Thermal behaviour of heat exchangers at subdesign conditions

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An analytical method has been developed to predict the thermal behaviour of an exchanger when a deviation occurs between the design-point and the actual working conditions. As long as the P_R, P_η curves of a particular exchanger are available, the method makes it possible to estimate the exchanger characteristic parameters at the modified working state. Moreover, at the design stage, the use of exchanger effectiveness gradient vector concept provides a basis for comparing the stability of several proposed exchangers under variable load conditions. Sample problems illustrate the application of the method and the use of presented charts.
(Keywords: heat exchangers; thermodynamics; mathematical models; design; working conditions)

Comportement thermique des échangeurs de chaleur dans des conditions inférieures à celles prévues

Une étude analytique a été mise au point pour prévoir le comportement thermique d'un échangeur lorsqu'il se produit un écart entre les conditions nominales et celles de fonctionnement réelles. Dans la mesure où l'on dispose de courbes P_R, P_η pour un échangeur particulier, cette méthode permet d'estimer les paramètres caractéristiques de l'échangeur pour l'état de fonctionnement modifié. De plus, au stade de la conception, l'utilisation du concept du vecteur de gradient d'efficacité de l'échangeur fournit une base de comparaison de la stabilité de plusieurs échangeurs proposés dans des conditions de puissance variable. Des problèmes donnés en exemple illustrent l'application de la méthode et l'utilisation des diagrammes présentés.
(Mots clés: échangeurs de chaleur; thermodynamique; modèles mathématiques; conception; conditions de fonctionnement)

Fundamental parameters which characterize a particular heat exchanger for a specified heat duty are the flow rates, inlet and outlet temperatures of the fluid streams, the flow arrangement, the required pressure drops and the surface areas on each side. Implication of the design methodology[1,2] provides numerical values to the principal parameters and the cluster of these numerals is often labelled as the design-point for the heat exchanger. Since a heat exchanger is generally a part of a system, it may be exposed to unpredicted loads such as a change in inlet temperatures or flow rates. Resulting response to such changes might cause the principal parameters to deviate considerably at the outlet. A drastic drop in exchanger performance might arise. Thus the exchanger behaviour when it operates at subdesign conditions should be known so that the initial design is made properly. Several optional solutions will be available when the thermal and mechanical designs are completed. To make a meaningful comparison between various exchangers, however, the designer also has to compare the candidate exchangers for their subdesign characteristics, rather than focusing exclusively on the design-point.

In contrast to other process equipment such as pumps, compressors and fans, the operating curves for heat exchangers are not available. This is partly because of the large number of variables involved in the exchanger design. Nevertheless, it is possible to derive explicit relationships to predict the exchanger behaviour when one or several principal variables are modified.

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In this study, owing to the variations in operating conditions around a design-point, the resulting effect on prime factors of the exchanger is analytically investigated. Results are certainly a great value to the control engineer to control the system or the process involved.

Fundamental relations

From a thermodynamic point of view, the exchanger effectiveness, ε, compares the actual heat transfer rate to the maximum possible heat transfer rate as would be attained only in a counter flow heat exchanger of infinite size, namely $q_{max} = C_h(T_1 - t_1)$, if $C_h < C_c$ or $q_{max} = C_c(T_1 - t_1)$, if $C_c < C_h$. Thus ε, is defined as:

$$\varepsilon = \frac{C_h(T_1 - T_2)}{C_{min}(T_1 - t_1)} = \frac{C_c(t_2 - t_1)}{C_{min}(T_1 - t_1)} \tag{1}$$

where C_{min} is the smaller of the C_h and C_c magnitudes. It is to be noted that ε becomes unity when both fluids, having identical heat capacity rates, exchange heat in a reversible manner.

In addition to flow geometry of a particular exchanger, considering the thermo-physical properties of the working fluid, the functional form of the effectiveness can be described as:

$$\varepsilon = \phi(\lambda, \eta, \text{flow geometry}) \tag{2}$$

Nomenclature

A	Heat transfer surface area (m^2)
C_c	Flow-stream capacity rate of cold-side fluid (W °C^{-1})
C_h	Flow-stream capacity rate of hot-side fluid (W °C^{-1})
C_{max}	Maximum of C_c or C_h
C_{min}	Minimum of C_c or C_h
C_{mix}	Mixed fluid capacity rate (W °C^{-1})
E	Magnitude of the effectiveness gradient vector
N	Normalized deviation vector, Equation (16)
P	Exchanger temperature effectiveness
R	Flow-stream capacity rate-ratio, C_c/C_h
dR, ΔR	Increment in R
T	Temperature of hot-side fluid (°C)
U	Overall heat transfer coefficient (W m^{-2} °C^{-1})
$\mathbf{e}_1$, $\mathbf{e}_2$	Unit vectors, see *Figure 4*
m	Mass flow rate (kg s^{-1})
dr	Deviation vector, $\mathbf{dr}=\mathrm{d}R\mathbf{e}_1+\mathrm{d}\eta\mathbf{e}_2$
t	Temperature of cold-side fluid (°C)

Greek letters

ε	Exchanger effectiveness, dimensionless
$\nabla\varepsilon$	Effectiveness gradient vector, Equation (18)
λ	Flow-stream capacity rate ratio, C_{min}/C_{max}
$\Delta\lambda$	Increment in λ
η	Number of heat transfer units, AU/C_{min}
dη, $\Delta\eta$	Increment in η

Subscripts

cs	Cross
i,j	Arguments denoting position on (R0η) plane
p	Parallel
s	Shell and tube
R, λ, η	Indicate partial derivatives
o	Exchanger design-point
1	Inlet
2	Outlet

where, $\lambda=C_{min}/C_{max}$, is the ratio of the smaller to the larger of two heat capacity rates C_h and C_c. The number of heat transfer units, $\eta=AU/C_{min}$, is the parameter which indicates the heat transfer size of the exchanger. The two additional parameters which possess physical significance for describing the thermal behaviour of an exchanger are the temperature effectiveness, and the heat capacity rate ratio which are respectively defined as:

$$P=\frac{t_2-t_1}{T_2-t_1},\quad R=\frac{C_c}{C_h}=\frac{T_1-T_2}{t_2-t_1} \tag{3}$$

The above mentioned non-dimensional parameters can be interrelated as follows:

$$\text{If } C_{min}=C_c,\quad \text{then } R\leqslant 1,\quad \varepsilon=P \quad\text{and}\quad \lambda=R \tag{4}$$

Similarly, if $C_{min}=C_h$ then,

$$R>1,\quad \varepsilon=PR \quad\text{and}\quad \lambda=R^{-1} \tag{5}$$

Specifying the flow arrangement of the exchanger under study, through Equations (2), (4) and (5), the functional dependence of P on η and R can be expressed as:

$$P=P(\eta, R) \tag{6}$$

Determination of exchanger parameters at subdesign conditions

Departure of exchanger parameters from their design-point values due to change in operating conditions can be categorized in two groups:

1. Deviation in temperatures

The flow rates of both fluids are kept constant. Loss in heat transfer area due to fouling is negligible. Only one or two terminal temperatures depart from their original values in such a way that the magnitude of deviations compared to their design-point values, i.e. $\delta T/T_o$, is always less that unity. Thus the effect of property variations on principal parameters can be neglected. The exchanger parameters maintain their original design values at the subdesign condition concerned. In other words, $\eta=\eta_o$, $C_c=(C_c)_o$, $C_h=(C_h)_o$ and by Equation (6), $P=P_o$ where the subscript (o) denotes the design point of the exchanger. Referring to Equation (3) then the following relations can be derived between the new terminal temperatures.

$$P_oT_1+(1-P_o)t_1-t_2=0 \tag{7}$$

$$T_1+R_ot_1-R_ot_2-T_2=0 \tag{8}$$

Pertaining to the change in terminal temperatures, four possible situations may take place in a single-phase, two fluid heat exchanger.

Case a. The inlet temperatures of both fluids are changed from the design-point values of (T_{1o}, t_{1o}) to (T_1, t_1).

Case b. The inlet temperature of the hot side (T_{1o}) and the outlet temperature of the cold side (t_{2o}) streams are altered to (T_1, t_2).

Case c. The outlet temperature of the hot side (T_{2o}) and the inlet temperature of the cold side (t_{1o}) streams are changed to (T_2, t_1).

Case d. The outlet temperatures of both fluids are changed from (T_{2o}, t_{2o}) to (T_2, t_2).

In all cases, the two terminal temperatures of the new operating condition are always prescribed. Accordingly, the other two are resolved by simultaneous solution of Equations (7) and (8). The resulting equations for all temperature related subdesign conditions are presented in *Table 1*.

Table 1 Concise equations to be used for temperature deviated subdesign conditions
Tableau 1 *Equations concises à utiliser pour des conditions inférieures à celles prévues de l'écart de température*

Subdesign conditions	Exchanger terminal temperatures at the modified state			
	t_1 (°C)	T_1 (°C)	T_2 (°C)	t_2 (°C)
Case a	Specified	Specified	$T_1-P_oR_o(T_1-t_1)$	$t_1+P_o(T_1-t_1)$
Case b	$\dfrac{P_oT_1-t_2}{(P_o-1)}$	Specified	$\dfrac{(R_oP_o+P_o-1)T_1-R_oP_ot_2}{(P_o-1)}$	Specified
Case c	Specified	$\dfrac{T_2-R_oP_ot_1}{(1-P_oR_o)}$	Specified	$\dfrac{P_oT_2-(R_oP_o+P_o-1)t_1}{(1-R_oP_o)}$
Case d	$\dfrac{(P_oR_o-1)t_2+P_oT_2}{(R_oP_o+P_o-1)}$	$\dfrac{P_oR_ot_2+(P_o-1)T_2}{(R_oP_o+P_o-1)}$	Specified	Specified

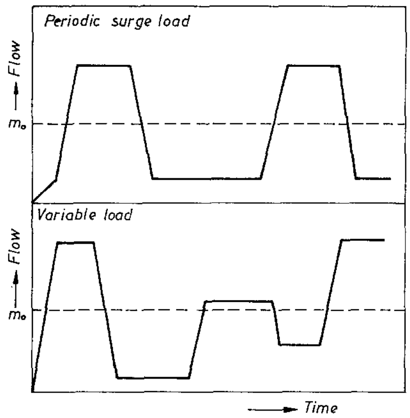

Figure 1 Simplified water demand patterns of an industrial installation
Figure 1 *Configurations simplifiées de la demande d'eau d'une installation industrielle*

2. Deviation in flow rates

Depending upon the system demands, the flow rate of a particular fluid might vary with time. As shown in *Figure 1*, due to shift washing, or specific processes, the hot water consumption of an installation may oscillate around a mean value. Moreover, the fouling layers accumulating on heat transfer surfaces might considerably reduce the heat exchange area. Because of the nature of fluids in use, partial clogging of exchanger tubes will modify the overall heat conductance, and reduce the surface area.

Occurrence of one or several of these instances in an exchanger results with deviations on the principal parameters; η and R. The corresponding change in temperature effectiveness however can be estimated by the Taylor's-series expansion of the relation given by Equation (6) centred about the design-point.

$$P(\eta_o+\Delta\eta, R_o+\Delta R)=P(\eta_o, R_o)+\left[\left(\Delta\eta\frac{\partial}{\partial\eta}+\Delta R\frac{\partial}{\partial R}\right)P\right]_{\substack{\eta=\eta_o\\R=R_o}}$$

$$+\ldots+\left[\frac{1}{n!}\left(\Delta\eta\frac{\partial}{\partial\eta}+\Delta R\frac{\partial}{\partial R}\right)^n P\right]_{\substack{\eta=\eta_o\\R=R_o}} \quad (9)$$

where the subscripts on the derivatives signify that they are to be evaluated at the design point, i.e., $\eta=\eta_o$, $R=R_o$.

For small perturbations around a design point, the higher order terms in the series expansion can be neglected. Linearized temperature effectiveness at the new operating condition then becomes:

$$P=P_o+\left(\frac{\partial P}{\partial\eta}\right)_o\Delta n+\left(\frac{\partial P}{\partial R}\right)_o\Delta R \quad (10)$$

With respect to Equations (4) and (5), the partial derivatives in Equation (10) are expressed in terms of the exchanger effectiveness as:

If $R\leqslant 1$ then $\lambda=R, P_R=\varepsilon_\lambda, P_\eta=\varepsilon_\eta$ (11)

If $R>1$ then $\lambda=R^{-1}$, $P_R=-\lambda^2(\varepsilon+\lambda\varepsilon_\lambda)$, $P_\eta=\lambda\varepsilon_\eta$ (12)

In the literature[3,5], analytical expressions for effectiveness and the number of heat transfer units relationships are available for various types of exchangers. Due to complexity of such expressions however, finite difference technique has been applied for estimating the required derivatives. For a two-dimensional solution domain in which the range of parameters are specified as, $0<\lambda<\lambda_m$, and $0<\eta<\eta_m$, the number of computational grid points will be $i_m\times j_m$ where $i_m=\lambda_m/\Delta\lambda$ and $j_m=\eta_m/\Delta\eta$. The implication of central difference approximation yields the partial derivatives of the exchanger effectiveness at a particular grid point (i, j) as follows:

$$(\varepsilon_\lambda)_{ij}=\frac{\varepsilon_{i+1,j}-\varepsilon_{i-1,j}}{2\Delta\lambda} \quad (13)$$

$$(\varepsilon_\eta)_{ij}=\frac{\varepsilon_{i,j+1}-\varepsilon_{i,j-1}}{2\Delta\eta} \quad (14)$$

Specifying the type of the exchanger under study, ε-effectiveness equations[3] have been utilized for evaluating ε_{ij}s. A square network of spacing is considered in computations, and increments in both λ and η are taken to be 0.05. Such a scheme of algorithm is implemented to shell and tube, and to cross flow heat exchangers, and the resulting (P_R, P_η) curves for these exchangers are presented in *Figures 2* and *3*. Thus, in predicting the exchanger temperature effectiveness a particular subdesign condition, together with the deviations; $\Delta\eta$ and ΔR, the partial derivatives; $(P_\eta, P_R)_o$ to be determined by *Figures 2* or *3*, are substituted into Equation (10).

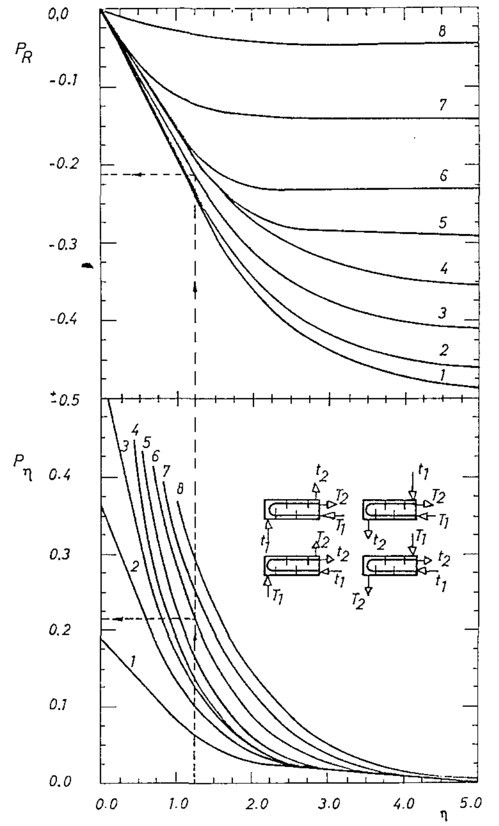

Figure 2 (P_R, P_η) curves for a shell-and-tube heat exchanger with one shell and any multiple of two tube passes. R: 1, 0.00; 2, 0.25; 3, 0.50; 4, 0.75; 5, 1.00; 6, 1.33; 7, 2.00; 8, 4.00

Figure 2 *Courbes P_R, P_η pour un échangeur de chaleur multitubulaire avec une calandre et un multiple de 2 passes dans les tubes*

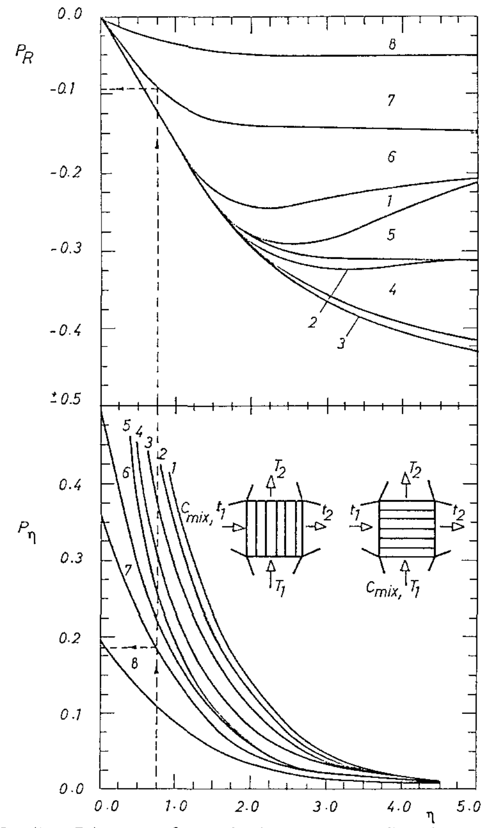

Figure 3 (P_R, P_η) curves for a single-pass, cross-flow heat exchanger with one fluid mixed and the other unmixed. R: 1, 0.00; 2, 0.25; 3, 0.50; 4, 0.75; 5, 1.00; 6, 1.33; 7, 2.00; 8, 4.00. In these curves R is defined as $C_{mixed}/C_{unmixed}$

Figure 3 *Courbes P_R, P_η pour un échangeur de chaleur à courant croisé à une seule passe entre un mélange de fluides et un fluide pur. Dans ces courbes R est défini comme $C_{mélangé}/C_{pur}$*

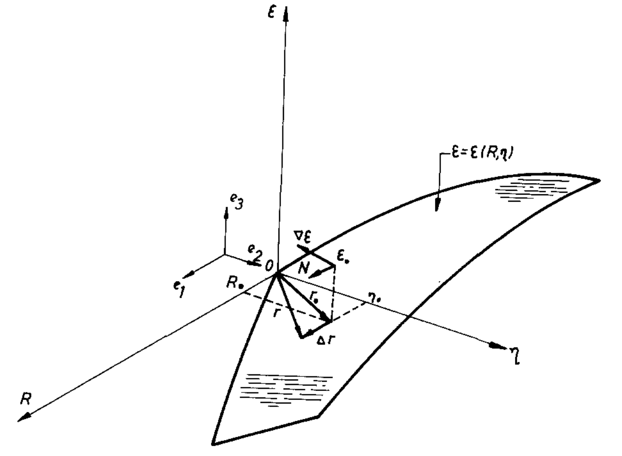

Figure 4 Representation of the ε-effectiveness surface on $(R\eta\varepsilon)$ coordinate system

Figure 4 *Représentation de la surface effective ε dans le système de coordonnées $R\eta\varepsilon$*

Thermal sensitivity of exchangers to subdesign deviations

It is essential for a process designer to be able to compare the performance of the candidate exchangers not only at the design-point but also at subdesign conditions. In *Figure 4*, the exchanger design-point is designated by the position vector, $\mathbf{r}_o = R_o\mathbf{e}_1 + \eta_o\mathbf{e}_2$, on (RO) plane and the corresponding effectiveness is shown on the ε-surface. A change in ε due to deviation in position vector by the amount of $\mathbf{dr}$ is:

$$d\varepsilon = \left(\frac{\partial \varepsilon}{\partial R}\right)dR + \left(\frac{\partial \varepsilon}{\partial \eta}\right)d\eta \tag{15}$$

If one defines the normalized deviation vector as:

$$\mathbf{N} = \left(\frac{dR}{R}\right)\mathbf{e}_1 + \left(\frac{d\eta}{\eta}\right)\mathbf{e}_2 \tag{16}$$

Then the deviation in effectiveness becomes:

$$d\varepsilon = \nabla\varepsilon \cdot \mathbf{N} \tag{17}$$

Where $\nabla\varepsilon$ is called the effectiveness gradient vector and according to Equations (15) and (16), it is expressed in the following form[4]:

$$\nabla\varepsilon = R\left(\frac{\partial \varepsilon}{\partial R}\right)\mathbf{e}_1 + \eta\left(\frac{\partial \varepsilon}{\partial \eta}\right)\mathbf{e}_2 \tag{18}$$

It is evident from Equation (17) that if the two vectors; $\nabla\varepsilon$ and $\mathbf{N}$ are in the same direction then a maximum deviation in performance will take place, i.e. $d\varepsilon_{max} = |\nabla\varepsilon| \cdot |\mathbf{N}|$. Hence the magnitude of $\nabla\varepsilon$ which is represented by E can be used as a measure of stability of the candidate exchangers for identical disturbances around a design-point. The smaller the value of E, the more reduced is the sensitivity of the exchanger to varying conditions, and no appreciable change in performance will be noticed. Depending upon the heat capacity rate ratio, the components of the effectiveness gradient vector are expressible as:

If $R \leqslant 1$ then $E_1 = RP_R$, $E_2 = \eta P_\eta$ (19)

If $R>1$ then $E_1=\varepsilon+R^2P_R$, $E_2=\eta RP_\eta$ (20)

and the magnitude of the gradient vector becomes:

$$E=\sqrt{E_1^2+E_2^2} \tag{21}$$

In addition to relations give above, (P_R, P_η) curves suffice in determining the E-values of an exchanger at a specified condition. Although the method is equally applicable to all types of exchanger, due to their industrial importance, E-values of shell-and-tube and cross-flow type exchangers are computed and tabulated in *Tables 2* and *3*. *Figure 5* also illustrates the typical trend of E curves for three different type heat exchangers. For equal disturbances around an identical operating point, *Figure 5* indicates that the condition $E_p<E_s<E_{cs}$ takes effect. Hence the least discernable change in performance is observed on parallel flow heat exchangers.

Applications

To demonstrate the use of the presented charts for predicting the thermal behaviour of heat exchangers, the following problems frequently raised at the design phase are depicted and numerically studied.

Example 1

The uncertainties in the thermal performance of the two exchangers, namely shell-and-tube, and cross-flow with one fluid mixed, are to be compared. Both exchangers operate at $R_o=0.25$, $\varepsilon_o=0.793$, and their estimated heat transfer coefficients present $\pm 10\%$ uncertainty.

Uncertainty in U singly affects the number of heat transfer units, and the heat capacity rate ratio is constant. From Equation (15) the uncertainty in thermal performance is: $d\varepsilon=(\varepsilon_\eta)_o\, d\eta$, where $d\eta=\pm\eta_o(dU/U_o)$. For $R<1$, $\varepsilon=P$, and the change in ε becomes:

$$d\varepsilon=\pm(P_\eta)_o\eta_o\left(\frac{dU}{U_o}\right) \tag{22}$$

For the cross-flow heat exchanger, at $R_o=0.25$, $\varepsilon_o=0.793$,

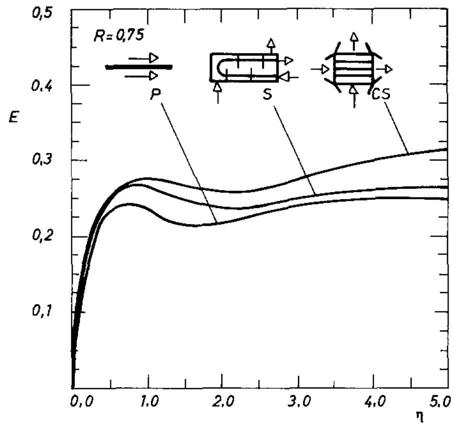

Figure 5 Effect of exchanger geometry on the magnitude of the effectiveness gradient vector at $R=0.75$. P = parallel-flow; s = shell-and-tube; cs = cross-flow

Figure 5 *Influence de la forme géométrique de l'échangeur sur la grandeur du vecteur de gradient d'efficacité pour* R = *0,75.* P = *écoulement parallèle,* s = *multitubulaire,* cs = *courant croisé*

Table 2 Stability of a shell-and-tube heat exchanger with one shell and any multiple of tube passes

Tableau 2 *Stabilité d'un échangeur de chaleur multitubulaire avec une calandre et plusieurs passes daus les tubes*

	E for indicated capacity rate ratios							
η	0.00	0.25	0.50	0.75	1.00	1.33	2.00	4.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.50	0.315	0.291	0.271	0.254	0.281	0.254	0.270	0.292
1.00	0.383	0.333	0.293	0.265	0.306	0.265	0.294	0.333
1.50	0.349	0.294	0.258	0.246	0.298	0.249	0.261	0.293
2.00	0.282	0.237	0.228	0.238	0.293	0.241	0.230	0.238
2.50	0.212	0.190	0.206	0.239	0.291	0.241	0.205	0.199
3.00	0.156	0.140	0.200	0.248	0.290	0.250	0.205	0.156
3.50	0.112	0.135	0.195	0.254	0.291	0.256	0.200	0.140
4.00	0.076	0.125	0.201	0.259	0.292	0.260	0.202	0.138
4.50	0.049	0.119	0.204	0.262	0.292	0.264	0.200	0.132
5.00	0.040	0.115	0.206	0.265	0.292	0.266	0.204	0.132

Table 3 Stability of a single-pass, cross-flow heat exchanger with one fluid mixed and the other unmixed

Tableau 3 *Stabilité d'un échangeur de chaleur à courant croisé à une seule passe entre un mélange de fluides et un fluide pur*

	E for indicated capacity rate ratios, $C_{mixed}/C_{unmixed}$							
η	0.00	0.25	0.50	0.75	1.00	1.33	2.00	4.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.50	0.315	0.293	0.274	0.256	0.282	0.256	0.274	0.292
1.00	0.383	0.340	0.304	0.277	0.316	0.277	0.305	0.338
1.50	0.348	0.313	0.277	0.266	0.316	0.265	0.281	0.311
2.00	0.275	0.270	0.256	0.254	0.308	0.263	0.262	0.265
2.50	0.209	0.223	0.242	0.263	0.310	0.273	0.249	0.224
3.00	0.134	0.193	0.225	0.279	0.310	0.283	0.229	0.186
3.50	0.110	0.162	0.221	0.279	0.311	0.289	0.214	0.166
4.00	0.076	0.142	0.220	0.296	0.311	0.302	0.206	0.137
4.50	0.049	0.122	0.208	0.308	0.313	0.304	0.210	0.126
5.00	0.040	0.087	0.213	0.316	0.314	0.314	0.212	0.128

η_o is 2.00 (Reference 3). From *Figure 3*, $(P_\eta)_o$ is 0.130 and Equation (22) yields: $d\varepsilon_{cs} = \pm 0.026$, or in percent notation, $d\varepsilon_{cs}/\varepsilon_o = \pm 3.2\%$.

Following the same procedure for the shell-and-tube exchanger, $\eta_o = 2.219$, $(P_\eta)_o = 0.095$ and the deviation in performance, $d\varepsilon_s$, becomes ± 0.021 which is $\pm 2.6\%$ of the design value. The results show that, due to 10% uncertainty in overall heat transfer coefficient, the thermal performance will be affected by about 20% less if a shell-and-tube heat exchanger is employed.

Example 2

A shell-and-tube heat exchanger with one shell and two tube passes is to be designed to operate at $R_o = 0.50$, $\eta_o = 0.75$. Depending upon the system needs, however, the cold-side flow rate may increase by 8%, and the surface fouling may reduce the heat transfer surface area by 5%. The exchanger effectiveness being a critical figure, a maximum of 10% deviation from its design value is permitted. Therefore, it is desired to specify the degree of accuracy needed for the overall heat transfer coefficient. For constant inlet temperatures, percent deviations in the outlet temperatures are also to be determined.

The maximum deviation in the performance is given as: $d\varepsilon_{max} = E.\ |\mathbf{N}|$, where $|\mathbf{N}| = [(d\eta/\eta_o)^2 + (dR/R_o)^2]^{0.5}$.

Solving for $d\eta/\eta_o$ yields:

$$\frac{d\eta}{\eta_o} = \left[\left(\frac{d\varepsilon_{max}}{E}\right)^2 - \left(\frac{dR}{R_o}\right)^2\right]^{0.5} \tag{23}$$

Due to fluctuations in the cold-side flow rate, percent variation in the heat capacity rate ratio becomes: $dR/R_o = dC_c/(C_c)_o = 0.08$, and determining E from *Table 2*, Equation (23) gives $d\eta/\eta_o$ as 0.129. Furthermore the following relationship holds between the variations in U, C_{min}, and A.

$$\frac{d\eta}{\eta_o} = \frac{dA}{A_o} + \frac{dU}{U_o} - \frac{dC_{min}}{(C_{min})_o} \tag{24}$$

Thus the required accuracy in U which is expressed by dU/U_o term in Equation (24) is 25.9%. The temperature effectiveness and the heat capacity rate ratio at the deviated state are:

$$P = P_o + (P_R)_o\, dR + (P_\eta)_o\, d\eta, \quad R = R_o + dR \tag{25}$$

Figure 2 provides the partial derivatives; $(P_R, P_\eta)_o$, and additionally substituting the increments in R and η into Equation (25) results as; $P = 0.495$, $R = 0.54$.

The specified inlet temperatures at the deviated state correspond to operational case *a* in *Table 1*. Then the amount of change in outlet temperatures are formulated as:

$$\frac{t_2 - t_{2o}}{T_1 - t_1} = P - P_o, \quad \frac{T_2 - T_{2o}}{T_1 - t_1} = P_o R_o - PR \tag{26}$$

Describing the results as the percent of the difference between the inlet temperatures, 3.2% rise in the cold-side and 3% decrease in the hot-side outlet temperatures are determined.

Example 3

It is desired to maintain the effectiveness of an air-cooled heat exchanger constant at the operating conditions. However it is known that, due to working conditions, the overall heat transfer coefficient will be reduced by 10% of that evaluated at the design-point. Therefore the required percent change in air flow rate has to be estimated. The exchanger is designed for $R_o = 0.30$, $\eta_o = 1.75$.

Since $R_o < 1$, then $\varepsilon = P$ and $C_{mix} = C_{min}$. Deviations in the number of heat transfer units and in the heat capacity rate ratio are:

$$d\eta = \eta_o\left(\frac{dU}{U_o} - \frac{dC_{mix}}{C_{mix_o}}\right), \quad dR = R_o\left(\frac{dC_{mix}}{C_{mix_o}}\right)$$

Substituting these relations into Equation (15) and letting $d\varepsilon$ be zero yields:

$$\frac{dC_{mix}}{(C_{mix})_o} = \frac{\eta_o P_\eta}{\eta_o P_\eta - R_o P_R}\left(\frac{dU}{U_o}\right) \tag{27}$$

After numerical evaluation, dC_{mix}/C_{mix_o} is determined to be -7.7%.

Concluding remarks

The problem of heat exchanger design is very intricate. Most probably a better design will be arrived at by considering the exchanger behaviour at subdesign conditions as well as at the design-point. The analytical method proposed in this study sheds light on the problem of predicting the exchanger governing parameters at conditions different from those for which the exchanger is designed. In the analysis, the modified operating regime is assumed to be at steady state. The transient response of exchangers to sudden changes in temperatures or in flow rates, is a totally distinct subject area and is well documented[6-8].

The partial derivatives P_R, and P_η are central to the characterization of the exchanger response in the vicinity of an operating point. The magnitude of the exchanger effectiveness gradient vector can be used as a criterion for comparing the thermal performance of several candidate exchangers under identical conditions. Due to their widespread use in industry, the method is applied to shell-and-tube and to cross-flow exchangers. However, the results are equally applicable to other direct transfer type (recuperative) exchangers.

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