GILBERT BOAKYE THE KUDRYASHOV METHODS FOR CONSTRUCTING SOLITARY WAVES OF SCHRÖDINGER EQUATIONS MASTER THESIS 2023

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M.Sc. THESIS

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Approval

We certify that we have read the thesis submitted by Gilbert Boakye titled "**THE KUDRYASHOV METHODS FOR CONSTRUCTING SOLITARY WAVES OF SCHRÖDINGER EQUATIONS**" and that in our combined opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Sciences.

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Declaration

I hereby declare that all information, documents, analysis and results in this thesis have been collected and presented according to the academic rules and ethical guidelines of the Institute of Graduate Studies, Near East University. I also declare that as required by these rules and conduct, I have fully cited and referenced information and data that are not original to this study.

Gilbert Boakye

14/08/2023

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Gilbert Boakye

Abstract

The Kudryashov Methods for Constructing Solitary Waves of Schrödinger Equations

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This thesis investigates the use of the Kudryashov methods for the construction of Solitary wave solutions in Schrödinger equations with applications in nonlinear optics.

More precisely, the Kudryashov methods, established in 2012 and 2022, are formally adopted to extract solitary wave solutions of generalized Schrödinger equations involving distinct laws, such as the Kerr law and the parabolic law.

The derived solitary wave solutions provide valuable insights into the dynamics of optical pulses and their behavior in different nonlinear media.

Chapter I introduces solitary waves, outlines the study's goals, and highlights the relevance of the Kudryashov methods among others.

Chapter II reviews key literature on nonlinear PDEs, focusing on solution techniques for the Schrödinger equation.

Chapter III details two versions of the Kudryashov method, applying them to the KdV equation to demonstrate their effectiveness.

In Chapter IV, both methods are used to derive solitary wave solutions for generalized Schrödinger equations, revealing how changes in nonlinearity coefficients affect wave amplitude and width.

Chapter V concludes that while both methods are effective, Method II is more flexible and is recommended for equations where Method I is limited.

Keywords: Schrödinger equations, Kerr law, parabolic law, Kudryashov methods, solitary waves

Özet

Bu tez, doğrusal olmayan optikte uygulamaları olan Schrödinger denklemleri için soliter dalga çözümlerinin elde edilmesinde Kudryashov yöntemlerinin kullanımını incelemektedir. 2012 ve 2022'de geliştirilen Kudryashov yöntemleri, Kerr ve parabolik yasalar içeren genelleştirilmiş Schrödinger denklemlerine uygulanmıştır.

Elde edilen çözümler, optik darbelerin farklı doğrusal olmayan ortamlardaki davranışlarını anlamada faydalıdır.

Birinci bölümde konunun amacı ve önemi açıklanmış, ikinci bölümde ilgili literatür incelenmiştir. Üçüncü bölümde Kudryashov yöntemleri tanıtılmış ve KdV denklemi üzerinde denenmiştir.

Dördüncü bölümde bu yöntemler Schrödinger denklemlerine uygulanarak çeşitli soliter çözümler elde edilmiştir. Son olarak, beşinci bölümde Yöntem II'nin daha esnek olduğu ve Yöntem I'in sınırlı kaldığı durumlarda tercih edilmesi gerektiği sonucuna varılmıştır.

Anahtar Kelimeler: Schrödinger denklemleri, Kerr yasası, parabolik yasa, Kudryashov yöntemleri, soliter dalgalar

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List of Abbreviations

BEC:	Bose Einstein Condensate
KdV:	Korteweg de Vries
KM:	Kudryashov Method
NLPDE:	Nonlinear Partial Differential Equation
NLSE:	Nonlinear Schrödinger Equation
ODE:	Ordinary Differential Equation
PDE:	Partial Differential Equation

CHAPTER I
Introduction

The study of solitary waves, due to their real applications (Wazwaz, 2009), has garnered significant attention in various scientific disciplines. These wave phenomena emerge in nonlinear systems and play a crucial role in fields such as fluid dynamics, quantum mechanics, and nonlinear optics (Wazwaz, 2009). More especially, each type of solitary wave possesses unique properties, making them fundamental objects of study in physics (Wazwaz, 2009).

In recent years, the Kudryashov methods (Ayati et al., 2017) have emerged as effective tools for constructing solitary wave solutions in nonlinear partial differential equations (Özişik et al., 2022). These methods have demonstrated their applicability across diverse domains, ranging from fluid dynamics to plasma physics (Özişik et al., 2022).

This thesis focuses on a comprehensive exploration of the Kudryashov methods in the context of Schrödinger equations. The Kudryashov methods provide innovative approaches to extracting solitary wave solutions of Schrödinger equations, shedding light on the underlying physics of solitary wave formation and propagation.

Background of the Study

Solitary waves discovery dates back to the 19th century when Scottish engineer John Scott Russell observed a solitary wave in a canal (Wazwaz, 2009). They have since been extensively studied in various disciplines, including physics and engineering.

Formulated in 1925 by Erwin Schrödinger, the Schrödinger equation concept was introduced by considering the de Broglie hypothesis (Figueiredo et al., 2019). According to this hypothesis, matter particles are characterized by a wave packet spread out over space (Figueiredo et al., 2019).

Schrödinger equations find wide-range applications in different scientific fields. Originally introduced as a key equation in quantum mechanics to describe the behavior of quantum particles, the Schrödinger equation has also found relevance beyond its original domain (*Schrödinger's Equation: Explained & Amp; How to Use It*, 2021). It arises as a model in nonlinear optics, Bose-Einstein condensates (Liu & Kengne, 2019), plasma physics, and other physical systems exhibiting wave-like behavior (Wazwaz, 2009).

The search for solitary wave solutions in Schrödinger equations is of great importance due to such waves possess unique characteristics (Arora et al., 2022 & Wazwaz, 2009). For example, solitons as a type of solitary waves can carry energy, information, and other conserved quantities without distortion (Turitsyn et al., 2012).

Several existing methods have been developed to construct solitary wave solutions for Schrödinger equations. In this context, the "Kudryashov Methods" offer new approaches to constructing solitary waves of Schrödinger equations. These methods, developed by Kudryashov (Özişik et al., 2022), introduce innovative ideas and techniques that expand the repertoire of available solitary wave solutions (Özişik et al., 2022. By applying the Kudryashov methods, this study aims to explore their advantages, limitations, and applications in generating solitary wave solutions.

Research Objectives

The main objectives of this research are twofold. First, we delve into a detailed examination of the Kudryashov methods, referred to as Method I and Method II. These methods are applied to nonlinear PDEs to extract solitary wave solutions, with a particular focus on their differences and advantages in handling different types of equations. Second, the thesis extends the application of these methods to more generalized forms of the Schrödinger equation, introducing different nonlinearities such as the Kerr and parabolic laws.

By doing so, we aim to provide a comprehensive understanding of the capabilities and limitations of the Kudryashov methods in capturing solitary wave dynamics in various nonlinear settings.

Significance of the Study

Solitary waves constitute a crucial component in describing a diverse array of nonlinear phenomena across various branches of applied sciences. Their inherent capacity to propagate energy, information, and other conserved quantities devoid of distortion underscores their significance (Turitsyn et al., 2012). These useful characteristics underscore the imperative to investigate solitary waves within the realm of NLPDs.

Research Questions

1. What are the merits of the Kudryashov methods in handling Schrödinger equations?
2. What are the demerits of the Kudryashov methods in handling Schrödinger equations?

Scope and Limitations of the Study

The scope of the study revolves around the Kudryashov methods (Ayati et al., 2017) and their applications in finding solitary waves of nonlinear PDEs. It focuses on the two versions of the method, proposed in 2012 and 2022.

Ethical Considerations

In the course of conducting this research, ethical considerations have been of paramount importance. Ensuring the ethical integrity of this study involves maintaining a commitment to objectivity, accuracy, and transparency in the presentation of findings and results. The use of existing literature and resources is appropriately attributed and referenced. Moreover, the research respects the intellectual property rights of other researchers and scholars by accurately citing their work.

Moreover, this research avoids any form of plagiarism, fabrication, or misrepresentation of information. The goal is to contribute to the academic community with the highest ethical standards, maintaining credibility and scholarly rigor throughout the study.

Organization of the Study

This thesis is structured as follows: In Chapter 1, there is a comprehensive introduction encompassing the study's significance, research questions, objectives, and ethical considerations. Chapter 2 delineates the examination of pertinent literature. Chapter 3 concentrates on the methodologies and approaches employed in the study. Specifically, it scrutinizes and draws comparisons between the Kudryashov Methods I and II, offering intricate explanations of how they are employed to determine solitary wave solutions for NLPDEs, notably focusing on the KdV equation. The fourth chapter delves into the efficacy of the Kudryashov methods in tackling a generalized Schrödinger equation with different nonlinearities.

The concluding chapter encapsulates a summary of the entire thesis along with presenting recommendations derived from the study's findings. It also serves to provide a concise overview of the key points discussed throughout the thesis, highlighting the main contributions and insights obtained from the research. In essence, this chapter brings closure to the thesis by reiterating its significance, summarizing its core contents, and suggesting avenues for future exploration and implementation.

CHAPTER II
Literature Review

This chapter presents a literature review aimed at providing a comprehensive overview of the existing research and advancements regarding Schrödinger equations, the Kudryashov methods, and solitary waves. Additionally, it encompasses a thorough examination of the literature on PDEs in general, delving into various definitions and approaches to studying such equations.

Theoretical Background

As mentioned in the earlier portions of this thesis by Wazwaz (2009), solitary waves have found significant applications in various fields of physics, such as optics, fluid dynamics, and quantum mechanics. The Schrödinger equations on the other hand are a class of nonlinear PDEs that describe wave propagation in quantum mechanics, nonlinear optics, and other wave-related phenomena (Liu & Kengne, 2019). Moreover, the Kudryashov methods (Ayati et al., 2017) are effective techniques used to construct exact solutions, particularly solitary waves, for a wide range of nonlinear PDEs (Ryabov et al., 2011).

Schrödinger Equations

Schrödinger equations have emerged in a wide range of phenomena from plasma physics to nonlinear optics (Wazwaz, 2009). However, unlike the linear Schrödinger equation, nonlinear Schrödinger equations involve additional nonlinear terms (Wazwaz, 2009), making them more complex and challenging to solve. Therefore, researchers often employ various numerical methods to obtain numerical solutions for them. In this thesis, the Kudryashov methods emerge as potent techniques employed to discover exact solitary wave solutions for a diverse array of NLPDEs.

Kudryashov Methods

The Kudryashov methods (Ayati et al., 2017), devised in 2012 and 2022 as method I and method II respectively, are influential mathematical techniques employed to discover exact solutions, including solitary wave solutions, for a diverse range of NLPDEs. In recent years, the Kudryashov method I has been used to find solitary wave solutions of many nonlinear PDEs by different authors (Ayati et al., 2017 & Mirzazadeh et al., 2014).

Partial Differential Equations

PDEs are equations in which the unknown function, often known as the dependent variable, and its partial derivatives are both present (Wazwaz, 2009; Exner et al., 2021 & Partial Differential Equations). ODEs only need the dependent variable to depend on one independent variable, but PDEs demand that the dependent variable depends on several independent variables (Wazwaz, 2009).

For instance, the function u relies on both x and t in PDEs like $u = u(x,t)$ or $u = u(x,y,t)$, or on x, y, and t, respectively (Wazwaz, 2009).

PDEs may explain a wide range of physical phenomena. For instance, according to Wazwaz (2009), equations that illustrate heat flow in one-, two-, and three-dimensional spaces include the following:

$$u_t = ku_{xx} \tag{1.1}$$

$$u_t = k\left(u_{xx} + u_{yy}\right) \tag{1.2}$$

and

$$u_t = k(u_{xx} + u_{yy} + u_{zz}) \tag{1.3}$$

The dependent variable $u = u(x,t)$ in Eq. (1) is dependent on the position x and the time t. $u = u(x,y,t)$ in Eq. (2) is reliant on the three independent variables x, y, and t (Wazwaz, 2009). The dependent variable $u = u(x,y,z,t)$ in Eq. (1.3) depends on the four independent variables x, y, z, and t (Wazwaz, 2009). The equations

$$u_{tt} = c_2 u_{xx} \tag{1.4}$$

$$u_{tt} = c_2\left(u_{xx} + u_{yy}\right) \tag{1.5}$$

$$u_{tt} = c_2(u_{xx} + u_{yy} + u_{zz}) \tag{1.6}$$

which describe one-dimensional, two-dimensional, and three-dimensional spaces, respectively, are examples of wave propagation equations (Wazwaz, 2009). The unknown functions are specified as $u = u(x,t)$, $u = u(x,y,t)$, and $u = u(x,y,z,t)$, respectively, in Eqs. (1.4), (1.5), and (1.6) (Wazwaz, 2009).

Furthermore, $u_t + uu_x - vu_{xx} = 0$ and $u_t + 6uu_x + u_{xxx} = 0$ respectively, are used to describe the Burgers equation and the Korteweg-de Vries (KdV) equation (Wazwaz, 2009). The variables x and t affect the function u in these equations.

The Order of PDEs

The order of a partial differential equation is determined by the highest order of the partial derivatives in the equation (Wazwaz, 2009). It provides information about the complexity and characteristics of the equation (Wazwaz, 2009). Detailed below is the concept of order in PDEs with some vivid examples:

First-Order PDE: A first-order PDE involves only first-order partial derivatives (Rhee et al., 2014 & Ghasemi, 2019). One example is the linear transport equation: $a(x,y)u_x + b(x,y)u_y = c(x,y)$. Here, $u(x,y)$ is the unknown function, and $a(x,y)$, $b(x,y)$, and $c(x,y)$ are given coefficients (Rhee et al., 2014 & Ghasemi, 2019). The first-order derivatives u_x and u_y appear in the equation.

Second-Order PDE: A second-order PDE contains second-order partial derivatives and is more complex than a first-order PDE. The Laplace equation is a classic example: $u_{xx} + u_{yy} = 0$ (Strauss, 2007).

This equation appears in various fields, including electrostatics and fluid dynamics (Vogt, 2007). It involves second-order derivatives u_{xx} and u_{yy} (Strauss, 2007).

Third-Order PDE: A third-order PDE includes third-order partial derivatives (Evans, 2010 & Mechee et al., 2014). One example is the KdV equation, $u_t + 6uu_x + u_{xxx} = 0$ which represents lengthy internal waves in a stratified ocean, weakly interacting shallow water waves, ion-acoustic waves in plasma, and acoustic waves on crystal lattice (Lewis et al., 2022).

This nonlinear equation describes certain types of waves and involves third-order derivative u_{xxx} (Wazwaz, 2009).

Higher-Order PDEs: The fourth, fifth, and even higher orders of PDEs are possible. For example, when considering both the momentum and the continuity equations simultaneously, the well-known Navier-Stokes equations, which describe fluid flow (Hosch, 2023), are fourth-order PDEs (Society for Industrial and Applied Mathematics, Hosch, 2023).

Linear and Nonlinear PDEs

Based on the structure of the equations, partial differential equations may be divided into two basic categories: linear and nonlinear (Wazwaz, 2009). The subsequent sections investigate each category using concrete instances.

Linear PDEs: A linear PDE is one in which the dependent variable and its derivatives appear linearly (Wazwaz, 2009). This means that the dependent variable and its derivatives are raised to the power of 1 and do not multiply or divide each other (Wazwaz, 2009). Linear PDEs have particularly nice mathematical properties and often have well-developed solution methods. A typical example is the Linear Heat Equation: $u_t = ku_{xx}$ (Wazwaz, 2009).

This equation describes the diffusion of heat and is a classic example of a linear PDE (Kovács, 2021). The dependent variable u and its derivatives appear linearly, with no nonlinear terms (Wazwaz, 2009). Other examples of linear PDEs according to Wazwaz (2009), are the wave equation ($u_{tt} = c^2u_{xx}$), the Laplace equation ($u_{xx} + u_{yy} = 0$), and the Klein-Gordon equation ($\nabla^2 u - \frac{1}{c^2}u_{tt} = \mu^2 u$), among others.

Nonlinear PDEs: When the dependent variable and its derivatives do not behave linearly, the PDE is considered nonlinear (Wazwaz, 2009). The dependent variable's derivatives, as well as any of its products, powers, or other nonlinear processes, are included in nonlinear terms (Wazwaz, 2009).

In comparison to linear PDEs, nonlinear PDEs are typically harder to solve and analyze. Wazwaz (2009) lists a few examples of nonlinear partial differential equations such as the Burgers equation ($u_t + uu_x = \alpha u_{xx}$), the KdV equation ($u_t + \alpha uu_x + bu_{xx} = 0$), the mKdV equation ($u_t - 6u^2u_x + u_{xxx} = 0$), and the Sine-Gordon equation ($u_{tt} - u_{xx} = \alpha \sin u$), etc. These types of equations model various phenomena, including fluid flow and traffic flow (Wazwaz, 2009).

Homogenous and Inhomogeneous PDEs

Homogeneous and inhomogeneous partial differential equations (PDEs) are classifications based on the nature of the forcing term or source in the equation (Wazwaz, 2009).

Homogeneous PDEs: Homogeneous PDEs are equations in which the dependent variable and its derivatives combine to form a homogeneous expression, meaning that the equation is equal to zero (Wazwaz, 2009 & Fog & Fog, 2017). In other words, the equation lacks any external sources or forcing terms (Wazwaz, 2009). The wave equation is a classic example of a homogeneous PDE. In one dimension, it is given by $u_{tt} - c^2u_{xx} = 0$ (Wazwaz, 2009). This means that when the source term is zero (0), the equation is homogeneous (Fog & Fog, 2017).

Here, $u(x,t)$ represents the displacement or amplitude of the wave, c is the wave speed, and u_{tt} and u_{xx} are the second partial derivatives of u with respect to time (t) and position (x), respectively (Fog & Fog, 2017). The absence of any external forcing term makes it a homogeneous PDE (Wazwaz, 2009). Notably, homogeneous PDEs have certain characteristics that cannot be overlooked. A few of such characteristics have been detailed below:

First, homogeneous PDEs have a special characteristic known as superposition (Linear PDEs and the Principle of Superposition, n.d. & Choksi, 2022), which states that if $u_1(x,t)$ and $u_2(x,t)$ are solutions to the homogeneous PDE, then any linear combination of them, such as $u_1(x,t) + u_2(x,t)$ is likewise a solution (Linear PDEs and the Principle of Superposition, n.d. & Choksi, 2022). Due to this characteristic, generic solutions can be created using linear combinations. Also, homogeneous PDEs can often be solved using separation of variables, Fourier series, Laplace transforms, or other analytical techniques (Wazwaz, 2009).

The superposition principle, combined with these methods, allows for the construction of general solutions (Wazwaz, 2009). Boundary and initial conditions are typically used to determine specific solutions within a given problem (Wazwaz, 2009). In terms of their applications, homogeneous PDEs find applications in various fields. For example, the wave equation is used to describe vibrations of strings, membranes, and other wave phenomena (Cas, 2022) while the Laplace equation appears in electrostatics and steady-state heat conduction problems (Holagh et al., 2019). The homogeneous nature of these equations simplifies their analysis and allows for the discovery of fundamental solutions and characteristic behaviors.

Inhomogeneous PDEs: Inhomogeneous PDEs include a non-zero source term or forcing function, representing external influences or interactions in the system being modeled (Wazwaz, 2009). This term causes the equation to be non-homogeneous. The heat equation with a heat source is an example of an inhomogeneous PDE (Huang et al., 2013). In one dimension, it is given by $u_t - ku_{xx} = f(x,t)$ where $u(x,t)$ represents the temperature distribution, k is the thermal diffusivity constant, u_t and u_{xx} are the first and second partial derivatives of u with respect to time (t) and position (x), respectively, and $f(x,t)$ is the heat source term (Hancock, n.d.). The presence of the non-zero function $f(x,t)$ makes it an inhomogeneous PDE (Wazwaz, 2009).

Solutions to PDEs

Different solutions for PDEs can be considered (Polyanin et al., 2008; Wazwaz, 2009).

Exact solutions: When it is possible to identify explicit mathematical formulations that fulfill the PDE, exact solutions are discovered. These solutions can include approaches like the variable separation (Wazwaz, 2009) and symmetry methods (Hydon, 2000). Since they give accurate formulae for the related dependent variables, exact solutions are greatly desired (Wazwaz, 2009).

It is however important to note that the choice of solution method depends on the specific characteristics of the PDE, boundary conditions, and the problem at hand. In some cases, a combination of different solution techniques may be necessary to obtain a complete understanding of the solution behavior.

By solving nonlinear differential equations exactly, we can gain a clearer understanding of complex effects such as spatial localization of transfer processes, the presence or absence of stationary states under certain conditions, blow-up solutions, and the possibility of no smoothness or discontinuity of unknown (Polyanin & Sorokin, 2021).

An essential step in comprehending the behavior of physical systems and foretelling their long-term evolution is the analytical solution of PDEs and the discovery of exact solutions. Exact solutions serve as standards for testing numerical techniques and indicate significant trends while also offering insights into the underlying dynamics (Roy & Sinclair, 2009). However, due to the complexity of the equations and the wide range of boundary or initial conditions that may be applied, finding exact solutions to PDEs is frequently a difficult task.

Here, a number of potent strategies and approaches have been proposed to deal with such problems, allowing the creation of exact mathematical expressions that fulfill the PDEs. Each method possesses unique characteristics and is suited to particular types of PDEs and boundary conditions. Detailed below are some of them.

Method of Separation of Variables: The key aspect of this method lies in its ability to transform the partial differential equation into a system of ordinary differential equations (Wazwaz, 2009). In this system, each ODE depends on just one variable, enabling independent solutions (Wazwaz, 2009). By involving the boundary conditions and initial conditions, the constants of integration can be determined, leading to a complete solution (Wazwaz, 2009).

Transform Methods: The PDE can be transformed into an algebraic or ordinary differential equation using transform techniques such as Fourier, Laplace, or Mellin transforms (Spence, 2015). Utilizing the appropriate transform and inverse transform can yield the exact solution. This is especially beneficial for linear partial differential equations with constant coefficients, where transform techniques have proven to be advantageous (Manssour et al., 2021).

Numerical Solutions: Numerical solutions are essential when exact analytical solutions are challenging or not feasible to obtain. Numerical methods allow for approximating the solution by discretizing the domain and applying computational techniques to solve the resulting system of algebraic equations (Wazwaz, 2009). The first step in obtaining a numerical solution is to discretize the PDE domain (Wikipedia contributors, 2023).

This involves dividing the continuous domain into a finite number of grid points or elements (Wikipedia contributors, 2023). There are several numerical methods available for solving PDEs, depending on the nature of the problem and the desired accuracy.

Finite Difference Method: In this approach, the derivatives in the PDE are approximated by finite difference formulas (Liu, 2018). The PDE is transformed into a system of algebraic equations, which can be solved using iterative techniques (Liu, 2018).

Finite Element Method: This method involves dividing the domain into smaller subdomains (elements) and approximating the solution within each element using a set of basic functions (Krishnamoorthi et al., 2013). The problem is then transformed into a system of equations that can be solved using numerical techniques (Krishnamoorthi et al., 2013).

Approximate Solutions: When it is possible to identify explicit mathematical formulations that satisfy the PDE approximately, approximate solutions are discovered. Some techniques for finding the approximate solution of partial differential equations are as follows:

Perturbation Method: The perturbation method (Nayfeh, 2000) is a systematic approach for finding approximate solutions to PDEs by expanding the solution as a series in terms of a small parameter. It is particularly useful for problems with small perturbations or nonlinearity (Shen & Huang, 2007).

Homotopy Analysis Method: The Homotopy analysis method (Liao, 2012) is a powerful analytical technique that has been drawing more and more attention from researchers in various fields. It serves as an effective approach to address strongly nonlinear problems by providing convergent series solutions (Yang & Lin, 2022).

Classification and Types of PDEs

PDEs can be broadly classified into three (3) main categories based on their properties, including elliptic, parabolic, and hyperbolic equations (Karapetyants & Kravchenko, 2022). For example,

Elliptic PDEs: For second-order elliptic PDEs in the form of $A\nabla^2 u + B\nabla u + Cu = f$, where A, B, and C are coefficients, f represents the source term, and the discriminant is given by the expression $\Delta = B^2 - 4AC$. The discriminant helps determine the nature of the solutions (Wazwaz, 2009). If $\Delta < 0$, the PDE is called elliptic and has complex conjugate solutions.

Parabolic PDEs: If $\Delta = 0$, the PDE is called parabolic (Wazwaz, 2009).

Hyperbolic PDEs: If $\Delta > 0$, the PDE is called hyperbolic PDE and has a real and distinct solutions (Wazwaz, 2009).

Boundary Conditions

Boundary conditions are essential in solving partial differential equations as they specify the behavior of the solution at the boundaries of the domain (Wazwaz, 2009). They play a crucial role in determining a unique solution to the PDE. Let's discuss some common types of boundary conditions:

Dirichlet Boundary Condition: A Dirichlet boundary condition specifies the exact value of the solution at the boundary (Galbusera & Niemeyer, 2018). Moreover, according to a study by Wazwaz (2009), Dirichlet boundary conditions are imposed when specifying the function u on the boundary of a bounded domain. For example, using a rod with length L, where $0 < x < L$, as a case study, Wazwaz

(2009) suggested that the boundary conditions are set as follows: $u(0) = \alpha$ and $u(L) = \beta$, where α and β represent constants.

Similarly, for a rectangular plate with dimensions $0 < x < L_1$ and $0 < y < L_2$, the boundary conditions are prescribed for the edges of the plate: $u(0, y)$, $u(L_1, y)$, $u(x, 0)$, and $u(x, L_2)$ (Wazwaz, 2009).

These boundary conditions are classified as homogeneous when the dependent variable u is equal to zero at any point on the boundary (Wazwaz, 2009). Conversely, if u is non-zero at any point on the boundary, the conditions are termed inhomogeneous (Wazwaz, 2009).

Neumann Boundary Condition: Here, the prescribed condition is related to the normal derivative $\frac{du}{dn}$ of function u along the outward normal direction of the boundary (Wazwaz, 2009). For instance, for a rod with length L, Wazwaz (2009) again recounts that the Neumann boundary conditions take the form of $u_x(0, t) = \alpha$ and $u_x(L, t) = \beta$, where α and β are specified constants.

Robin Boundary Condition: In the region Ω, for an elliptic partial differential equation, Robin boundary conditions are defined by prescribing the sum of αu and the normal derivative of $u \left(\frac{du}{dn} = f\right)$ at all points on the boundary of Ω (Wolfram Research, Inc., n.d.). Here, α and f are constants that are provided as part of the boundary conditions.

Mixed Boundary Condition: A mixed boundary condition involves a combination of different types of boundary conditions at different portions of the boundary (What Are Boundary Conditions? Numerics Background | SimScale, 2023). For example, a mixed boundary condition could specify a combination of Dirichlet and Neumann conditions at different sections of the boundary.

Initial Conditions

Initial conditions are still another essential element in the solution of partial differential equations, in addition to boundary conditions. Initial conditions describe how the solution behaves at the beginning of the issue or in its initial state (Wazwaz, 2009).

Related Research

Numerous branches of physics and mathematics contain the intriguing and important phenomenon known as the solitary wave. Solitary waves are extremely important for understanding complicated wave behaviors and nonlinear dynamics due to their distinctive properties.

The discovery of solitary waves traces back to the remarkable observations made by the Scottish scientist John Scott Russell in 1844 (Wazwaz, 2009). John Russell had the opportunity to witness a unique phenomenon during his observation of a boat being rapidly drawn along a narrow channel by a pair of horses (Wazwaz, 2009).

When the boat suddenly came to a stop, a bulge of water it had set in motion continued to travel along the channel with undiminished speed and unchanged form. Russell eloquently described this solitary elevation as "a rounded, smooth, and well-defined heap of water," which he dubbed a "Wave of translation" (Wazwaz, 2009).

It was this single humped wave of water, retaining its identity even after the interaction, that later became known as a soliton (Wazwaz, 2009). The experimental discovery of solitary waves and their remarkable properties was an important milestone in the study of nonlinear waves.

In 1895, Diederik Johannes Korteweg and his Ph.D. student, Gustav de Vries, mathematically derived the KdV equation, a nonlinear partial differential equation given by $\frac{\partial u}{\partial t} + au\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x^3} = 0$ where $u(x, t)$ is the dependent variable representing the wave amplitude as a function of spatial coordinate x and time t, and a is a constant representing the wave speed (Wazwaz, 2008).

The KdV equation describes the propagation of long waves with finite amplitude in dispersive media (Wazwaz, 2009). The KdV equation incorporated both nonlinearity and dispersion, accounting for the steepening and spreading effects of the wave, respectively (Wazwaz, 2009). Solitary waves emerged as a solution to this equation, where the competing effects of nonlinearity and dispersion strikingly balanced each other, resulting in highly stable and localized wave packets (Wazwaz, 2009).

Categories of Traveling Waves Solutions

According to Wazwaz (2009), studying equations that model wave phenomena necessitate an exploration of traveling wave solutions. A traveling wave solution denotes a steady pattern that moves at a constant speed (Wazwaz, 2009). These solutions are often obtained by simplifying nonlinear evolution equations into associated ordinary differential equations (Wazwaz, 2009). This is typically accomplished using the ansatz $u(x,t) = u(\xi)$, where $\xi = x - ct$. Here, c represents the wave speed, and this transformation converts the partial differential equation in terms of x and t into an ordinary differential equation in terms of ξ, which can be solved using various suitable methods (Wazwaz, 2009).

There are several types of traveling wave solutions which are of particular interest, particularly in the domain of solitary wave theory, which is rapidly advancing in various scientific fields, from shallow water waves to plasma physics (Wazwaz, 2009). Notably, traveling waves manifest in numerous forms, and only a few are solitary waves (described before) and periodic waves (traveling waves that exhibit periodicity).

Some Well-Known Solitary Waves

Here, some well-known solitary waves such as kink solitary wave, bright soliton, and dark soliton are introduced.

Kink Solitary Wave:

According to Zhu et al. (2022), the kink is a type of solitary wave that features a sharp bend in its waveform, transitioning between a stable base and several smaller oscillations.

A kink in a three-dimensional portrait has been represented in Figure 2.

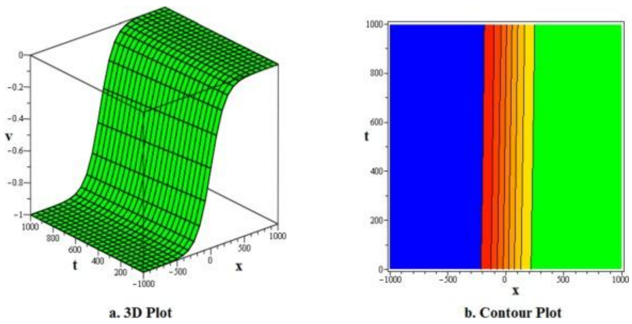


Figure 2: An example of a kink solitary wave (Alsheekhhussain et al., 2025)

Bright and Dark Soliton Waves: Bright solitons are identified by a concentrated peak of intensity against a uniform backdrop, whereas dark solitons are defined by a localized decrease in intensity within a continuous wave environment (Gandzha & Sedletsky, 2017)

However, dark solitons are deemed more suitable for implementation in optical communications compared to bright solitons (Ma et al., 2016). Most essentially, according to Horikis and Ablowitz (2015), dark solitons demonstrate greater resistance to perturbations during propagation when compared to bright solitons.

Bright and dark solitons have been given in Figures 3 and 4 respectively.

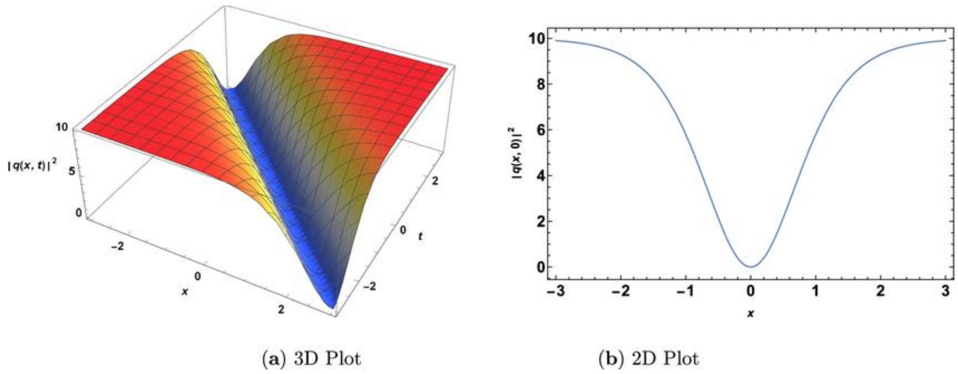


Figure 3: An example of a dark soliton (Yıldırım et al., 2023)

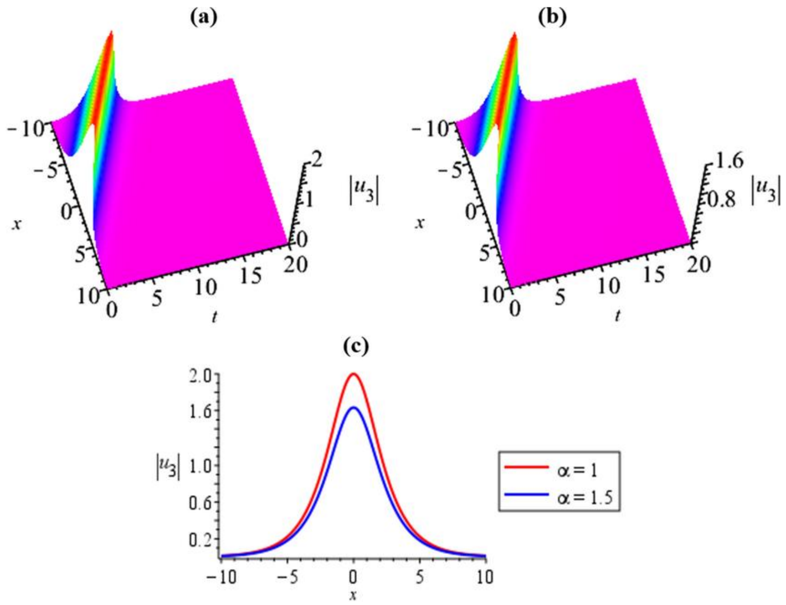


Figure 4: An example of a bright soliton (Hosseini et al., 2024)

CHAPTER III
Methodologies

In this chapter, we are interested in reviewing some types of the Kudryashov method which are useful in constructing solitary waves of nonlinear PDEs in applied sciences. More precisely, we review two types of the Kudryashov method proposed and established in 2012 and 2022 respectively, and compare their efficacies in handling nonlinear PDEs.

3.1. *Kudryashov Method I*

Here, the study proceeds with reviewing the basic ideas of the Kudryashov method I (Kudryashov, 2012) to find solitary wave solutions of nonlinear PDEs. To this end, we consider the following nonlinear PDE

$$P(u, u_x, u_t, \dots) = 0, \tag{3.1}$$

where P is a known function and $u = u(x,t)$ is unknown. Using the transformation $u = U(\epsilon)$ where $\epsilon = x - ct$ (c is the velocity of Solitary), we find

$$O(U(\epsilon), U'(\epsilon), U''(\epsilon), \dots) = 0. \tag{3.2}$$

The Kudryashov method I assumes the solution of Eq. (3.2) can be written as

$$U(\epsilon) = a_0 + a_1 K(\epsilon) + a_2 K^2(\epsilon) + \cdots + a_N K^N(\epsilon), \quad a_N \neq 0, \tag{3.3}$$

In (3.3), $a_i, i = 0,1,\dots,N$ are unknowns, N is derived by the balance approach, and $K(\epsilon)$ is

$$K(\epsilon) = \frac{1}{1+da^{\epsilon}},$$

satisfying

$$K'(\epsilon) = K(\epsilon)(K(\epsilon)-1)\ln(a).$$

Based on Eq. (3.2) and Eq. (3.3), a nonlinear algebraic system is generated whose solution yields solitary waves of Eq. (3.1).

The 2014 article ‘’The modified Kudryashov method for solving some fractional-order nonlinear equations’’ by Ege and Misirli provides more details on the Kudryashov method I.

3.2. Kudryashov method II

In the current subsection, the key concepts of the Kudryashov method II (Kudryashov, 2022) in establishing solitary waves of nonlinear PDEs are presented. Let’s consider a nonlinear PDE of the form

$$P(u, u_x, u_t, \dots) = 0, \tag{3.4}$$

where P is a known function and $u = u(x,t)$ is unknown. By applying the transformation $u = U(\epsilon)$ where $\epsilon = x - ct$, Eq. (3.4) can be reduced a nonlinear ODE as

$$O(U(\epsilon), U'(\epsilon), U''(\epsilon), \dots) = 0. \tag{3.5}$$

Now, suppose that Eq. (3.5) can be written as follows

$$U_\epsilon^2 = P(U)E(U), \tag{3.6}$$

where P and E are the polynomials of U and $U = U(\epsilon)$. The first step is considering

$$U(\epsilon) = F(\xi), \ \ \xi = \phi(\epsilon), \tag{3.7}$$

for Eq. (3.6) and looking for $F(\xi)$ and $\phi(\epsilon)$. By employing the chain rule to (3.7), we get

$$U_\epsilon = F_\xi \xi_\epsilon. \tag{3.8}$$

Substituting (3.7) and (3.8) into Eq. (3.6) yields

$$F_\xi^2 \xi_\epsilon^2 = P(F)E(F). \tag{3.9}$$

Now, letting $\xi_\epsilon^2 = P(F)$ and solving

$F_\xi^2 = E(F),$ (from Eq. (3.9))

$$\epsilon = \epsilon_0 + \int \frac{d\xi}{\sqrt{P(F)}},$$

Solitary wave solutions of Eq. (1) are derived.

3.3. Applications of Kudryashov methods

In this part of the study, we consider applying the Kudryashov methods to the KdV equation to generate solitary wave solutions. The KdV equation under consideration is of the form

$$\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x^3} - 6u\frac{\partial u}{\partial x} = 0 \tag{4.1}$$

which can be rewritten as

$$u_t + u_{xxx} - 6uu_x = 0 \tag{4.2}$$

By utilizing the travelling wave transformation given as

$$u = U(\epsilon), \ \ \epsilon = x - ct \tag{4.3}$$

where c is the solitary wave speed, we obtain

$$\frac{\partial u}{\partial t} = \frac{\partial U}{\partial \epsilon}\frac{\partial \epsilon}{\partial t} = U'(\epsilon)(-c) = -cU' \tag{4.4}$$

$$u_{xxx} = U'''(\epsilon) \tag{4.5}$$

$$uu_x = UU' \tag{4.6}$$

As a result, we get the reduced form of the KdV equation

$$-cU'(\epsilon) + U'''(\epsilon) - 6U(\epsilon)U'(\epsilon) = 0 \tag{4.7}$$

Now, by integrating Eq. (4.7) with respect to ϵ, we get the following ordinary differential equation

$$-cU' + U'' - 3U^2 = 0 \tag{4.8}$$

By employing the balance principle to Eq. (4.8), we get the balance number as

$$N + 2 = 2N \Rightarrow N = 2 \tag{4.9}$$

3.3.1. Applying the Kudryashov Method I

By taking $N = 2$ on Eq. (3.3), a finite series is derived as

$$U(\epsilon) = a_0 + a_1K(\epsilon) + a_2K^2(\epsilon) \tag{4.10}$$

where a_0, a_1, and a_2 are unknown constants, and

$$K'(\epsilon) = K(\epsilon)(K(\epsilon) - 1)\ln(a) \tag{4.11}$$

Based on Eq. (4.8), we need to find $U'(\epsilon)$ and $U''(\epsilon)$. It can be done by differentiating Eq. (4.10) and considering Eq. (4.11) as follows:

$$U'(\epsilon) = a_1K'(\epsilon) + 2a_2K'(\epsilon)(K(\epsilon) \Rightarrow U'(\epsilon) = a_1(K(\epsilon)(K(\epsilon) - 1)\ln(a)) + 2a_2K(\epsilon)(K(\epsilon)(K(\epsilon) - 1)\ln(a)) = 2a_2\ln(a)K^3(\epsilon) + \left(a_1\ln(a) - 2a_2\ln(a)\right) - K^2(\epsilon) - a_1\ln(a)K(\epsilon) \tag{4.12}$$

$$U''(\epsilon) = 6a_2(\ln(a))^2K^4(\epsilon) + \left(-6a_2(\ln(a))^2 + 2(\ln(a))^2(a_1 - 2a_2)\right)K^3(\epsilon) + (-2(\ln(a))^2(a_1 - 2a_2) - a_1(\ln(a))^2)K^2(\epsilon) + a_1(\ln(a))^2K(\epsilon) \tag{4.13}$$

Substituting (4.10), (4.12), and (4.13) into Eq. (4.8) and collecting the terms in different powers of $K(\epsilon)$, we find the following system of algebraic-type

$$a_0 + \frac{1}{3}c = 0$$

$$a_1(\ln(a))^2 - 3\left(a_0 + \frac{1}{3}c\right)a_1 - 3a_0a_1 = 0$$

$$6\left(-\frac{1}{2}a_1 + \frac{2}{3}a_2\right)(\ln(a))^2 - 3\left(a_0 + \frac{1}{3}c\right)a_2 3a_1^2 - 3a_0a_2 = 0$$

$$6\left(-\frac{5}{3}a_2 + \frac{1}{3}a_1\right)(\ln(a))^2 - 6a_1a_2 = 0$$

$$6a_2(\ln(a))^2 - 3a_2^2 = 0$$

By applying a symbolic computation like MAPLE, we will derive

$$a_1 = -2(\ln(a))^2$$

$$a_2 = 2(\ln(a))^2$$

$$c = (\ln(a))^2 - 6a_0$$

Therefore, the following Solitary solution to the KdV equation is constructed

$$u(x,t) = a_0 - 2(\ln(a))^2 \frac{1}{1+da^{x-((\ln(a))^2-6a_0)t}} + 2(\ln(a))^2 \left(\frac{1}{1+da^{x-((\ln(a))^2-6a_0)t}}\right)^2$$

The dynamic of the above dark solitary wave has been presented in Figure 5 for $a_0 = 0.1$, $a = 2.7$, and $d = 1$.

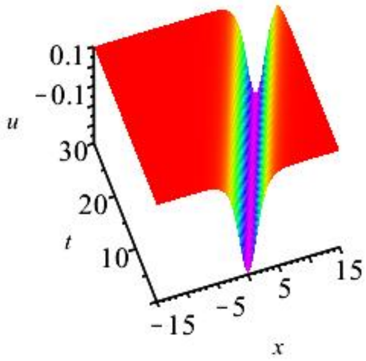


Figure 5: $u(x,t)$ for $a_0 = 0.1$, $a = 2.7$, and $d = 1$.

3.3.2. Applying the Kudryashov Method II

To derive Solitary waves of the KdV equation, we first try to rewrite Eq. (4.8) as follows

$$U_{\epsilon\epsilon} - AU + BU^2 = 0 \tag{4.14}$$

where $A = c,\ \ B = -3$

Multiplying Eq. (4.14) by U_ϵ and integrating with respect to ϵ yields

$$\frac{1}{2}U_\epsilon^2 - \frac{1}{2}AU^2 + \frac{1}{3}BU^3 = 0 \tag{4.15}$$

The Kudryashov method II seeks the solitary wave solution of Eq. (4.15) as

$$U(\epsilon) = F(\xi),\ \ \xi = \phi(\epsilon) \tag{4.16}$$

By applying the chain rule to (4.16), we derive

$$U_\epsilon = F_\xi \xi_\epsilon \tag{4.17}$$

Substituting (4.16) and (4.17) into Eq. (4.15) leads to

$$\frac{1}{2}F_\xi^2 \xi_\epsilon^2 = \frac{1}{2}AF^2 - \frac{1}{3}BF^3 \tag{4.18}$$

Now, by assuming $\xi_\epsilon = F(\xi)$, Eq. (4.18) is written as

$$3F_\xi^2 = 3A - 2BF \tag{4.19}$$

The exact solution of Eq. (4.19) is

$$U(\epsilon) = F(\xi) = \frac{-c^2B^2 + 2cB^2\xi - B^2\xi^2 + 9A}{6B} \tag{4.20}$$

By using the following integral

$$\epsilon = \epsilon_0 + \int \frac{d\xi}{F(\xi)}$$

we have

$$\epsilon = \epsilon_0 - \frac{\operatorname{arctanh}\left(\frac{2cB^2-2B^2\xi}{6B\sqrt{A}}\right)}{\sqrt{A}} \tag{4.21}$$

From Eq. (4.20), we find

$$\xi = \frac{cB+\sqrt{9A-6BF}}{B} \tag{4.22}$$

By inserting Eq. (4.22) into Eq. (4.21), we find

$$\epsilon = \epsilon_0 + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{9A-6BF}}{3\sqrt{A}}\right)}{\sqrt{A}} \tag{4.23}$$

By rewriting Eq. (4.23) as

$$\frac{\sqrt{A}}{2}(\epsilon-\epsilon_0) = \operatorname{arctanh}\left(\frac{\sqrt{9A-6BF}}{3\sqrt{A}}\right)$$

and solving it for F, we find

$$U(\epsilon) = \frac{6Ae^{\sqrt{A}(\epsilon-\epsilon_0)}}{B\left(1+2e^{\sqrt{A}(\epsilon-\epsilon_0)}+e^{2\sqrt{A}(\epsilon-\epsilon_0)}\right)}$$

Therefore, the Solitary solution of the KdV equation is constructed as

$$u(x,t) = -2c\frac{e^{\sqrt{c}(x-ct-\epsilon_0)}}{1+2e^{\sqrt{c}(x-ct-\epsilon_0)}+e^{2\sqrt{c}(x-ct-\epsilon_0)}}$$

or

$$u(x,t) = -2c\frac{e^{\sqrt{c}(x-ct-\epsilon_0)}}{\left(1+e^{\sqrt{c}(x-ct-\epsilon_0)}\right)^2}$$

The dynamic of the above dark solitary wave has been presented in Figure 6 for $c = 0.1$ and $\epsilon_0 = 0$.

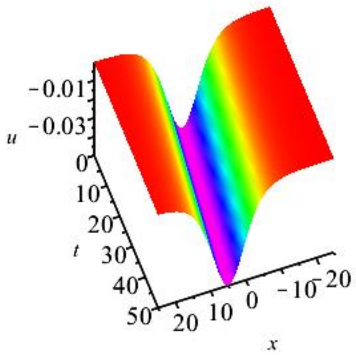


Figure 6: $u(x,t)$ for $c = 0.1$ and $\epsilon_0 = 0$

CHAPTER IV

Results and Analysis

In the current chapter, the efficiency of the Kudryashov methods in extracting solitary wave solutions of a generalized Schrödinger equation with different nonlinearities such as Kerr and parabolic laws is examined in detail. All computations have been performed by the MAPLE package.

4.1. Generalized Schrödinger equation involving the Kerr law

To start, consider the following generalized Schrödinger equation involving the Kerr law nonlinearity

$$i\frac{\partial u}{\partial t}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y\partial x}+\frac{\partial^2 u}{\partial y^2}+u+\alpha|u|^2u=0 \qquad (4.1)$$

Suppose a transformation such as

$u=U(\epsilon)e^{i(\kappa_2 x+\lambda_2 y-\omega t)},\ \ \epsilon=\kappa_1 x+\lambda_1 y-\nu t$

where ν and ω are the speed and frequency of the solitary wave respectively. After inserting the above transformation into Eq. (4.1) and distinguishing real and imaginary portions, we find

$$\left((2\lambda_2+\kappa_2)\lambda_1+\kappa_1\lambda_2+\kappa_1\kappa_2-\nu\right)\frac{dU(\epsilon)}{d\epsilon}=0$$

$$(\kappa_1^2+\kappa_1\lambda_1+\lambda_1^2)\frac{d^2U(\epsilon)}{d\epsilon^2}+(-\kappa_2^2-\kappa_2\lambda_2-\lambda_2^2+\omega+1)U(\epsilon)+\alpha U^3(\epsilon)=0 \qquad (4.2)$$

From the imaginary part, we have the speed of the Solitary as

$\nu=2\kappa_1\kappa_2+\kappa_1\lambda_2+\kappa_2\lambda_1+2\lambda_1\lambda_2$

By considering $U''(\epsilon)$ and $U^3(\epsilon)$ and applying the balance principle, we derive

$N+2=3N\Rightarrow N=1$

It is worth mentioning that the special case of Eq. (4.1) has been solved using the Kudryashov method I in (Hosseini et al., 2017).

4.1.1. Applying the Kudryashov method I

By effecting $N=1$ on Eq. (3.3), a finite series is acquired as

$$U(\epsilon)=a_0+a_1K(\epsilon) \qquad (4.3)$$

where a_0, a_1, and a_2 are unknown constants, and

$$K'(\epsilon)=K(\epsilon)(K(\epsilon)-1)\ln(a) \qquad (4.4)$$

Substituting (4.3) and (4.4) into Eq. (4.2) and collecting the terms in different powers of $K(\epsilon)$, we find the following system of algebraic-type

$$\alpha a_0^2 - \kappa_2^2 - \kappa_2\lambda_2 - \lambda_2^2 + \omega + 1 \;=\; 0 \tag{4.5}$$

$$a_1(\kappa_1^2 + \kappa_1\lambda_1 + \lambda_1^2)\ln(a)^2 + 2a_0^2\alpha a_1 + a_1(\alpha a_0^2 - \kappa_2^2 - \kappa_2\lambda_2 - \lambda_2^2 + \omega + 1) \;=\; 0 \tag{4.6}$$

$$-3a_1(\kappa_1^2 + \kappa_1\lambda_1 + \lambda_1^2)\ln(a)^2 + 3a_0\alpha a_1^2 \;=\; 0 \tag{4.7}$$

$$2a_1(\kappa_1^2 + \kappa_1\lambda_1 + \lambda_1^2)\ln(a)^2 + \alpha a_1^3 \;=\; 0 \tag{4.8}$$

By applying a symbolic computation like MAPLE, we will derive

$$a_0 = \sqrt{-\frac{\kappa_1^2+\kappa_1\lambda_1+\lambda_1^2}{2\alpha}}\ln(a) \tag{4.9}$$

$$a_1 = \frac{\ln(a)(\kappa_1^2+\kappa_1\lambda_1+\lambda_1^2)}{\alpha\sqrt{-\frac{\kappa_1^2+\kappa_1\lambda_1+\lambda_1^2}{2\alpha}}} \tag{4.10}$$

$$\omega = \frac{1}{2}(\ln(a))^2\kappa_1^2 + \frac{1}{2}(\ln(a))^2\kappa_1\lambda_1 + (\ln(a))^2\lambda_1^2 + \kappa_2^2 + \kappa_2\lambda_2 + \lambda_2^2 - 1 \tag{4.11}$$

Therefore, the following solitary wave solution to the generalized Schrödinger equation involving the Kerr law is constructed

$$u(x,t) = \left(\sqrt{-\frac{\kappa_1^2+\kappa_1\lambda_1+\lambda_1^2}{2\alpha}}\ln(a) + \frac{\ln(a)(\kappa_1^2+\kappa_1\lambda_1+\lambda_1^2)}{\alpha\sqrt{-\frac{\kappa_1^2+\kappa_1\lambda_1+\lambda_1^2}{2\alpha}}}\frac{1}{1+da^{\kappa_1x+\lambda_1y-vt}}\right)e^{i(\kappa_2x+\lambda_2y-\omega t)} \tag{4.12}$$

where

$$v = 2\kappa_1\kappa_2 + \kappa_1\lambda_2 + \kappa_2\lambda_1 + 2\lambda_1\lambda_2$$

$$\omega = \frac{1}{2}(\ln(a))^2\kappa_1^2 + \frac{1}{2}(\ln(a))^2\kappa_1\lambda_1 + (\ln(a))^2\lambda_1^2 + \kappa_2^2 + \kappa_2\lambda_2 + \lambda_2^2 - 1$$

The dynamic of the above dark Solitary has been presented in Figure 7 for $\kappa_1 = 0.5$, $\lambda_1 = 0.2$, $\kappa_2 = 0.2$, $\lambda_2 = 0.5$, $a = 2.7$, $d = 1$, $y = 0$ when (a) $\alpha = -1$ and (b) $\alpha = -1.5$. From Figure 1, it is obviously determined that by increasing the value of $|\alpha|$, the amplitude of the solitary waves decreases whiles its width increases.

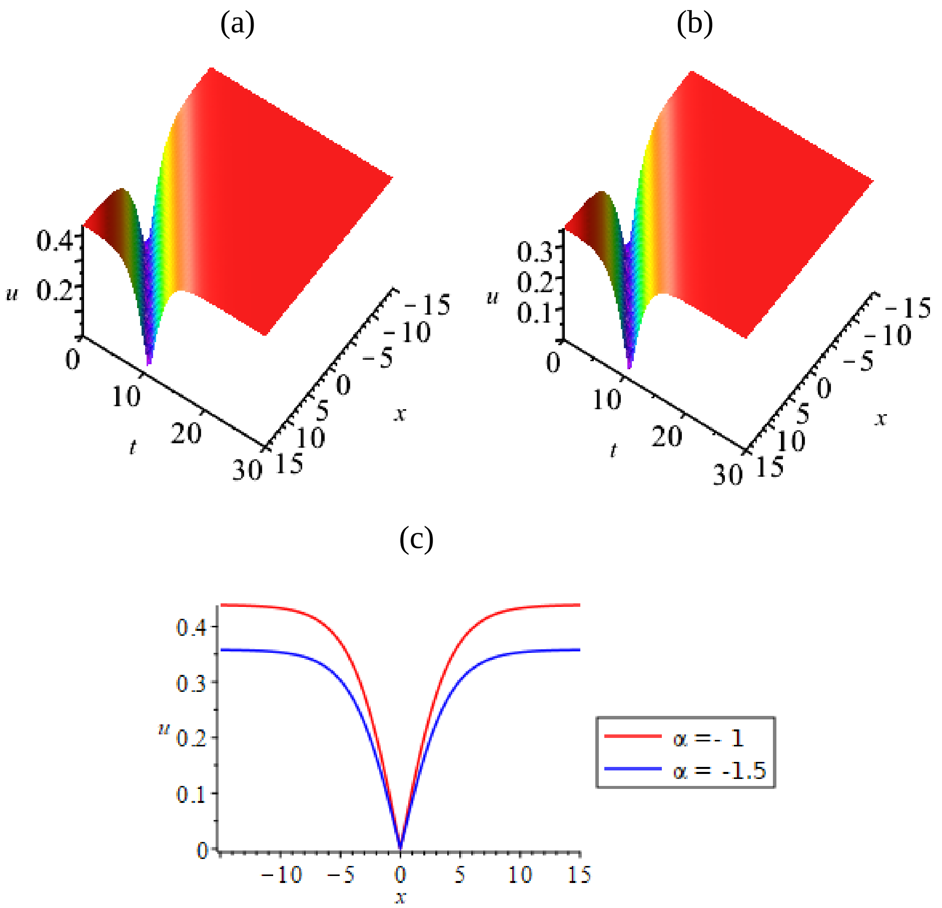


Figure 7: $u(x,t)$ for $\kappa_1 = 0.5, \lambda_1 = 0.2, \kappa_2 = 0.2, \lambda_2 = 0.5, a = 2.7, d = 1,$ $y = 0$ when (a) $\alpha = -1$ and (b) $\alpha = -1.5$.

4.1.2. Applying the Kudryashov method II

To extract solitary waves of the governing equation, we first try to rewrite Eq. (4.2) as follows

$$U_{\epsilon\epsilon} - AU + BU^3 = 0 \quad (4.13)$$

where

$$A = \frac{\kappa_2^2+\kappa_2\lambda_2+\lambda_2^2-\omega-1}{\kappa_1^2+\kappa_1\lambda_1+\lambda_1^2}, \quad B = \frac{\alpha}{\kappa_1^2+\kappa_1\lambda_1+\lambda_1^2}$$

Multiplying Eq. (4.13) by U_ϵ and integrating with respect to ϵ leads to

$$U_\epsilon^2 - AU^2 + \frac{1}{2}BU^4 = 0 \quad (4.14)$$

The generalized method seeks the solitary wave solution of Eq. (4.14) as

$$U(\epsilon) = F(\xi), \;\; \xi = \phi(\epsilon) \quad (4.15)$$

By applying the chain rule to (4.15), we derive

$$U_\epsilon = F_\xi \xi_\epsilon \quad (4.16)$$

Setting (4.15) and (4.16) in Eq. (4.14) results in

$$F_\xi^2 \xi_\epsilon^2 = AF^2 - \frac{1}{2}BF^4 \tag{4.17}$$

Now, by assuming $\xi_\epsilon = F(\xi)$, Eq. (4.17) is written as

$$F_\xi^2 = A - \frac{1}{2}BF^2 \tag{4.18}$$

The exact solution of Eq. (4.18) is

$$U(\epsilon) = F(\xi) = \sqrt{\frac{2A}{B}}\sin\left(\frac{\sqrt{2B}}{2}(\xi_0 - \xi)\right) \tag{4.19}$$

where ξ_0 denotes a free constant.

By using the following integral

$$\epsilon = \epsilon_0 + \int \frac{d\xi}{F(\xi)}$$

we have

$$\epsilon = \epsilon_0 - \frac{\ln\left(\csc\left(\frac{\sqrt{2B}}{2}(\xi - \xi_0)\right) - \cot\left(\frac{\sqrt{2B}}{2}(\xi - \xi_0)\right)\right)}{\sqrt{B}\sqrt{\frac{A}{B}}} \tag{4.20}$$

From Eq. (4.19), we find

$$\xi = \xi_0 - \sqrt{\frac{2}{B}}\sin^{-1}\left(\frac{1}{\sqrt{\frac{2A}{B}}}U(\epsilon)\right) \tag{4.21}$$

By inserting Eq. (4.21) into Eq. (4.20), we find

$$\epsilon = \epsilon_0 - \frac{\sqrt{B}}{\sqrt{AB}}\ln\left(\frac{\sqrt{AB}\left(-\sqrt{2}A + \sqrt{(-BU^2(\epsilon) + 2A)A}\right)}{ABU(\epsilon)}\right) \tag{4.22}$$

By rewriting Eq. (4.22) as

$$\frac{\sqrt{AB}}{\sqrt{B}}(\epsilon - \epsilon_0) = -\ln\left(\frac{\sqrt{AB}\left(-\sqrt{2}A + \sqrt{(-BU^2(\epsilon) + 2A)A}\right)}{ABU(\epsilon)}\right)$$

considering the base e to both sides of it and solving the resulting equation for $U(\epsilon)$, we obtain

$$U(\epsilon) = 2\sqrt{2}\sqrt{\frac{A}{B}}\frac{e^{\sqrt{A}(\epsilon - \epsilon_0)}}{1 + e^{2\sqrt{A}(\epsilon - \epsilon_0)}}.$$

So, the solitary wave solution of the generalized Schrödinger equation with the Kerr law is derived as

$$u(x,t) = 2\sqrt{2}\sqrt{\frac{A}{B}}\frac{e^{\sqrt{A}(\kappa_1 x + \lambda_1 y - vt - \epsilon_0)}}{1 + e^{2\sqrt{A}(\kappa_1 x + \lambda_1 y - vt - \epsilon_0)}} e^{i(\kappa_2 x + \lambda_2 y - \omega t)}$$

where

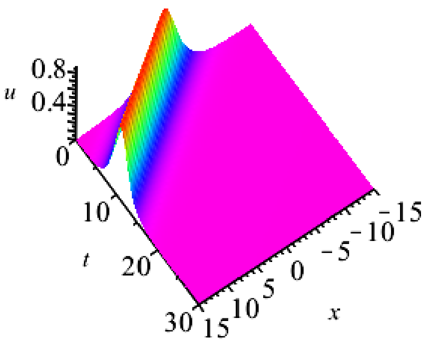
$$A = \frac{\kappa_2^2+\kappa_2\lambda_2+\lambda_2^2-\omega-1}{\kappa_1^2+\kappa_1\lambda_1+\lambda_1^2}$$

$$B = \frac{\alpha}{\kappa_1^2+\kappa_1\lambda_1+\lambda_1^2}$$

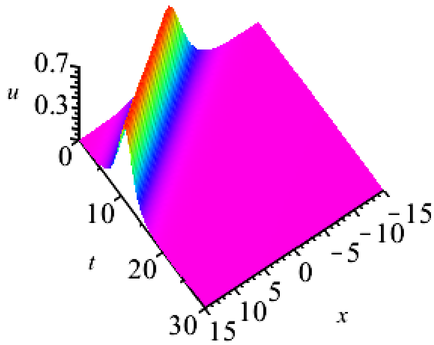
$$v = 2\kappa_1\kappa_2 + \kappa_1\lambda_2 + \kappa_2\lambda_1 + 2\lambda_1\lambda_2$$

Figure 8 signifies the dynamic of the above bright Solitary for $\kappa_1 = 0.5$, $\lambda_1 = 0.2$, $\kappa_2 = 0.2$, $\lambda_2 = 0.5$, $\omega = 1$, $y = 0$ when (a) $\alpha = 1$ and (b) $\alpha = 1.5$. From Figure 1, it is obviously determined that by increasing the value of α, both the amplitude and width of the solitary wave decrease.

(a)



(b)



(c)

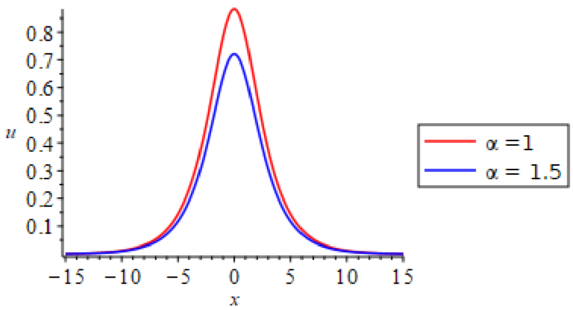


Figure 8: $u(x,t)$ for $\kappa_1 = 0.5$, $\lambda_1 = 0.2$, $\kappa_2 = 0.2$, $\lambda_2 = 0.5$, $\omega = 1$, $y = 0$ when (a) $\alpha = 1$ and (b) $\alpha = 1.5$.

4.2. Generalized Schrödinger equation involving the parabolic law

Foremost, we consider the following generalized Schrödinger equation involving the parabolic law nonlinearity

$$i\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y \partial x} + \frac{\partial^2 u}{\partial y^2} + u + \alpha|u|^2 u + \beta|u|^4 u = 0 \tag{4.23}$$

Assume a transformation such as

$u = U(\epsilon)e^{i(\kappa_2 x+\lambda_2 y-\omega t)}, \ \ \epsilon = \kappa_1 x + \lambda_1 y - vt$

where v and ω are the speed and frequency of the solitary wave respectively. After inserting the above transformation into Eq. (4.23) and distinguishing real and imaginary portions, we find

$$\left((2\lambda_2 + \kappa_2)\lambda_1 + \kappa_1\lambda_2 + \kappa_1\kappa_2 - v\right)\frac{dU(\epsilon)}{d\epsilon} = 0$$

$$(\kappa_1^2 + \kappa_1\lambda_1 + \lambda_1^2)\frac{d^2U(\epsilon)}{d\epsilon^2} + (-\kappa_2^2 - \kappa_2\lambda_2 - \lambda_2^2 + \omega + 1)U(\epsilon) + \alpha U^3(\epsilon) + \beta U^5(\epsilon) = 0$$

From the imaginary part, we have the speed of the Solitary as

$v = 2\kappa_1\kappa_2 + \kappa_1\lambda_2 + \kappa_2\lambda_1 + 2\lambda_1\lambda_2$

Now, the transformation $U(\epsilon) = \sqrt{\Psi(\epsilon)}$ results in

$$\frac{1}{2}(\kappa_1^2 + \kappa_1\lambda_1 + \lambda_1^2)\Psi(\epsilon)\frac{d^2\Psi(\epsilon)}{d\epsilon^2} - \frac{1}{4}(\kappa_1^2 + \kappa_1\lambda_1 + \lambda_1^2)\left(\frac{d\Psi(\epsilon)}{d\epsilon}\right)^2 + (-\kappa_2^2 - \kappa_2\lambda_2 - \lambda_2^2 + \omega + 1)\Psi^2(\epsilon) + \alpha\Psi^3(\epsilon) + \beta\Psi^4(\epsilon) = 0 \quad (4.24)$$

By considering $\Psi''(\epsilon)\Psi(\epsilon)$ and $\Psi^4(\epsilon)$ and applying the balance principle, we derive

$2N + 2 = 4N$

and so $N = 1$

4.2.1. Applying the Kudryashov Method I

By effecting $N = 1$ on Eq. (3.3), a finite series is acquired as

$$\Psi(\epsilon) = a_0 + a_1 K(\epsilon) \quad (4.25)$$

where a_0 and a_1 are unknown constants, and

$$K'(\epsilon) = K(\epsilon)(K(\epsilon) - 1)\ln(a) \quad (4.26)$$

Substituting (4.25) and (4.26) into Eq. (4.24) and collecting the terms in different powers of $K(\epsilon)$, we find a system of algebraic type whose solution gives

$a_0 = 0$

$a_1 = -\frac{3\alpha}{4\beta}$

$\omega = \frac{16\beta\kappa_2^2 + 16\beta\kappa_2\lambda_2 + 16\beta\lambda_2^2 + 3\alpha^2 - 16\beta}{16\beta}$

$\kappa_1 = -\frac{\beta\lambda_1\ln(a) + \sqrt{-3\beta^2\lambda_1^2(\ln(a))^2 - 3\alpha^2\beta}}{2\beta\ln(a)}$

Therefore, the following solitary wave solution to the generalized Schrödinger equation involving the parabolic law is constructed

$$u(x,t) = \sqrt{-\frac{3\alpha}{4\beta}\frac{1}{1 + da^{\kappa_1 x + \lambda_1 y - vt}}}\,e^{i(\kappa_2 x + \lambda_2 y - \omega t)}$$

where

$v = 2\kappa_1\kappa_2 + \kappa_1\lambda_2 + \kappa_2\lambda_1 + 2\lambda_1\lambda_2$, $\omega = \frac{16\beta\kappa_2^2+16\beta\kappa_2\lambda_2+16\beta\lambda_2^2+3\alpha^2-16\beta}{16\beta}$, $\kappa_1 = -\frac{\beta\lambda_1\ln(a)+\sqrt{-3\beta^2\lambda_1^2(\ln(a))^2-3\alpha^2\beta}}{2\beta\ln(a)}$

The dynamic of the above kink solitary wave has been presented in Figure 9 for $\kappa_1 = 0.5$, $\lambda_1 = 0.2$, $\kappa_2 = 0.2$, $\lambda_2 = 0.5$, $\alpha = -1$, $a = 2.7$, $d = 1$, $y = 0$ when (a) $\beta = -1$ and (b) $\beta = -1.1$. From Figure 9, it is obviously determined that by increasing the value of $|\beta|$, the amplitude of the solitary wave decreases.

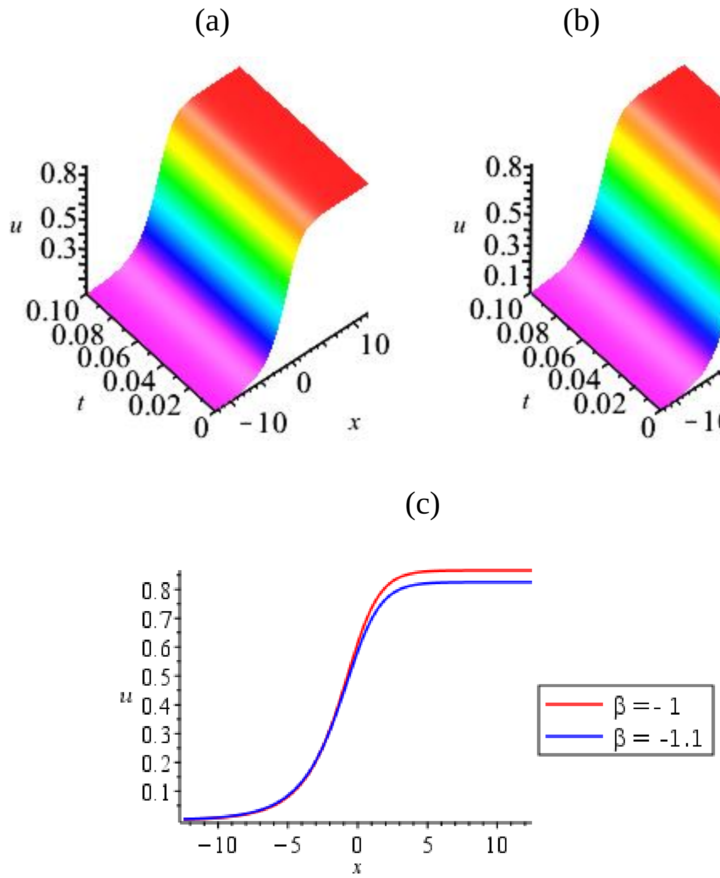


Figure 9: $u(x,t)$ for $\kappa_1 = 0.5$, $\lambda_1 = 0.2$, $\kappa_2 = 0.2$, $\lambda_2 = 0.5$, $\alpha = -1$, $a = 2.7$, $d = 1$, $y = 0$ when (a) $\beta = -1$ and (b) $\beta = -1.1$.

4.2.2. Applying the Kudryashov Method II

To construct the solitary wave solution of the governing equation, we first consider $\kappa_1 = 1$, $\kappa_2 = 1$, $\lambda_1 = 1$, and $\lambda_2 = 1$. Accordingly, we find

$$(v-6)\frac{dU(\epsilon)}{d\epsilon} = 0$$

$$\frac{d^2U(\epsilon)}{d\epsilon^2} + \frac{\omega-2}{3}U(\epsilon) + \frac{1}{3}\alpha U^3(\epsilon) + \frac{1}{3}\beta U^5(\epsilon) = 0 \quad (4.27)$$

From the imaginary part, we have the speed of the solitary wave as $v = 6$

We first rewrite Eq. (4.27) as follows

$$U_{\epsilon\epsilon} + \frac{\omega-2}{3}U(\epsilon) + \frac{1}{3}\alpha U^3(\epsilon) + \frac{1}{3}\beta U^5(\epsilon) = 0 \tag{4.28}$$

Letting $\omega - 2 = 0$ yields the simpler ODE

$$U_{\epsilon\epsilon} + \frac{1}{3}\alpha U^3(\epsilon) + \frac{1}{3}\beta U^5(\epsilon) = 0 \tag{4.29}$$

Multiplying Eq. (4.29) by U_ϵ and integrating w.r.t. ϵ leads to

$$U_\epsilon^2 + \frac{1}{6}\alpha U^4(\epsilon) + \frac{1}{9}\beta U^6(\epsilon) = 0. \tag{4.30}$$

The generalized method seeks the solitary wave solution of Eq. (4.30) as

$$U(\epsilon) = F(\xi),\ \ \xi = \phi(\epsilon). \tag{4.31}$$

By applying the chain rule to (4.31), we derive

$$U_\epsilon = F_\xi \xi_\epsilon \tag{4.32}$$

Setting (4.31) and (4.32) in Eq. (4.30) results in

$$F_\xi^2 \xi_\epsilon^2 = -\frac{1}{6}\alpha F^4(\epsilon) - \frac{1}{9}\beta F^6(\epsilon) \tag{4.33}$$

Now, by assuming $\xi_\epsilon = F(\xi)$, Eq. (4.33) is written as

$$F_\xi^2 = -\frac{1}{6}\alpha F^2(\epsilon) - \frac{1}{9}\beta F^4(\epsilon) \tag{4.34}$$

The exact solution of Eq. (4.34) is

$$U(\epsilon) = F(\xi) = -\frac{24\alpha e^{-\frac{\sqrt{-6\alpha}}{6}(\xi_0-\xi)}}{e^{-\frac{\sqrt{-6\alpha}}{3}(\xi_0-\xi)} - 96\alpha\beta} \tag{4.35}$$

where ξ_0 denotes a free constant.

By using the following integral

$$\epsilon = \epsilon_0 + \int \frac{d\xi}{F(\xi)}$$

we have

$$\epsilon = \epsilon_0 + \frac{\sqrt{6}\left(96\alpha\beta e^{-\frac{\sqrt{-6\alpha}}{6}(\xi_0-\xi)} + e^{-\frac{\sqrt{-6\alpha}}{6}(\xi_0-\xi)}\right)}{24(-\alpha)^{\frac{3}{2}}} \tag{4.36}$$

From Eq. (4.35), we find

$$\xi = \frac{\sqrt{6}\ln\left(\frac{4\left(-3\alpha+\sqrt{6\alpha\beta U^2+9\alpha^2}\right)+\xi_0\sqrt{-\alpha}}{U}\right)}{\sqrt{-\alpha}} \tag{4.37}$$

By inserting Eq. (4.37) into Eq. (4.36), we find

$$\epsilon = \epsilon_0 - \frac{\sqrt{6}\left(-2\beta U^2+\sqrt{3}\sqrt{2\alpha\beta U^2+3\alpha^2}-3\alpha\right)}{\left(\sqrt{3}\sqrt{2\alpha\beta U^2+3\alpha^2}-3\alpha\right)\sqrt{-\alpha}U} \tag{4.38}$$

From Eq. (4.38), U can be found as

$$U(\epsilon) = \frac{\sqrt{-6(\alpha^2\epsilon^2 - 2\alpha^2\epsilon_0\epsilon + \alpha^2\epsilon_0^2 + 4\beta)\alpha}}{\alpha^2\epsilon^2 - 2\alpha^2\epsilon_0\epsilon + \alpha^2\epsilon_0^2 + 4\beta}$$

So, the exact solution of the generalized Schrödinger equation with the parabolic law is derived as

$$u(x,t) = \frac{\sqrt{-6(\alpha^2(x+y-6t)^2 - 2\alpha^2\epsilon_0(x+y-6t) + \alpha^2\epsilon_0^2 + 4\beta)\alpha}}{\alpha^2(x+y-6t)^2 - 2\alpha^2\epsilon_0(x+y-6t) + \alpha^2\epsilon_0^2 + 4\beta} e^{i(x+y-2t)}$$

The dynamic of the above exact solution has been presented in Figure 10 for $\alpha = 10$, $\epsilon_0 = 0$, $y = 0$ when (a) $\beta = 10$ and (b) $\beta = 15$. From Figure 10, it is obviously determined that by increasing the value of β, both the amplitude and width of the solitary wave decrease.

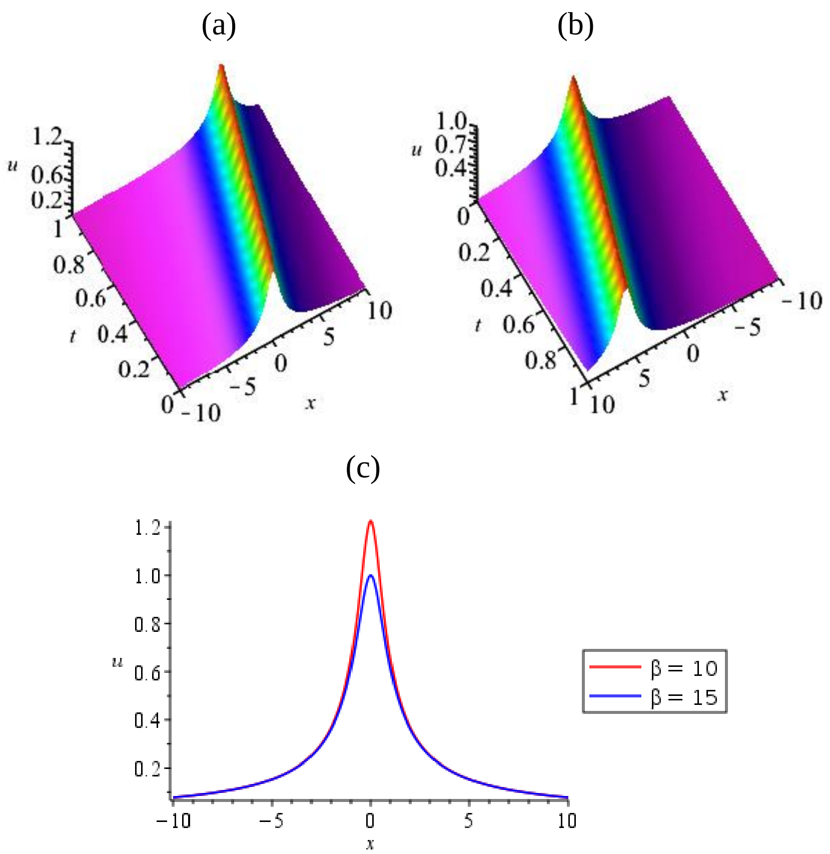


Figure 10: $u(x,t)$ for $\alpha = 10$, $\epsilon_0 = 0$, $y = 0$ when (a) $\beta = 10$ and (b) $\beta = 15$.

CHAPTER V
Conclusion and Recommendations

The primary focus of this thesis was on utilizing various forms of the Kudryashov methods to construct solitary waves for generalized Schrödinger equations involving the Kerr law and the parabolic law. Both methods demonstrated their efficacy in generating diverse solitary wave solutions for the nonlinear models under consideration.

In the first case, we considered the following generalized Schrödinger equation involving the Kerr law nonlinearity, i.e.

$$i\frac{\partial u}{\partial t}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y\partial x}+\frac{\partial^2 u}{\partial y^2}+u+\alpha|u|^2u=0$$

and derived its dark and bright solitary wave. We showed that

- by increasing the value of $|\alpha|$, the amplitude of the dark solitary wave decreases while its width increases;
- by increasing the value of $|\alpha|$, both the amplitude and width of the solitary wave decrease.

In the second case, we employed the following generalized Schrödinger equation involving the parabolic law nonlinearity, i.e.

$$i\frac{\partial u}{\partial t}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y\partial x}+\frac{\partial^2 u}{\partial y^2}+u+\alpha|u|^2u+\beta|u|^4u=0$$

and obtained its kink solution and rational solitary wave. We demonstrated that

- by increasing the value of $|\beta|$, the amplitude of the kink solitary wave decreases;
- by increasing the value of β, both the amplitude and width of the rational solitary wave decrease.

Regarding Kudryashov method I, it is evident that this approach adeptly addresses a wide array of nonlinear partial differential equations, particularly those of higher order, with notable efficiency. However, it is important to note that the applicability of this method hinges on the prerequisite that the assumed balance number is an integer; otherwise, it cannot be employed to address the governing model.

In contrast, Kudryashov method II presents an intriguing alternative, as it does not impose the constraint of the balance number being an integer. This method proves capable of handling equations in the form of $U_{\epsilon}^{2} = P(U)E(U)$, provided that the integral $\int \frac{d\xi}{\sqrt{P(F)}}$ exists or remains well-defined.

In light of these findings, it is recommended that researchers and practitioners consider the specific characteristics of the nonlinear equations at hand when selecting an appropriate method from the Kudryashov toolkit. Additionally, further exploration could be undertaken to investigate the potential applicability of Kudryashov method II in scenarios where the balance number constraint becomes a limiting factor for method I. This could contribute to expanding the versatility and utility of the Kudryashov techniques in tackling a broader range of nonlinear problems.

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APPENDICES

Appendix A

Similarity Report

THE KUDRYASHOV METHODS FOR CONSTRUCTING SOLITARY WAVES OF SCHRÖDINGER EQUATIONS

by Gibert Boakye

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