₃C phases



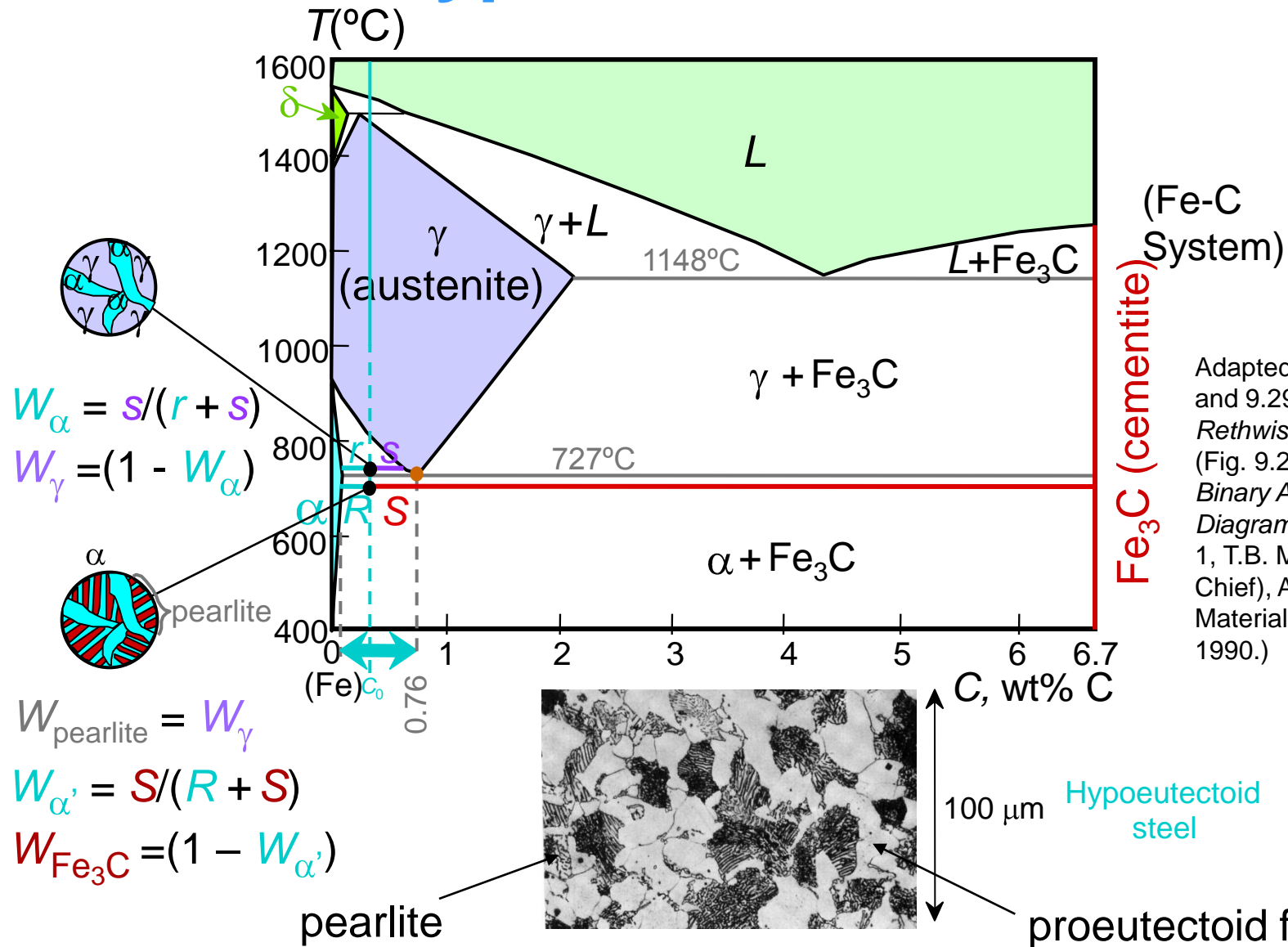
Hypoeutectoid Steel



Adapted from Fig. 9.30, *Callister & Rethwisch 8e*.



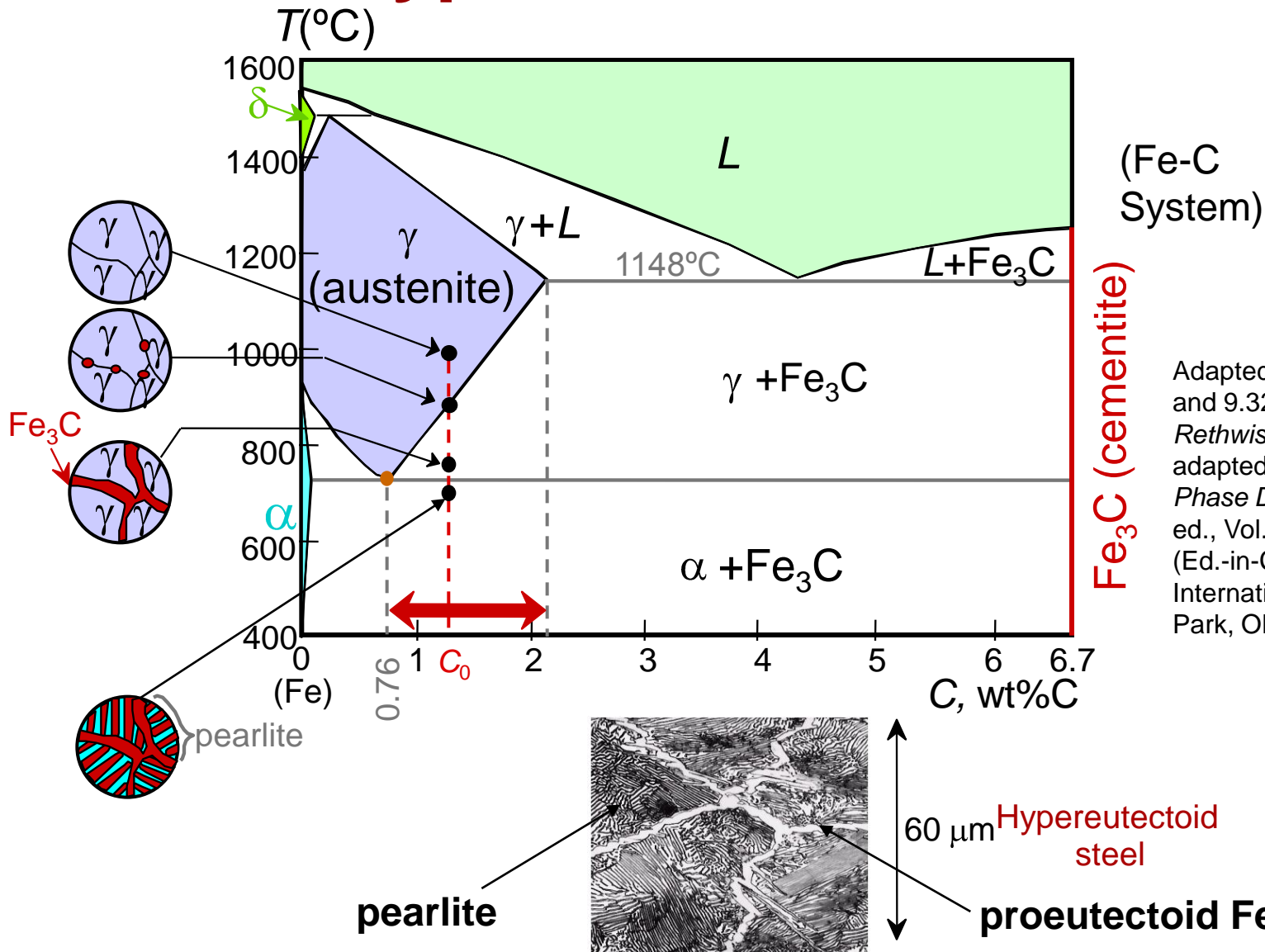
Hypoeutectoid Steel



Adapted from Fig. 9.30, Callister & Rethwisch 8e.



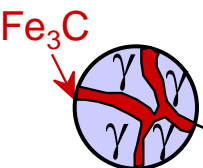
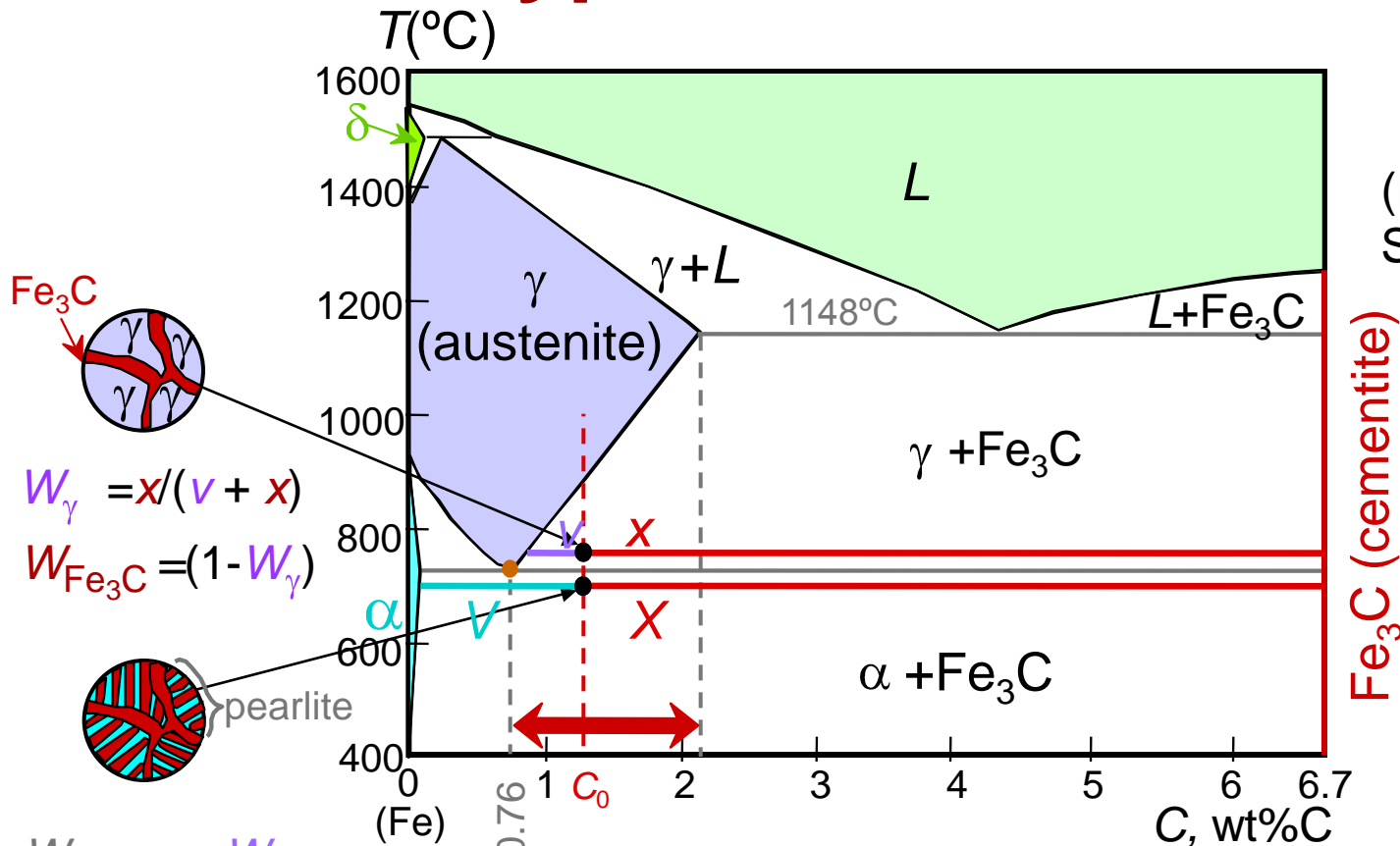
Hypereutectoid Steel



Adapted from Fig. 9.33, *Callister & Rethwisch 8e*.



Hypereutectoid Steel



$$W_{\gamma} = \frac{x}{v + x}$$

$$W_{Fe_3C} = (1 - W_{\gamma})$$



$$W_{pearlite} = W_{\gamma}$$

$$W_{\alpha} = \frac{X}{V + X}$$

$$W_{Fe_3C'} = (1 - W_{\alpha})$$



pearlite

60 μm Hypereutectoid steel

proeutectoid Fe₃C

Adapted from Fig. 9.33, Callister & Rethwisch 8e.



Example Problem

For a 99.6 wt% Fe-0.40 wt% C steel at a temperature just below the eutectoid, determine the following:

- a) The compositions of Fe_3C and ferrite (α).
- b) The amount of cementite (in grams) that forms in 100 g of steel.
- c) The amounts of pearlite and proeutectoid ferrite (α) in the 100 g.



Solution to Example Problem

a) Using the RS tie line just below the eutectoid

$$C_{\alpha} = 0.022 \text{ wt\% C}$$

$$C_{\text{Fe}_3\text{C}} = 6.70 \text{ wt\% C}$$

b) Using the lever rule with the tie line shown

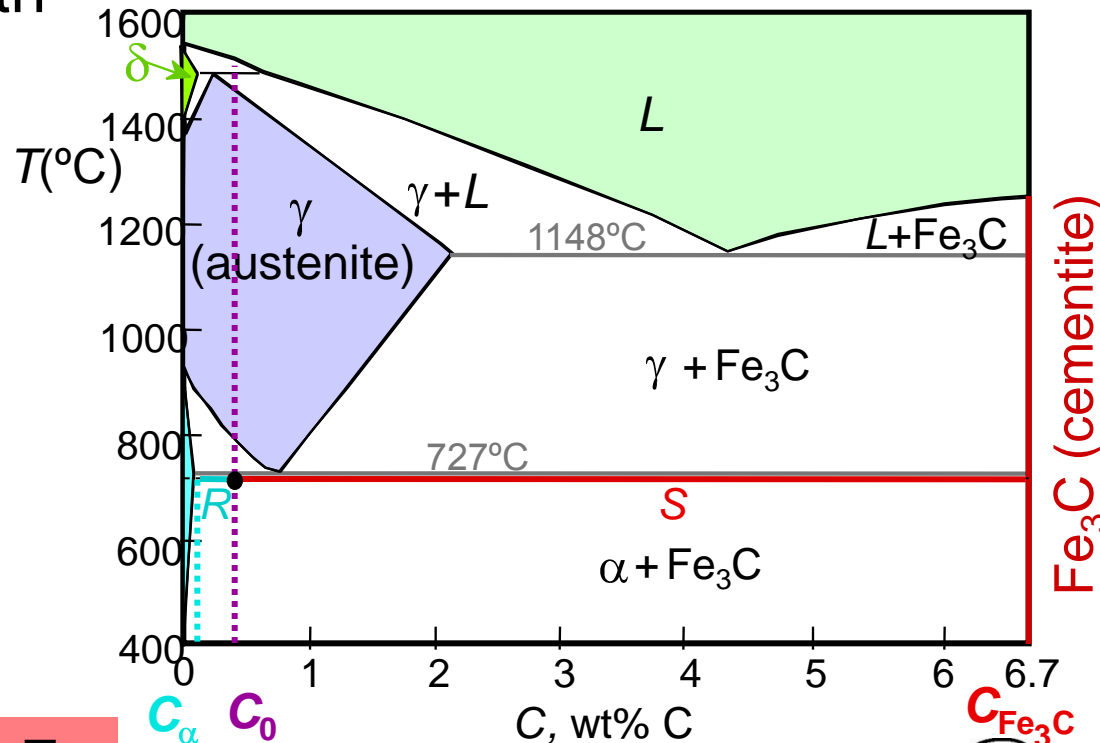
$$W_{\text{Fe}_3\text{C}} = \frac{R}{R+S} = \frac{C_0 - C_{\alpha}}{C_{\text{Fe}_3\text{C}} - C_{\alpha}}$$

$$= \frac{0.40 - 0.022}{6.70 - 0.022} = 0.057$$

Amount of Fe_3C in 100 g

$$= (100 \text{ g}) W_{\text{Fe}_3\text{C}}$$

$$= (100 \text{ g})(0.057) = \mathbf{5.7 \text{ g}}$$



Solution to Example Problem (cont.)

- c) Using the VX tie line just above the eutectoid and realizing that

$$C_0 = 0.40 \text{ wt\% C}$$

$$C_\alpha = 0.022 \text{ wt\% C}$$

$$C_{\text{pearlite}} = C_\gamma = 0.76 \text{ wt\% C}$$

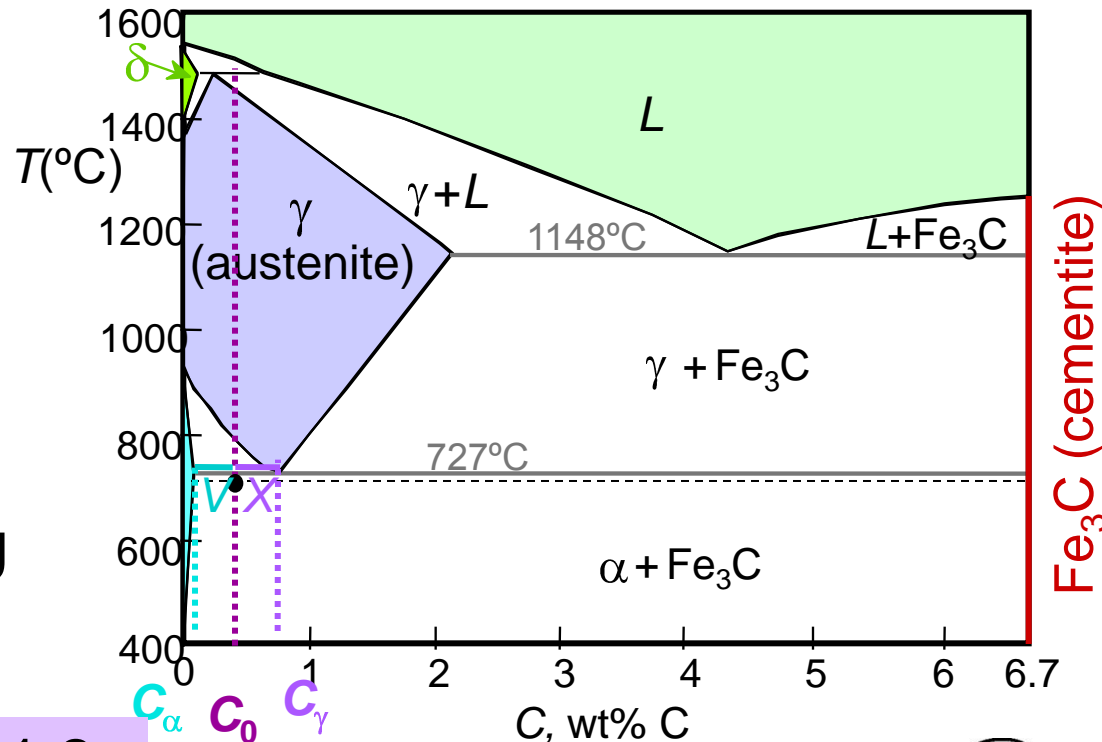
$$W_{\text{pearlite}} = \frac{V}{V+X} = \frac{C_0 - C_\alpha}{C_\gamma - C_\alpha}$$

$$= \frac{0.40 - 0.022}{0.76 - 0.022} = 0.512$$

Amount of pearlite in 100 g

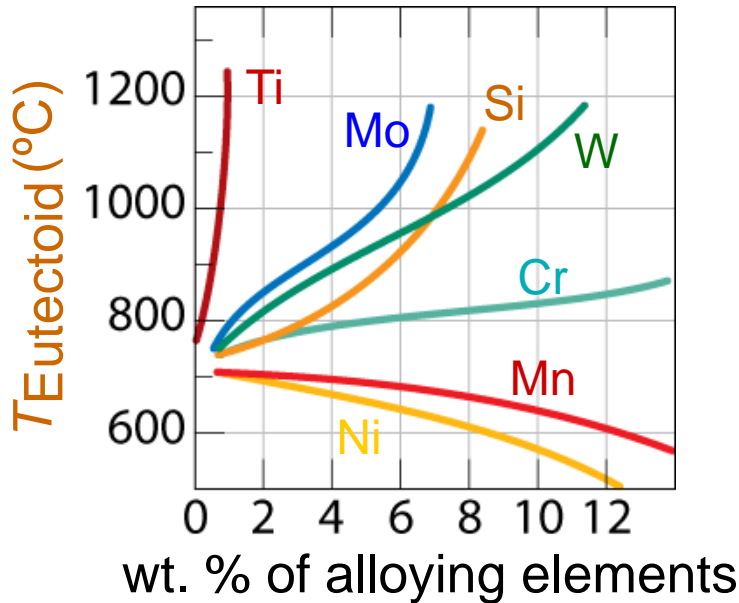
$$= (100 \text{ g}) W_{\text{pearlite}}$$

$$= (100 \text{ g})(0.512) = 51.2 \text{ g}$$



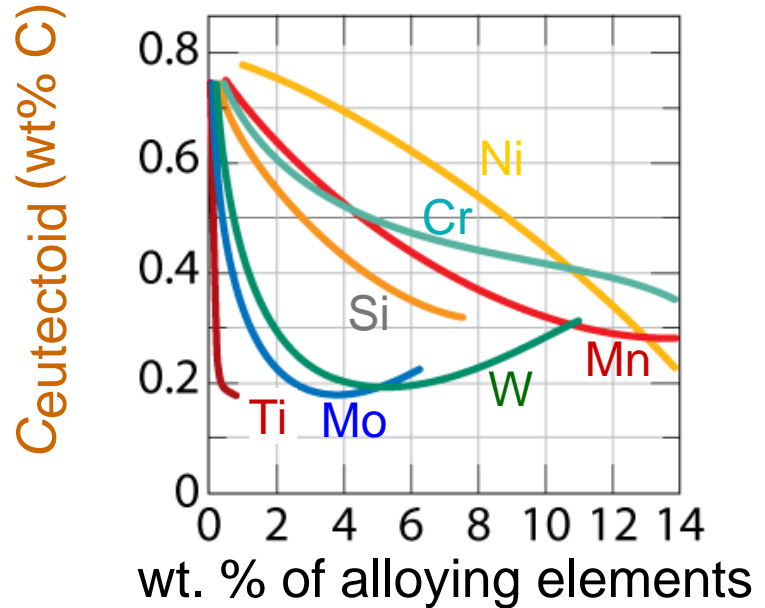
Alloying with Other Elements

- $T_{\text{Eutectoid}}$ changes:



Adapted from Fig. 9.34, Callister & Rethwisch 8e. (Fig. 9.34 from Edgar C. Bain, *Functions of the Alloying Elements in Steel*, American Society for Metals, 1939, p. 127.)

- $C_{\text{Eutectoid}}$ changes:



Adapted from Fig. 9.35, Callister & Rethwisch 8e. (Fig. 9.35 from Edgar C. Bain, *Functions of the Alloying Elements in Steel*, American Society for Metals, 1939, p. 127.)



Summary

- **Phase diagrams** are useful tools to determine:
 - the number and types of phases present,
 - the **composition** of each phase,
 - and the weight fraction of each phasegiven the temperature and composition of the system.
- The microstructure of an alloy depends on
 - its composition, and
 - whether or not cooling rate allows for maintenance of equilibrium.
- Important phase diagram phase transformations include **eutectic**, **eutectoid**, and **peritectic**.

