EE 472 POWER SYSTEM ANAYSIS II

LECTURE NOTES

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CONTENTS

1. **SYMMETRICAL COMPONENTS**

Three unbalanced phasors of a three-phase system can be resolved into three balanced systems of phasors. The balanced sets of components are:

1. Positive-sequence components consisting of three phasors equal in magnitude, displaced from each other by 120 degrees in phase, and **having the same phase sequence as the original phasors.**
2. Negative-sequence components consisting of three phasors equal in magnitude, displaced from each other by 120 degrees in phase, and **having the phase sequence opposite to that of the original phasors.**
3. Zero-sequence components consisting of three phasors equal in magnitude and with **zero phase displacement** from each other.

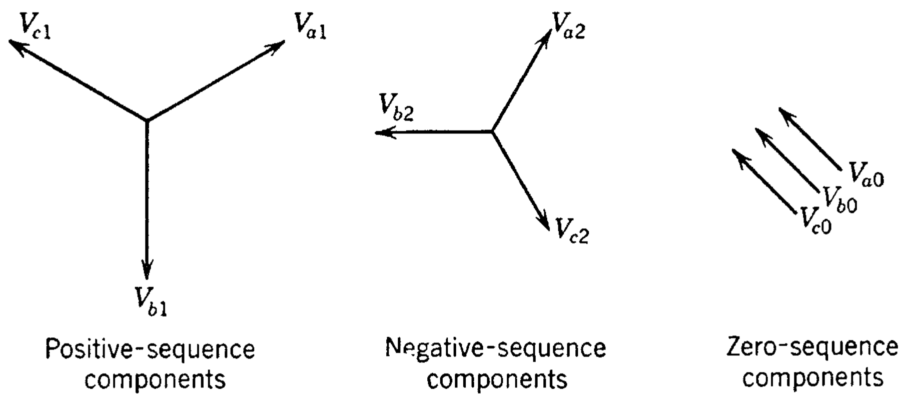


Fig. 1.1. Three sets of balanced phasors which are the symmetrical components of three unbalanced phasors.

The three sets of symmetrical components are designated by the additional subscript 1 for the positive-sequence components, 2 for the negative-sequence components, and 0 for the zero-sequence components. The positive-sequence components of V_a, V_b, and V_c are V_{a1}, V_{b1}, and V_{c1}. Similarly, the negative-sequence components are V_{a2}, V_{b2}, and V_{c2}, and the zero-sequence components are V_{a0}, V_{b0}, and V_{c0}. Phasors representing currents will be designated by I with subscripts as for voltages.

Since each of the original unbalanced phasors is the sum of its components, the original phasors expressed in terms of their components are:

$$V_a = V_{a1} + V_{a2} + V_{a0} \tag{1.1}$$

$$V_b = V_{b1} + V_{b2} + V_{b0} \quad (1.2)$$

$$V_c = V_{c1} + V_{c2} + V_{c0} \quad (1.3)$$

The phase displacement of the symmetrical components of the voltages and currents in a three-phase system by $120°$, it is convenient to have a shorthand method of indicating the rotation of a phasor through $120°$. The result of the multiplication of two complex numbers is the product of their magnitudes and the sum of their angles.

The letter a is commonly used to designate the operator which causes a rotation of $120°$ in the counterclockwise direction. Such an operator is a complex number of unit magnitude with an angle of $120°$ and is defined by the following expressions:

$$a = 1\angle 120^0 = 1e^{j2\pi/3} = -0.5 + j0.866$$

If the operator a is applied to a phasor twice in succession, is rotated through $240°$. Three successive a applications of a rotate the phasor through $360°$. Thus, the phasor

$$a^2 = 1\angle 240^0 = -0.5 - j0.866$$

and

$$a^3 = 1\angle 360^0 = 1\angle 0^0 = 1$$

$$1 + a + a^2 = 0 = 0 + j0$$

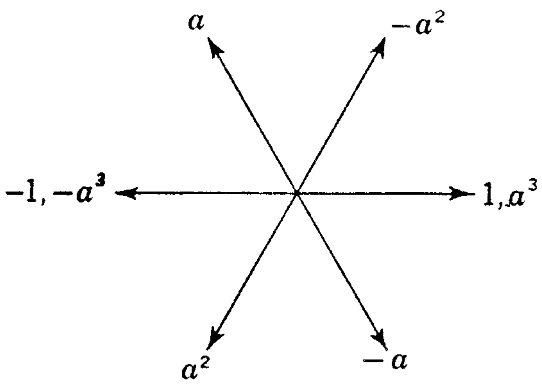


Fig. 1.2. Phasor diagram of the various powers of the operator a.

We note that the number of unknown quantities can be reduced by expressing each component of Va and Vb as the product of some function of the operator a and a component of Va. Reference to Fig. 1.1 verifies the following relations:

$$V_{b1} = a^2 V_{a1} \qquad V_{c1} = aV_{a1}$$

$$V_{b2} = aV_{a2} \qquad V_{c2} = a^2 V_{a2} \quad (1.4)$$

$$V_{b0} = V_{a0} \qquad V_{c0} = V_{a0}$$

Upon substitution of Eqs. (4) in Eqs. (1) to (3), we obtain

$$V_a = V_{a1} + V_{a2} + V_{a0} \tag{1.5}$$

$$V_b = a^2V_{a1} + aV_{a2} + V_{a0} \tag{1.6}$$

$$V_c = aV_{a1} + a^2V_{a2} + V_{a0} \tag{1.7}$$

Adding Eqs. (1.5), (1.6), and (1.7) gives

$$V_a + V_b + V_c = (1 + a + a^2)V_{a1} + (1 + a + a^2)V_{a2} + 3V_{a0} \tag{1.8}$$

and, since $1 + a + a^2 = 0$,

$$V_{a0} = 1/3(V_a + V_b + V_c) \tag{1.9}$$

Equation (1.9) enables us to find the zero-sequence components of three unsymmetrical phasors. We see that no zero-sequence components exist if the sum of the phasors is zero. Since the sum of the line-to-line voltage phasors in a three-phase system is always zero, zero-sequence components are never present in the line voltages, regardless of the amount of unbalance. The sum of the three line-to-neutral voltage phasors is not necessarily zero, and voltages to neutral may contain zero-sequence components.

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}}_{A} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}, A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}}_{\substack{symmetrical \\ components}} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \underbrace{\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}}_{\substack{unsymmetrical \\ phase system}}$$

The equations could have been written for any set of related phasors, and we might have written them for currents instead of for voltages. They may be solved either analytically or graphically. Because some of the preceding equations are so fundamental, they are summarized below for currents.

$$I_a = I_{a1} + I_{a2} + I_{a0} \tag{1.10}$$

$$I_b = a^2I_{a1} + aI_{a2} + I_{a0} \tag{1.11}$$

$$I_c = aI_{a1} + a^2I_{a2} + I_{a0} \tag{1.12}$$

$$I_{a1} = 1/3(I_a + aI_b + a^2 I_c) \tag{1.13}$$

$$I_{a2} = 1/3(I_a + a^2 I_b + aI_c) \tag{1.14}$$

$$I_{a0} = 1/3(I_a + I_b + I_c) \tag{1.15}$$

In a three-phase system the sum of the line currents is equal to the current I_n in the return path through the neutral. Thus,

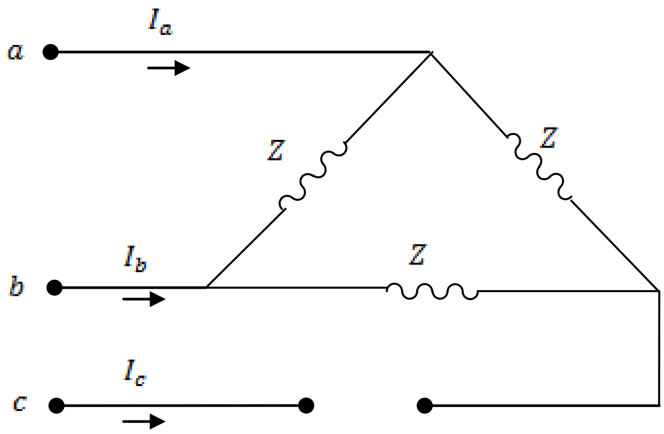
$$I_a + I_b + I_c = I_n \tag{1.16}$$

Comparing Eqs. (1.15) and (1.16) gives

$$I_n = 3I_{a0} \tag{1.17}$$

In the absence of a path through the neutral of a three-phase system, I_n is zero, and the line currents contain no zero-sequence components.

Example:



$I_a = 10\angle 0^0\ A$

$I_b = 10\angle 180^0\ A$

$I_c = 0\ A$

Find the symmetrical components of the current.

Answer:

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c) = 0$$

$$I_{a1} = \frac{1}{3}(I_a + aI_b + a^2 I_c) = 5.78\angle{-30^0}\ A$$

$$I_{a2} = \frac{1}{3}(I_a + a^2 I_b + aI_c) = 5.78\angle{30^0}\ A$$

$$I_{b1} = a^2 I_{a1} = 5.78\angle{-150^0}\ A$$

$$I_{b2} = aI_{a2} = 5.78\angle{150^0}\ A$$

$$I_{b0} = I_{c0} = I_{a0} = 0\ A$$

$$I_{c1} = aI_{a1} = 5.78\angle{90^0}\ A$$

$$I_{c2} = a^2 I_{a2} = 5.78\angle{-30^0}\ A$$

$$I_{b0} = I_{c0} = I_{a0} = 0\ A$$

$$I_c = I_{c1} + I_{c2} + I_{c0} = 0\ A$$

2. **SEQUENCE IMPEDANCES AND SEQUENCE NETWORKS**

The symmetrical components of unbalanced currents flowing in balanced impedances will produce voltage drops like sequence only.

The impedance of a circuit when positive-sequence currents alone are flowing is called the impedance to positive-sequence current. Similarly, when only negative-sequence currents are present, the impedance is called the impedance to negative-sequence current. When only zero-sequence currents are present, the impedance is called the impedance to zero-sequence current. These names of the impedances of a circuit to currents of the different sequences are usually shortened to the less descriptive terms, positive-sequence impedance, negative-sequence impedance, and zero-sequence impedance.

The analysis of an unsymmetrical fault on a symmetrical system consists of finding the symmetrical components of the unbalanced currents which are flowing. Since the component currents of one phase sequence cause voltage drops of like sequence only and are independent of currents of other sequences, in a balanced system, currents of any one sequence may be considered to flow in an independent network composed of the impedances to the current of that sequence only. The single-phase equivalent circuit composed of the impedances to current of anyone sequence only is called the sequence network for that particular sequence. The sequence network includes any generated emfs of like sequence. Sequence networks carrying the currents I_{a1}, I_{a2}, and I_{a0} are interconnected to represent various unbalanced fault conditions. Therefore, to calculate the effect of a fault by the method of symmetrical components, it is essential to determine the sequence impedances and to combine them to form the sequence networks.

The positive-sequence and negative-sequence impedances of linear, symmetrical, static circuits are identical, because the impedance of such circuits is independent of phase order provided the applied voltages are balanced. The impedance of such circuits to zero-sequence currents may differ from the impedance to positive- and negative-sequence currents. The impedances of rotating machines to currents of the three sequences will generally be different for each sequence.

In deriving the equations for inductance and capacitance of transposed transmission lines, we assumed balanced three-phase currents and did not specify phase order. Therefore, the resulting equations are valid for both positive- and negative-sequence impedances. The inductance and capacitance of transmission lines for zero-sequence currents will be discussed later.

For symmetrical three-phase static loads consisting of lumped constants or loads which can be analyzed as having lumped constants, the impedances to current of positive, negative, and zero sequences are the same because each phase is isolated from, and independent of, the other phases.

2.1. Sequence Networks of an Unloaded Generator

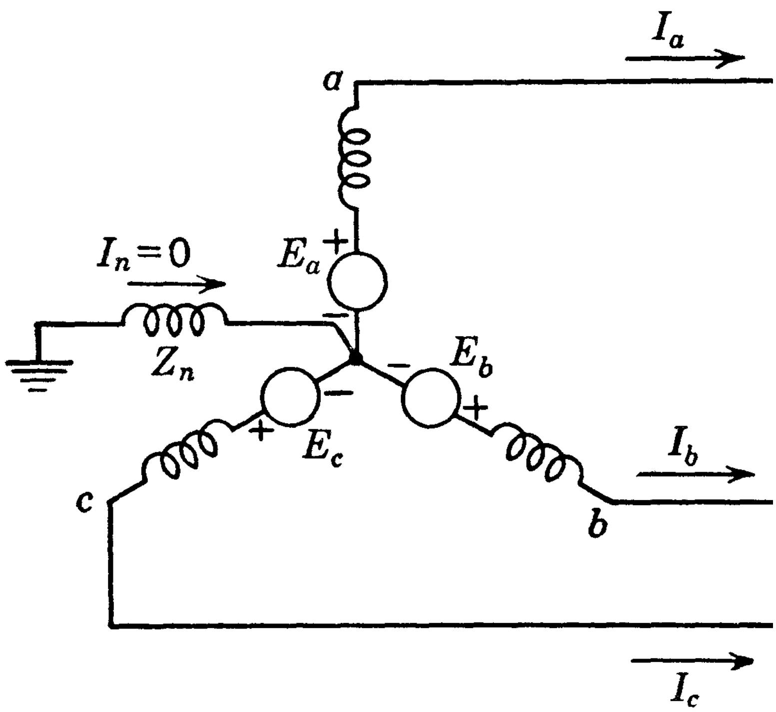
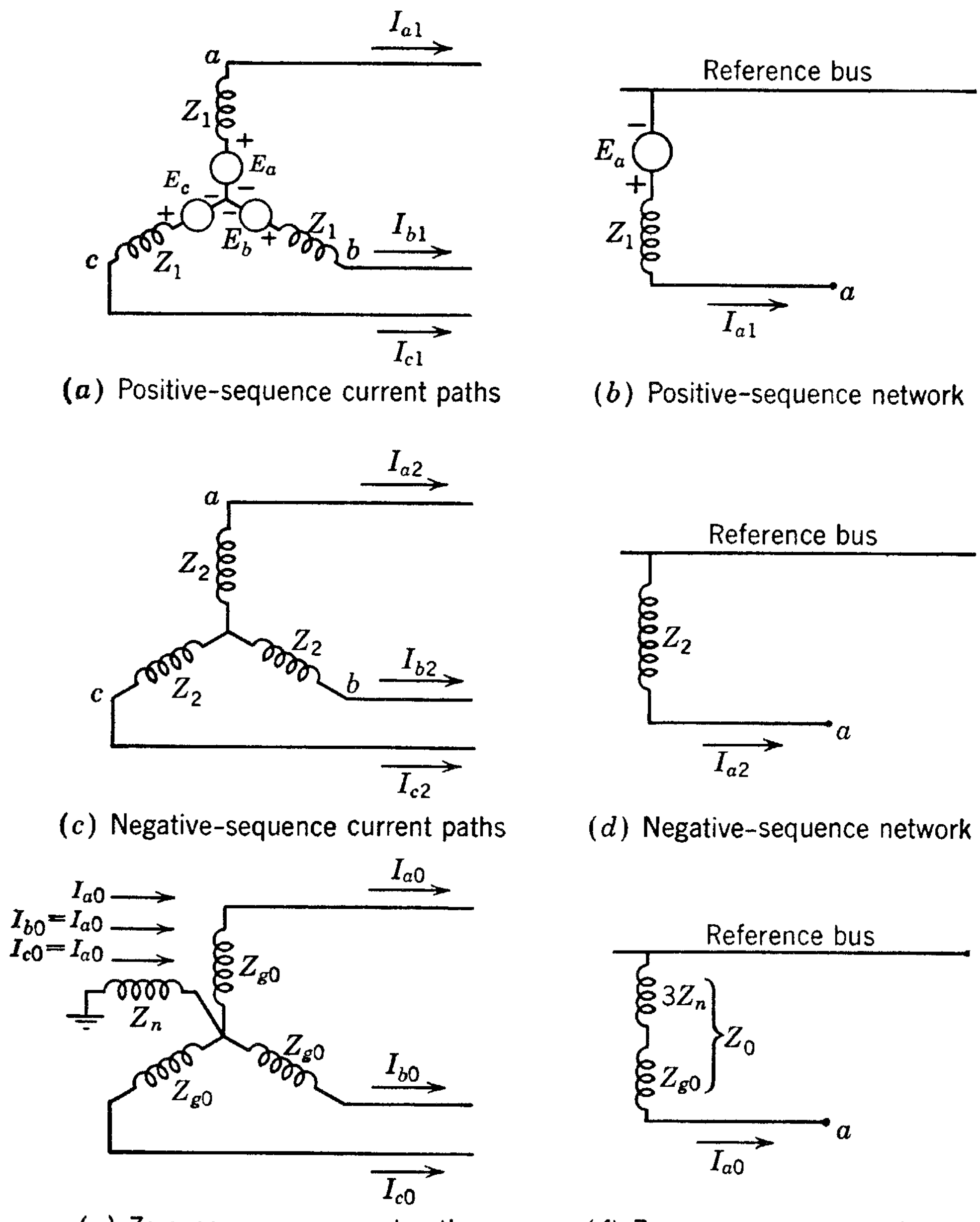


FIG. 2.1 Circuit diagram of an unloaded generator grounded through a reactance. The emfs of each phase are E_a, E_b , and E_c.

In this section our task is simple because one generator and perhaps an impedance in the neutral comprise the entire circuit. The generated voltages are of positive sequence only, since the generator is designed to supply balanced three-phase voltages. Therefore the positive-sequence network is composed of an emf in series with the positive-sequence impedance of the generator. The negative- and zero-sequence networks contain no emfs but include the impedances of the generator to negative- and zero-sequence currents, respectively. The sequence components of current are shown in Fig. 2.2. rrhey are flowing through impedances of their own sequence only, as indicated by the appropriate subscripts on the impedances shown in the figure. The sequence networks shown in Fig. 2.2 are the single-phase equivalent circuits of the balanced three-phase circuits through which the symmetrical components of the unbalanced currents are considered to flow. The generated emf in the positive-sequence network is the no-load terminal voltage to neutral, which is also equal to the voltages behind transient and subtransient reactances and to the voltage behind synchronous reactance since the generator is not loaded. The reactance in the positive-sequence network is the sub-transient, transient, or synchronous reactance, depending on whether subtransient, transient, or steady-state conditions are being studied.



(*a*) Positive-sequence current paths (*b*) Positive-sequence network

(*c*) Negative-sequence current paths (*d*) Negative-sequence network

(*e*) Zero-sequence current paths (*f*) Zero-sequence network

Fig. 2.2 Paths for current of each sequence in a generator, and the corresponding sequence networks.

The reference bus for the positive- and negative-sequence networks is the neutral of the generator. So far as positive- and negative-sequence components are concerned the neutral of the generator is at ground potential since only zero-sequence current flows in the impedance between neutral and ground. The reference bus for the zero-sequence network is the ground at the generator.

The current flowing in the impedance Z_n between neutral and ground is $3I_{a0}$. By referring to Fig. 2.2e, we see that the voltage drop of zero sequence from point a to ground is $-3I_{a0}Z_n - I_{a0}Z_{g0}$, where Z_{g0} is the zero-sequence impedance per phase of the generator. The zero-sequence network, which is a single-phase circuit assumed to carry only the zero-sequence current of one phase, must, therefore, have an impedance of $3Z_n + Z_{g0}$, as shown in Fig. 2.2f. The total zero-sequence impedance through which I_{a0} flows is

$$Z_0 = 3Z_n + Z_{g0} \tag{2.1}$$

Usually the components of current and voltage for phase a are found from equations determined by the sequence networks. The equations for the components of voltage drop from point a of phase a to the reference bus (or ground) are, as may be deduced from Fig. 2.2,

$$V_{a1} = E_a - I_{a1}Z_1 \tag{2.2}$$

$$V_{a2} = -I_{a2}Z_2 \tag{2.3}$$

$$V_{a0} = -I_{a0}Z_0 \tag{2.4}$$

Where E_a is the positive-sequence no-load voltage to neutral, Z_1 and Z_2 are the positive- and negative-sequence impedances of the generator, and Z_0 is defined by Eq. (2.1). The above equations, which apply to any generator carrying unbalanced currents, are the starting points for the derivation of equations for the components of current for different types of faults. They apply to the case of a loaded generator if E_a is given the value computed for the voltage behind subtransient, transient, or synchronous reactance for the load existing before the fault.

2.2. Zero-Sequence Networks of Transformers

The zero-sequence equivalent circuits of three-phase transformers deserve special attention. The various possible combinations of the primary and secondary windings in Y and Δ alter the zero-sequence network. Transformer theory enables us to construct the equivalent circuit for the zero-sequence net,vork. We remember that no current flows in the primary of a transformer unless current flows in the secondary, if we neglect the relatively small magnetizing current. We know, also, that the primary current is determined by the secondary current and the turns ratio of the windings, again with magnetizing current neglected. These principles guide us in the analysis of individual cases. Five possible connections of two-winding transformers will be discussed. These connections are shown in Fig. 2.3. The arrows on the connection diagrams show the possible paths for the flow of zero-sequence current. Absence of an arrow indicates that the transformer connection is such that zero-sequence current cannot flow. The zero-sequence approximately equivalent circuit, with resistance and a path for magnetizing current omitted, is shown in Fig. 2.3 for each connection. The letters P and Q identify corresponding points on the connection diagram and equivalent circuit. The reasoning to justify the equivalent circuit for each connection follows.

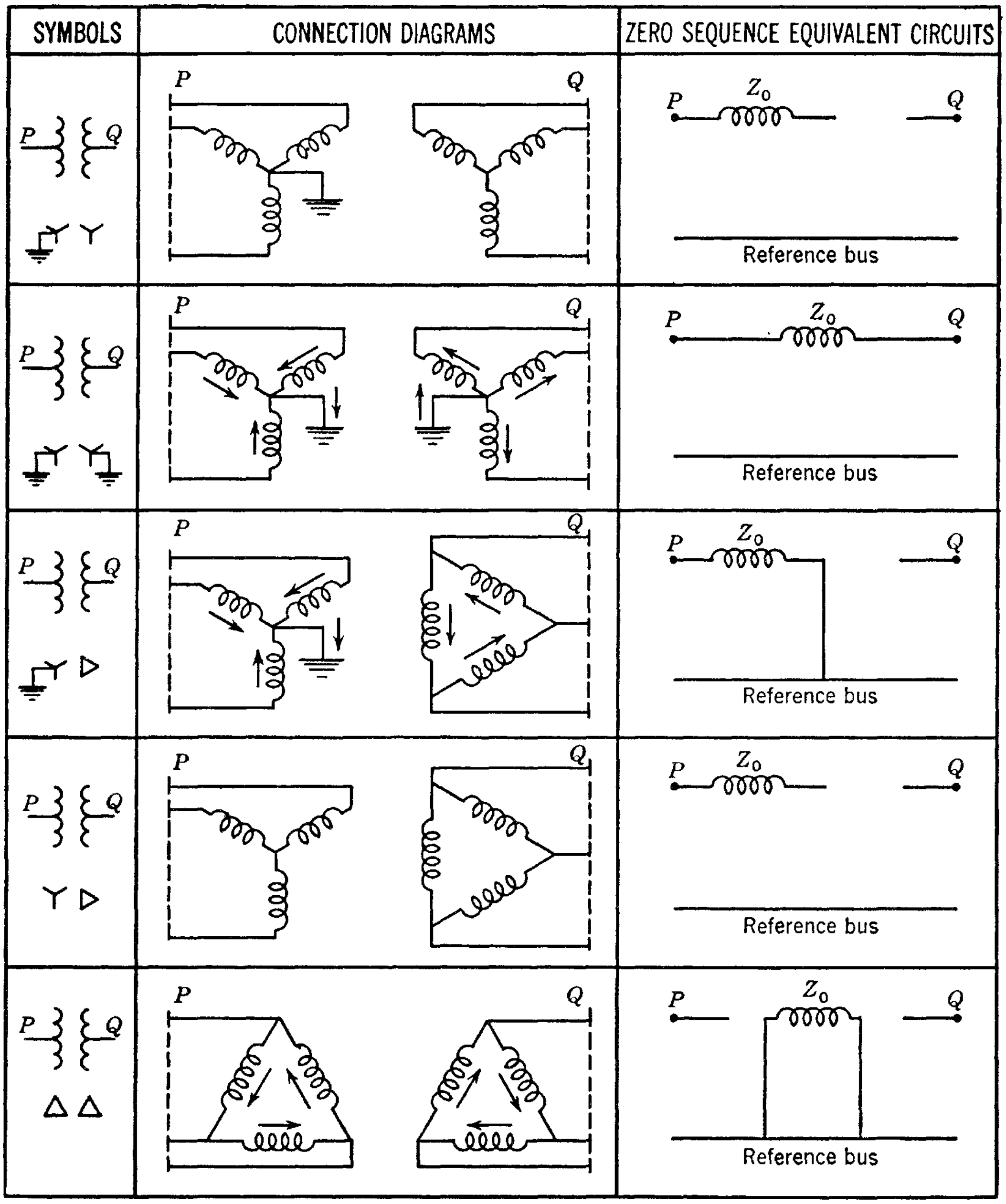
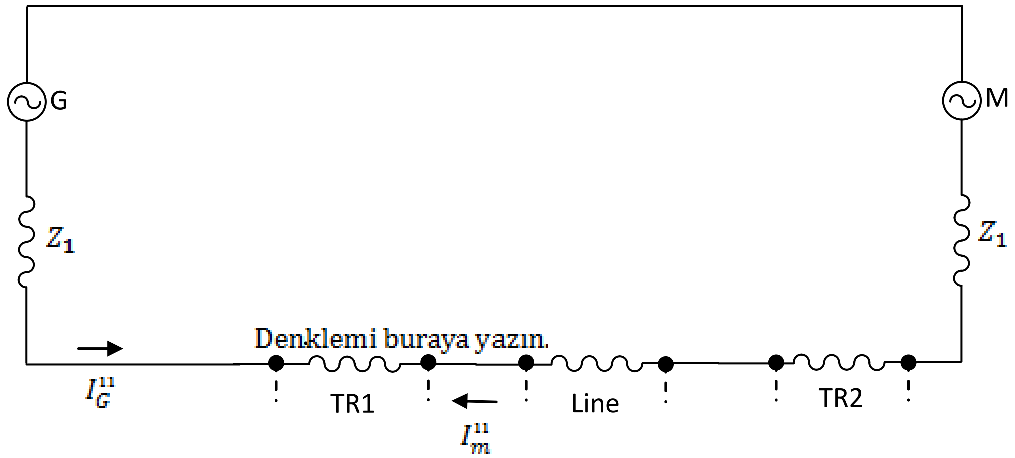


FIG. 2.3. Zero-sequence equivalent circuits of three-phase transformer banks, together with diagrams of connections and the symbols for one-line diagrams.

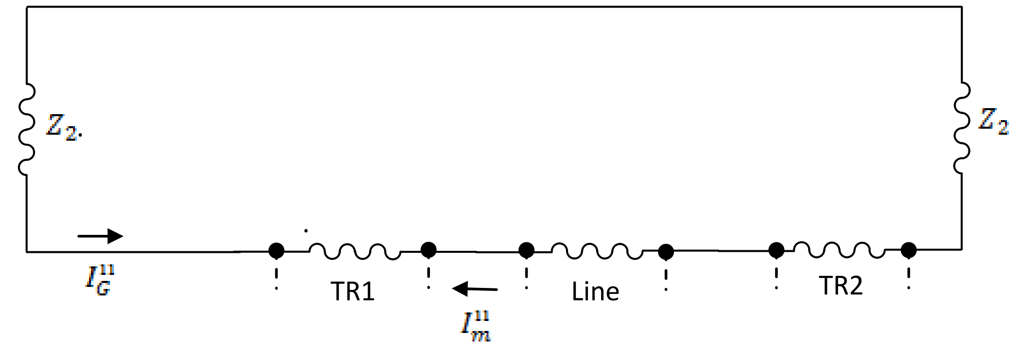
Example:

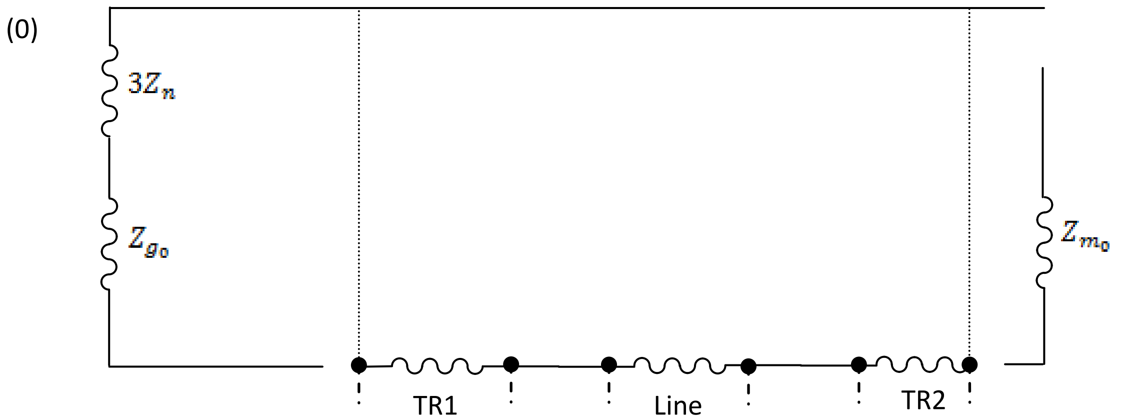
Solve the (+), (-) and (0) sequence networks of the above power system.

(+)



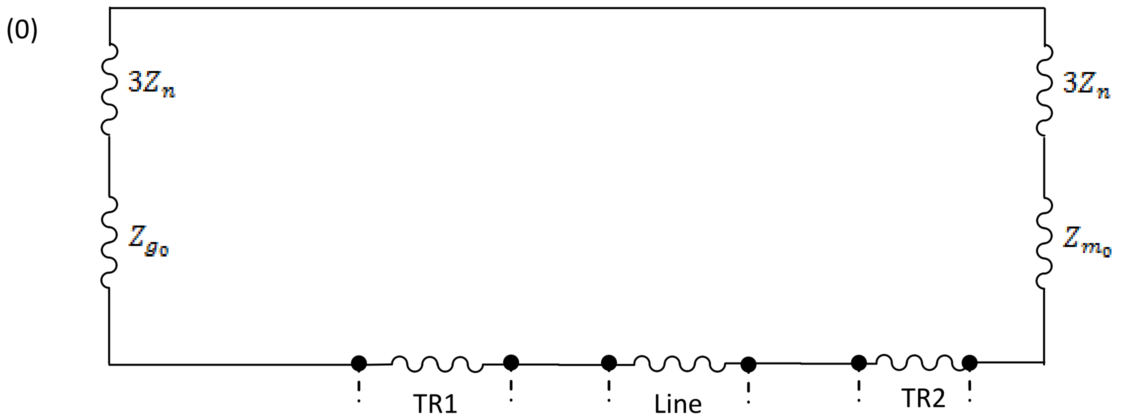
(-)





Example:

Zero sequence network,



2.3. Single line-to-ground Fault on an Unloaded Generator

The circuit diagram for a single line-to-ground fault on an unloaded Y-connected generator with its neutral grounded through a reactance is shown in Fig. 2.4 where phase a is the one on which the fault occurs. The relations to be developed for this type of fault will apply only when the fault is on phase a, but this should cause no difficulty since the phases are labeled arbitrarily and any phase may be designated as phase a. The conditions at the fault are expressed by the following equations:

$$I_b = 0 \qquad I_c = 0 \qquad V_a = 0$$

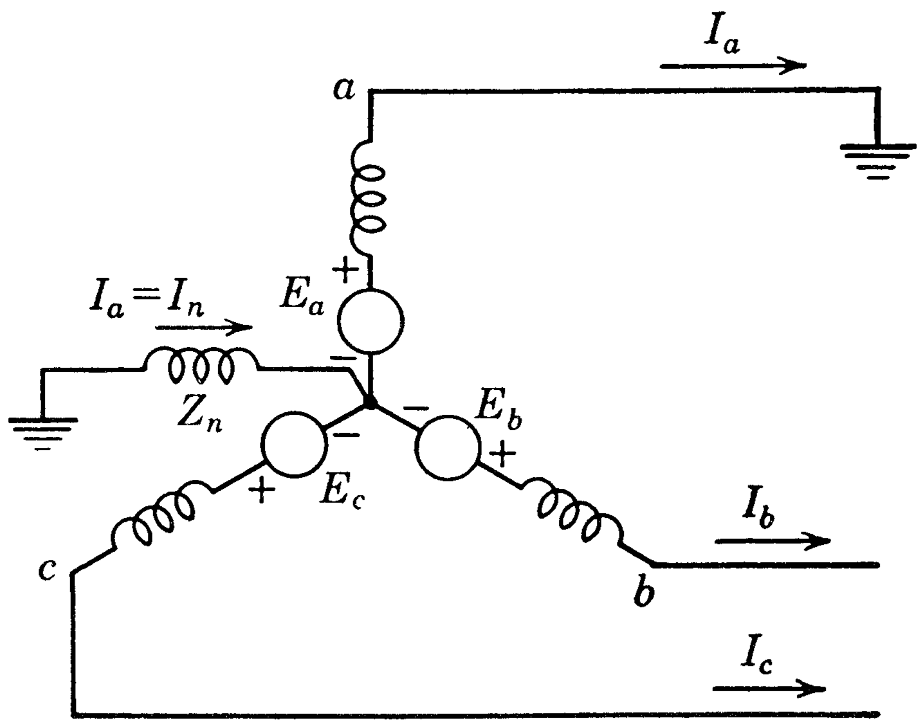


FIG. 2.4. Circuit diagram for a single line-to-ground fault on phase a at the terminals of an unloaded generator whose neutral is grounded through a reactance.

When $I_b = 0$ and $I_c = 0$ are substituted in Eqs. (1.13) to (1.15), we obtain

$$I_{a1} = 1/3(I_a + aI_b + a^2 I_c) = \frac{I_a}{3}$$

$$I_{a2} = 1/3(I_a + a^2 I_b + aI_c) = \frac{I_a}{3}$$

and

$$I_{a0} = 1/3(I_a + I_b + I_c) = \frac{I_a}{3}$$

Therefore,

$$I_{a1} = I_{a2} = I_{a0} \quad (2.5)$$

By Eq. (1.5), since $V_a = 0$,

$$V_a = V_{a1} + V_{a2} + V_{a0} = 0$$

and

$$V_{a1} = -V_{a2} - V_{a0}$$

Then, by Eq. (2.2),

$$V_{a1} = -V_{a2} - V_{a0} = E_a - I_{a1}Z_1$$

and from Eqs. (2.3) and (2.4)

$$I_{a2}Z_2 + I_{a0}Z_0 = E_a - I_{a1}Z_1$$

but, since $I_{a1} = I_{a2} = I_{a0}$,

$$I_{a1}Z_2 + I_{a1}Z_0 = E_a - I_{a1}Z_1$$

and, solving for I_{a1}, we obtain

$$I_{a1} = \frac{E_a}{Z_1+Z_2+Z_0} \quad (2.6)$$

Equations (2.5) and (2.6) are the special equations for a single line-to-ground fault. They are used with Eqs. (2.2) to (2.4), together with the symmetrical-component relations to determine all the voltages and currents at the fault. If the three sequence networks of Fig. 2.2 are connected in series as shown in Fig. 2.5, we see that the currents and voltages resulting therefrom satisfy the equations above, for the three sequence impedances are then in series with the voltage E_a. With the sequence networks so connected, the voltage across each sequence network is the symmetrical component of V_a of that sequence. The connection of the sequence networks as shown in Fig. 2.5 is a convenient means of remembering the equations for the solution of the single line-to-ground fault, for all the necessary equations can be determined from the sequence network connection.

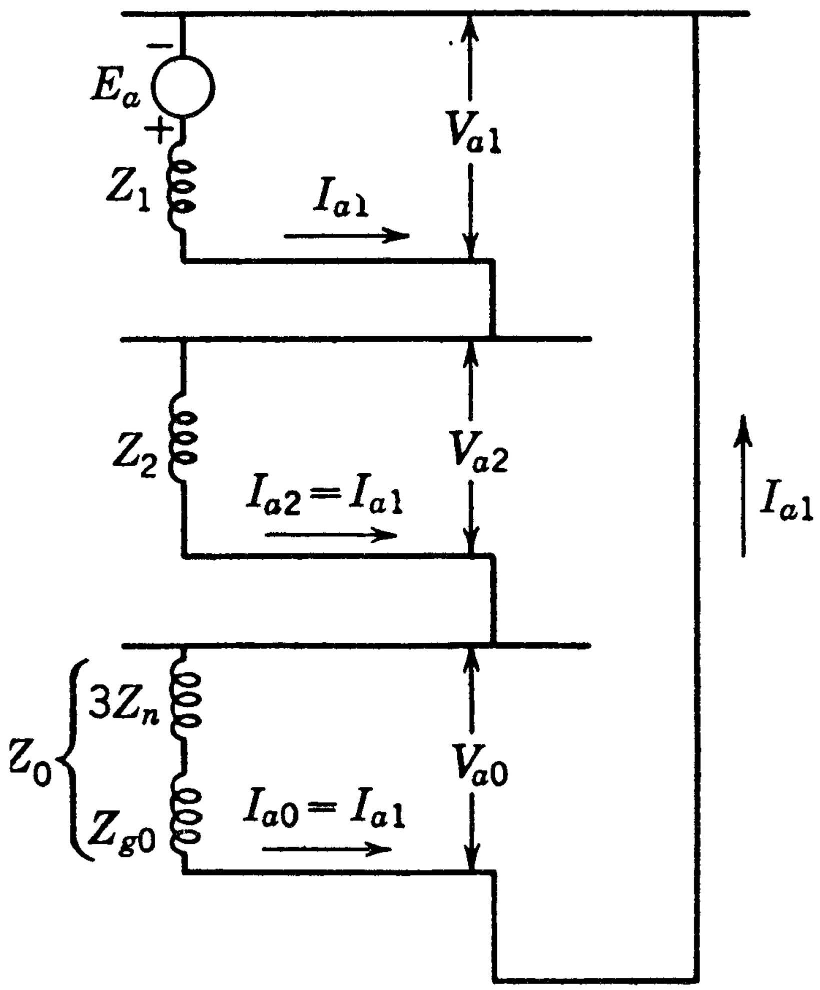


FIG. 2.5 Connection of the sequence networks of an unloaded generator for a single line-to-ground fault on phase a at the terminals of the generator

If the neutral of the generator is not grounded, the zero-sequence network is open-circuited, and Z_0 is infinite. Since Eq. 2.6 shows that I_{a1} is zero when Z_0 is infinite, I_{a2} and I_{a0} must be zero. Thus no current flows in line a since I_a is the sum of its components, all of which are zero. The same result can be seen without the use of symmetrical components since inspection of the circuit shows that no path exists for the flow of current in the fault unless there is a ground at the generator neutral.

Example: A 20.000-kva, 13.8 kv generator has a direct-axis sub transient reactance of 0.25 per unit. The negative- and zero-sequence reactances are, respectively, 0.35 and 0.10 per unit. The neutral of the generator is solidly grounded. Determine the subtransient current in the generator and the line-to-line voltages for subtransient conditions when a single line-to-ground fault occurs at the generator terminals with the generator operating unloaded at rated voltage. Neglect resistance.

Solution

On a base of 20.000 kva, 13.8 kv, $E_a = 1.0$ per unit, since the internal voltage is equal to the terminal voltage at no load.

Then, in per unit,

$$I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_0} = \frac{1.0 + j0}{j0.25 + j0.35 + j0.10} = -j1.43\ p.u.$$

$$I_F = I_a = 3I_{a1} = -j4.29\ p.u.$$

$$I_{base} = \frac{20.10^6}{\sqrt{3}(13.8)10^3} = 836\ A$$

Subtransient current in line a is,

$$I_F = I_a = (-j4.29)(836) = -j3585\ A$$

The symmetrical components of the voltage from point a to ground are:

$$V_{a1} = E_a - I_{a1}Z_1 = 1.0 - (-j1.43)(j0.25) = 0.643\ p.u.$$

$$V_{a2} = -I_{a2}Z_2 = -(-j1.43)(j0.35) = -0.50\ p.u.$$

$$V_{a0} = -I_{a0}Z_0 = -(-j1.43)(j0.10) = -0.143\ p.u.$$

Line-to-ground voltages are:

$$V_a = V_{a1} + V_{a2} + V_{a0} = 0.643 - 0.50 - 0.143 = 0\ p.u.$$

$$V_b = a^2V_{a1} + aV_{a2} + V_{a0} = 0.643(-0.5 - j0.866) - 0.50(-0.5 + j0.866) - 0.143$$

$$V_b = -0.215 - j0.989\ p.u.$$

$$V_c = aV_{a1} + a^2V_{a2} + V_{a0} = 0.643(-0.5 + j0.866) - 0.50(-0.5 - j0.866) - 0.143$$

$$V_c = -0.215 + j0.989\ p.u.$$

Line-to-line voltages are:

$$V_{ab} = V_a - V_b = 0.215 + j0.989 = 1.01\angle 77.7^0\ p.u.$$

$$V_{bc} = V_b - V_c = 0 - j1.978 = 1.978\angle 270^0\ p.u.$$

$$V_{ca} = V_c - V_a = -0.215 + j0.989 = 1.01\angle 102.3^0\ p.u.$$

Since the generated voltage-to-neutral E_a was taken as 1.0 per unit, the above line-to-line voltages are expressed in per unit of the base voltage-to-neutral. When expressed in volts the postfault line voltages are:

$$V_{ab} = 1.01 \times \frac{13.8}{\sqrt{3}}\angle 77.7^0 = 8.05\angle 77.7^0\ kV$$

$$V_{bc} = 1.978 \times \frac{13.8}{\sqrt{3}}\angle 270^0 = 15.73\angle 270^0\ kV$$

$$V_{ca} = 1.01 \times \frac{13.8}{\sqrt{3}}\angle 102.3^0 = 8.05\angle 102.3^0\ kV$$

If $Z_n = j5.0\ \Omega$,

$$Z_{base} = \frac{(13.8 * 10^3)^2}{20.10^6} = 9.5\ \Omega$$

$$Z_{react,p.u.} = \frac{3 \times j5.0}{9.5} = j1.58\ p.u.$$

$$I_{a1} = I_{a2} = I_{a0}$$

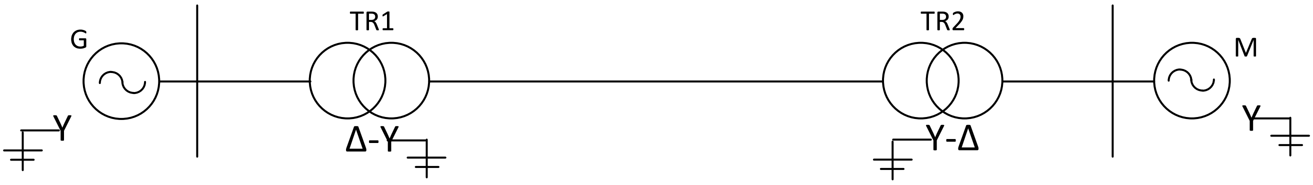
$$I_{a1} = \frac{1.0}{j(0.25 + 0.35 + 0.10 + 1.58)} = -j0.44\ p.u.$$

$$I_{F2} = I_a = 3I_{a1} = -j1.32\ p.u.$$

$$I_{base} = \frac{20.10^6}{\sqrt{3}(13.8)10^3} = 836\ A$$

$I_{F2} = I_a = (-j4.29)(836) = -j1.1\ kA$ (due to the reactance of $Z_n = j5.0\ \Omega$)

Tutorial 1.



$G \quad : 25\, MVA,\, 11\, kV,\, X_1 = X_2 = 0.20,\, X_0 = 0.06$

$M \quad : 20\, MVA,\, 11\, kV,\, X_1 = X_2 = 0.25,\, X_0 = 0.05$

$TR1 \quad : 30\, MVA,\, 11/66\, kV, X = 0.10$

$TR2 \quad : 25\, MVA,\, 66/11\, kV, X = 0.10$

$Line \quad : X_1 = X_2 = j40\ \Omega,\, X_0 = j100\ \Omega$

Select system common base values as 100 MVA and 11 kV at the generator side and marking each respective value in p.u. draw

Positive sequence network, negative sequence network and zero sequence network of the given power system.

Answer:

$G \quad : X_1 = X_2 = 0.2\frac{100}{25} = 0.8\, p.u.$

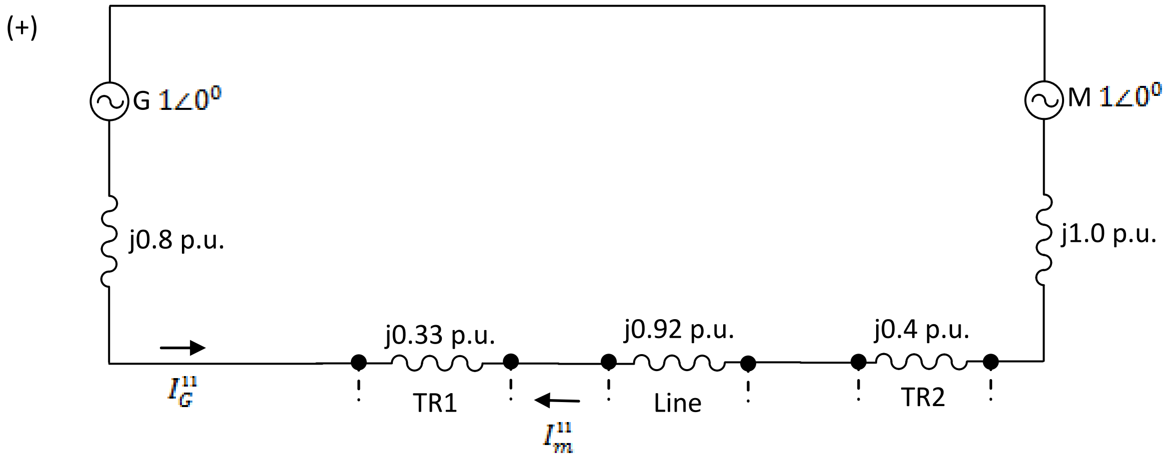
$M \quad : X_1 = X_2 = 0.25\frac{100}{20} = 1.2\, p.u.$

$TR1 \quad : X = 0.1\frac{100}{30} = 0.33\ p.u.$

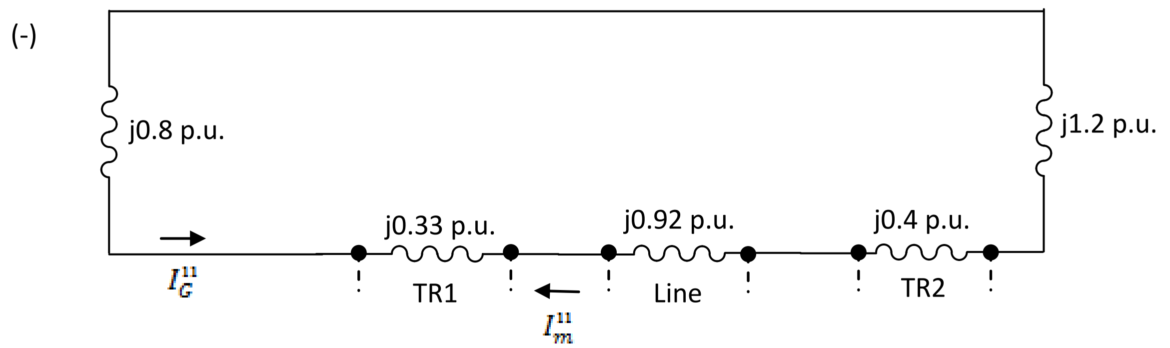
$TR2 \quad : X = 0.1\frac{100}{25} = 0.4\ p.u.$

$Line \quad : Z_{base} = \frac{66^2}{100} = 43.56\ \Omega,\, X_1 = X_2 = \frac{j40}{43.56} = 0.92\ p.u.$

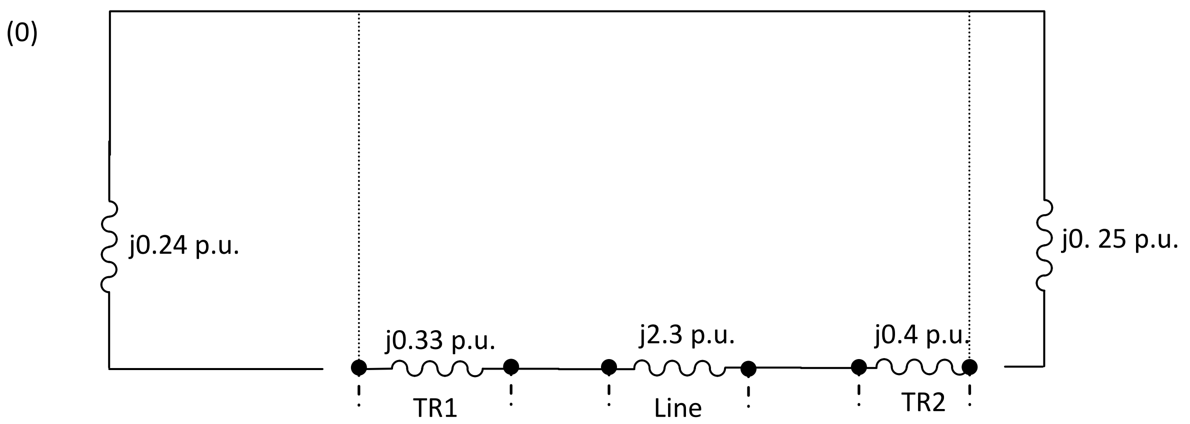
- *Positive Sequence Network,*



- *Negative Sequence Network,*



- Zero sequence network,

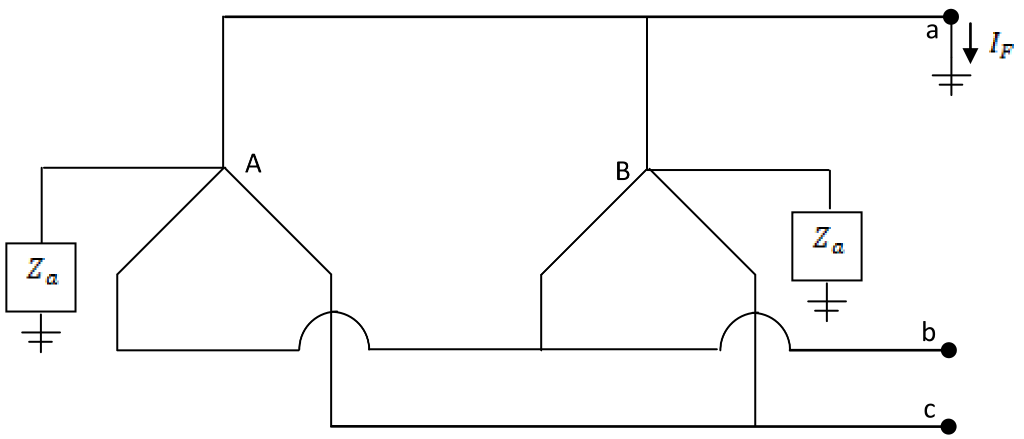


$G \quad : X_0 = 0.06\frac{100}{25} = 0.24\ p.u.$

$M \quad : X_0 = 0.05\frac{100}{20} = 0.25\ p.u.$

$Line \quad : X_0 = \frac{j100}{43.56} = j2.3\ p.u.$

Tutorial 2. Single line to ground fault at terminal a. Find the fault current value at point a?



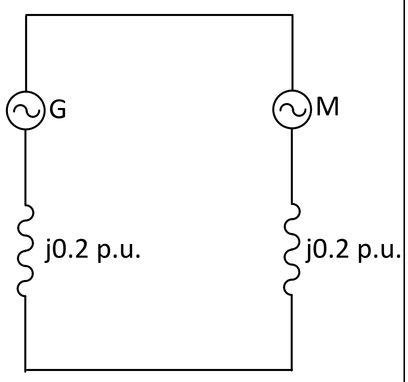
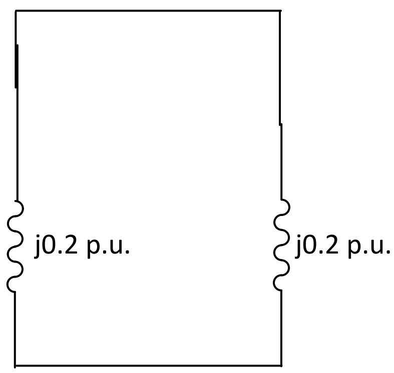
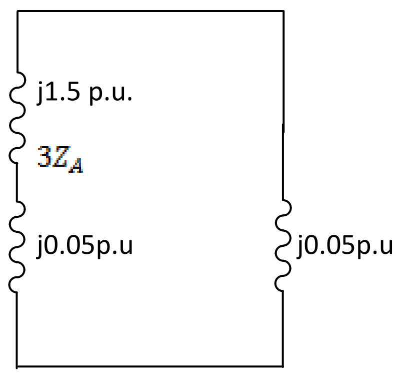
A and B transformers: $20\ MVA$, $11\ kV$, $X_1 = X_2 = 0.2$, $X_0 = 0.05$

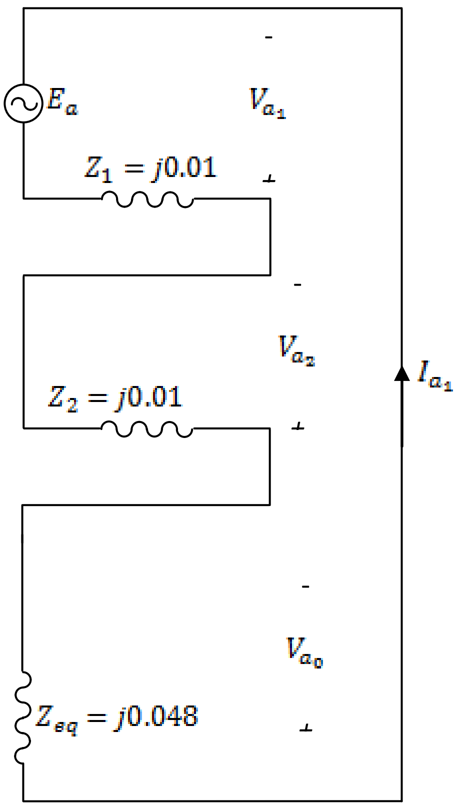
$Z_A = j3\ \Omega$, $Z_B = 0$

Answer:

$$Z_{base} = \frac{(11.10^3)^2}{20.10^6} = 6.05\ \Omega$$

$$Z_{A(p.u.)} = \frac{j3}{6.05} = j0.5\ p.u.$$

Positive Sequence Network	Negative Sequence Network	Zero Sequence Network
		



$$I_{a1} = I_{a2} = I_{a0} = \frac{1.0}{j\left(0.1 + 0.1 + \underbrace{0.048}_{j1.55//j0.05}\right)}$$

$$I_{a0} = \frac{1}{j0.248} = -j4.03\ p.u.$$

$$I_F = 3I_{a0} = -j12.09\ p.u.$$

2.4. Double line-to-ground Fault on an Unloaded Generator

The circuit diagram for a double line-to-ground fault on an unloaded, Y-connected generator having a grounded neutral is shown in Fig. 2.7. The faulted phases are b and c. The conditions at the fault are expressed by the following equations:

$$V_b = V_c = 0$$

$$I_a = 0$$

Substituting $V_b = 0$ and $V_c = 0$ in Eqs. (1.9), $V_{a1} = 1/3(V_a + aV_b + a^2V_c)$, and $V_{a2} = 1/3(V_a + a^2V_b + aV_c)$ gives

$$V_{a1} = 1/3(V_a + 0 + 0) = \frac{V_a}{3}$$

$$V_{a2} = 1/3(V_a + 0 + 0) = \frac{V_a}{3}$$

$$V_{a0} = 1/3(V_a + 0 + 0) = \frac{V_a}{3}$$

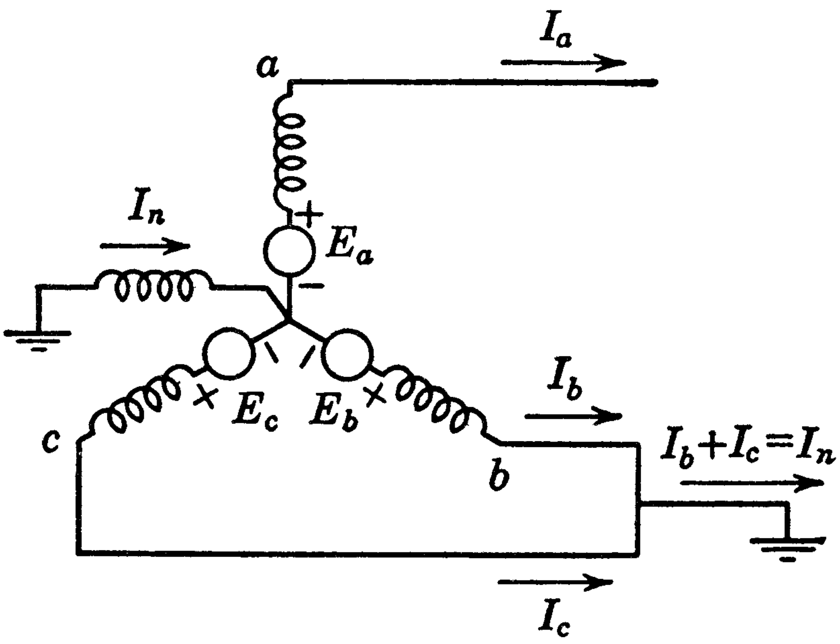


FIG. 2.7 Circuit for a double line-to-ground fault on phases b and c at the terminals of an unloaded generator whose neutral is grounded through a reactance.

Therefore,

$$V_{a1} = V_{a2} = V_{a0} \tag{2.7}$$

Solving Eqs. (2.3) and (2.4) for I_{a2} and I_{a0} and substituting V_{a1} for V_{a2} and V_{a0} , we obtain

$$I_{a2} = -\frac{V_{a2}}{Z_2} = -\frac{V_{a1}}{Z_2}$$

$$I_{a0} = -\frac{V_{a0}}{Z_0} = -\frac{V_{a1}}{Z_0}$$

Replacing V_{a1} by $E_a - I_{a1}Z_1$ gives

$$I_{a2} = -\frac{E_a - I_{a1}Z_1}{Z_2}$$

and

$$I_{a0} = -\frac{E_a - I_{a1}Z_1}{Z_0}$$

Since $I_a = 0$,

$$I_{a1} + I_{a2} + I_{a0} = 0$$

and

$$I_{a1} - \frac{E_a - I_{a1}Z_1}{Z_2} - \frac{E_a - I_{a1}Z_1}{Z_0} = 0$$

$$I_{a1}Z_2Z_0 - E_aZ_0 + I_{a1}Z_1Z_0 - E_aZ_2 + I_{a1}Z_1Z_2 = 0 \tag{2.8}$$

$$I_{a1} = \frac{E_a(Z_2 + Z_0)}{Z_1Z_2 + Z_1Z_0 + Z_2Z_0} = \frac{E_a}{Z_1 + Z_2Z_0/(Z_2 + Z_0)}$$

Equations (2.7) and (2.8) are the special equations for a double line-to-ground fault. They are used with Eqs. (2.2) to (2.4) and the symmetrical component relations to determine all the voltages and currents at the fault. Equation (2.7) indicates that the sequence networks should be connected in parallel, as shown in Fig. 2.8, since the positive-, negative-, and zero-sequence voltages are equal at the fault. Examination of Fig. 2.8 shows that all the conditions derived above for the double line-to-ground fault are satisfied by this connection.

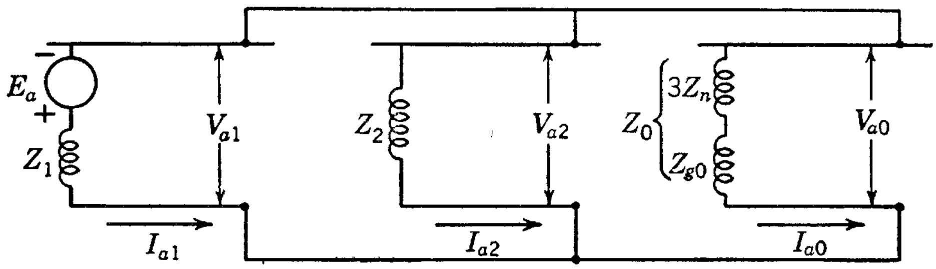


FIG. 2.8 Connection of the sequence networks of an unloaded genera tor for a double line-to-ground fault on phases b and c at the terminals of the generator

The diagram of network connections shows that the positive-sequence current I_{a1} is determined by the voltage E_a impressed on Z_1 in series with the parallel combination of Z_2 and Z_0. The same relation is given by Eq. (2.8).

In the absence of a ground connection at the generator no current can flow into the ground at the fault. In this case Z_0 would be infinite and I_{a0} would be zero. In so far as current is concerned the result would be the same as in a line-to-line fault. Equation (2.8) for a double line-to-ground fault approaches $I_{a1} = E_a/(Z_1 + Z_2)$ for a line-to-line fault as Z_0 approaches infinity, as may be seen by dividing the numerator and denominator of the second term in the denominator of Eq. (2.8) by Z_0 and letting Z_0 be infinitely large.

3. **UNSYMMETRICAL FAULTS ON POWER SYSTEMS**

In the derivation of equations for the symmetrical components of currents and voltages in a general network during a fault, we will designate as I_a, I_b, and I_c the currents flowing out of the original balanced system at the fault from phases a, b, and c, respectively. We can visualize the currents I_a, I_b, and I_c by referring to Fig. 3.1, which shows the three lines of the three-phase system at the part of the network where the fault occurs. The flow of current from each line into the fault is indicated by arrows shown on the diagram beside hypothetical stubs connected to each line at the fault location. Appropriate connections of the stubs represent various types of faults. For instance, connecting stubs b and c produces a line-to-line fault through zero impedance. The current in stub a is then zero, and I_b is equal to $-I_c$.

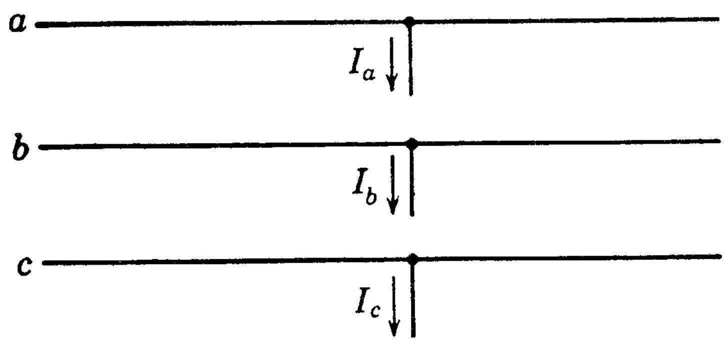


FIG. 3.1 Three conductors of a three-phase system. The stubs carrying currents I_a, I_b, and I_c may be interconnected to represent different types of faults

The line-to-ground voltages at the fault will be designated V_a, V_b, and V_c. Before the fault occurs, the line-to-neutral voltage of phase a at the fault will be called V_f, which is a positive-sequence voltage since the system is assumed to be balanced. We met the prefault voltage V_f previously in "*Symmetrical Three-Phase Faults on Synchronous Machines*" in calculations to determine the currents in a power system when a symmetrical three-phase fault occurred.

A single-line diagram of a power system containing three synchronous machines is shown in Fig. 3.2. Such a system is sufficiently general that equations derived therefrom are applicable to any balanced system regardless of the complexity. Figure 3.2 also shows the sequence networks of the system. The point where a fault is assumed to occur is marked P on the single-line diagram and on the sequence networks. As we saw in "*Symmetrical Three-Phase Faults on Synchronous Machines*", the load current flowing in the positive-sequence network is the same, and the voltages to ground external to the machines are the same, regardless of whether the machines are represented by their voltages behind subtransient reactance and their subtransient reactances, or by their voltages behind transient reactance and their transient reactances, or by their voltages behind synchronous reactance and their synchronous reactances.

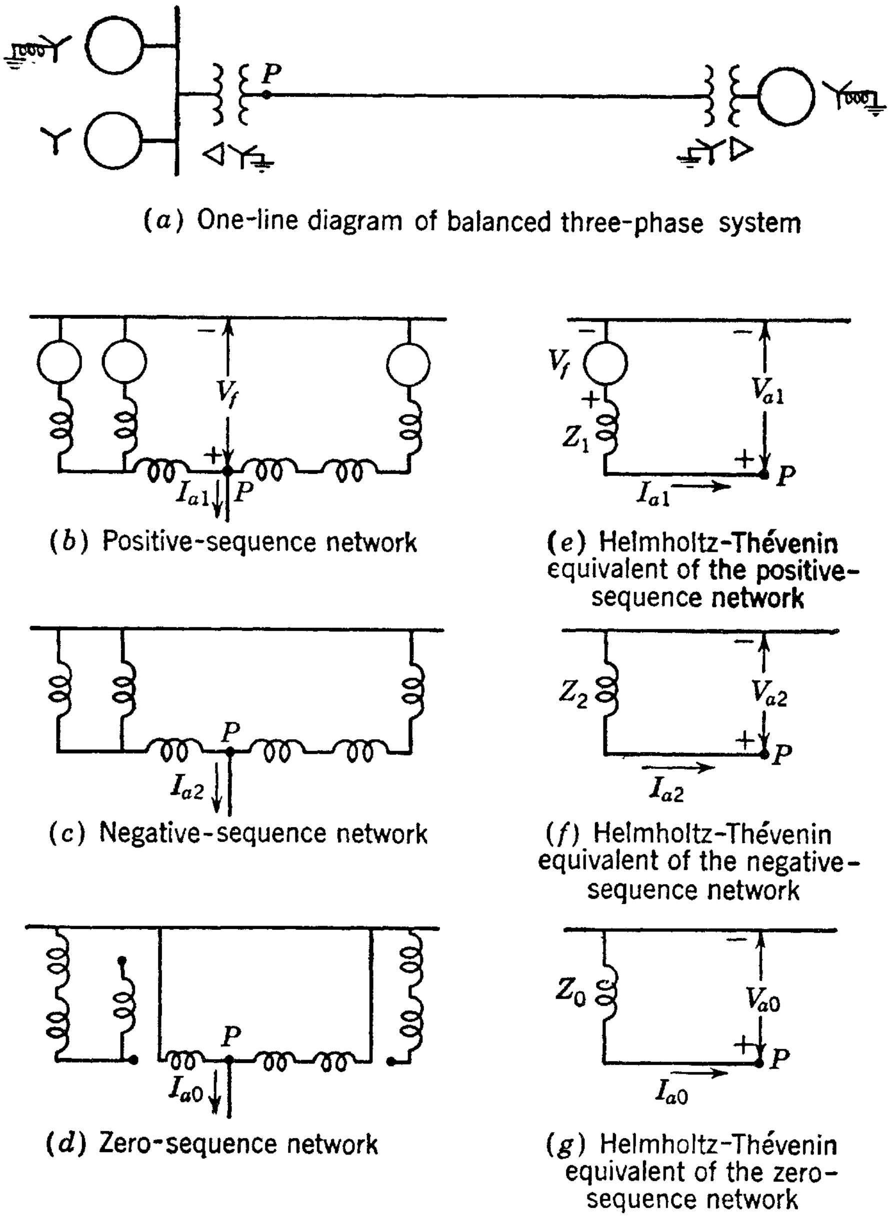


FIG. 3.2 One-line diagram of a three-phase system, the three sequence networks of the system, and the Helmholtz-Thevenin equivalent of each network for a fault at P

Since linearity is assumed in drawing the sequence networks, each of the networks can be replaced by its Helmholtz-Thevenin equivalent between the two terminals composed of its reference bus and the point of application of the fault. The Helmholtz-Thevenin equivalent circuit of each sequence network is shown adjacent to the diagram of the corresponding network in Fig. 3.2. The internal voltage of the single generator of the equivalent circuit for the positive-sequence network is V_f, the prefault voltage to neutral at the point of application of the fault. The impedance Z_1 of the equivalent circuit is the impedance measured between point P and the reference bus of the positive-sequence network with all the internal emfs short-circuited. The value of Z_1 is dependent on whether subtransient, transient, or synchronous reactance is used in the sequence network, which is, in turn, dependent on whether subtransient, transient, or steady-state currents are being computed.

Since no negative- or zero-sequence currents are flowing before the fault occurs, the prefault voltage between point P and the reference bus is zero in the negative- and zero-sequence networks. Therefore, no emfs appear in the equivalent circuits of the negative- and zero-sequence networks. The impedances Z_2 and Z_0 are measured between point P and the reference bus in their respective networks and depend on the location of the fault.

Since I_a is the current flowing from the system into the fault, its components I_{a1}, I_{a2} and I_{a0} flow out of their respective sequence networks and the equivalent circuits of the networks at P, as shown in Fig. 3.2. Examination of the equivalent circuits of the sequence networks shows that the voltages V_{a1}, V_{a2}, and V_{a0} at point P are expressed by the following equations:

$$V_{a1} = V_f - I_{a1}Z_1 \tag{3.1}$$

$$V_{a2} = -I_{a2}Z_2 \tag{3.2}$$

$$V_{a0} = -I_{a0}Z_0 \tag{3.3}$$

The only differences between Eqs. (3.1) to (3.3) and Eqs. (2.2) to (2.4) are the substitution of V_f for E_a and the interpretation of Z_1, Z_2, and Z_0. For a fault at the terminals of an isolated generator at no load, E_a and V_f are equal, and Eqs. (3.1) to (3.3) reduce to Eqs. (2.2) to (2.4).

Solution Algorithm for Unbalanced Faults:

Step 1. Draw the positive-, negative- and zero-sequence networks of the overall system.

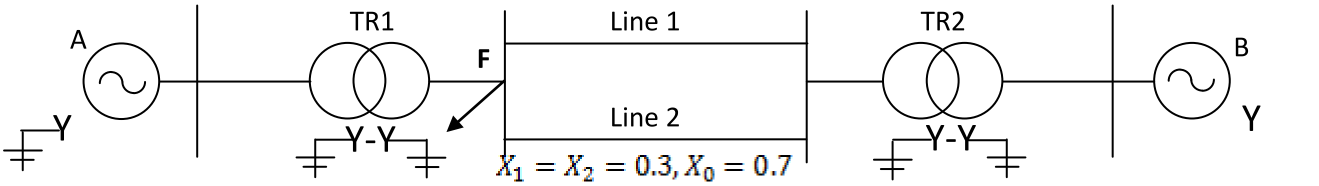
Step 2. Calculate and replace the sequence networks by Thevenin equivalence as seen from the fault points.

Step 3. For the given fault type find out the relationships amoung (+), (-) and (0) sequence are voltages and currents.

Step 4. Interconnect sequence network such that the relations in step 3 are satisfied.

Step 5. So that resulting network for the designed variables.

Example:



Unit A and B generate $1.0\,p.u.$ voltage,

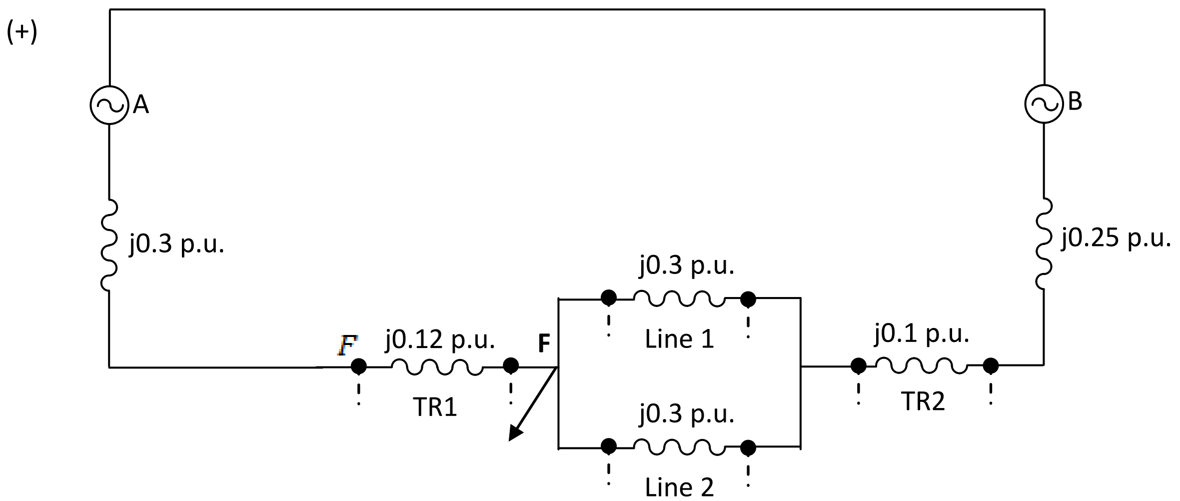
A : $X_1 = 0.3, X_2 = 0.2, X_0 = 0.05$

B : $X_1 = 0.25, X_2 = 0.15, X_0 = 0.03$

$TR1$: $X = 0.12$ $TR2$: $X = 0.10$

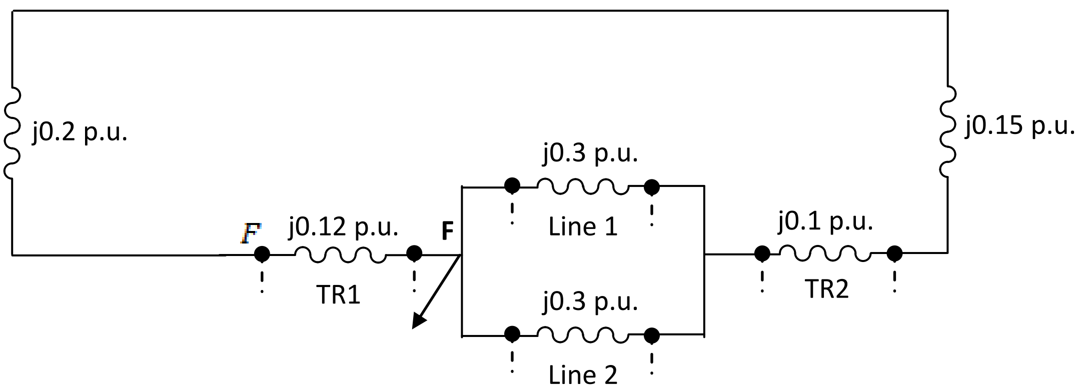
All data given in $p.u.$ with respect to select all common bases of the power system. $V_f = 1.0p.u.$: Voltage before the fault.

Double lin-to-ground fault occurs at F, find the I_F.



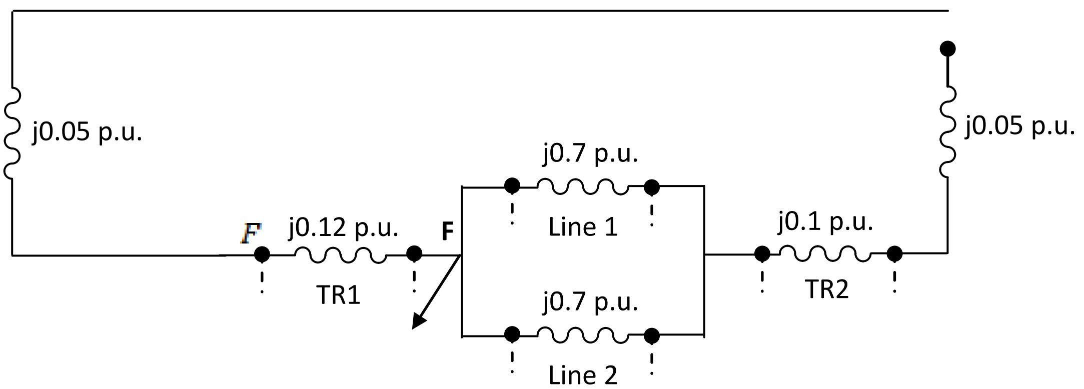
$$Z_1 = \frac{j0.42 \times j0.5}{j(0.42 + 0.5)} = j0.23\,p.u.$$

(-)

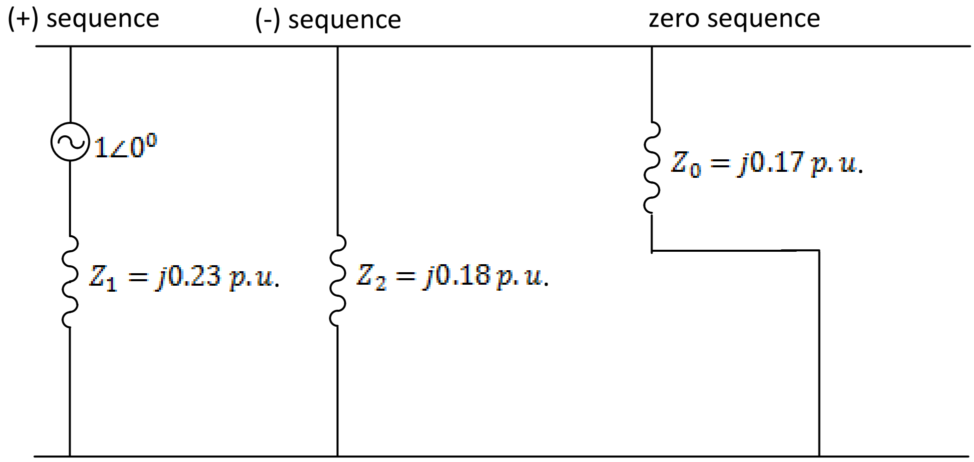


$$Z_2 = \frac{j0.32 \times j0.4}{j(0.32 + 0.4)} = j0.18\ p.u.$$

(0)



$$Z_0 = j0.17\ p.u.$$

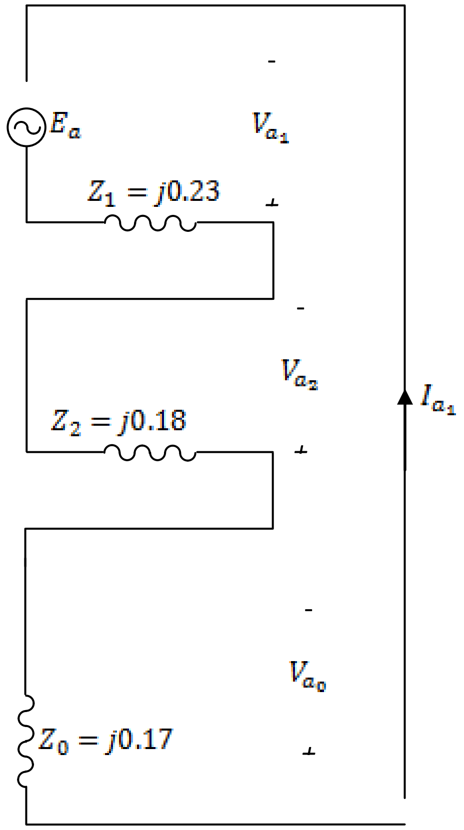


$$I_{a1} = \frac{1.0}{Z_1 + \frac{Z_2 Z_0}{Z_2 + Z_0}} = \frac{1.0}{j\left(0.23 + \frac{0.17 \times 0.18}{0.17 + 0.18}\right)} = -j3.15\ p.u.$$

$$I_{a0} = -\frac{V_{a0}}{Z_0} = -\frac{V_f - I_{a1}Z_1}{Z_0} = -\frac{0.2755}{j0.17} = j1.62\ p.u.$$

$$I_F = 3I_{a0} = j4.86\ p.u.$$

Let's solve the same problem with the same fault point location for a single line to ground fault.



$$I_{a1} = I_{a2} = I_{a0} = \frac{1.0}{j(0.23 + 0.18 + 0.17)} = -j1.72\ p.u.$$

$I_F = 3I_{a0} = -j5.16\ p.u.$

If fault through an impedance,

$$I_{a1} = I_{a2} = I_{a0} = \frac{E_a}{Z_1 + Z_2 + \underbrace{Z_0}_{|Z_{g_0}+3Z_n|} + 3Z_f}$$

4. **LOAD FLOW ANAYSIS**

A load flow study is the determination of voltage, current and p.f. at the given nodes of the power system network under normal operating conditions. Load flow studies are essential in planning and future development of the system.

P : Active power

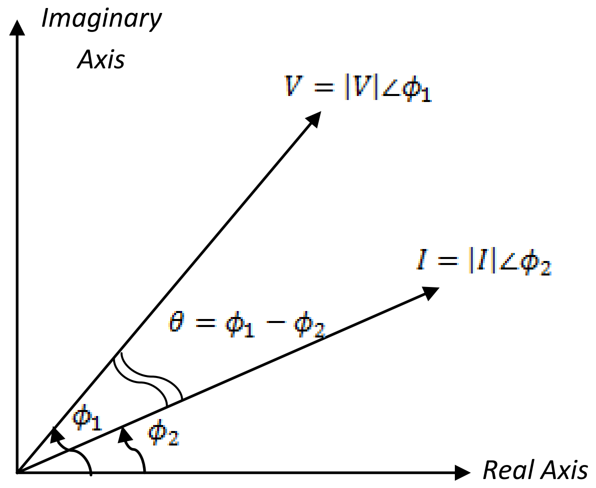
Q : Reactive power

V : Voltage magnitude

δ : Active power

$S = P + jQ$

$S = VI^*$



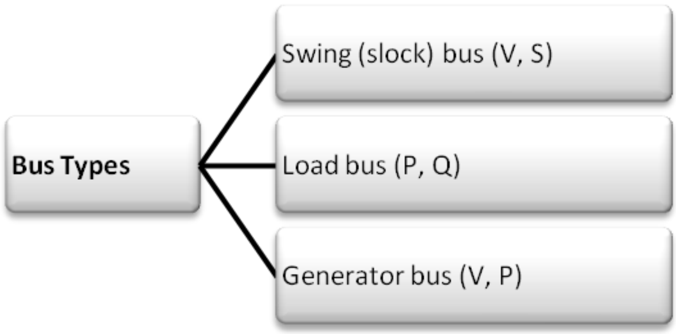
$P = |V||I|\cos\theta$

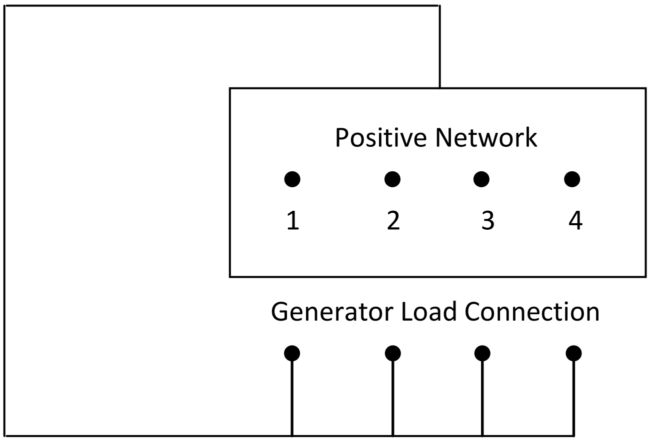
$Q = |V||I|\sin\theta$

$I^* = |I|\angle{-\phi_2}$

$S = VI^* = |V||I|\angle\phi_1 - \phi_2$

Hence we have to consider the conjugate vector.



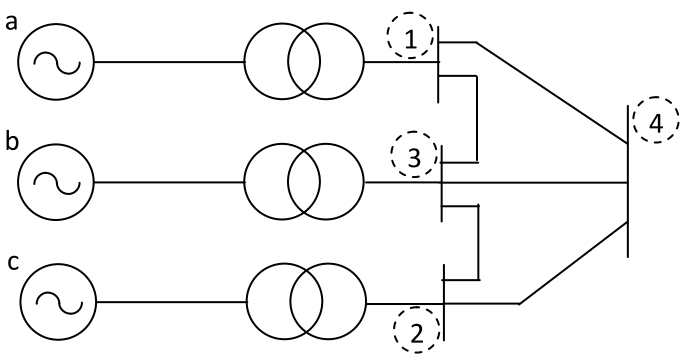


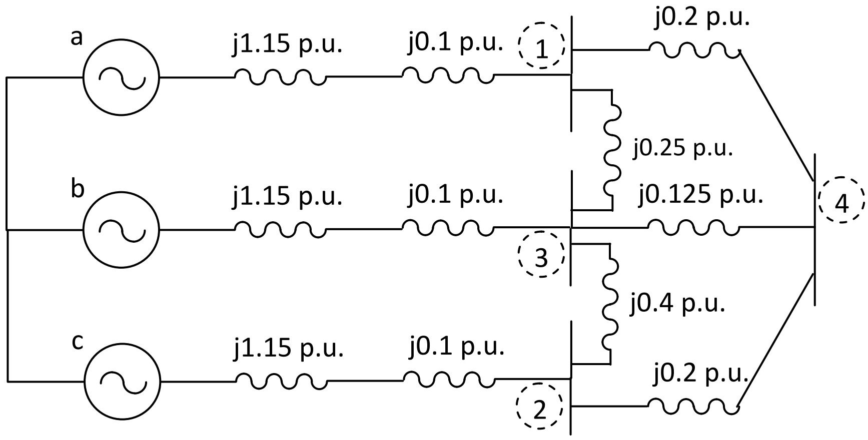
$$I = YV$$

Assume for a 4 bus system,

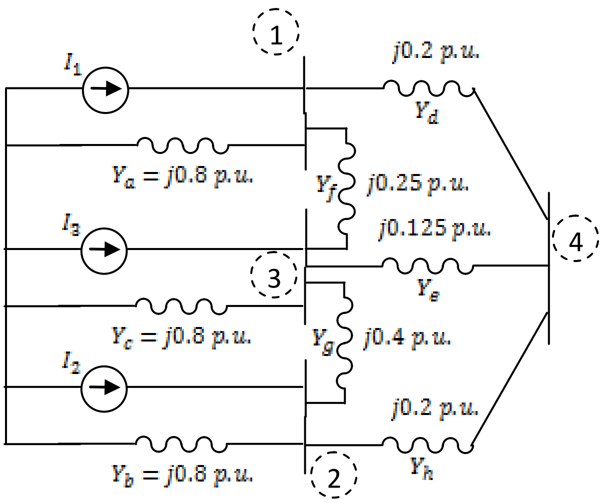
$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Y_{bus} : Bus Admittance Matrix





Corresponding reactance diagram all data given in p.u. With current sources replacing the equivalent voltage sources (values shown are admittance in p.u.).



$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$I_1 = V_1 Y_a + (V_1 - V_3) Y_f + (V_1 - V_4) Y_d$$

$$I_1 = V_1 (Y_a + Y_f + Y_d) - V_3 Y_f - V_4 Y_d$$

$$Y_{11} = Y_a + Y_f + Y_d = -j0.8 - j4.0 - j5.0 = -j9.8$$

$$Y_{12} = 0$$

$$Y_{13} = -Y_f = j4.0 = Y_{31}$$

$Y_{14} = -Y_d = j5.0 = Y_{41}$

$$\left.\begin{aligned} Y_{11} &= -j0.8 - j4.0 - j5.0 = -j9.8 \\ Y_{22} &= -j5.0 - j2.5 - j0.8 = -j8.3 \\ Y_{33} &= -j4.0 - j2.5 - j8.0 - j0.8 = -j15.3 \\ Y_{44} &= -j5.0 - j5.0 - j8.0 = -j18.0 \end{aligned}\right\} Self\ Admittance(Diagonal\ elements)$$

$$\left.\begin{aligned} Y_{12} &= Y_{21} = 0 \\ Y_{13} &= Y_{31} = j4.0 \\ Y_{14} &= Y_{41} = j5.0 \\ Y_{23} &= Y_{32} = j2.5 \\ Y_{24} &= Y_{42} = j5.0 \\ Y_{34} &= Y_{43} = j8.0 \end{aligned}\right\} Mutual\ off\ Diagonal$$

$$Y_{bus} = \begin{bmatrix} -j9.8 & 0 & j4.0 & j5.0 \\ 0 & -j8.3 & j2.5 & j5.0 \\ j4.0 & j2.5 & -j15.3 & j8.0 \\ j5.0 & j5.0 & j8.0 & -j18.0 \end{bmatrix}$$

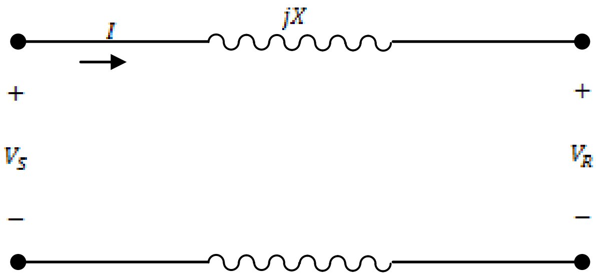
Properties of the Bus Admittance Matrix

- It is a source matrix of order $n \times n$
- It is symmetrical
- It is complex

Each off diagonal element Y_{km} is the negative of the branch admittance between busses k and m.

Each diagonal element is the sum of admittances the branches terminating on the bus k, including the branches to ground

5. **POWER FLOW IN A SHORT TRANSMISSION LINE**



$$S = P + jQ = VI^*$$

At the secondary end, on per phase basis

$$S_S = P_S + jQ_S = V_S I^*$$

$$I = \frac{1}{jX}(V_S - V_R), I^* = \frac{1}{-jX}(V_S{}^* - V_R{}^*)$$

Hence,

$$S_S = \frac{V_S}{-jX}(V_S{}^* - V_R{}^*)$$

Now suppose that,

$$V_R = |V_R|\angle 0^0 \; so \; V_R = V_R{}^*$$

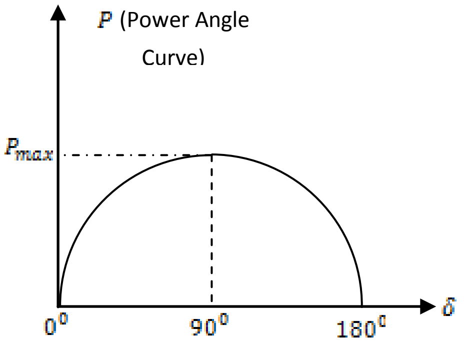
$$V_S = |V_S|\angle\delta$$

$$S_S = \frac{|V_S|^2 - [|V_R||V_S|\cos\delta + j|V_R||V_S|\sin\delta]}{-jX}$$

$$S_S = \frac{|V_R||V_S|}{X}\sin\delta + \frac{j}{X}(|V_S|^2 - |V_R||V_S|\cos\delta)$$

$$P_S = \frac{|V_R||V_S|}{X}\sin\delta \;\; W$$

$$Q_S = \frac{1}{X}(|V_S|^2 - |V_R||V_S|\cos\delta)\; VAR$$



$$P + jQ = VI^* \Rightarrow I = \frac{P - jQ}{V^*}$$

For a 4 bus system

$$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4$$

$$\frac{P_2 - jQ_2}{V_2{}^*} = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4$$

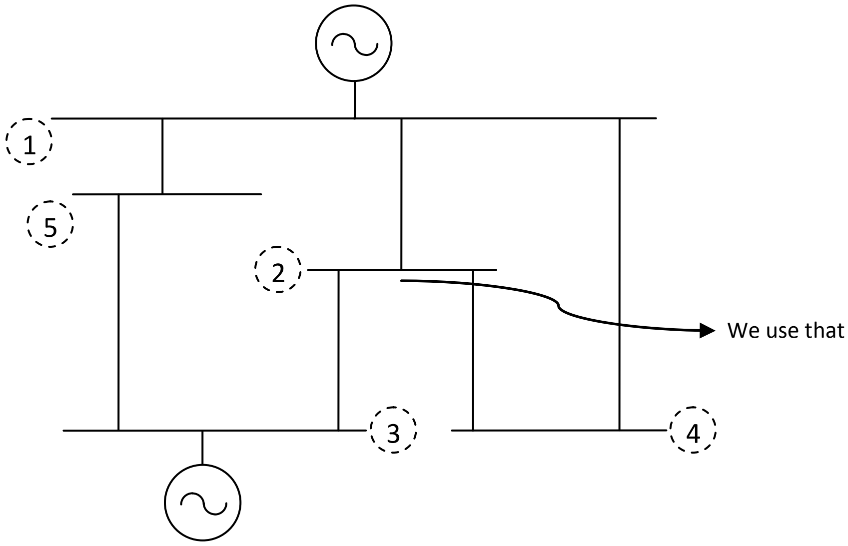
Gauss-Seidell Method

$$V_k = \frac{1}{Y_{kk}}\left[\frac{P_k - jQ_k}{V_k{}^*} - \sum_{n=1}^{N} Y_{kn}V_n\right], n \neq k$$

Bus system for bus 2,

$$V_2 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2{}^*} - Y_{21}V_1 - Y_{23}V_3 - Y_{24}V_4\right]$$

Example:



...	...	$1.02\angle 0^0$	Swing (Slack) Bus
-0.6	-0.3	$1.0\angle 0^0$ Initial	Load Bus
1.0	...	$1.04\angle 0^0$ Initial	Voltage Magnitude Constant
-0.4	-0.1	$1.0\angle 0^0$ Initial	Load Bus
-0.6	-0.2	$1.0\angle 0^0$ initial	Load Bus

$p.u.)$	$p.u.)$
0.588235	-2.352941
0.329157	-1.568627
1.176471	-4.705882
1.176471	-4.705882
0.588235	-2.352941
1.176471	-4.703882

Find $V_2^{(1)}$ using Gauss-Seidell?

Answer:

$$V_2 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - Y_{21}V_1 - Y_{23}V_3 - Y_{24}V_4 - Y_{25}V_5\right]$$

$$Y_{22} = Y_{21} + Y_{23} + Y_{24} = 2.352941 - j9.410764$$

$$Y_{21} = -0.588235 + j2.352941$$

$$Y_{23} = -1.176471 + j4.705882$$

$$Y_{24} = -0.588235 + j2.352941$$

$Y_{25} = 0$ (no connection between bus)

$$V_2^{(1)} = \frac{1}{Y_{22}}\left[\frac{-0.6 + j0.3}{1.0 + j0} - [1.02 \times (-0.588235 + j2.352941) + 1.04 \times (-1.176471 + j4.705882) + 1.0 \times (-0.588235 + j2.352941)]\right]$$

$V_2^{(1)} = 0.980635 - j0.052530\ p.u.$ (conjugate of V_2)

5.1. Load Flow with The Newton-Raphson Method

$$S = P + jQ = VI^* \Rightarrow I = \frac{P - jQ}{V^*}$$

For N bus system at bus k,

$$I_k = \sum_{n=1}^{N} Y_{kn} V_n$$

$$P_k - jQ_k = V_k^{\ *} I = V_k^{\ *} \sum_{n=1}^{N} Y_{kn} V_n$$

Let,

$$V_k = |V_k| \angle \delta_k \quad , \quad V_n = |V_n| \angle \delta_n \quad , \quad Y_{kn} = |Y_{kn}| \angle \theta_{kn}$$

Then,

$$P_k - jQ_k = \sum_{n=1}^{N} |V_k V_n Y_{kn}| \angle \theta_{kn} + \delta_n - \delta_k$$

$$P_k = \sum_{n=1}^{N} |V_k V_n Y_{kn}| \cos \theta_{kn} + \delta_n - \delta_k$$

$$Q_k = -\sum_{n=1}^{N} |V_k V_n Y_{kn}| \sin \theta_{kn} + \delta_n - \delta_k$$

Reminder

Consider two functions of two variables x_1 and x_2 such that,

$$f_1(x_1, x_2) = c_1$$

$$f_2(x_1, x_2) = c_2$$

where c_1 and c_2 are constants.

Let $x_1^{(0)}$ and $x_2^{(0)}$ be the initial estimates,

Let $\Delta x_1^{(0)}$ and $\Delta x_2^{(0)}$ be the values by which, We initial estimates differ from the correct solutions,

$$f_1\left(x_1^{(0)} + \Delta x_1^{(0)}, x_2^{(0)} + \Delta x_2^{(0)}\right) = c_1$$

$$f_2\left(x_1^{(0)} + \Delta x_1^{(0)}, x_2^{(0)} + \Delta x_2^{(0)}\right) = c_2$$

Expanding the left hand side in a Taylor series,

$$f_1\left(x_1^{(0)}, x_2^{(0)}\right) + \Delta x_1^{(0)} \left.\frac{\partial f_1}{\partial x_1}\right|_{x_1^{(0)}} + \Delta x_2^{(0)} \left.\frac{\partial f_1}{\partial x_2}\right|_{x_2^{(0)}} = c_1$$

$$f_2\left(x_1^{(0)}, x_2^{(0)}\right) + \Delta x_1^{(0)} \left.\frac{\partial f_2}{\partial x_1}\right|_{x_1^{(0)}} + \Delta x_2^{(0)} \left.\frac{\partial f_2}{\partial x_2}\right|_{x_2^{(0)}} = c_2$$

Matrix Form,

$$\begin{bmatrix} c_1 - f_1\left(x_1^{(0)}, x_2^{(0)}\right) \\ c_2 - f_2\left(x_1^{(0)}, x_2^{(0)}\right) \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix}$$

$$\begin{bmatrix} \Delta c_1^{(0)} \\ \Delta c_2^{(0)} \end{bmatrix} = \underbrace{J^{(0)}}_{\substack{Jacobian \\ Matrix}} \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix}$$

The solution of gives $\Delta x_1^{(0)}$ and $\Delta x_2^{(0)}$,

Then a better estimate at the solution,

$$x_1^{(1)} = x_1^{(0)} + \Delta x_1^{(0)}$$

$$x_2^{(1)} = x_2^{(0)} + \Delta x_2^{(0)}$$

The iterations are continued until Δx_1 and Δx_2 become smaller than a predetermined value.

__Finishing Reminder__

Corresponding to the Matrix Equation for a three bus system (with bus 1 as the slack (swing) bus)

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} & \frac{\partial P_2}{\partial |V_3|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} & \frac{\partial P_3}{\partial |V_3|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial |V_2|} & \frac{\partial Q_3}{\partial |V_3|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \\ \Delta |V_3| \end{bmatrix}$$