

NEAR EAST UNIVERSITY
FACULTY OF ENGINEERING



**DESIGN OF A RADAR BASED COLLISION
AVOIDANCE SYSTEM (RACAS) FOR VEHICLES**

Guner Bugrahan

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Avoidance System (RACAS) For
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**Approval of Director of the Institute of
Applied and Social Sciences**

Prof. Dr. Fakhreddin Mamedov



**We certify that this thesis is satisfactory for the award of the
degree of Master of Science in Electrical and Electronic
Engineering**

Examining Committee in Charge:

Assoc. Prof. Dr. Rahib Abiyev, Chairman of Committee, Computer
Engineering Department, NEU



Assist. Prof. Dr. Kadri Bürüncük, Electrical and Electronic
Engineering Department, NEU



Assist. Prof. Dr. Kamil Dimililer, Electrical Engineering Department,
GAU



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ABSTRACT

This thesis describes a novel approach to safe over-take by using special radar system, mounted on the vehicle. The system is capable of preventing potential accidents by utilizing the radar properties. Hence the system is named "*Radar Based Automatic Collision Avoidance System (RACAS)*".

RACAS is mounted on top of a vehicle, and it measures dynamically the relative speeds of both receding and approaching vehicles, as well as the relative distances from them. A formula has been derived to predetermine whether or not an accident can occur during an over-take. RACAS obtains the necessary data from the radar output, compares the conditions by using the derived formula for a safe over-take, and a decision is made to inform the driver whether or not to attempt to over-take the vehicle in the front.

The potential application of RACAS is very important as it can help to avoid collisions and thus reduce the number of accidents on roads.

This thesis covers the theory of the proposed novel system and makes recommendations on how it can be implemented in practice.

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INTRODUCTION

The risk of car collision has a complex nature but it can be assumed that the risk is directly related to the number of vehicles on the road. This risk is also related to the speeds of the cars, the age and experience of the drivers, the mental health and the general conditions of the drivers. For example, there is a higher risk when the drivers are exceeding the speed limit, or when they have taken excessive amounts of alcohol. It can be stressed that in nearly all cases, with some exceptions, it is one or more of the drivers' fault when a collision takes place. Accidents can in general be minimized by educating the drivers and also by introducing heavy fines when the law is broken. For example, in the U.K., a drunk driver may lose his or her driving license when caught driving with excessive levels of alcohol.

Car manufacturers have introduced or have thought of introducing various schemes in order to help minimize accidents. For example, some manufacturers thought of providing a key-lock system such that only a sober driver can allegedly remember the right key combinations and this is supposed to stop a drunk driver from starting the engine. Such methods do not help to minimize accidents and they fail in practice for several reasons.

Another method chosen by some car manufacturers is to provide Cruise Control Systems (CCS). In this scheme, the car runs at a pre-defined fixed speed set by the driver and the system helps the driver to concentrate on the road and not on the controls. CCS does not provide any safety to the driving and on the contrary, it can make the driving unsafe, as a tired driver will not have immediate and full control of the throttle and the brakes.

Another scheme adopted by some car manufacturers is the Adaptive Cruise Control Systems (ACCS). In this system, the vehicle is equipped with a millimeter wave radar system and the speed of the vehicle in front of the vehicle is constantly measured. The vehicle then automatically runs at a speed, which provides a safe distance to the vehicle in the front. In such systems, the throttle and the brakes of the vehicle are controlled continuously to assure that a safe distance, set by the driver, is kept to the vehicle ahead

at all times. The system can also warn the driver and alert the airbags just before a potential collision. Mercedes S-class vehicles and some other expensive luxury vehicles now provide the options of ACCS based

Control systems. Although ACCS can provide collision avoidance with the car ahead, Holger Meinel, a senior researcher at *Daimler Benz Aerospace A.G.* says that the ACCS is not a safety feature, but it is a comfort feature. This is the action taken so that the company will not be liable for any damages if a vehicle equipped with ACCS is involved in an accident. Mercedes-Benz system uses a 77 MHz Doppler radar linked into the electronic control and braking system to maintain a safe distance between a car with the system and the vehicle in front of it. The Mercedes ACCS system option will cost an extra \$1500.

In this research thesis, a novel approach to safety in driving is described. The system basically consists of a 77 MHz millimeter wave radar, which is mounted on top of the vehicle. The radar dynamically measures the distance to the vehicle ahead and also to the vehicle approaching us from the opposite lane. The speeds of all these vehicles involved are also determined. The system then performs calculations and informs the driver whether or not it is safe to over-take the vehicle in front of our vehicle, without having a collision with the vehicle coming from the opposite lane. As shown in the thesis, the calculation is based upon the relative distances and the speeds of all three cars. The system is named as RACAS (Radar based Automatic Collision Avoidance System).

A simulation model is developed using the Visual Basic programming language on a standard PC. This model is used to verify the correctness of the collision formula derived in the thesis.

The system developed in the thesis is analyzed from a theoretical point of view and any hardware has not been designed. It is hoped that the car manufacturers in their new model cars can use the theory developed.

Chapter 1 gives the theory of collision and equations are derived to describe the various

Condition under which a car collision can take place. A graphical, Visual Basic based simulation program is also described in this chapter, which is developed by the author in order to verify the correctness of various collisions conditions.

Chapter 2 is an introduction to the radar systems. The basic radar theory, range, noise, effects of weather and transmitter and propagation conditions are discussed in this chapter.

Chapter 3 describes the theory of one of the fundamental parts of any radar system, the antenna. Various antenna designed are described in detail.

Chapter 4 is an introduction to the types of radar systems. The theory of the Doppler effect, CW radars, and frequency modulated CW radar systems are described here.

Finally, Chapter 5 describes the design principles and the requirements of a radar based system for automatic collision detection.

1. THE THEORY OF COLLISION DETECTION

1.1 Traffic Accidents

Occurrence of traffic accidents depend on lots of parameters where most of them cannot be eliminated today. Perhaps, and it is hoped that, some or all of these parameters will be eliminated in future, and less lives will be lost on roads. Some of the main reasons of traffic accidents are:

- Speeding
- Driving when drunk or under the influence of alcohol
- Faulty brakes
- Driver errors
- Road conditions

Speeding is perhaps the most common reason for accidents. Speeding makes the handling of the car more difficult, and as a result, causes the driver errors to increase. It is also more difficult to stop when the car is at a high speed since the stopping-distance is directly proportional to the speed of the car.

Alcohol has a negative effect on driving as it makes the driver less alert. A driver under the influence of alcohol can not judge the distance and speed accurately and the end result is usually some kind of serious accident.

Collision with the vehicle coming from the opposite lane is also one of the biggest reasons of car accidents and deaths on roads. This usually happens when the driver attempts to over-take the vehicle in the front but can not judge the speed of the vehicle coming from the opposite lane. The end result is a head-on collision and the death rate is very high in such accidents.

In this thesis, equations are developed to describe the theory of head-on collision when the speeds and the distances of the vehicles involved in a collision are known, or can be

estimated. The correctness of these equations are verified by using a graphical simulation program developed by the author.

1.2 The Theory of Collision

The aim of collision avoidance is to prevent accidents while the driver attempts to over-take the vehicle in the front. For a safe over-take, the driver has to estimate the speed of his car, the speed of the front car on the same lane, and also the speed of the approaching car from the opposite lane. In addition, it is required to estimate the distances between all these three vehicles before a decision can be made on whether or not it is safe to over-take. However, from time to time, for one reason or another, the decision is wrong and accidents do happen. The main problem here is not to be able to estimate the speeds and the distances of the vehicles accurately. It is an actual fact that mankind naturally makes mistakes and also learns from these mistakes. This is one of the reasons why an experienced driver makes less accidents.

The main aim in this research is to estimate the speeds, the distances and the conditions of a collision electronically. Electronic systems make less or perhaps no errors and it is hoped that an electronic collision avoidance system will minimize most of the car accidents due to head-on collisions. The method chosen in this research work is to use an on-board radar system to estimate the speeds and the distances of all the vehicles involved in a potential collision course.

Figure 1.1 shows the positions of three cars before and after over-take. The top of the figure is the situation before the over-take, and the lower part of the figure shows a safe over-taking situation. Roads are shown in both cases with the two lanes divided with a line running through the centres of the roads. The vehicles on the left travel to the right using the left-hand lane, and the vehicle on the right also uses the left-hand lane and travels to the left. Here, it is assumed that car C is cruising at a constant speed V_C . Car A is going at a constant speed of V_A and is attempting to over-take car C. Car B is approaching from the opposite lane with a constant speed of V_B .

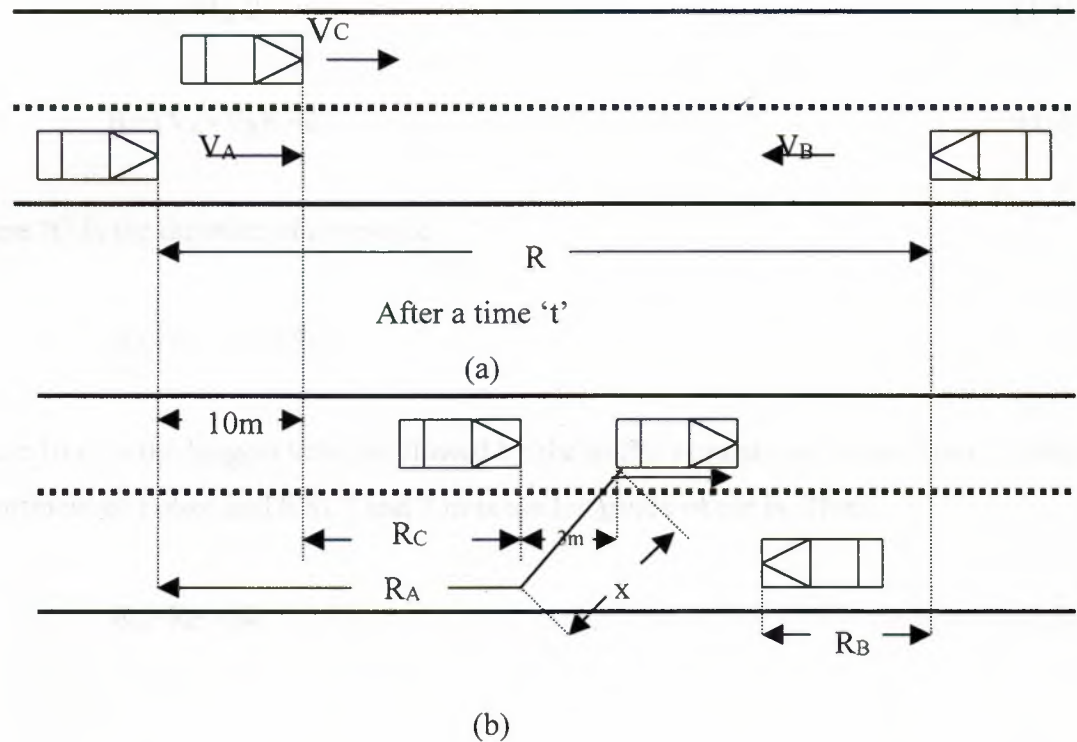


Figure 1.1 (a) Shows the position of three cars before over-take (b) shows the position of three cars after over-take.

1.2.1 Determination Of The Range.

If we assume a 3 m as a safe relative distance between A and C after the over-take, we can write the following relationship for an over-take at any time:

$$R = (R_A - x) + 3 + R_B$$

Where, R_A and R_B are the distances covered by car A and car B respectively during over-taking. x is the hypotenuse of the right triangle, from Fig. 1.1, with sides 3 and 4

m then x is 5 m. R is the distance between A and B before over-take. Using the speeds of A and B and the value of x, equations 1.1 and 1.2 can be written as:

$$R = R_A + R_B - 2 \quad (1.1)$$

$$R = (V_A + V_B)t - 2 \quad (1.2)$$

Where "t" is the duration of over-take

$$R_A = R_C + 10 + 5 + 3$$

Where 10 m is the longest vehicle allowed by the traffic regulations [taken from Traffic Department of Police in TRNC] and 5 m is the longitude of car B. Then

$$R_A = R_C + 18 \quad (1.3)$$

Substituting $R_A = V_A t$, $R_C = V_C t$ into equation 1.3 and solve for "t"

$$t = \frac{18}{V_A - V_C} \quad (1.4)$$

Putting the value of "t" in equation 1.2

$$R = 18 \frac{V_A + V_B}{V_A - V_C} - 2 \quad (1.5)$$

Equation 1.5 gives the minimum distance between car A and B for safe over-take, given the speeds and the relative distances between the cars. In practice, the speeds of three cars A, B and C can dynamically be measured and the distances for a safe over-take can be calculated at any time during the journey. Equation 1.5 is derived without considering the accelerations of the cars. In section 1.2.3, the accelerations of three cars

will be taken into account.

1.2.2 Relative Speed

A radar mounted on moving car will normally measure the relative speeds and equation 1.5 can be modified and written in terms of relative speeds.

From Fig. 1.1, the relative speeds of the cars can be expressed as follows:

$$V_{AC}=V_A - V_C \quad (1.6)$$

And

$$V_{AB}=V_A + V_B \quad (1.7)$$

Substituting Eq. 1.6 and 1.7 in Eq. 1.5, one can express the safe distance in terms of relative speed:

$$R=18 \frac{V_{AB}}{V_{AC}} - 2 \quad (1.8)$$

Where V_{AB} is the relative speed of car B with respect to car A (driver's car) and V_{AC} is the relative speed of the car C with respect to car A. The conditions for a safe over-taking can thus be summarized as:

1)

$$V_{AC}>0 \quad (1.9)$$

This condition implies that the speed of car A should be greater than the speed of car C.

2)

$$R \geq 18 \frac{V_{AB}}{V_{AC}} - 2 \quad (1.10)$$

Equation 1.9 and 1.10 can be used in a radar based intelligent system to provide some kind of visual or audio warning to the driver to avoid any collision before an over-take.

1.2.3 Determination of the Range With Acceleration.

The expression for range with constant speeds has been derived in section 1.2.2. Now, we try to find another expression for range including constant acceleration of the cars.

$$R_A = V_A t + 1/2 a_A t^2 \quad (1.11)$$

$$R_C = V_C t + 1/2 a_C t^2 \quad (1.12)$$

Inserting equation 1.11 and 1.12 into equation 1.3

$$1/2(a_A - a_C)t^2 + (V_A - V_C)t - 18 = 0$$

By using relative speed and acceleration, that equation 1.6 and $a_{AC} = a_A - a_C$ then we have

$$a_{AC}t^2 + 2V_{AC}t - 36 = 0$$

solving for t,

$$t = \frac{\sqrt{V_{AC}^2 + 36a_{AC}} - V_{AC}}{a_{AC}} \quad (1.13)$$

In the same way of equations 1.11 and 1.12 R_B can be written as:

$$R_B = V_B t + 1/2 a_B t^2 \quad (1.14)$$

Substituting equations 1.11 and 1.14 into 1.2 then

$$R = (V_A + V_B)t + (a_A + a_B)t^2 - 2$$

By using relative speed and acceleration, equation 1.7 and $a_{AB} = a_A + a_B$ then we have equation 1.15

$$R = V_{AB}t + a_{AB}t^2 - 2 \quad (1.15)$$

Put the value of t of Eq. 1.13 into Eq. 1.15, equation 1.16 is yielded

$$R = \left[\frac{\sqrt{V_{AC}^2 + 36a_{AC}} - V_{AC}}{a_{AC}} \right] \left[V_{AB} + a_{AB} \left(\frac{\sqrt{V_{AC}^2 + 36a_{AC}} - V_{AC}}{a_{AC}} \right) \right] - 2 \quad (1.16)$$

Where V_{AB} is the relative speed of car B with respect to car A (driver's car) and V_{AC} is the relative speed of the car C with respect to car A. a_{AB} and a_{AC} are the relative accelerations of the car B and the car C with respect to the car A respectively. The conditions for a safe over-taking can thus be summarized as:

1)

$$V_{AC} > 0 \quad (1.9)$$

This condition implies that the speed of car A should be greater than the speed of car C.

2)

$$R \geq \left[\frac{\sqrt{V_{AC}^2 + 36a_{AC}} - V_{AC}}{a_{AC}} \right] \left[V_{AB} + a_{AB} \left(\frac{\sqrt{V_{AC}^2 + 36a_{AC}} - V_{AC}}{a_{AC}} \right) \right] - 2 \quad (1.17)$$

In Table 1.1, the range (R) and time (t) of over-take is calculated at various speeds of cars "A", "B" and "D". Ranges have been calculated from the formulae 1.5 and 1.16 considering constant speeds and speeds with acceleration respectively as well as the times required have been calculated from the formulae 1.4 and 1.13 considering the constant speeds and speeds with acceleration respectively.

Table 1.1 gives us an idea about what the ranges will be and times required for over-take at various speeds.

Table 1.1 Range and time required for over-take at the various speeds

	$V_A(\text{Km/h})$	$V_B(\text{Km/h})$	$V_C(\text{Km/h})$	$R \text{ (m)}$	$t(\text{sec})$
Constant speed	120	120	110	365	2.96
Speed with Acceleration	120	120	110	296	3.36
Constant speed	110	120	80	170	1.80
Speed with Acceleration	110	120	80	129.95	1.63
Constant speed	100	110	95	635	10.72
Speed with Acceleration	100	110	95	362	4.26
Constant speed	85	120	80	620	10.79
Speed with Acceleration	85	120	80	362	4.26
Constant speed	100	200	95	904	10.79
Speed with acceleration	100	200	95	468	4.26

1.3 Collision Warning Simulation

A computer program is developed by the author in order to simulate the collision warning conditions. The program is written on a PC using the Visual Basic programming language. The reason of using Visual Basic was because of its power, ease of programming and debugging, and also because a visual graphical output was required.

The main visual form of the program is shown in Figure 1.2. Basically, the display

consists of a road layout with the lanes drawn in the middle of the road. Two cars (named as A and C; C in the front) are placed on the left hand side of the display, traveling on the left hand side of the road. Similarly, another car (named as B) is placed on the right hand side of the display, travelling to the left, in the left hand lane of the road. When the program starts, the user is required to enter the speeds of all the three vehicles and the distance (or range) between the vehicles traveling towards each other in the opposite lanes. Pressing the *Evaluate* push-button displays a message-box, which informs the user whether or not there is a potential collision situation. The user can then press the push-button *Simulate* to start the simulation. During the simulation, an animation technique is used to move the vehicles in their lanes. The range between the vehicles traveling towards each other is displayed dynamically and this range is reduced as the vehicles move close to each other. The vehicle at the back (vehicle A), and travelling to the right attempts to take-over the vehicle in front of it (vehicle C). If there is a collision situation, this is shown in the animation. If on the other hand there is no collision situation then vehicle A moves to the correct lane in front of vehicle C. The user can *Stop* the program, change the parameters and then re-start it at any time.

The animation is carried out in the program using a timer routine. A car moves by disabling its image on the form, and then re-enabling it at a slightly different position on the form.

The program listing of the simulation program is given in Figure 1.3.

The simulation program has been very useful in verifying the correctness of the equations derived to investigate the various collision situations.

1. The Theory Of Collision Detection

Form1

Enter speeds in km/hr

VC VA VB

Enter range

R

EVALUATE

SIMULATE

STOP

Range= meters

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COLLISION WARNING SYSTEM SIMULATION

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Figure 1.2 Collision warning system simulation form

'This subroutine implements the STOP button

```
Private Sub cmdend_Click()
```

```
End
```

```
End Sub
```

' This subroutine implements the EVALUATE button

```
Private Sub cmdevaluate_Click()
```

```

Dim range As Single
range = 5 + 15 * (txtva * 1000# + txtvb * 1000#) / (txtva * 1000# - txtvc * 1000#)
If txtr >= range Then
    MsgBox "Overtake with no collision"
Else
    MsgBox "COLLISION WARNING!"
End If
'
' 2 metre is 120 pixels.
'place car A 10 metre behind car C and car B txtr distance from car A.
imgva.Left = imgvc.Left - 120 * 10 / 2
imgvb.Left = imgva.Left + 120 * txtr / 2
evaluate_flag = 1
txtres = 2 * (imgvb.Left - imgva.Left) / 120

End Sub

'
' This subroutine implements the START button
'

Private Sub cmdstart_Click()
If evaluate_flag = 0 Then
    MsgBox "You must EVALUATE first"
    Exit Sub
End If

' speeds va,vb and vc are in km/hr
' calculate the distance it travels every second

```


1. The Theory Of Collision Detection

```
vadistance = txtva * 1000# / 3600#    'meters per second
vbdistance = txtvb * 1000# / 3600#    'meters per second
vcdistance = txtvc * 1000# / 3600#    'meters per second

vastep = vadistance * 120 / 2          'step in every second
vbstep = vbdistance * 120 / 2          'step in every second
vcstep = vcdistance * 120 / 2          'step every second

imgva.Top = imgva.Top + 360            'lower car A at the beginning
Timer1.Enabled = True
End Sub

'
' This subroutine is run only once during the program startup
'

Private Sub Form_Load()
'
' Cars move every second where each movement is 2 metre.
'

Form1.WindowState = 2
flag = 0
evaluate_flag = 0
End Sub

'
' This subroutine implements the timer routine
'

Private Sub Timer1_Timer()
imgvc.Left = imgvc.Left + vcstep
imgva.Left = imgva.Left + vastep
imgvb.Left = imgvb.Left - vbstep
```

```

txtres = 2 * (imgvb.Left - imgva.Left) / 120
If flag = 0 Then
If (imgva.Left - imgvc.Left) >= 5 * 120 / 2 Then
    imgva.Top = imgva.Top - 360
    flag = 1
End If
End If
End Sub

```

Figure 1.3 Simulation program listing

2. INTRODUCTION TO RADAR SYSTEMS

2.1 Presentation of Radar

Radar is an electromagnetic system for the detection and location of objects. It operates by transmitting a particular type of waveform, a pulse-modulated sine wave for example, and detects the nature of the echo signal. Radar is used to extend the capability of one's senses for observing the environment, especially the sense of vision. The value of radar lies not in being a substitute for the eye, but in doing what the eye cannot do. Radar cannot resolve detail as well as the eye, nor is it capable of recognising the 'colour' of objects to the degree of sophistication of which the eye is capable. However, radar can be designed to see through those conditions impervious to normal human vision, such as darkness, haze, fog, rain, and snow. In addition, radar has the advantage of being able to measure the distance or range to the object. This is probably its most important attribute. An elementary form of radar consists of a transmitting antenna emitting electromagnetic radiation generated by an oscillator of some sort, a receiving antenna and an energy-detecting device, or receiver. A portion of the transmitted signal is intercepted by a reflecting object (target) and is reradiated in all directions. It is the energy reradiated in the back direction that is of prime interest to the radar. The receiving antenna collects the returned energy and delivers it to a receiver, where it is processed to detect the presence of the target and to extract its location and relative velocity. The distance to the target is determined by measuring the time taken for the radar signal to travel to the target and back. The direction, or angular position, of the target may be determined from the direction of arrival of the reflected wave-front. The usual method of measuring the direction of arrival is with narrow antenna beams. If relative motion exists between target and radar, the shift in the carrier frequency of the reflected wave (Doppler effect) is a measure of the target's relative (radial) velocity and may be used to distinguish moving targets from stationary objects. In radars, which continuously track the movement of a target, a continuous indication of the rate of change of target position is also available.

The name radar reflects the emphasis placed by the early experimenters on a device to detect the presence of a target and measure its range. Radar is a contraction of the words

radio detection and ranging. It was first developed as a detection device to warn of the approach of hostile aircraft and for directing anti aircraft weapons. Although a well-designed modern radar can usually extract more information from the target signal than merely range, the measurement of range is still one of radar's most important functions. There seems to be no other competitive techniques, which can measure range, as the most common radar waveform is a train of narrow, rectangular-shape pulses modulating a sine wave carrier. The distance, or range, to the target is determined by measuring the time T_r taken by the pulse to travel to the target and return. Since electromagnetic energy propagates at the speed of light $c = 3 \times 10^8$ m/s, the range R is

$$R = \frac{cT_r}{2} \quad (2.1)$$

The factor 2 appears in the denominator because of the two-way propagation of radar. With the range in kilometer or nautical miles, and T_r in microseconds, equation (2.1) becomes

$$R(\text{km}) = 0.5T_r(\mu\text{s})$$

Each microsecond of round-trip travel time corresponds to a distance of 0.081 nautical mile, 0.093 statute mile, 150 meters, 164 yards, or 492 feet

Once the transmitted pulse is emitted by the radar, a sufficient length of time must elapse to allow any echo signals to return and be detected before the next pulse may be transmitted. Therefore the longest range at which targets are expected determines the rate at which the pulses may be transmitted. If the pulse repetition frequency is too high, echo signals from some targets might arrive after the transmission of the next pulse, and ambiguities in measuring range might result. Echoes that arrive after the transmission of the next pulse is called second time around (or multiple-time-around) echoes. Such an echo would appear to be at a much shorter range than the actual and could be misleading if it were not known to be a second-time-around echo. The range beyond which targets appear as second-time-around echoes is called the maximum unambiguous range and is

$$R_{unamb} = \frac{c}{2f_p} \quad (2.2)$$

where f_p = pulse repetition frequency in Hz. A plot of the maximum unambiguous range as a function of pulse repetition frequency is shown in Figure 2.1. Although the typical radar transmits a sample pulse-modulated waveform, there are a number of other suitable modulations that might be used. The pulse carrier might be frequency or phase-modulated to permit the echo signals to be compressed in time after reception. This achieves the benefits of high range-resolution without the need to resort to a short pulse. The technique of using a long, modulated pulse to obtain the resolution of a short pulse, but with the energy of a long pulse, is known as pulse compression. Continuous waveforms (CW) also can be used by taking advantage of the doppler frequency shift to separate the received echo from the transmitted signal and the echoes from stationary clutter. Unmodulated CW waveforms do not measure range, but a range measurement can be made by applying either frequency or phase modulation.

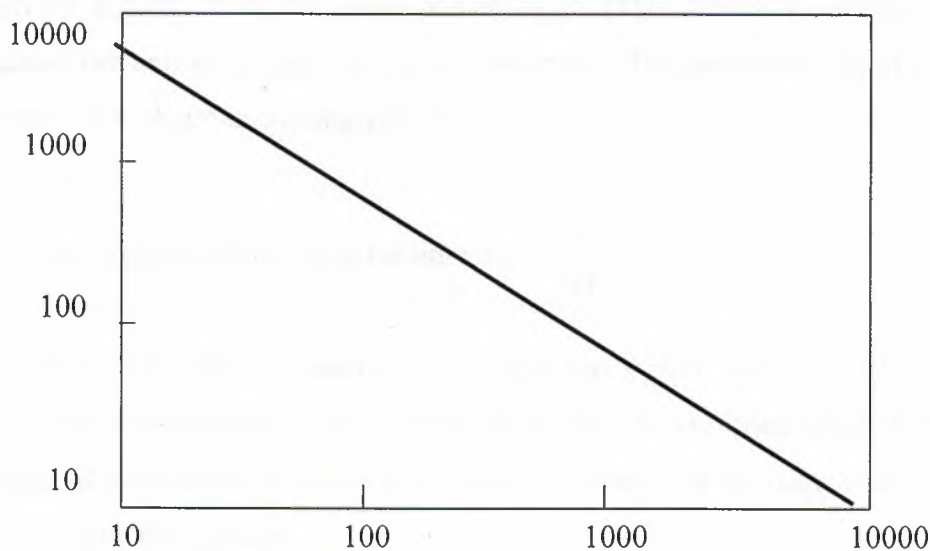


Figure 2.1 Plot of maximum unambiguous range as a function of the pulse repetition frequency

2.2 The Simple Form Of the Radar Equation

The radar equation relates the range of radar to the characteristics of the transmitter, receiver, antenna, target, and environment. It is useful not just as a means for

determining the maximum distance from the radar to the target, but it can serve both as a tool for understanding radar operation and as a basis for radar design. In this section, the simple form of the radar equation is derived. If the power of the radar transmitter is denoted by P , and if an isotropic antenna is used (one which radiates uniformly in all directions), the power density (watts per unit area) at a distance R from the radar is equal to the transmitter power divided by the surface area $4\pi R^2$ of an imaginary sphere of radius R , or

$$\text{Power density from isotropic antenna} = \frac{P_t}{4\pi R^2} \quad (2.3)$$

Radars employ directive antennas to channel, or direct, the radiated power P , into some particular direction. The gain G of an antenna is a measure of the increased power radiated in the direction of the target as compared with the power that would have been radiated from an isotropic antenna. It may be defined as the ratio of the maximum radiation intensity from the subject antenna to the radiation intensity from a lossless, isotropic antenna with the same power input. (The radiation intensity is the power radiated per unit solid angle in a given direction.) The power density at the target from an antenna with a transmitting gain G is

$$\text{Power density from directive antenna} = \frac{P_t G}{4\pi R^2} \quad (2.4)$$

The target intercepts a portion of the incident power and reradiates it in various directions. The measure of the amount of incident power intercepted by the target and reradiated back in the direction of the radar is denoted as the radar cross section σ , and is defined by the relation

$$\text{Power density of echo signal at radar} = \frac{\sigma}{4\pi R^2} \quad (2.5)$$

The radar cross section σ has units of area. It is a characteristic of the particular target and is a measure of its size as seen by the radar. The radar antenna captures a portion of

the echo power. If the effective area of the receiving antenna is denoted A_e , the power P_r received by the radar is

$$P_r = \frac{P_t G A_e \sigma}{(4\pi)^2 R^4} \quad (2.6)$$

The maximum radar range $R_{\max}$ is the distance beyond which the target cannot be detected. It occurs when the received echo signal power P_r , just equals the minimum detectable signal $S_{\min}$. Therefore

$$R_{\max} = \left[\frac{P_t G A_e \sigma}{(4\pi)^2 S_{\min}} \right]^{\frac{1}{4}} \quad (2.7)$$

This is the fundamental form of the radar equation. Note that the important antenna parameters are the transmitting gain and the receiving effective area. Antenna theory gives the relationship between the transmitting gain and the receiving effective area of an antenna as

$$G = \frac{4\pi A_e}{\lambda^2} \quad (2.8)$$

Since radars generally use the same antenna for both transmission and reception, equation (2.8) can be substituted into equation (2.7), first for A_e then for G , to give two other forms of the radar equation

$$R_{\max} = \left[\frac{P_t G \lambda^2 \sigma}{(4\pi)^3 S_{\min}} \right]^{1/4} \quad (2.9)$$

$$R_{\max} = \left[\frac{P_t A_e^2 \sigma}{4\pi \lambda^2 S_{\min}} \right]^{1/4} \quad (2.10)$$

These three forms (Equations. 2. 7, 2.9, and 2.10) illustrate the need to be careful in the interpretation of the radar equation. For example, from equation. (2.9) it might be

thought that the range of radar varies as $\lambda^{-1/2}$, but equation (2.10) indicates a $\lambda^{-1/2}$ relationship, and Equation (2.7) shows the range to be independent of J . The correct relationship depends on whether it is assumed the gain is constant or the effective area is constant with wavelength. Furthermore, the introduction of other constraints, such as the requirement to scan a specified volume in a given time, can yield different wavelength dependence. These simplified versions of the radar equation do not adequately describe the performance of practical radar. Many important factors that affect range are not explicitly included. In practice, the observed maximum radar ranges are usually much smaller than what would be predicted by the above equations, sometimes by as much as a factor of two. There are many reasons for the failure of the simple radar equation to correlate with actual performance, as discussed later.

2.3 Radar Block Diagram And Operation

The operation of typical pulse radar may be described with the aid of the block diagram shown in Figure 2.2. The transmitter may be an oscillator, such as a magnetron, that is "pulse" (turned on and off) by the modulator to generate a repetitive train of pulses. The magnetron has probably been the most widely used of the various microwave generators, for radar. A typical radar for the detection of aircraft at ranges of 100 or 200 nmi might employ a peak power of the order of a megawatt, an average power of several kilowatts, a pulse width of several microseconds, and a pulse repetition frequency of several hundred pulses per second. The waveform generated by the transmitter travels via a transmission line to the antenna, where it is radiated into space. A single antenna is generally used for both transmitting and receiving. The receiver must be protected from damage caused by the high power of the transmitter. This is the function of the duplexer. The duplexer also serves to channel the returned echo signals to the receiver and not to the transmitter. The duplexer might consist of two gas-discharge devices, one known as a TR (transmit-receive) and the other an ATR (anti-transmit-recessive). The TR protects the receiver during transmission and the TR directs the echo signal to the receiver during reception. Solid-state ferrite circulator and Receiver protectors with gas-plasma TR devices and/or diode limiters are also employed as duplexers.

The receiver is usually of the superheterodyne type. The first stage might be a low-noise RF amplifier, such as a parametric amplifier or a low-noise transistor. However, it is not always desirable to employ a low-noise first stage in radar. The receiver input could simply be the mixer stage, especially in military radars that must operate in a noisy environment. Although a receiver with a low-noise front-end will be more sensitive, the mixer input can have greater dynamic range, less susceptibility to overload, and less vulnerability to electronic interference.

The mixer and local oscillator (LO) convert the RF signal to an intermediate frequency (IF). A " typical " IF amplifier for an air-surveillance radar might have a centre frequency of 30 or 60 MHz and a bandwidth of the order of one megahertz. The IF amplifier should be designed as a *matched filter* i.e. its frequency-response function $H(f)$ should maximize the peak-signal-to-mean-noise-power ratio at the output. This occurs when the magnitude of the frequency-response function $|H(f)|$ is equal to the magnitude of the echo signal spectrum $|S(f)|$, and the phase spectrum of the matched filter is the negative of the phase spectrum of the echo signal. In radar whose signal waveform approximates a rectangular pulse, the conventional IF bandwidth B and the pulse width τ is of the order of unity, that is, $B\tau \approx 1$.

After maximizing the signal-to-noise ratio in the IF amplifier, the pulse modulation is extracted by the second detector and amplified by the video amplifier to a level where it can be properly displayed, usually on a cathode-ray tube (CRT).

Properly displayed, usually on a properly cathode-ray tube (CRT). Timing signals are also supplied to the indicator to provide the range zero. Angle information is obtained from the pointing direction of the antenna. The most common form of cathode-ray tube display is the plan position indicator, or PPI (Figure 2.3), which maps in polar coordinates the location of the target in azimuth and range. This is an intensity-modulated display in which the amplitude of the receiver output modulates the electron-beam intensity (z-axis) as the electron beam is made to sweep outward from the center of the tube. The beam rotates in angle in response to the antenna position. A *B-scope* display is similar to the *PPI* except that it utilizes rectangular, radar than polar,

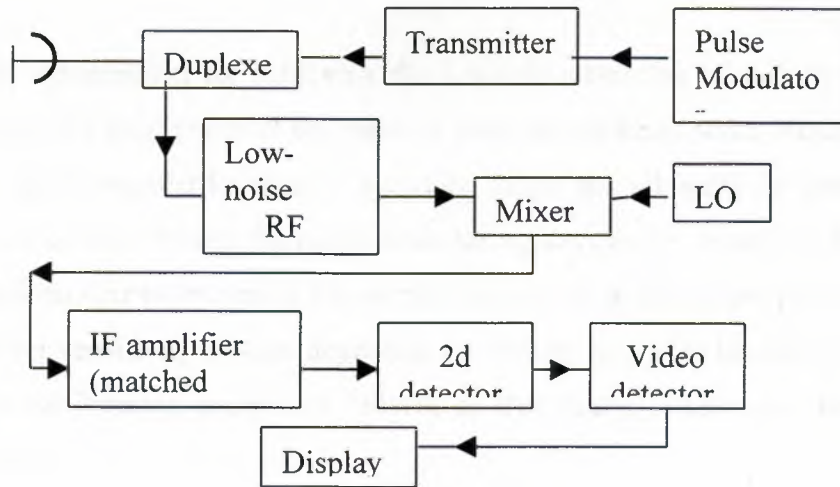


Figure 2.2 Block diagram of a pulse radar

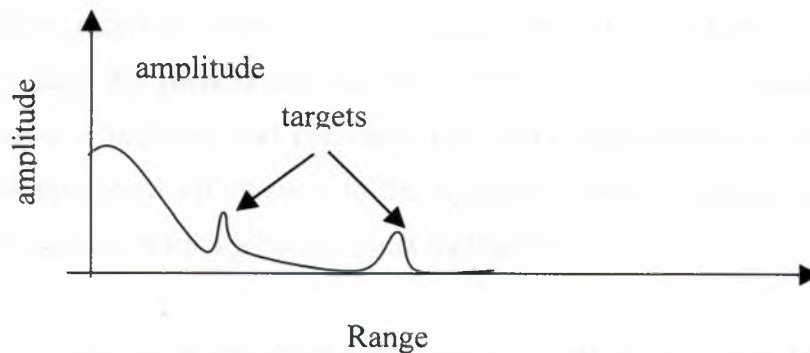


Figure 2.3 A scope presentation displaying amplitude vs. Range (deflection modulation)

coordinates to display ranges vs. angle. Both the B-scope and the *PPI*; being intensity modulated, have limited dynamic range. Another form of display is the *A-scope*, shown in Figure 2.3, which plots target amplitude (y axis) vs. range (x axis), for some fixed direction. This is a deflection-modulated display. It is more suited for tracking-radar application than for surveillance radar.

The block diagram of Figure 2.2 is a simplified version that omits many details. It does not include several devices often found in radar, such as means for automatically compensating the receiver for changes in frequency (AFC) or gain (AGC) receiver circuits for reducing interference from other radars and from unwanted signals, rotary joints in the transmission lines to allow movement of the antenna, circuitry for discriminating between moving targets and unwanted stationary objects (MTI) and

pulse compression for achieving the resolution benefits of a short pulse but with the energy of a long pulse. If the radar is used for tracking, some means are necessary for sensing the angular location of a moving target and allowing the antenna automatically to lock-on and to track the target. Monitoring devices are usually included to ensure that the transmitter is delivering the proper shape pulse at the proper power level and that the receiver sensitivity has not degraded. Provisions may also be easily incorporated in the radar for locating equipment failures so that faulty circuits can be easily found and replaced.

Instead of displaying the "raw-video" output of surveillance radar directly on the CRT, it might first be processed by an automatic detection and tracking (ADT) device that quantizes the radar coverage into range-azimuth resolution cells, adds (or integrates) all the echo pulses received within each cell, establishes a threshold (on the basis of these integrated pulses) that permits only the strong outputs due to target echoes to pass while rejecting noise, establishes and maintains the tracks (trajectories) of each target, and displays the processed information to the operator. These operations of an ADT are usually implemented with digital computer technology.

A common form of radar antenna is a reflector with a parabolic shape, fed (illuminated) from a point source at its focus. The parabolic reflector focuses the energy into a narrow beam, just as does a searchlight or an automobile headlamp. The beam may be scanned in space by mechanical pointing of the antenna. Phased-array antennas have also been used for radar. In a phased array, the beam is scanned by electronically varying the phase of the currents across the aperture.

2.4 Range Performance of The Radar

The simple form of the radar equation derived in Sec.2.2 expressed the maximum radar range $R_{\max}$ terms or radar and target parameters:

$$R_{\max} = \left[\frac{P_t G A_e \sigma}{(4\pi)^2 S_{\min}} \right]^{1/4} \quad (2.11)$$

Where P_t = transmitted power, watts

G = antenna gain

A_e = antenna effective aperture, m^2

σ = radar cross section, m^2

S_{min} = minimum detectable signal. Watts

All the parameters are to some extent under the control of the radar designer, except for the target cross-section σ . The radar equation states that if long ranges are desired, the transmitted power must be large, the radiated energy must be concentrated into a narrow beam (high transmitting antenna gain), the received echo energy must be collected with a large antenna aperture (also synonymous with high gain), and the receiver must be sensitive to weak signals.

In practice however, the simple radar equation does not predict the range performance or actual radar equipment to a satisfactory degree or accuracy. The predicted values or radar range are usually optimistic. In some cases the actual range might be only half that predicted. Part of this discrepancy is due to the failure of Equation (2.11) to explicitly include the various losses that can occur throughout the system or the loss in performance usually experienced when electronic equipment is operated in the field rather than under laboratory-type conditions. Another important factor that must be considered in the radar equation is the statistical or unpredictable nature of several of the parameters. The minimum detectable signal S_{min} and the target cross section σ are both statistical in nature and must be expressed in statistical terms. Other statistical factors which do not appear explicitly in equation (2.11) but which have an effect on the radar performance is the meteorological conditions along the propagation path and the performance of the radar operator, if one is employed. The statistical nature of these several parameters does not allow the maximum radar range to be described by a single number. Its specification must include a statement of the probability that the radar will detect a certain type of target at a particular range.

In this chapter, the simple radar equation will be extended to include most of the important factors that influence radar range performance. If all those factors affecting

radar range were known, it would be possible, in principle, to make an accurate prediction of radar performance. But, as is true for most endeavours, the quality of the prediction is a function of the amount of effort employed in determining the quantitative effects of the various parameters. Unfortunately, the effort required to specify completely the effects of all radar parameters to the degree of accuracy required for range prediction is usually not economically justified. A compromise is always necessary between what one would like to have and what one can actually get with reasonable effort. This will be better appreciated as we proceed through the chapter and note the various factors that must be taken into account.

A complete and detailed discussion of all those factors that influence the prediction of radar range is beyond the scope of a single chapter. For this reason many subjects will appear to be treated only lightly. This is deliberate and is necessitated by brevity. More detailed information will be found in some of the subsequent chapters or in the references listed at the end of the chapter.

2.5 Minimum Detectable Signal

The ability of a radar receiver to detect a weak echo signal is limited by the noise energy that occupies the same portion of the frequency spectrum as does the signal energy. The weakest signal the receiver can detect is called the minimum *detectable signal*. The specification of the minimum detectable signal is sometimes difficult because of its statistical nature and because the criterion for deciding whether a target is present or not may not be too well defined.

Detection is based on establishing a threshold level at the output of the receiver. If the receiver output exceeds the threshold, a signal is assumed to be present. This is called *threshold detection*. Consider the output of a typical radar receiver as a function of time figure 2.4. This might represent one sweep of the video output displayed on an A-scope. The envelope has a fluctuating appearance caused by the random nature or noise. If a large signal is present such as at A in figure 2.4, it is greater than the surrounding noise peaks and can be recognized on the basis of its amplitude. Thus, if

the threshold level were set sufficiently high, the envelope would not generally exceed the threshold if noise alone were present, but would exceed it if a strong signal were present. If the signal were small, however, it would be more difficult to recognize its presence. The threshold level must be low if weak signals are to be detected, but it cannot be so low that noise peaks cross the threshold and give a false indication of the presence of targets.

The voltage envelope of figure 2.4 is assumed to be matched-filter receiver. A matched filter is one designed to maximize the output peak signal to average noise (power) ratio. It has a frequency-response function, which is proportional to the complex conjugate of the signal spectrum. (This is not the same as the concept of "impedance match" of Circuit theory.) The ideal matched-filter receiver cannot always be exactly realized in practice, but it is possible to approach it with practical receiver circuits. A matched filter for a radar transmitting a rectangular-shaped pulse is usually characterized by a bandwidth B approximately the reciprocal of the pulse width τ , or $B\tau \cong 1$. The output of a matched-filter receiver is the cross correlation between the received waveform and a replica of the transmitted waveform. Hence it does not preserve the shape of the input waveform. (There is no reason to wish to preserve the shape of the received waveform so long as the output signal-to-noise ratio is maximized.)

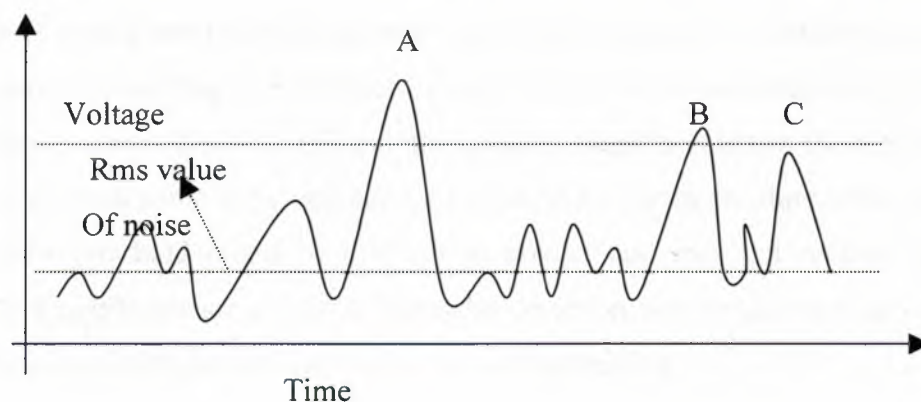


Figure 2.4 Typical envelope of the radar received output as a function of time.

A and B and C represent signal pulses noise. A and B would be valid detections, but C is a missed detection.

Let us return to the receiver output as represented in Figure 2.4. A threshold level is established, as shown by the dashed line. A target is said to be detected if the envelope crosses the threshold. If the signal is large such as at A, it is not difficult to decide that a target is present. But consider the two signals at B and C, representing target echoes of equal amplitude. The noise voltage accompanying the signal at B is large enough so that the combination of signal plus noise exceeds the threshold. At C, the noise is not as large and the resultant signal plus noise does not cross the threshold. Thus the presence of noise will sometimes enhance the detection of weak signals but it may also cause the loss of a signal, which would otherwise be detected.

Weak signals such as C would not be lost if the threshold level were lower. But too low a threshold increases the likelihood that noise alone will rise above the threshold and be taken for a real signal. Such an occurrence is called a *false alarm*. Therefore, if the threshold is set too low, false target indications are obtained, but if it is set too high, targets might be missed. The selection of the proper threshold level is a compromise that depends upon how important it is if a mistake is made either by (1) failing to recognize a signal that is present (probability of a miss) or by (2) falsely indicating the presence of a signal when none exists (probability of a false alarm).

When an operator viewing a cathode-ray-tube display makes the target-decision process; it would seem that the criterion used by the operator for detection ought to be analogous to the setting of a threshold, either consciously or subconsciously. The chief difference between the electronic and the operator thresholds is that the former may be determined with some logic and can be expected to remain constant with time, while the latter's threshold might be difficult to predict and may not remain fixed. The individual's performance as part of the radar detection process depends upon the state of the operator's fatigue and motivation, as well as training.

The capability of the human operator as part of the radar detection process can be determined only by experiment. Needless to say, in experiments of this nature there are likely to be wide variations between different experimenters. Therefore, for the purpose of the present discussion, the operator will be considered the same as an electronic threshold detector, an assumption that is generally valid for an alert, trained operator.

The signal-to-noise ratio necessary to provide adequate detection is one of the important parameters that must be determined in order to compute the minimum detectable signal. Although the detection decision is usually based on measurements at the video output, it is easier to consider maximizing the signal-to-noise ratio at the output of the IF amplifier rather than in the video. The receiver may be considered linear up to the output of the IF. It is shown by Van Vleck and Middleton that maximizing the signal-to-noise ratio at the output of the IF is equivalent to maximizing the video output. The advantage of considering the signal-to-noise ratio at the IF is that the assumption of linearity may be made. It is also assumed that the IF filter characteristic approximates the matched filter, so that the output signal-to-noise ratio is maximized.

2.6 Receiver Noise.

Since noise is the chief factor limiting receiver sensitivity, it is necessary to obtain some means of describing it quantitatively. Noise is unwanted electromagnetic energy, which interferes with the ability of the receiver to detect the wanted signal. It may originate within the receiver itself, or it may enter via the receiving antenna along with the desired signal. If the radar were to operate in a perfectly noise-free environment so that no external sources of noise accompanied the desired signal, and if the receiver itself were so perfect that it did not generate any excess noise, there would still exist an unavoidable component of noise generated by the thermal motion of the conduction electrons in the ohmic portions of the receiver input stages. This is called *thermal* noise, or *Johnson noise*, and is directly proportional to the temperature of the ohmic portions of the circuit and the receiver bandwidth. The available thermal-noise power, generated by a receiver of bandwidth B_n (in hertz) at a temperature T (degrees Kelvin) is equal to

$$\text{Available thermal-noise power} = kTB. \quad \text{Eq (2.12)}$$

where k = Boltzmann's constant = 1.38×10^{-23} J/deg. If the temperature T is taken to be 290 K, which corresponds approximately to room temperature (15°C), the factor kT is 4×10^{-21} W/Hz of bandwidth. If the receiver circuitry were at some other temperature, the thermal-noise power would be correspondingly different.

A receiver with a reactance input such as a parametric amplifier need not have any significant ohmic loss. The limitation in this case is the thermal noise seen by the antenna and the ohmic losses in the transmission line.

For radar receivers of the superheterodyne type (the type of receiver used for most radar applications), the receiver bandwidth is approximately that of the intermediate-frequency stages. It should be cautioned that the bandwidth B_n of Equation (2.12) is not the 3-dB, or half -power, bandwidth commonly employed by electronic engineers. It is an integrated bandwidth and is given by

$$B_n = \frac{\int_{-\infty}^{\infty} |H(f)|^2 df}{|H(f_0)|^2} \quad \text{Equation 2.13}$$

Where $H(f)$ = frequency-response characteristic of IF amplifier (filter) and f_0 = frequency of maximum response (usually occurs at midband).

When $H(f)$ is normalized to unity at midband (maximum- response frequency), $H(f_0)=1$. The bandwidth B_n is called the noise *bandwidth* and is the bandwidth of an equivalent rectangular filter whose noise-power output is the same as the filter with characteristic $H(f)$. The 3-dB bandwidth is defined as the separation in hertz between the points on the frequency-response characteristic where the response is reduced to 0.707 (3 dB) from its maximum value. The 3-dB bandwidth is widely used, since it is easy to measure. The measurement of noise bandwidth however, involves a complete knowledge of the response characteristic $H(f)$. The frequency-response characteristics of many practical radar receivers are such that the 3 dB and the noise bandwidths do not

differ appreciably. Therefore the 3 dB bandwidth may be used in many cases as an approximation to the noise bandwidth[16].

The noise power in practical receivers is often greater than can be accounted for by thermal noise alone. The additional noise components are due to mechanisms other than the thermal agitation or the conduction electrons. For purposes of the present discussion, however, the exact origin of the extra noise components is not important except to know that it exists. No matter whether the noise is generated by a thermal mechanism or by some other mechanism. The total noise at the output of the receiver may be considered to be equal to the thermal-noise power obtained from an "ideal" receiver multiplied by a factor called the noise figure. The noise figure F_n of a receiver is defined by the equation

$$F_n = \frac{N_o}{kT_o B_n G_n} = \frac{\text{noise out of practical receiver}}{\text{noise out ideal receiver at std temp } T_o} \quad (2.14a)$$

Where N_o = noise output from receiver, and G_a = available gain. The standard temperature T_o is taken to be 290 K. according to the Institute of Electrical and Electronics Engineers definition. The noise N is measured over the linear portion of the receiver input-output characteristic, usually at the output of the IF amplifier before the nonlinear second detector. The receiver bandwidth B_n is that of the IF amplifier in most receivers. The available gain G_a is the ratio of signal out S_o to the signal in S_i and $kT_o B_n$ is the input noise N_i in an ideal receiver. Equation (2.14a) may be rewritten as

$$F_n = \frac{S_i / N_i}{S_o / N_o} \quad (2.14b)$$

The noise figure may be interpreted, therefore, as a measure of the degradation of signal-to-noise-ratio as the signal passes through the receiver. Rearranging equation (2.14b), the input signal may be expressed as

$$S_i = \frac{k T_0 B_n F_n S_0}{N_0} \quad (2.15)$$

If the detectable signal $S_{\min}$ is that value of S_i corresponding to the minimum ratio of signal-to-noise ratio $(S_0 / N_0)_{\min}$ necessary for detection, then

$$S_{\min} = k T_0 B_n F_n \left(\frac{S_0}{N_0} \right)_{\min} \quad (2.16)$$

2.7 Probability Density Function

Substituting Equation 2.16 results in the following form of the radar equation:

$$R_{\max}^4 = \frac{P_t G A_e \sigma}{(4\pi)^2 k T_0 B_n F_n (S_0 / N_0)_{\min}} \quad (2.17)$$

Before continuing the discussion of the factors involved in the radar equation, it is necessary to digress and review briefly some topics in probability theory in order to describe the signal-to-noise ratio in statistic terms.

The basic concepts of probability theory needed in solving noise problems may be found in any of several references. In this section we shall briefly review probability and the probability-density function and cite some examples.

Noise is a random phenomenon. Predictions concerning the average performance of random phenomena are possible by observing and classifying occurrences, but one cannot predict exactly what will occur for any particular event. Phenomena of a random nature can be described with the aid of probability theory.

Probability is a measure of the likelihood of occurrence of an event. The scale of probability ranges from 0 to 1. An event, which is certain, is assigned the probability 1. An impossible event is assigned the probability 0. The intermediate probabilities are assigned so that the more likely an event, the greater is its probability.

One of the more useful concepts of probability theory needed to analyze the detection of signals in noise is the *probability-density function*. Consider the variable x as representing a typical measured value of a random process such as a noise voltage or current. Imagine each x to define a point on a straight line corresponding to the distance from a fixed reference point. The distance of x from the reference point might represent the value of the noise current or the noise voltage. Divide the line into small equal segments of length Δx and count the number of times that x falls in each interval. The probability-density function $p(x)$ is then defined as

$$\lim_{\Delta x \rightarrow 0} \frac{\text{total number of values}}{N} = N \quad (2.18)$$

The probability that a particular measured value lies within the infinitesimal width centered at x is simply $p(x)dx$. The probability that the value of x lies within the range from x_1 and x_2 is found by integrating $p(x)$ over the range of interest, or

$$\text{Probability } (x_1 < x < x_2) = \int_{x_1}^{x_2} p(x) dx \quad (2.19)$$

$$\int_{-\infty}^{\infty} p(x) dx = 1 \quad (2.20)$$

The average value of a variable function $\phi(x)$, that is described by the probability density function, $p(x)$, is

$$\langle \phi(x) \rangle_{av} = \int_{-\infty}^{\infty} \phi(x) p(x) dx \quad (2.21)$$

This follows from the definition of an average value and the probability-density

function. The mean, or average, value of x is

$$\langle x \rangle_{av} = m_1 = \int_{-\infty}^{\infty} x p(x) dx \quad (2.22)$$

And the mean square value is

$$\langle x^2 \rangle_{av} = m_2 = \int_{-\infty}^{\infty} x^2 p(x) dx \quad (2.23)$$

The quantities m_1 and m_2 are sometimes called the first and second moments of the random variable x if x represents an electric voltage or current, m_1 is the d-c component. It is the value read by a direct-current voltmeter or ammeter. The mean square value (m_2) of the current when multiplied by the resistance gives the mean power. The mean square value of voltage times the conductance is also the mean power. The *variance* is defined as

$$\mu_2 = \sigma^2 = \langle (x - m_1)^2 \rangle_{av} = \int_{-\infty}^{\infty} (x - m_1)^2 p(x) dx = m_2 - m_1^2 = \langle x^2 \rangle_{av} - \langle x \rangle_{av}^2 \quad (2.24)$$

The variance is the mean square deviation of x about its mean and is sometimes called the *second central* moment. If the random variable is a noise current, the product of the variance and resistance gives the mean power of the a-c component. The square root of the variance σ is called the *standard deviation* and is the root-mean-square (rms) value of the a-c component.

We shall consider four examples of probability-density functions: the uniform, gaussian, Rayleigh, and exponential. The uniform probability-density (Figure 2.5a) is defined as

$$P(x) = \begin{cases} K & \text{for } a < x < a+b \\ 0 & \text{for } x < a \text{ and } a+b < x \end{cases}$$

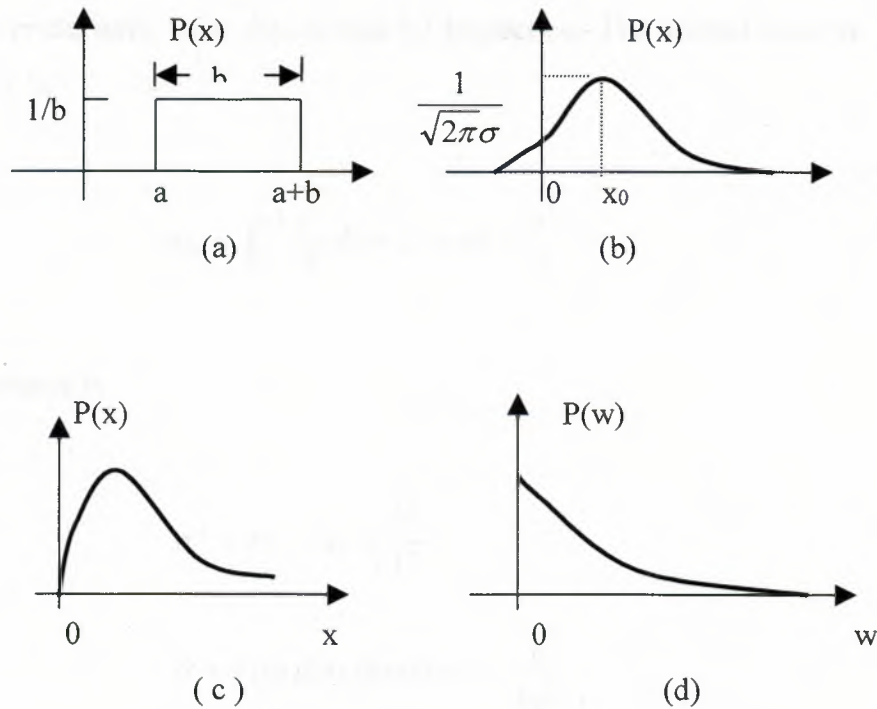


Figure 2.5 Examples of probability-density function (a) Uniform (b) Gaussian (c) Rayleigh (voltage) (d) Rayleigh (power) or exponential.

Where k is constant. A rectangular, or uniform, distribution describes the phase of a random sine wave relative to a particular origin of time; that is, the phase of the sine wave may be found equal probability, anywhere from 0 to 2π , with $k = 1/2\pi$. It also applies to the distribution of the found-off (quantizing) error in numerical computations and in analog-to-digital converters.

The constant k can be found by applying Equation (2.20); that is.

$$\int_{-\infty}^{\infty} p(x) dx = \int_a^{a+b} k dx = 1 \quad \text{or} \quad k = \frac{1}{b}$$

$$m_1 = \int_a^{a+b} \frac{1}{b} x dx = a + \frac{b}{2}$$

This result could have been determined by inspection. The second-moment, or mean square-value is

$$m_2 = \int_a^{a+b} \frac{x^2}{b} dx = a^2 + ab + \frac{b^2}{3}$$

And the variance is

$$\sigma^2 = m_2 - m_1^2 = \frac{b^2}{12}$$

$$\sigma = s \tan \text{dart deviation} = \frac{b}{2\sqrt{3}}$$

The Gaussian, or normal, probability density Figure 2.5b is one of the most important in noise theory since many sources of noise, such as thermal noise or shot noise, may be represented by gaussian statistics. Also, a gaussian representation is often more convenient to manipulate mathematically. The gaussian density function has a bell-shaped appearance and is determined by

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \frac{-(x - x_0)^2}{2\sigma^2} \quad (2.25)$$

Where exp is the exponential function, and the parameters have been adjusted to satisfy the normalizing condition of Equation 2.20. It can be shown that

$$m_1 = \int_{-\infty}^{\infty} x p(x) dx = x_0 \quad m_2 = \int x^2 p(x) dx = x_0^2 + \sigma^2 \quad \mu_2 = m_2 - m_1^2 = \sigma^2 \quad (2.26)$$

The probability density of the sum of a large number of independently distributed quantities approaches the gaussian probability-density function no matter what the individual distributions may be, provide that the contribution of any one quantity is not comparable with the result of all others. This is the *central limit theorem*. Another property of the gaussian distribution is that no matter how large a value x we may choose, there is always some finite probability of finding a greater value. If the noise at the input of the threshold detector were truly gaussian, the no matter how high the threshold were set, there would always be a chance that it would be exceeded by noise and appear as a false alarm. However, the probability diminishes rapidly with increasing x , and for all practical purposes the probability of obtaining an exceedingly high value of x is negligibly small.

The Rayleigh probability-density function is also of special interest to the radar system engineer. It describes the envelope of the noise output from a narrowband filter (such as the IF filter in a superheterodyn receiver), the cross-section fluctuations of certain types of complex radar targets, and many kinds of clutter and weather echoes. The Rayleigh density function is

$$p(x) = \frac{2x}{\langle x^2 \rangle_{av}} \exp\left(-\frac{x^2}{\langle x^2 \rangle_{av}}\right) \quad x \geq 0 \quad (2.27)$$

This is plotted in figure 2.5c. The parameter x might represent a voltage, and $\langle x^2 \rangle_{av}$ the mean, or average, value of the voltage squared. If x^2 is replaced by w represents power instead of voltage (assuming the resistance is 1 ohm), equation 2.27 becomes

$$P(w) = \frac{1}{w_0} \exp\left(-\frac{w}{w_0}\right) \quad w \geq 0 \quad (2.28)$$

Where W_0 is the average power. This is the exponential probability-density function, but it is sometimes called the Rayleigh-power probability-density function. It is plotted in Figure 2.5. The standard deviation of the Rayleigh density of Equation (2.27) is equal to $\sqrt{(4/\pi)-1}$ times the mean value, and for the exponential density of Equation (2.28) the standard deviation is equal to W_0 . There are other probability-density functions of

interest in radar, such as the Rice, log normal, and the chi square. These will be introduced as needed.

Another mathematical description of statistical phenomena is the *probability distribution function* $P(x)$, defined as the probability that the value x is less than some specified value

$$P(x) = \int_{-\infty}^x p(x) dx \quad \text{or} \quad p(x) = \frac{d}{dx} P(x) \quad (2.29)$$

In some cases, the distribution function may be easier to obtain from an experimental set of data than the density function. The density function may be found from the distribution function by differentiation.

2.8 Integration Of Radar Pulses

The relationship between the signal-to-noise ratio the probability of detection and the probability of false alarm applies for a single pulse only. However, many pulses are usually returned from any particular target on each radar scan and can be used to improve detection. The number of pulses n_B returned from a point target as the radar antenna scans through its beam-width is

$$n_B = \frac{\theta_B f_p}{\theta_s} = \frac{\theta_B f_p}{6w_m} \quad (2.30)$$

where θ_B = antenna beam-width in deg

f_p = pulse repetition frequency in Hz

θ_s = antenna scanning rate in deg/s

w_m = antenna scan rate in rpm

Typical parameters for aground-based search radar might be pulse repetition frequency

300 Hz. 1.5° beam width and antenna scan rate 5 rpm ($30^\circ/\text{s}$). These parameters result in 15 hits from a point target on each scan. The process of summing all the radar echo pulses for the purpose of improving detection is called *integration*. Many techniques might be employed for accomplishing integration. All practical integration techniques employ some sort of storage device. Perhaps the most common radar integration method is the cathode-ray-tube display combined with the integrating properties of the eye and brain of the radar operator. The discussion in this section is concerned primarily with integration performed by electronic devices in which detection is made automatically on the basis of a threshold crossing.

Integration may be accomplished in the radar receiver either before the second detector (in the IF) or after the second detector (in the video). A definite distinction must be made between these two cases. Integration before the detector is called *pre-detection*, or *coherent*, integration, while integration after the detector is called *post-detection*, or *noncoherent*, integration. Pre-detection integration requires that the phase of the echo signal be preserved if full benefit is to be obtained from the summing process. On the other hand, phase information is destroyed by the second detector; hence post-detection integration is not concerned with preserving RF phase. For this convenience, post-detection integration is not as efficient as pre-detection integration.

If n pulses, all of the same signal-to-noise ratio, were integrated by an ideal pre-detection integrator, the resultant, or integrated, signal-to-noise (power) ratio would be exactly n times that of a single pulse. If the same n pulses were integrated by an ideal post-detection device, the resultant signal-to-noise ratio would be less than n times that of a single pulse. This loss in integration efficiency is caused by the nonlinear action of the second detector, which converts some of the signal energy to noise energy in the rectification process.

The comparison of pre-detection and post-detection integration may be briefly summarized by stating that although post-detection integration is not as efficient as pre-detection integration, it is easier to implement in most applications. Post-detection integration is therefore preferred, even though the integrated signal-to-noise ratio may

not be as great, an alert, trained operator viewing a properly designed cathode-ray tube display is a close approximation to the theoretical post-detection integrator.

The efficiency of post-detection integration relative to ideal pre-detection integration has been computed by Marcum when all pulses are of equal amplitude. The integration efficiency may be defined as follows:

$$E_i(n) = \frac{(S/N)_1}{n(S/N)_n} \quad (2.31)$$

where,

n = number of pulses integrated

$(S/N)_1$ = value of signal-to-noise ratio of a single pulse required to produce given probability of detection (for $n=1$)

$(S/N)_n$ = value of signal-to-noise ratio per pulse required to produce same probability of detection when n pulses are integrated.

The improvement in the signal-to-noise ratio when n pulses are integrated post-detection is $nE_i(n)$ and is the *integration-improvement factor*. It may also be thought of as the effective number of pulses integrated by the post-detection integrator. The improvement with ideal pre-detection integration would be equal to n .

2.9 Effects Of Weather On Radar

It was stated that radar could see through weather effects such as fog, rain, or snow. This is not strictly true in all cases and must be qualified, as the performance of some radars can be strongly affected by the presence of meteorological panicles (hydrometeors). In general, radars at the lower frequencies are not bothered by meteorological or weather effects, but at the higher frequencies, weather echoes may be quite strong and mask the desired target signals just as any other unwanted clutter signal

Whether the radar detection of meteorological particles such as rain, snow, or hail is a

blessing or a curse depends upon one's point of view. Weather echoes are a nuisance to the radar operator whose job is to detect aircraft or ship targets. Echoes from a storm, for example, might mask or confuse the echoes from targets located at the same range and azimuth. On the other hand radar return from rain, snow, or hail is of considerable importance in meteorological research and weather prediction. Radar may be used to give an up-to-date pattern of precipitation in the area around the radar. It is a simple and inexpensive gauge or measuring the precipitation over relatively large expanses. As a rain gauge it is quite useful to the hydrologist in determining the amount of water falling into a watershed during a given period of time. Radar has been used extensively for the study of thunderstorms, squall lines, tornadoes, hurricanes, and in cloud-physics research. Not only is radar useful as a means of studying the basic properties of these phenomena, but it may also be used for gathering the information needed for predicting the course of the weather. Hurricane tracking and tornado warning are examples of applications in which radar has proved its worth in the saving of life and property. Another important application of radar designed for the detection of weather echoes is in airborne weather-avoidance radars, whose function is to indicate to the aircraft pilot the dangerous storm areas to be avoided.

The simple radar equation is

$$P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \quad (2.32)$$

The symbols are as defined before. In extending the radar equation to meteorological targets, it is assumed that rain, snow, hail, or other hydrometeors may be represented as a large number of independent scatterers or cross section σ_i located within the radar resolution cell. Let $\sum \sigma_i$ denoted the average total backscatter cross-section or the particles per unit or volume. The indicated summation of σ_i is carried out over the unit volume. The radar cross section may be expressed as $\sigma = V_m \sum \sigma_i$, where V_m is the volume or the radar resolution cell. The volume V_m occupied by a radar beam or vertical beam-width ϕ_B horizontal beam-width θ_B , and a pulse duration τ is approximately

$$V_m \approx \frac{\pi}{4} (R\theta_B)(R\phi_B)(c\tau/2) \quad (2.33)$$

Where c = velocity of propagation. In the radar-meteorology literature, the radar pulse-extent h (in units of length) is often used instead of $c\tau$ in this equation. The factor $\pi/4$ is included to account for the elliptical shape of the beam area. In some instances this factor is omitted for convenience; however, radar meteorologists almost always include it since they are concerned with accurate measurement of rainfall rate using the radar equation. In the interest of even further accuracy, a correction is usually made to Equation 2.33 to account for the fact that the effective volume of uniform rain illuminated by the two-way radar antenna pattern is less than that indicated when the half-power beam widths are used to define the volume. Assuming a gaussian-shaped antenna pattern, the volume given by Equation 2.33 must be reduced by a factor of $2 \ln 2$ to describe the equivalent volume that accounts for the echo power received by the two-way antenna pattern from distributed clutter. Thus the radar equation or Equation 2.32 can be written

$$\overline{P_r} = \frac{P_t G^2 \lambda^2 \theta_B \phi_B c \tau}{1024 (\ln 2) \pi^2 R^2} \sum_i \sigma_i = \frac{P_t G \lambda^2 c \tau}{1024 (\ln 2) R^2} \sum_i \sigma_i \quad (2.34)$$

In the above, the relationship $G = \pi^2 / \theta_B \phi_B$ for a gaussian beamshape was substituted. The bar over P_r denotes that the received power is averaged over many independent radar sweeps to smooth the signal fluctuations. This equation assumes that the volume of the antenna resolution cell is completely filled with uniform precipitation. If not, a correction must be made by introducing a dimensionless beam-filling factor ψ which is the fraction of the cross-sectional area of the beam intercepted by the region or scattering particles. It is difficult to estimate this correction. The resolution cell is not likely to be completely filled at long range or hence the beam is viewing the edge or a precipitation cell. If the "bright band" is within the radar resolution cell, the precipitation also will not be uniform.

When the radar wavelength is large compared with the circumference of a scattering particle of diameter D (Rayleigh scattering region the radar cross section)

$$\sigma_i = \frac{\pi^5 D^6}{\lambda^2} |K|^2 \quad (2.35)$$

Where $|K|^2 = (\epsilon - 1)/(\epsilon + 2)$, and ϵ = dielectric constant of the scattering particles. The value $|K|^2$ for water varies with temperature and wavelength. At 10 °C and 10 cm wavelength, it is approximately 0.93. Its value for ice at all temperatures is about 0.197 and is independent of frequency in the centimeter-wavelength region. Substituting equation 2.35 into 2.34 yields

$$p_r = \frac{\pi^5 P_t G^2 \tau}{1024 (\ln 2) R^2 \lambda^2} |K|^2 \sum_i D^6 \quad (2.36)$$

Since the particle diameter D appears as the sixth power, in any distribution of precipitation particles the small number of large drops will contribute most to the echo power.

Equation 2.36 does not include the attenuation of the radar energy by precipitation, which can be significant at the higher microwave frequencies and when accurate measurements are required. The two-way attenuation of the radar signal in traversing the range R and back is $\exp(-2\alpha R)$, where α is the one-way attenuation coefficient. If α is not a constant over the path R , the total attenuation must be expressed as the integrated value over the two-way path.

Scattering from rain. Equation 2.36, which applies for Rayleigh scattering, may be used as a basis for measuring with radar the sum of the sixth power of the raindrop diameters in a unit volume. The Rayleigh approximation is generally applicable below C band (5 cm wavelength) and, except for the heaviest rains, is a good approximation at

X band (3 cm). Rayleigh scattering usually does not apply above X band. Another complication at frequencies above X band is that the attenuation due to precipitation precludes the making of quantitative measurements conveniently.

The sum of the sixth power of the diameters per unit volume in equation 2.36 is called 2, the *radar reflectivity factor*, or

$$Z = \sum_i D^6 \quad (2.37)$$

In this form Z has little significance for practical application. Experimental measurements, however, show that Z is related to the rainfall rate r by

$$Z = ar^b \quad (2.38)$$

where a and b are empirically determined constants. With this relationship the received echo power can be related to rainfall rate. A number of experimenters have attempted to determine the constants in Equation 2.38, but considerable variability exists among the reported results. Part of this is probably due to the difficulty in obtaining quantitative measurements and the variability of rain with time and from one location to another. One form of Equation 2.38 that has been widely accepted is

$$Z = 200r^{1.6} \quad (2.39)$$

where Z is in mm^6/m^3 and r is in mm/h . This has been said to apply to stratiform rain. For orographic rain $Z = 31r^{1.71}$ and for thunderstorm rain $Z = 486r^{1.37}$. Thus a single expression need not be used, and the choice of a Z - r relationship can be made on the basis of the type of rain. Substituting equation 2.39 into (2.36) with $|K|^2 = 0.93$ yields

$$\bar{P}_r = \frac{2.4 P_t G \tau^{1.6}}{R^2 \lambda^2} \times 10^{-8} \quad (2.40)$$

where r is in mm/h, R and λ in meters, τ in seconds and P_t in watts. This indicates how the radar output can be made to measure rainfall.

The backscatter cross section per unit volume as a function of wavelength and rainfall rate is shown plotted in Figure 2.6. The dashed lines are plotted by summing the Rayleigh cross section of equation 2.55 over unit volume and substituting Equation 2.39 to give

$$\eta = \sum_i \sigma_i = 7f^4 r^{1.6} \times 10^{-12} \quad m^2 / m^3 \quad (2.41)$$

where f is the radar frequency in GHz and r the rainfall rate in mm/h. The solid curves are exact values computed by Haddock. The Rayleigh scattering approximation is seen to be satisfactory over most of the frequency range of interest to radar.

The reflectivity factor Z of equation 2.37, which was defined as the sum of the sixth power of the particles' diameter per unit volume, was based on the assumption of Rayleigh scattering. When the scattering is not Rayleigh, a quantity similar to Z is defined, which is called the *equivalent radar reflectivity factor* Z , given by

$$Z = \lambda^4 \eta / (\pi^5 |K|^2) \quad (2.42)$$

where f is the actual radar reflectivity, or backscatter cross section per unit volume, and $|K|^2$ is taken as 0.93.

Instead of the rainfall rate r , the intensity of precipitation is sometimes stated in terms of the dB reflectivity factor $Z = 200r^{1.6}$, or dBz = $10 \log Z$. A rainfall rate of 1 mm/h equals 23 dBz, 4 mm/h equals 33 dBz, and 16 mm/h equals 42 dBz. (This may be an incorrect usage of the precise definition of decibels as a power ratio, but it is the jargon used by the radar meteorologist.)

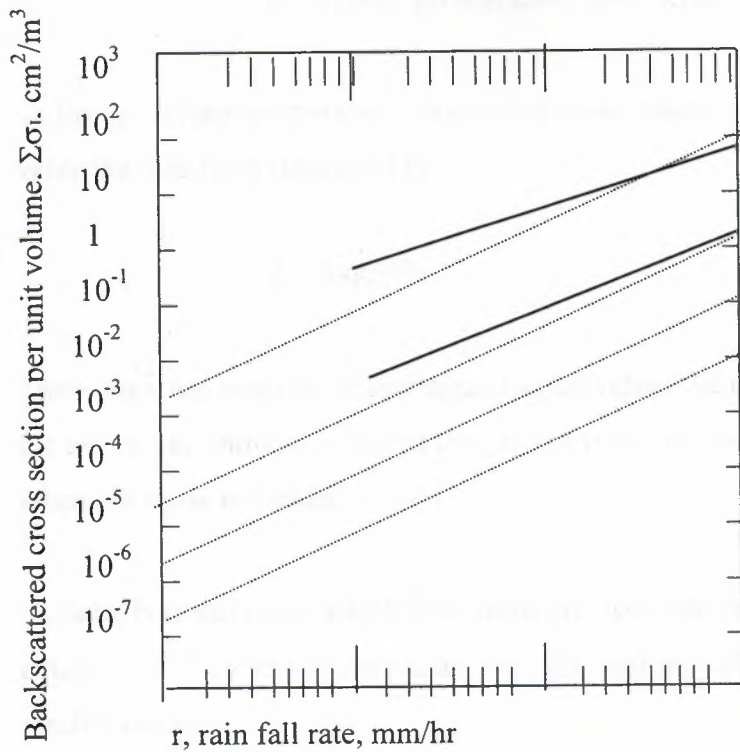


Figure 2.6 Exact (solid curves) and approximate (dash curves) back scattering cross section per unit volume of rain

Scattering from snow. Dry snow particles are essentially ice crystals, either single or aggregated. The relationship between Z and snowfall rate r is as given by equation 2.38 for rain, but with different constants a and b . There have been less measurements of the Z - r relationship for snow than for rain, and there have been several different values proposed for the constants a and b . The following two expressions have been suggested

$$Z = 2000r^2 \quad (2.43a)$$

$$Z = 1780r^{2.21} \quad (2.63b)$$

Measurements show a correlation between surface temperature and the coefficient a of the $Z = ar^b$ relationship, which suggest the following

$$Z = 1050r^2 \text{ for dry snow (ave. temp. } < 0^\circ\text{C)} \quad (2.44a)$$

$$Z = 1600r^2 \text{ for wet snow (ave. temp. } > 0^\circ\text{C)} \quad (2.44b)$$

A lower surface temperature results in a lower value of the coefficient a . Still another value that has been suggested is

$$Z = 1000r^{1.6} \quad (2.45)$$

There does not seem to be any agreed-upon value; the reader can take his pick. In all of the above, the snowfall rate r at the ground is in millimeters per hour of water measured when the snow is melted.

A radar is usually less affected by snow and ice than by rain because the factor $|K|^2$ in equation 2.57 is less for ice than for rain, and snowfall rates are generally less than rainfall rates.

Scattering from water-coated ice spheres. Moisture in the atmosphere at altitudes where the temperature is below freezing takes the form of ice crystals, snow or hail. As these particles fall to the ground they melt and change to rain in the warmer environment of the lower altitudes. When this occurs, there is an increase in the radar backscatter since water particles reflect, more strongly than ice. As the ice particles, snow, or hail begin to melt, they first become water-coated ice spheroids. At radar wavelengths, scattering and attenuation by water-coated ice spheroids the size of wet snowflakes is similar in magnitude to that of spheroidal water drops of the same size and shape. Even for comparatively thin coatings of water, the composite particle scatters nearly as well as a similar all-water particle.

Radar observations of light precipitation show a horizontal "bright band" at an altitude, at which the temperature is just above 0°C . The measured reflectivity in the center of the bright band is typically about 12 to 15 dB greater than the reflectivity from the snow above it and about 6 to 10 dB greater than the rain below. The center of the bright band is generally from about 100 to 400 m below the 0°C isotherm. Although the bright

band is relatively thin, considerable attenuation can occur when radar observations are made through it at low elevations.

The bright band is due to changes in snow falling through the freezing level. At the onset of melting the snow changes from flat or needle-shaped particles which scatter feebly to similarly shaped particles which, owing to a water coating, scatter relatively strongly. As melting progresses, the particles lose their extreme shapes, and their velocity of fall increases causing a decrease in the number of particles per unit volume and a reduction in the backscatter.

Scattering from clouds. Most cloud droplets do not exceed 100 μm in diameter ($1 \mu\text{m} = 10^{-6} \text{ m}$); consequently Rayleigh scattering may be applied at radar frequencies for the prediction of cloud echoes. In Rayleigh scattering, the backscatter is proportional to the sixth power of the diameter, Equation 2.35. Since the diameter of cloud droplets is about one-hundredth the diameter of raindrops, the echoes from fair-weather clouds are usually of little concern.

It is also possible to obtain weak echoes from a deep, intense fog at millimeter wavelengths, but at wavelengths of 3 cm and longer, echoes due to fog may generally be regarded as insignificant.

Attenuation by precipitation. In the frequency range for which Rayleigh scattering applies (particles small in size compared with the wavelength) the attenuation due to absorption is given by

$$\text{Attenuation (dB/km)} = 0.434 \left[\frac{\pi^2}{\lambda} \left(\sum_i D^3 \right) \text{Im}(-K) \right] \quad (2.46)$$

where the summation is over 1 m^3 , D is the particle diameter in centimeters, λ is the wavelength in centimeters. $\text{Im}(-K)$ is the imaginary part of $-K$, and K is a factor which depends upon the dielectric constant of the particle. At a temperature of 10°C , the value of $\text{Im}(-K)$ for water is 0.00688 when the wavelength is 10 cm (S band) and 0.0247 for 3.2 cm wavelength (X band). Equation (13.28) is a good approximation for rain

attenuation at S-band or longer wavelengths. Since rain attenuation is usually small and unimportant at the longer wavelengths where this expression is valid, the simplicity offered by the Rayleigh scattering approximation is of limited use for predicting the attenuation through rain.

The computation of rain attenuation must therefore be based on the exact formulation for spheres as developed by Mie[19]. The result of such computations are shown in figure 2.7 as a function of wavelength and the rainfall rate.

The attenuation produced by ice particles in the atmosphere, whether occurring as hail, snow, or ice-crystal clouds, is much less than that caused by rain of an equivalent rate of precipitation. Gunn and East [18] state that the attenuation in snow is

$$\text{Attenuation at } 0^{\circ} \text{ C (dB/Km)} = \frac{0.00349r^{1.6}}{\lambda^4} + \frac{0.00224r}{\lambda} \quad (2.47)$$

where r = snowfall rate (mm/h of melted water content). And λ = wavelength. cm.

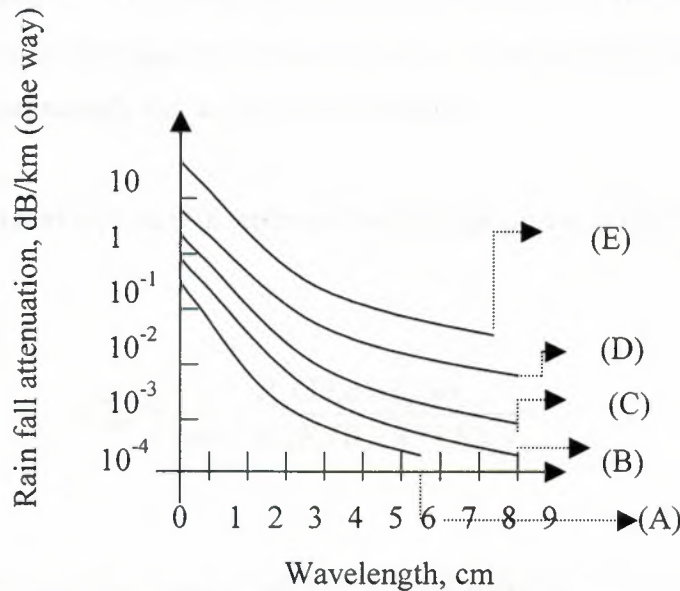


Figure 2.7 One-way attenuation (dB/Km) in rain at a temperature of 18°C .

(a) Drizzle 0.25 mm/h (b) light rain 1mm/h (c) moderate rain 4 mm/h (d) heavy rain 16 mm/h (e) excessive rain 40 mm/h

2.10 Transmitter Power

The power P_t in the radar equation (2.11) is called by the radar engineers the *peak* power. The peak pulse power as used in the radar equation is not the instantaneous peak power of a sine wave. It is defined as the power averaged over that carrier-frequency cycle which occurs at the maximum of the pulse of power. (Peak power is usually equal to one-half the maximum instantaneous power.) The *average* radar power P_{av} is also of interest in radar and is defined as the average transmitter power over the pulse-repetition period. If the transmitted waveform is a train of rectangular pulses of width τ and pulse-repetition period $T_p = 1/f_p$ the average power is related to the peak Power by

$$P_{av} = \frac{P_t \tau}{T_p} = P_t \tau f_p \quad (2.48)$$

The ratio P_{av}/P_t , τ/T_p , or τf_p is called the *duty cycle* of the radar. A pulse radar for detection of aircraft might have typically a duty cycle of 0.001, while a CW radar which transmits continuously has a duty cycle of unity.

Writing the radar equation in terms of *the* average power rather than the peak power, we get

$$R_{\max}^4 = \frac{P_{av} G A_e \sigma n E_i(n)}{(4\pi)^2 k T_0 F_n (B_n \tau) (S/N)_1 f_p} \quad (2.49)$$

The bandwidth and the pulse width are grouped together since the product of the two is usually of the order of unity in most pulse-radar applications.

If the transmitted waveform is not a rectangular pulse, it is sometimes more

convenient to express the radar equation in terms of the energy $E_t = P_{av} / f_p$ contained in the transmitted waveform:

$$R_{\max}^4 = \frac{E_t G A_e \sigma n E_t (n)}{(4\pi)^2 k T_0 F_n (B_n \tau) (S/N)_r} \quad (2.50)$$

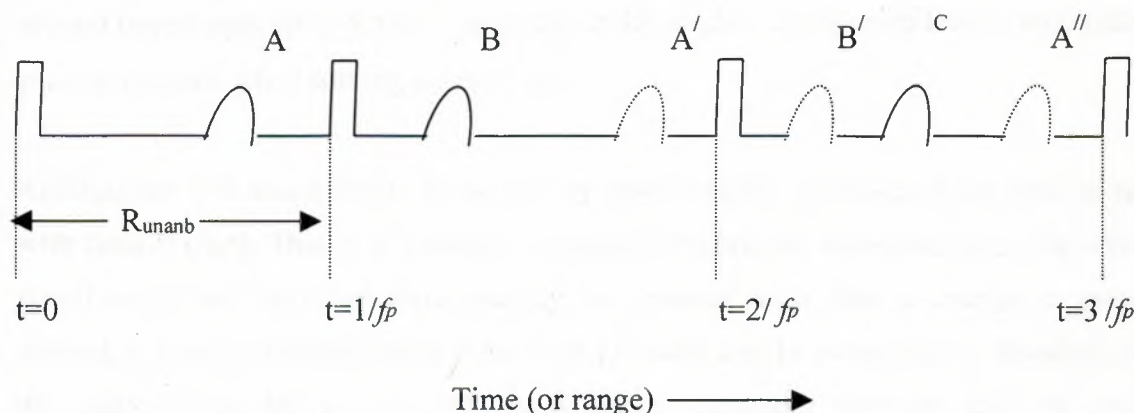
In this form, the range does not depend explicitly on either the wavelength or the pulse repetition frequency. The important parameters affecting range are the total transmitted energy nE_t , the transmitting gain G , the effective receiving aperture A_e , and the receiver noise figure F_n .

2.11 Pulse Repetition Frequency And Range Ambiguities

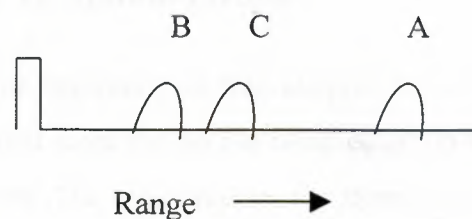
The pulse repetition frequency (prf) is determined primarily by the maximum range at which targets are expected. If the prf is made too high, the likelihood of obtaining target echoes from the wrong pulse transmission is increased. Echo signals received after an interval exceeding the pulse-repetition period are called *multiple-time-around* echoes. They can result in erroneous or confusing range measurements. Consider the three targets labeled A , B , and C in figure 2.8 a. Target A is located within the maximum unambiguous range R_{unamb} equation 2.2 of the radar, target B is at a distance greater than R_{unamb} but less than $2R_{\text{unamb}}$, while target C is greater $2R_{\text{unamb}}$ but less than $3R_{\text{unamb}}$. The appearance of the three targets on an A-scope is sketched in figure 2.8b. The multiple-time-around echoes on the A-scope cannot be distinguished from proper target echoes actually within the maximum unambiguous range. Only the range measured for target A is correct; those for B and C are not.

One method of distinguishing multiple-time-around echoes from unambiguous echoes is to operate with a varying pulse repetition frequency. The echo signal from an unambiguous range target will appear at the same place on the A-scope on each sweep no matter whether the prf is modulated or not. However, echoes from multiple-time-

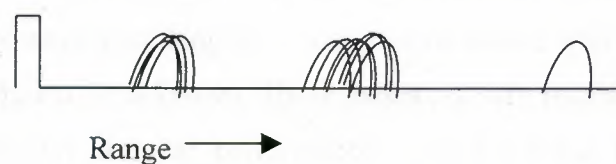
around targets will be spread over a finite range as shown in figure 2.8c. The prf may be changed continuously within prescribed limits, or it may be changed discretely among several predetermined values. The number of separate pulse repetition frequencies will depend upon the degree of the multiple time targets. Second-time targets need only two separate repetition frequencies in order to be resolved.



(a)

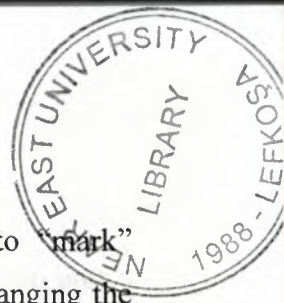


(b)



(c)

Figure 2.8 Multiple-time-around echoes that give rise to ambiguities in range. (a) Three targets A, B and C, where A is within R_{unamb} and B and C are multiple-time-around targets; (b) the appearance of the three on the A-scope; (c) appearance of the three targets on the A-scope with a changing prf.



Instead of modulating the prf, other schemes that might be employed to "mark" successive pulses so as to identify multiple-time-around echoes including changing the pulse amplitude, pulse width, frequency, phase, or polarization of transmission from pulse to pulse. Generally, such schemes are not so successful in practice as one would like. One the fundamental limitation is the foldover of nearby targets; that is, nearby strong ground targets (clutter) can be quite large and can mask weak multiple-time-around targets appearing at the same place on the display. Also, more time is required to process the data when solving ambiguities.

Ambiguities may theoretically be solved by observing the variation of the echo signal with time (range). This is not always a practical technique, however, since the echo-signal amplitude can fluctuated strongly for reasons other than a change in range. Instead, the range ambiguities in a multiple prf radar can be conveniently decoded and the range found by the use of the Chinese remainder theorem [20] or other computational algorithms[21].

2.12 System Losses

At the beginning of this chapter it was mentioned that one of the important factors omitted from the simple radar equation was the losses that occur throughout the radar system. The losses reduce the signal-to-noise ratio at the receiver output. They may be of two kinds, depending upon whether or not they can be predicted with any degree or precision beforehand. The antenna beam-shape loss, collapsing loss, and losses in the microwave plumbing are examples of losses which can be calculated if the system configuration is known. These losses are very real and cannot be ignored in any serious prediction of radar performance. Losses are not readily subject to calculation and which are less predictable include those due to field degradation and to operator fatigue or lack of operator motivation. Estimates of the latter type of loss must be made on the basis of prior experience and experimental observations. They may be subject to considerable variation and uncertainty. Although the loss associated with anyone factor may be small, there are many possible loss mechanisms in a complete radar system, and their sum total can be significant.

In this section, loss (number greater than unity) and efficiency (number less than unity) are used interchangeably, one is simply the reciprocal of the other.

Plumbing loss. There is always some finite loss experienced in the transmission lines which connect the output of the transmitter to the antenna. At the lower radar frequencies the transmission line introduces little loss, unless its length is exceptionally long. At the higher radar frequencies, attenuation may not always be small and may have to be taken into account. In addition to the losses in the transmission line itself, an additional loss can occur at each connection or bend in the line and at the antenna rotary joint if used. Connector losses are usually small, but if the connection is poorly made, it can contribute significant attenuation. Since the same transmission line is generally used for both receiving and transmission, the loss to be inserted in the radar equation is twice the one-way loss.

The signal suffers attenuation as it passes through the duplexer. Generally, the greater the isolation required from the duplexer on transmission, the larger will be the insertion loss. By insertion loss is meant the loss introduced when the component, in this case the duplexer is inserted into the transmission line. The precise value of the insertion loss depends to a large extent on the particular design. For a typical duplexer it might be of the order of 1 dB. A gas-tube duplexer also introduces loss when in the fired condition (arc loss); approximately 1 dB is typical.

In an S-band (300 MHz) radar, for example, the plumbing losses might be as follows:

100 ft of RG-13/U A1 waveguide transmission line (two-way)	1.0 dB
Loss due to poor connections (estimate)	0.5 dB
Rotary-joint loss	0.4 dB
Duplexer loss	<u>1.5 dB</u>
Total plumbing loss	3.4 dB

Beam-shape loss. The antenna gain that appears in the radar equation was assumed to be a constant equal to the maximum value. But in reality the train of pulses returned

from a target with a scanning radar is modulated in amplitude by the shape of the antenna beam. To properly take into account the pulse-train modulation caused by the beam shape, the computations of the probability of detection would have to be performed assuming a modulated train of pulses rather than constant-amplitude pulses. Some authors do indeed take account of the beam shape in this manner when computing the probability of detection. Therefore, when using published computations of probability of detection it should be noted whether the effect of the beam shape has been included. In this text, this approach is not used. Instead a *beam-shape loss* is added to the radar equation to account for the fact that the maximum gain is employed in the radar equation rather than a gain that changes pulse to pulse. This is a simpler, albeit less accurate, method. It is based on calculating the reduction in signal power and thus does not depend on the probability of detection. It applies for detection probabilities in the vicinity of 0.50, but it is used as an approximation with other values as a matter of convenience.

Let the one-way-power antenna pattern be approximated by the gaussian expression $\exp(-2.78\theta^2/\theta_B^2)$, where θ is the angle measured from the center of the beam and θ_B is the beam-width measured between half-power points. If n_B is the number of pulses received within the half-power beam-width θ_B , and n the total number of pulses integrated (n does not necessarily equal n_B), then the beam-shape loss (number greater than unity) relative to a radar that integrates all n pulses with an antenna gain corresponding to the maximum gain at the beam center is

$$\text{Beam-shape loss} = \frac{n}{1 + 2^{(n-1)/2} \sum_{k=1}^n \exp(-5.55k^2/n_B^2)} \quad (2.51)$$

For example, if we integrate 11 pulses, all lying uniformly between the 3-dB beam-width, the loss is 1.96 dB.

The beam-shape loss considered above was for a beam shaped in one plane only. It applies to a fan beam, or to a pencil beam if the target passes through its center. If the

target passes through any other point of the pencil beam, the maximum signal received will not correspond to the signal from the beam center. The beam-shape loss is reduced by the ratio of the square of the maximum antenna gain at which the pulses were transmitted divided by the square of the antenna gain at beam center. The ratio involves the square because of the, two-way transit.

Limiting loss. Limiting in the radar receiver can lower the probability of detection. Although a well-designed and engineered receiver will not limit the received signal under normal circumstances, intensity modulated CRT displays such as the PPI or the B-scope have limited dynamic range and may limit. Some receivers however, might employ limiting for some special purpose, eg for pulse compression processing for example.

When there are a large number of pulses per beam-width integrated, the scanning loss is generally taken to be 1.6 dB for a fan beam scanning in one coordinate and 3.2 dB when two-coordinate scanning is used.

When the antenna scans rapidly enough that the gain on transmit is not the same as the gain on receive, an additional loss has to be computed, called the *scanning loss*. The technique for computing scanning loss is similar in principle to that for computing beam-shape loss. Scanning loss can be important for rapid-scan antennas or for very long range radars such as those designed to view extraterrestrial objects. A similar loss must be taken into account when covering a search volume with a step-scanning pencil beam, as with a phased array, since not all regions of space are illuminated by the same value of antenna gain.

Limiting results in a loss of only a fraction of a decibel for a large number of pulses integrated, provided the limiting ratio (ratio of video limit level to rms noise level) is as large as 2 or 3^{10} . Other analyses of bandpass limiters show that for small signal-to-noise ratio, the reduction in the signal-to-noise ratio of a sine-wave imbedded in narrowband gaussian noise is $\pi/4$ (about 1 dB). However, by appropriately shaping the spectrum or the input noise, it has been suggested that the degradation can be made negligibly small.

Collapsing loss. If the radar were to integrate additional noise samples along with the wanted signal-to-noise pulses, the added noise results in a degradation called the *collapsing loss*. It can occur in displays which collapse the range information, such as the C-scope which displays elevation vs. azimuth angle. The echo signal from a particular range interval must compete in a collapsed-range C-scope display, not only with the noise energy contained within that range interval, but with the noise energy from all other range intervals at the same elevation and azimuth. In some 3D radars (range, azimuth, and elevation) that display the outputs at all elevations on a single PPI (range, Azimuth), display, the collapsing of the 3D radar information onto a 2D display results in a loss. A collapsing loss can occur when the output of a high-resolution radar is displayed on a device whose resolution is coarser than that inherent in the radar. A collapsing loss also results if the outputs of two (or more) radar receivers are combined and only one contains signal while the other contains noise.

The mathematical derivation of the collapsing loss, assuming a square-law detector, may be carried out as suggested by Marcum who has shown that the integration of m noise pulses, along with n signal-plus-noise pulses with signal-to-noise ratio per pulse $(S/N)_n$, is equivalent to the integration of $m + n$ signal-to-noise pulses each with signal-to-noise ratio $n(S/N)_n/(m + n)$. The collapsing loss in this case is equal to the ratio of the integration loss L_i for $m + n$ pulses to the integration loss for n pulses, or

$$L_i(m, n) = \frac{L_i(m + n)}{L_i(n)} \quad (2.52)$$

For example, assume that 10 signal-plus-noise pulses are integrated along with 30 noise pulses and that $P_d = 0.90$ and $n_f = 10^8$. $L_i(40)$ is 3.5 dB and $L_i(10)$ is 1.7 dB, so that the collapsing loss is 1.8 dB. It is also possible to account for the collapsing loss by substituting into the radar equation the parameter $E_i(m+n)$ for $E_i(n)$, since $E_i(n) = I/L_i(n)$.

The above applies for a square-law detector. Trunk has shown that the collapsing loss for a linear detector differs from that of the square-law detector, and it can be much greater. The comparison between the two is shown in Figure 2.8 as a function of the

collapsing ratio $(m + n)/n$. The difference between the two cases can be large. As the number of hits n increases, the difference becomes smaller.

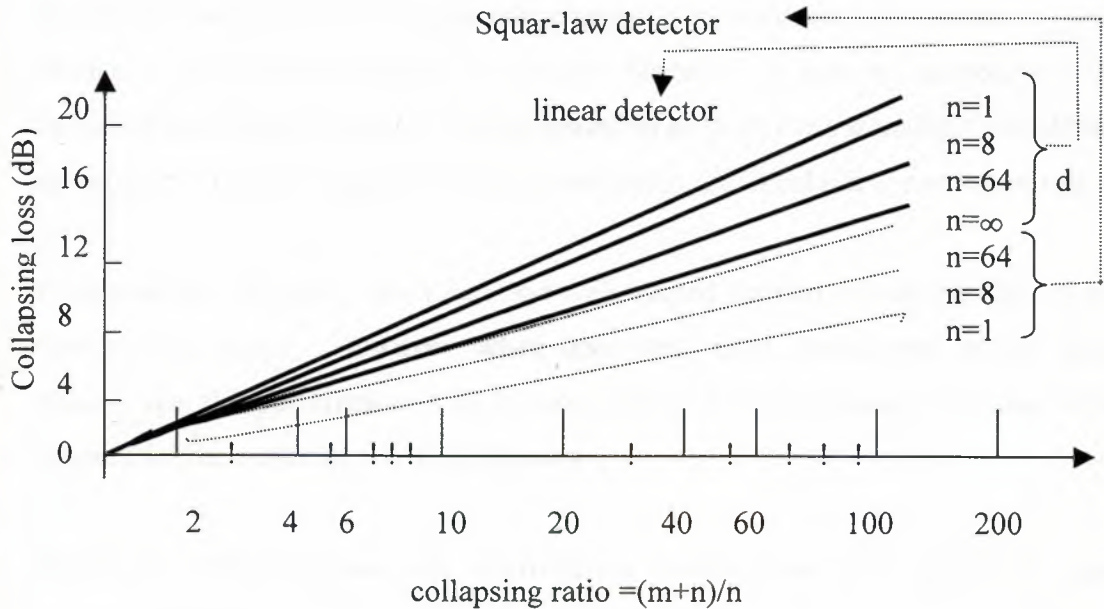


Figure 2.9 Collapsing loss versus collapsing ratio $(m + n)/n$, for a false alarm probability of 10^{-6} and a detection probability of 0.5. [From Trunk, courtesy Proc. I EEE.]

Nonideal equipment. The transmitter power introduced into the radar equation was assumed to be the output power (either peak or average). However, transmitting tubes are not all uniform in quality, nor should it be expected that any individual tube will remain at the same level of performance throughout its useful life. Also the power is usually not uniform over the operating band of the device. Thus, for one reason or another, the transmitted power may be other than the design value. To allow for this, a loss factor may be introduced. This factor can vary with the application, but lacking a better number, a loss of about 2 dB might be used as an approximation.

Variations in the receiver noise figure over the operating band also are to be expected. Thus, if the best noise figure over the band is used in the radar equation, a loss factor has to be introduced to account for its poorer value elsewhere within the band.

If the receiver is not the exact matched filter for the transmitted waveform, a loss in signal-to-noise ratio will occur. A typical value of loss for a nonmatched receiver might be about 1 dB. Because of the exponential relation between the false-alarm time and the threshold level equation 2.44, a slight change in the threshold can cause a significant change in the false alarm time. In practice, therefore, it may be necessary to set the threshold level slightly higher than calculated so as to insure a tolerable false alarm time in the event of circuit instabilities. This increase in the threshold is equivalent to a loss.

Operator loss. An alert, motivated, and well-trained operator should perform as well as described by theory. However, when distracted, tired, overloaded, or not properly trained, operator performance will decrease. There is little guidance available on how to account for the performance of an operator.

Based on both empirical and experimental results, one study gives the operator efficiency factor as

$$\rho = 0.7(P_d)^2 \quad (2.53)$$

where P_d is the single-scan probability of detection. This was said to apply to a good operator viewing a PPI under good conditions. Its degree of applicability, however, is not clear.

It is not unusual to find no account of the operator loss being taken in the radar equation. This is probably justified when operators are alert, motivated, and well trained. It is also justified when automatic (electronic) detections are made without the aid of an operator. When the operator does introduce loss into the system, it is not easy to select a proper value to account for it. The better action is to take steps to correct loss in operator performance rather than tolerate it by including it as a loss factor in the radar equation.

Field degradation: When a radar system is operated under laboratory conditions by engineering personnel and experienced technicians, the inclusion of the above losses

into the radar equation should give a realistic description of the performance of the radar under normal conditions (ignoring anomalous propagation effects). However, when a radar is operated under field conditions, the performance usually deteriorates even more than can be accounted for by the above losses, especially when the equipment is operated and maintained by inexperienced or unmotivated personnel. It may even apply, to some extent, to equipment operated by professional engineers under adverse field conditions. Factors which contribute to field degradation are poor tuning, weak tubes, water in the transmission lines, incorrect mixer-crystal current, deterioration or receiver noise figure, poor TR tube recovery, loose cable connections, etc.

To minimize field degradation, radars should be designed with built-in automatic performance-monitoring equipment. Careful observation or performance-monitoring instruments and timely preventative maintenance can do much to keep radar performance up to design level. Radar characteristics that might be monitored include transmitter power, receiver noise figure, the spectrum and/or shape of the transmitted pulse, and the decay time of the TR tube.

A good estimate of the field degradation is difficult to obtain since it cannot be predicted and is dependent upon the particular radar design and the conditions under which it is operating. A degradation of 3 dB is sometimes assumed when no other information is available.

Other loss factors. A radar designed to discriminate between moving targets and stationary objects (MTI radar) may introduce additional loss over a radar without this facility. The MTI discrimination technique results in complete loss of sensitivity for certain values of target velocity relative to the radar. These are called *blind speeds*.

The additional noise introduced by the non-optimum gate width will result in some degradation. The *straddling loss* accounts for the loss in signal-to-noise ratio for targets not at the center of the range gate or at the center of the filter in a multiple-filter-bank processor.

Another factor that has a profound effect on the radar range performance is the propagation medium discussed briefly.

There are many causes for inefficiency in a radar. Not all have been included here. Although they may each be small, the sum total can result in a significant reduction in radar performance. It is important to understand the origins of these losses, not only for better predictions of radar range, but also for the purpose of keeping them to a minimum by careful radar design.

2.12.1 Propagation Effects

In analyzing radar performance it is convenient to assume that the radar and target are both located in free space. However, there are very few radar applications which approximate free-space conditions. In most cases of practical interest, the earth's surface and the medium in which radar waves propagate can have a significant effect on radar performance. In some instances the propagation factors might be important enough to overshadow all other factors that contribute to abnormal radar performance. The effects of non-free-space propagation on the radar are of three categories: (1) *attenuation* of the radar wave as it propagates through the earth's atmosphere, (2) *refraction* of the radar wave by the earth's atmosphere, and (3) *lobe structure* caused by interference between the direct wave from radar to target and the wave which arrives at the target via reflection from the ground.

In general, for most applications of radar at microwave frequencies, the attenuation in propagating through either the normal atmosphere or through precipitation is usually not sufficient to affect radar performance. However, the *reflection* of the radar signal from rain (clutter) is often a limiting factor in the performance of radar in adverse weather.

The decreasing density of the atmosphere with increasing altitude results in a bending, or refraction, of the radar waves in a manner analogous to the bending of light by an optical prism. This bending usually results in an increase in the radar line of sight. Normal atmospheric conditions can be accounted for in a relatively simple manner by considering the earth to have a larger radius than actual. A " typical " earth radius for

refractive effect is four-thirds the actual radius. At times, atmospheric conditions might cause more than usual bending of the radar rays, with the result that the radar range will be considerably increased. This condition is called *superrefraction*, or *ducting*, and is a form of *anomalous propagation*. It is not necessarily a desirable condition since it cannot be relied upon. It can degrade the performance of MTI radar by extending the range at which ground clutter is seen.

The presence of the earth's surface not only restricts the line of sight, but it can cause major modification of the coverage within the line of sight by breaking up the antenna elevation pattern into many lobes. Energy propagates directly from the radar to the target, but there can also be energy that travels to the target via a path that includes a reflection from the ground. The direct and ground-reflected waves interfere at the target either destructively or constructively to produce nulls or reinforcements (lobes). The lobing that results causes non-uniform illumination of the coverage, and is an important factor that influences the capability of a radar system.

Most propagation effects that are of importance cannot be easily included into the radar equation. They must be properly taken into account, however, since they can have a major impact on performance.

2.12.2 Other Considerations

Prediction of radar range. In this chapter, some of the more important factors that enter into the radar equation for the prediction of range were briefly considered. The radar equation (2.1), with the modifications indicated in this chapter, becomes

$$R_{\max}^4 = \frac{P_{av} G A \rho_a \sigma n E_i(n)}{(4\pi)^2 k T_0 F_n (B\tau) f_p (S/N)_1 L_s} \quad (2.44)$$

where $R_{\max}$ = maximum radar range, m

G = antenna gain

A = antenna aperture, m^2

ρ_a = antenna efficiency

n = number of hits integrated

$E_i(n)$ = integration efficiency (less than unity)

L_s = system losses (greater than unity) not included in other parameters

σ = radar cross section of target, m^2

F_n = noise figure

k = Boltzmann's constant = 1.38×10^{-23} J/deg

T_o = standard temperature = 290 K

B = receiver bandwidth, Hz

τ = pulse width, s

fp = pulse repetition frequency, Hz

$(S/N)_1$ = signal-to-noise ratio required at receiver output (based on single-hit detection)

This equation can also be written in terms of energy rather than power. The energy in the transmitted pulse is $E_t = P_{av}/fp = P_t \tau$ and the signal-to-noise power ratio $(S/N)_1$ can be replaced by the signal-to-noise energy ratio $(E/N_o)_1$, where $E = S/\tau$ is the energy in the received signal, and $N_o = N/B$ is the noise power per unit bandwidth, or the noise energy. Also note that $B\tau \cong 1$, and $T_o F_n = T_s$ is defined as the system noise temperature. Then the radar equation becomes

$$R_{\max}^4 = \frac{E_t G A \rho_a \sigma n E_i(n)}{(4\pi)^2 k T_s (E/N_o)_1 L_s} \quad (2.55)$$

Although equation 2.55 was derived for a rectangular pulse, it can be applied to other waveforms as well, provided matched-filter detection is employed. Most calculations for probability of detection with signal-to-noise ratio as the parameter apply equally well when the ratio of signal-energy-to-noise-power-per-hertz is used instead. The radar equation developed in this chapter for pulse radar can be readily modified to accommodate CW, FM-CW, pulse-doppler, MTI, or tracking radar.

Radar performance figure. This is a figure of merit sometimes used to express the relative performance of radar. It is defined as the ratio of the pulse power of the radar transmitter to the minimum detectable signal power of the receiver. It is not often used.

Blip-scan ratio, This is the same as the single-scan probability of detection. It predates the widespread use of the term probability of detection and came about by the manner in which the performance of ground-based search-radars was checked. An aircraft would be flown on a radial course and on each scan of the antenna it would be recorded whether or not a target blip had been detected on the radar display. This was repeated many times until sufficient data was obtained to compute, as a function of range, the ratio of the average number of scans the target was seen at a particular range (blips) to the total number of times it could have been seen (scans). This is the blip-scan ratio and is the probability of detecting a target at a particular range, altitude, and aspect. The head-on and tail-on are the two easiest aspects to provide in field measurements. The experimentally found blip-scan ratio curves are subject to many limitations, but it represents one of the few methods for evaluating the performance of an actual radar equipment against real targets under somewhat controlled conditions.

Cumulative probability of detection. If the single scan probability of detection for a surveillance radar is P_d , the probability of detecting a target at least once during N scans is called the cumulative probability of detection, and may be written

$$P_c = 1 - (1 - P_d)^N \quad (2.56)$$

The variation of P_d with range might have to be taken into account when computing P_c . The variation of range based on the cumulative probability of detection can be as the third power rather than the more usual fourth power variation based on the single scan probability.

The cumulative probability has sometimes been proposed as a measure of the detectability of a radar rather than the single-scan probability of detection, which is more conservative. In practice the use of the cumulative probability is not easy to apply. Furthermore, radar operators do not usually use such a criterion for reporting detections. They seldom report a detection the first time it is observed, as is implied in the definition of cumulative probability. Instead, the criterion for reporting a detection might be a threshold crossing on two successive scans, or threshold crossings on two out of three scans, three out of five, or so forth. In track-while-scan radars the measure

of performance might be the probability of initiating a target track rather than a criterion based on detection alone.

Surveillance-radar range equation. The form of the radar equation described in this chapter applies to a radar that dwells on the target for n pulses. It is sometimes called the *searchlight* range equation. In a search or surveillance radar there is usually an additional constraint imposed that modifies the range equation significantly. This constraint is that the radar is required to search a specified volume of space within a specified time. Let Ω denote the angular region to be searched in the scan time t_s . (For example, Ω might represent a region 360° in azimuth and 30° in elevation.) The scan time is $t_s = t_o \Omega / \Omega_0$, where t_o is the time on target $= n / f_p$, and Ω_0 is the solid angular beamwidth which is approximately equal to the product of the azimuth beamwidth θ_a times the elevation beamwidth θ_e . (This assumes that θ_A / θ_e and θ_E / θ_e are integers, where θ_A is the total azimuth coverage and θ_E the total elevation coverage, such that $\Omega \cong \theta_A / \theta_E$.) The antenna gain can be written as $G = 4\pi / \Omega_0$. With the above substitutions into equation (2.54) the radar equation for a search radar becomes

$$R_{\max}^4 = \frac{P_{av} A_e \sigma E_i(n)}{4\pi k T_{s0} F_n (S/N)_1 L_s} \frac{t_s}{\Omega} \quad (2.57)$$

This indicates that the important parameters in a search radar are the average power and the antenna effective aperture. The frequency does not appear explicitly in the search-radar range equation. However, the lower frequencies are preferred for a search radar since large power and large aperture are easier to obtain at the lower frequencies, it is easier to build a good MTI capability, and there is little effect from adverse weather.

Different radar range equations can be derived for different applications, depending on the particular constraints imposed. The radar equation will also be considerably different if clutter echoes or external noise, rather than receiver noise, determine the background with which the radar signal must compete. Some of these other forms of the radar equation are given elsewhere in this text.

Accuracy of the radar equation. The predicted value of the range as found from the radar equation cannot be expected to be checked experimentally with any degree of accuracy. It is difficult to determine precisely all the important factors that must be included in the radar equation and it is difficult to establish a set of controlled, realistic experimental conditions in which to test the calculations. Thus it might not be worthwhile to try to obtain too great a precision in the individual parameters of the radar equation. Nevertheless, if a particular range is required of a radar, the systems engineer must provide it. The safest means to achieving a specified range performance is to design conservatively and add a safety factor. The inclusion of a safety factor in design is not always appreciated, especially in competitive procurements, but it is a standard procedure in many other engineering disciplines. In the few cases where this luxury was permitted, fine radars were obtained since they accomplished what was needed even under degraded conditions.

3. ANTENNAS

3.1 Introduction To Antennas

The purpose of radar antenna is to act as a transducer between free space propagation and guide wave (transmission line) propagation. The function of the antenna during transmission is to concentrate the radiated energy into a shaped beam which points in the desired direction in space. On reception, the antenna collects the energy contained in the echo signal and delivers it to the receiver.

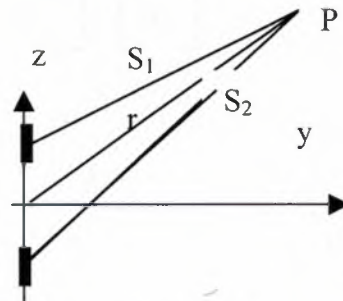
For our case, a signal and small antenna serve to the purpose of transmission and reception in the radar. The big size of antenna gives us one of the handicaps of assembling it on vehicles. Hence, the antenna size should be small. Because of that, the bandwidth of the transmission signal should be greater than 77 GHz.

$$\text{Length of antenna } L: \lambda = \frac{c}{f} = \frac{3 \times 10^8}{77 \times 10^9} = 0.00389 \text{ m} = 4 \text{ mm}.$$

Where c is the speed of light $3 \times 10^8 \text{ m/s}$.

λ is the wave length [m].

3.2 Horizontal Dipole



The current at 'p' is given as:

$$[I] = I_0 e^{i\omega(t-r/c)} \quad \text{Where } I_0 \text{ is the current at antenna in the direction of 'z'.$$

ω is the angular frequency.

r is the distance from origin to any point 'p'.

c is the propagation speed of wave and is $3 \times 10^8 \text{ m/s}$.

The infinitesimal current of antenna is delay by r/c at point 'p'.

The vector potential 'A' is given below and has only 'z' component.

$$A_z' = \frac{\mu_0}{4\pi} \int_{-l/2}^{l/2} \frac{[I_0]}{s} dz = \frac{\mu_0 I_0}{4\pi r} e^{i\omega(t-r/c)} \text{ the wave number } k=\omega/c \text{ then.}$$

$$A_z' = \frac{\mu_0 I_0}{4\pi r} e^{-i\omega r/c} e^{i\omega t} = \frac{\mu_0 I_0}{4\pi r} e^{i\omega t} e^{-ikr}$$

$$A_z' = A_z e^{i\omega t} \quad \text{Where } A_z = \frac{\mu_0 I_0}{4\pi r} e^{-ikr} \quad \text{Phasors form of } A_z^2$$

We apply the spherical coordinates transformation to rectangular coordinates we have:

$$\begin{bmatrix} A_r \\ A_\Theta \\ A_\Phi \end{bmatrix} = \begin{bmatrix} \sin\Theta \cos\phi & \sin\Theta \sin\phi & \cos\Theta \\ \cos\Theta \cos\phi & \cos\Theta \sin\phi & -\sin\Theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

By multiplying two matrices and $A_x=0$ and $A_y=0$.

$$A_r = \sin\Theta \cos\phi A_x + \sin\Theta \sin\phi A_y + \cos\Theta A_z = \cos\Theta A_z$$

$$A_\Theta = \cos\Theta \cos\phi A_x + \cos\Theta \sin\phi A_y - \sin\Theta A_z = -\sin\Theta A_z$$

$$A_\Phi = -\sin\phi A_x + \cos\phi A_y = 0$$

By substituting the value of A_z into A_r and A_Θ we have.

$$A_r = \cos\Theta A_z = \frac{\mu_0 I l_0}{4\pi r} e^{-ikr} \cos\Theta$$

$$A_\Theta = -\frac{\mu_0 I l_0}{4\pi r} e^{-ikr} \sin\Theta$$

$$A_\phi = 0$$

By using Maxwell equation.

$$\vec{H} = \frac{1}{\mu_0} \text{rot} \vec{A} = \frac{1}{\mu_0} \vec{\nabla} \times \vec{A}$$

$\vec{\nabla}$ Operator for spherical coordinate is.

$$\vec{H} = \frac{1}{\mu_0} \vec{\nabla} \times \vec{A} = \begin{bmatrix} \mathbf{e}_r & r\mathbf{e}_\Theta & r\sin\Theta\mathbf{e}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \Theta} & \frac{\partial}{\partial \Phi} \\ A_r & rA_\Theta & r\sin\Theta A_\phi \end{bmatrix}$$

$$H_r = \frac{1}{\mu_0 r^2 \sin\Theta} \left[\frac{\partial(r\sin\Theta A_\phi)}{\partial \Theta} - \frac{\partial(rA_\Theta)}{\partial \Phi} \right]$$

Since $A_\phi = 0$ and the partial derivative (rA_Θ) is zero with respect to ϕ therefore

$$H_r = 0$$

$$H_{\Theta} = -\frac{1}{\mu_0 r \sin \Theta} \left[-\frac{\partial(r \sin \Theta A_{\Phi})}{\partial r} - \frac{\partial A_r}{\partial \Phi} \right]$$

Since A_{Φ} is zero and by substituting the value of A_r then as we know the partial derivative of $\cos \Theta A_z$ is zero then:

$$H_{\Theta} = -\frac{1}{\mu_0 r \sin \Theta} \left[-\frac{\partial(\cos \Theta A_z)}{\partial \Phi} \right] = 0$$

$$H_{\Phi} = \frac{r \sin \Theta}{\mu_0 r^2 \sin \Theta} \left[\frac{\partial(r A_{\Theta})}{\partial r} - \frac{\partial A_r}{\partial \Theta} \right]$$

Substitute the value of A_{Θ} and A_r

$$H_{\Phi} = \frac{1}{\mu_0 r} \left[\frac{\partial}{\partial r} (-r \sin \Theta \frac{\mu_0 I_0 l_0}{4\pi r} e^{-ikr}) - \frac{\partial}{\partial r} (\cos \Theta A_z) \right]$$

By taking the partial derivatives, we have:

$$H_{\Phi} = \frac{1}{\mu_0 r} \left[\frac{r \sin \Theta \mu_0 I_0 l_0}{4\pi r} (ik) e^{-ikr} + \sin \Theta A_z \right]$$

Substitute the A_z instead of the value of A_z and rearrange the formula then

$$H_{\Phi} = \frac{ik \sin \Theta A_z}{\mu_0} \left[1 + \frac{1}{ikr} \right] \quad H_r = H_{\Theta} = 0$$

The second equation of Maxwell can be written in phasor format as:

$$\vec{\nabla} \times \vec{H} = \vec{J} + i\omega \epsilon_0 \vec{E} \quad \text{since} \quad \vec{J} = 0$$

$$\vec{E} = \frac{1}{i\omega\epsilon_0} \vec{\nabla} \times \vec{H}$$

$$\vec{E} = \frac{1}{i\omega\epsilon_0 r^2 \sin\Theta} \begin{bmatrix} e_r & re_\Theta & r\sin\Theta e_\Phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \Theta} & \frac{\partial}{\partial \Phi} \\ H_r & rH_\Theta & r\sin\Theta H_\Phi \end{bmatrix}$$

$$E_r = \frac{1}{i\omega\epsilon_0 r^2 \sin\Theta} \left[\frac{\partial}{\partial r} (r \sin\Theta H_\Phi) - \frac{\partial}{\partial \Phi} (r H_\Theta) \right]$$

By putting the values of H_Φ and H_Θ and after taking the partial derivatives. Let

Wave number $K = \omega/c$, $c = 1/\sqrt{\mu_0\epsilon_0}$, $k = \sqrt{\mu_0\epsilon_0} \omega$, $\eta = \frac{k}{\omega c} = \sqrt{\frac{\mu_0}{\epsilon_0}}$ then we have

$$E_r = \frac{\eta k l I_0}{2\pi} e^{-ikr} \left(\frac{1}{r^2} + \frac{1}{ikr^2} \right)$$

$$E_\Theta = \frac{-r}{i\omega\epsilon_0 r \sin\Theta} \left[\frac{\partial}{\partial r} (r \sin\Theta H_\Phi) - \frac{\partial}{\partial \Phi} H_r \right]$$

By putting the values of H_Φ and H_r and after taking the partial derivatives Let

$\sqrt{\mu_0\epsilon_0}$ Intrinsic Impedence, wave number: $K = \omega/c$, $c = 1/\sqrt{\mu_0\epsilon_0}$

$k = \sqrt{\mu_0\epsilon_0} \omega$, $\eta = \frac{k}{\omega c} = \sqrt{\frac{\mu_0}{\epsilon_0}}$ then we have

$$E_\Theta = \frac{\eta k l I_0}{4\pi r} e^{-ikr} \left(1 + \frac{1}{ikr} + \frac{1}{k^2 r^2} \right)$$

The *Far Field* Of antenna is the field distribution essentially independent of the distance from the antenna .

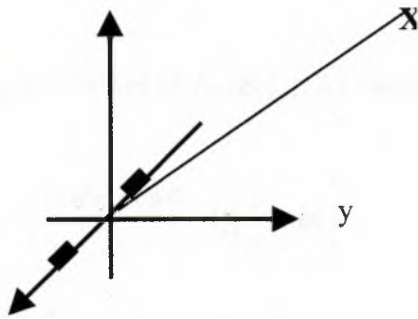
For *Far Field* $kr \gg 1$ then

$$E_r = E_\phi = 0$$

$$E_\theta = i\eta \frac{klI_0}{4\pi r} e^{-ikr} \sin \Theta$$

3.3 Vertical Dipole

As we did in “Z” direction, we can write the potential vector for “X” axis as follows:



$$A_r = \sin \Theta \cos \Phi A_x$$

$$A_\Theta = \cos \Theta \sin \Phi A_x$$

$$A_\Phi = -\sin \Phi A_x$$

By using $\vec{H} = \frac{1}{\mu_0} \vec{\nabla} \times \vec{A}$ We can write the components of magnetic intensity vector as follows

$$H_r = \frac{1}{\eta_0 r^2 \sin \Theta} \left[\frac{\partial}{\partial \Theta} (-r \sin \Theta \sin \Phi A_x) - \frac{\partial}{\partial \Phi} (r \cos \Theta \cos \Phi A_x) \right]$$

Inserting the values of A_x and after taking the partial derivative

$$H_r = \frac{1}{\eta_0 r^2 \sin \Theta} [(-r \cos \Theta \sin \Phi A_x) + (r \cos \Theta \sin \Phi A_x)] = 0$$

$$H_r = 0$$

$$H_\Theta = \frac{1}{\eta_0 r \sin \Theta} \left[\frac{\partial}{\partial r} (-r \sin \Theta \sin \Phi A_x) - \frac{\partial}{\partial \Phi} (\sin \Theta \cos \Phi A_x) \right]$$

Inserting the values of A_x and after taking the partial derivative

$$H_\Theta = \frac{\sin \Phi}{\eta_0} A_x \left[\frac{1}{r} + ik \right]$$

$$H_\Phi = \frac{1}{\eta_0 r} \left[\frac{\partial}{\partial r} (r \cos \Theta \cos \Phi A_x) - \frac{\partial}{\partial \Theta} (\sin \Theta \cos \Phi A_x) \right]$$

Inserting the values of A_x and after taking the partial derivative

$$H_\Phi = -\frac{\cos \Theta \cos \Phi}{\eta_0} A_x \left[\frac{1}{r} + ik \right]$$

The second equation of Maxwell can be written in phasor format as:

$$\vec{\nabla} \times \vec{H} = \vec{J} + i\omega \epsilon_0 \vec{E} \quad \text{since} \quad \vec{J} = 0$$

$$\vec{E} = \frac{1}{i\omega \epsilon_0} \vec{\nabla} \times \vec{H}$$

$$\vec{E} = \frac{1}{i\omega \epsilon_0 r^2 \sin \Theta} \begin{bmatrix} e_r & r e_\Theta & r \sin \Theta e_\Phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \Theta} & \frac{\partial}{\partial \Phi} \\ H_r & r H_\Theta & r \sin \Theta H_\Phi \end{bmatrix}$$

$$E_r = \frac{i}{\eta_0 \omega_0 r^2 \sin \Theta} \left[\frac{\partial}{\partial \Theta} (r \sin \Theta H_\Phi) - \frac{\partial}{\partial \Theta} (r H_\Theta) \right]$$

Inserting the values of H_Θ , H_Φ and after taking the partial derivative

$$E_r = \frac{2i \cos \Theta \sin \Phi A_x}{\eta_0 \varepsilon_0 \omega} \left[\frac{ik}{r} + \frac{1}{r^2} \right]$$

$$E_\Theta = \frac{i}{\mu_0 \varepsilon_0 \omega r \sin \Theta} \left[\frac{\partial}{\partial r} (r \sin \Theta H_\Phi) - \frac{\partial}{\partial \Phi} (H_r) \right] \text{ Inserting the values of}$$

H_r , H_Φ and after taking the partial derivative

$$E_\Theta = \frac{i \cos \Theta \cos \Phi A_x}{\mu_0 \varepsilon_0 \omega} \left[k^2 - \frac{ik}{r} - \frac{1}{r^2} \right]$$

$$E_\Phi = \frac{i}{\omega \varepsilon_0 r} \left[\frac{\partial}{\partial r} (H_\Theta) - \frac{\partial}{\partial \Theta} (H_r) \right] \text{ Inserting the values of}$$

H_Θ , H_r and after taking the partial derivative

$$E_\Phi = \frac{i \sin \Phi}{\mu_0 \varepsilon_0 \omega} \left[k^2 - \frac{ik}{r} - \frac{1}{r^2} \right]$$

By putting the values of H_Φ and H_r and after taking the partial derivatives. Let $\sqrt{\mu_0 \varepsilon_0}$ Intrinsic Impedence, wave number: $K = \omega/c$, $c = 1/\sqrt{\mu_0 \varepsilon_0}$

, $k = \sqrt{\mu_0 \varepsilon_0} \omega$, $\eta = \frac{k}{\omega c} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ and the *Far Field* Of antenna is the field distribution essentially independent of the distance from the antenna .

For *Far Field* $kr \gg 1$ then

$$E_r = 0$$

$$E_\Theta = \frac{i\eta k I_0 l}{4\pi r} \cos\Theta \sin\Phi e^{-ikr}$$

$$E_\Phi = \frac{i\eta k I_0 l}{4\pi r} \sin\Phi$$

3.4 Antenna Parameters

Radiation Power Density

This is used for power pattern and equals to time average of poynting vector

$$\vec{S}_{av} = \frac{1}{2} \text{Re}[\vec{E} \times \vec{H}^*] \text{ w/m}^2$$

$$\vec{S}_{av} = \frac{1}{2} \text{Re} \frac{(\vec{e}_r |E|^2)}{\eta} = \frac{\eta}{2} \left(\frac{k I_0 l}{4\pi r} \right)^2 [(\sin\Theta + \cos\Theta \cos\phi)^2 + \sin^2\phi] \vec{e}_r$$

Radiated Power

The total average power radiated by antenna.

$$P_{rad} = \oint \vec{S}_{av} \cdot \vec{n} \, ds = \oint \text{Re} \left\{ \vec{E} \times \vec{H}^* \right\} d\vec{s} = \iint U \, d\Omega$$

$$P_{rad} = \xi \int_0^{2\pi} \int_0^\pi [(\sin\theta + \cos\theta \cos\phi)^2 + \sin^2\phi] r^2 \sin\theta \, d\theta \, d\phi$$

Where $\xi = \frac{\eta}{2} \left(\frac{k I_0 l}{4\pi} \right)^2$ substituting the value ξ and k in above equation and taking

the integration then we have

$$P_{\text{rad}} = 395 \left(\frac{I_0 l}{\lambda} \right)^2$$

Directivity

The ratio of the radiation, intensity in a given direction from the antenna to the radiation intensity average over all direction.

$$D = 4\pi \frac{U}{P_{\text{rad}}} \quad \text{Putting the value of U and } P_{\text{rad}}$$

$$D = \frac{3}{4} [(\sin \theta + \cos \theta \cos \phi)^2 + \sin^2 \phi]$$

For the maximum directivity $\theta = \frac{\pi}{2}$ and $\phi = \frac{\pi}{2}$

$$D_{\text{max}} = 1.5 \quad A_a \frac{\lambda^2}{4\pi} D_0 = \frac{\lambda^2}{4\pi} 1.5 = 0.12 \lambda^2$$

The directivity is of importance for coverage accuracy, or resolution considerations and is more closely related to the antenna bandwidth. The difference between the tow antenna gain is usually small. The power gain and directivity my be related by the radiation efficiency factor

$$G = \delta D$$

δ : Radiation efficiency factor

Effective Aperture

Another useful parameter is the antenna aperture or effective area

$$G = \frac{4\pi A_e}{\lambda^2}$$

$$A_e = \delta_a A$$

Where, A is the Physical area of Antenna and δ_a is Antenna aperture efficiency

$$A_e = 0.12\lambda^2$$

$$G = \frac{4\pi}{\lambda^2} 0.12\lambda^2 = 1.5$$

3.5 Reflector Of Antenna

One of the most widely used microwave antenna reflector is Parabolic Reflector. The parabola is illuminated by the source of energy called 'FEED' placed at focus of the parabola and directed to the reflector surface. The parabola is well suited for microwave antennas because:

- 1- Any ray from focus is reflected in a direction parallel to the axis of the parabola.
- 2- The distance travelled by any ray from the focus to the parabola and by reflection to a plane perpendicular to the parabola axis is independent of its path.

Therefore a point source of energy located at the focus is converted into a plane waveform of uniform phase.

4. TYPES OF RADAR SYSTEMS

4.1 DOPPLER EFFECT

A radar detects the presence of objects and locates their position in space by transmitting electromagnetic energy and observing the returned echo. A pulse radar transmits a relatively short burst of electromagnetic energy, after which the receiver is turned on to listen for the echo. The echo not only indicates that a target is present, but the time that elapses between the transmission of the pulse and the receipt of the echo is a measure of the distance to the target. Separation of the echo signal and the transmitted signal is made on the basis of differences in time.

The radar transmitter may be operated continuously rather than pulsed if the strong transmitted signal can be separated from the weak echo. The received-echo-signal power is considerably smaller than the transmitter power; it might be as little as 10^{-18} that of the transmitted power-sometimes even less. Separate antennas for transmission and reception help segregate the weak echo from the strong leakage signal, but the isolation is usually not sufficient. A feasible technique for separating the received signal from the transmitted signal when there is relative motion between radar and target is based on recognizing the change in the echo-signal frequency caused by the Doppler effect.

It is well known in the fields of optics and acoustics that if either the source of oscillation or the observer of the oscillation is in motion, an apparent shift in frequency will result. This is the *doppler* -effect and is basis of CW radar. If R is the distance from the radar to target, the total number of wavelengths λ contained in the two-way path between the radar and the target is $2R/\lambda$. The distance R and the wavelength λ , are assumed to be measure in the same units. Since one wavelength corresponds to an angular excursion of 2π radians, the total angular excursion made by the electromagnetic wave during its transit to and from the target is $2R/\lambda$ radians. If the target is in motion, R and the phase ϕ are continually changing, A change in ϕ with respect to time is equal to a frequency. This is the doppler angular frequency ω_d , given

by

$$w_d = 2\pi f_d = \frac{d\phi}{dt} = \frac{4\pi}{\lambda} \frac{dR}{dt} = \frac{4\pi v_r}{\lambda} \quad (4.1)$$

Where f_d = doppler frequency shift and v_d = relative (or radial) velocity of target with respect to radar. The Doppler frequency shift is

$$f_d = \frac{2v_r}{\lambda} = \frac{2v_r f_0}{c} \quad (4.2a)$$

Where f_0 = transmitted frequency and c = velocity of propagation = 3×10^8 m/s. If f_d is in hertz, v_r in knots, and λ in meters,

$$f_d = \frac{1.03v_r}{\lambda} \quad (4.2b)$$

A plot of this equation is shown in Figure 4.1

The relative velocity may be written $v_d = v \cos\theta$, where v is the target speed and θ is the angle made by the target trajectory and the line joining radar and target. When $\theta=0$, the doppler frequency is maximum. The doppler is zero when the trajectory is perpendicular to the radar line of sight ($\theta=90^\circ$).

The type of radar which employs a continuous transmission, either modulated, has had wide application. Historically, the early radar experiments worked almost exclusively with continuous rather than pulsed transmission. Two of the more important early applications of the CW radar principle were the proximity (VT) fuzzy and the FM-CW altimeter. The CW proximity fuzzy was first employed in artillery projectiles during World War II and greatly enhanced the effectiveness of both field and antiaircraft artillery. The first practical model of the FM-CW altimeter was developed by the Western

Electric company in 1938, although the principle of altitude determination using radio-wave reflection was known ten years earlier, in 1928[22].

The CW radar is of interest not only because of its many applications, but its study also serves as a means for better understanding the nature and use of the doppler information contained in the echo signal, whether in a CW or a pulse radar (MTI) application. In addition to allowing the received signal to be separated from the transmitted signal, the CW radar provides a measurement of relative velocity, which may be used to distinguish moving targets from stationary objects or clutter.

4.2 CW Radars

Consider the simple CW radar as illustrated by the block diagram of figure 4.2a. The transmitter generates (unmodulated) oscillation of frequency f_0 , which is radiated by the antenna. A portion of radiated energy is intercepted by the target and is scattered, some of it in the direction of the radar, where it is collected by the receiving antenna. If the target is in the motion with a velocity v_r relative to the radar, the received signal will be shifted in frequency from the transmitted frequency f_0 by an amount $\pm f_d$ as given by equation (4.2). The plus sign associated with the doppler frequency applies if distance between target and radar is decreasing (closing target), that is, when the received signal frequency is greater than the transmitted signal frequency. The minus sign applies if the distance is increasing (receding target). The received echo signal at a frequency $f_0 \pm f_d$ enters the radar via the antenna and is heterodyned in the detector (mixer) with a portion of the transmitter signal f_0 to produce a doppler beat note of frequency f_d . The sign of f_d is lost in this process.

The purpose of the doppler amplifier is to eliminate echo from stationary targets and to amplify the doppler echo signal to a level where it can operate an indicating device. It might have a frequency-response characteristic similar to that of figure 4.2b. The low-frequency cutoff must be high enough to reject the d-c component caused by stationary

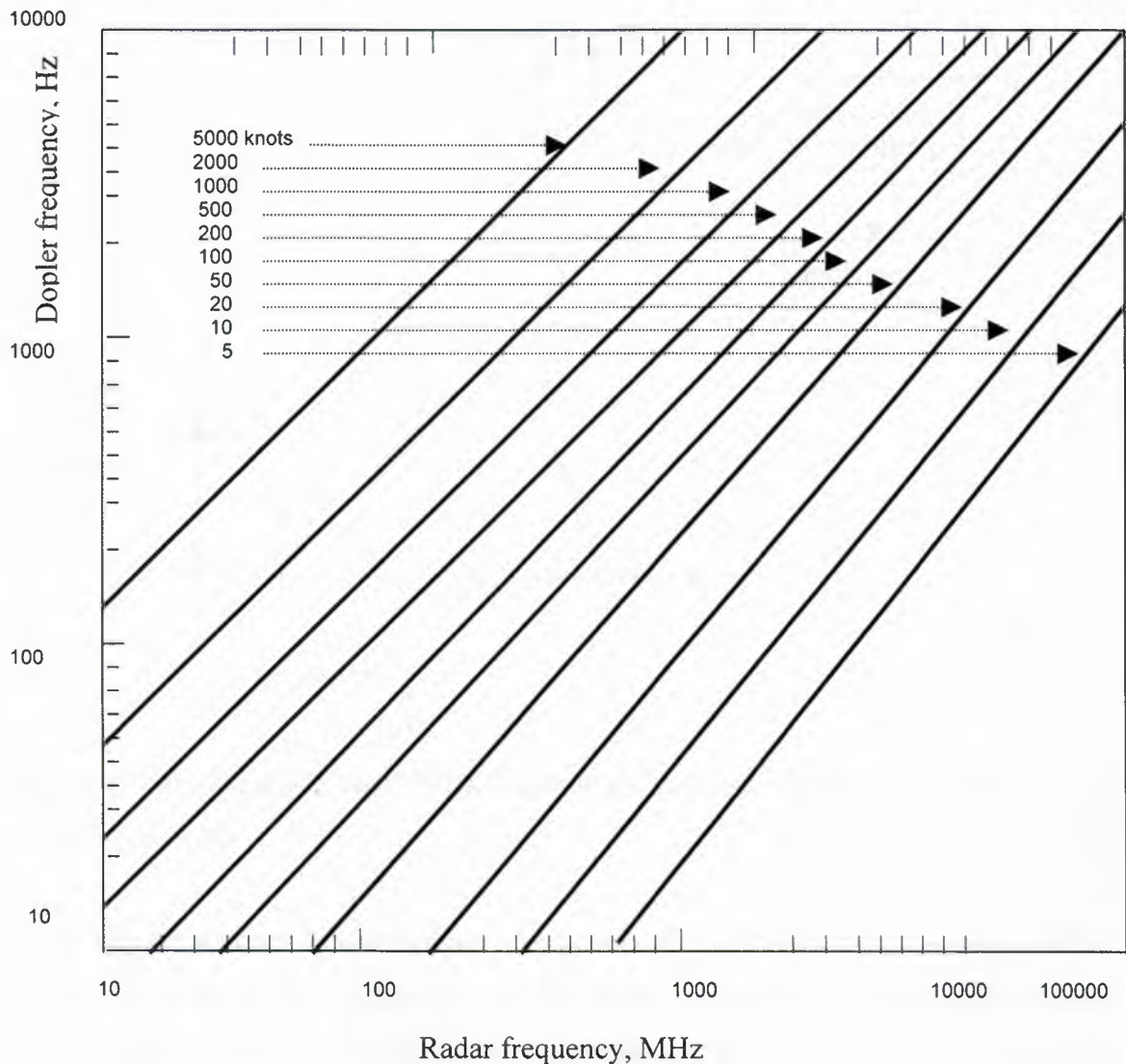


Figure 4.1 Doppler frequency [Equation (4.2b)] as a function of radar frequency and target relative velocity.

targets, but yet it must be low enough to pass the smallest Doppler frequency expected. Sometimes both conditions cannot be met simultaneously and a compromise is necessary. The upper cutoff frequency is selected to pass the highest doppler frequency expected.

The indicator might be a pair of earphones or a frequency meter. If exact knowledge of

4. Types Of Radar System

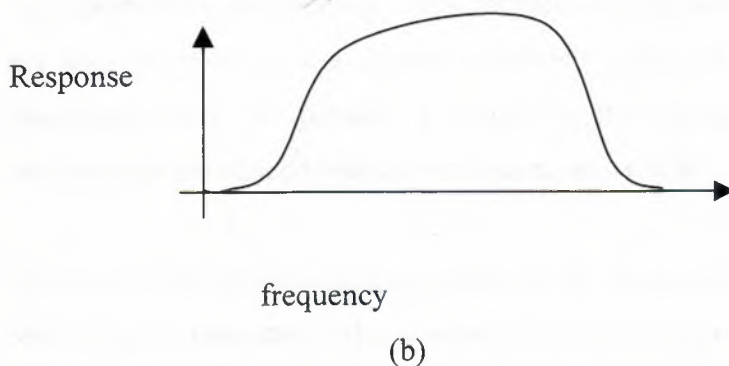
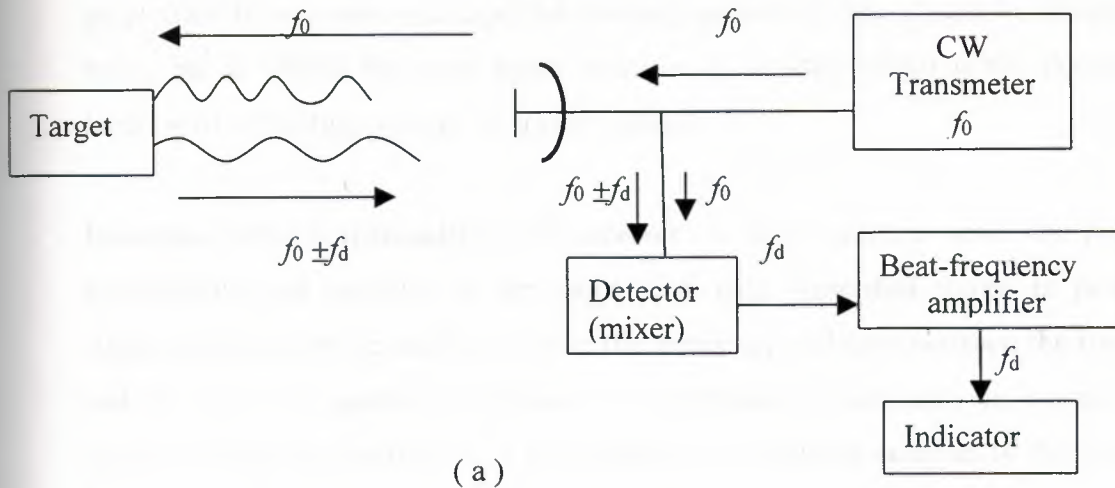


Figure 4.2 (a) Simple CW radar block diagram; (b) response characteristic of beat-frequency amplifier.

the Doppler frequency is not necessary, earphones are especially attractive provided the Doppler frequencies lie within the audio-frequency response of the ear. Earphones are not only simple devices, but the ear acts as a selective bandpass filter with a passband of the order of 50 Hz centered about the signal frequency. The narrow-bandpass characteristic of the ear results in an effective increase in the signal-to-noise ratio of the echo signal. With subsonic aircraft target and transmitter frequencies in the middle range of the microwave frequency region, the Doppler frequency usually falls within the passband of the ear. If audio detection were desired for those combinations of target velocity and transmitter frequency which do not result in audible Doppler frequencies, the Doppler signal could be heterodyned to the audible range. The Doppler frequency can also be detected and measured by conventional frequency meters, usually one that counts cycles. An example of the CW radar principle is the radio proximity fuze used with great success during World War II for the fuzing of artillery

projectiles. It may seem strange that the radio proximity fuze should be classified as a radar, but it fulfills the same basic function of a radar, which is the detection and location of reflecting objects by "radio" means.

Isolation between transmitter and receiver. A single antenna serves the purpose of transmission and reception in the simple CW radar described above. In principle, a single antenna may be employed since the necessary isolation between the transmitted and the received signals is achieved via separation in frequency as a result of the doppler effect. In practice, it is not possible to eliminate completely the transmitter leakage. However, transmitter leakage is not always undesirable. A moderate amount of leakage entering the receiver along with the echo signal supplies the reference necessary for the detection of the doppler frequency shift. If a leakage signal of sufficient magnitude were not present, a sample of the transmitted signal would have to be deliberately introduced into the receiver to provide the necessary reference frequency.

There are two practical effects which limit the amount of transmitter leakage power which can be tolerated at the receiver. These are (1) the maximum amount of power the receiver input circuitry can withstand before it is physically damaged or its sensitivity reduced (burnout) and (2) the amount of transmitter noise due to hum, microphonics, stray pick-up, and instability which enters the receiver, from the transmitter. The additional noise introduced by the transmitter reduces the receiver sensitivity. Except where the CW radar operates with relatively low transmitter power and insensitive receivers, additional isolation is usually required between the transmitter and the receiver if the sensitivity is not to be degraded either by burnout or by excessive noise.

The amount of isolation required depends on the transmitter power and the accompanying transmitter power noise as well as the ruggedness and the sensitivity of the receiver. For example, if the safe value of power which might be applied to a receiver were 10 mW and if the transmitter power were 1kW, the isolation between transmitter and receiver must be at least 50 dB.

The amount of isolation needed in a long-range CW radar is more often determined by the noise that accompanies the transmitter leakage signal rather than by any damage

caused by higher power. For example, suppose the isolation between the transmitter and receiver were such that 10 mW of leakage signal appeared at the receiver. If the minimum detectable signal were 10^{-13} watt (100 dB below 1 mW), the transmitter noise must be at least 110 dB (preferably 120 or 130 dB) below the transmitted carrier.

The transmitter noise of concern in doppler radar includes those noise components that lie within the range as the doppler frequencies. The greater the desired radar range, the more stringent will be the need for reducing the noise modulation accompanying the transmitter signal. If complete elimination of the *direct* leakage signal at the receiver could be achieved, it might not entirely solve the isolation problems since echoes from nearby fixed target (clutter) can also contain the noise components of the transmitted signal.

It will be recalled that the receiver of a pulsed radar is isolated and protected from the damaging effects of the transmitted pulse by the duplexer, which short circuits the receiver input period. Turning off the receiver during the transmission with a duplexer is not possible in a CW radar since the transmitter is operated continuously. Isolation between transmitter and receiver might be obtained with a single antenna by using a hybrid junction, circulator, turnstile junction, or with separate polarizations. Separate antennas for transmitting and receiving might also be used. The amount of isolation which can be readily achieved between the arms of practical hybrid junctions such as the magic T, rat race, or short-slot coupler is of the order of 20 to 30 dB. In some instances, when extreme precision is exercised, an isolation of perhaps 60 dB or more might be achieved. One limitation of the hybrid junction is the 6-dB loss in overall performance which results from the inherent waste of half the transmitted power and half the received signal power. Both the loss in performance and the difficulty in obtaining large isolations have limited the application of the hybrid junction to short-range radars.

Ferrite isolation devices such as the circulator do not suffer the 6-dB loss inherent in the hybrid junction. Practical devices have isolation of the order of 20 to 50 dB, Turnstile junctions achieve isolations as high as 40 to 60 dB. The use of orthogonal polarizations

for transmitting and receiving is limited to short range radars because of the relatively small amount of isolation that can be obtained.

An important factor which limits the use of isolation devices with a common antenna is the reflections produced in the transmission line by the antenna. The antenna can never be perfectly matched to free space, and there will always be some transmitted signal reflected back toward the receiver. The reflection coefficient from a mismatched antenna with a voltage standing-wave ratio σ is $|\rho|=(\sigma-1)/(\sigma+1)$. Therefore, if an isolation of 20 dB is to be obtained, the VSWR must be less than 1.22. If 40 dB of isolation is required, the VSWR must be less than 1.02.

The largest isolations are obtained with two antennas—one for transmission, the other for reception—physically separated from one another. Isolations of the order of 80 dB or more are possible with high-gain antennas. The more directive the antenna beam and the greater the spacing between antennas, the greater will be the isolation. When the antenna designer is restricted by the nature of the application, large isolations may not be possible. For example, typical isolations, between transmitting and receiving antennas on missiles might be about 50 dB at X band, 70 dB at K band and as low as 20 dB at L band. Metallic baffles, as well as absorbing material, placed between the antennas can provide additional isolation.

It has been reported that the isolation between two X-band horn antennas of 22 dB gain can be increased from a normal, value of 70 dB to about 120 dB by separating the two with a smooth surface covered by a sheet of radar-absorbing material and providing screening ridges at the edges of the horns. A common random enclosing the two antennas should be avoided since it limits the amount of isolation that can be achieved.

Additional isolation can be obtained by properly introducing a controlled sample of the signal directly into the receiver. The phase and amplitude of this “buck-off” signal are adjusted to cancel the portion of the transmitter signal that leaks into the receiver. An additional 10 dB of isolation might be obtained. The phase and amplitude of the leakage signal, however, can vary as the antenna scans, which results in varying cancellation. Therefore, when additional isolation is necessary, in the high-power CW

tracker illuminator, a dynamic canceller can be used that senses the proper phase and amplitude required of the nulling signal. Dynamic cancellation of the leakage by this type of the leakage by this type of "feed-through nulling" can exceed 30 dB[23].

The transmitter signal is never a pure CW waveform. Minute variations amplitude (AM) and phase (FM) can result in sideband components that fall within the doppler frequency band. These can generate raise targets or mask the desired signals. Therefore both AM and FM modulations can result in undesired sidebands. AM sidebands are typically 120 dB below the carrier, as measured in a 1 kHz band, and are relatively constant across the usual doppler of interest. The normal antenna isolation plus feed-through nulling usually reduces the AM component below receiver noise in moderate power radars. FM sideband are usually significantly greater than AM, but decrease with increasing offset from the carrier. The character of PM noise in the leakage signal is also affected by stabilizing the output frequency of the CW transmitter and by active noise degeneration using a microwave bridge circuit to extract the FM noise components. These are then fed back to the transmitter in such a manner as to reduce the original frequency deviation [23]. It has said that experience indicates that a satisfactory measurements of AM noise over the doppler frequency band provides assurance that both AM and PM noise generated by the tube are within required limits.

The transmitter noise that enters the radar receiver via backscatter from the clutter is sometimes called *transmitted clutter*. It can appear at the same frequencies as the doppler shifts from moving targets and can mask desired targets or cause spurious responses. This extraneous noise is produced by ion oscillations in the tube (usually a klystron amplifier) rather than by than by thermal noise, or noise fissure. When the ion oscillations appear, they usually have a magnitude about 40 dB below the carrier, or else they are not measurable. Thus there is no need to specify a measurement of this noise to a level better than 40 dB below the carrier to insure the required noise levels. Since ion oscillations may occur at some combination of tube parameters and not at others, the CW radar tube should be tested for noise-free operation over the expected range of beam voltage, heater voltage, RF drive level, and load VSWR. Noise free operation also requires well-filtered beam power-supplies and a dc heater supply.

Since ion oscillations require a finite time to develop (tens of microseconds), a pulse doppler radar with a pulse width of less than $10\ \mu\text{s}$ should not experience this form of noise.

Intermediate frequency receiver. The receiver of the simple CW radar of Figure 4.2 is in some respects analogous to a superheterodyne receiver. Receivers of this type are called *homodyne* receivers, or superheterodyne receivers with zero IF. The function of the local oscillator is replaced by the leakage signal from the transmitter, such a receiver is simpler than one with a more conventional intermediate frequency since no IF amplifier or local oscillator is required. However, the simpler receiver is not as sensitive because of increased noise at the lower intermediate frequencies caused by flicker effect. Flicker-effect noise occurs in semiconductor devices such as diode detectors and cathodes of vacuum tubes. The noise power produced by the flicker effect varies as $1/f^\alpha$, where α is approximately unity. This is in contrast to shot noise or thermal noise, which is independent of frequency. Thus, at the lower range of frequencies (audio or video region), where the doppler frequencies usually are found, the detector of the CW receiver can introduce a considerable amount of flicker noise, resulting in reduced receiver sensitivity. For short-range, low-power applications this decrease in sensitivity might be tolerated since it can be compensated by a modest increase in antenna aperture and/or additional transmitter power. But for maximum efficiency with CW radar, the reduction in sensitivity caused by the simple doppler receiver with zero IF cannot be tolerated.

The effects of flicker noise are overcome in the normal superheterodyne receiver by using an intermediate frequency high enough to render the flicker noise small compared with the normal receiver noise. This results from the inverse frequency dependence of flicker noise. Figure 4.4 shows a block diagram of the CW radar whose receiver operates with a nonzero IF. Separate antennas are shown for transmission and reception. Instead of the usual local oscillator found in the conventional superheterodyne receiver, the locally generated (or reference signal) is derived in this receiver from a portion of the transmitted signal mixed with a locally generated signal of frequency equal to that of the receiver IF. Since the output of the mixer consists of two sidebands on either side of the carrier plus higher harmonics, a narrowband filter

selects one of the sidebands as the reference signal. The improvement in receiver sensitivity with an intermediate-frequency superheterodyne might be as much as 30 dB over the simple receiver of figure 4.2

Received bandwidth. One of the requirements of the doppler-frequency amplifier in the simple CW radar figure 4.2 or the IF amplifier of the sideband superheterodyne figure 4.3 is that it be wide enough to pass the expected range of doppler frequencies. In most cases of practical interest the expected range of doppler frequencies will be much wider than the frequency spectrum occupied by the signal energy. Consequently, the use of a wideband amplifier covering the expected doppler range will result in an increase in noise and a lowering of the receiver sensitivity. If the frequency of the doppler-shifted echo signal were known beforehand. A narrowband filter—one just wide enough to reduce the excess noise without eliminating a significant amount of signal energy—might be used, If the waveform of the echo signal

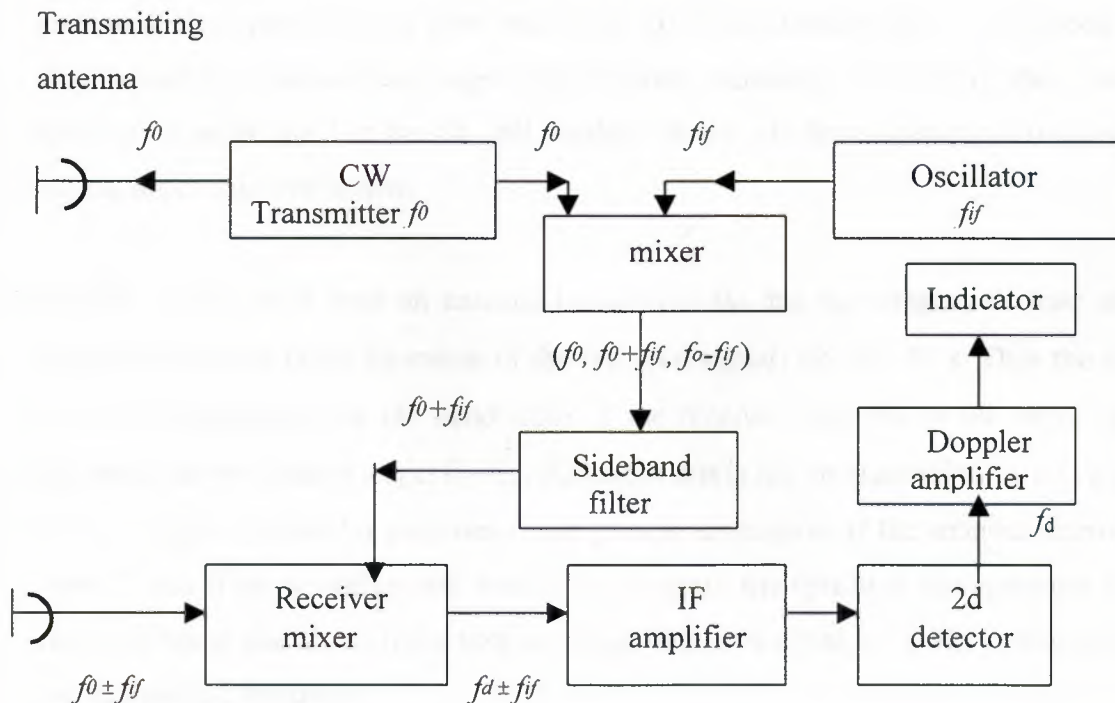


Figure. 4.3 Block diagram of CW doppler radar with nonzero IF receiver, sometimes called *sideband superheterodyne*.

were known, as well as its carrier frequency, the matched filter could be specified.

Several factors tend to spread the CW signal energy over a finite frequency band. These must be known if an approximation to the bandwidth required for the narrowband Doppler filter is to be obtained.

If the received waveform were a sine wave of infinite duration, its frequency spectrum would be a delta function figure 4.4a and the receiver bandwidth would be infinitesimal. But a sine wave of infinite duration and an infinitesimal bandwidth cannot occur in nature. The more normal situation is an echo signal which is a sine wave of finite rather than infinite duration. The frequency spectrum of a finite-duration sine wave has a shape of the form $[\sin\pi(f-f_0)\delta] / \pi(f-f_0)$ where f_0 and δ are the frequency and duration of the sine wave, respectively, and f is the frequency variable over which the spectrum is plotted figure 3.4b. Practical receivers can only approximate this characteristic. (Note that this is the same as the spectrum of a pulse of sine wave, the only difference being the relative value of the duration δ .) In many instances, the echo is not a pure sine wave of finite duration but is perturbed by fluctuations in cross section, target accelerations, scanning fluctuations, etc., which tend to broaden the bandwidth still further. Some of these spectrum-broadening effects are considered below.

Assume a CW radar with an antenna beamwidth θ_B deg scanning at the rate of θ_s deg/s. The time on target (duration of the received signal) is $\delta = \theta_B / \theta_s$ s. Thus the signal is of finite duration and the bandwidth of the receiver must be or the order of the reciprocal or the time on target θ_B / θ_s . Although this is not an exact relation, it is a good enough approximation for purposes of the present discussion. If the antenna beamwidth were 2° and if the scanning rate were $36^\circ/\text{s}$ (6 rpm), the spread in the spectrum or the received signal due to the finite time on target would be equal to 18 Hz, independent of the transmitted frequency.

In addition to the spread of the received signal spectrum caused by the finite time on target, the spectrum may be further widened if the target cross section fluctuates. The fluctuations widen the spectrum by modulating the echo signal. In a particular case, it

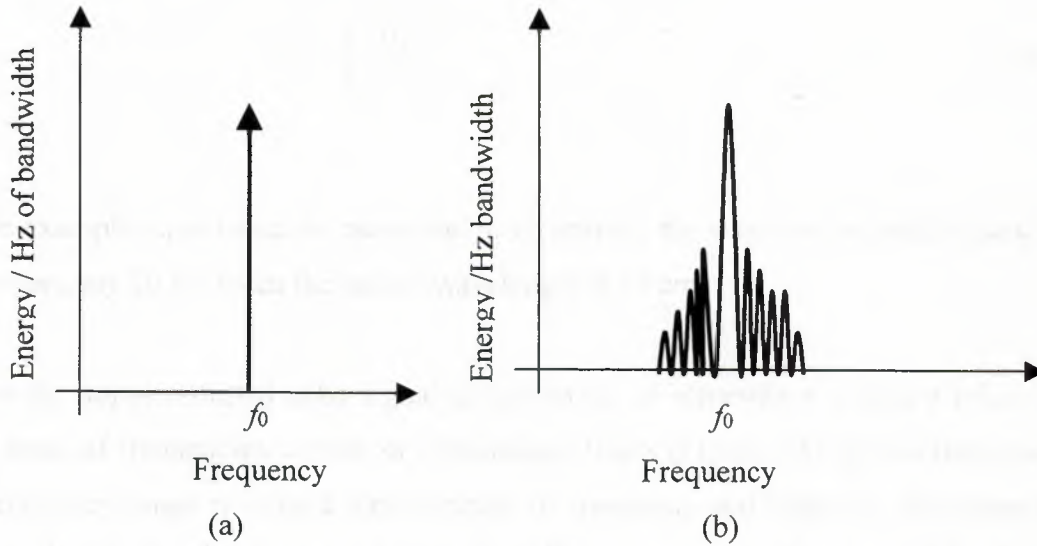


Figure 4.4 Frequency spectrum of CW oscillation or (a) infinite duration and (b) finite duration.

has been reported that the aircraft cross section can change by 15 dB for a change in target aspect of as little as $1/3^\circ$. The echo signal from a propeller-driven aircraft can also contain modulation components at a frequency proportional to the propeller rotation. The spectrum produced by propeller modulations is more like that produced by a sine-wave signal and its harmonics rather than a broad, white-noise spectrum. The frequency range of propeller modulation depends upon the shaft-rotation speed and the number of propeller blades. It is usually in the vicinity of 50 to 60 Hz for World War II aircraft engines. This could be a potential source of difficulty in a CW radar since it might mask the target's doppler signal or it might cause an erroneous measurement of doppler frequency. In some instances, propeller modulation can be an advantage. It might permit the detection of propeller-driven aircraft passing on a tangential trajectory, even though the doppler frequency shift is zero. The rotating blades of a helicopter and the compressor stages of a jet engine can also result in a modulation of the echo and a widening of the spectrum that can degrade the performance of CW doppler radar.

If the target's relative velocity is not constant, a further widening of the received signal spectrum can occur. If a_r is the acceleration of the target with respect to the radar, the signal will occupy a bandwidth

$$\nabla f_d = \left(\frac{2a_r}{\lambda} \right)^{1/2} \quad (4.4)$$

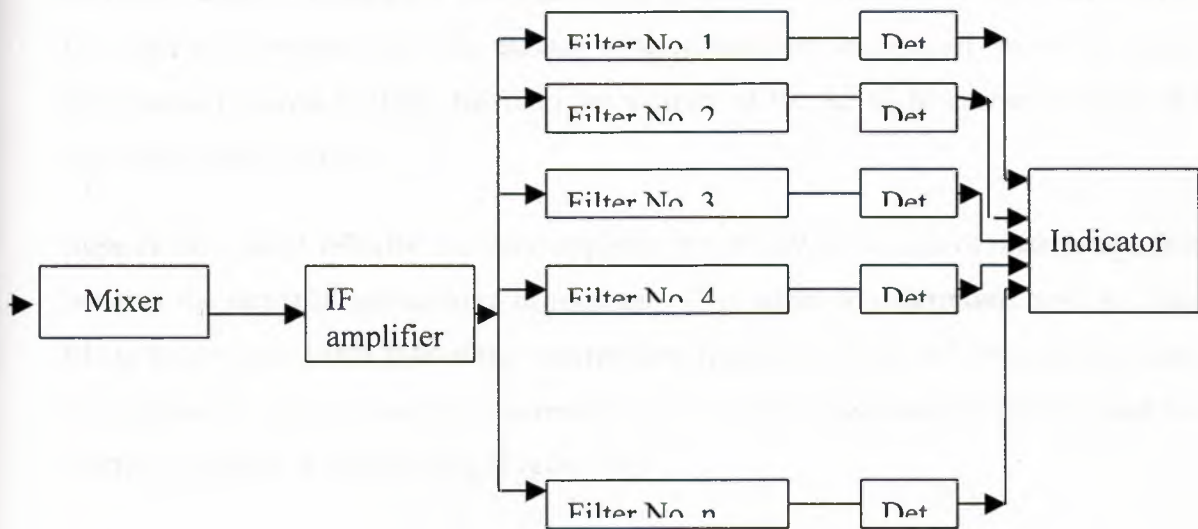
If, for example, a_r is twice the acceleration or gravity, the receiver bandwidth must be approximately 20 Hz when the radar's wavelength is 10 cm.

When the doppler-shifted echo signal is known to lie somewhere within a relatively wide band of frequencies, a bank or narrowband filters (Figure 4.5) spaced throughout the frequency range permits a measurement of frequency and improves the signal-to-noise ratio. The bandwidth of each individual filter is wide enough to accept the signal energy, but not so wide as to introduce more noise than need be. The center frequencies of the filters are staggered to cover the entire range of doppler frequencies. If the filters are spaced with their half-power points overlapped, the maximum reduction in signal-to-noise ratio of a signal which lies midway between adjacent channels compared with the signal-to-noise ratio at midband is 3 dB. The more filters used to cover the band, the less will be the maximum loss experienced, but the greater the probability of false alarm.

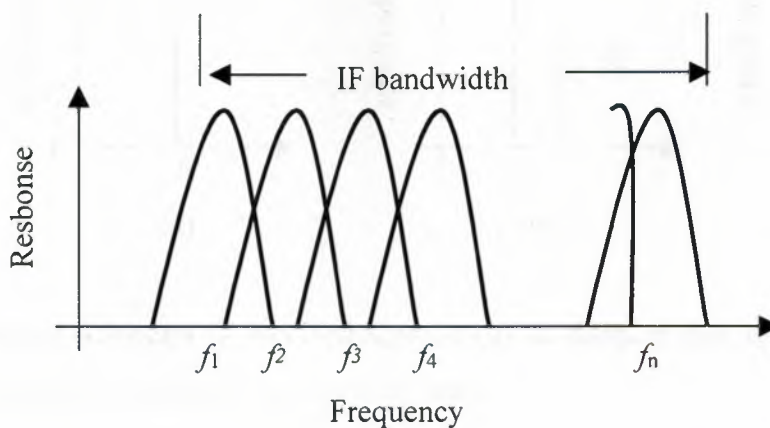
A bank of narrowband filters may be used after the detector in the video of the simple CW radar of Figure 4.2 instead of in the IF. The improvement in signal-to-noise ratio with a video filter bank is not as good as can be obtained with an IF filter bank, but the ability to measure the magnitude of doppler frequency is still preserved. Because of foldover, a frequency which lies to one side of the IF carrier appears, after detection, at the same video frequency as one which lies an equal amount on the other side of the IF. Therefore the sign of the doppler shift is lost with a video filter bank, and it cannot be directly determined whether the Doppler frequency corresponds to an approaching or to a receding target. (The sign of the doppler may be determined in the video by other means, as described later.) One advantage of the foldover in the video is that only half the number of filters are required than in the IF filter bank. The equivalent of a bank of contiguous bandpass filters may also be obtained by converting the analog IF or video signal to a set of sampled, quantized signals which are processed with digital circuitry by

means of the fast Fourier transform algorithm.

A bank of overlapping doppler filters, whether in the IF or video, increases the complexity of the receiver. When the system requirements permit a time sharing or the doppler frequency range, the bank of doppler filters may be replaced by a single narrowband tunable filter which search in frequency over the band of expected doppler frequencies until a signal is found.



(a)



(b)

Figure 4.5 (a) Block diagram of IF doppler filter bank; (b) frequency-response characteristic of Doppler filter bank.

After detecting and recognizing the signal, the filter may be programmed to continue its search in frequency for additional signals. The phase-locked filter, or the phase-locked loop.

If, in any of the above techniques, moving targets are to be distinguished from stationary objects, the zero-doppler-frequency component must be removed. The zero-doppler-frequency component has, in practice, a finite bandwidth due to the finite time on target, clutter fluctuations, and equipment instabilities. The clutter-rejection band of the doppler filter must be wide enough to accommodate this spread. In the multiple-filter bank, removal of those filters in the vicinity of the RF or IF carrier removes the stationary-target signals.

Sign of the radial velocity. In some applications of CW radar it is of interest to know whether the target is approaching or receding. This might be determined with separate filters located on either side of the intermediate frequency. If the echo-signal frequency lies below the carrier, the target is receding; if the echo frequency is greater than the carrier, the target is approaching (Figure 4.6).

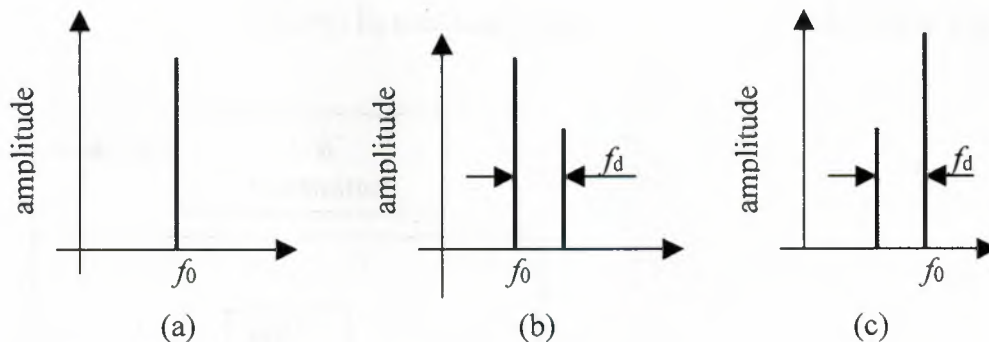


Figure 4.6 Spectra of received Signals. (a) No doppler shift, no relative target motion; (b) approaching target; (c) receding target.

Although the doppler-frequency spectrum "folds over" in the video because of the action of the detector, it is possible to determine its sign from a technique borrowed from single-sideband communications. If the transmitter signal is given by

4. Types Of Radar System

$$E_t = E_0 \cos \omega_0 t \quad (4.4)$$

the echo signal from a moving target will be

$$E_r = k_1 E_0 \cos[(\omega_0 \pm \omega_d) t + \phi] \quad (4.5)$$

where E_0 = amplitude of transmitter signal

k_1 = a constant determined from the radar equation

ω_0 = angular frequency of transmitter. rad/s

ω_d = doppler angular frequency shift

ϕ = a constant phase shift, which depends upon range of initial detection

The sign of the doppler frequency, and therefore the direction of target motion, may be found by splitting the received signal into two channels as shown in figure 4.7. In channel the signal is processed as in the simple CW radar of figure 4.2. The received signal and a portion of the transmitted heterodyne in the detector (mixer) to yield a difference signal

$$E_A = k_2 E_0 \cos(\pm \omega_d t + \phi) \quad \text{Equation 4.6}$$

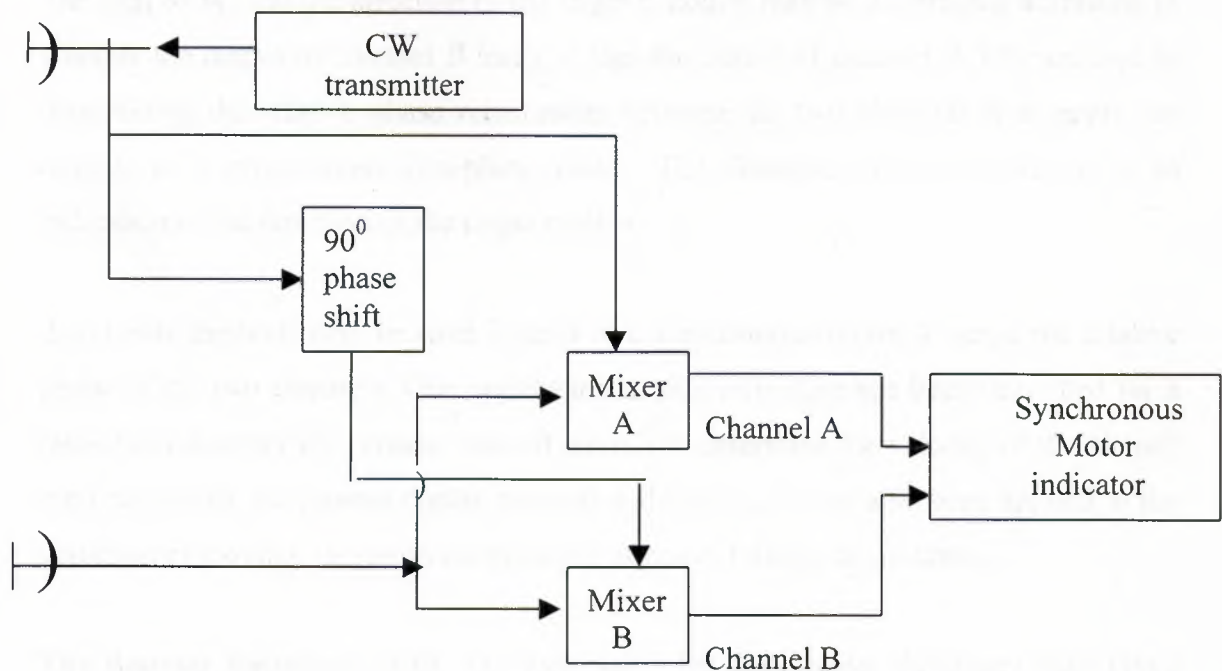


Figure 4.7 Measurement of doppler direction using synchronous two-phase motor.

The other channel is similar. except for a 90° phase delay introduced in the reference signal.

The output of the channel B mixer is

$$E_B = k_2 E_0 \cos(\pm \omega_d t + \phi + \pi/2) \quad (4.7)$$

If the target is approaching (positive doppler), the outputs from the two channels are

$$E_A(+) = k_2 E_0 \cos(\omega_d t + \phi) \quad E_B(+) = k_2 E_0 \cos(\omega_d t + \phi + \pi/2) \quad (4.8.a)$$

On the other hand, if the targets are receding (negative doppler).

$$E_A(-) = k_2 E_0 \cos(\omega_d t - \phi) \quad E_B(-) = k_2 E_0 \cos(\omega_d t - \phi - \pi/2) \quad (4.8.b)$$

The sign of W , and the direction of the target's motion may be determined according to whether the output of channel B leads or lags the output of channel A . One method of determining the relative phase relationship between the two channels is to apply the outputs to a synchronous two-phase motor. The direction of motor rotation is an indication of the direction of the target motion.

Electronic methods may be used instead of a synchronous motor to sense the relative phase of the two channels. One application of this technique has been described for a rate-of climb meter for vertical take-off aircraft to determine the velocity of the aircraft with respect to, the ground during take-off and landing. It has also been applied to the detection of moving, targets in the presence of heavy foliage, as discussed.

The doppler frequency shift. The expression for the doppler frequency shift given

previously by Equation (4.2) is an approximation that is valid for most radar applications. The correct expression for the frequency f^* from a target moving with a relative velocity v , when the frequency f transmitted is

$$f^* = f \frac{(1 + v/c)}{(1 - v/c)} \quad (4.9)$$

where c is the velocity of propagation. When, as is usually the case, $v \ll c$, Equation (4.9) reduces to the classical form of the doppler frequency shift. The phase shift associated with the return signal is $(4\pi/R_0/c)/(1-v/c)$, where R_0 is the range at time $t = 0$.

Applications of CW radar. The chief use of the simple, unmodulated CW radar is for the measurement of the relative velocity of moving target, as in the police speed monitor or in the previously mentioned rate-of-climb meter for vertical-take-off aircraft. In support of automobile traffic, CW radar has been suggested for the control of traffic lights, regulation of toll booths, vehicle counting, as a replacement for the "fifth-wheel" speedometer in vehicle testing, as a sensor in antilock braking systems, and for collision avoidance. For railways, CW radar can be used as a speedometer to replace the conventional axle-driven tachometer. In such an application it would be unaffected by errors caused by wheelslip on accelerating or wheelslide when braking. It has been used for the measurement of railroad-freight-car velocity during humping operations in marshalling yards, and as a detection device to give track maintenance personnel advance warning of approaching trains. CW radar is also employed for monitoring the docking speed of large ships. It has also seen application for intruder alarms and for the measurement of the velocity of missiles, ammunition, and baseballs.

The principal advantage of a CW doppler radar over other (nonradar) methods of measuring speed is that there need not be any physical contact with the object whose speed is being measured. In industry this has been applied to the measurement of turbine-blade vibration, the peripheral speed of grinding wheels, and the monitoring of vibrations in the cables of suspension bridges.

Most of the above applications can be satisfied with a simple, solid-state CW source

with powers in the tens of milliwatts. High-power CW radars for the detection of aircraft and other targets have been developed and have been used in such systems as the Hawk missile systems. However, the difficulty of eliminating the leakage of the transmitter signal into the receiver has limited the utility of unmodulated CW radar for many long-range applications. A notable exception is the Space Surveillance System (Spasur) for the detection of satellites.

The CW transmitter of Spasur at 216 MHz radiates a power of up to one megawatt from an antenna almost two miles long to produce a narrow, vertically looking fan beam. The receiver is separated from the transmitter by a distance of several hundred miles. Each receiver site consists of an interferometer antenna to obtain an angle measurement in the plane of the fan beam. There are three sets of transmitter-receiver stations to provide fence coverage of the southern United States.

The CW radar, when used for short or moderate ranges, is characterized by simpler equipment than a pulse radar. The amount of power that can be used with a CW radar is dependent on the isolation that can be achieved between the transmitter and receiver since the transmitter noise that finds its way into the receiver limits the receiver sensitivity. (The pulse radar has no similar limitation to its maximum range because the transmitter is not operative when the receiver is turned on.)

Perhaps one of the greatest shortcomings of the simple CW radar is its inability to obtain a measurement of range. This limitation can be overcome by modulating the CW carrier, as in the frequency-modulated radar described in the next section.

Some anti-air-warfare guided missile systems employ semiactive homing guidance in which a receiver in the missile receives energy from the target, the energy having been transmitted from an "illuminator" external to the missile. The illuminator, for example, might be at the launch platform. CW illumination has been used in many successful systems. An example is the Hawk tracking. It is a tracking radar as well as an illuminator since it must be able to follow the target as it travels through space. The doppler discrimination of a CW radar allows operation in the presence of clutter and has been well suited for low altitude missile defense systems. A block diagram or a

CW tracking illuminator is shown in Figure 4.7. Note that following the wide-band doppler amplifier is a *speed gate*, which is a narrow-band tracking filter that acquires the target's doppler and tracks its changing doppler frequency shift.

4.3 Frequency Modulated CW Radar

The inability of the simple CW radar to measure range is relatively spectrum (bandwidth) its transmitted waveform. Some sort of timing mark must be applied to a CW carrier if range is to be measured. The timing mark permits the time of transmission and the time of return to be recognized. The sharper or more distinct the mark, the more accurate the measurement or the transit time. But the more distinct the timing mark, the broader will be the transmitted spectrum. This follows from the properties of the Fourier transform. Therefore a finite Spectrum must of necessity be transmitted if transit time or range is to be measured.

The spectrum of a CW transmission can be broadened by the application of modulation, either amplitude, frequency, or phase. An example of an amplitude modulation pulse radar. The narrower the pulse, the more accurate the measurement of range and the broader the transmitted spectrum. A widely used technique to broaden the transmitter spectrum of CW radar is to frequency-modulate the carrier. The timing mark is the changing frequency. The transit time is proportional to the difference in frequency between the echo signal and the transmitter signal. The greater the transmitter frequency deviation in a given time interval, the more accurate the measurement of the transit time and the greater will be the transmitted spectrum.

Range and doppler measurement. In the frequency modulated CW radar (abbreviated FM-CW), the transmitter frequency is changed as a function of time in a known manner. Assume that the transmitter frequency increases linearly with time, as shown by the solid line in Figure 4.9a. If there is a reflecting object at a distance R , an echo signal will return after a time $T = 2R/c$. The dashed line in the figure represents the echo signal. If the echo signal is heterodyned with a portion of the transmitter signal in a Scan

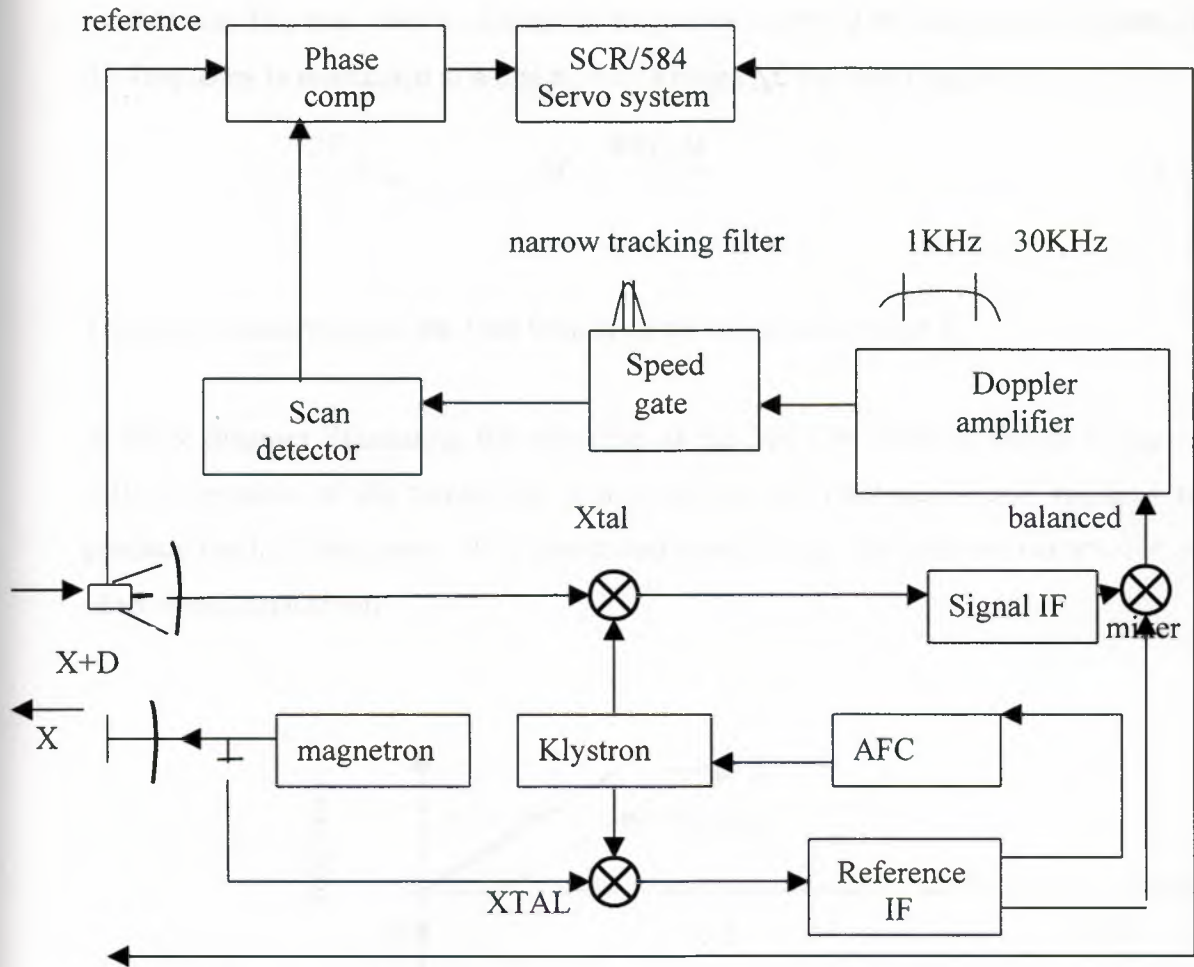


Figure 4.8 Block diagram of a CW tracking-illuminator. [Courtesy IEEE.]

nonlinear element such as a diode, a beat note f_b will be produced. If there is no doppler frequency shift, the beat note (difference frequency) is a measure of the target's range and, $f_b = f_r$, where f_r is the beat frequency due only to the target's range. If the rate of change of the carrier frequency f_0 , the beat frequency is

$$f_r = f_0 T = \frac{2R}{c} f_0 \quad (4.10)$$

In any practical CW radar, the frequency cannot be continually changed in one direction only. Periodicity in the modulation is necessary, as in the triangular frequency-modulation waveform shown in figure 4.9b. The modulation need not necessarily be triangular; it can be sawtooth, sinusoidal; or some other shape. The resulting beat frequency) as a function of time is shown in figure 4.9c for triangular

modulation. The beat note is of constant frequency except at the turn-around region. If the frequency is modulated at a rate f_m over a range Δf , the beat frequency is

$$f_r = \frac{2R}{c} 2f_m \quad \Delta f = \frac{4Rf_m \Delta f}{c} \quad (4.11)$$

Thus the measurement of the beat frequency determines the range R.

A block diagram illustrating the principle of the FM-CW radar is shown in figure 4.10. A portion of the transmitter signal acts as the reference signal required to produce the beat frequency. It is introduced directly into the receiver via a cable or other direct connection.

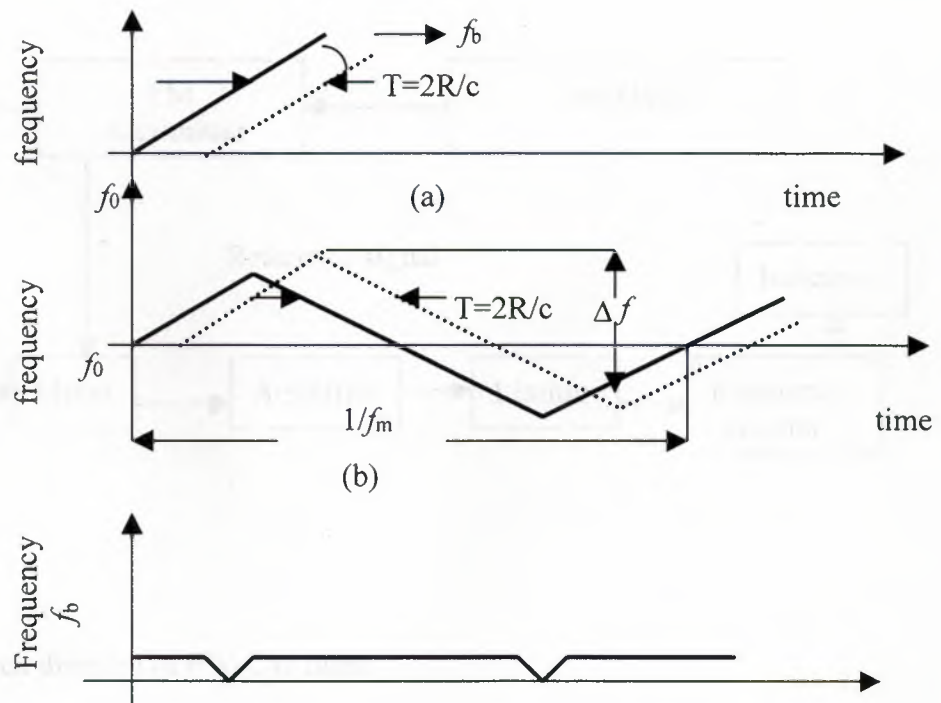


Figure 4.9 Frequency-time relationships in FM-CW radar. Solid curve represents transmitted signal; dashed curve represents echo. (a) Linear frequency modulation; (b) triangular frequency modulation; (c) beat note of (b).

Ideally, the isolation between transmitting and receiving antennas is made sufficiently large so as to reduce to a negligible level the transmitter leakage signal which arrives at the receiver via the coupling between antennas. The beat frequency is amplified and limited to remove any amplitude fluctuations. The frequency of the amplitude-limited beat note is measured with a cycle-counting frequency meter calibrated in distance. In the above, the target was assumed to be stationary. If this assumption is not applicable, a doppler frequency shift will be superimposed on the FM range beat note and an erroneous range measurement results. The doppler frequency shift causes the frequency-time plot of the echo signal to be shifted up or down figure 4.11a. On one portion of the frequency-modulation cycle, the beat frequency figure 4.11b is increased by the doppler shift, while on the other portion, it is decreased. If, for example, the target is approaching the radar, the produced during the increasing, or up, portion of the Transmitted antenna.

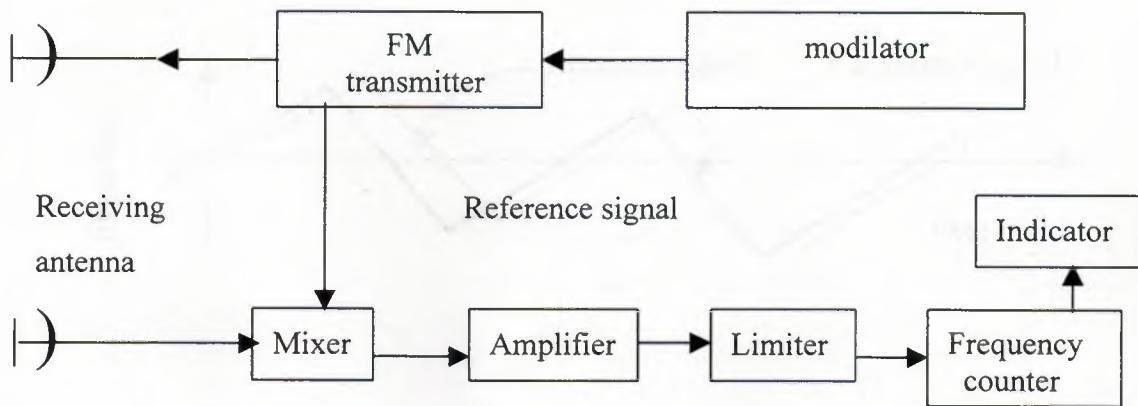


Figure 4.10 Block diagram of FM-CW radar

FM cycle will be the difference between the beat frequency due to the range f_r and the doppler frequency shift f_d equation 4.12a. Similarly, on the decreasing portion, the beat frequency $f_{(down)}$ is the sum of the two equation 4.12b.

$$f_b(\text{up}) = f_r - f_d \quad (4.12a)$$

$$f_b(\text{down}) = f_r + f_d \quad (4.12b)$$

The range frequency f_r may be extracted by measuring the average beat frequency; that is, $1/2[f_b(\text{up}) + f_b(\text{down})] = f_r$. If $f_b(\text{up})$ and $f_b(\text{down})$ are measured separately, for example, by switching a frequency counter every half modulation cycle, one-half the difference between the frequencies will yield the doppler frequency. This assumes $f_r > f_d$. If, on the other hand, $f_r < f_d$ such as might occur with a high-speed target at short range, the roles of the averaging and the difference-frequency measurements are reversed; the averaging meter will measure Doppler velocity, and the difference meter, range. If it is not known that the roles of the meters are

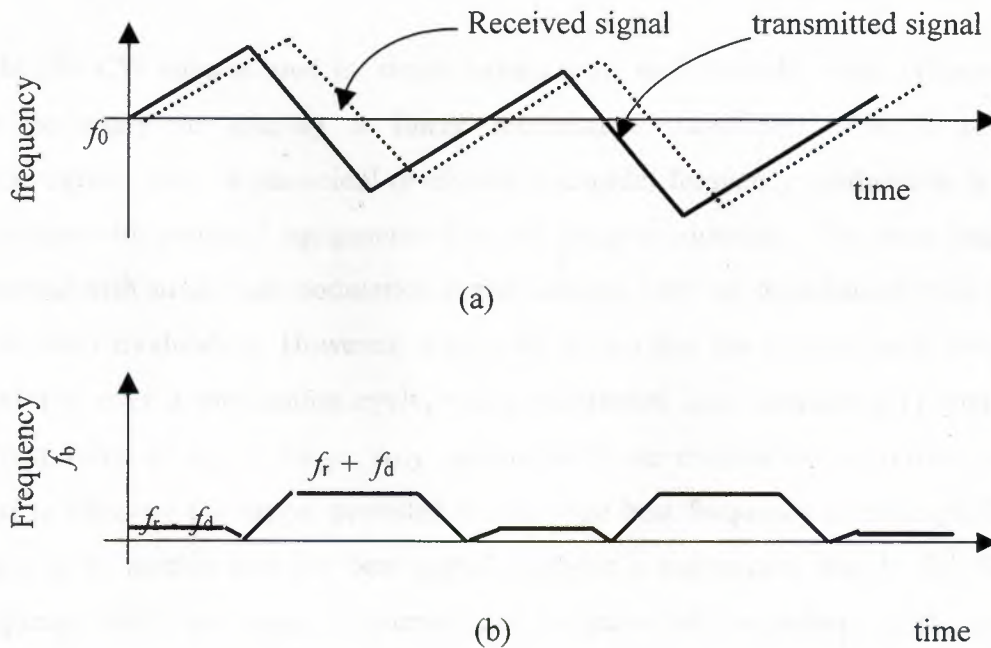


Figure 4.11 Frequency-time relationships in FM-CW radar when the received signal is shifted in frequency by the doppler effect (a) Transmitted (solid curve) and echo (dashed curve) frequencies; (b) beat frequency.

reversed because of a change in the inequality sign between f_r and f_d , an incorrect interpretation of the measurements may result.

When more than one target is present within the view of the radar, the mixer output will contain more than one difference frequency. If the system is linear, there will be a frequency component corresponding to each target. In principle, the range to each target may be determined by measuring the individual frequency components and applying equation 4.11 to each. To measure the individual frequencies, they must be separated from one another. This might be accomplished with a bank of narrowband filters. or alternatively, a single frequency corresponding to a single target may be singled out and continuously observed with a narrow band tunable filter. But if the motion of the targets were to produce a doppler frequency shift, or if the frequency-modulation waveform were nonlinear, or if the mixer were not operated in its linear region, the problem of resolving targets and measuring. the range of each becomes more complicated.

If the FM-CW radar is used for single targets only, such as in the radio altimeter, it is not necessary to employ a linear modulation waveform. This is certainly advantageous since a sinusoidal or almost sinusoidal frequency modulation is easier to obtain with practical equipments than are linear modulations. The beat frequency obtained with sinusoidal modulation is not constant over the modulation cycle as it is with linear modulation. However, it may be shown that the *average* beat frequency measured over a modulation cycle, when substituted into equation 4.11 yields the correct value of target range. Any reasonable-shape modulation waveform can be used to measure the range, provided the average beat frequency is measured. If the target is in motion and the beat signal contains a component due to the doppler frequency shift, the range frequency can be extracted, as before, if the average frequency is measured. To extract the doppler frequency, the modulation waveform must have equal upswing and downswing time intervals. The FM-CW radar principle was known and used at about the same time as pulse radar, although the early development of these two radar techniques seemed to be relatively independent of each other. FM-CW was applied to the measurement of the height of the ionosphere in the 1920 [24] and as an aircraft altimeter in the 1930 [25].

FW-CW altimeter. The FM-CW radar principle is used in the aircraft radio altimeter to measure height above the surface of the earth. The large backscatter cross section and the relatively short ranges required of altimeters permit low transmitter power and low antenna gain. Since the relative motion between the aircraft and ground is small, the effect of the doppler frequency shift may usually be neglected.

The band from 4.2 to 4.4 GHz is reserved for radio altimeters, although they have in the past operated at UHF. The transmitter power is relatively low and can be obtained from a CW magnetron, a backward-wave oscillator, or a reflex klystron, but these have been replaced by the solid state transmitter.

The altimeter can employ a simple homodyne receiver, but for better sensitivity and stability the superheterodyne is to be preferred whenever its more complex construction can be tolerated. A block diagram of the FM-CW radar with a sideband superheterodyne receiver is shown in Figure 4.11. A portion of the frequency-modulated transmitted signal is applied to a mixer along with the oscillator signal. The selection of the local-oscillator frequency is a bit different from that in the usual superheterodyne receiver. The local-oscillator frequency f_{IF} should be the same as the intermediate frequency used in the receiver, whereas in the conventional superheterodyne the LO frequency is of the same order of magnitude as the RF signal. The output of the mixer consists of the varying transmitter frequency $f_o(t)$ plus two sideband frequencies, one on either side of $f_o(t)$ and separated from $f_o(t)$ by the local-oscillator frequency f_{IF} . The filter selects the lower sideband $f_o(t) - f_{IF}$ and rejects the carrier and the upper sideband. The sideband that is passed by the filter is modulated in the same fashion as the transmitted signal. The sideband filter must have sufficient bandwidth to pass the modulation, but not the carrier or other sideband. The filtered sideband serves the function of the local oscillator.

When an echo signal is present, the output of the receiver mixer is an IF signal of frequency $f_{IF} + f_b$ where f_b is composed of the range frequency f_r and the Doppler velocity frequency f_d . The IF signal is amplified and applied to the balanced detector

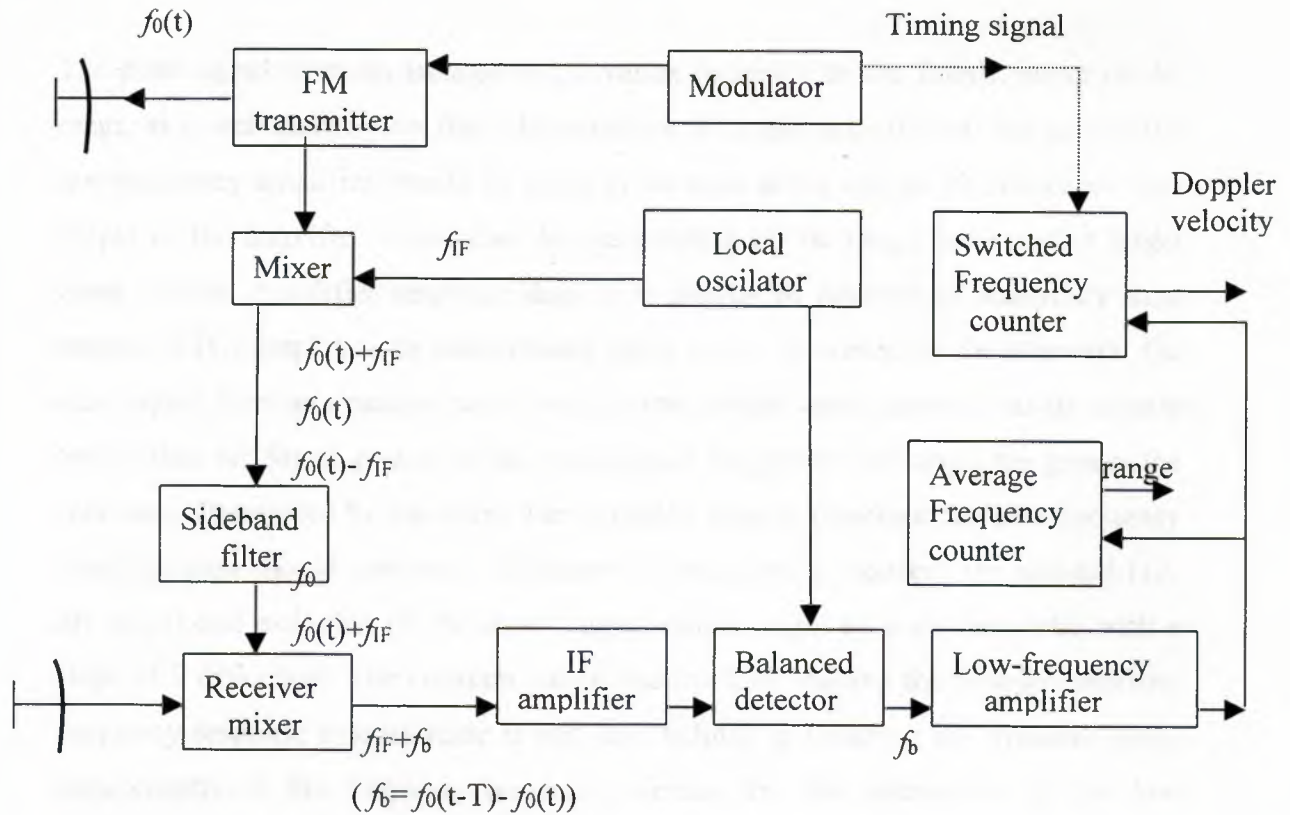


Figure 4.12 Block diagram of FM-CW radar using sideband perheterodyne receiver.

along with the local-oscillator signal f_{IF} . The output of the detector contains the beat frequency (range frequency and the doppler velocity frequency), which is amplified to a level where it can actuate the frequency-measuring circuits.

In figure 4.12, the output of the low-frequency amplifier is divided into two channels: one feeds an average-frequency counter to determine range, the other feeds a switched frequency counter to determine the doppler velocity (assuming $f_r > f_d$). Only the averaging frequency counter need be used in an altimeter application, since the rate of change of altitude is usually small.

A target at short range will generally result in a strong signal at low frequency, while one at long range will result in a weak signal at high frequency. Therefore the frequency characteristic of the low-frequency amplifier in the FM-CW radar may be shaped to provide attenuation at the low frequencies corresponding to short ranges and large echo signals. Less attenuation is applied to the higher frequencies, where the echo signals are weaker.

The echo signal from an isolated target varies inversely as the fourth power of the range, as is well known from the radar equation. With this as a criterion, the gain of the low-frequency amplifier should be made to increase at the rate of 12 dB/octave. The output of the amplifier would then be independent of the range, for constant target cross section. Amplifier response shaping is similar in function to sensitivity time control (STC) employed in conventional pulse radar. However, in the altimeter, the echo signal from an extended target such as the ground varies inversely as the square (rather than the fourth power) of the range, since the greater the range, the greater the echo area illuminated by the beam. For extended targets, therefore, the low-frequency amplifier gain should increase 6 dB/octave. A compromise between the isolated (12-dB slope) and extended (6-dB slope) target echoes might be a characteristic with a slope of 9 dB/octave. The constant output produced by shaping the doppler-amplifier frequency-response characteristic is not only helpful in lowering the dynamic range requirements of the frequency-measuring device, but the attenuation of the low frequencies effects a reduction of low-frequency interfering noise. Lowered gain at low altitudes also helps to reduce interference from unwanted reflections. The response at the upper end of the frequency characteristic is rapidly reduced for frequencies beyond that corresponding to maximum range. If there is a minimum target range, the response is also cut off at the low-frequency end, to further reduce the extraneous noise entering the receiver.

Another method of processing the range or height information from an altimeter so as to reduce the noise output from the receiver and improve the sensitivity uses a narrow-bandwidth low-frequency amplifier with a feedback loop to maintain the beat frequency constant. When a fixed-frequency excursion (or deviation) is used, as in the usual altimeter, the beat frequency can vary over a considerable range of values. The low-frequency-amplifier bandwidth must be sufficiently wide to encompass the expected range of beat frequencies. Since the bandwidth is broader than need be to pass the signal energy, the signal-to-noise ratio is reduced and the receiver sensitivity degraded. Instead of maintaining the frequency excursion Δf constant and obtaining a varying beat frequency, Δf can be varied to maintain the beat frequency constant. The beat-frequency amplifier need only be wide enough to pass the received signal energy.

thus reducing the amount of noise with which the signal must compete. The frequency excursion is maintained by a servomechanism to that value which permits the beat frequency to fall within the passband of the narrow filter. The value of the frequency excursion is then a measure of the altitude and may be substituted into equation 4.11

When used in the FM altimeter, the technique of servo -controlling the frequency excursion is usually applied at all altitudes above a predetermined minimum. Since the frequency excursion Δf is inversely proportional to range, the radar is better operated at very low altitudes in the more normal manner with a fixed M : and hence a varying beat frequency.

Measurement errors. The absolute accuracy of radar altimeters is usually of more importance at low altitudes than at high altitudes. Errors of a few meters might not be of significance when cruising at altitudes of 10 km, but are important if the altimeter is part of a blind landing system.

The theoretical accuracy with which distance can be measured depends upon the bandwidth of the transmitted signal and the ratio of signal energy to noise energy. In addition, measurement accuracy might be limited by such practical restrictions as the accuracy of the frequency-measuring device, the residual path-length error caused by the circuits and transmission lines, errors caused by multiple reflections and transmitter leakage, and the frequency error due to the turn-around of the frequency modulation.

A common form of frequency-measuring device is the cycle counter, which measures the number of cycles or half cycles of the beat during the modulation period. The total cycle count is a discrete number since the counter is unable to measure fractions of a cycle. The discreteness of the frequency measurement gives rise to an error called the *fixed* error, or *step* error. It has also been called the *quantization* error, a more descriptive name. The average number of cycles N of the beat frequency f_b in one period of the modulation cycle f_m is $\bar{f}_b / f_m$, where the bar over f_b denotes time average. equation 4.11 may be written as

4. Types Of Radar System

$$R = \frac{cN}{4\Delta f} \quad (4.13)$$

where R = range (altitude), m

c = velocity of propagation, m/s

Δf = frequency excursion, Hz

Since the output of the frequency counter N is an integer, the range will be an integral multiple of $c/(4\Delta f)$ and will give rise to a quantization error equal to

$$\delta R = \frac{c}{4\Delta f} \quad (4.14 a)$$

or
$$\delta R(m) = \frac{75}{\Delta f(MHz)} \quad (4.14 b)$$

Note that the fixed error is independent of the range and carrier frequency and is a function of the frequency excursion only. Large frequency excursions are necessary if the fixed error is to be small.

Since the fixed error is due to the discrete nature of the frequency counter, its effects can be reduced by wobbling the modulation frequency or the phase of the transmitter output. Wobbling the transmitter phase results in a wobbling of the phase or the beat signal so that an average reading of the cycle counter somewhere between N and $N + 1$ will be obtained on a normal meter movement. In one altimeter the modulation frequency was varied at a 10-Hz rate, causing the phase shift of the beat signal to vary cyclically with time. The indicating system was designed so that it did not respond to the 100Hz modulation directly, but it caused the fixed error to be averaged. Normal fluctuations in aircraft altitude due to uneven terrain, waves on the water, or turbulent air can also average out the fixed error provided the time constant or the indicating

device is large compared with the time between fluctuations. Over smooth terrain, such as an airport runway, the fixed error might not be averaged out. Note that even if the fixed error were not present, the accuracy with which the height can be measured will depend on the signal-to-noise ratio that can be comparable to the fixed error as given by equation 4.13.

Other errors might be introduced in the CW radar if there are uncontrolled variations in the transmitter frequency, modulation frequency, or frequency excursion. Target motion can cause an error in range equal to $v_r T_o$, where v_r is the relative velocity and T_o is the observation time. At short ranges the residual path error can also result in a significant error unless compensated for. The residual path error is the error caused by delays in the circuitry and transmission lines. Multipath signals also produce error. Figure 4.13 shows some of the unwanted signals that might occur in the FM altimeter[26-27]. The wanted signal is shown by the solid line while the unwanted signals are shown by the broken arrows. The unwanted signals include:

1. The reflection of the transmitted signals at the antenna caused by impedance mismatch.
2. The standing-wave pattern on the cable feeding the reference signal to the receiver, due to poor mixer match.
3. The leakage signal entering the receiver via coupling between transmitter and receiver antennas. This can limit the ultimate receiver sensitivity, especially at high altitudes.
4. The interference due to power being reflected back to the transmitter, causing a change in the impedance seen by the transmitter. This is usually important only at low altitudes. It can be reduced by an attenuator introduced in the transmission line at low altitude or by a directional coupler or an isolator.
5. The double-bounce signal.

Reflections from the landing gear can also cause errors.

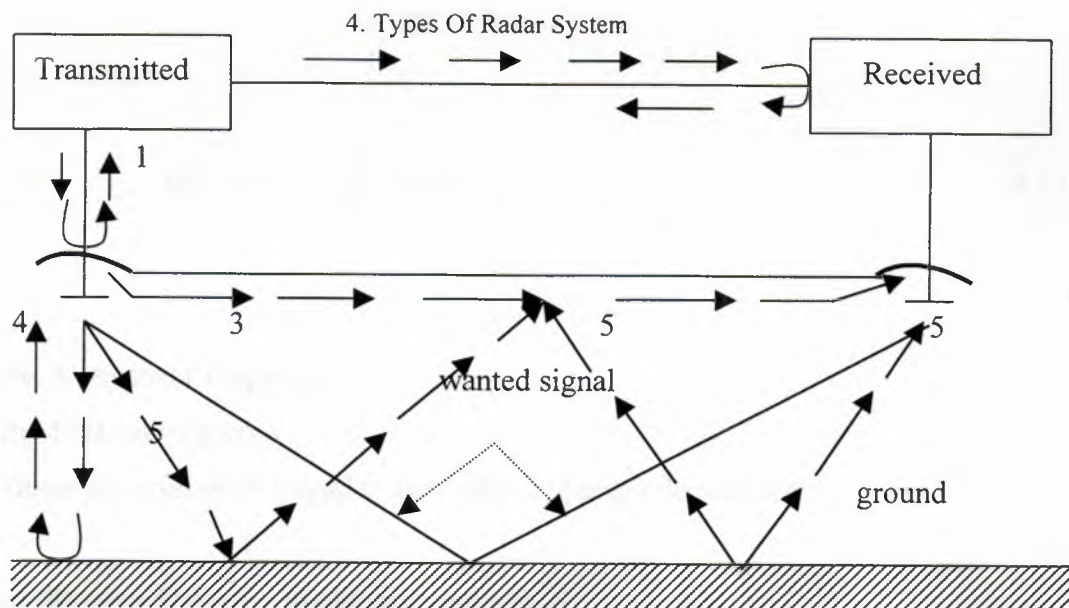


Figure 4.13 Unwanted signals in FM altimeter. [from Capelli, IRE Trans.]

Transmitter leakage. The sensitivity of FM-CW radar is limited by the noise accompanying the transmitter signal which leaks into the receiver. Although advances have been made in reducing the AM and FM noise generated by high-power CW transmitters, the noise is usually of sufficient magnitude compared with the echo signal to require some means of minimizing the leakage that finds its way into the receiver. The techniques described previously for reducing leakage in the CW radar apply equally well to the FM-CW radar. Separate antennas and direct cancellation of the leakage signal are two techniques which give considerable isolation.

Sinusoidal modulation. The ability of the FM-CW radar to measure range provides an additional basis for obtaining isolation. Echoes from short-range targets-including the leakage signal-may be attenuated relative to the desired target echo from longer ranges by properly processing the difference-frequency signal obtained by heterodyning the transmitted and received signals.

If the CW carrier is frequency-modulated by a sine wave, the difference frequency obtained by heterodyning the returned signal with a portion of the transmitter signal may be expanded in a trigonometric series whose terms are the harmonics or the modulating frequency f_m . Assume the form of the transmitted signal to be

$$\sin\left(2\pi f_o t + \frac{\Delta f}{2f_m} \sin 2\pi f_m t\right) \quad (4.15)$$

where, f_o = carrier frequency

f_m = modulation frequency

Δf = frequency excursion (equal to twice the frequency derivation)

The difference frequency signal may be written

$$\begin{aligned} f_D = & J_0(D) \cos(2\pi f_d t - \phi_0) + 2J_1(D) \sin(2\pi f_d t - \phi_0) \cos(2\pi f_m t - \phi_m) \\ & - 2J_2(D) \cos(2\pi f_d t - \phi_0) \cos 2(2\pi f_m t - \phi_m) \\ & - 2J_3(D) \sin(2\pi f_d t - \phi_0) \cos 3(2\pi f_m t - \phi_m) \\ & + 2J_4(D) \sin(2\pi f_d t - \phi_0) \cos 4(2\pi f_m t - \phi_m) + 2J_5(D) \dots \dots \dots \end{aligned} \quad (4.16)$$

where J_0, J_1, J_2 etc = Bessel functions of first kind and order 0, 1, 2, etc.. respectively

$$D = (\Delta f / f_m) \sin 2\pi f_m R_0 / c$$

R_0 = distance to target at time $t = 0$ (distance that would have been measured if target were stationary)

c = velocity of propagation

$$f_d = 2v_r f_o / c = \text{doppler frequency shift}$$

v_r = relative velocity of target with respect to radar

ϕ_0 = phase shift approximately equal to angular distance $4\pi f_o R_0 / c$

ϕ_m = phase shift approximately equal to $2\pi f_m R_0 / c$

The difference-frequency signal of equation 4.16 consists of a doppler-frequency component of amplitude $J_0(D)$ and a series of cosine waves of frequency $f_m, 2f_m, 3f_m$ etc. Each of these harmonics of f_m is modulated by a doppler-frequency component with amplitude proportional to $J_n(D)$. The product of the doppler-frequency factor times the n th harmonic factor is equivalent to a suppressed-carrier double-sideband modulation – see Figure 4.14.

In principle, any of the J_n components of the difference-frequency signal can be extracted in the FM-CW radar. Consider first the d-c term $J_0(D) \cos(2\pi f_d t - \phi_0)$. This is a cosine wave at the doppler frequency with an amplitude proportional to $J_0(D)$. Figure 4.14 shows a plot of several of the Bessel functions. The argument D of the Bessel function is proportional to range. The J_0 amplitude applies maximum response to signals at zero range in a radar that extracts the d-c doppler-frequency component. This is the range at which the leakage signal and its noise components (including microphony and vibration) are found. At greater ranges, where the target is expected, the effect of the J_0 Bessel function is to reduce the echo-signal amplitude in comparison with the echo at zero range (in addition to the normal range attenuation). Therefore, if the J_0 term were used, it would enhance the leakage signal and reduce the target signal, a condition opposite to that desired.

An examination of the Bessel functions figure 4.15 shows that if one of the modulation frequency harmonics is extracted (such as the first, second, or third harmonic), the amplitude of the leakage signal at zero range may theoretically be made equal to zero. The higher the number of the harmonic, the higher will be the order of the Bessel function and the less will be the amount of microphonism-leakage feedthrough. This results from the property that $J_n(x)$ behaves as x^n for small x . Although higher-order Bessel functions may reduce the zero-range response, they may also reduce the response at the desired target range if the target happens to fall at or near a range corresponding to a zero of the Bessel function. When only a single target is involved, the frequency excursion δf can be adjusted to obtain that value of D which places the maximum of the Bessel function at the target range.

The technique of using higher-order Bessel functions has been applied to the type of doppler-navigation radar discussed in the next section. A block diagram of a CW radar using the third harmonic (J_3 term) is shown in figure 4.16. The transmitter is sinusoidally frequency modulated at a frequency f_m to generate the waveform given by equation 4.15. The doppler-shifted echo is heterodyned with the transmitted signal to produce the beat-frequency signal of ; equation 4.16. One of the harmonics of f_m is selected (in this case the third) by a filter centered at the harmonic. The filter bandwidth is wide enough to pass both doppler-frequency sidebands. The filter output is mixed

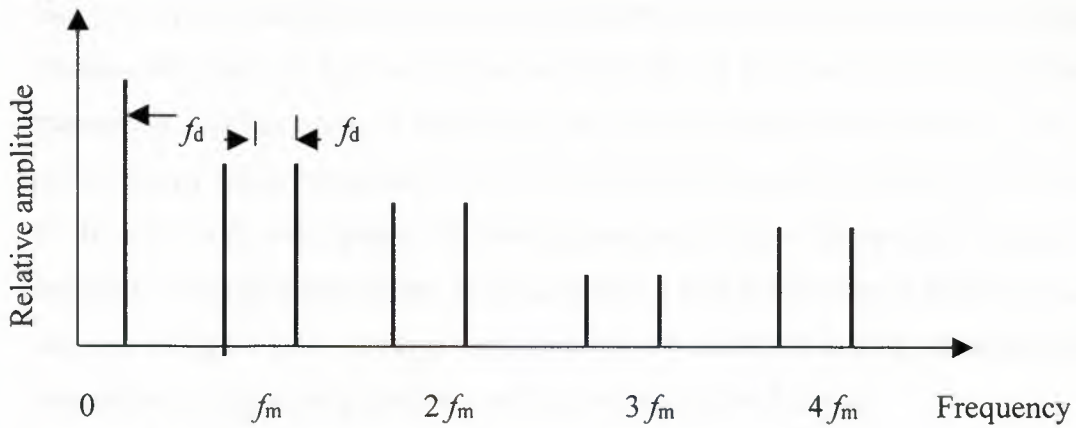


Figure 4.14 Spectrum of the difference frequency signal obtained from an FM-CW radar sinusoidally modulated at a frequency f_m when the target motion produces a doppler frequency shift f_d [After Saunders, IRE Tran].

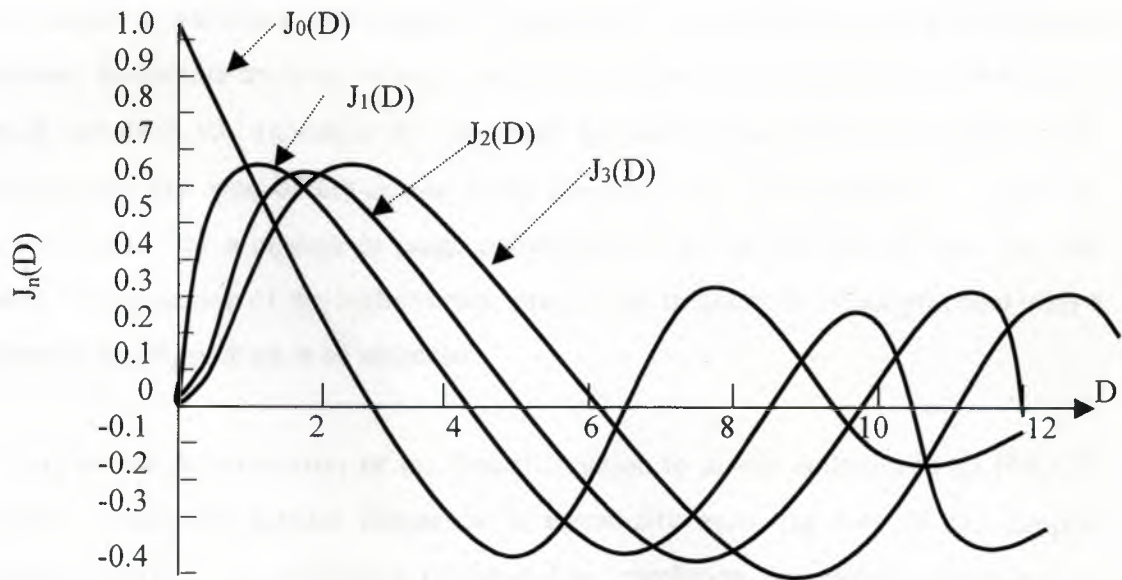


Figure 4.15 Plot of Bessel functions of order, 0, 1, 2, and 3; $D = (\Delta f / f_m) \sin 2\pi f_m R_0 / c$ of f_m . The doppler frequency is extracted by the low-pass filter.

with the (third) harmonic

Since the total energy contained in the beat-frequency signal is distributed among all the harmonics, extracting but one component wastes signal energy contained in the other harmonics and results in a loss of signal as compared with an ideal CW radar.

However, the signal-to-noise ratio is generally superior in the FM radar designed to operate with the n th harmonic as compared with a practical CW radar because the transmitter leakage noise is suppressed by the n th order Bessel function. The loss in signal energy when operating with the J_3 Bessel component is reported to be from 4 to 12 dB. Although two separate transmitting and receiving antennas may be used, it is not necessary in many applications. A single antenna with a circulator is shown in the block diagram of figure 4.16. Leakage introduced by the circulator and by reflections from the antenna are at close range and thus are attenuated by the J_3 factor.

A plot of $J_3(D)$ as a function of distance is shown in Figure 4.17. The curve is mirrored because of the periodicity of D . The nulls in the curve suggest that echoes from certain ranges can be suppressed if the modulation parameters are properly selected.

If the target is stationary (zero doppler frequency), the amplitudes of the modulation frequency harmonics are proportional to either $J_n(D) \sin \phi_0$ or $J_n(D) \cos \phi_0$, where $\phi_0 = 4\pi f_0 R_0 / c = 4\pi R_0 / \lambda$. Therefore the amplitude depends on the range to the target in RF wavelengths. The sine or the cosine terms can take any value between +1 and -1, including zero, for a change in range corresponding to one RF wavelength. For this reason, the extraction of the higher-order modulation frequencies is not practical with a stationary target, such as in an altimeter.

In order to use the properties of the Bessel function to obtain isolation in an FM-CW altimeter, when the doppler frequency is essentially zero, the role of the doppler frequency shift may be artificially introduced by translating the reference frequency to some different value. This might be accomplished with a single-sideband generator (frequency translator) inserted between the directional coupler and the RF mixer of figure 4.16. The frequency translation in the reference signal path is equivalent to a doppler shift in the antenna path. The frequency excursion of the modulation waveform can be adjusted by a servomechanism to maintain the maximum of the Bessel function at the aircraft's altitude. The frequency translator is not needed in an airborne doppler navigator since the antenna beam is directed at a depression angle other than 90° and a doppler-shifted echo is produced by the motion of the aircraft.

Matched filter detection. The operation of the FM-CW radar described earlier in this section does not employ optimum signal processing, such as described previously. The receiver is not designed as a *matched filter* for the particular transmitted waveform. Therefore, the sensitivity of the FM-CW receiver described here is degraded and the ability to operate with multiple targets is usually poor. For a radio altimeter whose target is the earth, these limitations usually present no problem. When used for the long-range detection of air targets, for example, the non optimum detection of the classical FM-CW radar will seriously binder its performance when compared to a properly designed pulse radar .It is possible, however, to utilize matched, filter processing in the

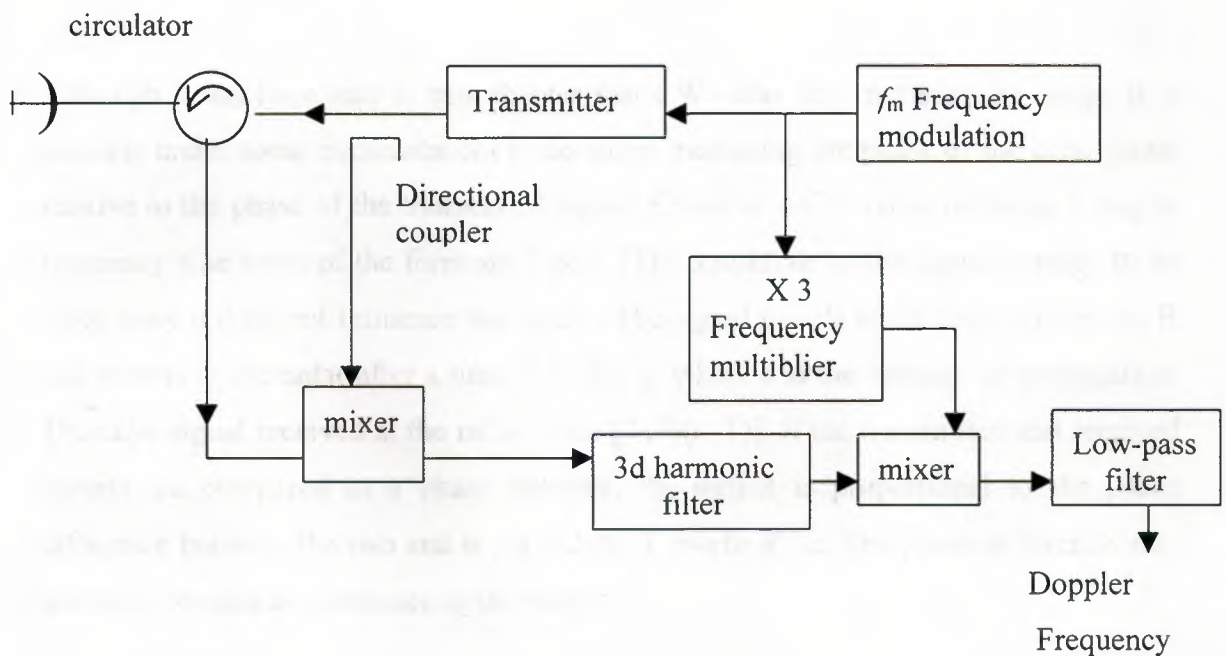


Figure 4.16 Sinusoidally modulated FM-CW radar extracting the third harmonic (J_3 Bessel component).

FM-CW radar in a manner similar to that employed with FM (chirp) pulse compression radar as described previously, and thus overcome the lower sensitivity and multiple-target problems. As the duty cycle of an FM pulse compression waveform increases it becomes more like the FM-CW waveform of unity duty cycle. Hence, the processing techniques can be similar. Pseudorandom phase-coded waveforms [28] and random noise waveforms [29-30] may also be applied to CW transmission. When the

modulation period is long, it may be desirable to utilize correlation detection instead of matched filter detection.

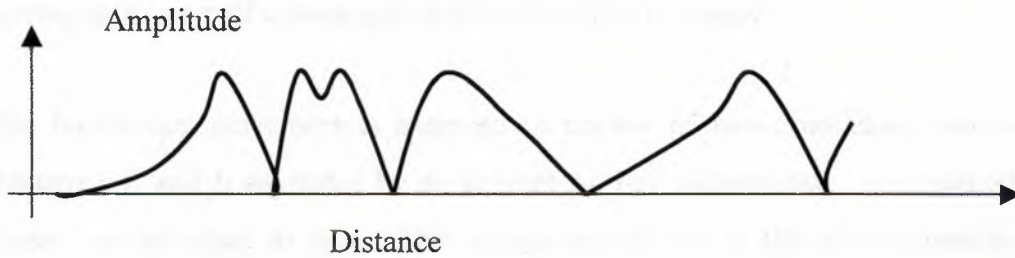


Figure 4.17 Plot of $J_3(D)$ as a function of distance.[From Saunders, IRE Trans.]

4.4 Multiple-frequency CW Radar

Although it has been said in this chapter that CW radar does not measure range, it is possible under some circumstances to do so by measuring the phase of the echo signal relative to the phase of the transmitted signal. Consider a CW radar radiating a single-frequency sine wave of the form $\sin 2\pi f_0 t$. (The amplitude of the signal is taken to be unity since it does not influence the result.) The signal travels to the target at a range R and returns to the radar after a time $T = 2R / c$, where c is the velocity of propagation. The echo signal received at the radar is $\sin [2\pi f_0(t - T)]$. If the transmitted and received signals are compared in a phase detector, the output is proportional to the phase difference between the two and is $\Delta\phi = 2\pi f_0 T = 4\pi f_0 R / c$. The phase difference may therefore be used as a measure of the range, or

$$R = \frac{c\Delta\phi}{4\pi f_0} = \frac{\lambda}{4\pi} \Delta\phi \quad (4.17)$$

However, the measurement of the phase difference $\Delta\phi$ is unambiguous only if $\Delta\phi$ does not exceed 2π radians. Substituting $\Delta\phi = 2\pi$ into Equation (4.11) gives the maximum unambiguous range as $\lambda/2$. At radar frequencies this unambiguous range is much too small to be of practical interest.

The region of unambiguous range may be extended considerably by utilizing two separate CW signals differing slightly in frequency. The unambiguous range in this case corresponds to a half wavelength at the difference frequency.

The transmitted waveform is assumed to consist of two continuous sine waves of frequency f_1 and f_2 separated by an amount Δf . For convenience, the amplitudes of all signals are set equal to unity. The voltage waveforms or the two components of the transmitted signal v_{1T} and v_{2T} may be written as

$$v_{1T} = \sin(2\pi f_1 t + \phi_1) \quad (4.18 a)$$

$$v_{2T} = \sin(2\pi f_2 t + \phi_2) \quad (4.18 b)$$

where ϕ_1 and ϕ_2 are arbitrary (constant) phase angles. The echo signal is shifted in frequency by the doppler effect. The form of the doppler-shifted signals at each of the two frequencies f_1 and f_2 may be written

$$v_{1R} = \sin \left[2\pi (f_1 \pm f_{d1})t - \frac{4\pi f_1 R_0}{c} + \phi_1 \right] \quad (4.19 a)$$

$$v_{2R} = \sin \left[2\pi (f_2 \pm f_{d2})t - \frac{4\pi f_2 R_0}{c} + \phi_2 \right] \quad (4.19 b)$$

where R_0 = range to target at a particular time $t = 0$ (range that would be measured if target were not moving)

f_{d1} = doppler frequency shift associated with frequency f_1

f_{d2} = doppler frequency shift associated with frequency f_2

Since the two RF frequencies f_1 and f_2 are approximately the same (that is $f_2 = f_1 + \Delta f$, where $\Delta f \ll f_1$) the doppler frequency shifts f_{d1} and f_{d2} are approximately equal to one another. Therefore we may write $f_{d1} = f_{d2} = f_d$.

The receiver separates the two components of the echo signal and heterodynes each received signal component with the corresponding transmitted waveform and extracts the two doppler-frequency components given below:

$$v_{1D} = \sin\left(\pm 2\pi f_d t - \frac{4\pi f_1 R_0}{c}\right) \quad (4.20a)$$

$$v_{2D} = \sin\left(\pm 2\pi f_d t - \frac{4\pi f_2 R_0}{c}\right) \quad (4.20b)$$

The phase difference between these two components is

$$\Delta\phi = \frac{4\pi(f_2 - f_1) R_0}{c} = \frac{4\pi\Delta f R_0}{c} \quad (4.21)$$

Hence

$$R_0 = \frac{c\Delta\phi}{4\pi\Delta f} \quad (4.22)$$

which is the same as that of Equation (4.17), with Δf substituted in place of f_0 .

The two-frequency CW technique for measuring range was described as using the doppler frequency shift. When the doppler frequency is zero, as with a stationary target, it is also possible, in principle, to extract the phase difference. If the target carries a beacon or some other form of echo-signal augmentor, the doppler frequency shift may be simulated by translating the echo frequency, as with a single-sideband modulator.

The two frequencies of the two-frequency radar were described as being transmitted simultaneously. They may also be transmitted sequentially in some applications by rapidly switching a single RF source.

A large difference in frequency between the two transmitted signals improves the accuracy of the range measurement since large Δf means a proportionately large change in $\Delta\phi$ for a given range. However, there is a limit to the value of Δf , since $\Delta\phi$ cannot be greater than 2π radians if the range is to remain unambiguous. The maximum unambiguous range R_{unamb} is

$$R_{unamb} = c/2\Delta f \quad (4.23)$$

Therefore Δf must be less than $c/2R_{unamb}$. Note that when Δf is replaced by the pulse repetition rate, Equation (4.39) gives the maximum unambiguous range of a pulse radar.

A qualitative explanation of the operation of the two-frequency radar may be had by considering both carrier frequencies to be in phase at zero range. As they progress outward, from the radar, the relative phase between the two increases because of their difference in frequency. This phase difference may be used as a measure of the elapsed time. When the two signals slip in phase by 1 cycle, the measurement of phase, and hence range, becomes ambiguous.

The two-frequency CW radar is essentially a single-target radar since only one phase difference can be measured at a time. If more than one target is present, the echo signal becomes complicated and the meaning of the phase measurement is doubtful. The theoretical accuracy with which range can be measured with the two-frequency CW radar can be found from the methods described. It can be shown that the theoretical rms range error is

$$\delta R = \frac{c}{4\pi\Delta f (2E / N_0)^{1/2}} \quad (4.24)$$

Where E = energy contained in received signal and N_0 = noise power per hertz of bandwidth. If this is compared with the rms range error theoretically possible with the linear FM pulse-compression waveform whose spectrum occupies the same bandwidth Δf , the error obtained with the two-frequency CW waveform is less by the factor 0.29.

Equation (4.24) indicates that the greater the separation Δf between the two frequencies, the less will be the rms error. However, the frequency difference must not be too large if unambiguous measurements are to be made. The selection of Δf represents a compromise between the requirements of accuracy and ambiguity. Both accurate and unambiguous range measurements can be made by transmitting three or more frequencies instead of just two. For example, if the three frequencies f_1 , f_2 , and f_3 are such that $f_3 - f_1 = k(f_2 - f_1)$ where k is a factor of the order of 10 or 20, the pair of frequencies f_3 , f_1 gives an ambiguous but accurate range measurement while the pair of frequencies f_2 , f_1 are chosen close to resolve the ambiguities in the f_3 , f_1 measurement. Likewise, if further accuracy is required a fourth frequency can be transmitted and its ambiguities resolved by the less accurate but unambiguous measurement obtained from

the three frequencies f_1, f_2, f_3 . As more frequencies are added, the spectrum and target resolution approach that obtained with a pulse or an FM-CW waveform

The measurement of range by measuring the phase difference between separated frequencies is analogous to the measurement of angle by measuring the phase difference between widely spaced antennas, as in an interferometer antenna. The interferometer antenna gives an accurate but ambiguous measurement of angle. The ambiguities may be resolved by additional antennas spaced closer together. The spacing between the individual antennas in the interferometer system corresponds to the separation between frequencies in the multiple-frequency distance-measuring technique. The minitrack system is an example of an interferometer in which angular ambiguities are resolved in a manner similar to that described.

The multiple-frequency CW radar technique has been applied to the accurate measurement of distance in surveying and in missile guidance. The *Tellurometer* is the name given to a portable electronic surveying instrument which is based on this principle. It consists of a master unit at one end of the line and a remote unit at the other end. The master unit transmits a carrier frequency of 3,000 MHz, with four single-sideband modulated frequencies separated from the carrier by 10.000, 9.990, 9.900, and 9.000 MHz. The 10-MHz difference frequency provides the basic accuracy measurement, while the difference frequencies of 1 MHz, 100 kHz, and 10 kHz permit the resolution of ambiguities.

The remote unit at the other end of the line receives the signals from the master unit and amplifies and retransmits them. The phases of the returned signals at the master unit are compared with the phases of the outgoing signals. Since the master and the remote units are stationary, there is no doppler frequency shift. The function of the doppler frequency is provided by modulating the retransmitted signals at the remote unit in such a manner that a 1-kHz beat frequency is obtained from the heterodyning process at the receiver of the master unit. The phase of the 1-kHz signals contains the same information as the phase of the multiple frequencies.

4. Types Of Radar System

The MRB 201 Tellurometer is capable of measuring distances from 200 to 250 km, assuming reasonable line of sight conditions, with an accuracy of $\pm 0.5 \text{ m} \pm 3 \times 10^{-6} d$, where d is the distance being measured. The transmitter power is 200 mW and the antenna is a small paraboloid with crossed feeds to make the polarizations of the transmitted and received signals orthogonal. The MRA 5 operates within the 10 to 10.5 GHz band, from 100 m to 50 km range with an accuracy or $\pm 0.5 \text{ m} \pm 3 \times 10^{-5} d$, assuming a correction is made for meteorological conditions.

In addition to its use in surveying, the multiple CW frequency method or measuring range has been applied in range-instrumentation radar for the measurement of the distance to a transponder-equipped missile; the distance to satellites; in satellite navigation systems based on range measurement; and for detecting the presence or an obstacle in the path or a moving automobile by measuring the distance, the doppler velocity, and the sign or the doppler (whether the target is approaching or receding).

5. DESIGN OF THE COLLISION AVOIDANCE SYSTEM

5.1 BACKGROUND

One of the first compact traffic radar systems was built in 1947 by the Connecticut Firm (Automatic Signal Co.). The first traffic radar system used 2.455 GHz in the S-band, antenna band widths varied from 15 to 20 degrees, operated only in stationary position and measured speed of the target to an accuracy of ± 3 Km/hr. The maximum detection range was an unimpressive 45 to 150 meters; vacuum-tube receivers do not have the sensitivity of solid-state receivers. A radar with a 45 m detection range would have less than 1.5 second to measure a target traveling 109 Km/hr. Hence S-band radars are obsolete [5].

In September 1994 Volvo presented the first prototype car radar at a European conference in Paris. New generation of car radars were developed in late 1990's, using 77 GHz. With this new technology, the size was only 20 cm width, 10 cm height, and 11.8 cm depth and having a weight of 1.5 Kg. A new generation of prototype is now available in quantity, single-unit, with all signal processing performed inside the antenna unit box.

Another scheme adapted by some car manufacturers is the Adaptive Cruise Control Systems (ACCS). In this system, the vehicle is equipped with a millimeter wave radar system and the speed of the vehicle in front is measured. The vehicle then automatically runs at a speed, which provides a safe distance to the vehicle in front. In such systems, the throttle and brakes of the vehicle are controlled continuously to the vehicle ahead at all times. The system can also warn the driver and alert the airbags just before a potential collision. Some expensive and luxury vehicles now provide the options of ACCS based control systems. The ACCS systems uses a 77 GHz Doppler radar, linked into the electronic control and braking system to maintain a safe distance between a car with the system and the vehicle in front of it. The cost of such systems are around \$1500.

The Celsius Tech Company introduced a 77 GHz FMCW car radar for general purpose applications. This system is based on CW radar and it has a few advantages. First of all it does not use a lot of bandwidth. It is a simple design and therefore is cheap. In order to overcome the problem of ranging and improving the performance, it is possible to add a distinct time mark to the CW. If the time mark is a small pulse, then the transmitted bandwidth would increase dramatically. Hence, it would not be a CW any longer, because it would have more of the pulse radar properties.

SAAB car manufacturers use a car radar system known as the SIRS77 (Small Intelligent Radar Sensor). This system is developed by the Swedish military supplier Celsius Tech. Company. SIRS77 is based on FMCW radar with 77 GHz operation. The radar is used on some SAAB cars in the ACCS systems. It is also used in underground rail tracking networks.

5.2 THE DESIGN

This thesis has investigated the theory of radars and derived equations for the possible avoidance of collision by informing the driver of a potential collision situation. The thesis is theoretical and it will be necessary to design hardware in order to verify the theory and also see the effectiveness of the system. This is outside the scope of this study, but some design guidelines are give in this section.

There are two approaches to the implementation of RACAS in practise. The first approach is to design the complete system from the first principles, and the second approach, which is preferable, is to purchase the major system components ready built and combine them into a working system. Both approaches are described below.

5.2.1 Designing the Radar System From First Principles

This approach is not easy since the design of a radar system requires extensive knowledge of electronics and communications. Such a design would also be expensive. What is required is to design a complete small radar system and install it in a car for testing. Figure 6.1 shows the block diagram of such a system proposed by the author.

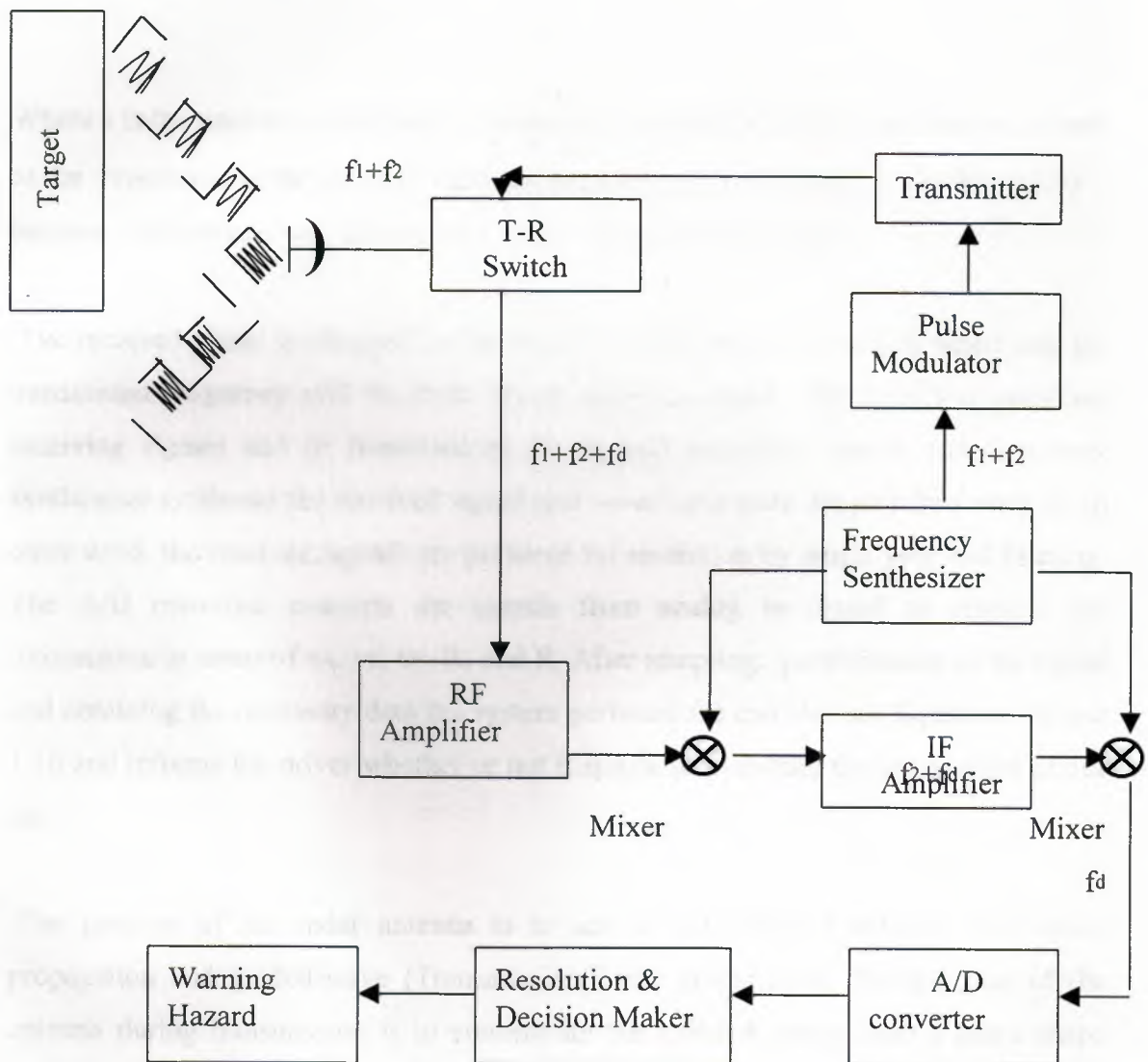


Figure 5.1 Block diagram of the proposed radar system

As we can see from the figure 5.1, Principal Radio Frequency (RF) of radar signal is set by the frequency synthesizer. The continuous signal is pulsed "on" and "off" by the modulator and is sent to the T-R switch. The T-R switch gives the opportunity to

achieve the train wave as well as the duration (τ) of the trip of the wave to cover the way of going to the target and coming back to the receiver of the radar. The transmitter sends out a radio signal by the aid of antenna, which will scatter off by the target and a small amount of the energy will come back to the receiver of the radar. The range (The distance from the target to the radar) can be calculated as:

$$R = c\tau/2$$

Where τ is the time taken for the transmitted signal to travel to the target and come back to the receiver, c is the speed of light (propagation speed of wave). $c\tau$ is divided by 2 because τ covers two way distance. i.e. going out and coming back.

The received signal is changed by the Doppler effect from the moving target and the transmitted frequency will be $f_1 + f_2 + f_d$ as received signals. RF amplifier amplifies receiving signals and IF (Intermediate Frequency) amplifier, mixers and frequency synthesizer synthesizes the received signal and we achieve pure doppler frequency f_d . In other ward, the received signals are prepared for resolution by amplifying and filtering. The A/D converter converts the signals from analog to digital to resolute the information in terms of v_A , v_B , v_C , R_1 and R . After sampling, quantification of the signal and obtaining the necessary data the system performs the calculations Equation 1.8 and 1.10 and informs the driver whether or not it is safe to over-take the car in front of our car.

The purpose of the radar antenna is to act as a transducer between free space propagation and guided-wave (Transmission Line) propagation. The function of the antenna during transmission is to concentrate the radiated energy into a beam shape which points in the desired direction in space. For obtaining this shape a special array of antennas can be used. In principle, a single antenna may be employed since the necessary isolation is done between the transmitted and the received signal. On reception, the antenna collects the energy contained in the echo signals and delivers it to the receiver.

As far as the size of the antenna is concerned, it should be very small in size because it is more convenient to mount a small antenna on top of a vehicle. The length of the required antenna can be calculated as:

$$\text{The length of antenna} = \lambda = c/f = 3 \times 10^8 / 77 \times 10^9 = 0.0038 \text{m} = 0.4 \text{ cm}$$

5.2.2 Purchasing a Ready-Built Radar

This is the second and the preferred option. Several manufacturers supply 77GHz radar systems, mainly designed for car applications. As discussed in section 5.1, these radars are used in ACCS based car cruise control applications, and not in collision detection. Such systems are small in size and weight and are designed mainly for car based radar applications.

5.2.3 Processing and Collision Warning

Whether the car system is designed from the first principles or a ready-built one is obtained it will be necessary to design a processing and a warning system around it. What will be required here is a microprocessor based interface to the radar system. As shown in the block diagram in Figure 5.2, the microprocessor will receive the range and the speed data from the radar system. It will then use the collision equations derived in Chapter 1 to determine whether or not there is a risk of collision. The microprocessor should then provide an output warning if there is a risk of collision. There is the choice of using either a visual or an audio warning system. It is recommended by the author to implement an audio warning system since a visual system may distract the attention of the driver and cause accidents. The audio warning could either be a simple buzzer, or a more sophisticated voice system can be used to inform the driver that there is a risk of collision if an overtake is attempted.

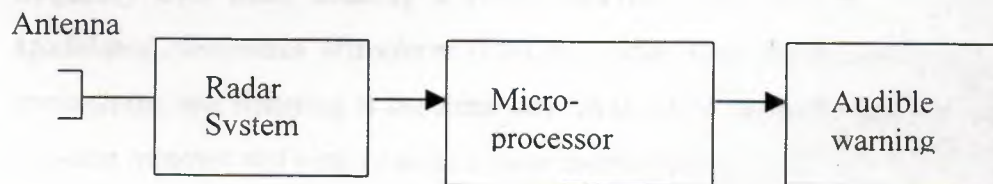


Figure 5.2 Block diagram of the proposed system

CONCLUSIONS

This project has discussed a novel approach to the safety of driving. It showed that a simple radar based system and a processor can be used on-board a vehicle to warn the driver of any collision risks before over-taking the vehicle in the front.

Chapter 1 described the basic principles of collision during an over-take and various formulae have been derived. A simulator program has also been developed in this chapter in order to simulate and check the validity of the formulae. The simulation program was PC based was developed using the Visual Basic programming language. The simulator results agree with the theory and it was possible to observe various collision scenarios using the simulator.

Chapter 2 was an introduction to the radar systems and various formulae have been derived which describes the basic principles of radar systems.

Chapter 3 was about the antenna systems used in radars. The basic principles and the theory of different antenna systems have been discussed in this section.

Chapter 4 the various types of radar have been discussed. It was determined that the most convenient type of radar for this project is *Frequency Modulated Continuous Waveform* (FMCW) radar. The plain Continuous Wave (CW) radar does have a few advantages. It does not use a lot of bandwidth. It is simple designed and therefore the cost is very low. In order to overcome the problem of ranging and improving the performance, it is possible to add a distinct time mark to the conventional CW radar. If this time mark were a small pulse, then the transmitted bandwidth would increase dramatically. Hence, it would not be a CW any longer, because it would have more of the pulse radar properties. The solution to this problem is to modulate the transmitted frequency over time, creating a sweep function. This method is called *Frequency Modulated continuous Waveform* (FMCW) radar. One should note that the radar is transmitting and listening at the same time as in a CW radar system, and not switching between transmit and receive as in a pulse radar system.

By changing the transmission frequency over time it is possible to measure the propagation time of the wave when it returns to the receiver. Knowing the propagation time it is simple to calculate the distance. It is also possible to exact the Doppler velocity of the target. The sweep function can have almost any form. However, it is quite common to use linear sweep function, due to the simplicity and its many pleasant characteristics. This will create a triangular or a sawtooth waveform in the time domain.

Chapter 5 discussed various hardware options which can be used to design the system in practice. It has been shown that there were two options: The first option was to purchase a ready-built radar system and develop only the processing part of the system. The second option was to develop the complete system from the first principles using electronic components. It is the author's opinion that the first option should be selected since it provides a low-cost solution to the problem. Designing a complete radar system is a very complex and a specialized field and it may be desirable to purchase a ready-built and tested system instead.

The project has been theoretical so far and the next stage is to develop a radar based hardware to test and evaluate the overall system response and usefulness in practice. Based upon the result and the cost of the final prototype system, it is hoped that the car manufacturers could be convinced to provide such systems in their new designs.

Suggestion For Future Work

1- RACAS measures the radial velocity as well as the angle between RACAS mounted vehicle and the other vehicles [6]. Hence, the system will be developed in such a way that the components of velocity can be calculated and the result will be more sensitive.

2-The output of the radar system and the attitude of driver can be taken as data and by the aid of an Expert System a decision will be more reliable.

3-According to Elizabeth A. Bert, car's hydraulic systems can be replaced by wires and microcontrollers [9]; in other words, all the control system of the cars such as breaks, throttle and steering will be operated electronically. If this is done RACAS can be used to give instructions to the necessary electronics systems for preventing accidents instead of warning the driver.

[3] SIRST (Signal Intelligence) Radar Network

See [2] for details

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