# Chapter 5

- ***The Time Value of Money***



## Key Concepts and Skills

Be able to Compute:

- the **Future Value** of an investment made today
- the **Present Value** of cash to be received at some future date
- the **Return on an Investment**
- the **Number of periods**



## Time Value of Money

- A $ in hand today is worth more than a $ tomorrow. Why?
  - Interest Rate Factor
  - **We know that receiving $1 today is worth more than $1 in the future. This is due to <u>OPPORTUNITY COSTS</u>.**
    - **The opportunity cost of receiving $1 in the future is the <u>interest</u> we could have earned if we had received the $1 sooner**



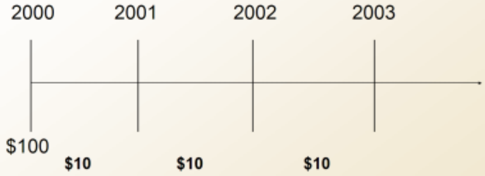
## Compound Interest vs. Simple Interest

- Simple interest
  - Interest earn only on the original investment, no interest is earned on interest
- Compound interest
  - Interest is earned on both the initial principal and the interest reinvested from prior periods



- Suppose you invest $100 for three years at 10% per year. What is the future value in three year?
- A) Simple Interest Rate
- B) Compound Interest

## Simple Interest Rate




At the end of 3rd year you will receive $130

**$100 * 0.10 = $10**
**$100 + ($10) *3 = $130**

**Is this the way that we can calculate i.r. in the real life?**

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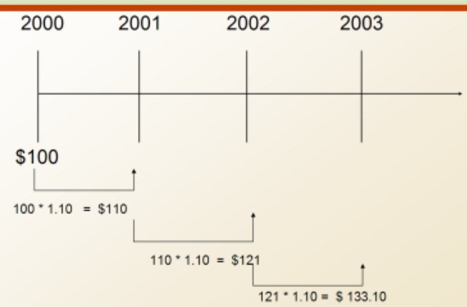
## Compound Interest Rate




**or/ 100*1.10*1.10*1.10 = $ 133.10**
**Is this the way that you are going to calculate the compounding rate?**

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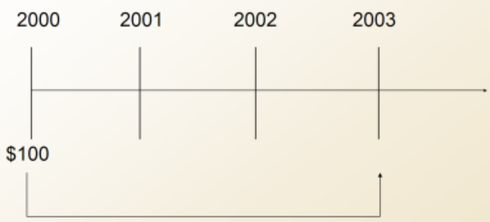
## Compound Interest Rate




$100 (1 + 0.10)^3 = \$ 133.10$

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## Future Value Equation - Formula

- $FV = PV(1 + r)^t$
  - FV = future value
  - PV = present value
  - r = period interest rate
  - t = number of periods
- Future Value Interest Factor = $(1 + r)^t$

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## 1. Future Value and Compounding

- Cash value of an investment at some time in the future
- When we talk about compounding, we mean interest earned on interest

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## Future Value of a Single Amount ($100 Invested for 3 years at 10%)

**Algebraic solution**

$$FV_t = PV_0(1+r)^t$$
$$= \$100\,(1.10)^3 = \$133.10$$

**Variation with time value tables**

$$FV_t = PV_0\left(FVIF_{r,t}\right)$$
$$= \$100\,(1.3310) = \$133.10$$

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## Future Values

- Suppose you invest $1000 for one year at 5% per year. What is the future value in one year?
- Simple interest
  - Interest = 1000(.05) = 50
    Value in one year = principal + interest
    = 1000 + 50 = 1050
- Compound interest
  - Future Value (FV) = 1000(1 + 0.05) = 1050



## Effects of Compounding

- Suppose you leave the money in for another year. How much will you have two years from now?



## Answer

- Simple interest
  - FV with simple interest = 1000 + 50 + 50 = 1100
- Compound interest
  - FV with compound interest
    = $1000(1 + 0.05)^2$ =1102.50
  - The extra 2.50 comes from the interest of .05(50) = 2.50 earned on the first interest payment



## Future Values

- Suppose you invest the $1000 from the previous example for 5 years. How much would you have?
  - FV = $1000(1.05)^5$ = 1276.28
- The effect of compounding is small for a small number of periods, but increases as the number of periods increases. (Simple interest would have a future value of $1250, for a difference of $26.28.)



## Future Values

- Suppose you had a relative deposit $10 at 5.5% interest 200 years ago. How much would the investment be worth today?
  - FV = $10(1.055)^{200}$ = 447,189.84
- What is the effect of compounding?
  - Simple interest = 10 + 200(10)(.055) = 120.00
  - Compounding added $447,069.84 to the value of the investment



## The Basic PV Equation - Refresher

- $FV = PV(1 + r)^t$
- Rearrange to solve for PV
- $PV = \frac{FV}{(1 + r)^t}$
- $PV = FV * \frac{1}{(1 + r)^t}$

## 1. Present Value and Discounting

- Current value of the future CF discounted at the appropriate discount rate
- When we talk about discounting, we mean finding the present value of some future amount.
- How much do I have to invest today to have some amount in the future?



## Present Value of a Single Amount ($100 Received in 3 years at 10%)

**Algebraic solution**

$$PV_0 = FV_t\left(\frac{1}{(1+r)^t}\right) = \$100\left(\frac{1}{(1+0.10)^3}\right) = \$75.13$$

**Variation with time value tables**

$$PV_0 = FV_t\left(PVIF_{r,t}\right) = \$100(0.7513) = \$75.13$$



## Present Value – Example 5.1

- Suppose you need $10,000 in one year for the down payment on a new car. If you can earn 7% annually, how much do you need to invest today?



## Present Values – Example 5.2

- You want to begin saving for you daughter's college education and you estimate that she will need $150,000 in 17 years. If you feel confident that you can earn 8% per year, how much do you need to invest today?



## Present Values – Example 5.3

- Your parents set up a trust fund for you 10 years ago that is now worth $19,671.51. If the fund earned 7% per year, how much did your parents invest?



## Example 5.4

- You need $50,000 in 10 years. If you can earn 6% interest, how much do you need to invest today?
- You should get $27,919.74

## Present Value – Important Relationship I

- For a given interest rate – the longer the time period, the lower the present value
  - What is the present value of \$500 to be received in 5 years? 10 years? The discount rate is 10%
  - 5 years: PV = 500 / $(1.1)^5$ = 310.46
  - 10 years: PV = 500 / $(1.1)^{10}$ = 192.77



## Present Value – Important Relationship II

- For a given time period – the higher the interest rate, the smaller the present value
  - What is the present value of \$500 received in 5 years if the interest rate is 10%? 15%?
    - Rate = 10%: PV = 500 / $(1.1)^5$ = 310.46
    - Rate = 15%; PV = 500 / $(1.15)^5$ =248.59



## Discount Rate

- Often we will want to know what the implied interest rate is in an investment
- Rearrange the basic PV equation and solve for r
  - $FV = PV(1 + r)^t$
  - $r = (FV / PV)^{1/t} - 1$
- If you are using formulas, you will want to make use of both the $y^x$ and the 1/x keys



## Discount Rate – Example 5.5

- You are looking at an investment that will pay \$1200 in 5 years if you invest \$1000 today. What is the implied rate of interest?



## Discount Rate – Example 5.6

- Suppose you are offered an investment that will allow you to double your money in 6 years. You have \$10,000 to invest. What is the implied rate of interest?



## Finding the Number of Periods

- Start with basic equation and solve for t (remember you logs)
  - $FV = PV(1 + r)^t$
  - $t = \ln(FV / PV) / \ln(1 + r)$
- You can use the financial keys on the calculator as well; just remember the sign convention.

## Number of Periods – Example 5.8

- You want to purchase a new car and you are willing to pay $20,000. If you can invest at 10% per year and you currently have $15,000, how long will it be before you have enough money to pay cash for the car?



## Number of Periods – Example 5.9

- Suppose you want to buy a new house. You currently have $15,000 and you figure you need to have a $ 21,750. Assume that you can earn 7.5% per year, how long will it be before you have enough money to buy the house?



## Table

**I. Symbols:**

PV = Present value, what future cash flows are worth today
$FV_t$ = Future value, what cash flows are worth in the future
$r$ = Interest rate, rate of return, or discount rate per period—typically, but not always, one year
$t$ = Number of periods—typically, but not always, the number of years
$C$ = Cash amount

**II. Future Value of *C* Invested at *r* Percent for *t* Periods:**

$FV_t = C \times (1 + r)^t$
The term $(1 + r)^t$ is called the *future value factor.*

**III. Present Value of *C* to Be Received in *t* Periods at *r* Percent per Period:**

$PV = C/(1 + r)^t$
The term $1/(1 + r)^t$ is called the *present value factor.*

**IV. The Basic Present Value Equation Giving the Relationship between Present and Future Value is:**

$PV = FV_t/(1 + r)^t$

## Suggested Problems

- 1-15, 17-20.