Transmission Line Modelling

parameters: ℓ (series inductance)
C (shunt capacitance)

in the sinusoidal steady state

$z = r + j\omega \ell c$: series impedance per meter

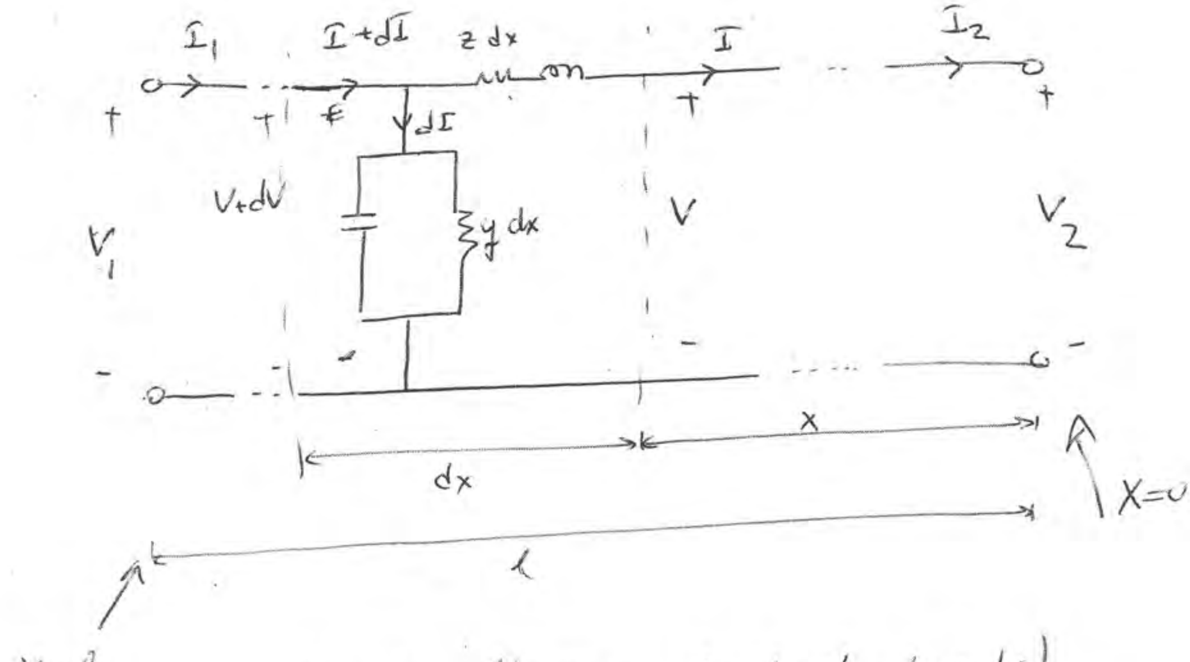
$y = g + j\omega c$: shunt admittance per meter to neutral

lower case: distributed

c : distributed inductance

ℓ : total length of line

Per phase equivalent:



$I_2 \neq I_1$ (because of shunt elements)

Side ① sending end
Side ② receiving end.

Kirchhoff's voltage law (KVL) and KCL:

$$dV = I \cdot z\, dx$$
$$dI = (V + dV)\, y\, dx \approx V y\, dx \quad (y\, dV\, dx \to 0)$$

$$\Rightarrow \left.\begin{aligned} \frac{dV}{dx} &= zI \\ \frac{dI}{dx} &= yV \end{aligned}\right\} \Rightarrow \begin{aligned} \frac{d^2V}{dx^2} &= z\frac{dI}{dx} = zyV = \gamma^2 V \\ \frac{d^2I}{dx^2} &= y\frac{dV}{dx} = yz\, I = \gamma^2 I \end{aligned}$$

$$\gamma^2 = yz$$
$$\Rightarrow \gamma = \sqrt{yz} : \text{ propagation constant}$$

without resistance: $z = j\omega L$, $y = j\omega C$ $\Rightarrow \gamma = \sqrt{j\omega L\, j\omega C} = j\omega\sqrt{LC}$

with resistance $\quad \gamma = \alpha + j\beta$

Solve: $\dfrac{d^2V}{dx^2} = \gamma^2 V \Rightarrow s^2 - \gamma^2 = 0$

$\Rightarrow$ ~~$s = \pm j\alpha$~~ $s = \pm\gamma$

$\cancel{\dfrac{d^2I}{dx^2} = \gamma^2 I}$

$$\Rightarrow V = k_1 e^{\gamma x} + k_2 e^{-\gamma x}$$
$$= (k_1 + k_2)\frac{e^{\gamma x} + e^{-\gamma x}}{2} + (k_1 - k_2)\frac{e^{\gamma x} - e^{-\gamma x}}{2}$$
$$= K_1 \cosh(\gamma x) + K_2 \sinh(\gamma x)$$

$K_1 = k_1 + k_2$
$K_2 = k_1 - k_2$

Boundary conditions:

$x = 0 \to V = V_2 \Rightarrow V_2 = K_1 + 0 \to \boxed{K_1 = V_2}$
$I = I_2 \qquad \dfrac{dV(0)}{dx} = z I_2$

$$\frac{dV}{dx} = -K_1 \gamma \sinh\gamma x + K_2 \gamma \cosh\gamma x$$
$$\Rightarrow \frac{dV(0)}{dx} = K_2\gamma = z I_2 \Rightarrow K_2 = \frac{z}{\gamma} I_2 = \quad \Rightarrow$$

$$V_2 = \frac{z}{\gamma} I_2 = \frac{z}{\sqrt{zy}} I_2 = \sqrt{\frac{z}{y}} I_2 = Z_c \; I_2$$

$Z_c \triangleq \sqrt{\frac{z}{y}}$: characteristic impedance

$$\Rightarrow \boxed{\begin{aligned} V &= V_2 \cosh \gamma x + Z_c I_2 \sinh \gamma x \\ I &= I_2 \cosh \gamma x + \frac{V_2}{Z_c} \sinh \gamma x \end{aligned}}$$

$$x = \ell \Rightarrow \boxed{\begin{aligned} V_1 &= V_2 \cosh \gamma \ell + Z_c I_2 \sinh \gamma \ell \\ I_1 &= I_2 \cosh \gamma \ell + \frac{V_2}{Z_c} \sinh \gamma \ell \end{aligned}}$$

relationship between the per phase voltages and currents at the two ends of the transmission lines

why V_1 and I_1 are defined by using V_2 and I_2?
V_2, I_2 are load quantities.
We need to determine what supply values are needed?

Example: 60 Hz, 138-kV, 3ϕ 225 mile
$r = 0.169\ \Omega/mi$ $\ell = 2.093\ mH/mi$
$c = 0.01427\ \mu F/mi$ $g = 0$
line delivers 40 MW at 132 kV with 95% power factor lagging.
sending end voltage? current?
transmission line efficiency?

Solution: $z = r + j\omega L = 0.169 + j2\pi 60 \; 2.093 \; 10^{-3}$

$$= 0.169 + j\,0.789$$

$$= 0.807 \angle 77.9^\circ \; \Omega/mi$$

$$y = g + j\omega C = 0 + j\omega \; 0.01427 \; 10^{-6} = 5.38 \; 10^{-6} \angle 90^\circ \; mhos/mi$$

$$\Rightarrow Z_c = \sqrt{\frac{z}{y}} = 387.3 \angle -6.05^\circ \; \Omega$$

$$\gamma l = \sqrt{zy}\, l = \sqrt{0.807 \angle 77.9 \;\; 5.38\, 10^{-6} \angle 90} \;\; 225$$

$$= 0.4688 \angle 83.95^\circ$$

$$= 0.0494 + j\,0.466$$

$$\sinh \gamma l = \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \frac{e^{0.0494} e^{j0.466} - e^{-0.0494} e^{-j0.466}}{2}$$

$$\Rightarrow \sinh \gamma l = 0.452 \angle 84.4^\circ$$

$\cosh^2 - \sinh^2 =$

$$\Rightarrow \cosh \gamma l = 0.895 \angle 1.42^\circ$$

$$\left[\cosh^2 \theta - \sinh^2 \theta = 1 \right]$$

$$|V_2| = \frac{132}{\sqrt{3}} kV = 76.2 \; kV$$

$$\angle V_2 = 0 \quad (\text{picked})$$

per phase: $P_{load} = \frac{40 \; 10^6}{3} = 13.33 \; MW$

$$\Rightarrow |I_2| = \frac{13.33 \; 10^6}{76.2 \; 10^3 * 0.95} = 184.14 \; A$$

$$\Rightarrow I_2 = 184.14 \angle -18.195^\circ \quad \left(\cos\theta = 0.95 \;\Rightarrow \theta = \right.$$

$$\Rightarrow \quad V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l$$

$$= 76{,}2 \; 10^3 * 0{,}895 \angle 1.42^\circ + 387{,}3 \angle -6.05^\circ * 184.14 \angle 18{,}05^\circ * 0.452 \angle 84.4^\circ$$

$$\boxed{V_1 = 89{,}28 \angle 19{,}39^\circ \text{ kV}}$$

$$V_1^{l-l} = 154.64 \text{ kV}$$

$$I_1 = I_2 \cosh \gamma l + \frac{V_2}{Z_c} \sinh \gamma l$$

$$= 162.42 \angle 14.76^\circ$$

$$P_{12} = \mathrm{Re}\{V_1 I_1^*\} = 14.45 \text{ MW}$$

$$\text{efficiency: } \eta = \frac{13.33}{14.45} = 0.92$$

Example: (4.2)

if $Z_L = Z_c$, find driving point impedance $\dfrac{V_1}{I_1}$

voltage gain $\dfrac{|V_2|}{|V_1|}$

current gain $\dfrac{|I_2|}{|I_1|}$

complex power gain $\dfrac{-S_{21}}{S_{12}}$

real power efficiency $\dfrac{-P_{21}}{P_{12}}$

Solution: $V_2 = Z_c I_2$

$$\Rightarrow \cancel{V_2 = Z_c I_2}$$

$$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l$$
$$= Z_c I_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l$$
$$= Z_c I_2 (\cosh \gamma l + \sinh \gamma l)$$
$$= Z_c I_2 e^{\gamma l} = V_2 e^{\gamma l} = V_2 e^{\alpha l} \cdot e^{j\beta l}$$

$$I_1 = I_2 \cosh \gamma l + \frac{V_2}{Z_c} \sinh \gamma l$$
$$= I_2 (\cosh \gamma l + \sinh \gamma l) = I_2 e^{\gamma l} = I_2 e^{\alpha l} e^{j\beta l}$$

$$\Rightarrow \frac{V_1}{I_1} = \frac{V_2 e^{\alpha l} e^{j\beta l}}{I_2 e^{\alpha l} e^{j\beta l}} = \frac{V_2}{I_2} = Z_c$$

driving point impedance $= Z_c$

$$|V_1| = |V_2| e^{\alpha l} \Rightarrow \frac{|V_2|}{|V_1|} = e^{-\alpha l}$$

$$|I_1| = |I_2| e^{\alpha l} \Rightarrow \frac{|I_2|}{|I_1|} = e^{-\alpha l}$$

$$-S_{21} = V_2 I_2^* = V_1 e^{-\alpha l} e^{-j\beta l} \left(I_1 e^{-\alpha l} e^{-j\beta l} \right)^*$$

$$= V_1 I_1^* e^{-2\alpha l}$$

$$= S_{12} e^{-2\alpha l}$$

$$\Rightarrow \boxed{\frac{-S_{21}}{S_{12}} = e^{-2\alpha l}}$$

Or

$$V_1 = Z_c I_1 \Rightarrow V_1 I_1^* = Z_c |I_1|^2$$

$$V_2 I_2^* = Z_c |I_2|^2$$

$$\Rightarrow \frac{-S_{21}}{S_{12}} = \left(\frac{|I_2|}{|I_1|} \right)^2 = e^{-2\alpha l}$$

$$\eta = \frac{-P_{21}}{P_{12}} = e^{-2\alpha l} \quad (\alpha \text{ is real})$$

Example 4.3) Repeat 4.2 if the line is lossless.
Find Z_c, γ, P_{12}.

Solution: $r = g = 0 \Rightarrow Z_c = \sqrt{\frac{j\omega \ell}{j\omega c}} = \sqrt{\frac{\ell}{c}} = \sqrt{\frac{L}{C}}$ } real

$\ell l = L$: total inductance

$c l = C$: total capacitance

$$\gamma = \sqrt{zy} = \sqrt{j\omega \ell \, j\omega c} = j\omega\sqrt{\ell c} \Rightarrow \begin{array}{l} \alpha = 0 \\ \beta = \omega\sqrt{\ell c} \end{array}$$

$$\Rightarrow \frac{|V_2|}{|V_1|} = \frac{|I_2|}{|I_1|} = \frac{-S_{21}}{S_{12}} = \frac{-P_{21}}{P_{12}} = \eta = 1$$

$P_{12} = Re[V_1 I_1^*] = Re\left[Z_c |I_1|^2\right] = Z_c |I_1|^2$ since Z_c is real

also. $I_1 = \frac{V_1}{Z_c} \rightarrow P_{12} = \frac{|V_1|^2}{Z_c}$

~~a power~~. a lossless line loaded by characteristic impedance is said to be loaded by surge impedance.

Z_c : also named surge impedance

A lossless line, operating at its nominal voltage terminated in its surge impedance is said to be surge impedance loaded (SIL)

P_{SIL} : Transmitted per-phase power

$$P_{SIL} = \frac{|V_1|^2}{Z_c}$$

$$P^{3-\phi}_{SIL} = 3\frac{|V_1|^2}{Z_c} = \frac{|V_{LL}|^2}{Z_c}$$

Waves on transmission lines:

general solution was found to be

$$V = k_1 e^{\gamma x} + k_2 e^{-\gamma x}$$

a voltage wave traveling to the right (incident)

a voltage wave traveling to the left (reflected)

<u>Note</u> $\gamma = \alpha + j\beta$

$\alpha \geq 0$: attenuation constant

β : phase constant

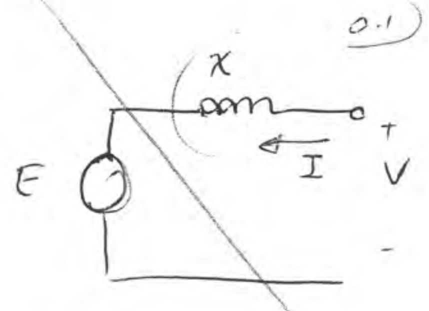
$$\Rightarrow \quad V = k_1 e^{\alpha x} e^{j\beta x} + k_2 e^{-\alpha x} e^{-j\beta x}$$

$$\Rightarrow \quad v(t,x) = \sqrt{2} Re\left[V e^{j\omega t}\right]$$

$$= \sqrt{2} Re\left\{k_1 e^{\alpha x} e^{j(\omega t + \beta x)}\right\} + \sqrt{2} Re\left\{k_2 e^{-\alpha x} e^{j(\omega t - \beta x)}\right\}$$

$$= v_1(t,x) + v_2(t,x)$$

70



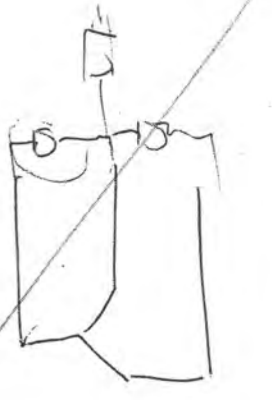
$S_R = V_R \cdot I_e$

$I_R = \frac{S}{V}$

$Z_B = \frac{V_B}{I_B} \quad \frac{2400}{10} = 24\ \Omega$

$Z I = \quad V = X I + E$

$\boxed{1.1}$



Assume $\alpha \to 0$

Consider $v_2(t,x) = \sqrt{2}\ Re\left[V_2\ e^{j(\omega t - \beta x)}\right]$

for fixed x: v_2 is a sinusoidal function of t

for fixed t: v_2 is a sinusoidal function of x

Scan the voltage at "x" points while "t" is increasing with the condition ~~at~~

$$\omega t - \beta x = \text{constant}$$

$\Rightarrow$ v_2 remains constant

$\Rightarrow$ We are looking at a fixed point on a voltage wave which is traveling to the left with a velocity of: (x is increasing from right to left)

$$\frac{dx}{dt} = \frac{\omega}{\beta} = \frac{\omega}{Im(\sqrt{zy})}$$

This is reflected wave.

when α is considered, the wave is attenuated as it moves to left.

$v_1(t,x)$ can be similarly analyzed.

For infinite long line with $\alpha > 0$:

$$e^{-\alpha x}\, e^{j(\omega t - \beta x)} \longrightarrow 0$$

$\Rightarrow$ There will be no reflected waves

~~Also if the infinite line is ended with Z_c:~~

Also, for infinite line; driving point impedance is Z_c.

Moreover, if a line is ended with $Z_c \Rightarrow$ driving point impedance is Z_c. $\Rightarrow$ no reflected waves

⚠ If an incident lightning voltage pulse traveling down a line hits the open-circuited end of the line (or an open switch) there will be a reflected wave generated such that the voltage at the line termination will approximately double. ⇒ Proper insulation must be done for the transformers.

Transmission Matrix

$$V_1 = \cosh \gamma l \; V_2 + Z_c \sinh \gamma l \; I_2$$

$$I_1 = \frac{1}{Z_c} \sinh \gamma l \; V_2 + \cosh \gamma_L \; I_2$$

Define $A = \cosh \gamma l$ $\quad B = Z_c \sinh \gamma l$ $\quad C = \frac{1}{Z_c} \sinh \gamma l$ $\quad D = \cosh \gamma l$ $\Big\}$ transmission parameters

$$\Rightarrow \quad V_1 = A \; V_2 + B \; I_2$$

$$I_1 = C \; V_2 + D \; I_2$$

(if γ is complex, so are A, B, C and D)

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$ transmission (or chain) matrix

$$\det T = AD - BC = \cosh^2 \gamma l - \sinh^2 \gamma l = 1$$

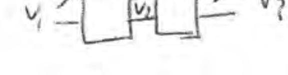
$$\Rightarrow \quad T^{-1} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = T_1 \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

for ~~chain~~ cascade connections

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = T_1 \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = T_1 T_2 \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

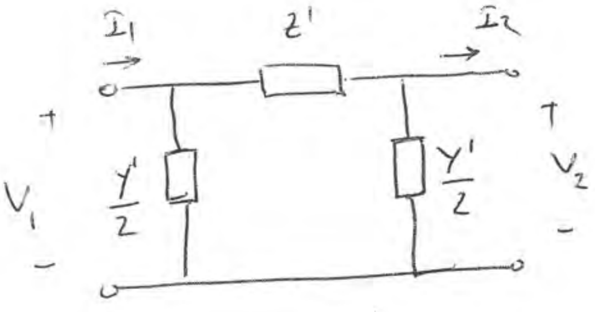
$$\Rightarrow T = T_1 T_2$$



Lumped Circuit Equivalent

Purpose: To find a π or T circuit that has the same A, B, C, D parameters.

Choice – π circuit



$$V_1 = V_2 + Z'\left(I_2 + \frac{Y'}{2}V_2\right)$$

$$\boxed{V_1 = \left(1 + \frac{Z'Y'}{2}\right)V_2 + Z' I_2}$$

$$I_1 = \frac{Y'}{2}V_1 + \frac{Y'}{2}V_2 + I_2$$

$$= \frac{Y'}{2}V_2 + \frac{Y'}{2}\cdot\left[\left(1 + \frac{Z'Y'}{2}\right)V_2 + Z' I_2\right] + I_2$$

$$\Rightarrow \boxed{I_1 = Y'\left[1 + \frac{Y'Z'}{4}\right]V_2 + \left(\frac{Y'Z'}{2} + 1\right) I_2}$$

$$\Rightarrow \quad A = 1 + \frac{Z'Y'}{2} \qquad B = Z'$$

$$C = \left(1 + \frac{Z'Y'}{4}\right)Y' \qquad D = 1 + \frac{Y'Z'}{2}$$

$$B = Z' = Z_c \sinh \gamma l = \sqrt{\frac{z}{y}} \sinh \gamma l$$

$$= \sqrt{\frac{z}{y}}\ \gamma l\ \frac{\sinh \gamma l}{\gamma l} \qquad \gamma = \sqrt{zy}$$

$$= z l\ \frac{\sinh \gamma l}{\gamma l} = Z \qquad zl = Z : \text{total series impedance of line}$$

$$= Z\ \frac{\sinh \gamma l}{\gamma l}$$

for power lines, usually, $|\gamma l| \ll 1$

$$\Rightarrow \frac{\sinh \gamma l}{\gamma l} \simeq 1 \Rightarrow Z' \simeq Z$$

($\frac{\sinh \gamma l}{\gamma l}$ may be viewed as a correction factor)

$$A = 1 + \frac{Z'Y'}{2} = \cosh \gamma l \qquad Z' = Z_c \sinh \gamma l$$

$$\Rightarrow \frac{Y'}{2} = \frac{(\cosh \gamma l - 1)}{Z_c \sinh \gamma l} = \frac{1}{Z_c} \tanh \frac{\gamma l}{2}$$

$$\left[\frac{\cosh \gamma l - 1}{\sinh \gamma l} = \frac{e^{\gamma l} + e^{-\gamma l} - 2}{e^{\gamma l} - e^{-\gamma l}} = \frac{(e^{\gamma l/2} - e^{-\gamma l/2})^2}{(e^{\gamma l/2} + e^{-\gamma l/2})(e^{\gamma l/2} - e^{-\gamma l/2})} = \frac{e^{\gamma l/2} - e^{-\gamma l/2}}{e^{\gamma l/2} + e^{-\gamma l/2}} = \tanh \gamma l/2 \right]$$

Also $\frac{1}{Z_c} = \frac{1}{\sqrt{\frac{z}{y}}} = \frac{y}{\underbrace{\sqrt{zy}}_{\gamma}} = \frac{yl}{\gamma l} = \frac{Y}{\gamma l}$ $\qquad Y$: total line-neutral admittance

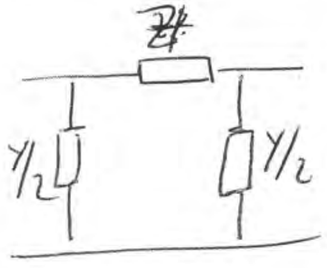
$$\Rightarrow \frac{Y'}{2} = \frac{Y}{2}\ \frac{\tanh \gamma l/2}{\gamma l/2}$$

$$|\gamma l| \ll 1 \Rightarrow \frac{\tanh(\gamma l/2)}{\gamma l/2} \simeq 1 \Rightarrow Y'/2 \simeq Y/2$$

<u>So</u>: $Z' = Z$, $Y' \doteq Y$

$$\Rightarrow Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{zl}{yl}} = \sqrt{\frac{Z}{Y}}$$

$$\gamma l = \sqrt{zyl^2} = \sqrt{zl \cdot yl} = \sqrt{ZY}$$



<u>Example:</u> (Cont'd)

Previously: $Z_c = 387.3 \angle -6.05^\circ \ \Omega$

$$\gamma l = 0.4688 \angle 83.95^\circ = 0.0494 + j0.466$$

$$\Rightarrow \cancel{Z' = Z \frac{\sinh \gamma l}{\gamma l} = 181.57 \angle 77.9}$$

$$Z' = Z_c \sinh \gamma l = 387.3 \angle -6.04 \times 0.452 \angle 84.4$$
$$= 175.06 \angle 78.35^\circ$$

alternatively:
$$Z' = Z \frac{\sinh \gamma l}{\gamma l} = 181.57 \angle 77.9 \times 0.9642 \angle 0.45$$
$$= 175.07 \angle 78.35^\circ$$

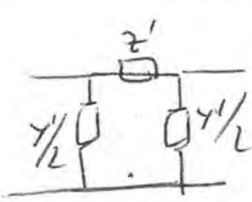
very close.

correction factor $= 0.9642 \angle 0.45 \approx 1$

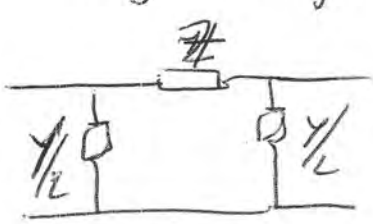
$$\frac{Y'}{2} = 614.97 \ 10^{-6} \angle 89.8^\circ \text{ mho}$$

for comparison:

$$\frac{Y}{2} = 605.25 \ 10^{-6} \angle 90^\circ \text{ mho}$$ very close again.



or

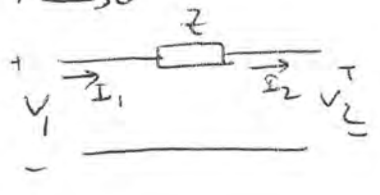


Simplifications

for long lines $|\gamma l| << 1$ is not used $\rightarrow$ exact model (Z', Y')
$l > 250$ km

for medium length lines $|\gamma l| << 1 \Rightarrow$ $Z' \to Z$, $Y' \to Y$ nominal π equivalent
75 km < l < 250 km

for short lines: $Y' \rightarrow 0$
$l < 75$ km $\Rightarrow$



Example:

a lossless line, open circuit end
V_1 is fixed.
calculate V_2, and compare for each model.

Solution:

lossless line $\rightarrow \alpha = 0$, $\gamma = j\beta$
open circuit $\rightarrow I_2 = 0$

Model 1: Long line (exact model)

$$V_1 = V_2 \cosh \gamma l = V_2 \cosh(j\beta l) = V_2 \cos(\beta l)$$

Model 2 Medium line

$$V_1 = \left[1 + \frac{ZY}{2}\right] V_2 = \left[1 + \frac{(\gamma l)^2}{2}\right] V_2 = \underbrace{\left[1 - \frac{(\beta l)^2}{2}\right]}_{\text{the first two terms of } \cos\beta l \text{ expansion}} V_2$$

Model 3 Short line

$$V_1 = V_2$$

Actually, voltage at receiving (open) end is higher than the voltage at sending end.

numerical calculations:

$\beta = 0.002$ rad/mi , 60 Hz open-wire line

for a 50-mile line:

$$\beta l = 0.002 * 50 \cong 0.1 \text{ rad.}$$

Model 1: $V_1 = V_2 \cos(0.1) = 0.995004\ V_2$

Model 2: $V_1 = \left[1 - \frac{(\beta l)^2}{2}\right] V_2 = 0.995000 V_2$

Model 3: $V_1 = V_2$

negligible error.

for a 200 mile line

$\beta l \approx 0.4$

$\Rightarrow$ Model 1: $V_1 = 0.92106\ V_2$

Model 2: $V_1 = 0.92\ V_2$

Model 3: $V_1 = V_2$

Model 1 and 2 are very close.
Short line model causes an error of % 8

for a 600 mile line

$\beta l \approx 1.2$

Model 1: $V_1 = 0.36235 V_2$

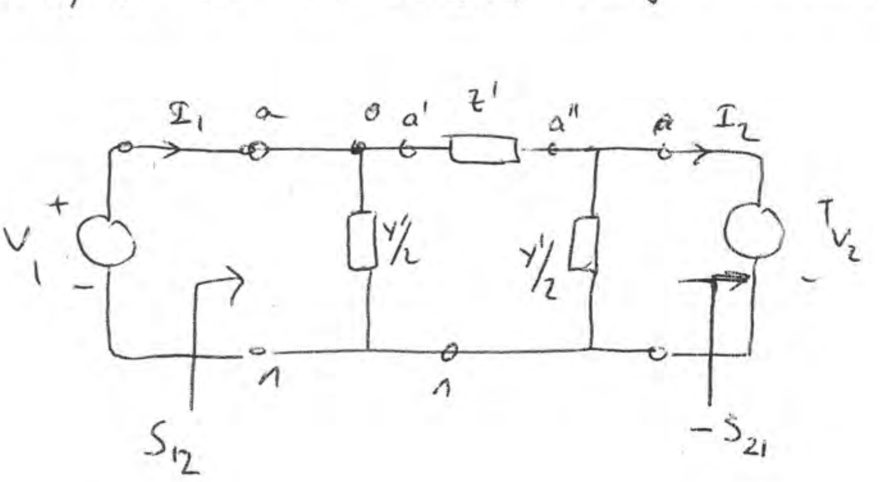
Model 2: $V_1 = 0.280\ V_2$

Model 3: $V_1 = V_2$

significant error
short line model is completely inaccurate.

! May be Skipped

Complex Power Transmission (Long or Medium Line)



- observation:

Sending-end power (S_{12}) = power consumed in $\left(\frac{Y'}{2}\right)$ + power supplied to the rest of the network through the terminal $\underline{\underline{a'n}}$.

$$\Rightarrow \quad S_{12} = \frac{Y'^*}{2}|V_1|^2 + \frac{|V_1|^2}{Z'^*} - \frac{|V_1||V_2|}{Z'^*}e^{j\theta_{12}}$$

power recieved:

$$-S_{21} = \underbrace{-\frac{Y'^*}{2}|V_2|^2}_{\text{power consumed on } Y'/2} \underbrace{- \frac{|V_2|^2}{Z'^*} + \frac{|V_1||V_2|}{Z'^*}e^{-j\theta_{12}}}_{\left(\text{power received through } a''n\right)}$$

Power Handling Capability of Lines

Thermal effects and stability limit the power handling capability of power lines.

I^2R losses $\rightarrow$ heat $\rightarrow$ transmission efficiency

$\hookrightarrow$ temperature rise of lines

$\downarrow$

line sagging.

irreversible stretching. (around $100°C$)

$\Rightarrow$ limitation on maximum current that can be carried

Bundling helps. Greater spacing between subconductors and increased surface area helps the heat dissipation

Problem is more serious with cables. Limited possibility of heat dissipation.

$\Rightarrow$ Lines have ~~defined~~ "rated current" values above which operation is not safe.
(specially extended ~~___~~ time)

Also, lines are designed for specific voltage levels.

Spacing between phases
conductor size and geometry
insulation
} selected depending on the intended voltage level

~~The~~ Also, consumers should be able to receive voltage values ~~within~~ within reasonable ~~___~~ neighborhood of the promised voltage level.

Maximum current + maximum voltage $\longrightarrow$ limitation on the MVA rating of the line (also on MW)

For example:

345 kV (line-to-line) transmission line may have a thermal rating of 1600 MVA.

At 100% power factor, it can transmit 1600 MW. This is the maximum active power that can be transmitted.

⚠ Thermal limitations may be more restricting on terminating equipment (transformers, etc)

Consider a lossless, long line with equal voltage magnitudes at each end.

$$\Rightarrow Z_c = \sqrt{\frac{L}{C}} \text{ is real}$$

$$\gamma = \alpha + j\beta = j\beta \quad \text{imaginary}$$

$$\Rightarrow Y' = Y \frac{\tanh \gamma l/2}{\gamma l/2} = j\omega C \frac{\tan \beta l/2}{\beta l/2} \quad \text{: admittance of pure capacitance}$$

$$Z' = Z_c \sinh \gamma l = j Z_c \sin \beta l \quad \text{impedance of pure inductance}$$

active power transmitted:

$$P_{12} = Re\{S_{12}\} = \frac{|V_1|^2}{Z_c} \frac{\sin \theta_{12}}{\sin \beta l}$$

remember: $P_{SIL} = \frac{|V_1|^2}{Z_c}$

$$\Rightarrow P_{12} = P_{SIL} \frac{\sin \theta_{12}}{\sin \beta l}$$

for fixed θ_{12}, as $l \nearrow$, $\beta l \nearrow$
$P_{12} \searrow$

for very long lines, with $\beta l = \frac{\pi}{2}$, even when θ_{12} is chosen $\pi/2$

$P_{12} = P_{SIL}$ (P_{SIL} is the maximum in this case

$P_{12} < P_{SIL}$ when θ_{12} is kept within safe limits

Example:

$\beta = 0.002$ rad/mi $\qquad \theta_{12} = 45^\circ$

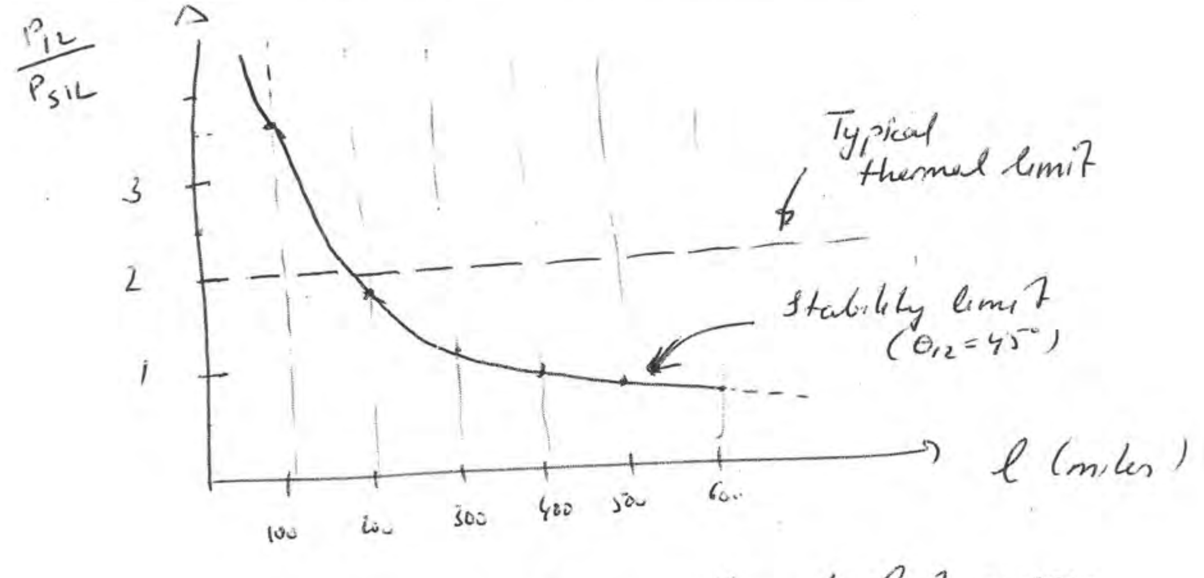
Find P_{12}/P_{SIL} as a function of line length

Solution

$$P_{12} = P_{SIL} \frac{\sin \theta_{12}}{\sin \beta l}$$

$$\rightarrow \frac{P_{12}}{P_{SIL}} = \frac{\sin 45}{\sin 0.002 l} = 0.707 \frac{1}{\sin}$$

l (miles)	100	200	300	400	500	600
	3.56	1.82	1.25	0.986	0.84	0.76



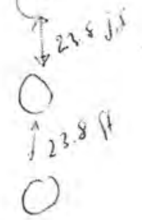
for short lines: thermal limit governs
for long lines: stability limit governs

Example (5.1) (Stevenson)

~~Line~~ 60 Hz, $l = 370$ km (230 mi)

Conductors: Rook, flat horizontal spacing

distance between conductors 7.25 m (23.8 ft)



Load: 125 MW at 215 kV, 100% power factor

find the voltage, current and power at the sending end

voltage regulation of the line

wavelength and velocity of propagation

Solution:

$$D_m = (23.8 \times 23.8 \times 47.6)^{1/3} \quad (D_{eq} \text{ in Stevenson})$$

$$\cong 30 \text{ ft}$$

for the conductor:

from table A.1:

At 60 Hz:

resistance: $0.1603 \ \Omega/mi$ (at 50°C)

inductive reactance (X_a) $0.415 \ \Omega/mi$

capacitive reactance (X_a') $0.0950 \ M\Omega \cdot mi$

} reactance per conductor 1-ft spacing 60 Hz

from table A.2. (Inductive reactance spacing factor (X_d at 60 Hz))

$D_m = 30$ ft $\longrightarrow X_d = 0.4127 \ \Omega/\text{mile}$ per conductor

from table A.3 (shunt capacitive reactance spacing factor X_d' at 60 Hz

$D_m = 30$ ft $\rightarrow X_d' = 0.1009 \ M\Omega - \text{miles}$ per conductor

~~total~~ series line impedance $= X_a + X_d$

shunt line capacitance $= X_a' + X_d'$

$$\Rightarrow \quad z = 0.1603 + j(0.415 + 0.4127) = 0.8431 \angle 79.04^\circ \ \Omega/mi$$

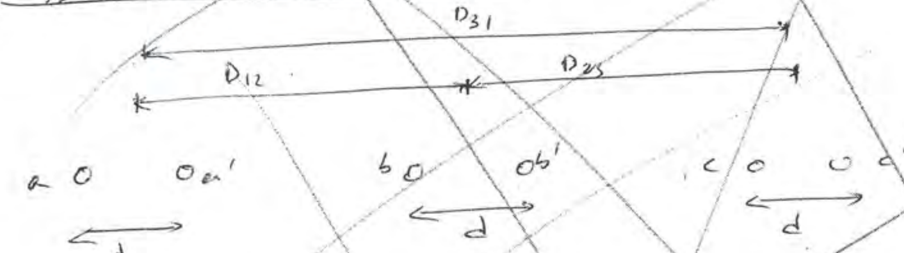
$$y = j \left[\frac{1}{0.0950 + 0.1009} \right] 10^{-6} = 5.105 \ 10^{-6} \angle 90^\circ \ \mho/mi$$

charging current:

single phase: $I_{chg} = j\omega C_{ab} V_{ab}$

3-ϕ $\qquad I_{chg} = j\omega C_n V_{an}$

Bundled Conductors:



charge per bundle divides equally between conductors.

also: $D_{12} >> d \Rightarrow D_{12} - d \rightarrow D_{12}$ etc.

$D_{12} + d \rightarrow D_{12}$

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left[\frac{q_a}{2}\Big(\underbrace{\ln\frac{D_{12}}{r}}_{a} + \underbrace{\ln\frac{D_{12}}{d}}_{a'}\Big) + \frac{q_b}{2}\Big(\underbrace{\ln\frac{r}{D_{12}}}_{b} + \underbrace{\ln\frac{d}{D_{12}}}_{b'}\Big) + \frac{q_c}{2}\Big(\underbrace{\ln\frac{D_{23}}{D_{31}}}_{c} + \underbrace{\ln\frac{D_{23}}{D_{31}}}_{c'}\Big)\right]$$

$$\Rightarrow V_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D_{12}}{\sqrt{rd}} + q_b \ln\frac{\sqrt{rd}}{D_{12}} + q_c \ln\frac{D_{23}}{D_{31}}\right]$$

~~By~~ for a transposed line:

$$\Rightarrow C_n = \frac{2\pi\varepsilon}{\ln\left(\frac{D_{eq}}{\sqrt{r\,d}}\right)} \text{ F/m to neutral}$$

$\gamma l = \sqrt{yz}\ l = \cancel{230} = 0.4772 \underline{/84.52^\circ} = 0.0456 + j\,0.475$

$Z_c = \sqrt{\frac{z}{y}} = 406.4 \underline{/-5.48^\circ}$

$V_R = V_2 = \frac{215\ 10^3}{\sqrt{3}} = 124.130 \underline{/0^\circ}$ V to neutral

receiving end

$I_R = I_2 = \frac{125\ 10^6}{\sqrt{3}\ 215\ 10^3} = 335.7 \underline{/0^\circ}$ A

$\Rightarrow \cosh \gamma l = 0{,}8904 \underline{/1.34^\circ}$

$\sinh \gamma l = 0.4596 \underline{/84.94^\circ}$

$V_S = V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l$

sending

$\Rightarrow \boxed{V_1 = 0.13785 \underline{/27.77^\circ}\ MV}$

$= 137.85 \underline{/27.77^\circ}$ kV

$I_1 = I_2 \cosh \gamma l + \frac{V_2}{Z_c} \sinh \gamma l = \cancel{0.3}$

$= 332.27 \underline{/26.33^\circ}$ A

at the sending end:

line voltage $= \sqrt{3}\ 137{,}85 = 238{,}8$ kV

line current : 332.3 A

power factor $= \cos(27{,}77 - 26{,}33) = $ ~~...~~

$= 0{,}9997 \approx 1$

Power : $\sqrt{3}\ 238{,}8 * 332.3 * 1 = \cancel{137.440\ kW}$

$= 137{,}440$ MW

at no load $(I_2=0) \Rightarrow V_2 = \dfrac{V_1}{\cosh \gamma l} = \dfrac{137\,851 \angle 27.77}{0.8904 \angle 1.34}$

$$= 154\,819 \angle 26.4°$$

no load   full load

regulation: $\dfrac{154{,}819 - 124.13}{124.13} = 0.247 \to 24.7\ \%$

wavelength? ~~β~~

$\gamma = \alpha + j\beta$

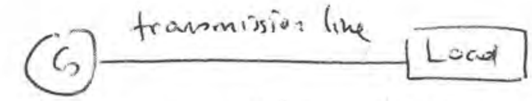
$\gamma l = \alpha l + j\beta l \to \beta = \dfrac{\mathrm{Im}[\gamma l]}{l} = \dfrac{0.4750}{230} = 0.002065\ \mathrm{rad/mi}$

$$\lambda = \frac{2\pi}{\beta} = 3043\ \mathrm{mi}$$

velocity $= f\lambda = 60 \times 3043 = 182580\ \mathrm{mi/s}$

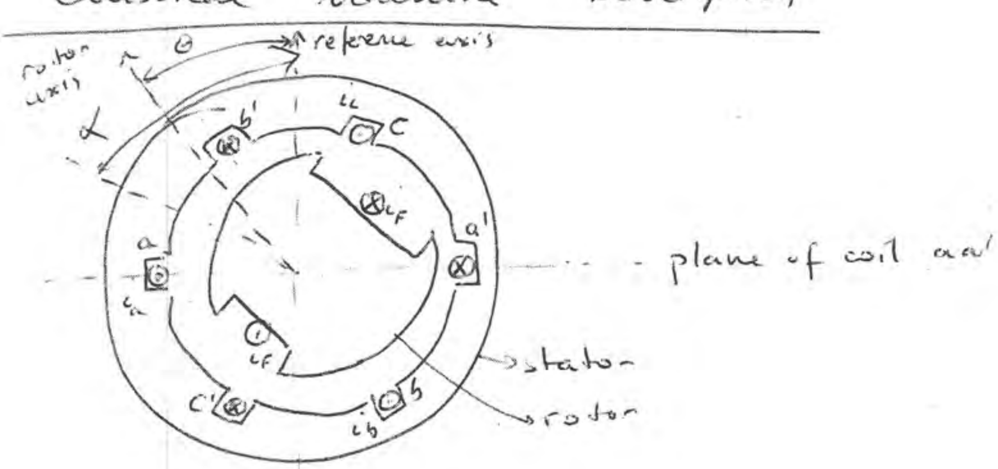
~~Transformer Modelling and the Per Unit~~ System.

Generator Modeling



generators supply complex power to loads through transmission lines.

Classical Machine Description



Rotor and stator are made of high-permeability materials for maximum flux density with the available MMF.

~~If the above motor machine~~
if the above generator is rotated at 3600 revolution per minute, it generates at 60 Hz.

Rotors → cylindrical (smooth surface)
salient pole (for lower speeds)

Voltage generation

voltage is induced across a coil if a time varying flux links that coil.

In the machine model shown, the magnetic flux ~~cros~~ crosses the air gap from rotor to stator, and back from stator to rotor. This creates a voltage called air gap voltage.

Air gap flux is generated by the field current (i_f) and stator currents (i_a, i_b, i_c).

Assumption

1. Mag. circuit is linear → superposition
2. $B_{ag} = B_{max} \cos(\alpha - \theta)$ .. Flux density in air gap due to i_f is radial.
   B is directed outwards.
3. Each coil consists of N turns concentrated in a single slot.

Superposition lets us calculate the air gap flux linkages due to i_f, i_a, i_b, i_c separately.

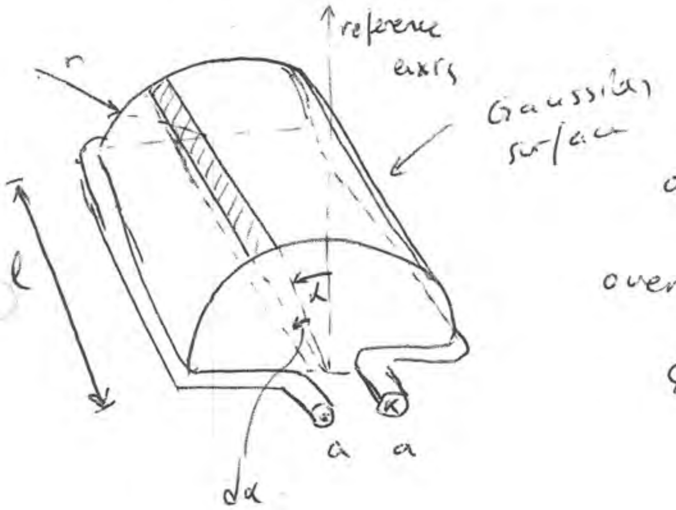
First $i_a = i_b = i_c = 0$, i_f exists → open circuit voltage

then $i_f = 0$, i_a, i_b, i_c alone. → armature reaction voltage

air gap voltage due to total air gap flux

Open circuit voltage $(i_a = i_b = i_c = 0)$

Consider coil aa'.



$$d\phi = lr B_{max} \cos(\alpha - \theta)\, d\alpha$$

over the whole Gaussian surface:

$$\phi = \int B \cdot ds = lr \int_{-\pi/2}^{\pi/2} B_{max} \cos(\alpha - \theta)\, d\alpha$$

$$= 2 l r B_{max} \cos\theta$$

$$= \phi_{max} \cos\theta$$

$(\theta = 0 \rightarrow \phi = \phi_{max})$ (stator and rotor are aligned)

For an N-turn concentrated coil:

$$\lambda_{aa'} = N \cdot \phi = N \phi_{max} \cos\theta = \lambda_{max} \cos\theta$$

Since flux linkages of bb' and cc' are maximum when $\theta = 120°$ and $240°$

$$\Rightarrow \lambda_{bb'} = \lambda_{max} \cos(\theta - 120°)$$

$$\lambda_{cc'} = \lambda_{max} \cos(\theta + 120°)$$

if the rotor has a uniform rotating $\rightarrow \theta = \omega_s t + \theta_o$

$$\Rightarrow \lambda_{aa'} = \lambda_{max} \cos(\omega_s t + \theta_o)$$

$$\Rightarrow e_{a'a} = \frac{d\lambda_{aa'}}{dt} = -\omega_s \lambda_{max} \sin(\omega_s t + \theta_o)$$

$$\Rightarrow e_{aa'} = \omega_s \lambda_{max} \sin(\omega_s t + \theta_o) = E_{max} \sin(\omega_s t + \theta_o)$$

phasors:

$$\Lambda_{aa'} = \frac{\lambda_{max}}{\sqrt{2}} e^{j\theta_0}$$

$$E_{aa'} = \frac{E_{max}}{\sqrt{2}} e^{j(\theta_0 - \frac{\pi}{2})} = -j \frac{\omega_0 \lambda_{max}}{\sqrt{2}} e^{j\theta_0}$$

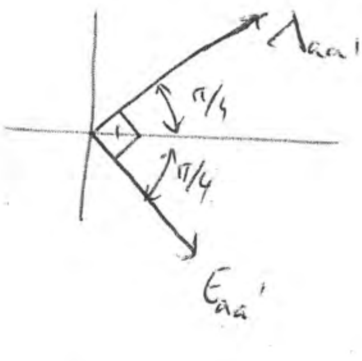
$$= -j\omega_0 \Lambda_{aa'}$$

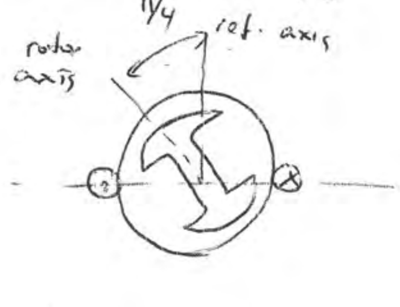
$\Rightarrow$ $E_{aa'}$ lags $\Lambda_{aa'}$ by 90°

For example, for $\theta_0 = \frac{\pi}{4}$, at $t=0$:

$$\Lambda_{aa'} = \frac{\lambda_{max}}{\sqrt{2}} e^{j\pi/4}$$

$$E_{aa'} = \frac{E_{max}}{\sqrt{2}} e^{j(\frac{\pi}{4} - \frac{\pi}{2})} = \frac{E_{max}}{\sqrt{2}} e^{-j\frac{\pi}{4}}$$





$E_{aa'}$, $E_{bb'}$, $E_{cc'}$ form a balanced 3-ϕ voltage set.

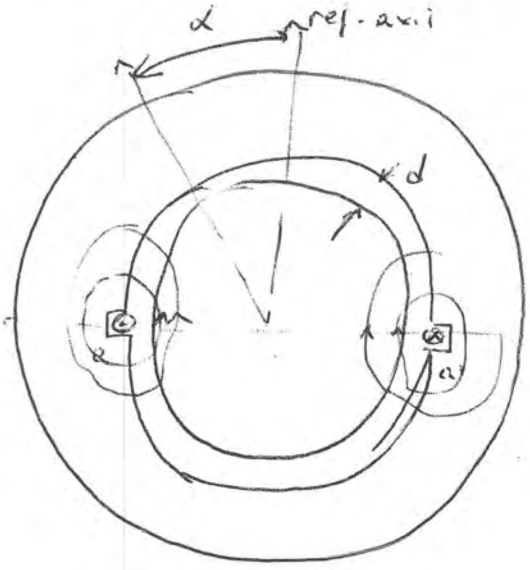
Assumption: a', b', c' are connected to the same neutral $\rightarrow$ E_a, E_b, E_c are used

Armature Reaction

Assume i_a, i_b, i_c are balanced 3-ϕ

$i_F = 0$

For simplicity, assume round rotor. Since $\hat{i}_F = 0$, it doesn't matter which way the rotor faces



plane of aa'

Ampere's Law (on a flux line)

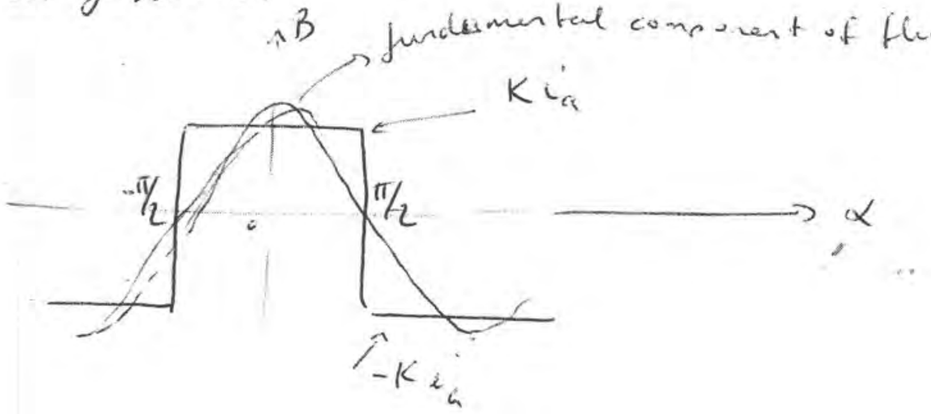
$$F = \oint_{\Gamma} \bar{H} \cdot d\bar{l} = \cancel{\frac{1}{\mu_r \mu_0} \int_{\Gamma_{iron}} \bar{B} \cdot d\bar{l}} + \frac{1}{\mu_0} \int_{\Gamma_{air}} \bar{B} \cdot d\bar{l} = N i_a$$

$$\Rightarrow \quad \frac{1}{\mu_0} \int_{\Gamma_{air}} \bar{B} \cdot d\bar{l} \simeq 2 \frac{B \cdot d}{\mu_0} = N i_a$$ (radial flux, uniform air gap)

$$\Rightarrow \quad B = \frac{\mu_0 N i_a}{2d} = K i_a$$

$0 < \alpha < \frac{\pi}{2} \Rightarrow B$ doesn't depend on α

Spatial distribution of flux can be found as follows:



$$B_a = \frac{4}{\pi} K i_a \cos\alpha$$

if $i_a = \sqrt{2}\,|I_a| \cos(\omega_s t + \angle I_a)$

$$\Rightarrow \quad B_a = \frac{4}{\pi} K i_a(t) \cos\alpha$$

$$= B'_{max} \cos\alpha \cos(\omega_s t + \angle I_a)$$

$$(B'_{max} \sim |I_a|)$$

when i_b and i_c are considered, similar ~~ott~~ results are obtained for other phases with corresponding phase shifts.

$$\Rightarrow \quad B_{abc} = B_a + B_b + B_c = B'_{max}\Big[\cos\alpha \cos(\omega_s t + \angle I_a) + \cos(\alpha - \tfrac{2\pi}{3}) \cos(\omega_s t + \angle I_a - \tfrac{2\pi}{3}) + \cos(\alpha + 2\pi/3) \cos(\omega_s t + \angle I_a + \tfrac{2\pi}{3})\Big]$$

$$\Rightarrow \quad B_{abc} = \frac{3}{2} B'_{max} \cos(\alpha - \omega_s t - \angle I_a)$$

Sinusoidal traveling wave in the air gap

3-ϕ voltages can generate this kind of traveling waves.

Corresponding armature reaction flux linkages

$$\lambda_{ar}(t) = N\,\phi(t) = N l r \int_{-\pi/2}^{\pi/2} \frac{3}{2} B'_{max} \cos(\alpha - \omega_s t - \angle I_a)\, d\alpha$$

$$= 3 N l r\, B'_{max} \cos(\omega_s t + \angle I_a)$$

$$= \sqrt{2}\, L_{s1}\, |I_a| \cos(\omega_s t + \angle I_a)$$

$$= L_{s1}\, i_a(t)$$

$$\Rightarrow \Lambda_{ar} = L_{s1}\, I_a$$

Terminal Voltage

i_a, i_b, i_c, i_f are present

$$\lambda_{ag} = \lambda_{aa'} + \lambda_{ar}$$

$$\Lambda_{ag} = \Lambda_{aa'} + \Lambda_{ar}$$

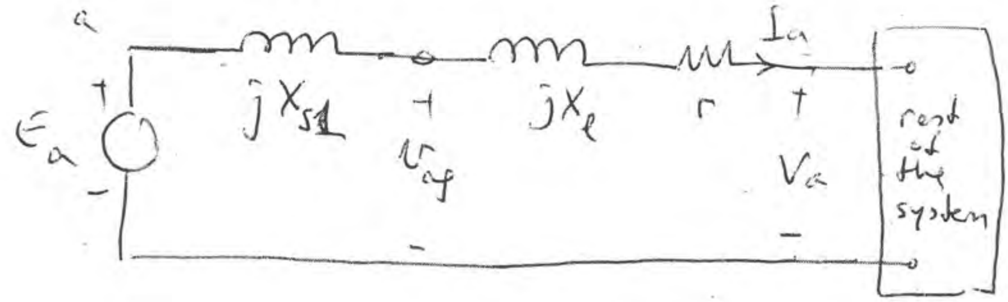
$$v_{ag} = -\frac{d\lambda_{ag}}{dt} \rightarrow V_{ag} = -j\omega_s \Lambda_{ag}$$

$$-\frac{d\lambda_{ag}}{dt} = -\frac{d\lambda_{aa'}}{dt} - \frac{d\lambda_{ar}}{dt}$$

$$\Rightarrow v_{ag} = e_a - L_{s1} \frac{di_a}{dt}$$

or $\quad V_{ag} = E_a - j\omega_s L_{s1} I_a$: There is a fictitious inductance L_{s1} which is responsible for the voltage difference between open circ. voltage and actual generated voltage.

There are additional elements:

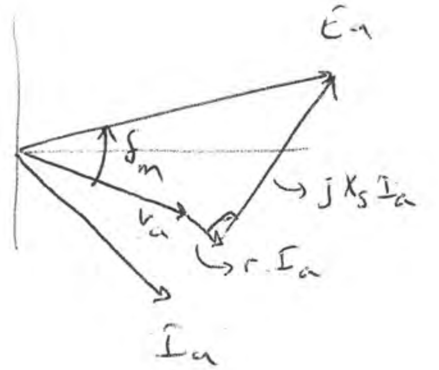


X_ℓ: $\approx$ 10% of X_{s1}

r: < 1% of X_{s1}

$$\Rightarrow \quad X_s \triangleq X_{s1} + X_{\ell}$$

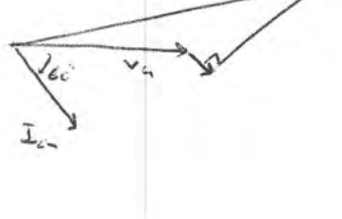
$$\Rightarrow \quad V_a = E_a - r I_a - j X_s I_a$$



δ_m: power angle

$= \angle E_a - \angle V_a$

Example 8.2: Given a round-rotor generator with $V_a = 1.0$, $X_s = 1.6$, $r = 0.004$ and $I_a = 1 \angle -60^\circ$ find E_a.



$$E_a = V_a + r I_a + j X_s I_a$$
$$= 1 + 0.004 \angle -60^\circ + 1.6 \angle +30^\circ$$
$$= 2.517 \angle 18.45^\circ$$

neglecting $r_a \rightarrow 2.516 \angle 18.45^\circ$

Example 8.3 — 3-ϕ short circuit on Example 8.2

I_f doesn't change

$\Rightarrow$ $|E_a|$ stays the same as it is proportional to I_f.

$$\Rightarrow \quad I_a = \frac{E_a}{r + jX_s} \rightarrow |I_a| \doteq \frac{2.517}{1.6} = 1.573$$

$$\angle I_a = 89.86 \text{ lagging } (\approx 90^\circ)$$

Example 8.4

In example 8.3, in the steady state, the rotor is turning at a uniform rate with $\theta = \omega_s t + \theta_0$.
Assume I_a lags E_a by 90°.
Consider the rotating fluxes in the air gap

We know that:

$$E_{aa'} = \frac{E_{max}}{\sqrt{2}} e^{j(\theta_0 - \frac{\pi}{2})}$$

So, ~~E_a lags~~ $\angle E_a = \theta_0 - \frac{\pi}{2}$

Since I_a lags E_a by 90° $\rightarrow$ $\angle I_a = \theta_0 - \frac{\pi}{2} - \frac{\pi}{2} = \theta_0 - \pi$

~~We k~~ The flux due to I_F : $\Lambda_{aa'} = \frac{\lambda_{max}}{\sqrt{2}} e^{j\theta_0}$

Flux due to I_a, i_b, i_c (armature reaction) $\Lambda_{ar} = L_s I_a$

Since I_a lags θ_0 by 180°, these two fluxes are in opposite direction (when there is a short circuit)

$\Rightarrow$ Short circuit currents set up a demagnetizing flux which opposes the flux due to i_F.

Salient-pole case

$\angle I_a$ relative to θ_0 must be considered.

Suppose $\angle I_a = \theta_0 \Rightarrow \Lambda_{ar}$ is aligned with rotor.

if $\angle I_a = \theta_0 - 180 \rightarrow \Lambda_{ar}$ is in opposite direction, (demagnetizing)

In both cases, the air gap seen by Λ_{ar} is small.
$\Rightarrow$ relatively large inductance

If $\angle I_a = \theta_0 \pm 90° \rightarrow \Lambda_{ar}$ is pointed in a perpendicular direction to the rotor axis.

$\Rightarrow$ Air gap is larger

$\Rightarrow$ i_a, i_b, i_c are less effective in producing flux

$\Rightarrow$ small inductance

In the general case, I_a, I_b, I_c phasors are resolved into two groups of components.

I_{ad}, I_{bd}, I_{cd} : set up a rotating flux with centerline aligned with the "direct axis" of the rotor

I_{aq}, I_{bq}, I_{cq} : flux in the quadrature axis direction.

by superposition

$$\Rightarrow \quad \Lambda_{ar} = \Lambda_{ad} + \Lambda_{aq}$$

$I_{ad}, I_{bd}, I_{cd} \longrightarrow$ create Λ_{ad}

Λ_{ad} is proportional to I_{ad}

$(|I_{ad}| = |I_{bd}| = |I_{cd}|$, $120°$ phase shifts$)$

$I_{aq}, I_{bq}, I_{cq} \longrightarrow$ create Λ_{aq}

Λ_{aq} is proportional to I_{aq}

$$\Rightarrow \quad \Lambda_{ad} = L_{d1} I_{ad}$$

$$\Lambda_{aq} = L_{q1} I_{aq}$$

remember

$$\Rightarrow \quad \Lambda_{ag} = \Lambda_{aa'} + \Lambda_{ar}$$

$$\Rightarrow \quad \Lambda_{ag} = \Lambda_{aa'} + \Lambda_{ad} + \Lambda_{aq}$$

$$\Rightarrow \quad \underbrace{-j\omega_0 \Lambda_{ag}}_{V_{ag}} = \underbrace{-j\omega_0 \Lambda_{aa'}}_{E_a} - j\omega_0 \Lambda_{ad} - j\omega_0 \Lambda_{aq}$$

$$\Rightarrow \quad V_{ag} = E_a - j\underbrace{\omega_0 L_{d1}}_{X_{d1}} I_{ad} - j\underbrace{\omega_0 L_{q1}}_{X_{q1}} I_{aq}$$

$$V_{ag} = E_a - jX_{d1} I_{ad} - jX_{q1} I_{aq}$$

$$\Rightarrow \quad V_a = V_{ag} - rI_a - jX_\ell I_a$$

$$= V_{ag} - rI_a - jX_\ell (I_{aq} + I_{ad})$$

$$\Rightarrow \quad \cancel{V_{ag}} \; V_a = E_a - jX_{d1} I_{ad} - jX_{q1} I_{aq} - rI_a - jX_\ell (I_{ad} + I_{aq})$$

$$V_a = E_a - rI_a - j\underbrace{(X_\ell + X_{d1})}_{X_d} I_{ad} - j\underbrace{(X_\ell + X_{q1})}_{X_q} I_{aq}$$

$$\Rightarrow \quad V_a = E_a - rI_a - jX_d I_{ad} - jX_q I_{aq}$$

X_d : direct axis reactance

X_q : quadrature axis reactance

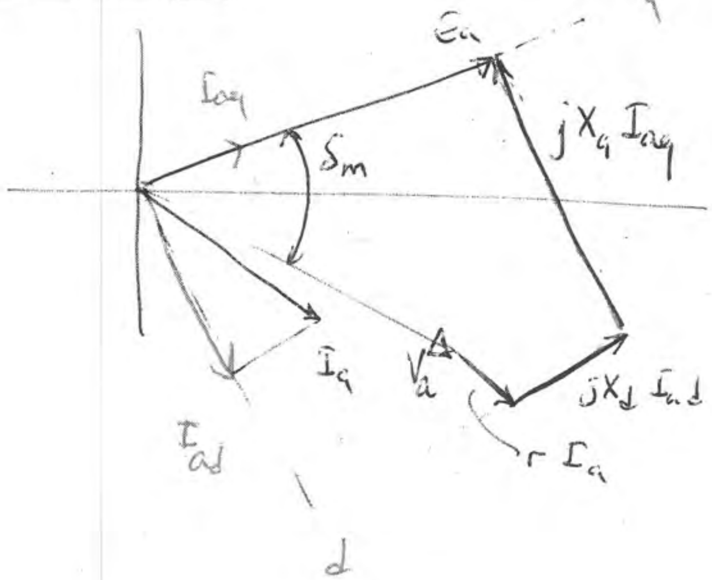
Generator model using X_d, X_q is called "two-reaction model" ∯

Typical values of X_d and X_q (in p.u)

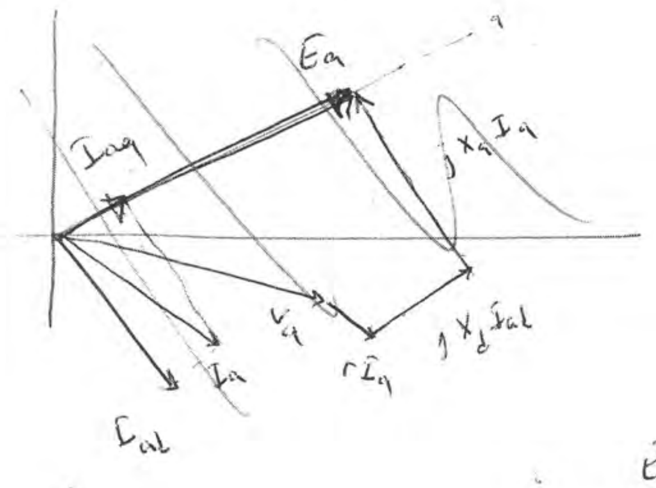
	2-pole turbine generators	Salient pole machines (with dampers)
X_d	1.20	1.25
X_q	1.16	0.70

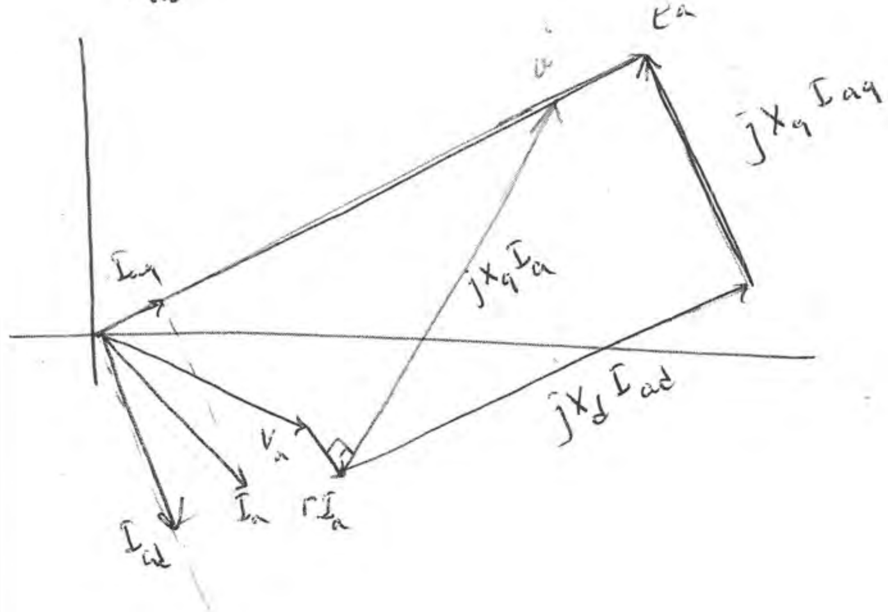
very close $X_d \simeq X_q$ for round rotor.

Phasor diagram.



Alternative phasor diagram.





Claim: $V_a + rI_a + jX_qI_a$ gets to the same direction as E_a.

proof:
$$a' = V_a + rI_a + jX_qI_a$$
$$a' = V_a + rI_a + jX_q(I_{ad} + I_{aq})$$

also
$$E_a = V_a + rI_a + jX_dI_{ad} + jX_qI_{aq}$$

$$\Rightarrow E_a - a' = j(X_d - X_q)I_{ad} + j\cancel{(X_q - X_q)}I_{aq}$$

$$\Rightarrow E_a - a' = j(X_d - X_q)I_{ad}$$

$\Rightarrow$ E_a and a' are collinear

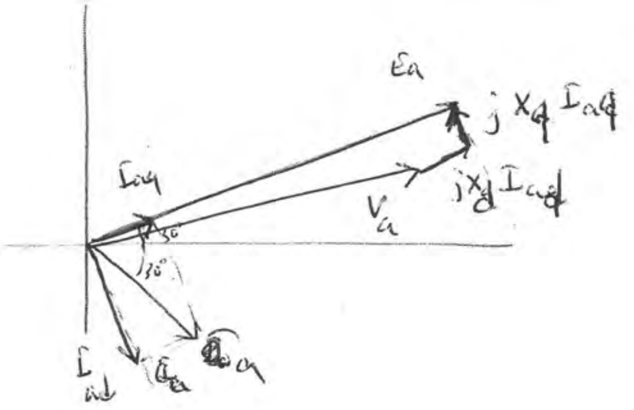
$\Rightarrow$ Given E_a, I_a we can find V_a

Given V_a, I_a we can find E_a

Example 8.5

$E_a = 1.5\angle 30^\circ$, $I_a = 0.5\angle -30^\circ$, $X_d = 1$, $X_q = 0.6$ neglect r

find V_a:



$$|I_{aq}| = |I_a| \cdot \cos 60 = \tfrac{1}{2} \cdot 0.5 = 0.25$$

$$\Rightarrow I_{aq} = 0.25\angle 30^\circ$$

$$|I_{ad}| = |I_a| \sin 60 = \tfrac{\sqrt{3}}{2} \cdot 0.5 = 0.433$$

$$\Rightarrow I_{ad} = 0.433\angle -60^\circ$$

$$\Rightarrow V_a = E_a - jX_d I_{ad} - jX_q I_q$$

$$= 1.5\angle 30^\circ - j\cancel{1} \cdot 0.433\angle -60 - j\,0.6 \times 0.25\angle 30$$

$$= 1.077\angle 22^\circ$$

Example 8.6

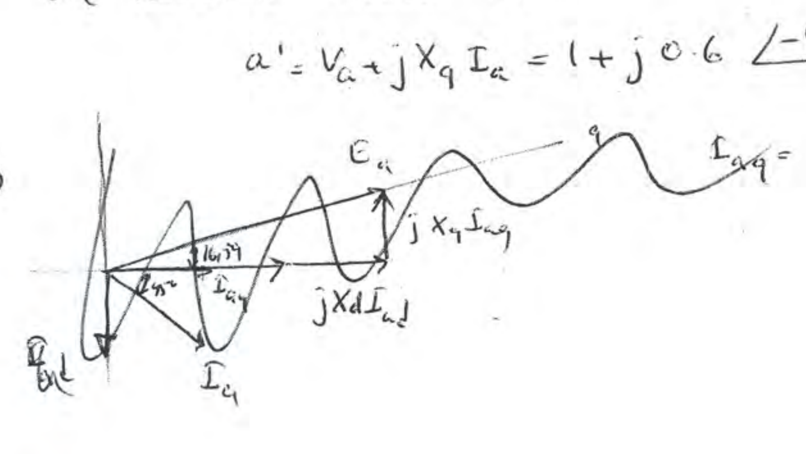
$V_a = 1\angle 0^\circ$, $I_a = 1\angle -45^\circ$, $X_d = 1$, $X_q = 0.6$, $r = 0$

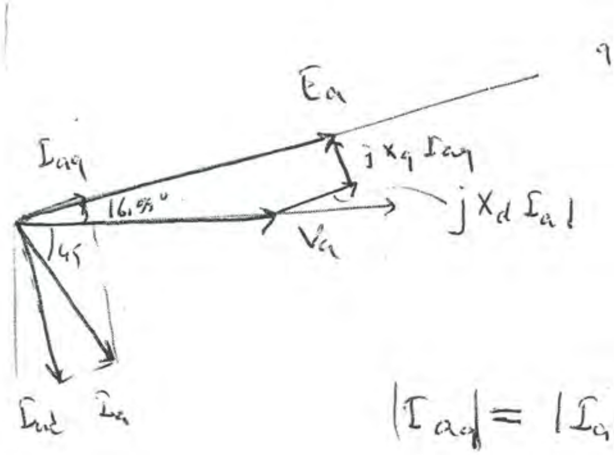
$E_a = ?$

We need to know a':

$$a' = V_a + jX_q I_a = 1 + j\,0.6\angle -45^\circ = 1.486\angle 16.59^\circ$$

$\Rightarrow$





$$|I_{aq}| = |I_a| \cos(61.59) = 0.4758$$

$$|I_{ad}| = |I_a| \sin 61.59 = 0.88$$

$$I_{aq} = 0.4758 \angle 16.59$$

$$I_{ad} = 0.88 \angle -73.41$$

$$\Rightarrow E_a = V_a + jX_d I_{ad} + jX_q I_{aq}$$

$$= 1 + j\,0.88 \angle -73.41 + j\,0.6 * 0.4758 \angle 16.59$$

$$\boxed{E_a = 1.838 \angle 16.59}$$

Example 8.7

In Ex. 8.5, there is a symmetric 3-ϕ short. i_f is constant.

$\Rightarrow |E_a| = 1.5$

$V_a = 0$ (short)

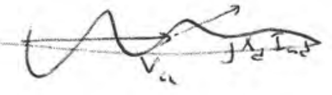
$$\Rightarrow \quad E_a = jX_d I_{ad} + jX_q I_{aq}$$

E_a and I_{aq} are on the same axis.

$\Rightarrow jI_{aq} \perp E_{aq}$

$jI_{ad} \parallel E_a$

Normally:



Short circuit current is independent from X_q.

$\Rightarrow I_{aq}$ must be zero.

$$\Rightarrow |E_a| = X_d |I_{ad}| \rightarrow |I_{ad}| = \frac{|E_a|}{X_d} = \frac{1.5}{1} = 1.5$$

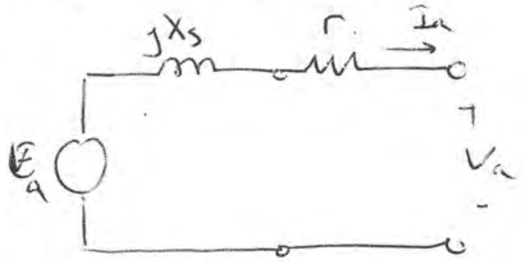
Power Delivered by Generator

S_G : complex power delivered by a generator

E_a : internal voltage

V_a : terminal voltage.

Case 1 : Round rotor



$$S_G = V_a I_a^* = V_a \left(\frac{E_a - V_a}{Z_G}\right)^*$$

$$Z_G = r + jX_s$$

neglecting r :

$\Rightarrow$ and assuming: $E_a = |E_a|$

$$V_a = |V_a| \angle -\delta_m$$

$$\Rightarrow \quad S_G = |V_a|\, e^{-j\delta_m} \left(\frac{|E_a| - |V_a|\, e^{-j\delta_m}}{jX_s}\right)^*$$

$$= |V_a|\, e^{-j\delta_m} \left(\frac{|E_a|}{X_s} e^{j\pi/2} - \frac{|V_a|}{X_s} e^{j\left(\delta_m + \frac{\pi}{2}\right)}\right)$$

$$S_G = \frac{|V_a|\,|E_a|}{X_s} e^{j\left(\frac{\pi}{2} - \delta_m\right)} - \frac{|V_a|^2}{X_s} e^{j\pi/2}$$

$$= \frac{|V_a|\,|E_a|}{X_s}\left(\underbrace{\cos\left(\frac{\pi}{2} - \delta_m\right)}_{\sin\delta_m} + j\underbrace{\sin\left(\frac{\pi}{2} - \delta_m\right)}_{\cos\delta_m}\right) - \frac{|V_a|^2}{X_s}\, j \sin\frac{\pi}{2}$$

$$\Rightarrow \quad S_G = \frac{|V_a|\,|E_a|}{X_s} \sin\delta_m + j\frac{|V_a|}{X_s}\Big[|E_a| \cos\delta_m - |V_a|\Big]$$

$$\Rightarrow \quad P_G = \underbrace{\frac{|V_a|\,|E_a|}{X_s}}_{\text{max power that can be delivered}} \sin\delta_m \, , \quad Q_G = |V_a| \frac{|E_a| \cos\delta_m - |V_a|}{X_s}$$

Case 2 Salient-pole generator

assume: $(r=0)$

$$E_a = |E_a| \angle 0^\circ$$

$$V_a = |V_a| \angle -\delta_m = |V_a|(\cos\delta_m - j\sin\delta_m)$$

$$\Rightarrow \quad \text{at } t=0,\ \theta = 90^\circ \ (\theta_0 = 90^\circ)$$

$$S_G = V_a I_a^* = V_a (I_{ad} + I_{aq})^*$$

$$|E_a| = V_a + jX_d I_{ad} + jX_q I_{aq}$$

$$= |V_a|\cos\delta_m - j|V_a|\sin\delta_m + jX_d I_{ad} + jX_q I_{aq}$$

Since $|E_a|$ is a real number, then I_{aq} and $jX_d I_{ad}$ are also real. Also, $jX_q I_{aq}$ is purely imaginary.

$$\Rightarrow \quad |E_a| = |V_a|\cos\delta_m + \cancel{j}X_d I_{ad}$$

$$0 = -\cancel{j}|V_a|\sin\delta_m + \cancel{j}X_q I_{aq}$$

$$\Rightarrow \quad I_{ad} = \frac{|E_a| - |V_a|\cos\delta_m}{X_d}$$

$$I_{aq} = \frac{|V_a|\sin\delta_m}{X_q}$$

$$\Rightarrow \quad S_G = |V_a| e^{-j\delta_m}\left(j\frac{|E_a| - |V_a|\cos\delta_m}{X_d} + \frac{|V_a|\sin\delta_m}{X_q}\right)^*$$

$$= |V_a|(\cos\delta_m - j\sin\delta_m)\left(\frac{|V_a|\sin\delta_m}{X_q} + \bar{j}\,\frac{|E_a| - |V_a|\cos\delta_m}{X_d}\right)$$

$$\Rightarrow P_G = |V_a|\left[\frac{|V_a|\cos\delta_m\sin\delta_m}{X_q} + \frac{|E_a| - |V_a|\cos\delta_m}{X_d}\sin\delta_m\right]$$

$$= \frac{|V_a||E_a|}{X_d}\sin\delta_m + \frac{|V_a|^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta_m$$

Note: if $X_d = X_q = X_s$

$$\Rightarrow \quad P_G = \frac{|E_a|\,|V_a|}{X_s} \sin \delta_m \quad \text{(same as before)}$$

Similarly:

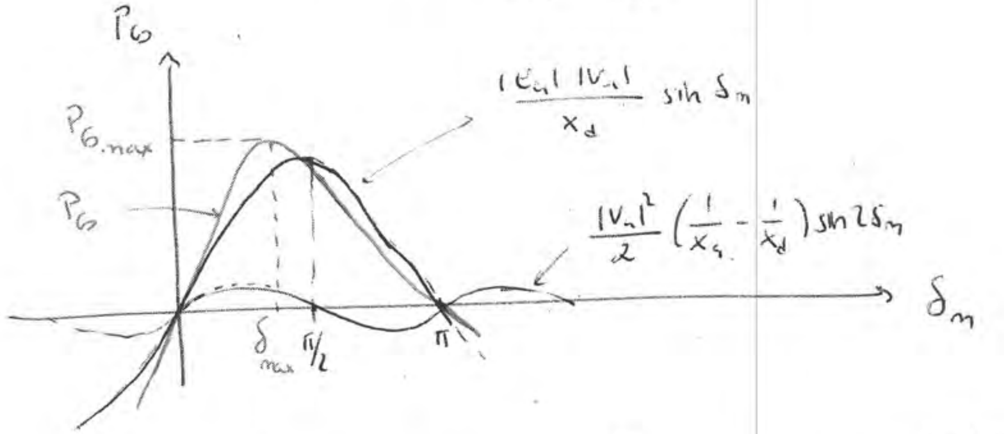
$$Q_G = \frac{|E_a|\,|V_a|}{X_d} \cos \delta_m - |V_a|^2 \left(\frac{\cos^2 \delta_m}{X_d} + \frac{\sin^2 \delta_m}{X_q} \right)$$

$X_d = X_q = X_s \Rightarrow$

$$Q_G = \frac{|E_a|\,|V_a|}{X_s} \cos \delta_m - \frac{|V_a|^2}{X_s}$$

Effect of saliency:

$$X_d > X_q \Rightarrow \frac{1}{X_q} - \frac{1}{X_d} > 0$$

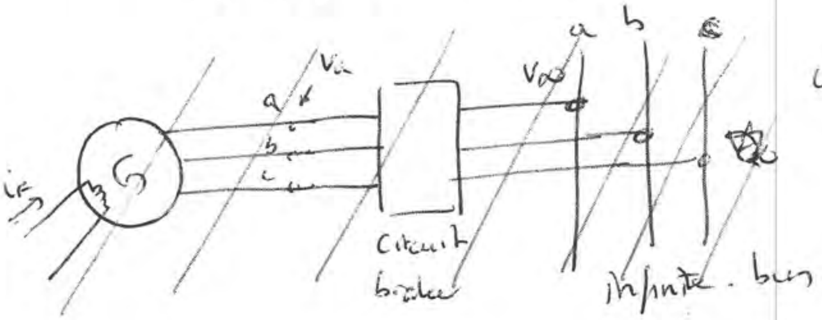


because of saliency, $P_{G\,max}$ occurs before $\delta_m = \frac{\pi}{2}$

Synchronizing Generator to an Infinite Bus.

* Generator power is small compared to power system
* It is synchronized first, then necessary adjustments are made ~~for~~ to deliver specified complex power.
* Bus voltage is not effected by this generator.

$\Rightarrow$ infinite bus (Ideal voltage source)



use the fig. in from the book.

Initially circuit braker is open.

$V_a = E_a \quad (I_a = 0)$

- frequency is the same
- phase sequence is the same
- phase is the same
- $|V_a| = |E_a| = |V_\infty|$

$\Big\} \; V_a = E_a = V_\infty$

When these conditions are satisfied, breaker may be closed. Then the system is "floating" $(I_a = 0)$ because $(E_a = V_\infty)$ and $S_G = 0$

Now, the valve is slowly opened.

P_{mech}^{in} slowly increases

Generator and turbine rotor tend to accelerate $\Rightarrow \delta_m$ increases $\Rightarrow P_G(\delta_m)$ increases. (V_∞ is fixed)

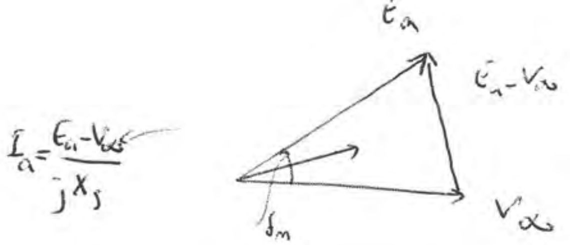
When steady state is reached (neglecting losses)

electrical power out = mech. power in.

(10)

if I_f = constant at its synchronization value,
$|E_a|$ stays the same. But, with increasing P_{mech}
~~$|E_a|$ E_a increases as $\angle E_a$ changes~~
$\angle E_a$ increases.

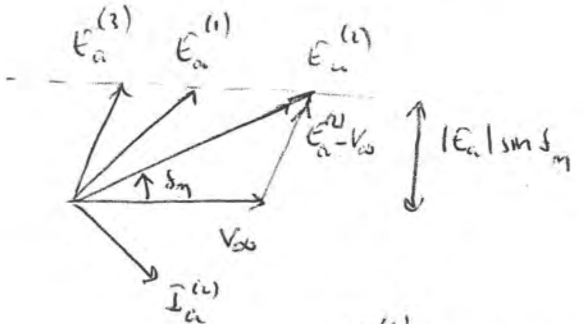
(assuming round rotor, with r=0)



if I_f is changed, $|E_a|$ changes proportionally.
If P_{mech} is kept constant meanwhile,

$$\Rightarrow \quad P_G(\delta_m) = \frac{|E_a|\,|V_\infty|}{X_s} \sin \delta_m = \text{constant}$$

$$\Rightarrow \quad |E_a| \sin \delta_m = \text{constant}$$



$E_a^{(1)}$ - at synchronization
$E_a^{(2)}$: increased i_F
$E_a^{(3)}$: decreased i_F

If $|E_a|$ is increased beyond $E_a^{(2)} \longrightarrow I_a \longrightarrow$ beyond $|I_a|^2$
power-factor ↘
so that
$P_G = |V_\infty|\,|I_a| \cos\phi = \text{const.}$

Example:

$i_F = 1000$ A at synch.

$V_\infty = 1\angle 0°$

$X_s = 1.5$

With i_F unchanged, the steam values at the turbine are adjusted until $P_G = 0.2$

a) Find I_a

b) With P_G unchanged, i_F is increased to 1600 A.
Find I_a

Solution:

a) At synch.

$$E_a = V_a = V_\infty = 1\angle 0$$

$\Rightarrow |E_a| = 1$ corresponds to $i_F = 1000$ A

$$\Rightarrow P_G = 0.2 = \frac{|E_a||V_\infty|}{X_s}\sin\delta_m = \frac{1}{1.5}\sin\delta_m$$

$$\rightarrow \delta_m = 17.46°$$

$$E_a = 1\angle 17.46°$$

$$I_a = \frac{E_a - V_\infty}{jX_s} = \frac{1\angle 17.46 - 1}{j1.5} = 0.202\angle 8.73°$$

(b) $i_F' = 1.6\, i_F \Rightarrow |E_a|' = 1.6\,|E_a|$

$$\Rightarrow P_G = 0.2 = \frac{1.6}{1.5}\sin\delta_m \rightarrow \delta_m = 10.81°$$

$$E_a = 1.6\angle 10.81°$$

$$\Rightarrow \quad I_a = \frac{1.6\angle 10.81° - 1}{j1.5} = 0.430\angle -62.31°$$

Synchronous Condenser

Consider round-rotor machine with $r=0$

Power delivered by the machine to a bus with voltage V_a is:

$$P_G = Re\, V_a I_a^* = \frac{|E_a|\,|V_a|}{X_s} \sin \delta_m$$

$\delta_m > 0 \Rightarrow P_G > 0$ (generator)

$\delta_m < 0 \rightarrow P_G < 0$ (motor)

$\delta_m = 0 \rightarrow P_G = 0$ (unloaded)

When $\delta_m = 0$, even though $P_G = 0$, $Q_G \neq 0$

It delivers reactive power.

$$Q_G = \frac{|V_a| \left(|E_a| - |V_a|\right)}{X_s}$$

So, if $|E_a| \geq |V_a|$ supplies reactive power like a capacitor bank connected to a bus

$\cancel{|E_a| < |V_a|}$

a generator ~~opera~~ designed to operate at this mode is called synch. condenser.

At large power levels, they are cheaper than capacitor banks, and ~~cea~~ continuous control of reactive power is possible through I_F.