NEAR EAST UNIVERSITY

Engineering Faculty
Civil Engineering Department

"Reinforced Concrete Structures Design"

By:

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Chapter 1. Introduction

This course discusses how a designer can approach a design problem in general and how a structural designer can design a structural system. It contains the design approach for different elements of reinforced concrete structures.

Design of Reinforced Concrete Structures

Different Kinds of Structures:

1- Reinforced Concrete structures: cheaper than steel, bigger sizes of elements, cannot be modified (properly and easily) well against fire, heavy.

2- Steel Structures: expensive, smaller elements, can be modified, weak against fire (can be prevented by adding concrete gypsum around the element) more ductile, light.

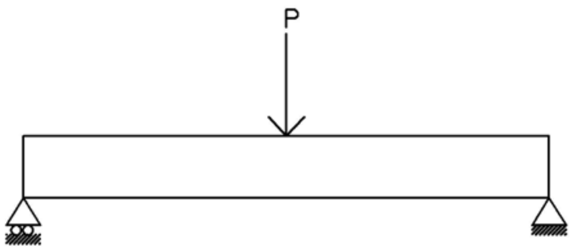
3- Timber Structure: for low rise buildings (up to 3 storeys), weak against fire, good against earthquakes (because it is light).

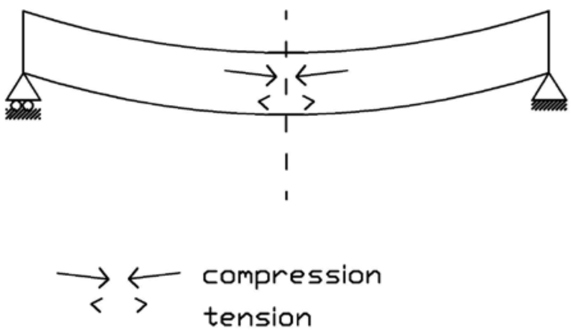
Concrete:

Concrete is a non-homogeneous manufactured stone composed of aggregate, granular inert materials which are held together by action of cement and water. aggregate is composed of gravel and sand .

Concrete is a good material for compressive forces and weak against tensile forces (stresses). We use reinforcement (steel bar) where we have tensile stresses for modifying the concrete structures.

The most important in concrete is rate of gravel/water.





Reinforced concrete is a composite material which utilizes the concrete in resisting compression forces and some other material, usually steel bar or wire (bridges), to resist the tension forces.

Codes and Standards for Reinforced Concrete Structures Calculations:

- ACI: American Concrete Institute
- Euronorm: European Code (European/French Code)
- BS: British Standard, CP: Code of Practice
- BAEL: French Code (Bases of Euronorm)
- TS (Turkish Standard)
- ABA (Iranian Reinforced Concrete Code)

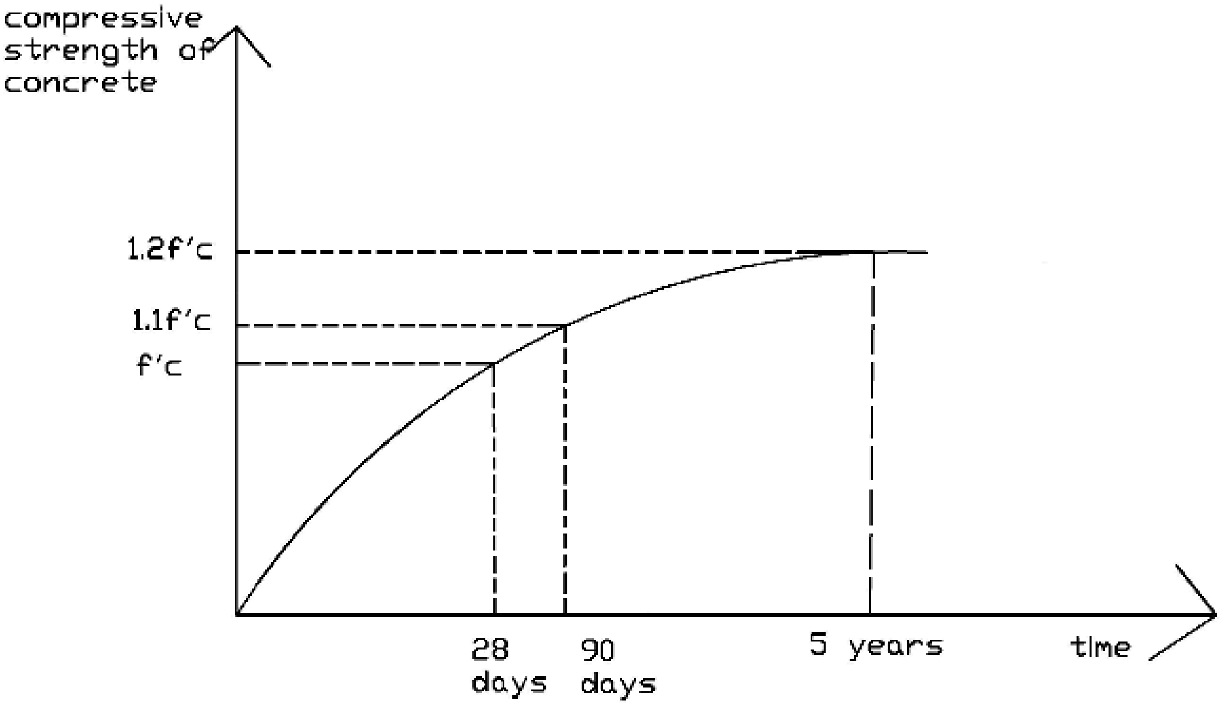
Admixture

A material other than Portland cement, aggregate or water added to concrete to modify its properties. (Especially for casting concrete in very cold or hot weathers)

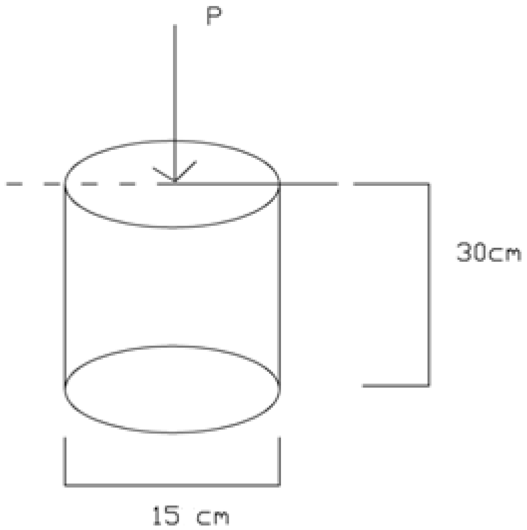
Compressive Stress of Concrete:

f'c is specified as compressive strength of concrete in pound per square inch (PSI) (Mega Pascal, MPa in SI Unit Systems or in Kgf/cm^2).

Compressive strength is determined by test of standard cylinder Samples and tested in accordance with ASTM (American Society for Testing and Materials). Specifications at 28 days.

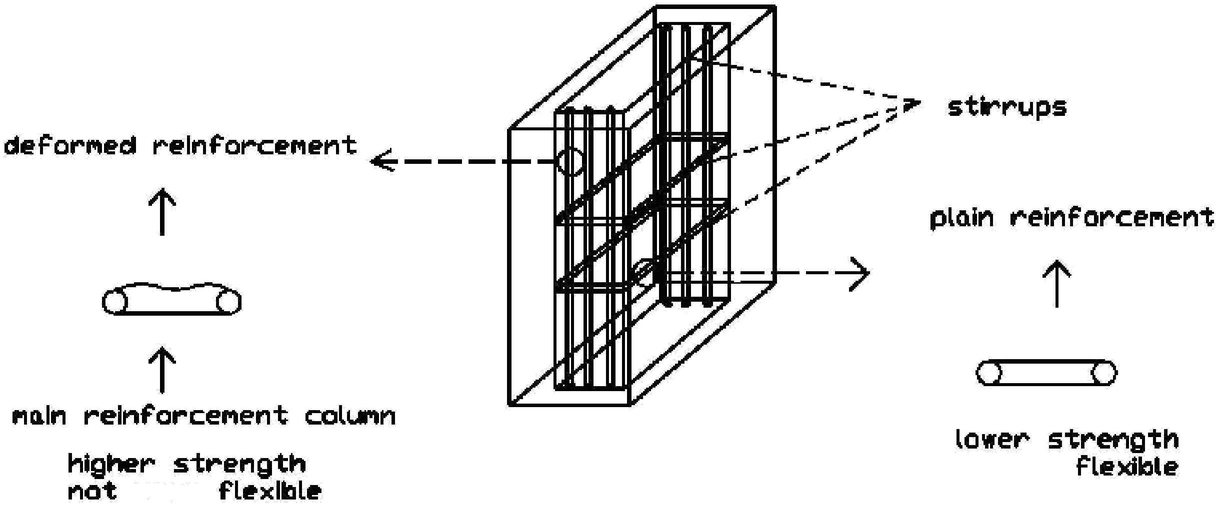


The height and diameter of standard cylinder samples are 6" x12" (about 15cm x 30cm).



<u>Deformed Bar</u>

A reinforcing bar which is used in reinforced concrete structures as main reinforcement.



The most important factor in concrete is the ratio of water over cement (W/C).

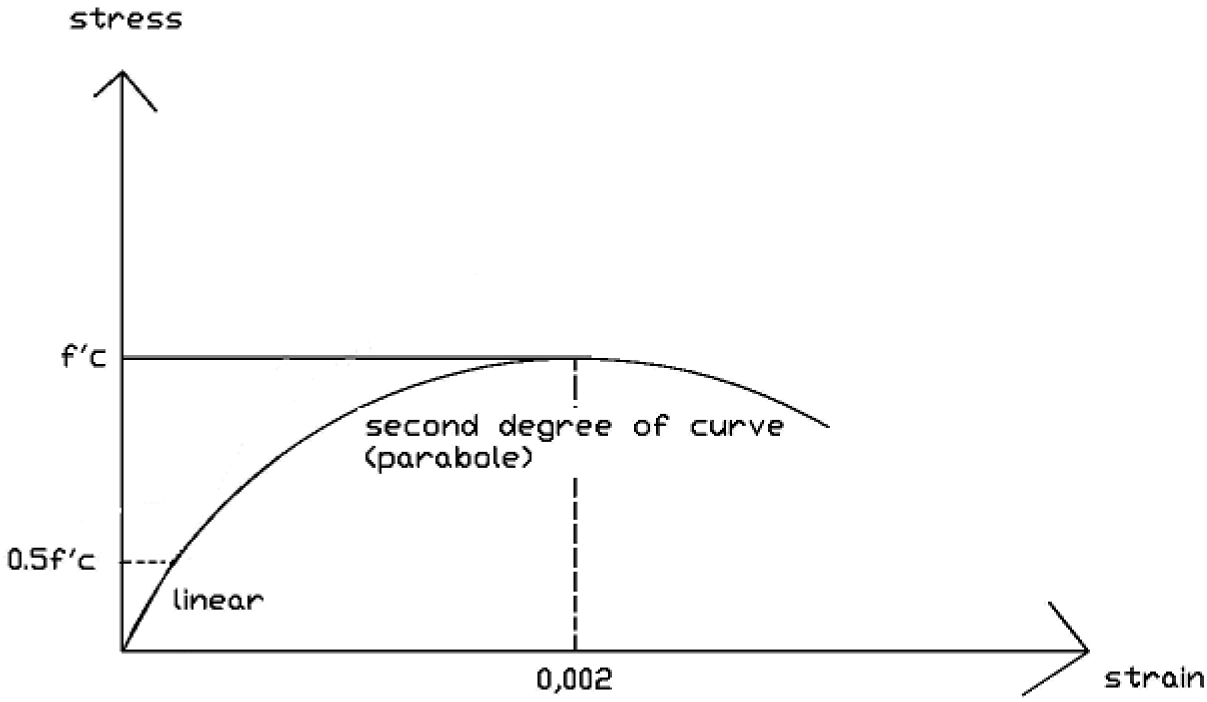
Water Ratio:
W/C ≈ 0.35 (low) – (more strength in concrete but low workability)
W/C ≈ 0.55 (normal)
W/C ≈ 0.7 (high) – (less strength but high workability)

Curing of Concrete

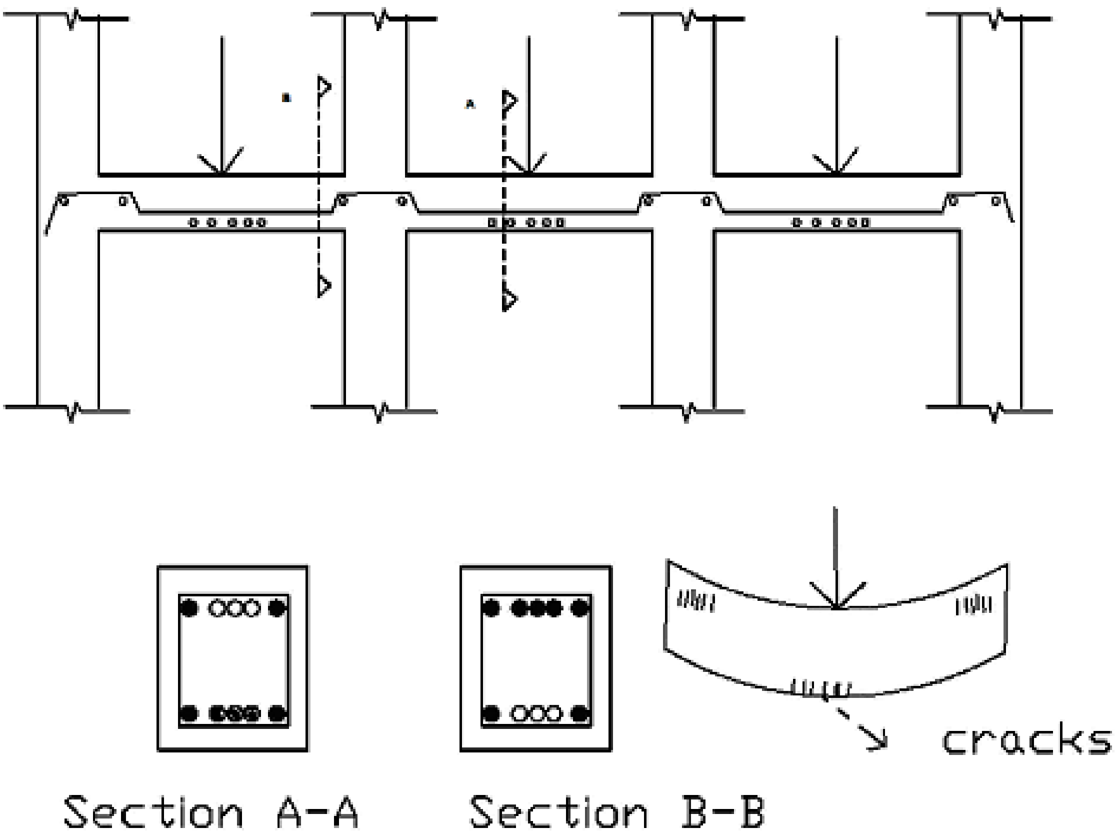
After casting of concrete it is needed concrete be cured for at least 42 days. Concrete should be covered from sun rays and from wind. Concrete should be watered every 6 hours, if we do not cure concrete during 42 first days the strength of concrete may be reduced. Sometimes curing compounds are used for curing the concrete.

Mechanical Properties of Concrete

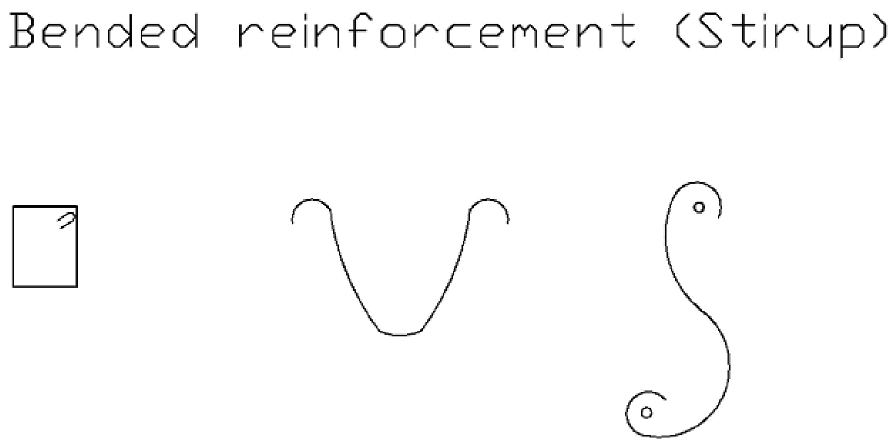
Stress-strain curve of compressible concrete:



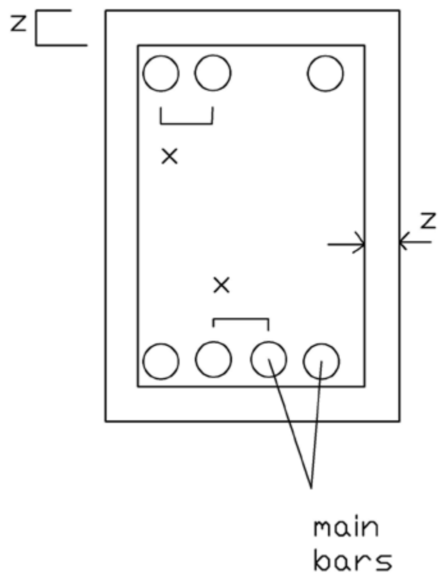
Bending of Reinforced Bars for Structural Construction



Bended Reinforcement (stirrup)



Bar Spacing and Concrete Covers



- x ≥ diameter of main bar
- x ≥ 1.33 x max aggregate size → for normal concrete max. of aggregate size may be 1” (2.5 cm).
  (Aggregate →gravel + sand)
- x ≥ 1” (2.5 cm).
- z ≥ diameter of main bars , always

Important Components of Structures

- columns
- beams and girders
- foundations(footings)
  - a- Surface foundations (single footing,…….)
  - b- Deep foundations (piles)

Analysis + Design

a- Diagram:

- axial force
- shear force
- bending moment

b- Material (determine the sizes of elements)

- steel
- concrete
- timber

Approximate Methods

- gravity loads
- horizontal loads
- portal method

Exact Solution

- slope deflection
- three moments
- moment distribution methods

Concrete Elements Design

Design of rectangular cross section beams with tensile reinforcement

1 – WSD: Working Stress Design
2 – USD: Ultimate Strength Design
3- Limit State Design

Design of concrete columns

a- Design of short columns
b- Design of long columns

CHAPTER -2 Gravity Loads

Gravity Loads (Structural analysis and design)

The analysis of structures deals with the determination of the loads, reactions, shear and bending moment.
Structural design deals with the proportion of members to resist the applied forces.
The consequence involved in creating a structure, then involves analysis first and then design.

Design

- Elastic design
- Plastic (post-elastic design)
- Elasto-Plastic Design

The ACI code requires the analysis be made by using the elastic theory where as structural design may be accomplished by using alternate design method (working stress design) or the strength design method (ultimate stress design)

Exact and Approximate Methods of Analysis

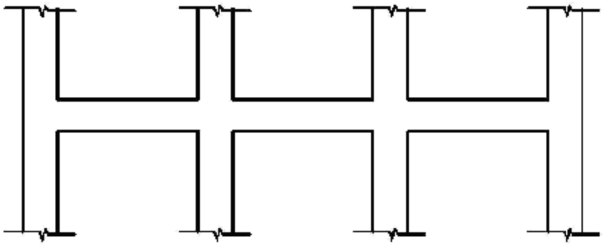
There exist methods which provide for an exact mathematical analysis of structures, such methods as slope-deflection and moment distribution may always be used to analyze concrete structures.

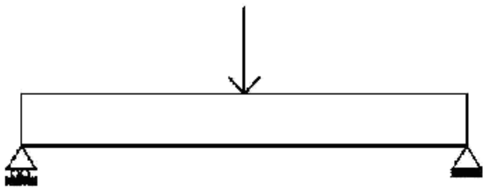
In some cases it is absolutely necessary to use the exact methods. In most common cases, however it is sufficiently accurate to use approximate methods.

The ACI code contains approximate coefficients calculating shears and moments which can be used (and only when) specified conditions have been satisfied. The Conditions normally are as follows:
1 - The differences between different spans < 20% and
2 – Ratio of live load over dead load < 3)
3 – The load is the uniformly distributed load.

The approximate method will be discussed in this course.

Continuous Beams and One-Way Slabs



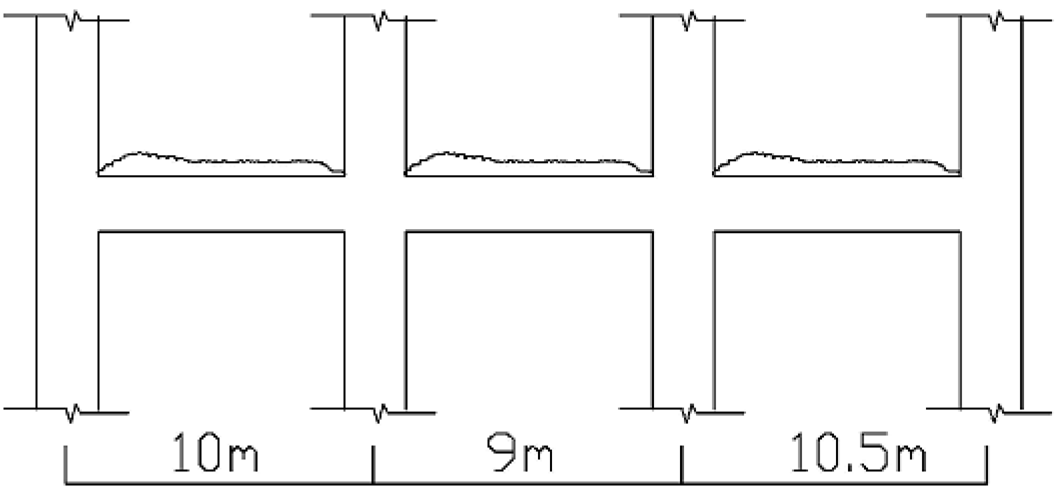


Approximate coefficients of shear and bending moment may be used when the following conditions are satisfied:

a. Adjacent clear spans may not differ in length by more than 20% of shorter span.
b. The ratio of live load to dead load may not exceed 3.
c. The loads must be uniformly distributed.

Example:

Please verify using the ACI approximate method for the continuous beam shown in the figure. (Dead load =1000kgf/m²x5m, live load=300kgf/m²x5m)



1. (10.5-9)/9< 20% → (1.5)/9 < 20% → 16.67%< 20%
2. Live load/dead load < 3 →(300x5)/(1000x5)<3

3. Yes, we don't have concentrated force and un-uniformly distributed loads and we have only uniformly distributed load on the beam.

- We can use ACI approximate coefficient for this example.

Beams and Slabs (One-way slab coefficients)

End Spans:

- If discontinuous end is unrestrained $W(L')^2/11$
- If discontinuous end is integral with interior spans $W(L')^2/14$ or $W(L')^2/16$

For Negative Moment:

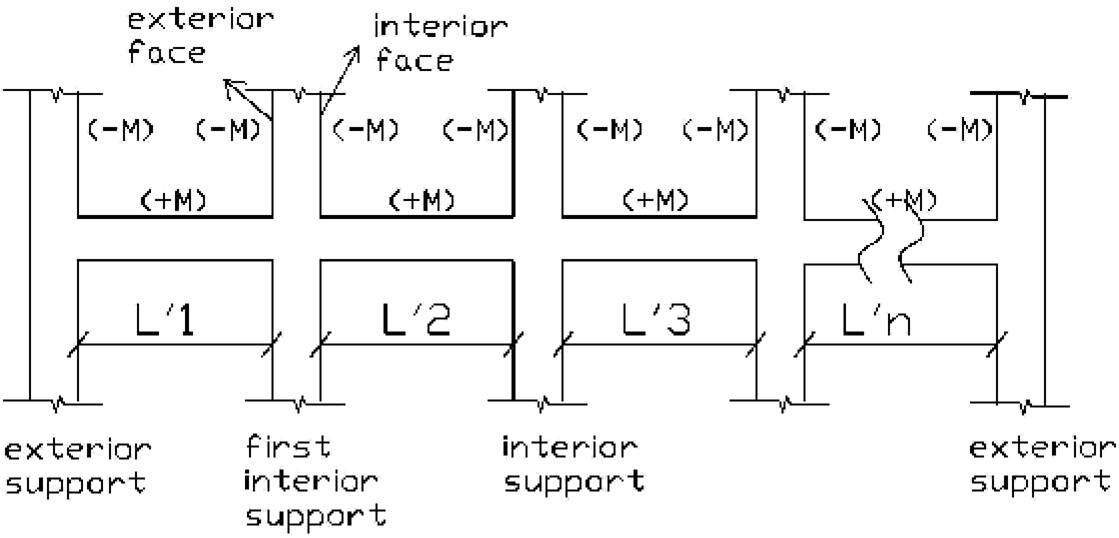
i. Two spans $W(L')^2/9$
ii. More than two spans $W(L')^2/10$
iii. Negative moment at other faces of interior supports $W(L')^2/11$

Negative Moment at Face of All Supports For:

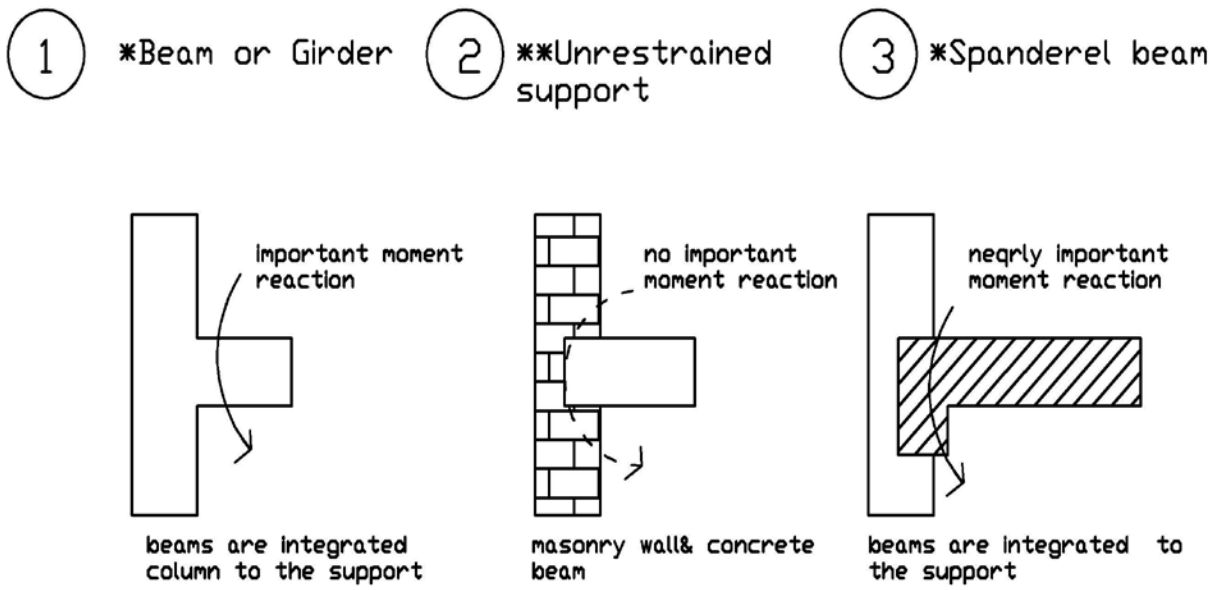
a- Slabs with spans not exceeding 10ft (3m) and
b- Beam and girders where the ratio of sum of columns stiffness to beam stiffness exceeds 8 at each end of span $W(L')^2/12$

Negative Moments at Interior Faces of Exterior Supports for Members Built Integrity with Their Supports:

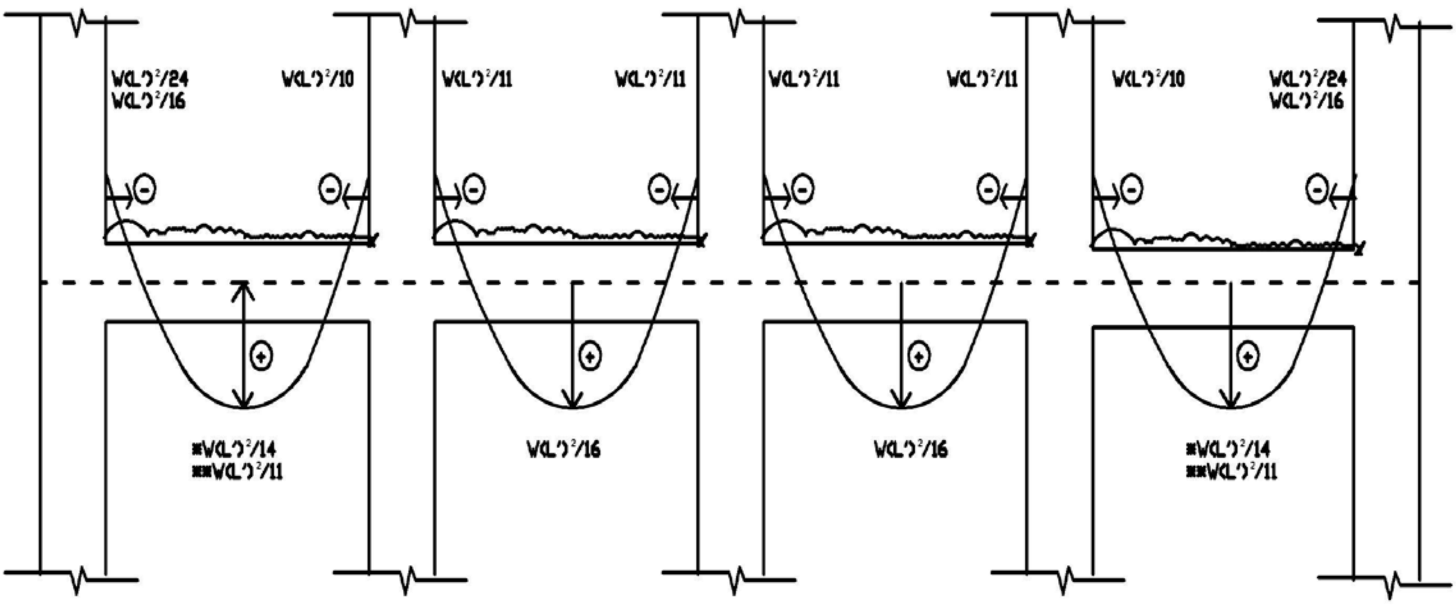
a- Where the support is a spandrel beam or girder $W(L')^2/24$
b- Where the support is a column $W(L')^2/16$



EXTERIOR SUPPORT



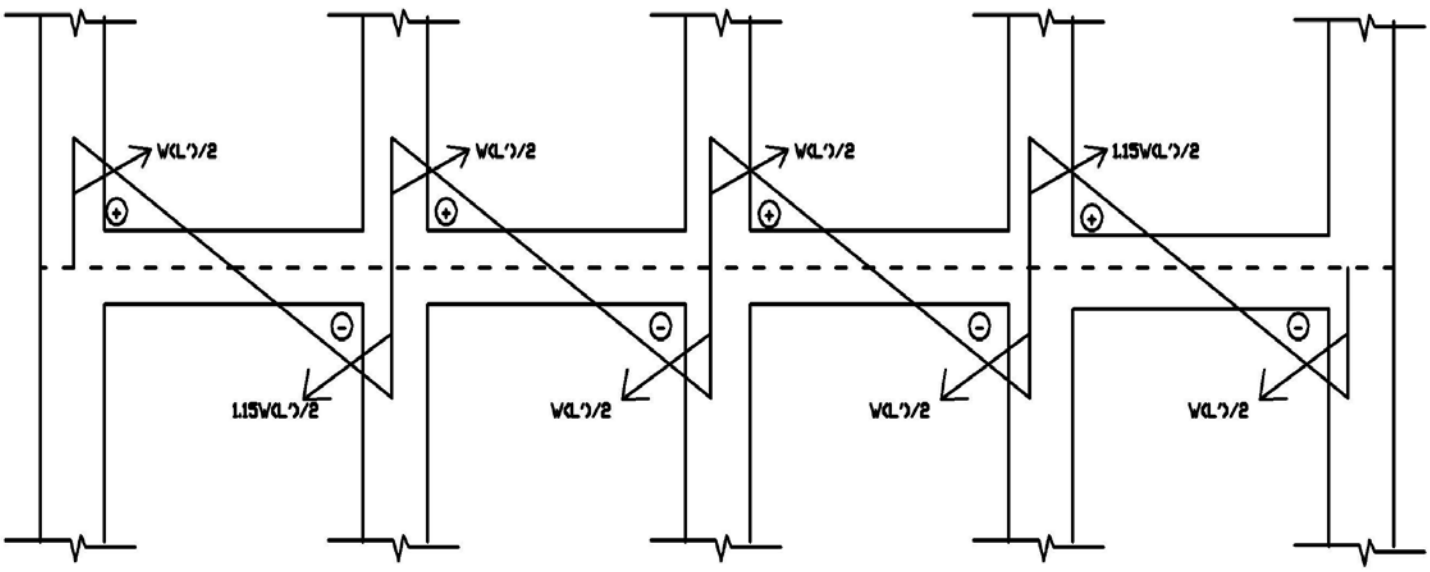
Bending Moment Diagram



$W(L')^2/24$ where the support is a spanderel beam or girder

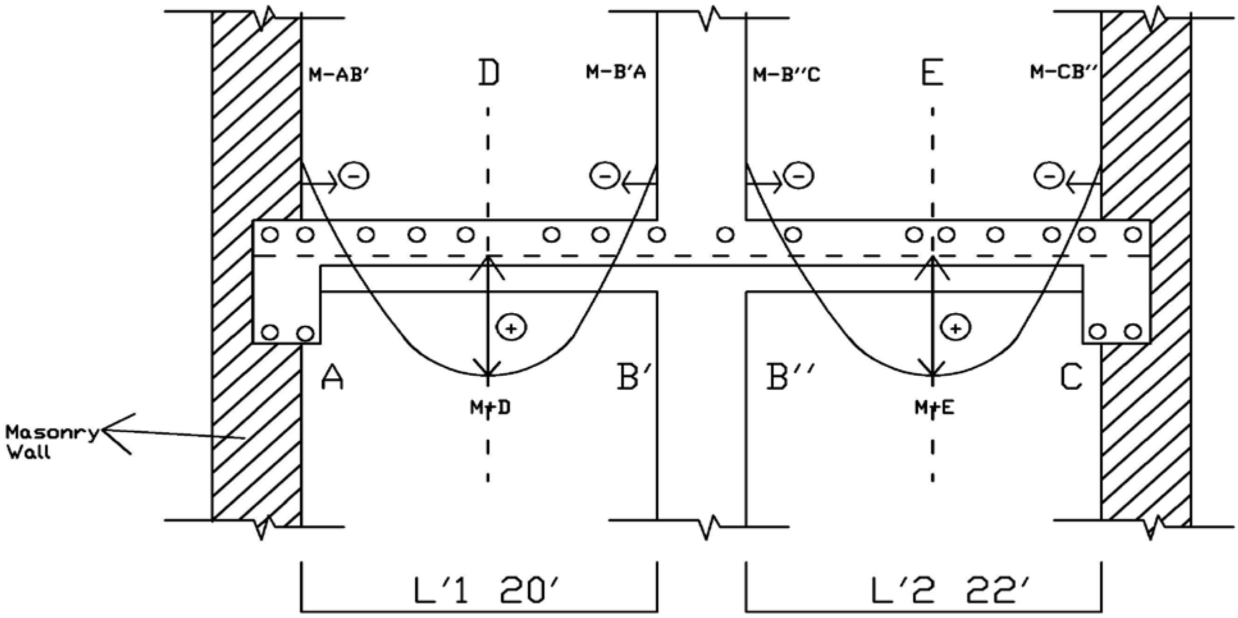
$W(L')^2/16$ where the support is column

Shear Force Diagram



Example:

A two span beam is supported by a spandrel beams at the outer edge and by a column in the centre. Dead load is (including beam weight) 1.0 kips/ft and live load is 2.0 kips/ft on both beams. Calculate all critical service load shear forces and bending moment for the beam. (Kips: kilo pounds per inch)



Solution:
Conditions for applying the ACI factors:

a- Loads are uniform distributed → ok
b- Live load /dead load =2.0(kips/ft)/1.0(kips/ft)=2 2<3→ok
c- (L'2 – L'1)/L'1= 22'-20'/20'=2/20=0.10<20% →ok

W=D.L. + L.L. =1+2 =3 kips/ft

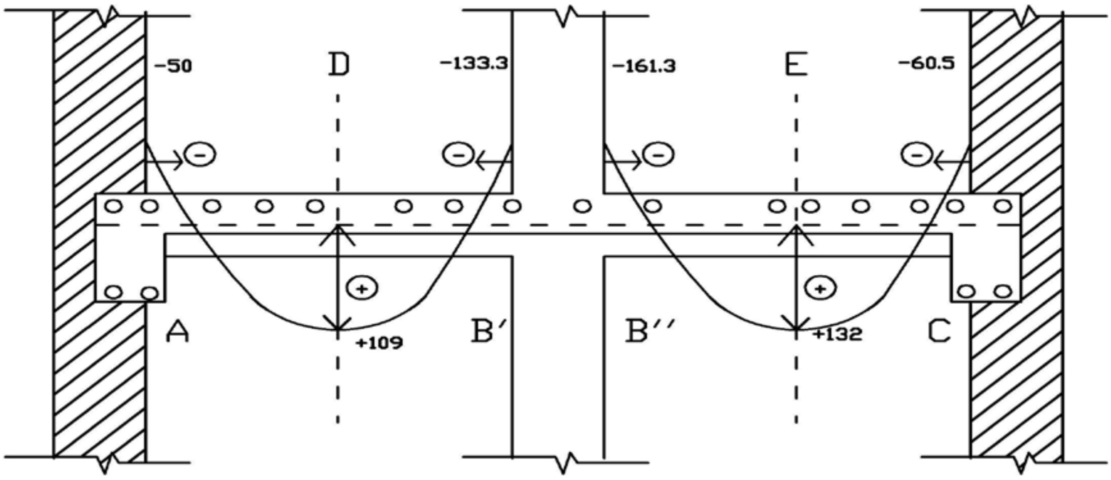
Moments:
*** Special case 2 spans

$M^{-}AB' = -W\,(L'1)^2/24 = -3(20)^2/24 = -50$ kips/ft
$M^{-}B'A = -W\,(L'1)^2/9 = -3(20)^2/9 = -133.3$ kips/ft
$M^{-}B''C = -W\,(L'2)^2/9 = -3(22)^2/9 = -161.3$ kips/ft
$M^{-}C\,B'' = -W\,(L'2)^2/24 = -3(22)^2/24 = -60.5$ kips/ft

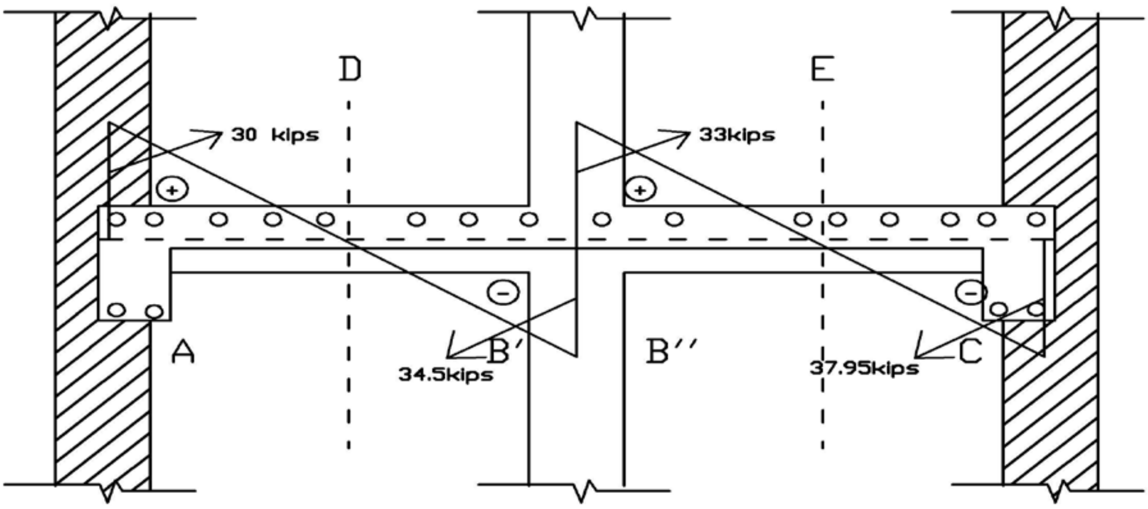
$M+D = W\,(L'1)^2/11 = 3(20)^2/11 = 109$ kips/ft
$M+E = W\,(L'2)^2/11 = 3(22)^2/11 = 132$ kips/ft

**** The masonry wall as support does not restrain the beam.

Bending Moment Diagram



Shear Force Diagram



$V_A = WL1'/2 = 30$ kips

V_C = WL2'/2 =33 kips

$V_{B'}$ = 1.15 WL1'/2 =34.5 kips

$V_{B''}$ =1.15 WL2'/2 =37.95 kips

Reactions:

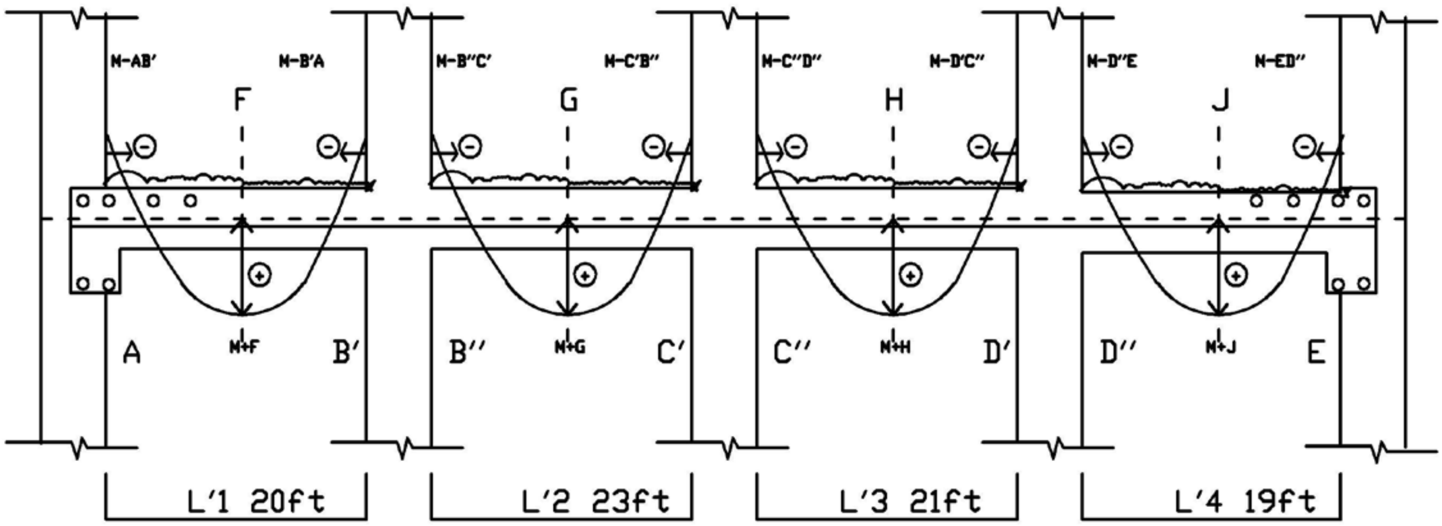
R_A = V_A=30 kips

R_C = V_C= 33 kips

R_B=|$V_{B'}$|+|$V_{B''}$|= 34.5 +37.95=72.45 kips

Example:

a given span shown in the figure has a total dead load of 1.5 kips/ft and live load of 2.5 kips/ft. Calculate the critical moments , shear forces and reactions .The beams built into at A and a column at E. The masonry wall at A does not offer restraint to beam AB.



Solution:

W=D.L. + L.L. =1.5(kips/ft) + 2.5 (kips/ft) = 4 kips/ft

First we should investigate weather the ACI coefficient can be applied or not.

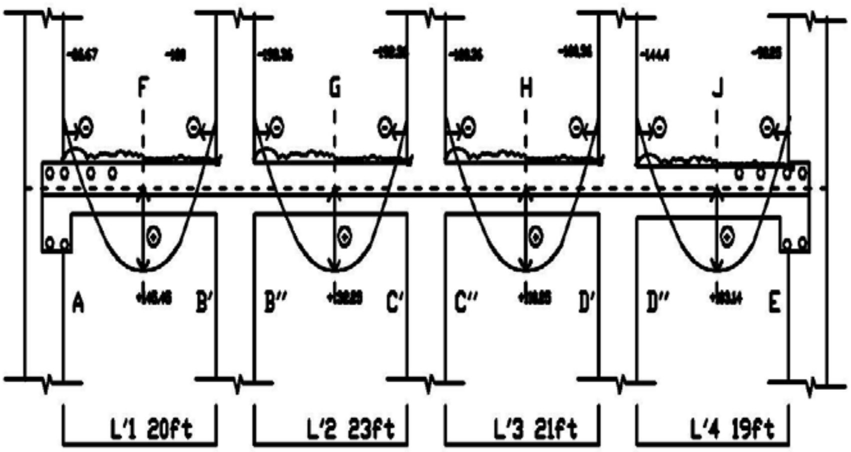
a- We have uniformly distributed load → ok
b- L.L./D.L. =2.5/1.5 =1.67 <3 → ok
c- (L'2-L'4)/L'4 = (23ft-19ft)/19ft =o.21 ≈ 20% →ok

Moments:
$M^{-}AB' = -W (L'1)^2 /24 = -4(20)^2/24 = -66.67$ kips/ft
$M^{-}B'A = -W (L'1)^2 /10 = -4(20)^2/10 = -160$ kips/ft
$M^{-}B''C' = -W (L'2)^2/11 = -4(23)^2/11 = -192.36$ kips/ft
$M^{-} C'B'' = -W (L'2)^2 /11 = -4(23)^2/11 = -192.36$ kips/ft
$M^{-} C''D' = -W (L'3)^2 /11 = -4(21)^2/11 = -160.36$ kips/ft
$M^{-} D'C'' = -W (L'3)^2 /11 = -4(21)^2/11 = -160.36$ kips/ft
$M^{-} D''E = -W (L'4)^2 /10 = -4(19)^2/10 = -144.4$ kips/ft
$M^{-} E D'' = -W (L'4)^2 /16 = -4(19)^2/16 = -90.25$ kips/ft

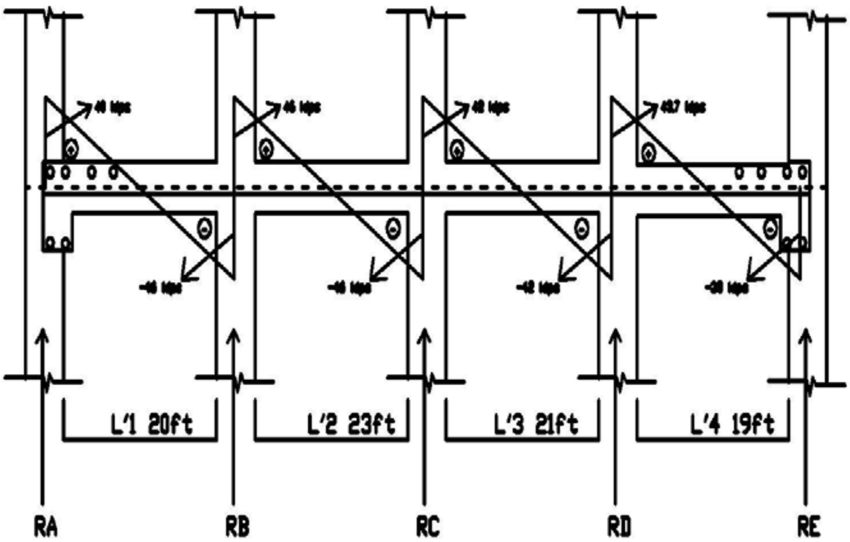
$M+F = W (L'1)^2 /11 = 4(20)^2/11 = 145.45$ kips/ft
$M+G = W (L'2)^2 /16 = 4(23)^2/16 = 132.25$ kips/ft
$M+H = W (L'3)^2 /16 = 4(21)^2/16 = 110.25$ kips/ft
$M+J = W (L'4)^2 /14 = 4(19)^2/14 = 103.14$ kips/ft
$V_A = WL1'/2 = 40$ kips
$V_{B'} = 1.15\ WL1'/2 = 46$ kips
$V_{B''} = WL2'/2 = 46$ kips
$V_{C'} = WL2'/2 = 46$ kips
$V_{C''} = WL3'/2 = 42$ kips
$V_{D'} = WL3'/2 = 42$ kips
$V_{D''} = 1.15WL4'/2 = 43.7$ kips
$V_E = WL4'/2 = 38$kips

Reactions:
$R_A = V_A = 40$ kips
$R_B = |V_{B'}|+|V_{B''}| = 46 + 46 = 92$ kips
$R_C = |V_{C'}|+|V_{C''}| = 46 + 42 = 88$ kips
$R_D = |V_{D'}|+|V_{D''}| = 42 + 43.7 = 85.7$ kips
$R_E = V_E = 38$kips

Bending Moment Diagram



Shear Force Diagram



CHAPTER 3- Lateral loads

CHAPTER 4 - Alternate Design Method

Notation

Ag = gross area of concrete cross section, in², (mm²)
As = area of tension reinforcement, in², (mm²)
A's = area of compression reinforcement, in², (mm²)
A**u** = total area of web reinforcement for shear Vs, measured in a direction parallel to the longitudinal reinforcement, in², (mm²)
a = angle between inclined stirrups or bent-up bars and longitudinal axis of a member, degrees (radians)
b = width of compression face of a flexural member, and smaller dimension of a column cross section, in (mm)
bo = perimeter of critical section for punching shear in slabs and footing, in (mm)
bw = width of web of a T-beam, I-beam or spanderel beam (inverted L-beam), in, (mm)
Bc = aspect ratio of a slab or column cross section long side / short side

c = a factor modifying the stiffness K of a member

Cc = compression force in concrete, kips (kN)
Cs = compression force in reinforcement, kips (kN)
d = distance from extreme compression fiber to centroid of tension steel, in (mm)
d' = distance from extreme compression fiber to centroid of compression steel, in (mm)
Ec = modulus of elasticity of concrete, psi (MPa)
Es = modulus of elasticity of steel (29x 1'000'000 psi or 200'000MPa)¨
fb = bearing stress, psi (MPa)
fs = stress in tension reinforcement or web reinforcement, psi (MPa)
f'_s = stress in compression reinforcement, psi (MPa)
f_y = yield strength of reinforcement, psi (MPa)
h = total thickness or total depth of a slab or beam, and larger dimension of a column cross section, in (mm)
h_f = flange thickness for T- beams, I – beams and spandrel beams (inverted L – beams) in, (mm)
I = moment of inertia of a cross section, in4, (mm4)
k = a factor related to the location of the neutral axis of a cross section
K = stiffness factor, E, I / L, in kips (kNm)
L = span length, ft (m)

M = bending moment, in kips (kNm)
n = modular ratio, Es / Ec
N = load normal to a cross section (axial load), to be taken as positive for compression and negative for tension, psi (MPa)
p = steel ratio for tension reinforcement, A / bd
p' = steel ratio for compression reinforcement, A's / bd
pf = steel ratio for tension reinforcement required to balance the compression force in overhanging flanges of T – beams, I – beams or spandrel beam (inverted L – beams), Asf / bwd
pw = steel ratio for tension reinforcement required to balance the compression force in the webs of T – beams, I – beams or spandrel beams, Asw / bwd

Introduction

The vast majority of reinforced concrete structures built in America before 1963 we proportioned based on a straight line theory called working stress design (referenced as the alternate stress design method in the 1983 ACI code).
When using working stress design techniques, members are proportioned so that they may sustain the anticipated real loads included (working or service loads) without the stresses in the concrete or reinforcing exceeding the proportional limits of the individual materials. Although the stress-strain diagram of concrete does not exhibit on-initial straight line proportion, it is still assumed that Hooke's Law does apply to concrete at stresses below 0.45f 'c.

This leads to the basic assumptions in working stress design required for the following derivations and discussion:

a. Plane sections before bending remain plane after bending.
b. Both the concrete and reinforcing steel obey Hooke's Law in the regions considered.
c. Strain is proportional to the distance from the neutral axis.
d. Tensile strength of the concrete is neglected.
e. Perfect bond or adhesion is developed between the concrete and reinforcing steel so that there is no slippage between the two materials.
f. The other basic assumptions concerning deformation and flexure of homogeneous members are valid.
g. The modulus of elasticity of concrete is giben by equation.
h. The modulus of elasticity of steel as given above.

Permissible Service Load Stresses:
Stresses in concrete shall not exceed the following:

Flexure
Extreme fiber in compression   0.45 f 'c

Shear

Beams and one-way slabs and footings:

Shear carried by concrete, VC $1.1\sqrt{f'c}$ ($\sqrt{f'c}$ / 11)

Maximum shear carried by concrete:

Plus shear reinforcement VC + $4.4\sqrt{f'c}$ (VC + $3\sqrt{f'c}/8$)

Joists

Shear carried by concrete, VC $1.2\sqrt{f'c}$ ($\sqrt{f'c}$ / 10)

Two way slabs and footings:

Shear carried by concrete, VC

$(1+ 2/\beta c)\sqrt{f'c}((1+ 2/\beta c)\sqrt{f'c}/12)$

but not greater than $2\sqrt{f'c}(\sqrt{f'c}/6)$

Bearing

On loaded area (columns) 0.3 f 'c

Tensile stress in reinforcement fs, shall not exceed the following:

Grade 40 or Grade 50 (Grade 300M) reinforcement

20'000psi (140MPa)

Grade 60 (Grade 400m) reinforcement or greater and welded wire fabric (smooth or deformed)

24'000psi (170MPa)

For flexural reinforcement, 3/8 in (10mm) or less in diameter, in one way slabs of not more than 12ft (4m) span 0.50 fu but not greater than 30'000 psi (200MPa)

Flexure

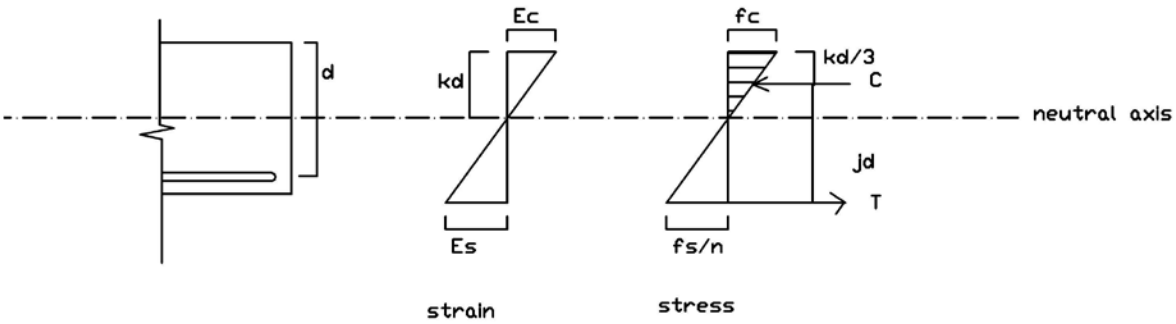
For investigation of stresses at service loads, straight-line theory (for flexure) shall be used with the following assumptions:

1. Strains vary linearly as the distance from the neutral axis, except, for the deep flexural members with overall depth-span ratios greater than 2/5 for continuous spans and 4/5 for simple spans, a non-linear distribution of strain shall be considered.
2. The stress-strain relationship of concrete is a straight line under service loads within permissible service load stresses.
3. In reinforced concrete members, concrete resists no tension.
4. The modular ratio, n = Es/Ec may be taken as the nearest whole number (but not less than 6). Except in calculations for deflections, the value of n for low-density concrete shall be assumed to be the same as for normal density concrete of the same strength.

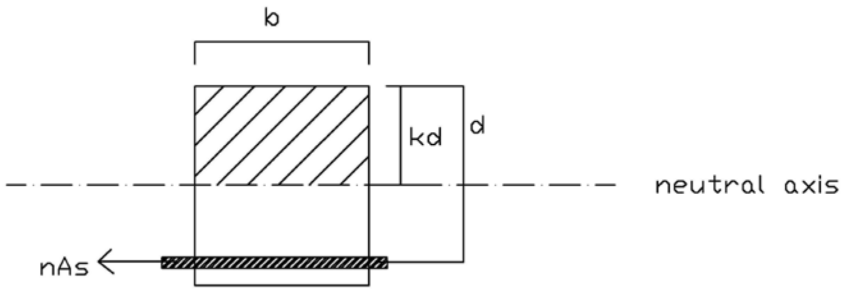
5. In doubly reinforced flexural members, an effective modular ratio of 2Es/Ec shall be used to transform compression reinforcement for stress computations. Compressive stress in such reinforcement shall not exceed permissible tensile stress.

Rectangular Beams

Distribution of strain & stress for a rectangular beam:



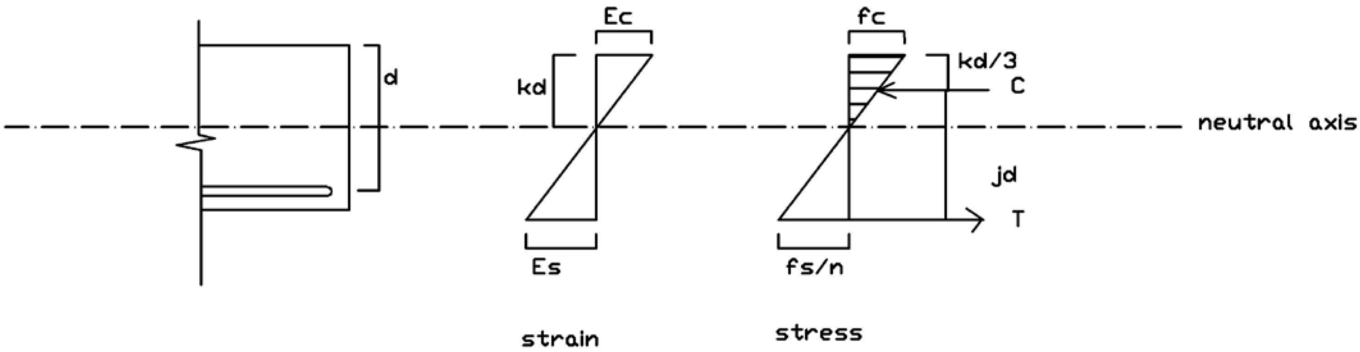
Ec = max strain of concrete
Es = max strain of steel
n = modular ratio
fc = max stress in concrete
fs = max stress in steel



Formulae:

fc = fs.k / n (1-k)
fs = n.fc(1-k) / k
k = √ (2p.n + (p.n)²) –p.n
j = 1- k/3
R = fc.j.k / 2
M = R.b.d²
d = √ (M / Rb)
p = kfc / 2fs
As = M / fsjd

Solved Problems



- T = area of steel x stress
- C = total compressive force = area of triangle = (f 'c x kd) / 2

Summation of horizontal forces must be equal to zero, and summation of moments must be equal zero.
Then C = T or 1 /2 fckdb = Asfs

And if the distance from centroid of the compressive stress to the centroid of tensile stress is jd, then

Mc = Cjd = 1 / 2fckd^{2}bj and ms = Asfsjd
jd = (d-kd) / 3 or j = 1-k / 3
Ec / Es = (kd) / (d-kd)
n = Es / Ec
fc = fsk / n(1-k)
fs = nfc(1-k) / k
nfc = fsk+nfck = k(nfc+fs)
or k = nfc / (nfc+fs) = 1 / (1+ (fs / nfc))
k = $\sqrt{}$ (2pn + (pn) ^{2}) –pn
R = 1 / 2fcjk
fc = 0.45 f 'c (allowable stress)
d = r$\sqrt{}$(M / bfc)
r = $\sqrt{}$(2 / jk)

$Mc = Ms = Rbd^2$

$d = \sqrt{(m / Rb)}$

$Mc = C \times jd$ (moment from steel bar)

$Ms = T \times jd$ (resisting moment of steel)

Failure (Collapse):

1. Steel fails before the failure of concrete (under reinforcement)
2. Concrete fails before the failure of steel (over reinforcement)
3. Concrete fails in the same time of steel failure (balanced)

Under reinforcement: $Ms = Tjd$

Over reinforcement: $Mc = Cjd$

Balanced: $Mc = Ms = Tjd = Cjd$

$fc = 0.4\ f'c$

$fs = 0.55\ fy$ (for buildings)

$fs = 0.5\ fy$ (for bridges)

Formulae:

$k = nfc / (nfc + fs)$

$fc = 0.45\ f'c$

$fs = 0.55\ fy$

$j = 1\text{-}k / 3$

$R = 1 / 2fcjk$

$d = \sqrt{(m / Rb)}$

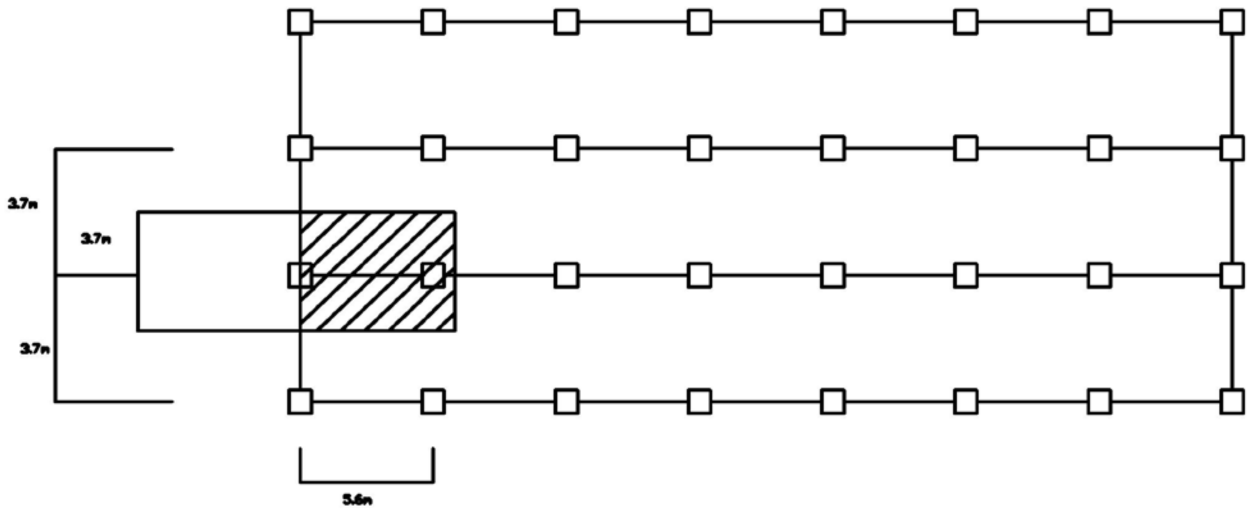
$As = M / fsjd$

Example:

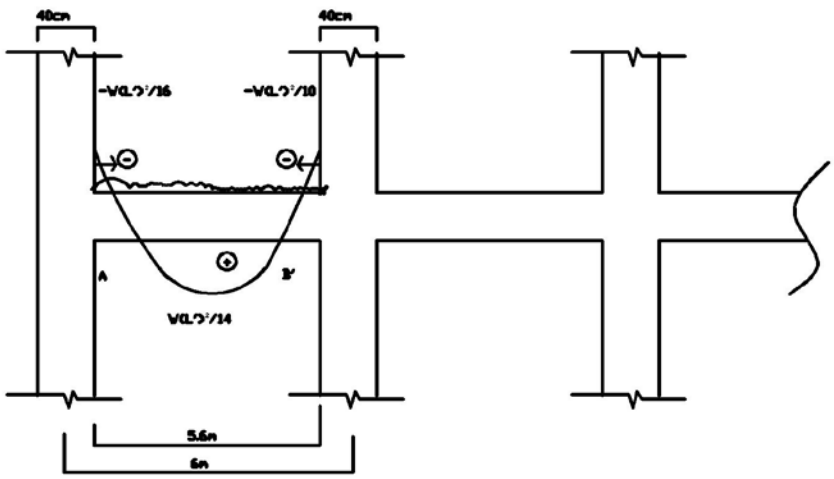
Find the dimensions and reinforcement of a beam which is under uniformly distributed loads of (Dead Load + Live Load).

Dead Load = 800 kgf/m², Live Load = 300 kgf/m². The length of the beam is 5.6m, and the spacing between the beams is 3.70m. $f'c$ = 250 kgf/cm², fy = 3000 kgf/cm². This beam is the first span of a continuous beam (more than 2 spans) and the exterior support is a concrete column.

Plan



Bending Moment Diagram



Step 1: Finding the bending moment

$W_{\square}$ = D.L + L.L = 800 kgf/m² + 300 kgf/m² = 1100 kgf/m²
W = S x $W_{\square}$ = 3.7m x 1100 kgf/m² = 4070 kgf/m

Moments:

M^{-}AB' = (-WL'1)² / 16 = (-4070kgf/m x (5.6m)²) / 16
= -7977 kgf/m
M^{-}B'A = (-WL'1)² / 10 = (-4070kgf/m x (5.6m)²) / 10
= -12763 kgf/m
ME = (WL'1)² / 14 = (4070kgf/m x (5.6m)²) / 14
= 9116 kgf/m

- Minimum dimensions (for controlling the deflection)
  h min = L / 18.5

For simple beam	h min = L / 16
For continuous beam (from one side)	h min = L / 18.5
For continuous beam (from two sides)	h min = L / 21

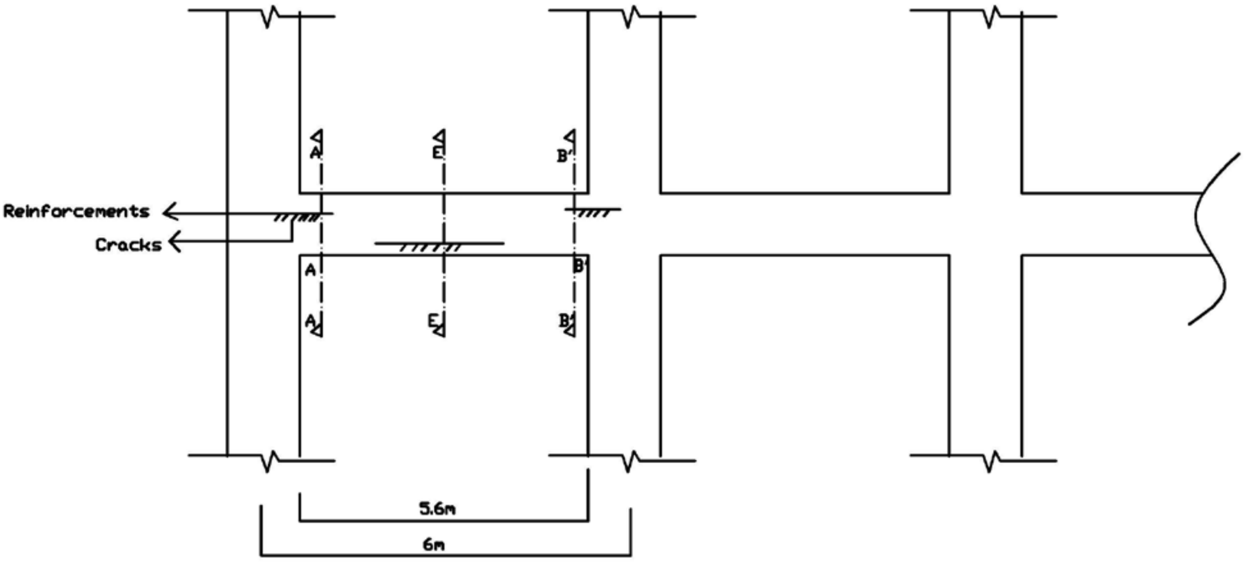
Step 2: Estimation for minimum dimensions

h min = L / 18.5 = 600 / 18.5 = 33cm → (take 40cm)
b min = h min / 2 = 40 / 2 = 20cm

Concrete cover = 2“ for beams
1” for slabs
2” foundation having lean concrete
3” foundation not having lean concrete
4” foundation on harmful soils

Step 3: Finding the beam height for applied bending moment
fc = 0.45 f ‘c = 0.45 x 250 = 112.5 kgf/cm²
fs = 0.55 fy = 0.55 x 3000 = 1650 kgf/cm²
n = Es / Ec = 2100000 / (15’000 $\sqrt{250}$) = 8.85 (take 9)
k = nfc / (nfc + fs) = (9 x 112.5) / (9 x 112.5 + 1650) = 0.38
j = 1-k / 3 = 1 – 0.38 / 3 = 0.87
R = 1 / 2fcjk = 1 / 2 x 112.5 x 0.87 x 0.38 = 18.5 kgf/cm²
d = √ (m / Rb) = √ ((12763 x 100) / (18.5 x 20)) = 58.7 cm
h = d + concrete cover = 58.7 + 5 cm = 63.7cm (take h = 65cm)
h = 65 > h min = 40 → OK

d = 65 – 5 = 60cm



Step 4: Finding the reinforcements

$M^{-}AB'$ = -7977 kgfm

$As^{-})_{AB'}$ = M / fsjd =(7977 x 100)/(1650 x 0.87 x 60)= 9.26cm²

$M^{-}B'A$ = -12763 kgfm

$As^{-})_{B'A}$ = M / fsjd =(12763 x 100)/(1650 x 0.87 x 60)= 14.81cm²

ME = 9116 kgfm

$As)_{E\,=}$ M / fsjd = (9116 x 100)/ (1650 x 0.87 x 60) = 10.58cm²

$As^{-})_{B'A}$ = 14.81cm² → 4 x (π Φ²) / 4 = 14.81cm²

Φ = √ ((14.81) / π) = 2.17cm → USE 4Φ22

$As^{-})_{A\,B'}$ = 9.26cm² → 3 x (π Φ²) / 4 = 9.26cm²

Φ = √ ((9.25) / 3 π)

or n_1 x (π Φ²) / 4 = 9.26cm²

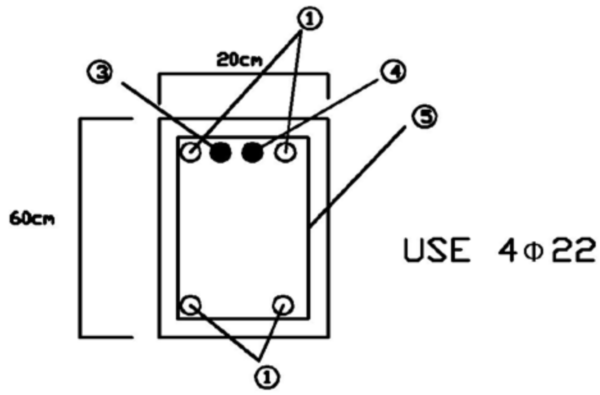
n_1 x (π (2.2)²) / 4 = 9.26cm²

$n_1 = 2.43 \sim 3 \quad \rightarrow \quad$ USE $3\Phi22$

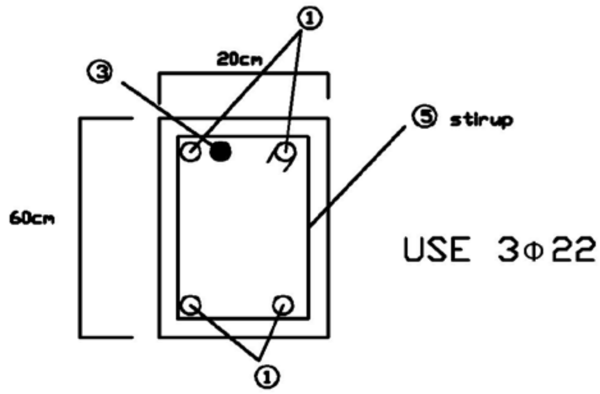
$As)_E = 10.58cm^2 \quad \rightarrow \quad n_2 \times (\pi\, \Phi^2) / 4 = 10.58cm^2$

$n_2 \times (\pi\, (2.2)^2) / 4 = 10.58cm^2$

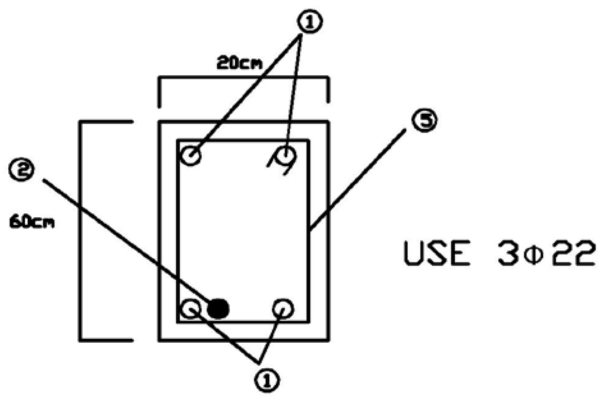
$n_2 = 2.79 \sim 3 \quad \rightarrow \quad$ USE $3\Phi22$



Section B′–B′



Section A–A



Section E–E

position	size	number	form	length
1	2.2	4		6000
2	2.2	1		4000
3	2.2	1		3000
4	2.2	1		3000
5	10	23		1500

1 for every 23 cm

CHAPTER 5 - Ultimate Strength Design: USD

Differences between WSD (Working Stress Design) and USD (Ultimate Stress Design)

1- Load Factors

In WSD: Load factors = 1

In USD: Load factor for Dead Load = 1.4
Load factor for Live Load = 1.7

We use reduction factors (ø) ø = 0.9 for bending
ø = 0.85 for shear
ø = 0.7 &
ø = 0.75 for columns

2- The Allowable Stresses and Strain

In WSD: fc = 0.45 f ‘c, fs = 0.55 fy

In USD: $fs = fy$, $\varepsilon u = 0.003$
εu (ultimate strength in concrete)

Ultimate Strength Design (USD):

Notation

As = area of tension reinforcement
A’s = area of compression reinforcement
Asf = area of reinforcement to develop compressive strength of overhanging flanges in I and T – sections
a =depth of equivalent regular stress block = β1
b = width of compression face of flexural member
bw = width of web in I and T section
c = distance from extreme compression fiber to neutral axis at ultimate strength
D = dead load
d = distance from extreme compression fiber to centroid of compression reinforcement
d’ = distance from extreme compression fiber to centoid of compression reinforcement
E = earthquake load
f ‘c = compressive strength of concrete
fy = yield strength of reinforcement
hf = flange thickness in I and T – sections
L = specified live load plus impact
Mn = nominal (or design) resisting moment of cross section

= Mu / ø ø =bending 0.9 ø =shear 0.8 ø =compression 0.7
p = As / bd δ = % of steel
p' = A's / bd
pb = reinforcement ratio producing balanced conditions at ultimate strength (reinforcement & concrete at same failure)
pf = Asf / bwd
pw = As / bwd
w = Asfy / bdf 'c
U = required ultimate load capacity of section
W = wind load
Ø = capacity reduction factor

Introduction

Strength design procedures differ from the alternate design procedures of Chapter 4. In the former it is recognized that at high stress levels in concrete, stress is not proportional to strain, and second, in strength design procedures design loads are multiplies of anticipated service loads. In the alternate design method, stress is assumed to be proportional to strain and design loads are to service loads.

Before derivation of the basic equation and relationships it is necessary to delineate the basic assumptions, which are:

1. Plane sections before bending remain plane after bending
2. At ultimate capacity, strain and stress are not proportional
3. Strain in the concrete is proportional to the distance from the neutral axis
4. Tensile strength of concrete is neglected in flexural computations
5. Ultimate concrete strain is 0.003
6. The modulus of elasticity of the reinforcing steel is 29'000'000 psi (200'000 MPa)
7. The average compressive stress in the concrete is 0.85 f 'c
8. The average tensile stress in the reinforcement does not exceed fy

The mechanics and derivations using the equivalent rectangular stress distribution, as assumed in the 1983 ACI Code, are somewhat simpler. It is further assumed that
a = βc and that β1 be taken as 0.85 for concrete strengths of 400 psi (30 MPa) or less; whereas, for greater strength, β1 shall be reduced at the rate of 0.05 for each 1000 psi of strength in excess of 4000 psi (at the rate of 0.008 for each 1.0 MPa of strength in excess of 30MPa), provided β1 does not fall below 0.65.

Load Factors and Under Strength Factors

Idealized capacity (u) = ultimate load (C1D + C2L)
ø x idealized capacity (øu) = ultimate load (C3D + C4L)

For bending moment, the nominal moment is Mu / ø = Mn

ø = 0.90 for flexure
ø = 0.85 for diagonal tension, bond and anchorage
ø = 0.75 for spirally reinforced compression members
ø = 0.70 for tied compression members
ø = 0.70 for bearing on concrete
ø = 0.65 for flexure in plain concrete (not reinforced)

U = 1.4D + 1.7L
U = 0.75 (1.4D + 1.7L + 1.87E) for earthquake
U = 0.75 (1.4D + 1.7L + 1.7W) for wind loading
U = 0.9D + 1.3W for high rise buildings

Flexural Computations by Strength Design

If the beam is under reinforced so that the failure in flexure is due to a yielding of steel, the expression for the nominal resisting moment assuming a rectangular stress distribution is

$$Mn = f'cbd^2w\,(1-0.59w)$$

In the ASCE-ACI Report and in the 1956 Building Code, "w" was limited to a maximum value of 0.40; in the 1983 Code an expression for the balanced steel ratio was presented:

$$pb = ((0.85f'c\,\beta 1) / (87'000 + fy)) \times ((0.85f'c\,\beta 1) / fy\,(600 / (600 + fy)))$$

% of steel due to balanced collapse

Compression Reinforcement

The moment of additional compression steel would be
M'n = A'sfy (d-d') if the compression steel is stressed to the yielding. Adding these two moments and deducting the effect of the concrete in compression occupied by the compressive reinforcement,

$$Mn = (As-A's)\,fy\,(d-a/2) + A'sfy\,(d-d')$$

For the expression to be valid, it is necessary that the compressive reinforcement reach its yield strength at the ultimate strength of the member. This is satisfied if:

$$p-p' \geq ((0.85f'c\beta 1d') / fyd) \times (87'000 / (87'000-fy))$$

I and T sections

The nominal capacity of an I or T section is:

$$Mn = (As-Asf)\ fy\ (d-0.5a) + Asffy\ (d-0.5hf)$$

This expression assumes that the section acts as and I or T section and that the neutral axis of the sections falls without the flanged section. The neutral axis falls within the flange if:

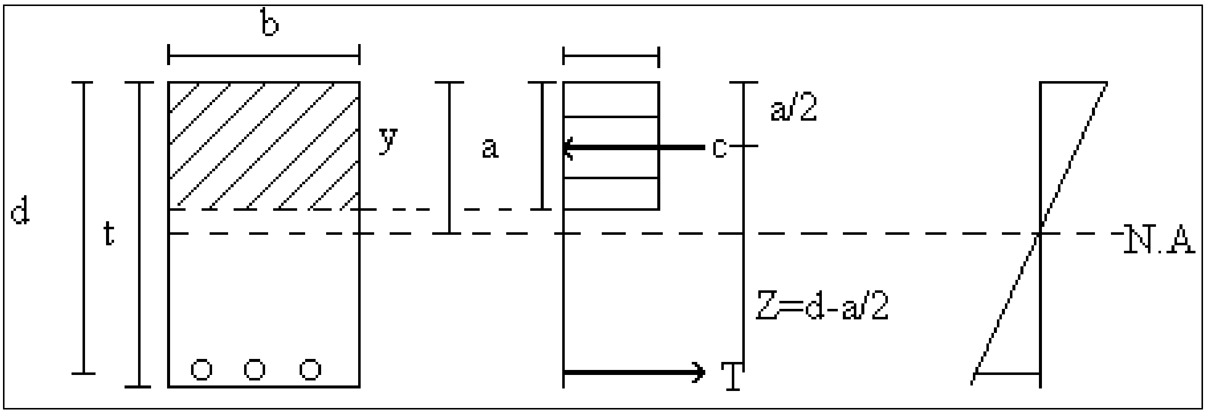
$$hf \geq 1.18wd / \beta 1$$

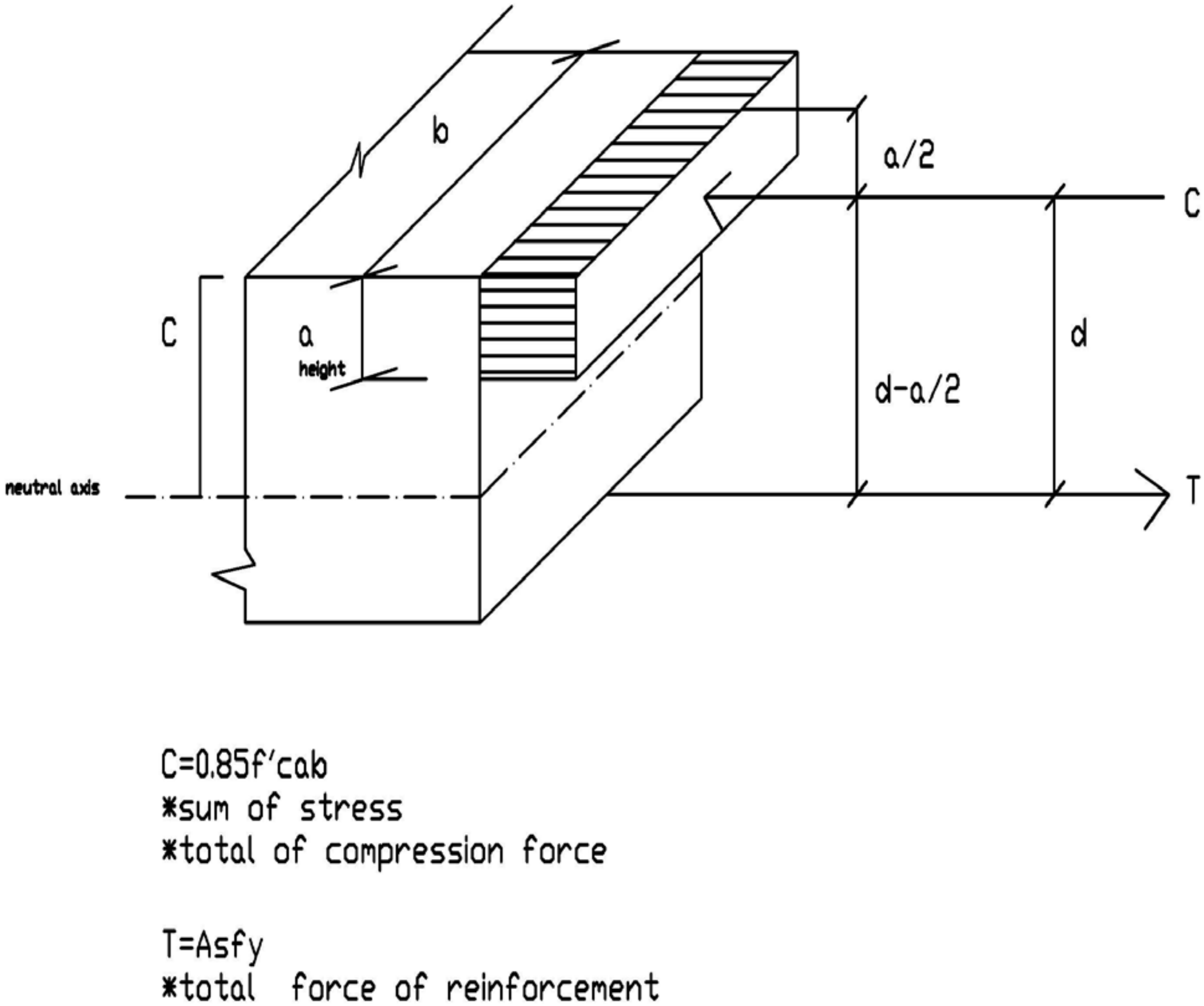
If the nominal moment capacity of the flange, is greater than the nominal applied moment, then the member is proportioned as a rectangular beam.

$$Mnf = 0.85f\text{‘}cbhf\ (d-0.5hf)$$

In I and T sections, the area of tension steel available to develop the concrete of the web in compression is limited to 0.75pb, that is

$$As / bwd – Asf / bwd \leq 0.75pb \quad \text{or} \quad pw-p\text{`}f \leq 0.75pb$$



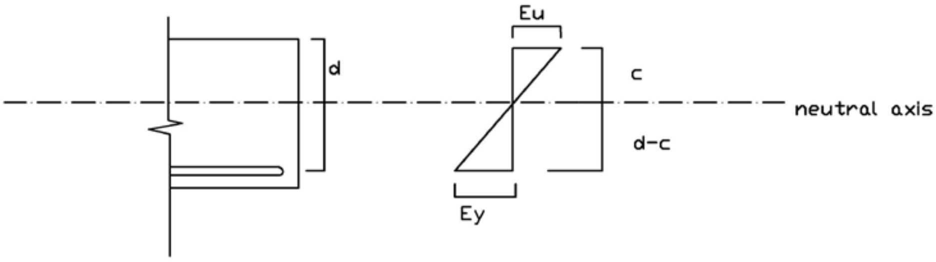


Summing the horizontal forces, T = C, then

$A_sf_y = 0.85f'_cab$ and $a = A_sf_y / (0.85f'_cb)$

Summing the moments about the centroid of the compressive force, $M_n = A_sf_y(d-a/2)$ Substituting for a,

$M_n = A_sf_yd\,(1-0.59\rho f_y/f'_c)$ or $M_n = f'_cbd^2w\,(1-0.59w)$

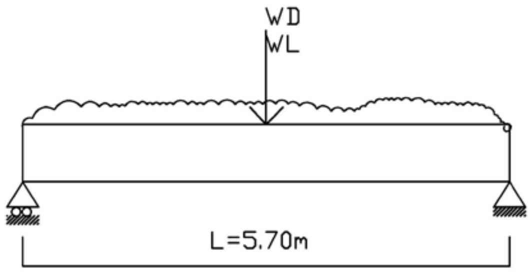


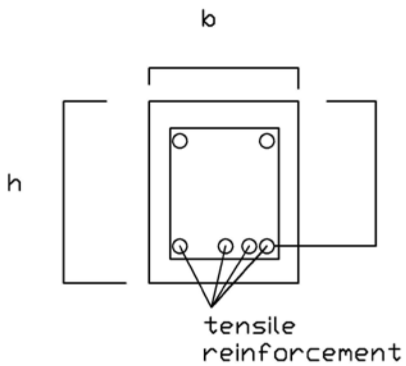
Relation between Mn & Mu : Mn = Mu / ø

Example:

(For design of a beam of a rectangular section with tensile reinforcement by using USD). Suppose a simple beam under Dead Load of 750 kgf/m² and Live Load of 250 kgf/m². Design the section of the beam (rectangular section). The length of the beam is 5.70m, and the spacing between the beams is 5.5m.

f 'c = 250 kgf/cm², fy = 3000 kgf/cm². Please design this beam using USD method (Ultimate Strength Design). In Dead Load, the weight of the beam is included.





Solution:

WD = 750 kgf/m²
WL = 250 kgf/m²
L = 5.70 m
S = 5.50 m
f ‘c = 250 kgf/cm²
fy = 3000 kgf/cm²

WD = WD▫ x S = 750 kgf/m² x 5.50 m = 4125 kgf/m
WL = WL▫ x S = 250 kgf/m² x 5.50 m = 1375 kgf/m

WD = 4.125 ton/m
WL = 1.375 ton/m

Step 1: Controlling the minimum dimensions of the beam

h min = L / 16 = 570 / 16 = 35.6cm → (take 40cm)
b min = h min / 2 = 40 / 2 = 20cm

Step 2: Calculation of loads and bending moment

Here Dead Load and Live Load are uniform distributed, therefore:

Wu = 1.4WD + 1.7WL
= 1.4(4.125) + 1.7(1.375) = 8.1 ton/m

Mu = WuL² / 8 = (8.1 ton/m x (5.7)²) / 8 = 32.8 ton m

Now we are looking for finding the resistant ultimate bending moment of the beam which can bear the applied ultimate bending moment

Mn = Asfyd(1-0.59pfy/f ‘c)
Mn = Mu / ø
Mu= ø Asfyd(1-0.59pfy/f ‘c)
ø = 0.9
Mu = 0.9 Asfyd(1-0.59pfy/f ‘c)

This is one equation with two unknowns, we should here use a kind of trial and error method (for example by fibbing a value of p and d and find Mu and compare it with the applied Mu)

p = As / bd

As per ACI code:

pmin = 14.1 / f ‘c (where f ‘c is in kgf/cm)

pmax ≤ 0.75 pb
where pb = ((0.85f ‘c β1) / fy) x (0.003 / (0.003 +єy)

$\beta 1 = 0.85$
$\epsilon y = fy / Es$

$p = 14.1 / fy = 14.1 / 3000 \rightarrow pmin = 0.0047$
$pmax = 0.75fb \rightarrow$
$pmax = 0.75 \times 0.85 f`c \times 0.85 / fy \times 0.003/ (0.003 + 3000/2.1x10000000)$
$pmax = 0.00143 \rightarrow$ take 0.0014
As = pbd
As = 0.0014 x 20 x 35 = $9.8cm^2 \rightarrow$ put in Mu
Mu = 0.9 As.fy.d (1 – 0.59p fy / f `c)
Mu = 0.9 x $9.8cm^2$ x 3000kgf / cm^2 x 35cm(1 – 0.59 x 0.0014 x 3000/250) = 834305 kgf .cm
Mu = 8.34305 t.m.

8.34305 t.m. < 32.8t.m → not good

Second Cycle of Trial and Error:

Try : h=70cm, b=35cm, d=65cm

pmin = 0.0047
pmax = 0.0014
Try p= 0.01
As = pbd
As = $22.75cm^2$
Mu = 0.9 x $22.75cm^2$ x 3000kgf / cm^2 x 65cm(1 – 0.59 x 0.01x 3000/250) = 370995 kgf .cm
37.0995 t.m > 32.8t.m → not good

Third Cycle of Trial and Error:

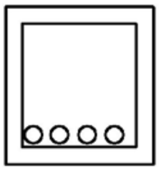
Try : h=90cm, b=35cm, d=65cm

pmin = 0.0047
pmax = 0.0014
Try p= 0.009
As = pbd
As = $20.48cm^2$
Mu = 0.9 x $20.48cm^2$ x 3000kgf / cm^2 x 65cm(1 – 0.59 x 0.009x 3000/250) = 3365215 kgf .cm
33.65t.m - 32. 33.65t.m / 32.8t.m = %2.5 < % 5

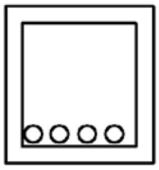
33.65t.m > 32.8t.m → O.k

<u>Design of Rectangular Section Reinforced Concrete Beams with Tensile and Compression Reinforcement:</u>

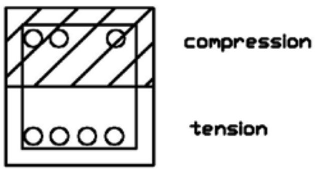
1-Rectangular section by using W.S.D. (working Stress Design)(for only tensile reinforcement in section) Alternative Method



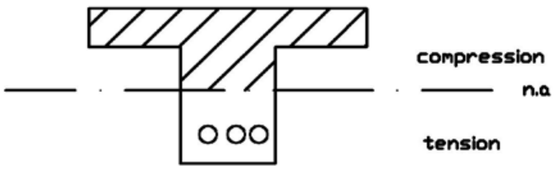
2-Rectangular section by using U.S.D.(Ultimate Stress Design)(for only tensile reinforcement in section) Strength Design



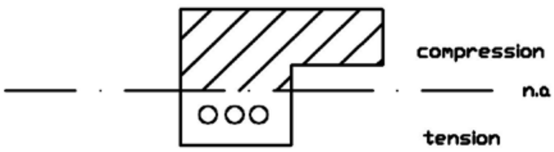
3-Rectangular Section by using U.S.D. (for the case that we have tensile and compression reinforcement in section)



4-T-section by using U.S.D

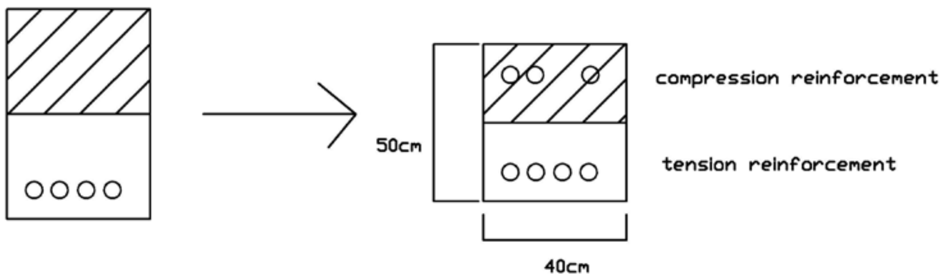


5-L-section by using U.S.D



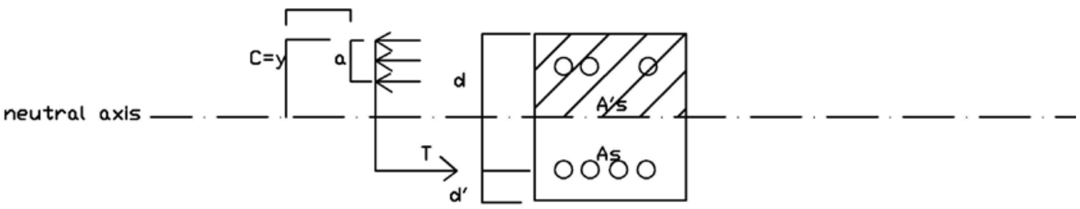
Q: When we use compression reinforcement in section in addition to tensile reinforcement?

A: When the dimensions are limited by architecture or designer for example height should be limited to a value, in this case for tension we can add the tensile reinforcement but for compression, the compression stress value exceeds the acceptable stress and therefore we should add compression reinforcement to help and reduce the compressive stress.



The Useful Formulas:

Mn = (As – A`s) fy (d – a/2) +A `s fy (d – d`)
Mu = øMn
Mu = ø [(As – A`s) fy (d – a/2) +A `s fy (d – d`)]

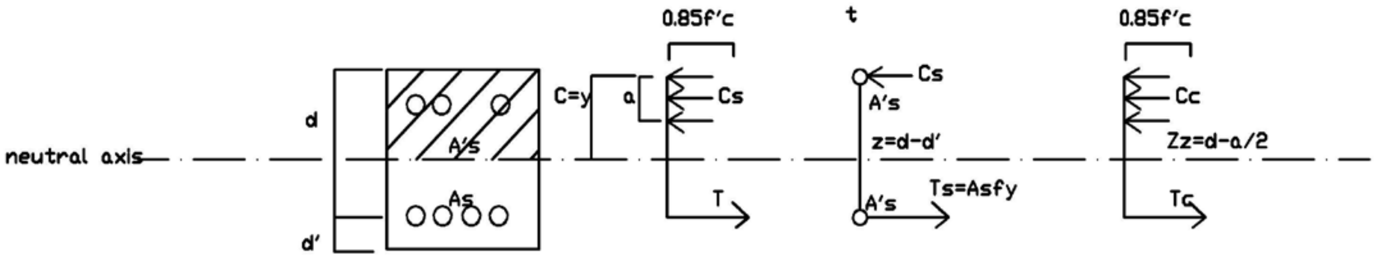


p- p` ≤ 0.75 pb → (p- p`) max = 0.75 pb
pb = 0.85 x 0.85 x f `c/fy x 0.003 / 0.003 x fy/ Es

p = As/bd

p` = A`s/bd

p- p`≥ ((0.85f ‘cβ1d’) / fyd) x 0.003 / 0.003 x fy/ Es



Tensile As → A`s

→ (As - A`s)

Ts =Asfy

Z = (d – d`)

Cc = 0.85 f `c(a.b)

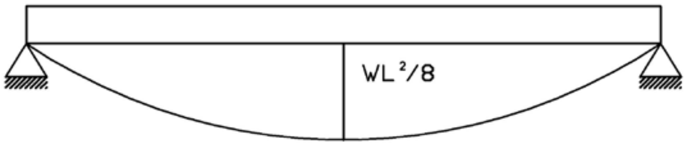
Mn1 = Ts x Z1

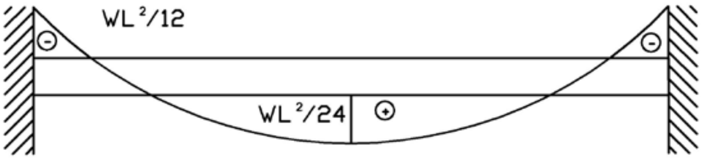
= Asfy x (d – d`)

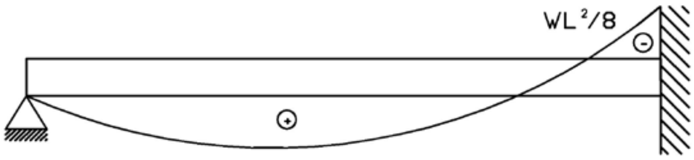
Mn2 = To x Z2

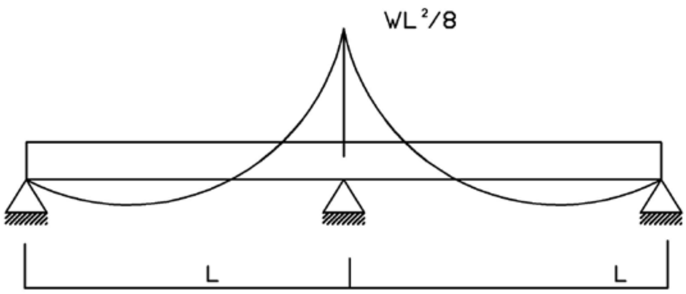
= (As - A`s) fy (d –a/2)

Mn = Mn1 + Mn2 = Asfy x (d – d`) + (As - A`s) fy (d –a/2)



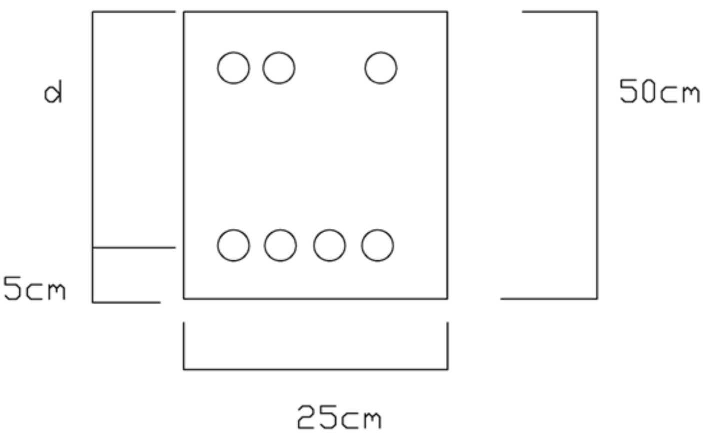






Example:

Please find reinforcements for a beam having rectangular section of 50cm x 25cm. This beam is under an ultimate bending moment of 36t.m. Assume f `c =280 kgf/cm², fy = 4200 kgf/cm²,
Es =2.1x10000000 kgf/cm².



Solution:

In this example we have limitation in height (because the height of 50cm is imposed to us by architectures) and perhaps we need to use compressive reinforcement in section in addition to tensile reinforcement. For this reason and for respecting the economic aspect first we try to put the maximum tensile reinforcement in section only and then evaluate the resisting bending moment in section for this case and compare with the imposed (applied) ultimate bending moment.

1 – We assume that we have only tensile reinforcement in section. The maximum reinforcement → pmax = 0.75 pb

pb = 0.85 x 0.85 x f \`c/fy x 0.003 / 0.003 x fy/ Es
=0.0289 → %2 ok
pmax =0.75 pb = 0.75 x 0.0289 =0.0216

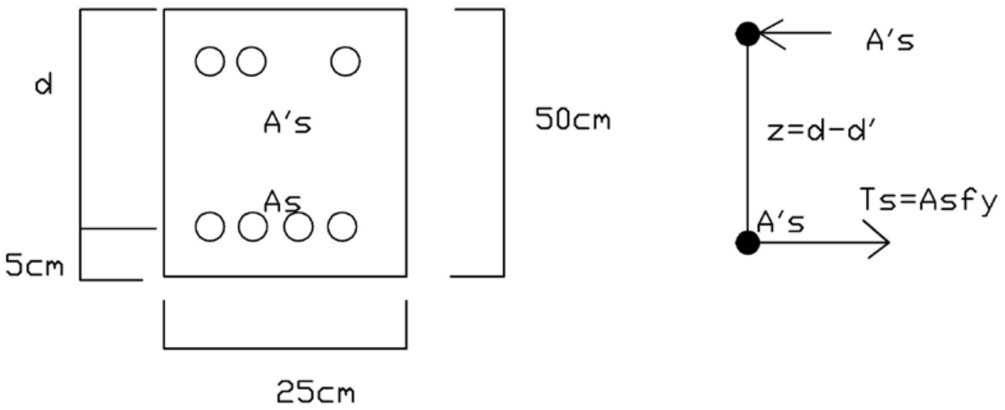
If we apply only p=pmax =0.0216 in section we will have:

Mu = 0.9 [As.fy.d (1 – 0.59p fy / f \`c)] (because we assume that we have only tensile reinforcement)
As = p bd
Mu = 0.9 [p bd.fy.d (1 – 0.59p fy / f \`c)]
Mu = 0.9 [0.0216 x 25 x 4200 45^2x (1 – 0.59x0.0216 4200/ 280)
= 3343383 kgf.m
33.43t.m < 36t.m → O.k

Using only tensile reinforcement is not enough and we should use extra compression reinforcement.

2 – In the section we consider A \`s and As :

Mu = ø [(As – A\`s) fy (d – a/2) +A \`s fy (d – d\`)] → A
Mu (applied) –Mu (resisting) = 36t.m. –33.433 = 2.567t.m.
Mu1 - Mu = T x Z= Asfy x (d – d\`) → B
A\`s = Mu(applied) –Mu(resisting) / fy (d – d\`) → B
=(2.67) x 1000000 /4200(45 – 5) =1.53



As you observe, in formula A we have two un-knowns (As, A\`s), in other words we have one equation A and two unknowns for finding As and A\`s we should use a trial& error method which takes time.
In the section we consider A\`s and As but we know that A\`s is imposed to the section due to extra moments (Mu (applied), Mu (resisting)), therefore we can use formula B for finding directly A\`s.
For finding (As – A\`s) we use the same pmax =0.0216 which we applied in formula for finding Mu (resisting) , therefore ;

(As – A\`s) = pmax.bd
(As – A\`s) = 0.216 x 25 x 45
(As – A\`s) = 24.30 cm^2
As = (As – A\`s) + A\`s = 24.30 cm^2 + 1.53 cm^2 = 25.83 cm^2

2 – Verification of p and p\`

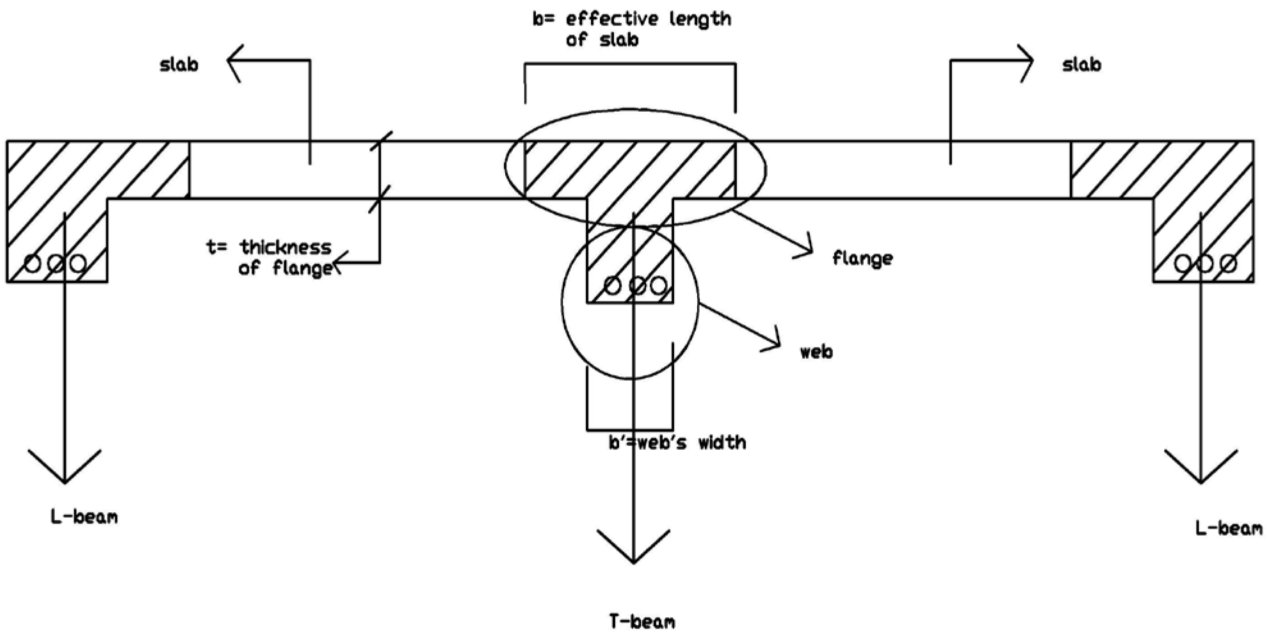
a- p- p\` ≤ pmax
p = As /bd =25.83/25x45 = 0.0229
p\` = A\`s /bd =1.53/25x45 = 0.00136
pb= 0.0288 → pmax = 0.75 pb = 0.0216
p- p\` = 0.0229 – 0.00136 < pmax = 0.0216
(p- p\` is lesser than pmax) → OK

b- p- p\`≥ (0.85f ‘cβ1) / fyd) x 0.003 / 0.003 x fy/ Es
=(0.85 x 280 x 0.85) / 4200 x 45) x 0.003 / 0.003 x 4200/ 2.1 x 1000000) = 0.0032→ OK

<u>Design of T-Beams:</u>

If N.A. is on the flange → Regular Beam
If N.A. is on the web → T-Beam

If the slab and rectangular beam casted (fabricated) monotically (in the same time) therefore we have T-beams, and for corner beams we have L-beams:



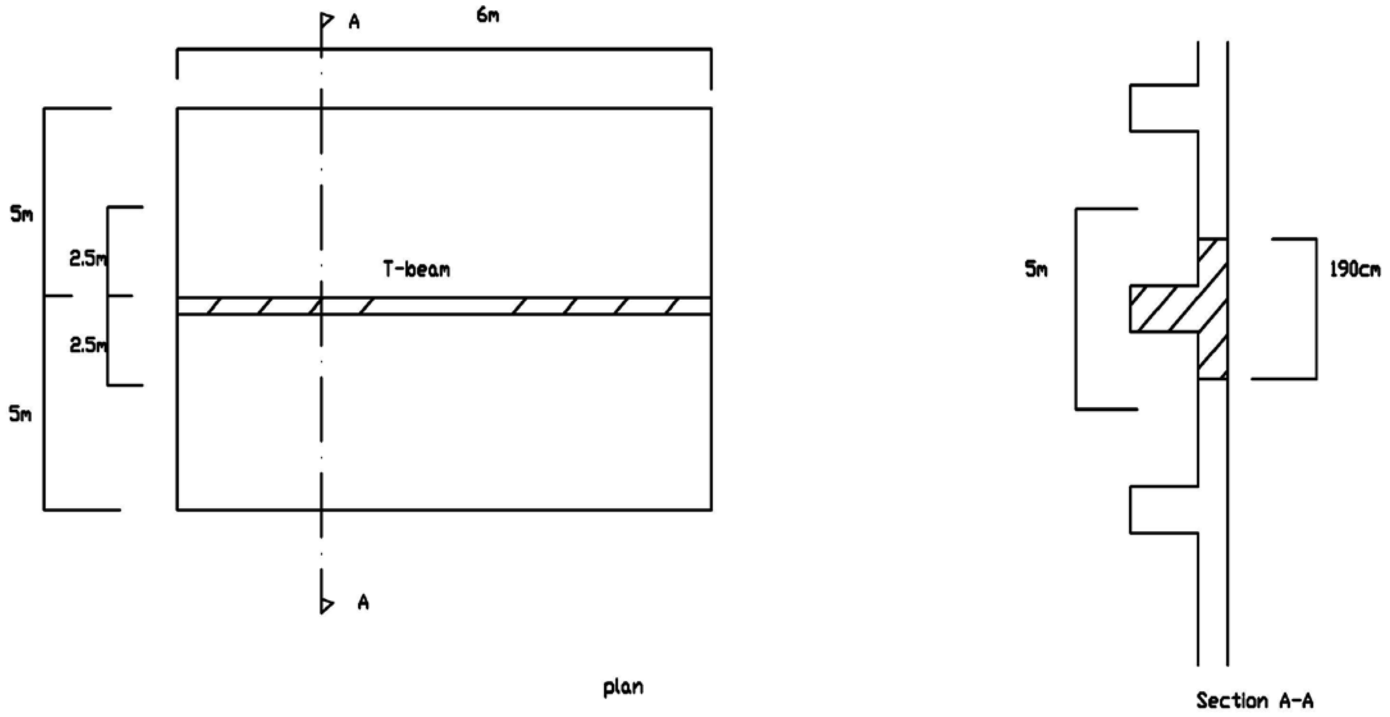
1-According to the ACI code the minimum of the following values is considered as b:

b= min 16t+b'
L/2
S

Where:
b= effective width of flange
t= thickness of flange (slab)
L= length of T-beam
S= spacing between beams

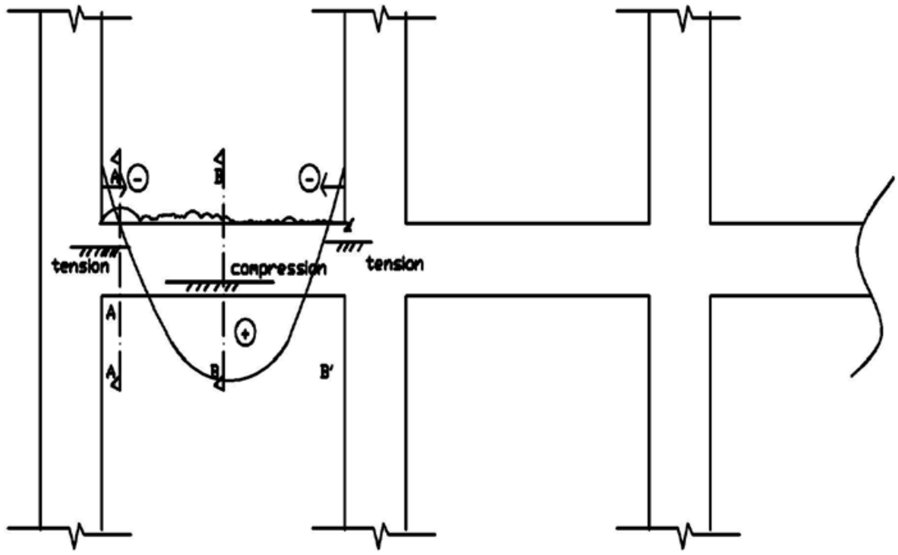
Example: L=6m, S=5m, t=10cm, b'=30cm

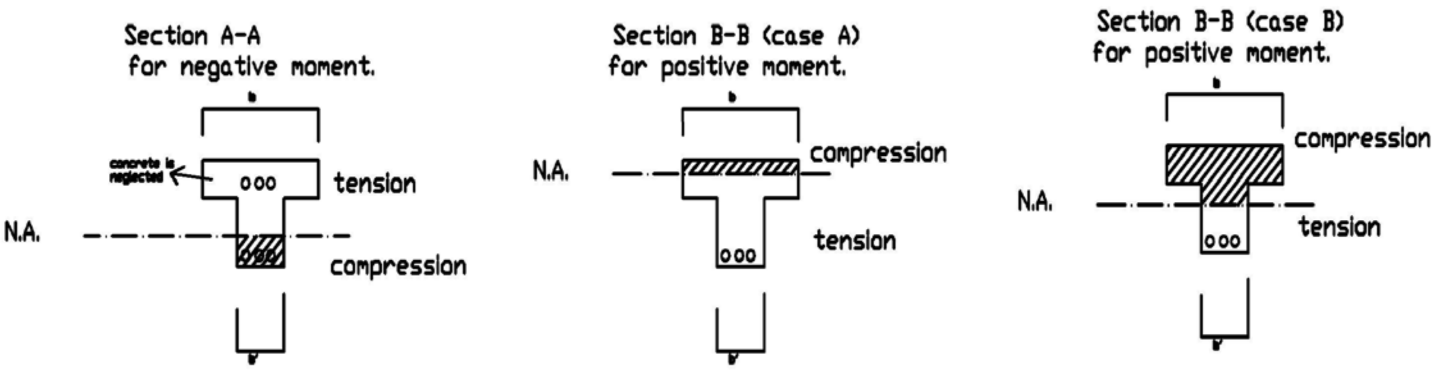
b= min 16t+b'=16(10)+30=190cm
L/2=600/2=300cm
S=500cm
b=190cm



2-Behaviour of Beam as T or Rectangular Beam:

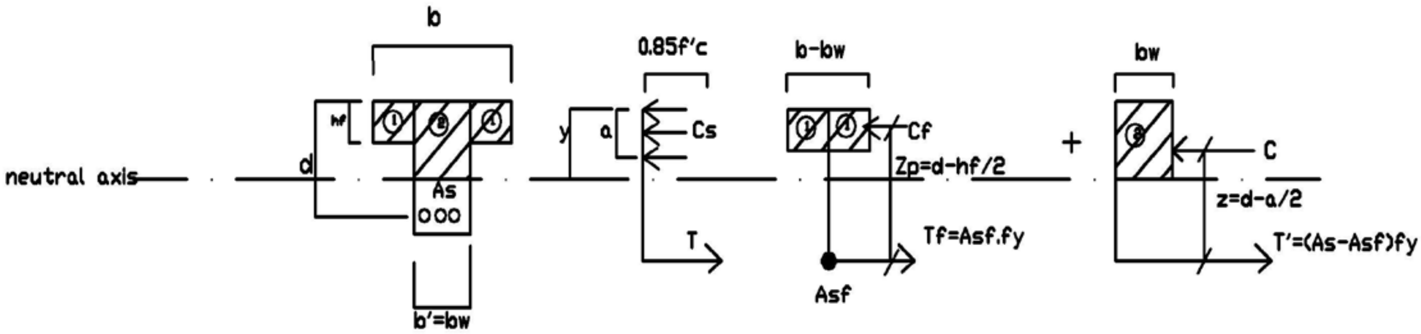
Behaviour of beam depends on the situation of neutral axis and the sign of bending moment. We can identify four cases for situation of the neutral axis and the sign of bending moment as follows.





In design of R/C elements we neglect the tensions concrete and consider the compression concrete along with the reinforcement.
The design of T-beam is like design of a rectangular beam with width of b'.

T-Beams:



Tensile Reinforcement As → Asf

(As-Asf)

Mn = Mnf + M'n

Mu = Muf + M'u

Mnf = tf x Zf = Asfy x (d – hf/2)

M'n = (As – Asf) fy (d - a/2)

Mn = Mnf + M'n

Mn = Asf.fy (d – hf/2) + (As –Asf) fy (d – a/2)

Mu = φMn

Mu = φ [Asf.fy(d – hf/2) + (As –Asf) fy (d – a/2)]

Mnf = 0.85 f 'c bhf (d – 0.5hf)

Cf = 0.85 f 'c (b – bw) hf (d – hf/2)

(b – bw) →for hanged part

b →for all flange

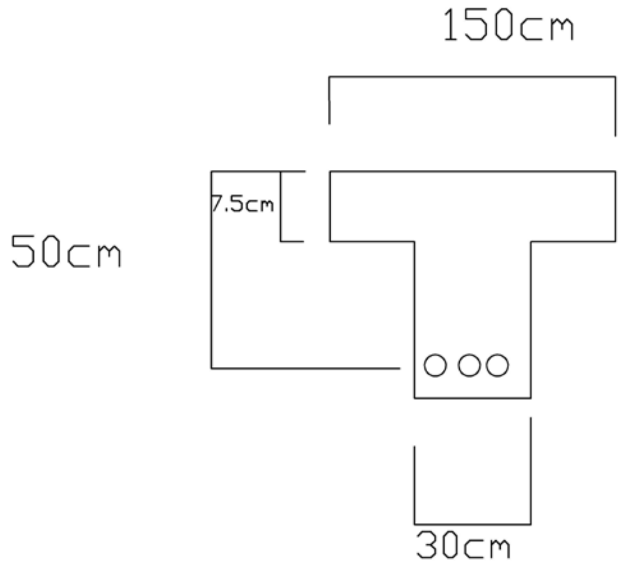
Zf = d – hf/2

Mnf = 0.85 f ‘c (b – bw) hf (d – hf/2) (for hanged par of the flange)

Example:

For the mid span of a beam , the shown section is proposed if the applied ultimate bending moment this section is equal to 32t.m. .Please determine the required reinforcement.

f ‘c = 280 kgf/cm², fy = 4200 kgf/cm²



Solution:

1- Here we have b = 150 cm (other wise b should be calculated)
2- Here because we have positive moment we should only verify the position of neutral axis. For this we should calculate the resisting moment of flange and compare with the given moment.

Mnf = 0.85 f ‘c bhf (d – 0.5hf)

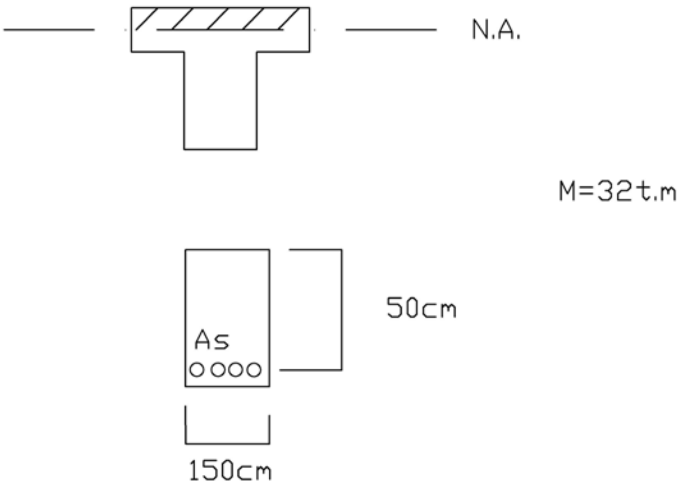
Mu = φMn

Mu = φ[Asf.fy(d – hf/2) + (As –Asf) fy (d – a/2)]

= 0.9 x 0.85 x 250 x 150 x 7.5(50 – 7.5/2)

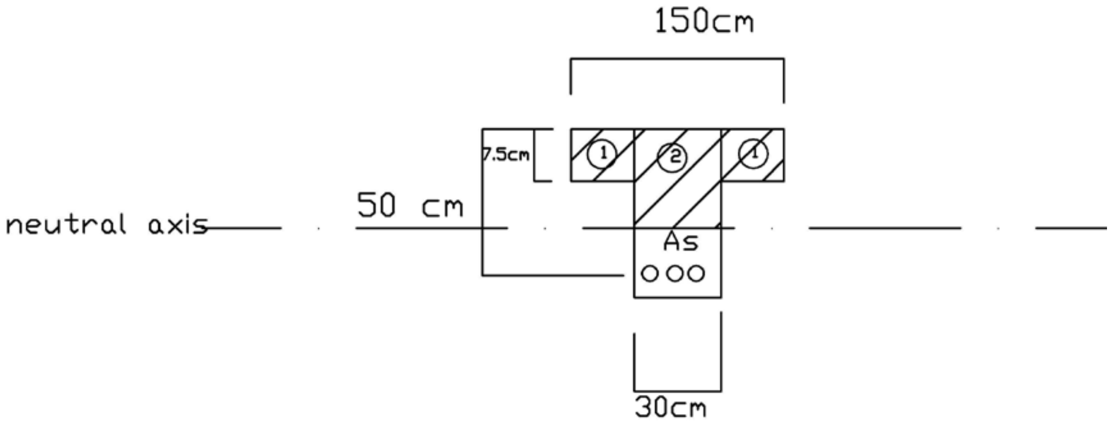
= 11145093kgf/cm

Mu = 111.451 t/m > 32 t.m. (neutral axis is in the flange)



Ultimate resisting moment of flange is bigger than applied ultimate bending moment, therefore neutral axis is situated at the flange and the design is like a design of rectangular section width of b.

Example:
Exactly like the previous example except that applied ultimate moment = 120t.m.



Solution:

1- b
2- position N.A. → Mnf → Muf = 111.451 t/m

Mn = (As – Asf) fy (d - a/2) + Asfy (d – hf/2)
Mu = φMn

$Mu = \varphi [(As – Asf) fy (d - a/2) + Asfy (d – hf/2)]$

$a = (As – Asf) fy/0.85f'c.bw$

$Asf = C'/ fy \rightarrow Asf=0.85f'c (b – bw) hf/fy$

$Asf = 0.85f'c (b – bw) hf/fy$

$=0.85 \times 280(150 -30)7.5/4200 =51cm^2$

$a = (As – Asf) fy/0.85f'c.bw$

$= (As – 51)4200/0.85 \times 280 \times 30$

$= (As-51)x0.58 = 0.58As – 30$

$Mu = \varphi [(As – Asf) fy (d - a/2) + Asfy (d – hf/2)]$

$120 \times 1000000 = 0.9(As – 51)4200(50 – 0.58As – 30 /2) + As4200 (50 – 7.5 /2)$

$120 \times 1000000 /0.9=(As – 51)4200(50 – 0.58As – 30 /2) + As4200(50 – 7.5 /2)$

$13333333.3 = (4200As – 214200) (50 – 0.29As – 15) + As4200 (50 – 3.75)$

$13333333.3 = - 1218As^2 + 209118As – 7497000 + 194250As$

$0 = - 1218As^2 + 403368As – 20830333.3$

$As = - b \pm \sqrt{}b^2 - 4ac / 2a$

$As = -403368 \pm (247427.5)/-2436$

$As1 = 64cm^2$

$As2 = 267cm^2$

$Asf = 51 cm^2$ verification with $p - pf' \leq 0.75 fb$

Try 1

$p = As / bwd$, $pf = Asf / bwd$

$p - pf \leq 0.75 pb$

$p = 64.01 / 30.50 = 0.0426$

$pf = 51 / 30.50 = 0.034$

$p - pf = 0.0426 – 0.034 = 0.0086$

$0.75pb = (0.75x0.85x0.85x280)/4200 – 0.003/(0.003 + 2100000$

$= 0.0216$

$0.0086 < 0.0216 \rightarrow$ OK

$As = 64.01cm^2$ is correct

Try 2

$pf = 0.0034$

$p = 267 / 30.50 = 0.178$

$p - pf = 0.144$

$0.144 > 0.0216 \rightarrow$ NO GOOD

$As = 267.16cm^2$ is not correct

$Asf = 51cm^2$

$As = 64.01cm$

$1\Phi20 = As = \pi = 9.14$

$\pi r^2 = \pi(1cm) = \pi$

$\Phi36mm \rightarrow 1\Phi36 = \pi d^2/4 = \pi(3.6)^2/4$

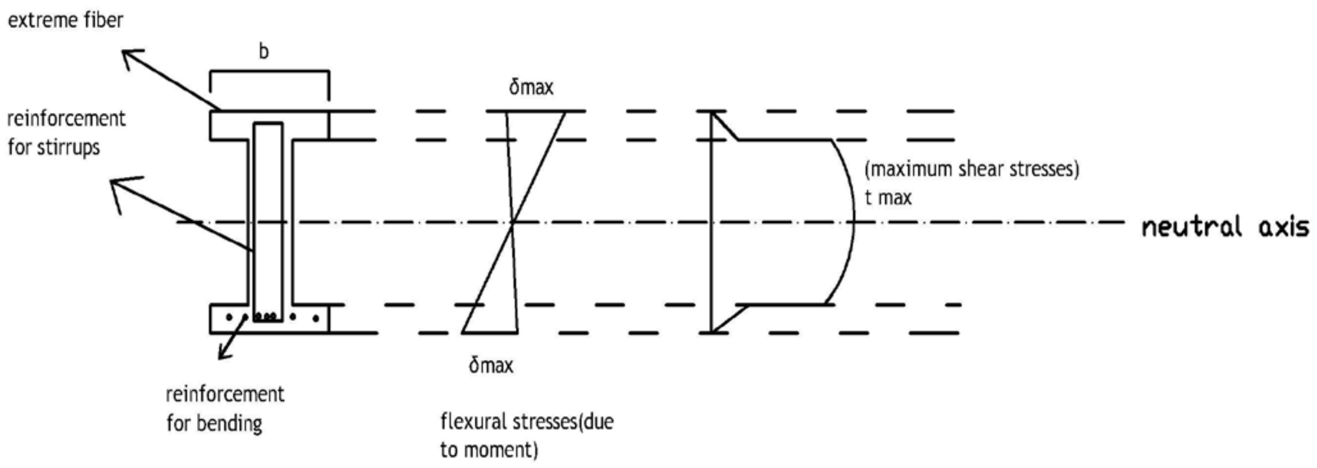
$= 10.17cm^2$

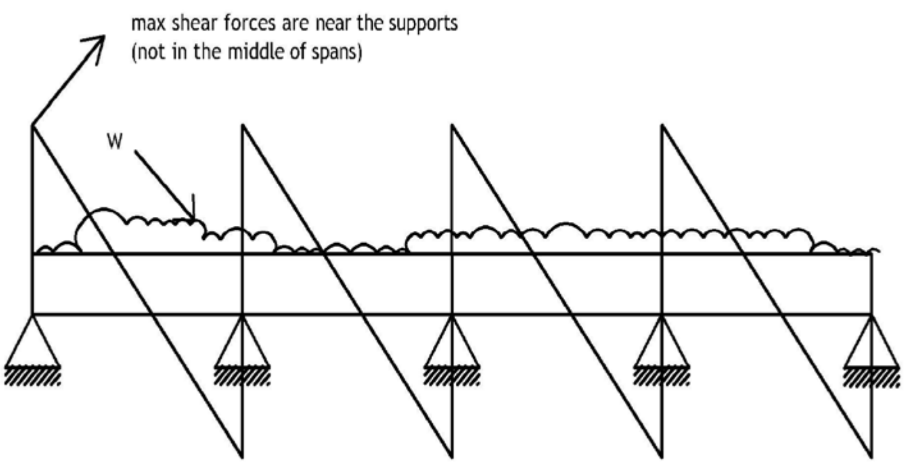
Try $7\Phi36 \rightarrow As = 7 \times 10.17cm^2 = 71.19cm^2 \rightarrow 64.01$

USE $7\Phi36$

$As = 71.19cm^2$

Chapter 6. Shear in Reinforced Concrete Beams

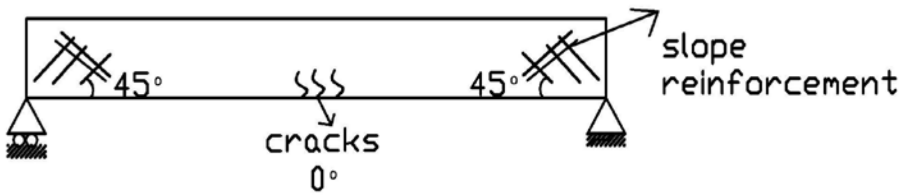


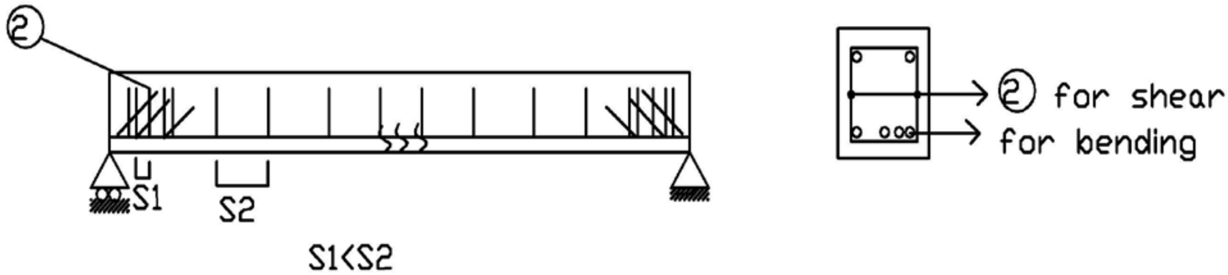


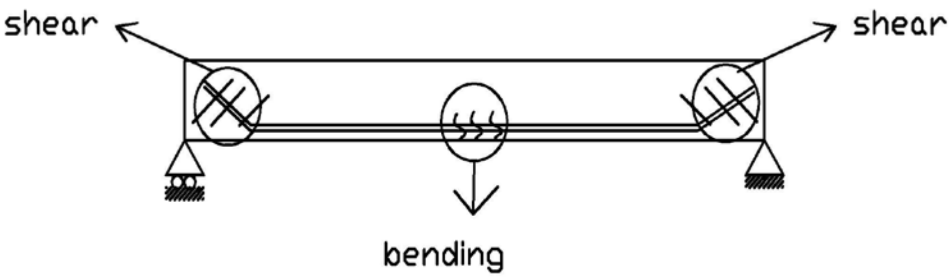
The zones of effect of bending moment (flexure stress) and maximum of shear stress are different (are at supports for negative moment)

1. Bending moment (flexural stress)
   → max along the beam →is at mid spans
   → max at sections → is at extreme fibers
2. Shear Stress
   → max at the beam → are at supports
   → max at the sections → is at extreme fibers

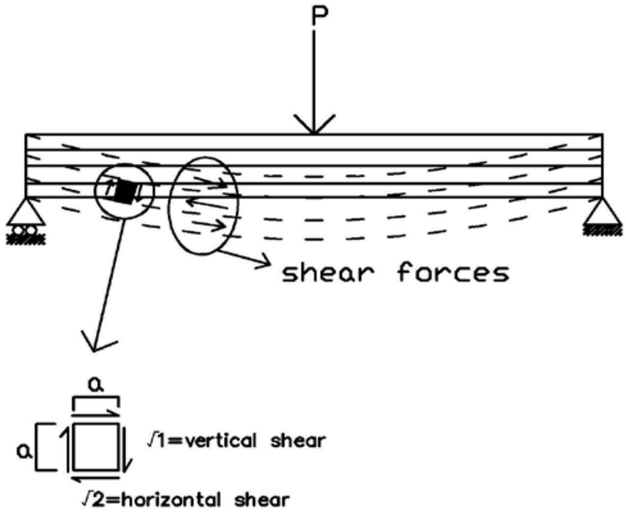
   Different Types of Shear Reinforcements

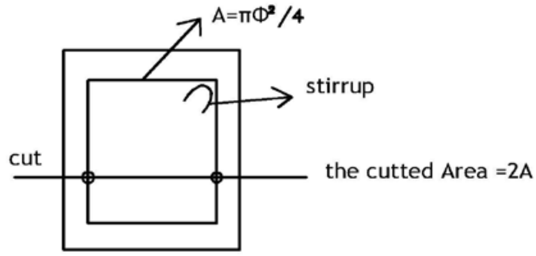






Design of Beams for Shear Stresses





$\varepsilon MA = -\sqrt{1} \text{ x } a + \sqrt{2} \text{ x } a = 0$ $\varepsilon MA = 0 \rightarrow \chi$

Shear Resistance:

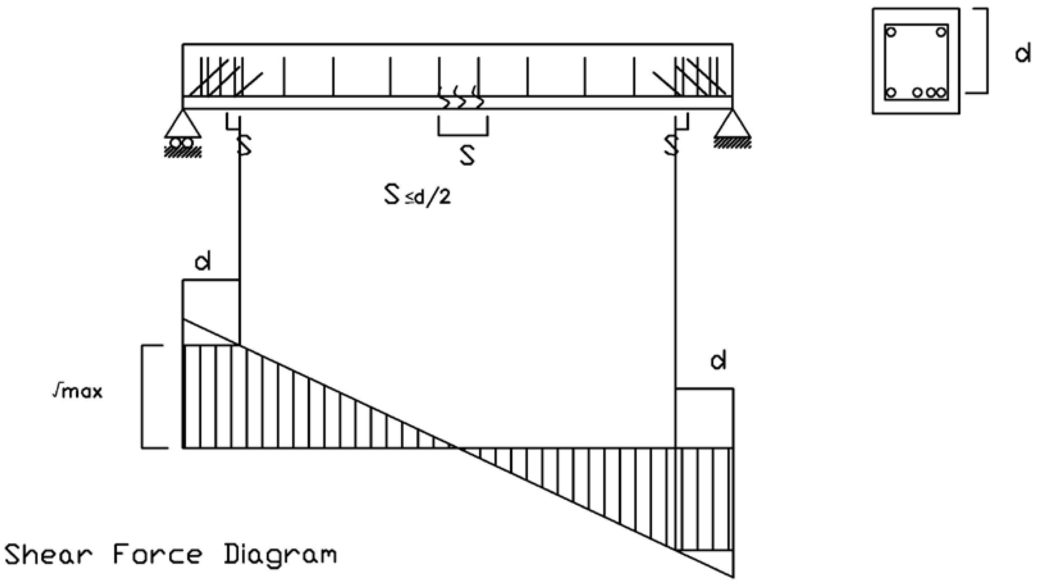
1. Concrete Section → (shear force resistance Tc due to concrete section)
2. Reinforcement (stirrup) section Vs = (shear force resistance due to stirrup)
   AV = 2A

Vs – force	Vc – force
vs – stress	vc - stress

allowable shear stress of concrete:

$vc = 0.292\sqrt{f'c}$ → when design is based on WSD method
$vc = 0.530\sqrt{f'c}$ → when design is based on USD method
(f ‘c is in kgf/ cm^2, vc is in kgf/cm)
Vc = vc x bw x d

<u>Design of Stirrups for a Beam</u>



$S = A_v f_y d / V_s$
$V_s = V_n - V_c$
$V_s = (V_u / \varphi) - V_c$

$V_c = v_c \times b_w \times d$ $\quad\quad$ $V_c = 0.292\sqrt{f'_c} \times b_w \times d \rightarrow$ for WSD
$V_c = 0.530\sqrt{f'_c} \times b_w \times d \rightarrow$ for USD