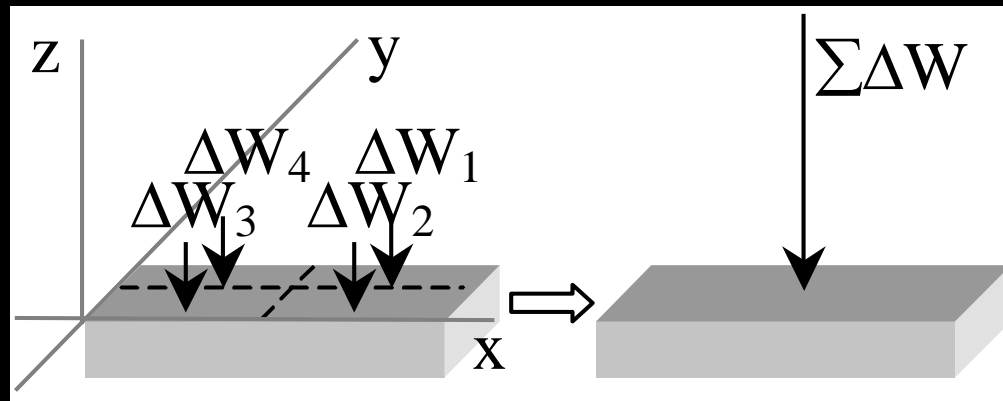




# Center of Gravity

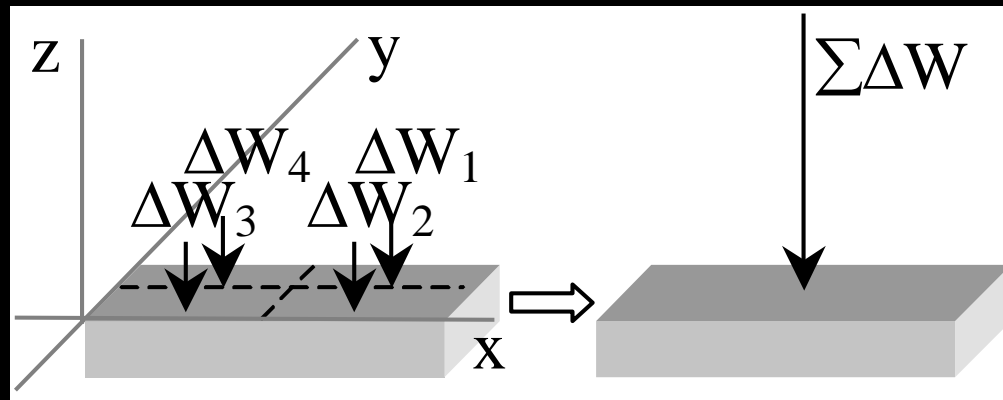
- *location of equivalent weight*
- *determined with calculus*



- *sum element weights*      $W = \int dW$

# Center of Gravity

- “average”  $x$  &  $y$  from moment



$$\sum M_y = \sum_{i=1}^n x_i \Delta W_i = \bar{x} W \Rightarrow \bar{x} = \frac{\sum (x \Delta W)}{W}$$

“bar” means average

$$\sum M_x = \sum_{i=1}^n y_i \Delta W_i = \bar{y} W \Rightarrow \bar{y} = \frac{\sum (y \Delta W)}{W}$$

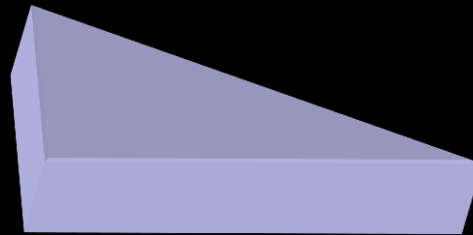
# Centroid

---

- “average”  $x$  &  $y$  of an area
- for a volume of constant thickness
  - $\Delta W = \gamma t \Delta A$  where  $\gamma$  is weight/volume
  - center of gravity = centroid of area

$$\bar{x} = \frac{\sum(x\Delta A)}{A}$$

$$\bar{y} = \frac{\sum(y\Delta A)}{A}$$

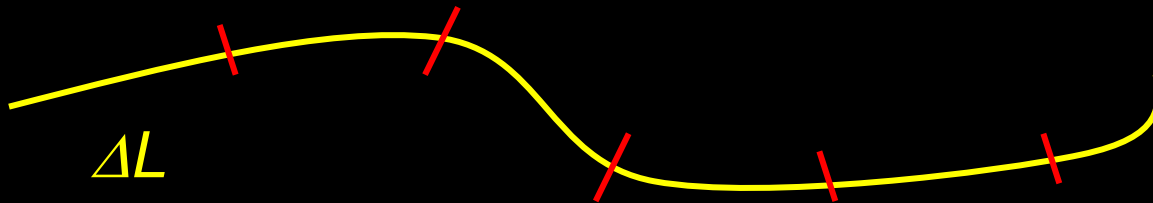


# Centroid

- for a line, sum up length

$$\bar{x} = \frac{\sum(x\Delta L)}{L}$$

$$\bar{y} = \frac{\sum(y\Delta L)}{L}$$

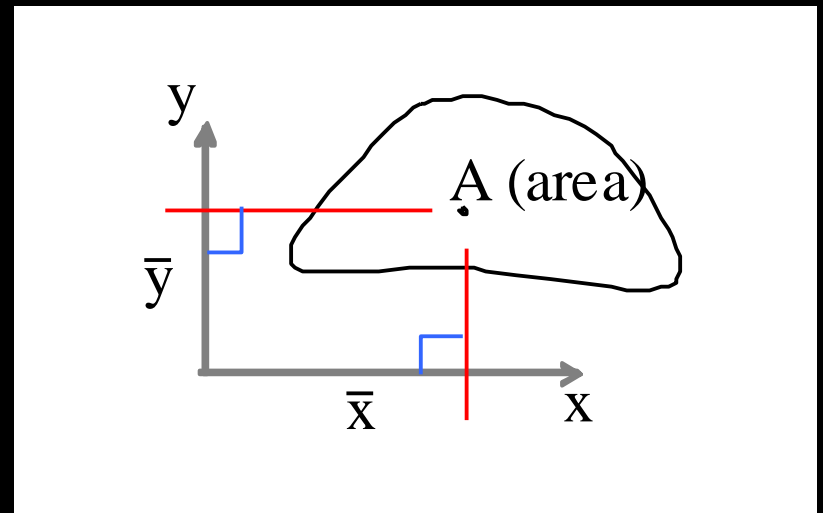


# 1<sup>st</sup> Moment Area

- *math concept*
- *the moment of an area about an axis*

$$Q_x = \bar{y}A$$

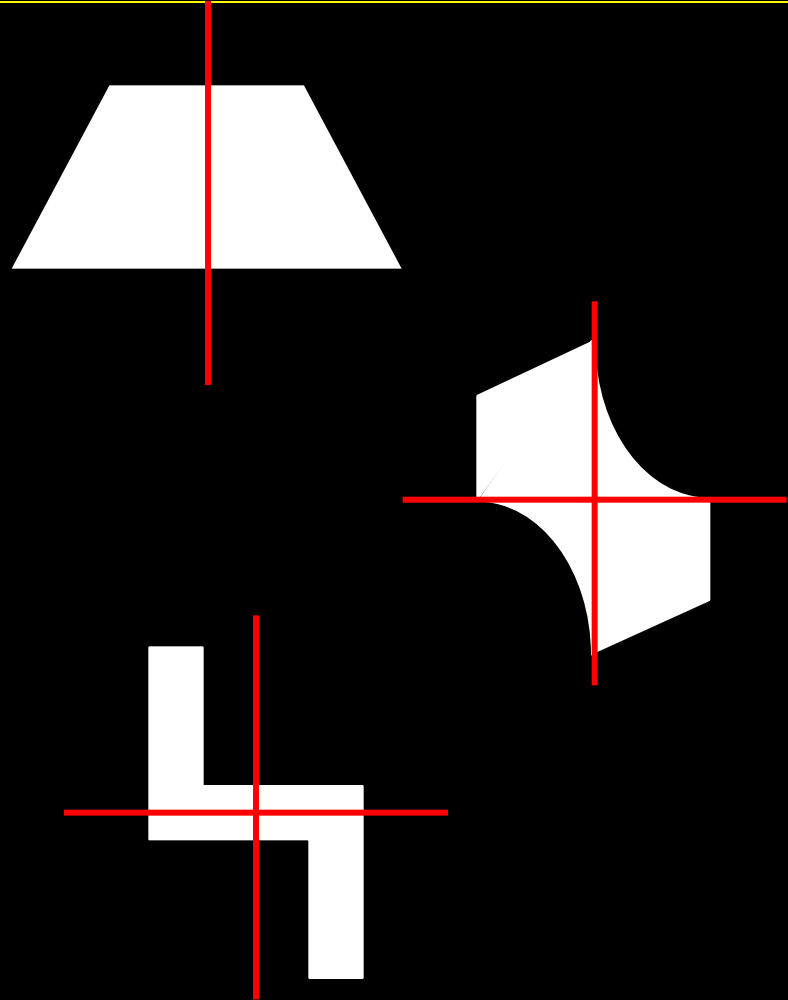
$$Q_y = \bar{x}A$$



# Symmetric Areas

---

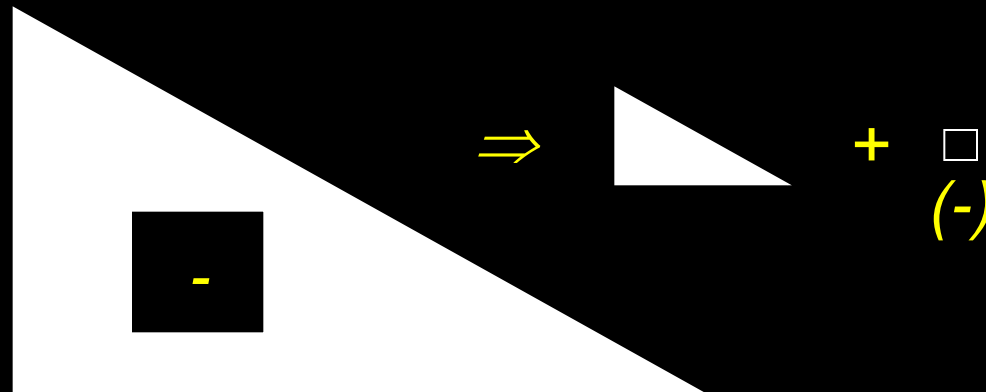
- *symmetric about an axis*
- *symmetric about a center point*
- *mirrored symmetry*



# Composite Areas

---

- *made up of basic shapes*
- *areas can be negative*
- *(centroids can be negative for any area)*





# Basic Procedure

---

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate  $\hat{x}$  and  $\hat{y}$

Component	Area	$\bar{x}$	$\bar{x}A$	$\bar{y}$	$\bar{y}A$
$\Sigma$					

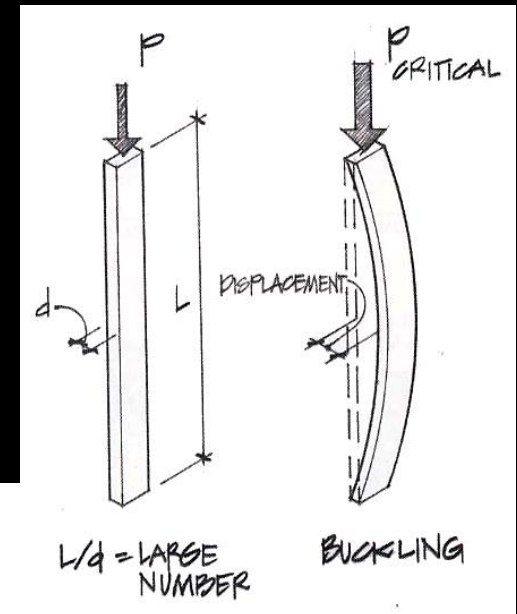
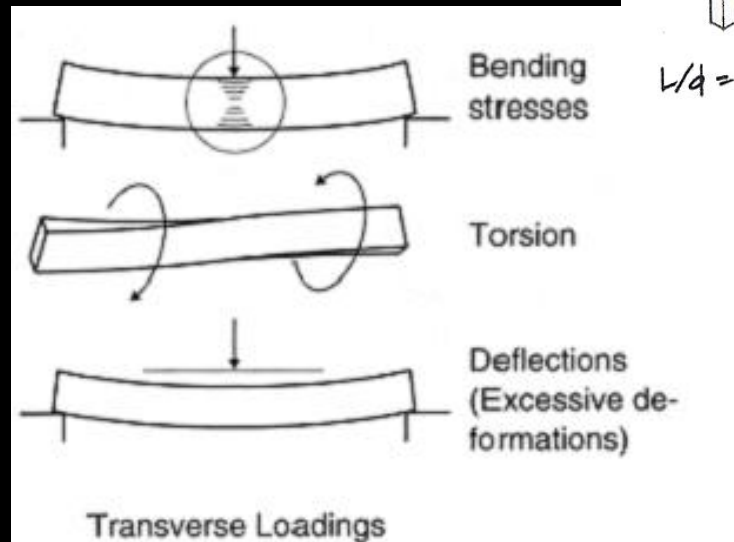
# Area Centroids

- *Table 7.1 – pg. 242*

Centroids of Common Shapes of Areas and Lines			
Shape		$\bar{x}$	$\bar{y}$
Triangular area		$\frac{b}{3}$	$\frac{h}{3}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Semicircular area		0	$\frac{4r}{3\pi}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$
Parabolic area		0	$\frac{3h}{5}$

# Moments of Inertia

- 2<sup>nd</sup> moment area
  - math concept
  - $\text{area} \times (\text{distance})^2$
- need for behavior of
  - beams
  - columns

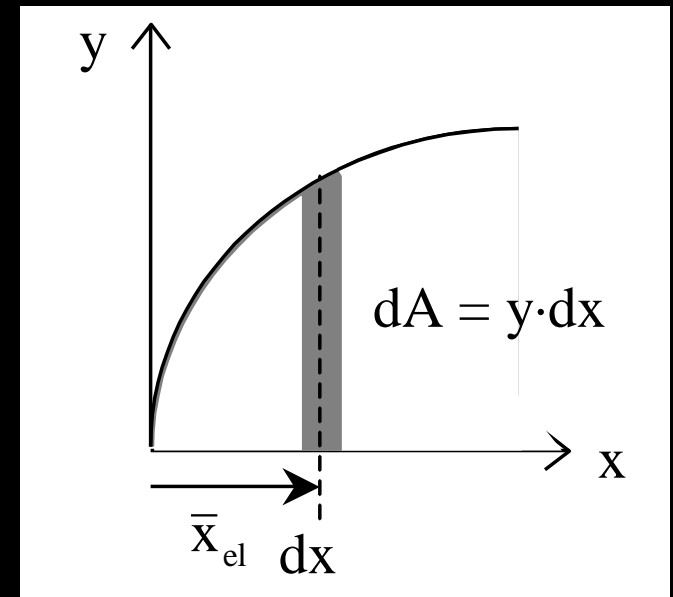


# Moment of Inertia

- about any reference axis
- can be negative

$$I_y = \int x^2 dA$$

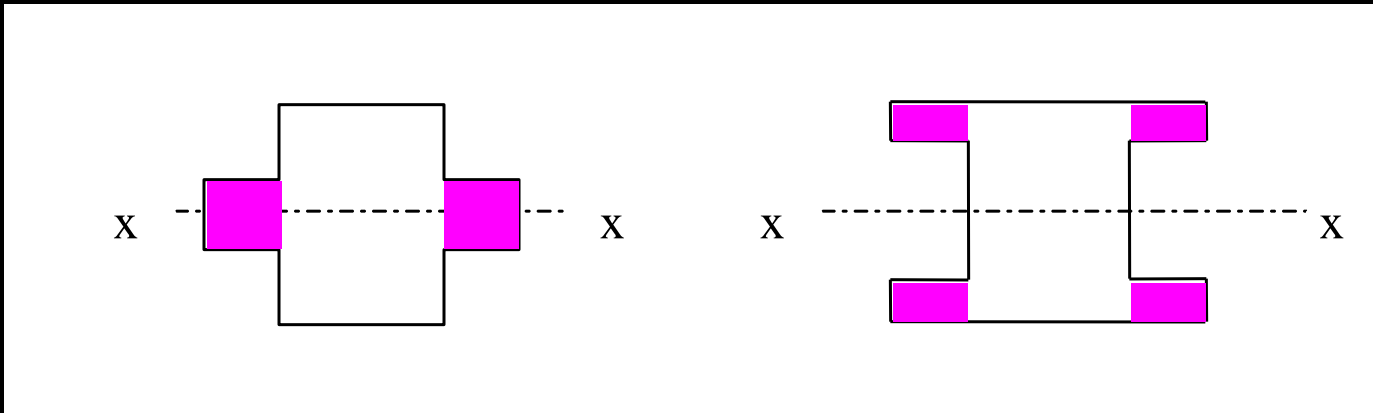
$$I_x = \int y^2 dA$$



- resistance to bending and buckling

# Moment of Inertia

- *same area moved away a distance*  
– *larger  $I$*

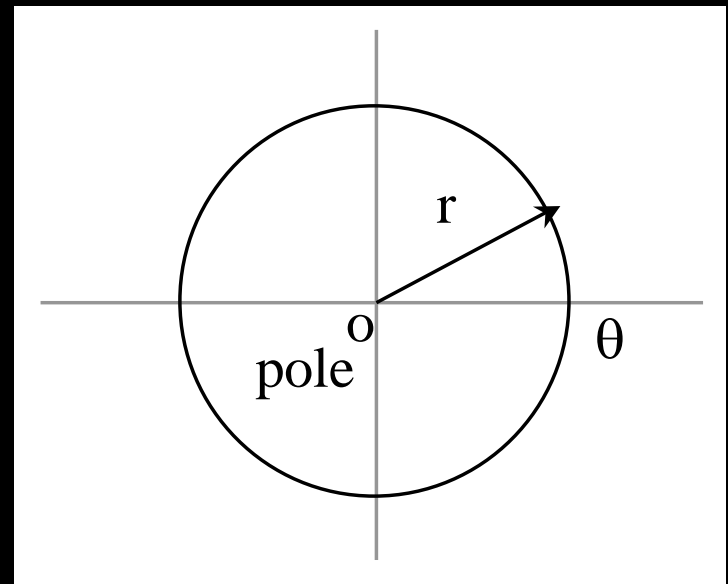


# Polar Moment of Inertia

---

- *for roundish shapes*
- *uses polar coordinates ( $r$  and  $\theta$ )*
- *resistance to twisting*

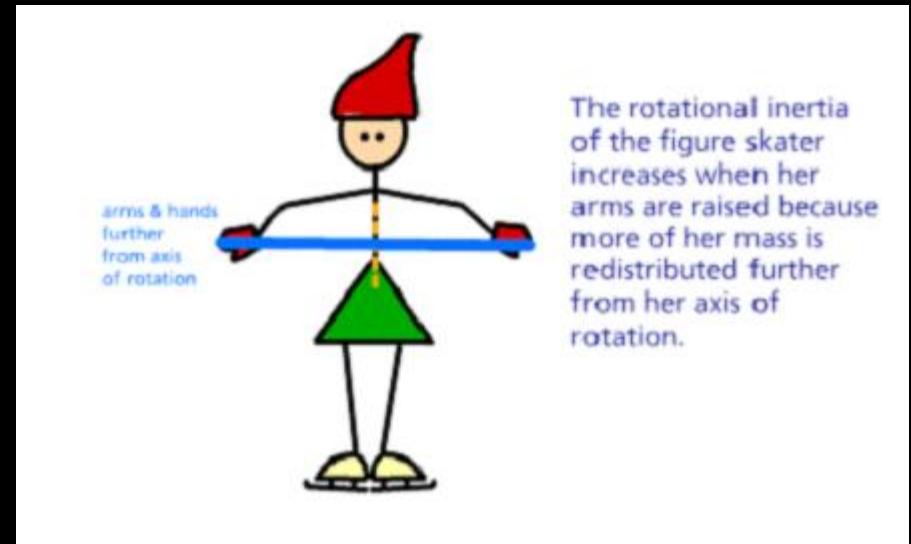
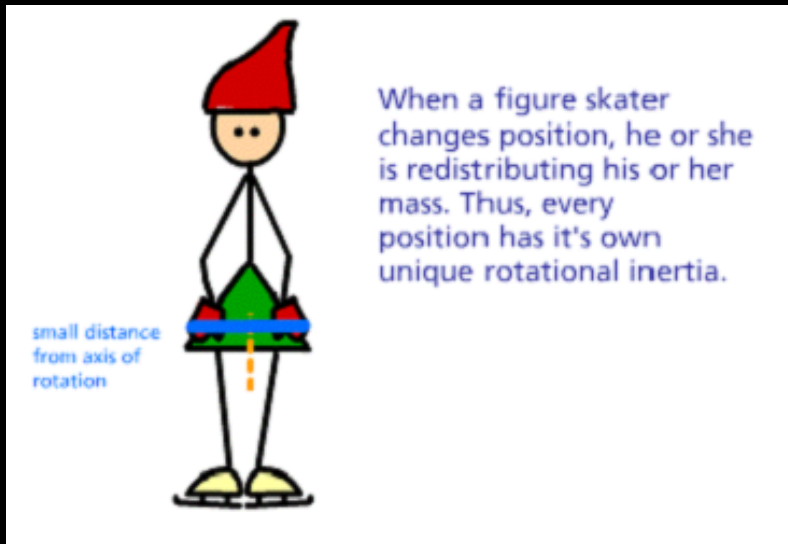
$$J_o = \int r^2 dA$$



# Radius of Gyration

- *measure of inertia with respect to area*

$$r_x = \sqrt{\frac{I_x}{A}}$$



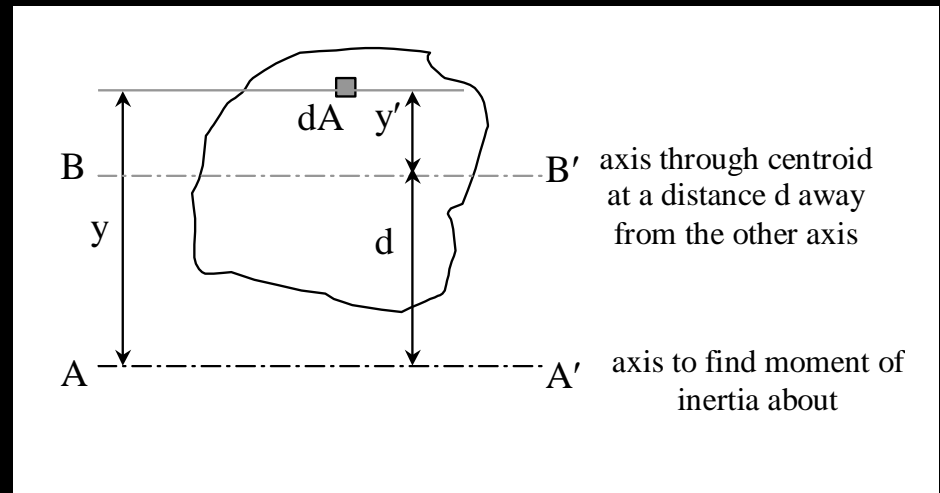
# Parallel Axis Theorem

- can find composite  $I$  once composite centroid is known (basic shapes)

$$\begin{aligned} I_x &= I_{cx} + Ad_y^2 \\ &= \bar{I}_x + Ad_y^2 \end{aligned}$$

$$I = \sum \bar{I} + \sum Ad^2$$

$$\bar{I} = I - Ad^2$$





# Basic Procedure

---

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with  $A$ ,  $\bar{x}$ ,  $\bar{x}A$ ,  $\bar{y}$ ,  $\bar{y}A$ ,  $\bar{I}$ 's,  $d$ 's, and  $Ad^2$ 's
5. Fill in table and get  $\hat{x}$  and  $\hat{y}$  for composite
6. Sum necessary columns
7. Sum  $\bar{I}$ 's and  $Ad^2$ 's

$$\begin{pmatrix} d_x \\ d_y \end{pmatrix} = \begin{pmatrix} \hat{x} - \bar{x} \\ \hat{y} - \bar{y} \end{pmatrix}$$

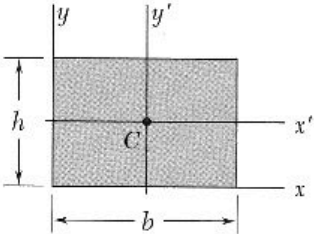
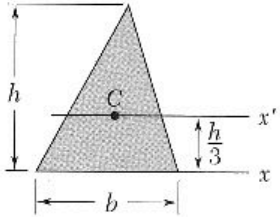
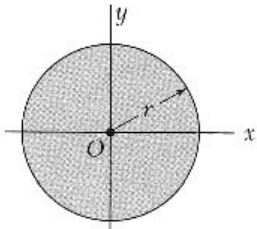
# Area Moments of Inertia

- Table 7.2 – pg. 252: (bars refer to centroid)

–  $x, y$

–  $x', y'$

–  $C$

<p>Rectangle</p>		$\bar{I}_x = \frac{1}{12}bh^3$ $\bar{I}_y = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
<p>Triangle</p>		$\bar{I}_x = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
<p>Circle</p>		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$