lecture
SIX

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mechanics of materials

Mechanics of Materials

- *MECHANICS*

- *MATERIALS*

STRENGTH OF MATERIALS
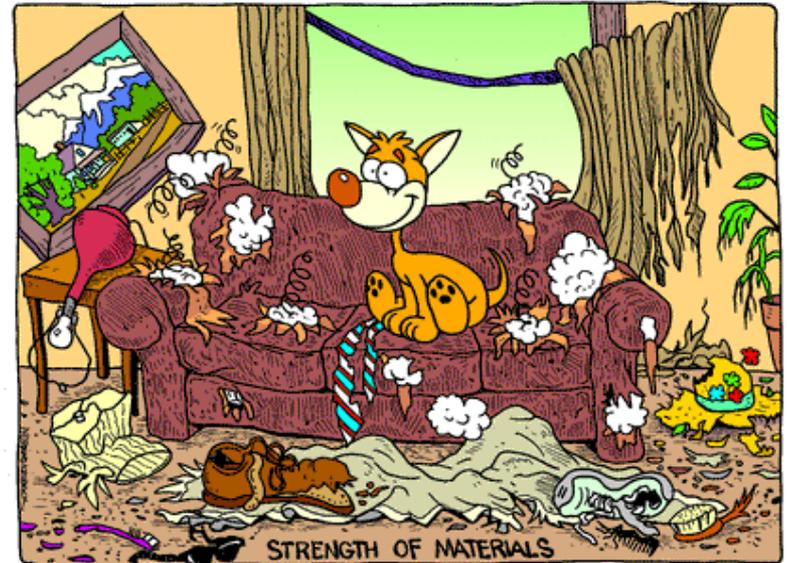
Mechanics of Materials

- *external loads and their effect on deformable bodies*

- *use it to answer question if structure meets requirements of*
  - *stability and equilibrium*
  - *strength and stiffness*

- *other principle building requirements*
  - *economy, functionality and aesthetics*

Knowledge Required

- *material properties*

- *member cross sections*

- *ability of a material to resist breaking*

- *structural elements that resist excessive*
 - *deflection*
  - *deformation*
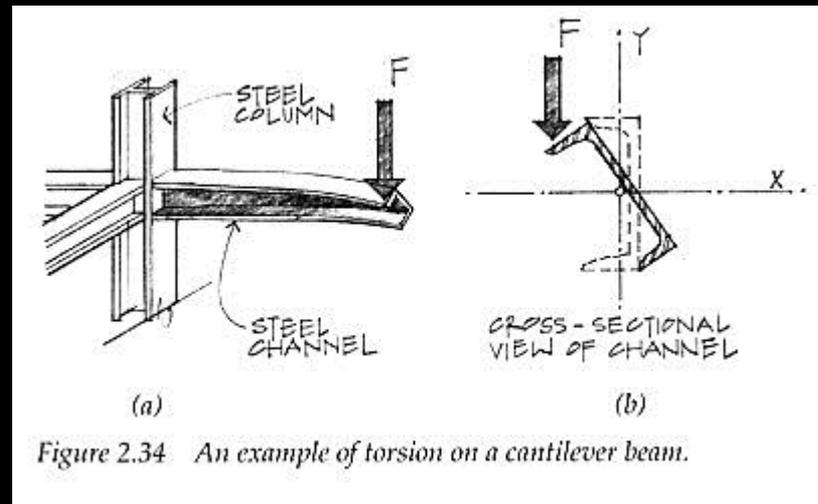
Figure 2.34   An example of torsion on a cantilever beam.

Problem Solving

1. *STATICS:*

   equilibrium of external forces, internal forces, <u>stresses</u>

2. *GEOMETRY:*

   cross section properties, deformations and conditions of geometric fit, <u>strains</u>

3. *MATERIAL PROPERTIES:*

   <u>stress-strain relationship</u> for each material obtained from testing

Stress

- *stress is a term for the <u>intensity</u> of a force, like a pressure*

- *internal <u>or</u> applied*

- *force per unit area*

$$stress = f = \frac{P}{A}$$

Design

- *materials have a critical stress value where they could break or yield*
  - *ultimate stress*
  - *yield stress*
  - *compressive stress*
  - *fatigue strength*
  - *(creep & temperature)*

 *acceptance vs. failure*

Design (cont)

- *we'd like*

$$f_{actual} << F_{allowable}$$

- *stress distribution may vary: <u>average</u>*

- *uniform distribution exists IF the member is loaded axially (concentric)*

Figure 5.3   Centric loads.
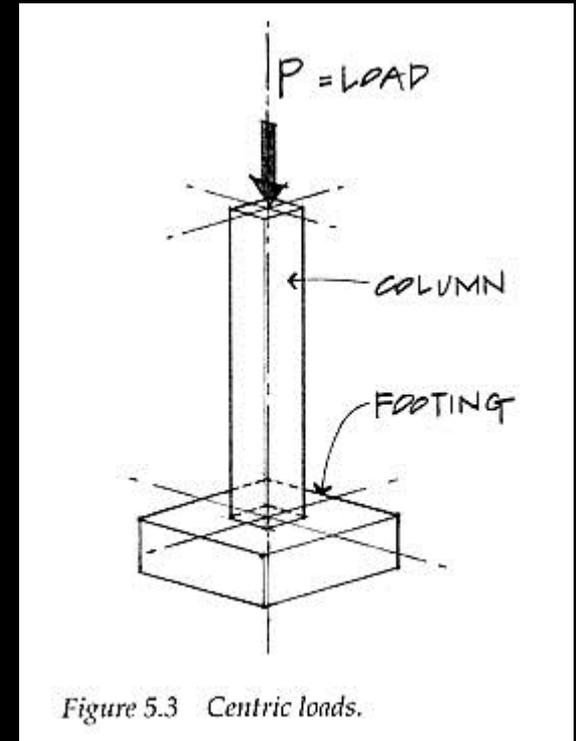
Scale Effect

- *model scale*
  - *material weights by volume, small section areas*

- *structural scale*
  - *much more material weight, bigger section areas*

- *scale for strength is not proportional:*

$$\frac{\gamma L^3}{L^2} = \gamma L$$
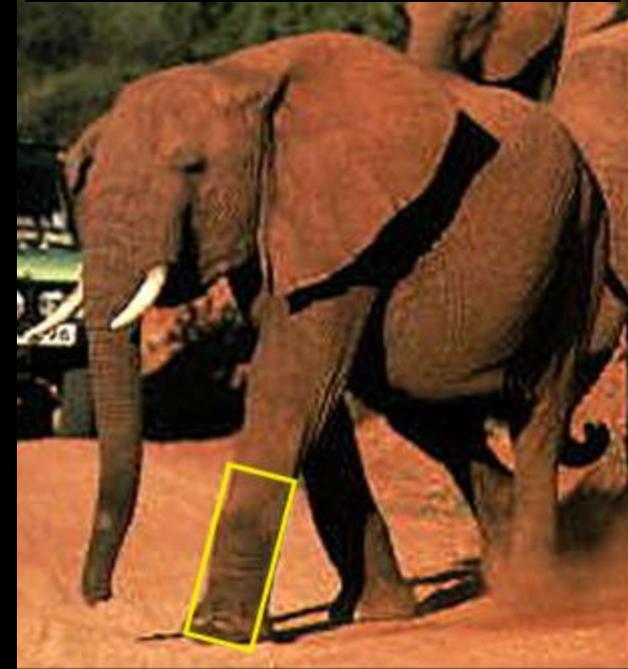
Normal Stress (direct)

- *normal stress is normal to the cross section*
 - *stressed area is perpendicular to the load*

$$f_{t\,or\,c} = \frac{P}{A}$$
$$(\sigma)$$
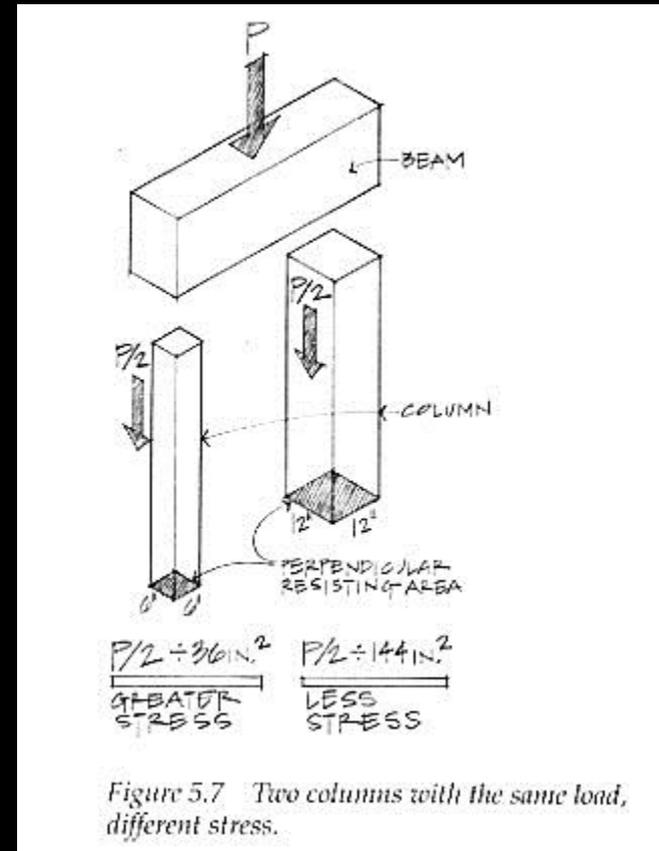
Figure 5.7 Two columns with the same load, different stress.

Shear Stress

- *stress parallel to a surface*

$$f_v \atop (\tau_{ave})$$ $$= \frac{P}{A} = \frac{P}{td}$$

Figure 5.10   Shear stress between two glued blocks.
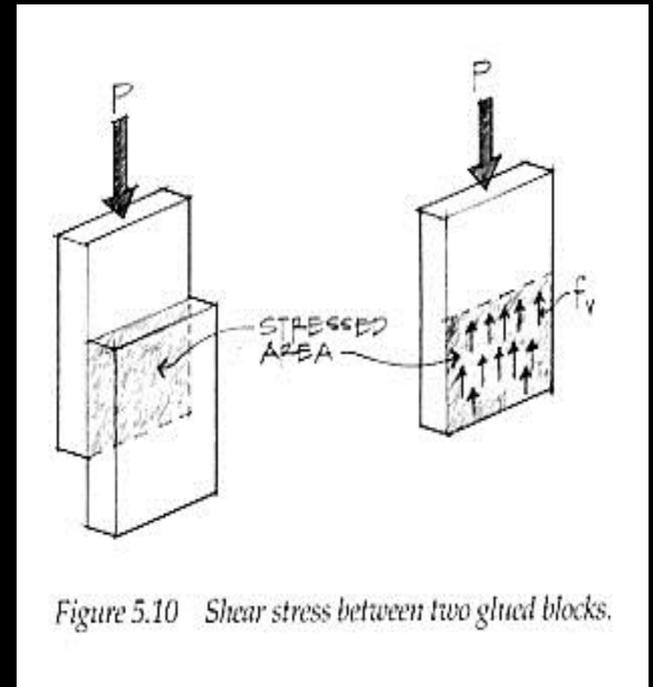
Bearing Stress

- *stress on a surface by <u>contact</u> in compression*

$$f_p \atop (\sigma) = \frac{P}{A} = \frac{P}{td}$$

Figure 5.3 *Centric loads.*
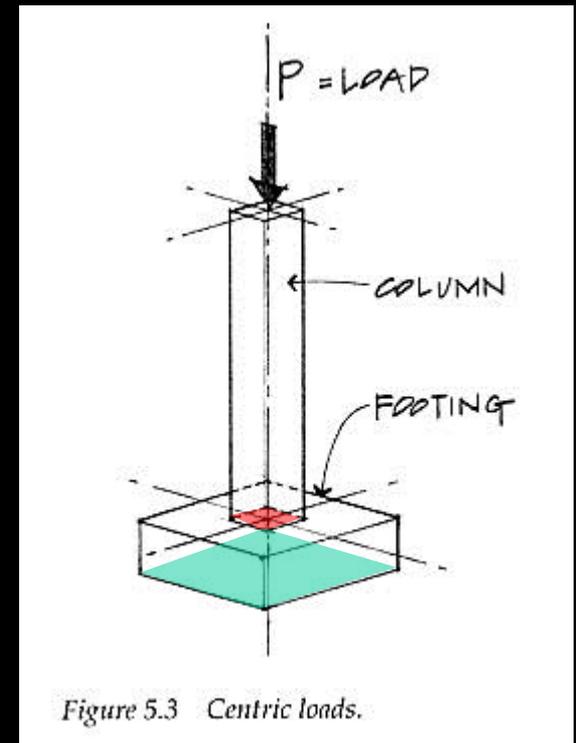
Bending Stress

- *normal stress caused by bending*

$$f_b = \frac{Mc}{I} = \frac{M}{S}$$
$$(\sigma)$$

Figure 8.8   Bending stresses on section b-b.
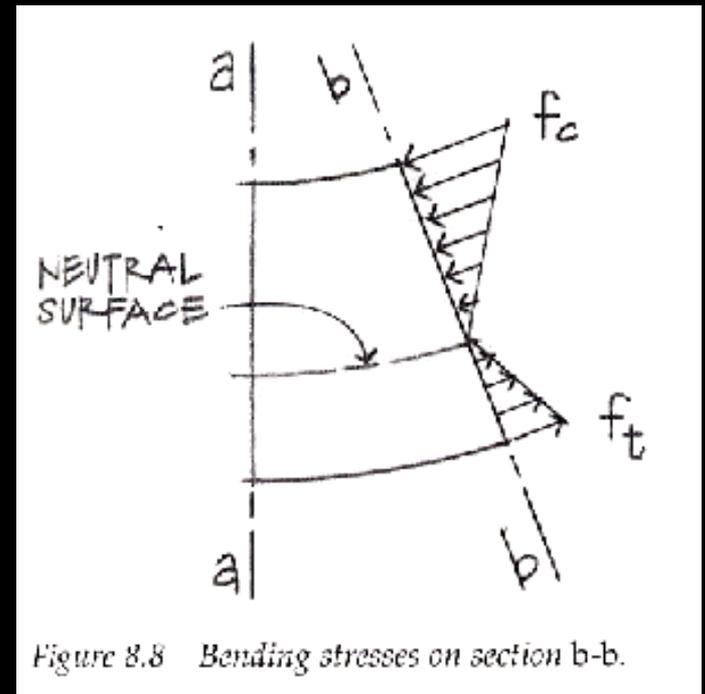
Torsional Stress

- *shear stress caused by twisting*

$$f_v = \frac{T\rho}{J}$$
$$(\tau)$$
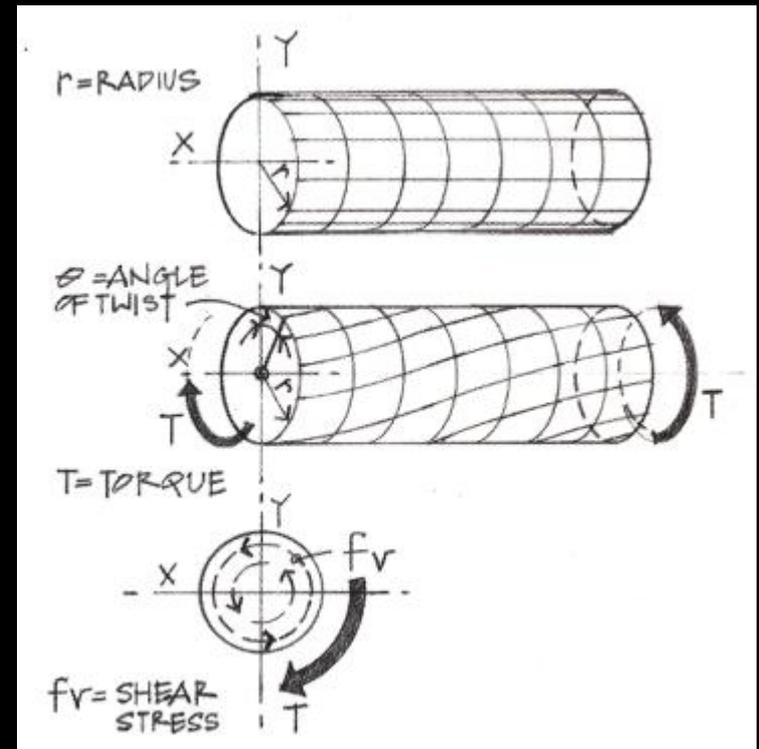
Structures and Shear

- *what structural elements see shear?*
  - *beams* ⎫
  - *bolts* ⎬ *connections*
  - *splices* ⎭
 - *slabs*
  - *footings*
  - *walls*
    - *wind*
    - *seismic loads*
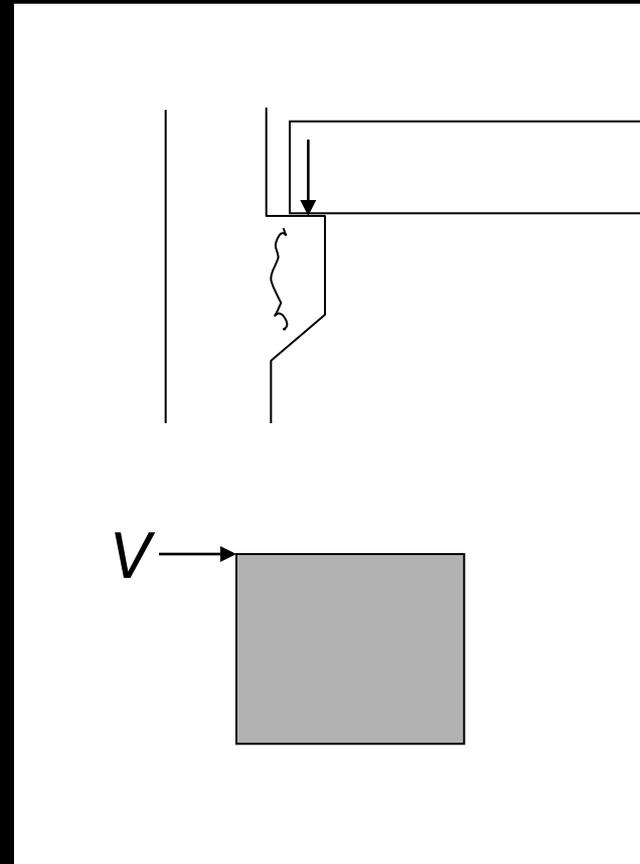
Bolts

- *connected members in tension cause shear stress*
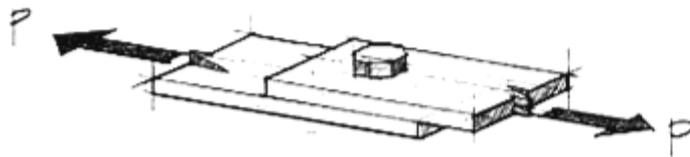

(a) Two steel plates bolted using one bolt.
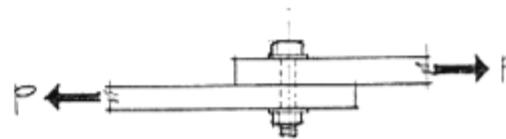
(b) Elevation showing the bolt in

- *connected members in compression cause bearing stress*

CENTER PLATE
BEARING STRESS
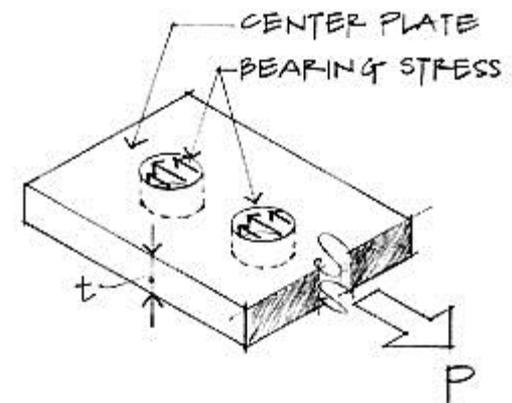
Bearing stress on plate.

Single Shear

- *seen when 2 members are connected*

(a) Two steel plates bolted using one bolt.

(b) Elevation showing the bolt in shear.

ONE SURFACE IN SHEAR

(c)

(d)

f_v = Average shear stress through bolt cross section

A = Bolt cross-sectional area

$$f_v = \frac{P}{A}$$

$$f_v = \frac{P}{A} = \frac{P}{\pi \, d^2/4}$$

Figure 5.11   A bolted connection—single shear.
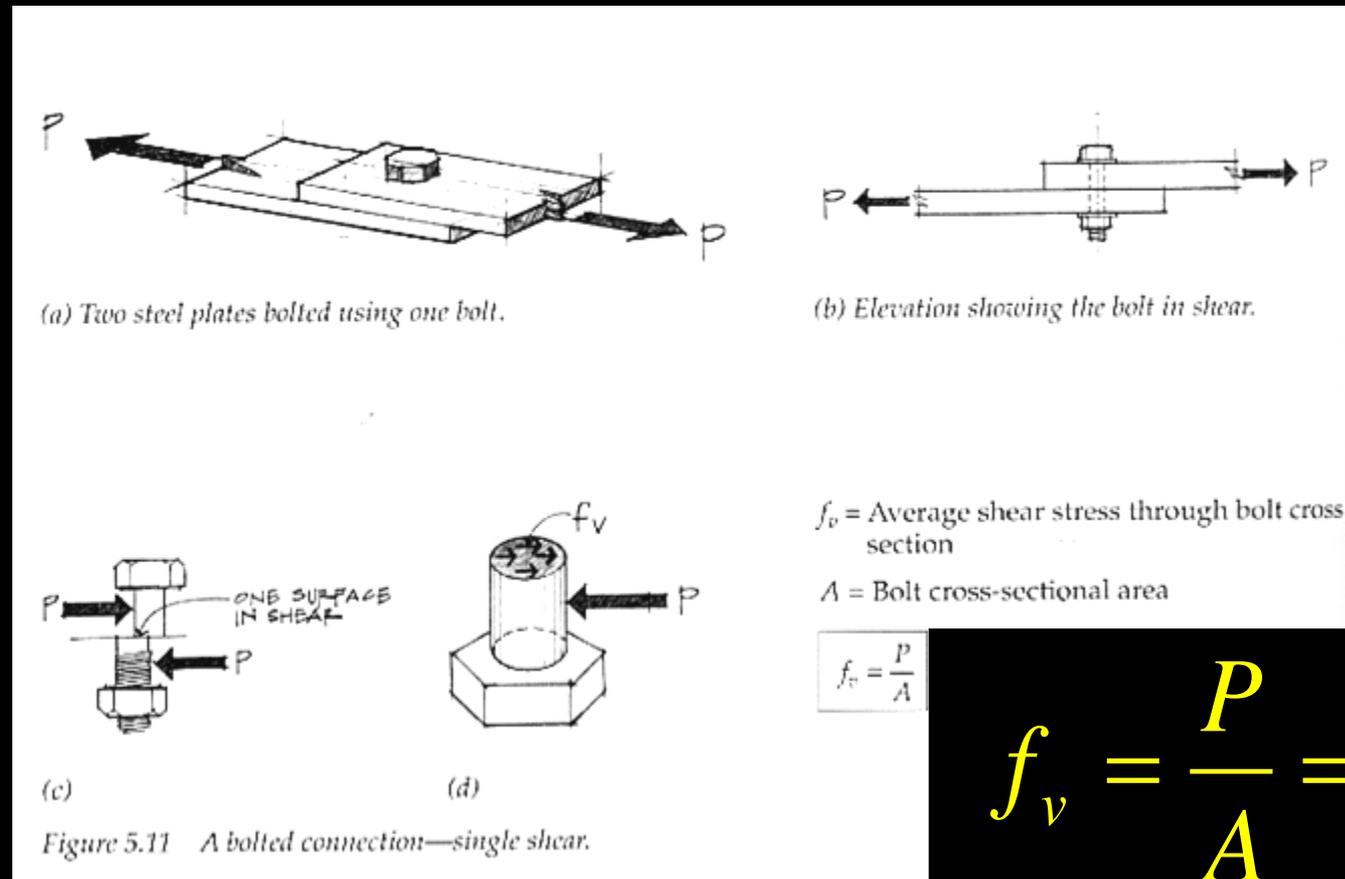
Double Shear

- seen when 3 members are connected

- *two* areas

$$f_v = \frac{P}{2A}$$

(two shear planes)
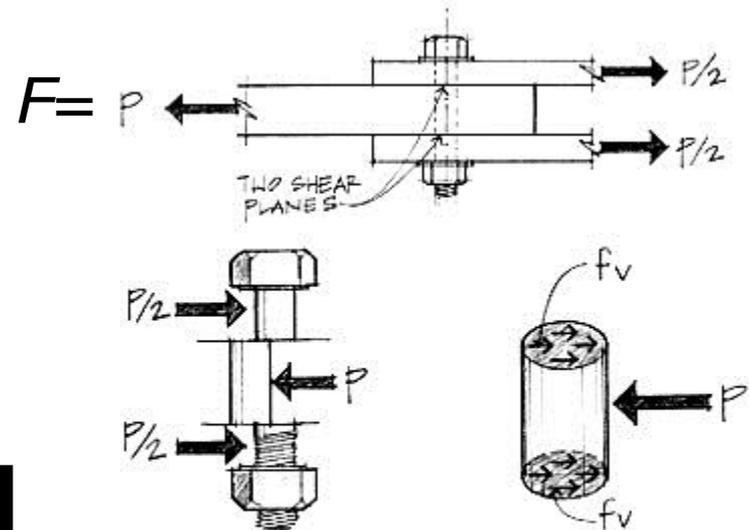
Free-body diagram of middle section of the bolt in shear.

Figure 5.12   A bolted connection in double shear.

$$f_v = \frac{P}{2A} = \frac{P/2}{A} = \frac{P/2}{\pi\, d^2/4}$$

Bolt Bearing Stress

- *compression & contact*

- *projected area*

Bearing stress on plate.
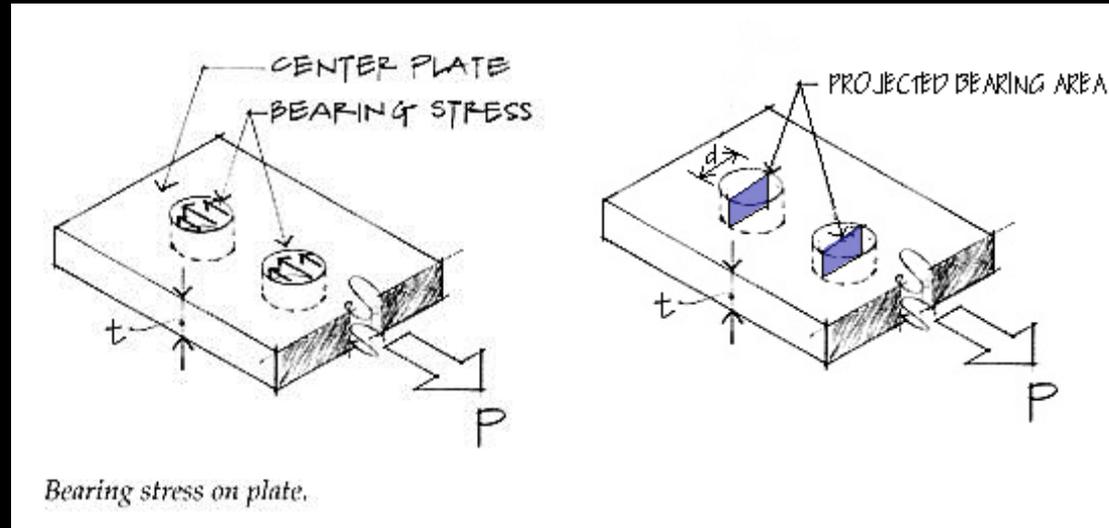
$$f_p = \frac{P}{A_{projected}} = \frac{P}{td}$$

Strain

- *materials deform*

- *axially loaded materials change length*

- *bending materials deflect*
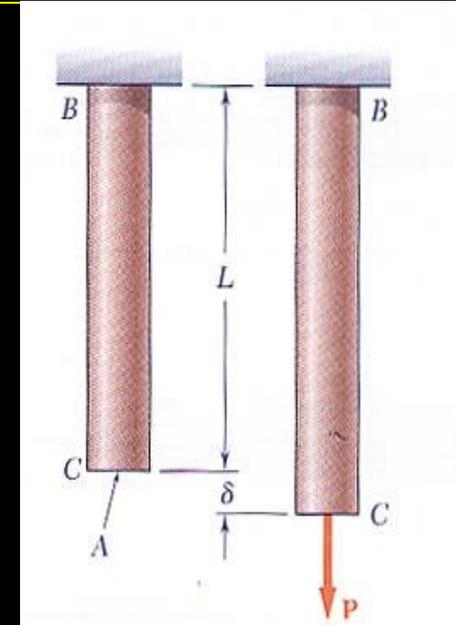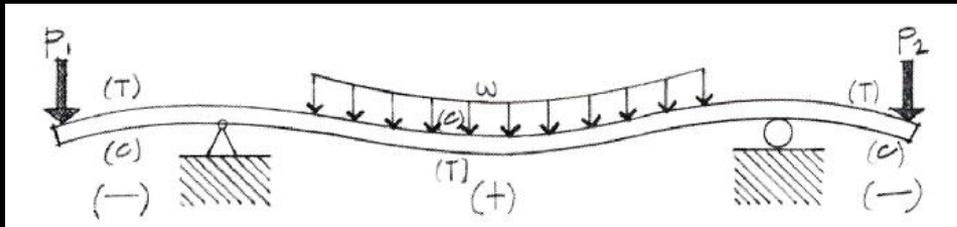
- *STRAIN:*
  - *change in length over length + UNITLESS*

$$strain = \varepsilon = \frac{\Delta L}{L}$$

Shearing Strain

- *deformations with shear*

- *parallelogram*

- *change in angles*

- *stress:* τ

- *strain:* γ
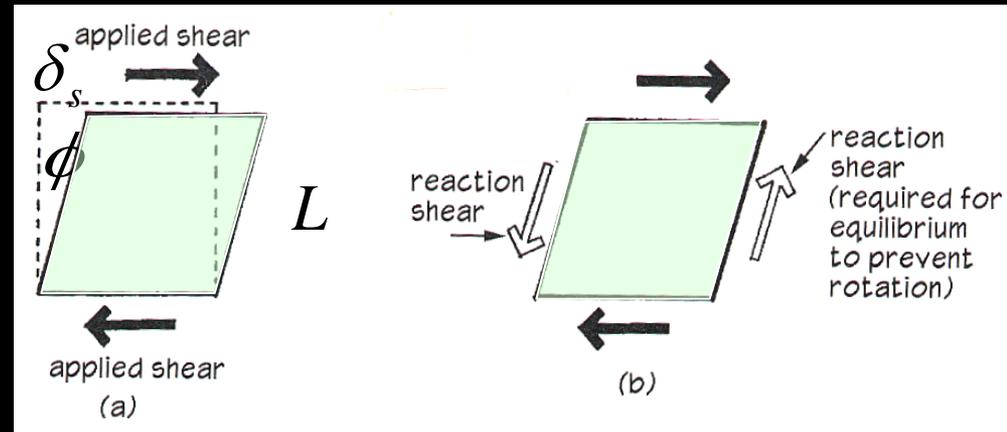
  – *unitless (radians)*

$$\gamma = \frac{\delta_s}{L} = \tan \phi \cong \phi$$

Shearing Strain

- *deformations with torsion*

- *twist*

- *change in angle of line*

- stress: τ

- strain: γ

  $$\gamma = \frac{\rho\phi}{L}$$

  – *unitless (radians)*
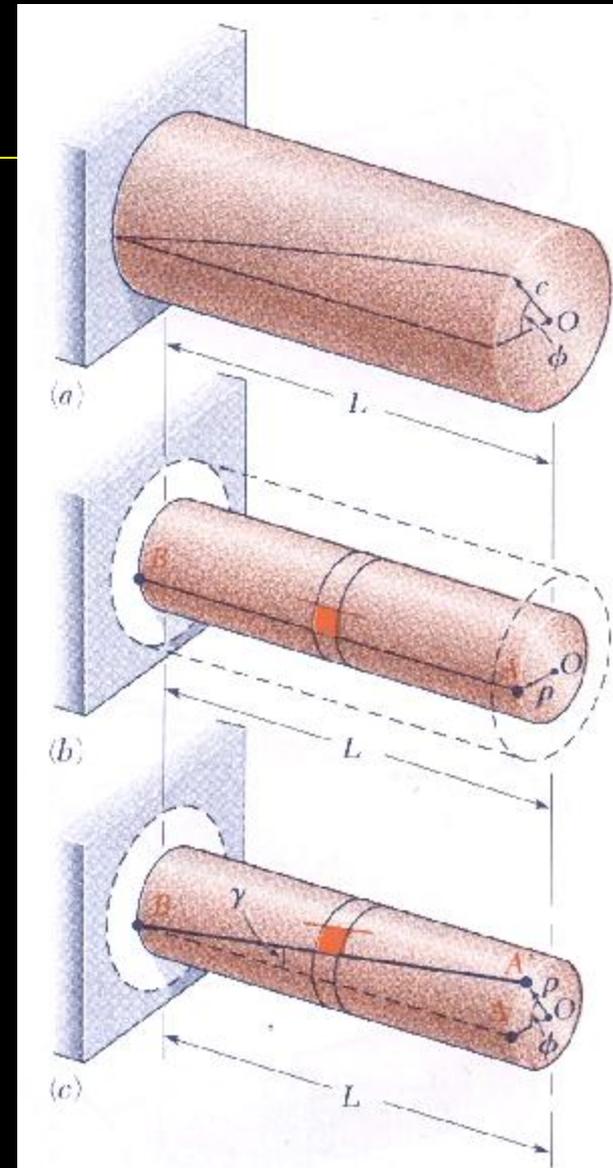
Load and Deformation

- *for stress, need P & A*
- *for strain, need δ & L*
  - *how?*
  - *TEST with load and measure*
  - *plot P/A vs. ε*
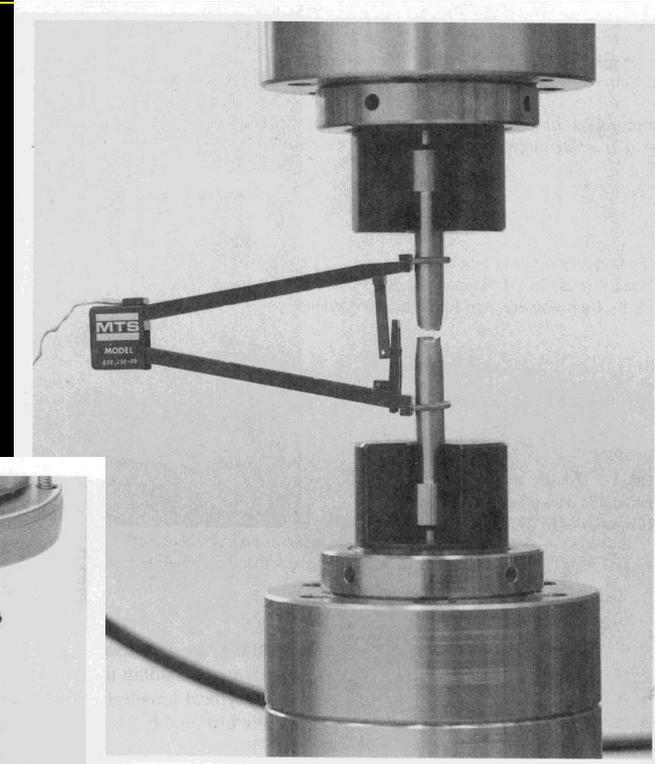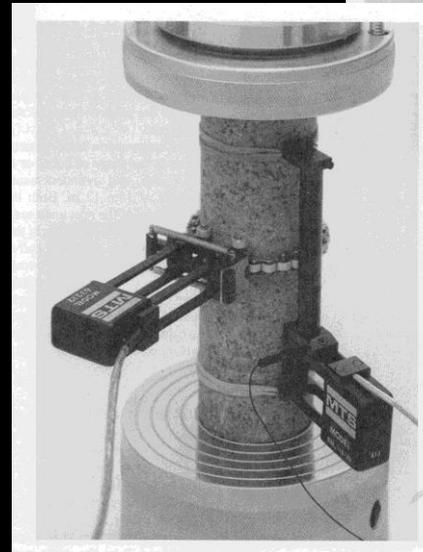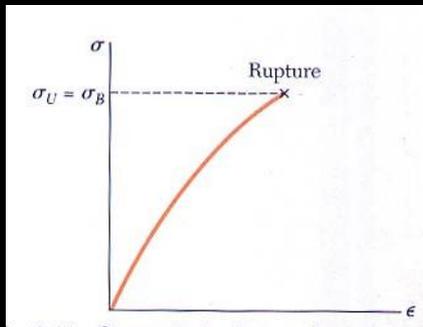
Material Behavior

- *every material has its own response*
  - *10,000 psi*
  - *L = 10 in*
  - *Douglas Fir vs. steel?*
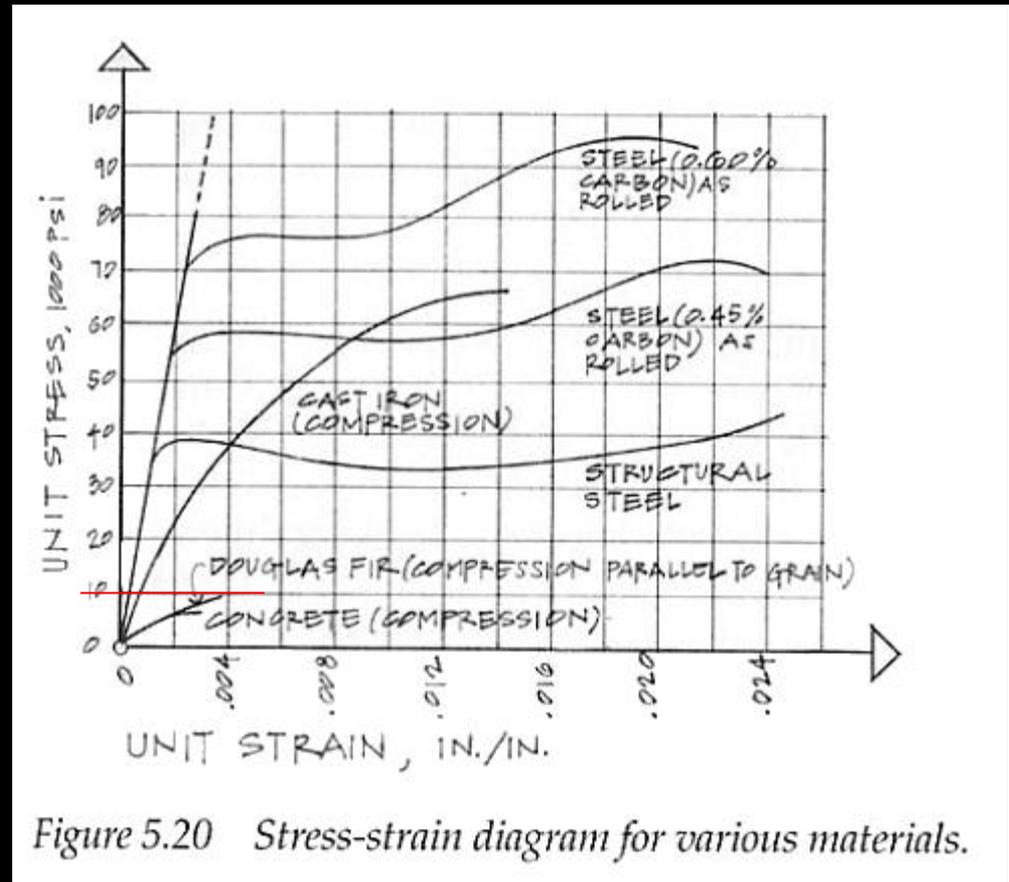
Figure 5.20   Stress-strain diagram for various materials.

Behavior Types

- *ductile - "necking"*

- *true stress*

$$f = \frac{P}{A}$$

- *engineering stress*
  – *(simplified)*

$$f = \frac{P}{A_o}$$
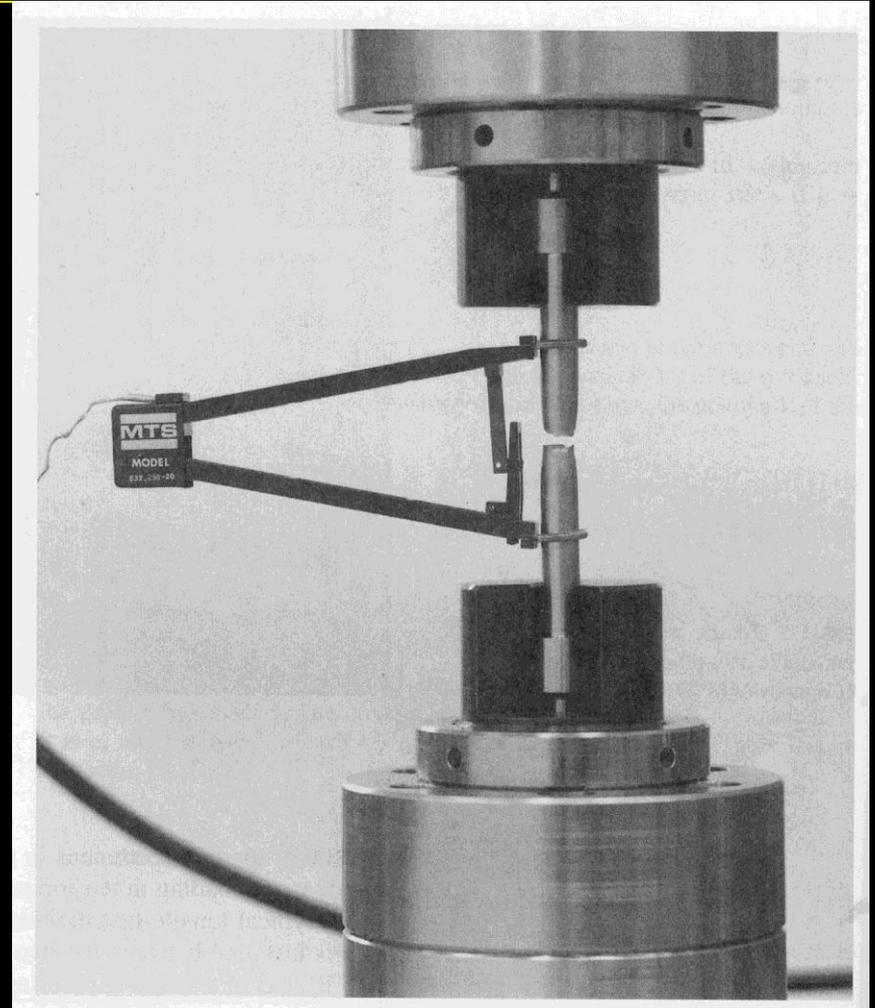
Behavior Types

- *brittle*
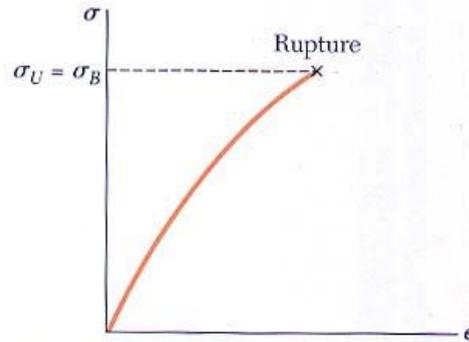
Fig. 2.11 Stress-strain diagram for a typical brittle material.

- *semi-brittle*
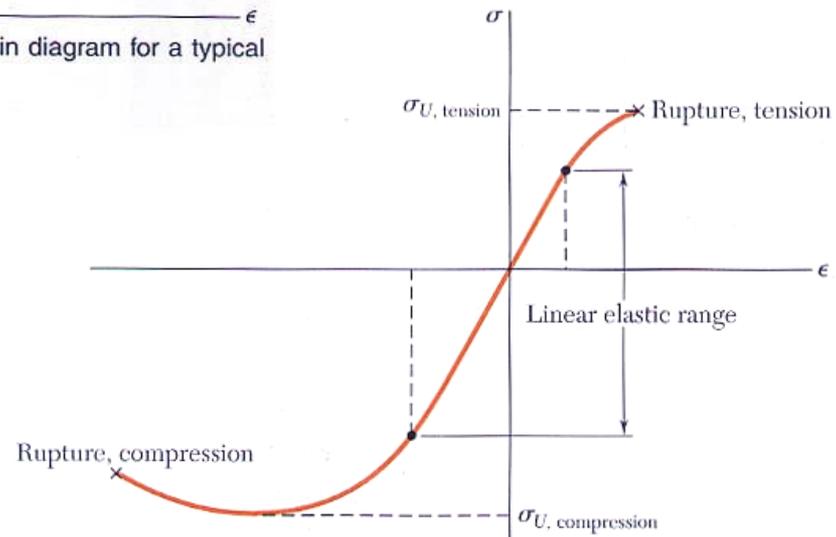
Fig. 2.14 Stress-strain diagram for concrete.

Stress to Strain

- *important to us in f-ε diagrams:*
  - *straight section*
  - *LINEAR-ELASTIC*
 - *recovers shape (no permanent deformation)*
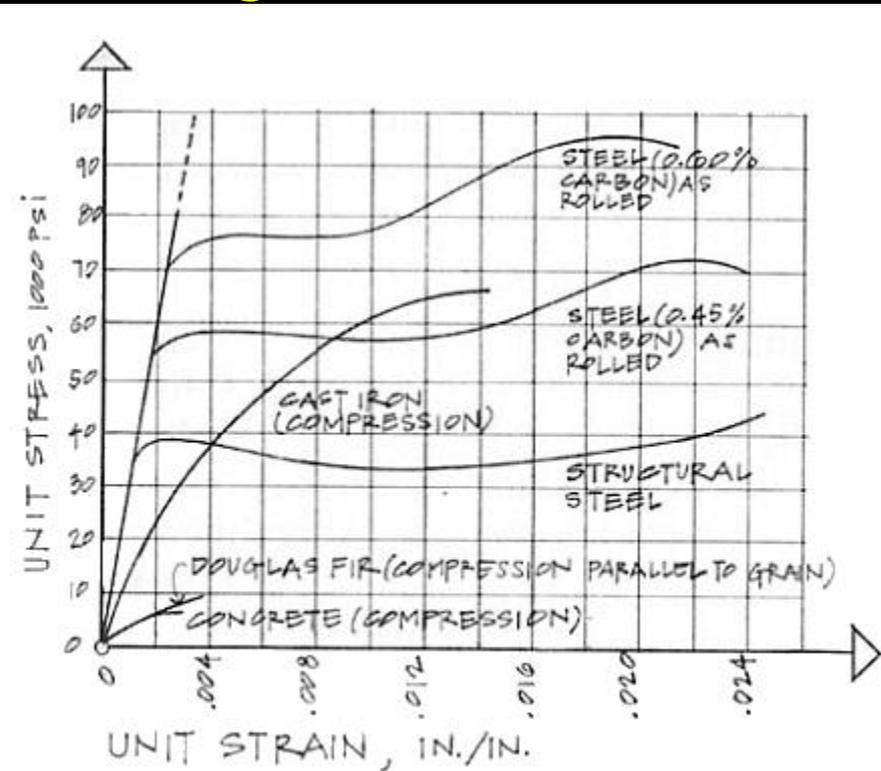
Figure 5.20   Stress-strain diagram for various materials.

Hooke's Law

- *straight line has constant slope*

- *Hooke's Law*

$$f = E \cdot \varepsilon$$
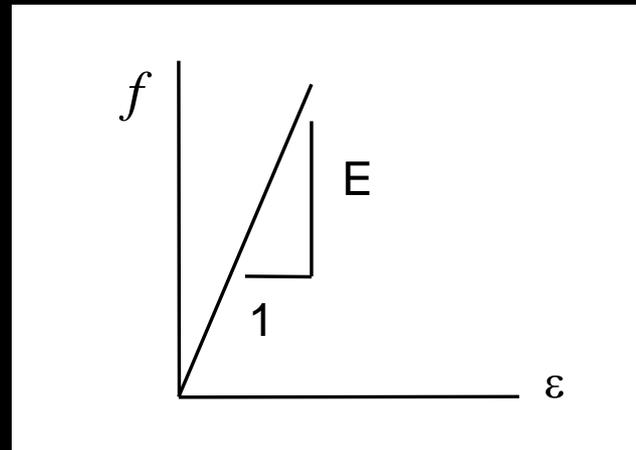
- *E*

  – *Modulus of elasticity*

 – *Young's modulus*

  – *units just like stress*
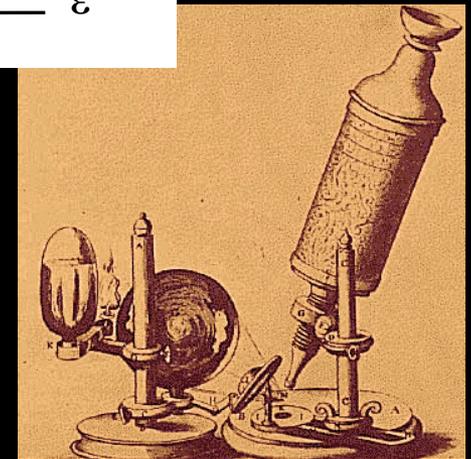
Stiffness

- *ability to resist strain*

- *steels*
  - *same E*
 - *different yield points*
  - *different ultimate strength*

f_u
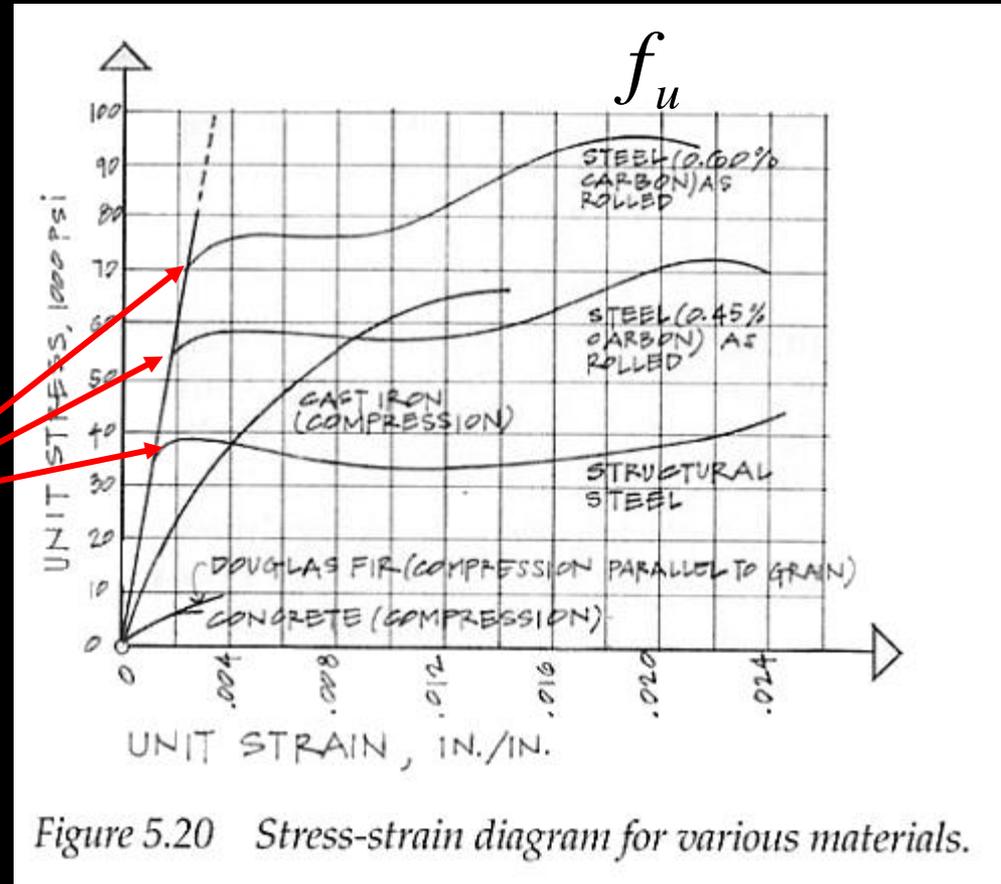
Figure 5.20   Stress-strain diagram for various materials.

Isotropy & Anisotropy

- ## *ISOTROPIC*
  - *materials with E same at any direction of loading*
  - *ex. steel*

- ## *ANISOTROPIC*
  - *materials with different E at any direction of loading*
  - *ex. wood is <u>orthotropic</u>*
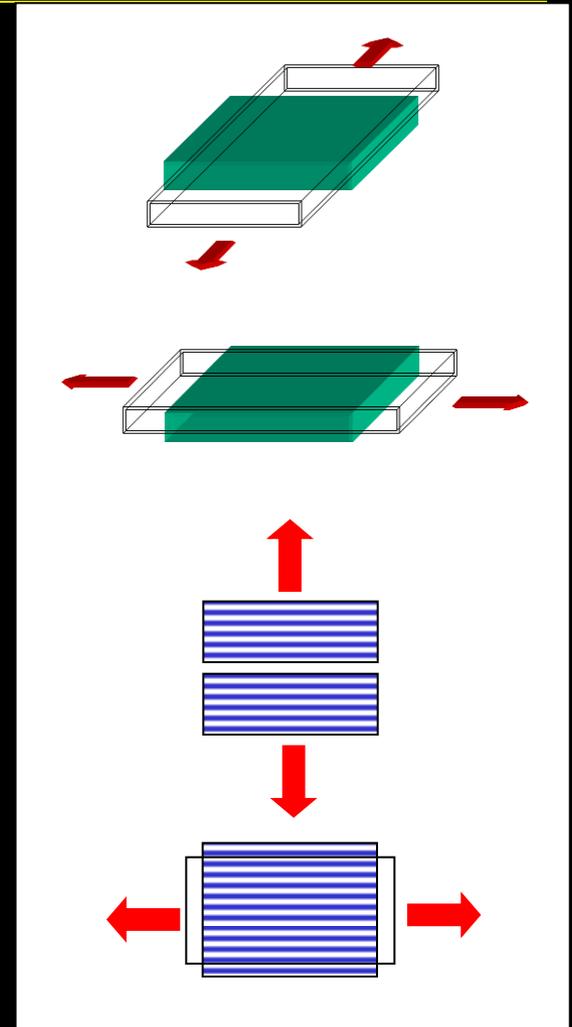
Elastic, Plastic, Fatigue

- *elastic springs back*
- *plastic has permanent deformation*
- *fatigue caused by reversed loading cycles*

Fig. 2.18

Fig. 2.20
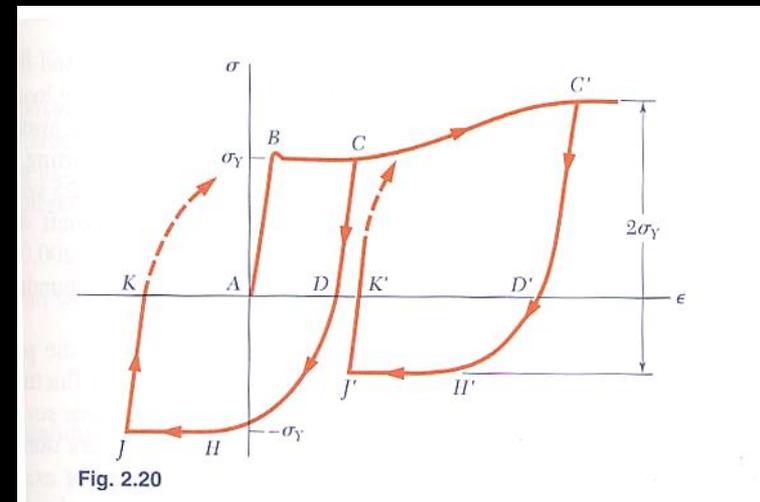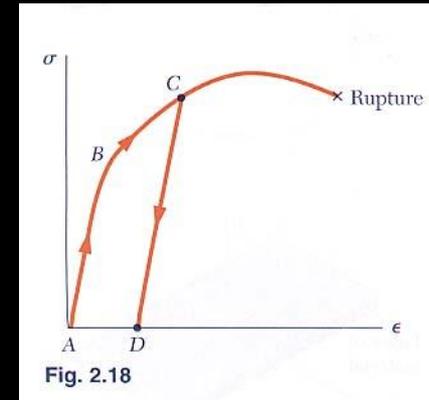
# *Plastic Behavior*

- *ductile*
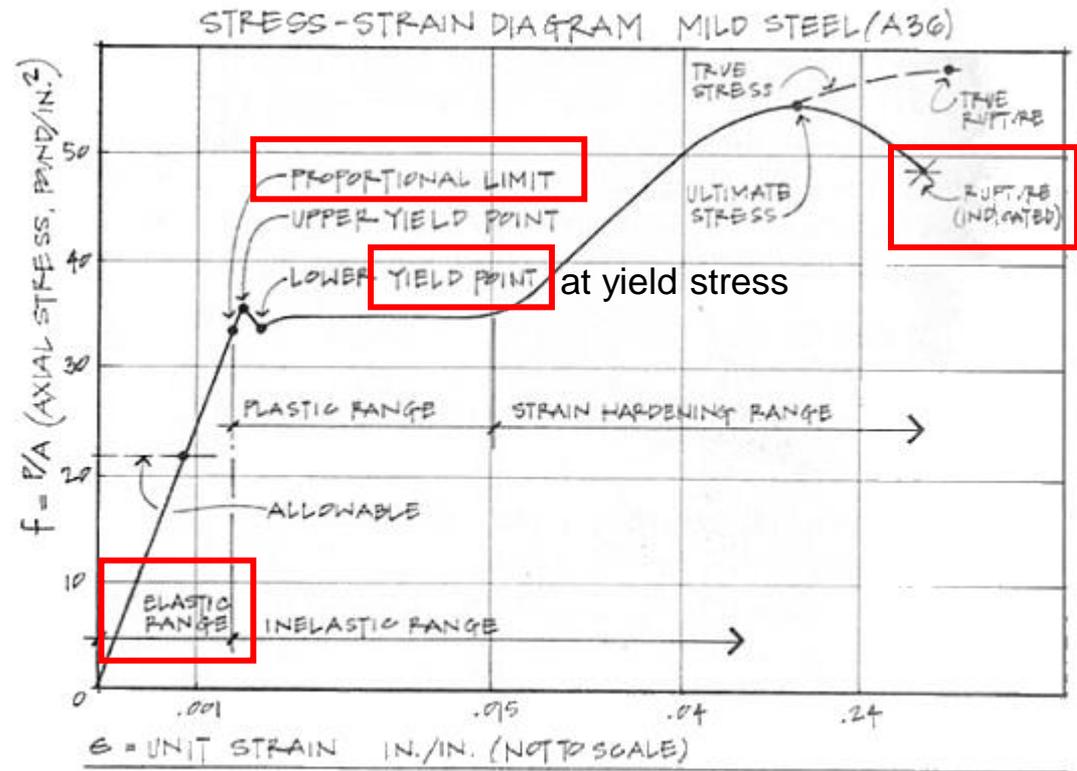
Figure 5.22   Stress-strain diagram for mild steel (A36) with key points highlighted.

Lateral Strain

- *or "what happens to the cross section with axial stress"*

$$\varepsilon_x = \frac{f_x}{E}$$

$$f_y = f_z = 0$$

- *strain in lateral direction*
  - *negative*
  - *equal for isometric materials*

$$\varepsilon_y = \varepsilon_z$$
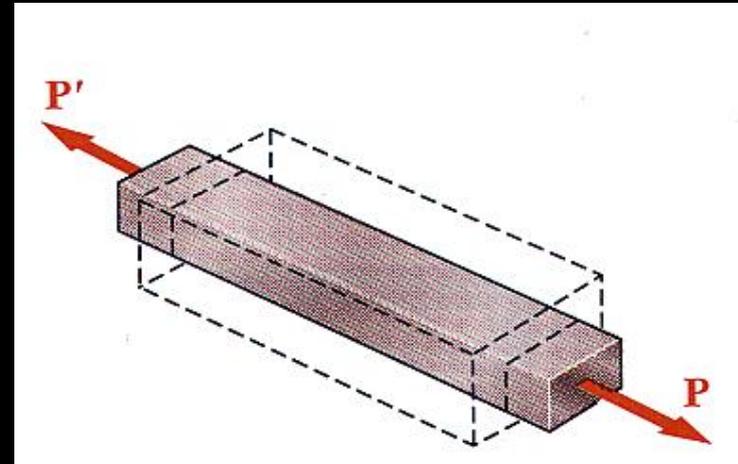
Poisson's Ratio

- *constant relationship between longitudinal strain and lateral strain*

$$\mu = -\frac{lateral\ strain}{axial\ strain} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

$$\varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E}$$

- *sign!* $0 < \mu < 0.5$
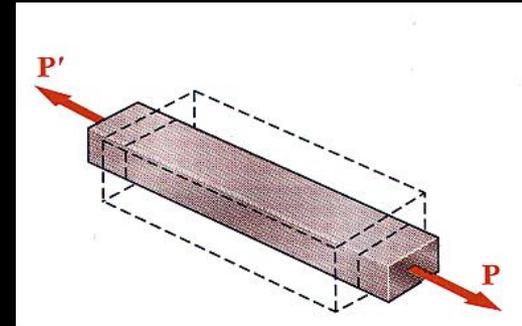
Calculating Strain

- *from Hooke's law*

$$f = E \cdot \varepsilon$$

- *substitute*

$$\frac{P}{A} = E \cdot \frac{\delta}{L}$$

- *get* $\Rightarrow$ $\qquad \delta = \dfrac{PL}{AE}$
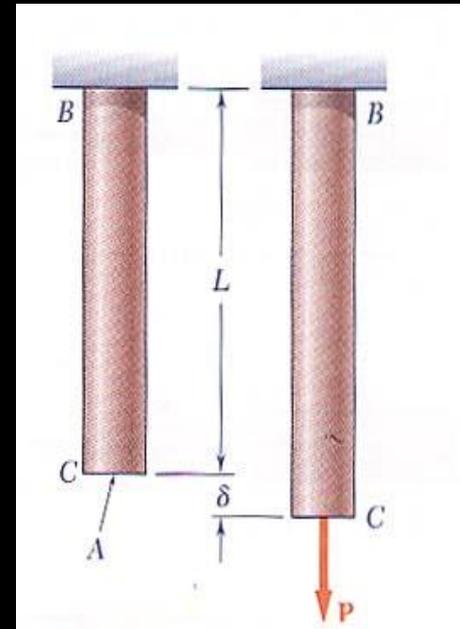
Orthotropic Materials

- *non-isometric*

- *directional values of E and μ*

- *ex:*

  - *plywood*
 - *laminates*
 - *polymer composites*
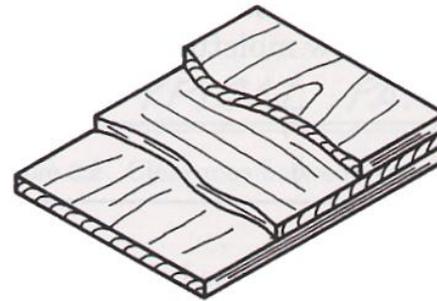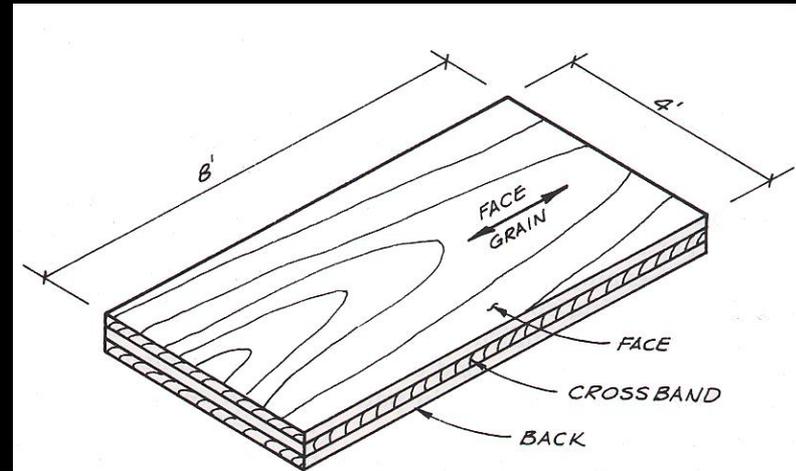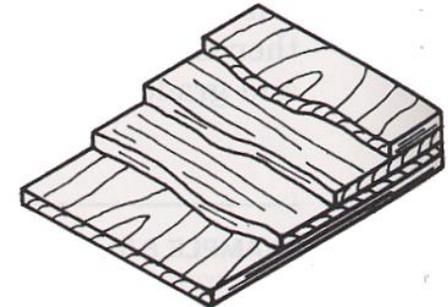
# Stress Concentrations

- *why we use f_{ave}*

- *increase in stress at changes in geometry*
  - *sharp notches*
  - *holes*
  - *corners*

STEEL HAS YIELDED

Figure 5.35   Stress trajectories around a hole.
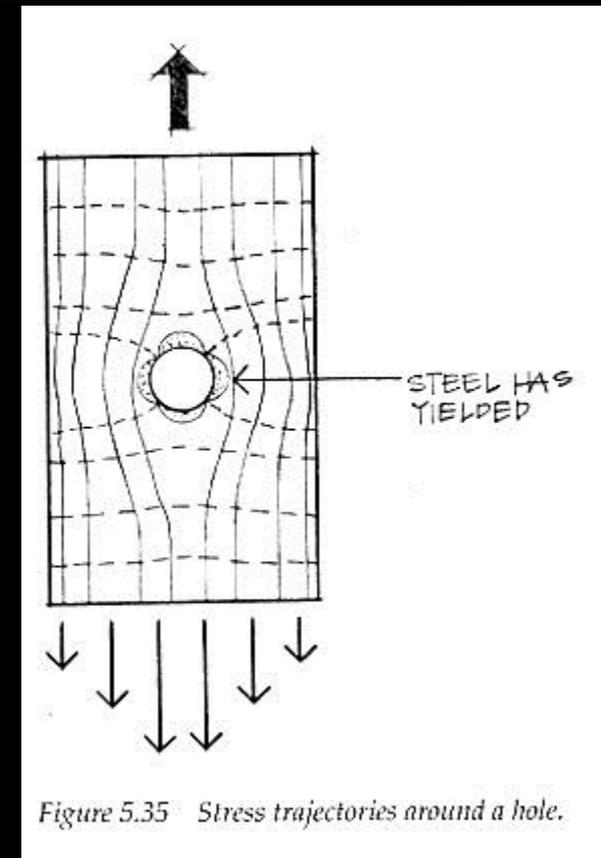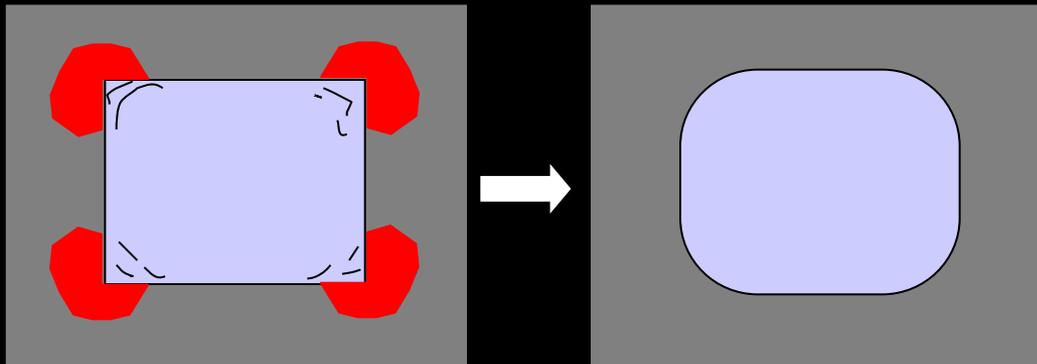
Maximum Stresses

- *if we need to know where max f and f_v happen:*
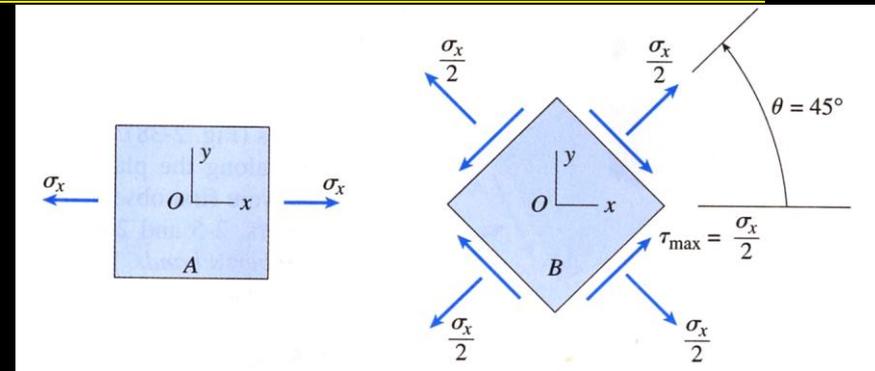
$$\theta = 0° \rightarrow \cos\theta = 1 \qquad f_{\max} = \frac{P}{A_o}$$

$$\theta = 45° \rightarrow \cos\theta = \sin\theta = \sqrt{0.5}$$

$$f_{v-\max} = \frac{P}{2A_o} = \frac{f_{\max}}{2}$$

Maximum Stresses

FIG. 2-37 Shear failure along a 45° plane of a wood block loaded in compression

FIG. 2-38 Slip bands (or Lüders' bands) in a polished steel specimen loaded in tension

Deformation Relationships

- *physical movement*
    - *axially (same or zero)*
    - *rotations from axial changes*
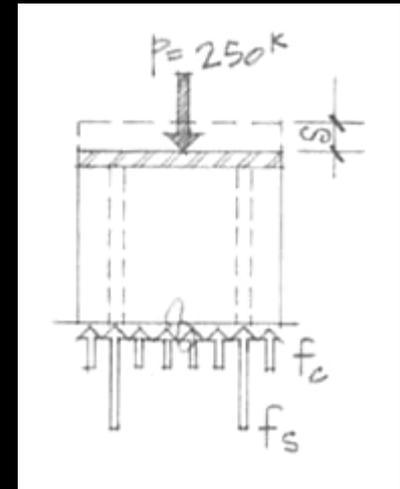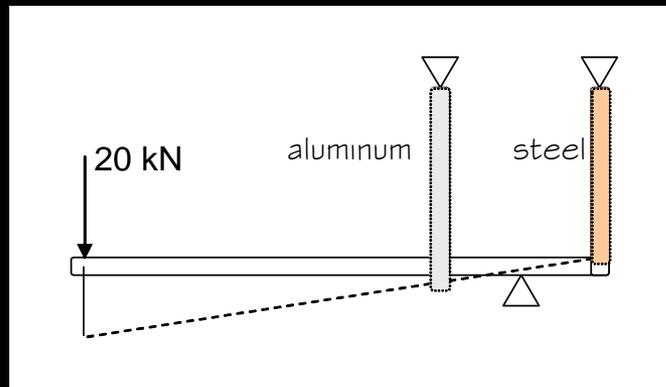
- $\delta = \dfrac{PL}{AE}$ *relates δ to P*

Deformations from Temperature

- *atomic chemistry reacts to changes in energy*

- *solid materials*

  - *can contract with decrease in temperature*

  - *can expand with increase in temperature*

- *linear change can be measured per degree*
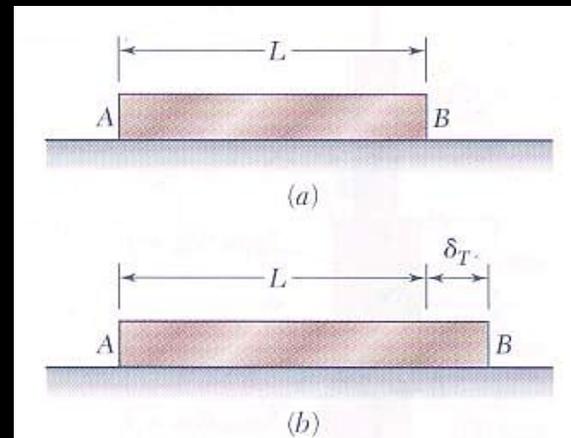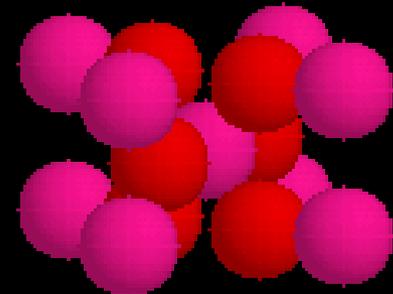
Thermal Deformation

- α - *the rate of strain per degree*

- *UNITS :* $/°F$, $/°C$

- *length change:* $$\delta_T = \alpha (\Delta T) L$$

- *thermal strain:* $$\varepsilon_T = \alpha (\Delta T)$$

  – *<u>no stress</u> when movement allowed*

Coefficients of Thermal Expansion

| Material	Coefficients (α) [in./in./°F]
Wood	3.0×10^{-6}
Glass	4.4×10^{-6}
Concrete	5.5×10^{-6}
Cast Iron	5.9×10^{-6}
Steel	6.5×10^{-6}
Wrought Iron	6.7×10^{-6}
Copper	9.3×10^{-6}
Bronze	10.1×10^{-6}
Brass	10.4×10^{-6}
Aluminum	12.8×10^{-6}
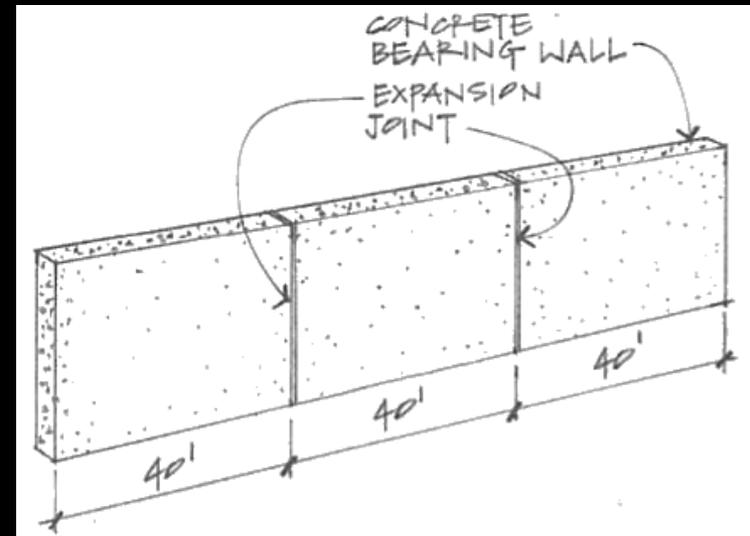
Stresses and Thermal Strains

- if thermal movement is restrained <u>stresses</u> are induced

1. bar pushes on supports

2. support pushes back

3. reaction causes internal stress

$$f = \frac{P}{A} = \frac{\delta}{L} E$$
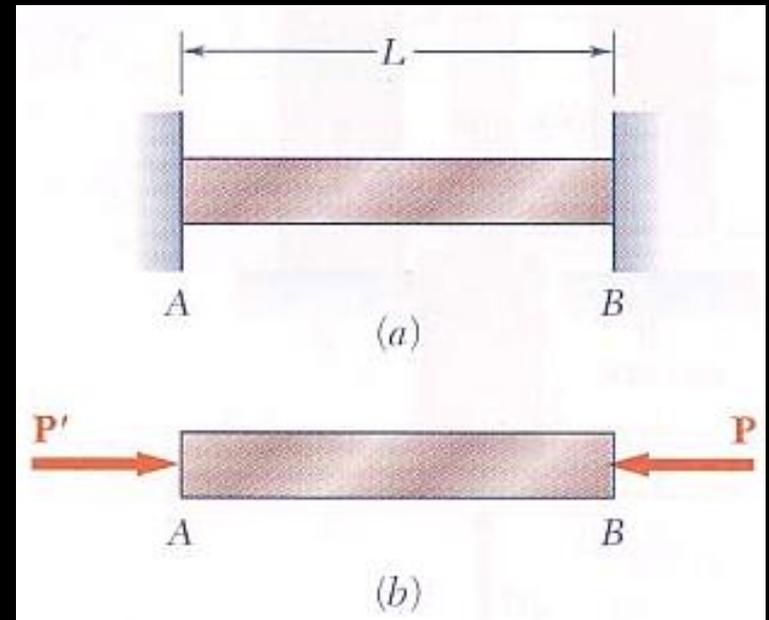
Superposition Method

– *can remove a support to make it look determinant*

– *replace the support with a reaction*

– *enforce the geometry constraint*
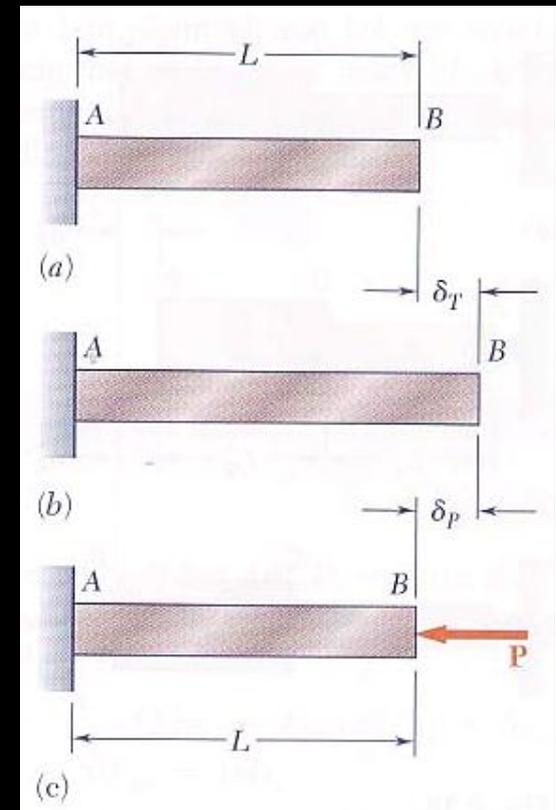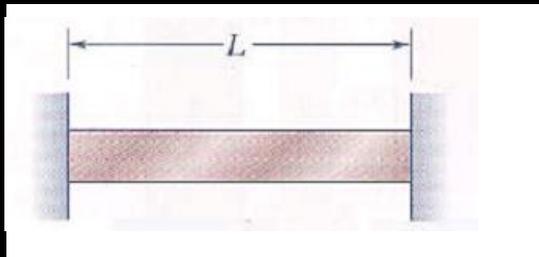
Superposition Method

– *total length change restrained to* *zero*

constraint: $\delta_P + \delta_T = 0$

$$\delta_p = -\frac{PL}{AE} \qquad \delta_T = \alpha(\Delta T)L$$

sub: $\quad -\dfrac{PL}{AE} + \alpha(\Delta T)L = 0$

$$f = -\frac{P}{A} = -\alpha(\Delta T)E$$
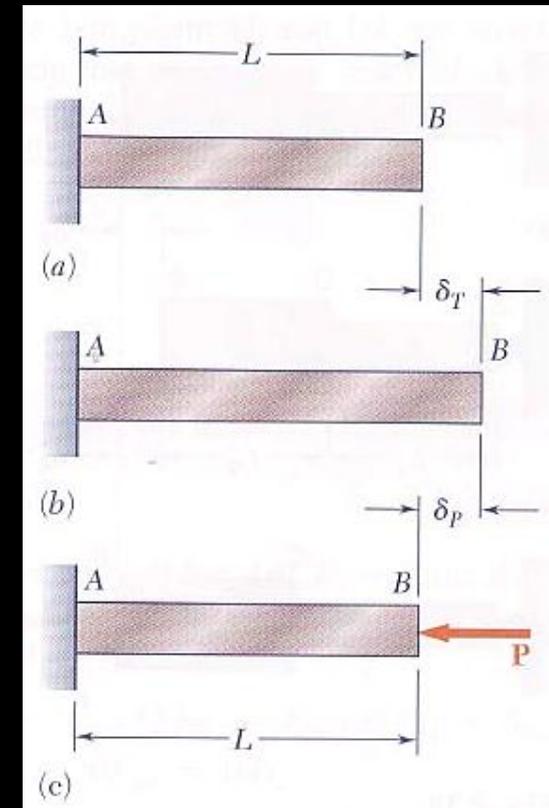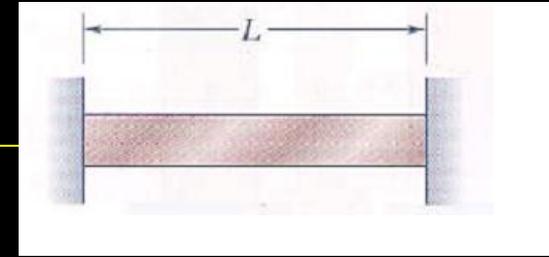
Design of Members

- *beyond allowable stress...*

- *materials aren't uniform 100% of the time*

  - *ultimate strength or capacity to failure may be different and some strengths hard to test for*

- *RISK & UNCERTAINTY*
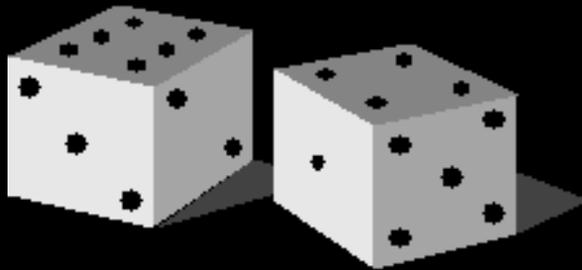
$$f_u = \frac{P_u}{A}$$

Factor of Safety

- *accommodate uncertainty with a safety factor:*

$$\text{allowable load} = \frac{\text{ultimate load}}{F.S}$$

- *with linear relation between load and stress:*

$$F.S = \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

Load and Resistance Factor Design

- *loads on structures are*
  - *not constant*
  - *can be more influential on failure*
  - *happen more or less often*
  - *UNCERTAINTY*
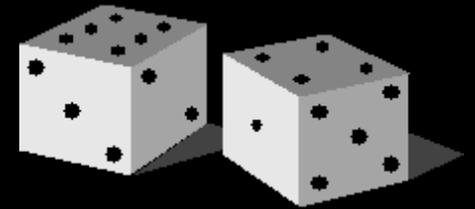
$$R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n$$

ϕ - *resistance factor*

γ - *load factor for (D)ead & (L)ive load*