3-ϕ Transformers

3 X 1-ϕ transformers can be connected as

Δ-Y
Y-Δ
Δ-Δ
Y-Y

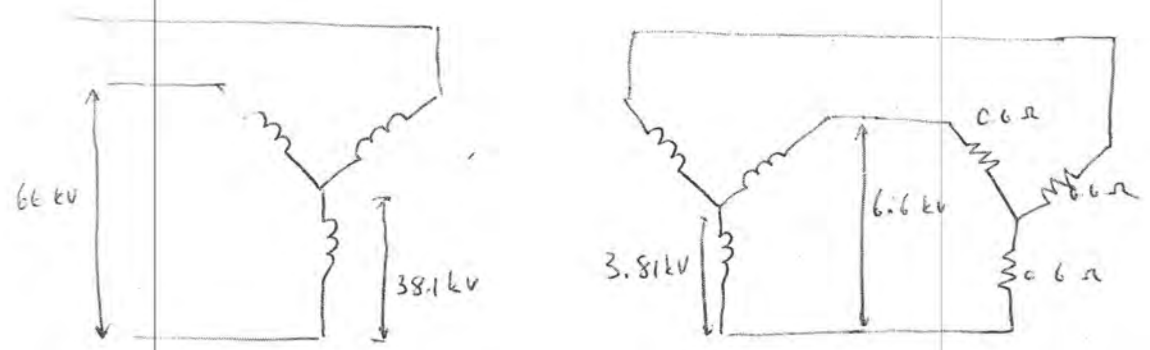
to form 3-ϕ transformers.

A transformer with all 3-ϕ wounded on the same core is preferred to 3 X 1-ϕ transformers.

Consider: 3 X 1-ϕ each 25 MVA, 38.1/3.81 kV
Y-Y connected.

For the 3-ϕ transformer, rating: 75 MVA
66/6.6 kV

($38.1\sqrt{3} = 66$)

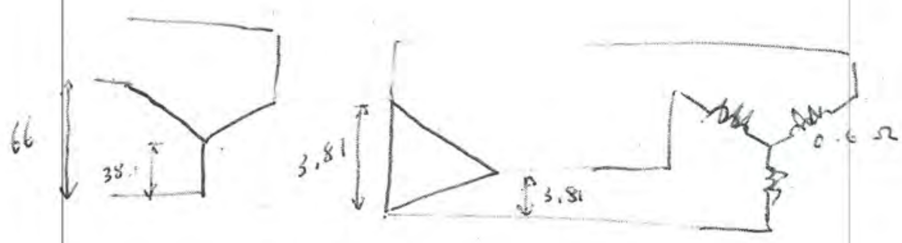


0.6 Ω/phase load is connected Y also

Because the system is balanced, neutrals of the load and low-voltage side are at the same potentials

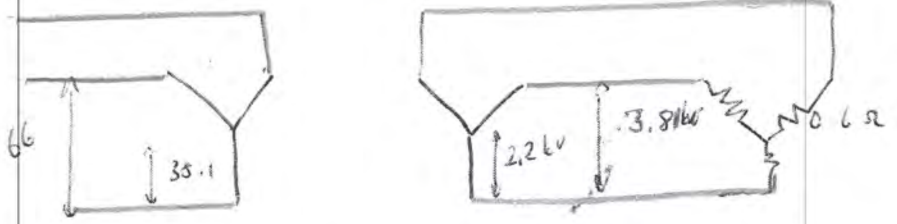
Z_{l-n} (measured from the high-tension side)
$$= 0.6\left(\frac{66}{6.6}\right)^2 = 60\ \Omega$$

Other connections:



Y — Δ

This connection and the following give the same output voltage magnitude (there is a phase shift)



$$Z_L' = 0.6\left(\frac{38.1}{2.2}\right)^2 = 0.6\left(\frac{66}{3.81}\right)^2 = 180\ \Omega$$

square of the ratio of the l-l voltages, not the square of the # of turns of each winding.

→ in 3-φ circuits

$$\frac{V_{B1}}{V_{B2}} = \frac{V_{LL-1}}{V_{LL-2}}$$ regardless of the connection.

Example $3 \times 1-\phi$ transformer. rated 25 MVA, 38.1/3.81 kV

Y-Δ connected.

0.6 Ω/phase load. (Y)

Choose $S_B = 75$ MVA

$V_B = 66$ kV for high-tension side

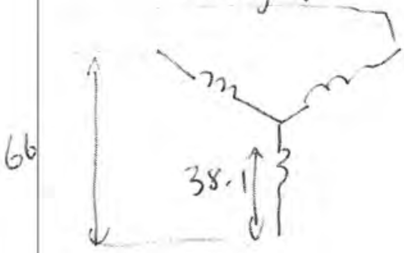
Find R_L^{pu} on the base for low-tension side

Determine R_L referred to high-tension side

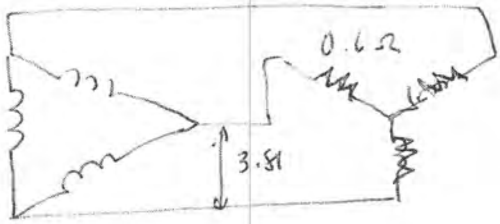
and p-u value of this resistance

Solution:

~~Rating of~~



75 MVA



$V_B = 3.81$ kV

$$Z_B = \frac{(3.81\ 10^3)^2}{75\ 10^6} = 0.1935\ \Omega$$

$$R_L^{pu} = \frac{0.6}{0.1935} = 3.1 \text{ pu}$$

$$Z_B = \frac{(66)^2}{75} = 58.1\ \Omega$$

$$R_L' = 0.6 * \left(\frac{66}{3.81}\right)^2 = 180\ \Omega$$

$$R_L'^{,pu} = \frac{180}{58.1} = 3.1 \text{ pu}$$

⟵⟶

⇒ p.u value of impedances (R, X) are the same whether they are reflected to high or low tension sides.

Bad Example

Example:

3-ϕ-transformer
400 MVA, 220 Y/22Δ kV

Short-circuit impedance measured ~~from~~ on the low-voltage side : 0.121 Ω

This is a low-valu $\Rightarrow$ it can be considered as leakage reactance

? p.u.-reactence of transformer

? the value to be used to represent this transformer in a system whose base on the high-tension side of the transformer is 100 MVA, 230 kV

Solution

transformer reactance on its own base $= \dfrac{0.121}{\frac{(22)^2}{400}} = 0.1$ p.u.

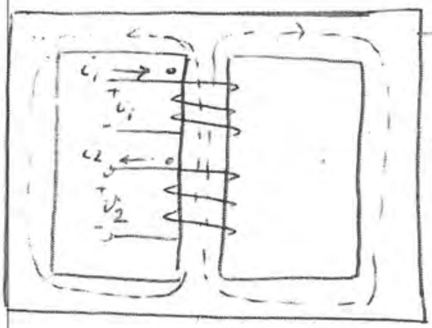
On the chosen base: $0.1 * \left(\dfrac{220}{230}\right)^2 \dfrac{100}{400} = 0.0228$ pu.

$\left[0.1 * \left(\dfrac{\frac{(22)^2}{400}}{\frac{(230)^2}{100}}\right) \right.$ doesn't give the same value $\left.\right]$

$\left[\left(\dfrac{(220)^2}{400}\right) \right.$ must be used $\left.\right]$

reflected value?

Ideal Transformer

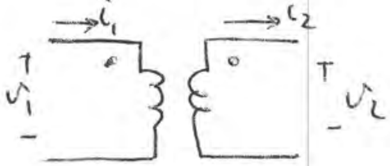


$\to \mu \to \infty$

flux: sinusoidal

$R_{winding} \to 0$

$$\left.\begin{aligned} v_1 = e_1 = N_1 \frac{d\phi}{dt} \\ v_2 = e_2 = N_2 \frac{d\phi}{dt} \end{aligned}\right\} \Rightarrow \frac{V_1}{V_2} = \frac{N_1}{N_2}$$

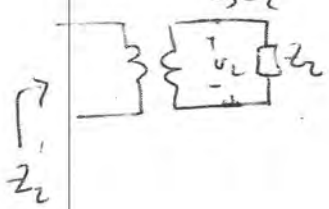


Ampere's Law.

$$\oint \bar{H} \cdot d\bar{l} = i_{net} = N_1 i_1 - N_2 i_2 = 0 \quad (\mu \to \infty)$$

$$\Rightarrow \quad N_1 I_1 = N_2 I_2$$

$$\Rightarrow \quad \boxed{\frac{I_1}{I_2} = \frac{N_2}{N_1}}$$



$$V_2 = Z_2 I_2$$

$$\Rightarrow Z_2 = \frac{V_2}{I_2} = \frac{\frac{N_2}{N_1} V_1}{\frac{N_1}{N_2} I_1} = \left(\frac{N_2}{N_1}\right)^2 \frac{V_1}{I_1}$$

$$Z_2' = \frac{V_1}{I_1} = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

Also. $V_1 I_1 = \frac{N_1}{N_2} V_2 \times \frac{N_2}{N_1} I_2 = V_2 I_2$

Similarly $\quad V_1 I_1^* = V_2 I_2^*$ for ideal transformer

Example: $N_1 = 2000$, $N_2 = 500$

$V_1 = 1200\angle 0^\circ$ V $\qquad I_1 = 5\angle -30^\circ$

$Z_2, Z_2', V_2, I_2 = ?$

$$\cancel{V_2 = \frac{N_1}{N_2} V_1 = \frac{2000}{500} 1200 = 4800\angle 0^\circ}$$

$$V_2 = \frac{N_2}{N_1} V_1 = \frac{500}{2000} 1200 = 300\angle 0^\circ$$

$$I_2 = \frac{N_1}{N_2} I_1 = \frac{2000}{500} 5\angle -30^\circ = 20\angle -30^\circ$$

$$\Rightarrow Z_2 = \frac{V_2}{I_2} = \frac{300\angle 0^\circ}{20\angle -30^\circ} = 15\angle 30^\circ \ \Omega$$

$$Z_2' = \left(\frac{N_1}{N_2}\right)^2 Z_2 = 16\ Z_2 = 240\angle 30^\circ \ \Omega$$

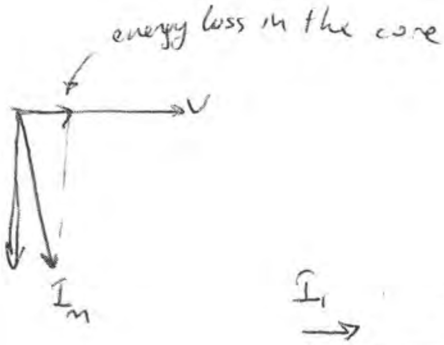
Equivalent circuit of a practical transformer

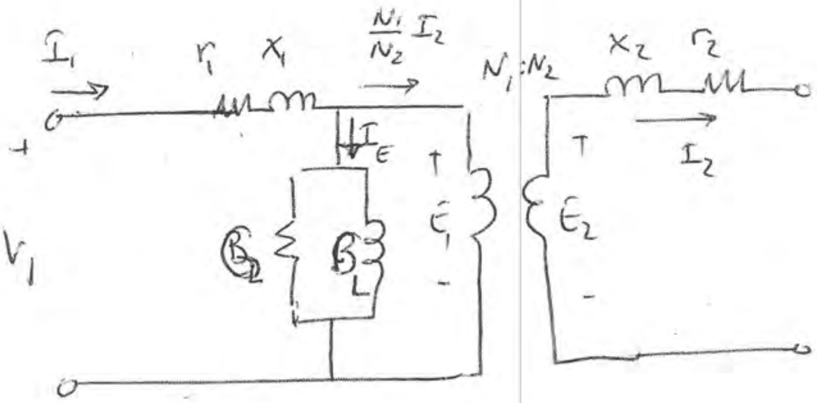
- μ is not infinite
- winding resistance is zero
- core losses
- leakage flux

when the secondary winding is open, primary current is not zero. A small current, called "magnetizing current" flows in primary.

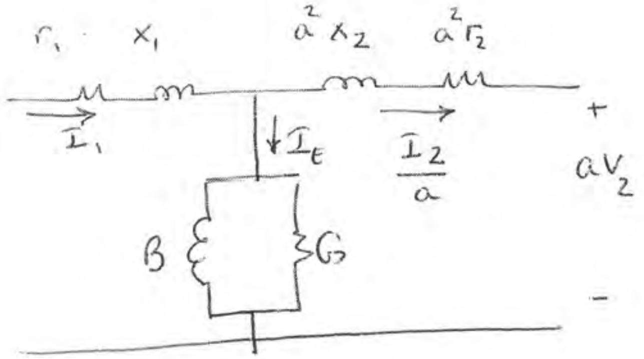
Iron losses : hysteresis losses
eddy-current losses → laminated sheets of steel

Magnetizing current is a highly inductive current.





referred to high voltage side:

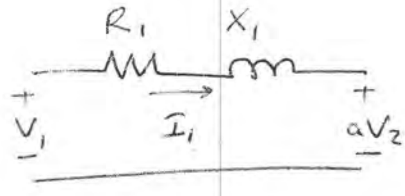


X : leakage reactance

$a = \frac{N_1}{N_2}$

I_E is usually ignored.

$$\Rightarrow \quad R_1 = r_1 + a^2 r_2$$
$$X_1 = x_1 + a^2 X_2$$



Example: 1-ϕ transformer.

$N_1 = 2000$, $N_2 = 500$

$r_1 = 2\ \Omega$ $r_2 = 0.125\ \Omega$

$x_1 = 8\ \Omega$ $x_2 = 0.50\ \Omega$

$z_2 = 12\ \Omega$ $V_1 = 1200\,V$

$V_2 = ?$ voltage regulation = ? neglect I_E.

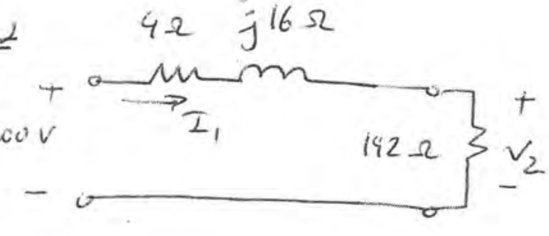
Solution: $a = \frac{N_1}{N_2} = 4$

$R_1 = r_1 + a^2 r_2 = 2 + 16 \times 0.125 = 4\ \Omega$

$X_1 = 8 + 16 * 0.5 = 16\ \Omega$

$z_2' = 16 * z_2 = 16 \times 12 = 192\ \Omega$

Ex



$$I_1 = \frac{1200}{4 + 192 + j16} = 6.10 \angle -4.67^\circ$$

$$aV_2 = 192\, I_1 = 1171.6 \angle -4.67^\circ$$

$$V_2 = \frac{1171.6}{4} \angle -4.67 = 292.9 \angle -4.67 \text{ V}$$

voltage regulation : $\dfrac{\frac{1200}{4} - 292.9}{292.9} = 0.0242$

2.42%

$G - jB_c$ can be measured by using open-circuit test.
$R + jX$ " " " short-circuit test

Per-Unit Impedances in Single-Phase Transformer Circuits

S_B : kVA rating of the transformer.

V_B { low voltage side value; if the parameters are referred to low voltage
high voltage side value; if the parameters are referred to high

per-unit impedance is the same

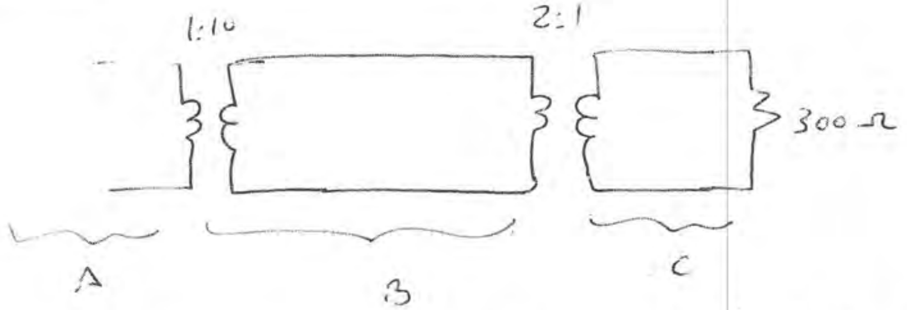
Example. 1-ϕ transformer, 110/440 V, 2.5 kVA
Leakage reactance measured from the low-tension side : 0.06 Ω ⇒ leakage reactance in p.u.

$S_B = 2.5\ 10^3$
$V_B = 110$ V } $Z_B = \dfrac{V_B^2}{S_B} = \dfrac{(110)^2}{2.5\ 10^3} = 4.84\ \Omega$

$$\Rightarrow X_{pu} = \frac{0.06}{4.84} = 0.0124 \text{ pu}$$

If many ~~transformers~~ circuits are connected ~~in~~ through transformers, different voltage bases are chosen for each part. Voltage base is calculated by using turns ratios. Same VA base is used for each part.

Example:



A-B transformer. 10000 kVA 138/13.8 kV leakage reactance: 10 %

B-C 10000 kVA 138/69 kV leakage reactance 8 %

Base in B is chosen 10000 kVA, 138 kV
find the p.u. value of 300 Ω resistive load referred to parts B and A.
Draw the impedance diagram neglecting magnetizing current, transformer resistance and line impedances.
Determine the voltage regulation if the voltage at the load is 66 kV with the assumption that the voltage input to circuit A remains consant.

Solution: $V_B^A = 0.1 * V_B^B = 0.1 * 138 = 13.8$ kV

$$V_B^C = 0.5 * V_B^B = 0.5 * 138 = 69 \text{ kV}$$

$$Z_B^C = \frac{(V_B^C)^2}{S_B} = \frac{(69\ 10^3)^2}{10000\ 10^3} = 476.1\ \Omega$$

$$\Rightarrow X^C = \frac{300}{476.1} = 0.63 \text{ pu}$$

$$Z_B^B = \frac{(V_B^B)^2}{S_B} = \frac{(138\ 10^3)^2}{10\ 10^6} = 1904\ \Omega$$

$\cancel{X^B =}$

Load impedance referred to B : $300 \times 2^2 = 1200\ \Omega$

$$\Rightarrow X^B = \frac{1200}{1904} = 0.63\ pu \quad (\text{same})$$

$$Z_B^A = \frac{(V_B^A)^2}{S_B} = \frac{(13{,}8\ 10^3)^2}{10\ 10^6} = 19.\ \Omega$$

Load imp referred to A : $300 * 2^2 \times (0.1)^2 = 12\ \Omega$

$$X^A = \frac{12}{19} = 0.63\ pu.$$

$\Rightarrow$

voltage at load : $\dfrac{66\ kV}{69\ kV} = 0{,}957$ pu.

Load current : $\dfrac{0{,}957}{0.63} = 1.52$ p.u.

voltage input ~~Input voltage~~ (voltage at load with load removed) $= (1.52) * j(0.1 + 0.08)) + 0.957$

$= 0{,}957 + j\,0.274 \rightarrow 0.995$ pu.

$$\Rightarrow r = \frac{0{,}995 - 0.957}{0{,}957} = 0.0397$$

(3.97 %)

ONE-LINE DIAGRAM.

Standard symbols:

Machine Fig. 6.25 / p 156 (Stevenson)

(Fig 10.1, page 493 of Chapman)

One-line diagram supplies the significant information about the system.

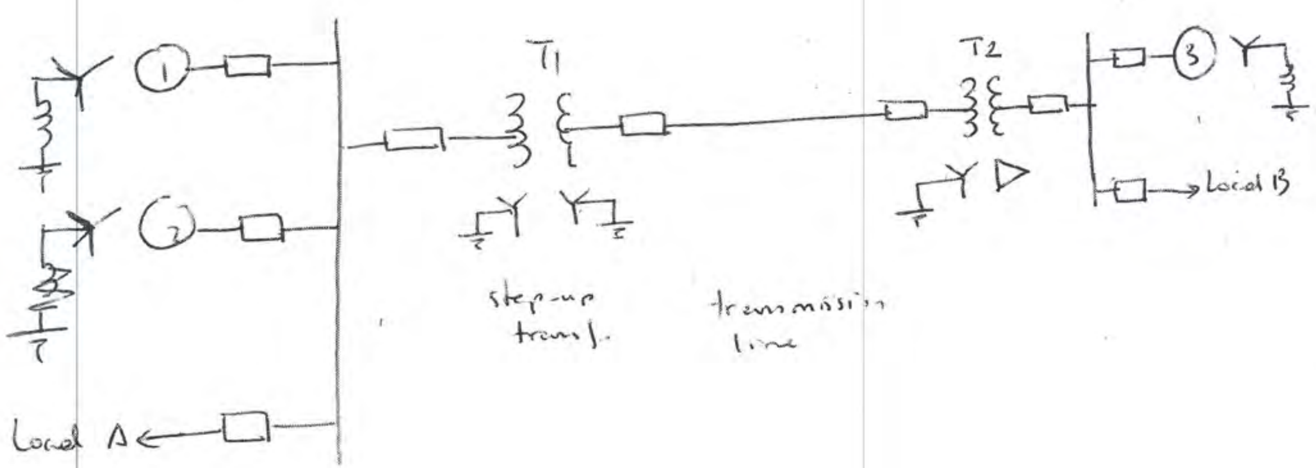
- Parameters are not shown
- 3-ϕ → 1-ϕ (Balanced system)
- Amount of information depends on the purpose.
  Circuit breakers may be omitted if load study is important.

Ground point is important to calculate unsymmetrical fault currents.

Most transformer neutrals are solidly grounded.

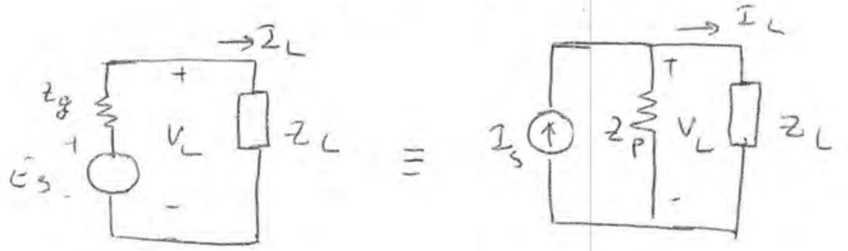
Generator neutrals are usually grounded through very large resistors (sometimes inductance coils)

A very simple power system:



Network Calculations

Constant emf $\equiv$ constant current source in parallel with impedance



$$V_L = E_g - Z_g I_L \quad = \quad V_L = (I_s - I_L) Z_p = Z_p I_s - Z_p I_L$$

$$\Rightarrow E_g - Z_g I_L = Z_p I_s - Z_p I_L$$

$$Z_p = Z_g \Rightarrow Z_p I_s = E_g$$

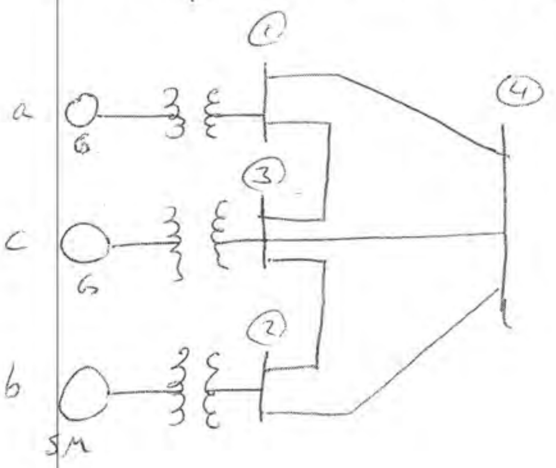
$$\Rightarrow I_s = E_g / Z_p$$

Same can be done for active loads

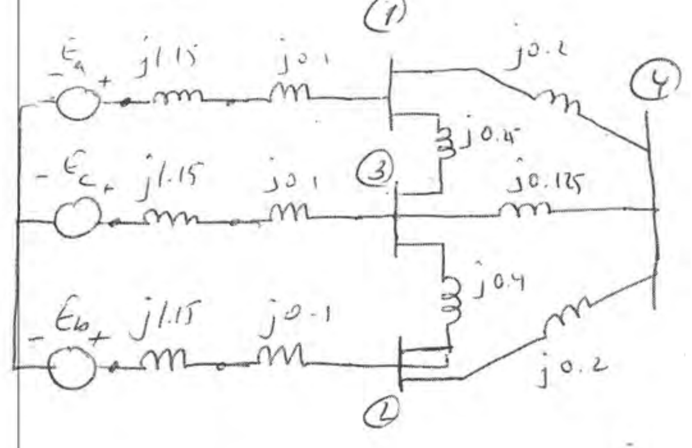
Node Equations

if more than two components are connected to a node, it is called "major node".

a sample one-line diagram:

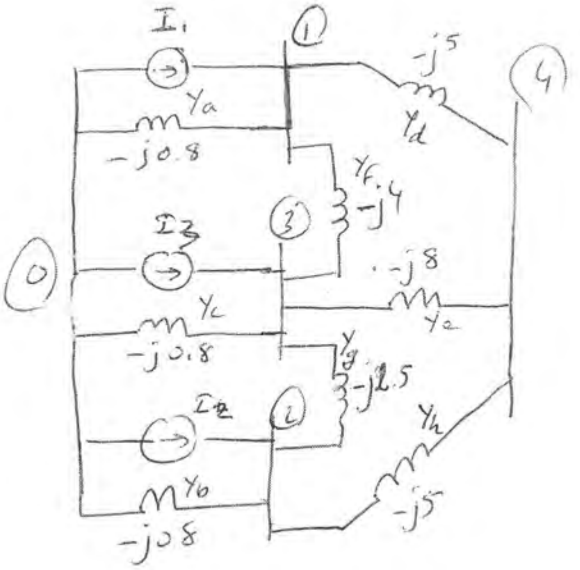


Reactance diagram -



$$\frac{1}{j(1.15+0.1)} = \frac{1}{j1.25} = 0,8(-j)$$

E → replaced by I
Z → replaced by Y



KCL (Node 1) $\quad I_1 = Y_a . V_1 + Y_f . (V_1 - V_3) + Y_d (V_1 - V_4)$

(Node 4) $\quad 0 = Y_d (V_4 - V_1) + Y_h (V_4 - V_2) + Y_e (V_4 - V_3)$

$$\Rightarrow \quad I_1 = V_1 [Y_a + Y_f + Y_d) - V_3 Y_f - V_4 Y_d$$

$$0 = -V_1 Y_d - V_2 Y_h - V_3 Y_e + V_4 (Y_d + Y_e + Y_h)$$

Same can be written for each node.
A matrix form set of equations is obtained.
The solution yields the currents when V_1, V_2, V_3, V_4 are known.

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \underbrace{\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}}_{Y_{bus}} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Y_{ij} — i: effect, j: cause

i: the node at which the current is expressed

j: the voltage that causes that component of current

Y_{bus} bus admittance matrix

*- Symmetrical

Y_{ii} : self admittances (driving point ad.)

others mutual admittances (transfer adm.)

For a network with N independent nodes

$$I_k = \sum_{n=1}^{N} Y_{kn} V_n$$

Example: $E_a = 1.5 \angle 0$

$E_b = 1.5 \angle -36.87$

$E_c = 1.5 \angle 0^\circ$

$$\Rightarrow \quad I_1 = I_3 = \frac{1.5\angle 0^\circ}{j1.25} = 1.2 \angle -90^\circ = -j1.2 \text{ pu}$$

$$I_2 = \frac{1.5 \angle -36.87}{j1.25} = 1.2 \angle -126.87 = -0.72 - j0.96 \text{ pu}$$

self adm.

$Y_{11} = -j0.8 - j4 - j5 = -j9.8$

$Y_{22} = -j0.8 - j2.5 - j5 = -j8.3$

$Y_{33} = -j4 - j0.8 - j2.5 \overset{-j8}{=} -j15.3 =$

$Y_{44} = -j5 - j8 - j5 = -j18$

$Y_{12} = Y_{21} = 0$

$Y_{13} = Y_{31} = +j4$ $\qquad Y_{23} = Y_{32} = -j2.5$ $\qquad Y_{34} = Y_{43} = +j8$

$Y_{14} = Y_{41} = +j5$ $\qquad Y_{24} = Y_{42} = +j5$

$\Rightarrow$

$$\begin{bmatrix} 0-j1.2 \\ -0.72-j0.96 \\ 0-j1.2 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} -j9.8 & j0 & j4 & j5 \\ j0 & -j8.3 & j2.5 & j5 \\ j4 & j2.5 & -j15.3 & j8 \\ j5 & +j5 & j8 & -j18 \end{bmatrix}}_{Y_{bus}} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$Y_{bus}^{-1} \begin{bmatrix} -j1.2 \\ -0.72-j0.96 \\ -j1.2 \\ 0 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 1.4111 - j0.2668 \\ 1.3830 - j0.3508 \\ 1.4059 - j0.2824 \\ 1.4009 - j0.2971 \end{bmatrix}$$

$$\Rightarrow V_1 = 1.436 \angle -10.71$$
$$V_2 = 1.427 \angle -14.24$$
$$V_3 = 1.434 \angle -11.36$$
$$V_4 = 1.432 \angle -11.97$$

$$Y_{bus}^{-1} = Z_{bus} = \begin{bmatrix} j0.4774 & j0.3706 & j0.4020 & j0.4142 \\ j0.3706 & j0.4872 & j0.3922 & j0.4126 \\ j0.4020 & j0.3922 & j0.4558 & j0.4232 \\ j0.4142 & j0.4126 & j0.4232 & j0.4733 \end{bmatrix}$$

Matrix Partitioning

$$A = \left[\begin{array}{cc|c} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ \hline a_{31} & a_{32} & a_{33} \end{array}\right] = \begin{bmatrix} D & E \\ F & G \end{bmatrix}$$

$$D = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad E = \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}$$

$$F = \begin{bmatrix} a_{31} & a_{32} \end{bmatrix} \quad G = a_{33}$$

if $AB = C$ where

$$B = \left[\begin{array}{c} b_{11} \\ b_{21} \\ \hline b_{31} \end{array}\right] = \begin{bmatrix} H \\ J \end{bmatrix}$$

$$\Rightarrow C = \begin{bmatrix} D & E \\ F & G \end{bmatrix} \begin{bmatrix} H \\ J \end{bmatrix} = \begin{bmatrix} DH + EJ \\ FH + GJ \end{bmatrix} = \begin{bmatrix} M \\ N \end{bmatrix}$$

where $M = DH + EJ$
$N = FH + GJ$

$$N = \begin{bmatrix} a_{31} & a_{32} \end{bmatrix} \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} + a_{33} b_{31}$$

$$= a_{31} b_{11} + a_{32} b_{21} + a_{33} b_{31}$$

⚠ row - column dimensions must be properly chosen.

Node Elimination by Matrix Algebra

Nodes at which current doesn't enter or leave the network, can be eliminated.

$$I = Y_{bus} V$$

Nodes to be eliminated are put to lower rows of the matrices.

$$\begin{bmatrix} I_A \\ I_x \end{bmatrix} = \begin{bmatrix} K & L \\ L^T & M \end{bmatrix} \begin{bmatrix} V_A \\ V_x \end{bmatrix}$$

I_x, V_x: Matrices of nodes to be eliminated

$[I_x \to 0]$

$$I_A = KV_A + LV_x$$

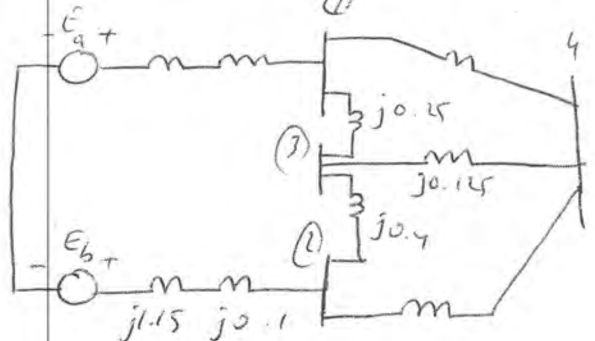
$$I_x = L^T V_A + MV_x \implies -L^T V_A = MV_x$$

$$-M^{-1} L^T V_A = V_x$$

$$\implies I_A = KV_A + L\left[-M^{-1} L^T V_A\right]$$

$$= \underbrace{\left[K - LM^{-1}L^T\right]}_{Y_{bus}} V_A$$

Example: G and T are removed from bus 3



Eliminate node 3, 4

$$Y_{bus} = \left[\begin{array}{cc|cc} -j9.8 & 0 & +j4 & j5 \\ 0 & -j8.3 & j2.5 & j5 \\ \hline j4 & j2.5 & -j14.5 & j8 \\ j5 & j8 & j8 & -j18 \end{array}\right] = \begin{bmatrix} K & L \\ L^T & M \end{bmatrix}$$

$$M = \begin{bmatrix} -j14.5 & j8 \\ j8 & -j18 \end{bmatrix}$$

$$\det M = -14.5 * 18 + 64 = -197$$

$$M^{-1} = \frac{1}{-197}\begin{bmatrix} -j18 & -j8 \\ -j8 & -j14.5 \end{bmatrix} = \begin{bmatrix} j0.0914 & j0.0406 \\ j0.0406 & j0.0736 \end{bmatrix}$$

$$LM^{-1}L^T = \begin{bmatrix} j4 & j5 \\ j2.5 & j5 \end{bmatrix}\begin{bmatrix} j0.0914 & j0.0406 \\ j0.0406 & j0.0736 \end{bmatrix}\begin{bmatrix} j4 & j2.5 \\ j5 & j5 \end{bmatrix}$$

$$= -\begin{bmatrix} j4.9264 & j4.0736 \\ j4.0736 & j3.4264 \end{bmatrix}$$

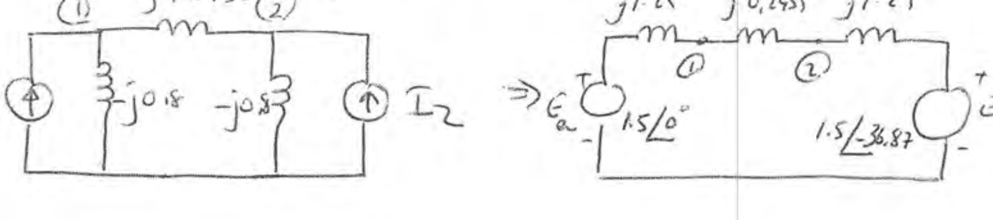
$$Y_{bus} = K - LM^{-1}L^T = \begin{bmatrix} -j4.8736 & j4.0736 \\ j4.0736 & -j4.8736 \end{bmatrix}$$

$\Rightarrow$ admittance between 1 and 2 = $\cancel{-j4.8736}$
$-j4.0736$

p.u. impedance between 1 & 2 $\quad j\frac{1}{4.0736}$

admittance between each of these nodes and reference node:

$$-j4.8736 - (-j4.0736) = -j0.8 \text{ pu}$$



$$I = \frac{E_a - E_b}{j(1.25 + 1.25 + 0.2455)} = \frac{1.5 - 1.5\angle -36.87}{j\,2.7455}$$ (132)

$$= 0.3278 - j0.1093 = 0.3455\angle -18.44 \text{ pu}$$

Power out of source a is:

$$1.5\angle 0° * 0.3455\angle 18.44 = 0.492 + j0.164 \text{ pu}$$

Power into source b

$$1.5\angle -36.87 * 0.3455\angle 18.44 = 0.492 - j0.164 \text{ pu.}$$

Note: Reactive VA values are equal.

$$(0.3455)^2\, 2.7455 = 0.328 = 0.164 + 0.164$$

The voltage at node 1:

$$1.50\angle 0 - j1.25 * 0.3455\angle -18.44$$

$$= 1.363 - j0.41 \text{ pu}$$

Node elimination is very important ~~for~~ ~~high~~ when the number of nodes is high. Then, matrix inversion which is very complicated, ~~becomes~~ is avoided.

Elimination: One node at a time, start from the highest node number.

$$Y_{bus} = \left[\begin{array}{cccc|c} Y_{11} & Y_{12} & \cdots & Y_{1j} \cdots & Y_{1n} \\ Y_{21} & Y_{22} & & Y_{2j} & Y_{2n} \\ \vdots & & & & \vdots \\ \hline Y_{n1} & Y_{n2} & \cdots & Y_{nj} & Y_{nn} \end{array}\right]_{n \times n}$$

K, Y, L^T, M

$$\Rightarrow Y_{bus} = \begin{bmatrix} Y_{11} & \cdots Y_{ij} & Y_{1,n-1} \\ \vdots & & \\ Y_{n1} & Y_{nj} \cdots & Y_{n,n-1} \end{bmatrix} - \frac{1}{Y_{nn}} \begin{bmatrix} Y_{1n} \\ Y_{2n} \\ Y_{kn} \\ Y_{n-1,n} \end{bmatrix} \begin{bmatrix} Y_{n1} & Y_{nj} & Y_{n,n-1} \end{bmatrix}$$

$$\Rightarrow Y_{kj(new)} = Y_{kj(orig)} - \frac{Y_{kn} Y_{nj}}{Y_{nn}}$$

Example:

$$Y_{bus} = \left[\begin{array}{ccc|c} -j9.8 & 0.0 & j4.0 & j5.0 \\ 0.0 & -j8.3 & j2.5 & j5.0 \\ j4.0 & j2.5 & -j14.5 & j8. \\ \hline j5 & j5 & j8 & -j18 \end{array}\right]$$

First remove node 4. $(n=4)$

$$Y_{31(new)} = Y_{31(orig)} - \frac{Y_{34} Y_{41}}{Y_{44}} = j4 - \frac{j5 \times j8}{-j18}$$

$$= j\left(4 + \frac{40}{18}\right) = j6.22$$

$$Y_{32}^{new} = j2.5 - \frac{j5 \times j8}{-j18} = j4.72$$

$$Y_{33}^{new} = -j14.5 - \frac{j8 \times j8}{-j18} = -j10.94$$

$$Y_{21}^{new} = 0 - \frac{j5 \times j5}{-j18} = j1.3889$$

$$Y_{22}^{new} = -j8.3 - \frac{j5 \times j5}{-j18} = -j6.91$$

$$Y_{23}^{new} = j2.5 - \frac{j8 \times j5}{-j18} = j4.72$$

$$Y_{11}^{new} = -j9.8 - \frac{j5 \times j5}{-j18} = -j8.41$$

$$Y_{12}^{new} = 0 - j\frac{5 \times j5}{-j18} = j1.389$$

$$Y_{13}^{new} = j4 - j\frac{8 \times j5}{-j18} = \cancel{j1.778}\ j6.22$$

$$\Rightarrow Y_{bus} = \left[\begin{array}{cc|c} -j8.41 & j1.889 & j6.22 \\ j1.3889 & -j6.91 & j4.72 \\ \hline j6.22 & j4.72 & -j10.94 \end{array}\right]$$

Eliminate node 3.

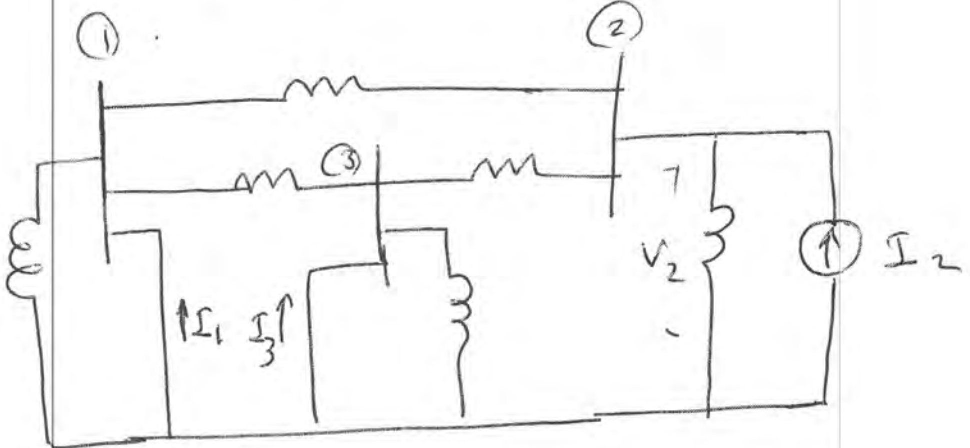
$$\Rightarrow Y_{bus} = \begin{bmatrix} -j4.87 & -j4072 \\ j4.072 & -j4.87 \end{bmatrix}$$

Bus Admittance and Impedance Matrix

$$Z_{bus} = Y_{bus}^{-1}$$

$$Z_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$$ for a 3-node system (3 independent)

$$I = Y_{bus} V$$



For a 3-node system:

$$I_2 = Y_{21} V_1 + Y_{22} V_2 + Y_{23} V_3$$

if $V_1 = V_3 = 0$ (short nodes 1 and 3)
inject I_2 to node 2

$\Rightarrow$ $\cancel{I_2 = Y_{22} V_2}$

$$Y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1 = V_3 = 0}$$

I_{inj}

$$Y_{22} = -\frac{\text{current leaving the node}}{\text{node voltage}}$$

The resultant admittance is the negative of the admittance directly connected between nodes 1 and 2

$$I = Y_{bus} V$$

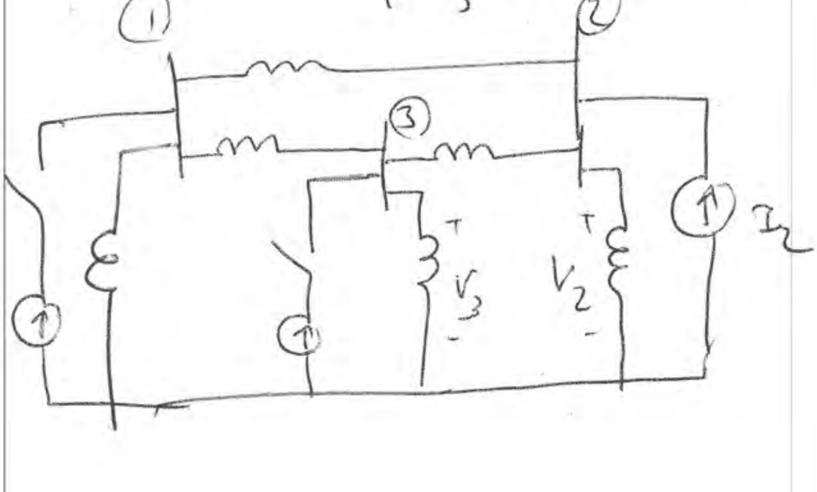
$$Y_{bus}^{-1} I = Z_{bus} I = V$$

V and I are column ~~vectors~~ matrices of the node voltages and currents entering the nodes from current sources.

$$V_1 = Z_{11} I_1 + Z_{12} I_2 + Z_{13} I_3$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 + Z_{23} I_3$$
$$V_3 = Z_{31} I_1 + Z_{32} I_2 + Z_{33} I_3$$

$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1 = I_3 = 0}$$

open current sources 1 and 3, inject I_2 to 2
apply ~~V_2 to node 2~~



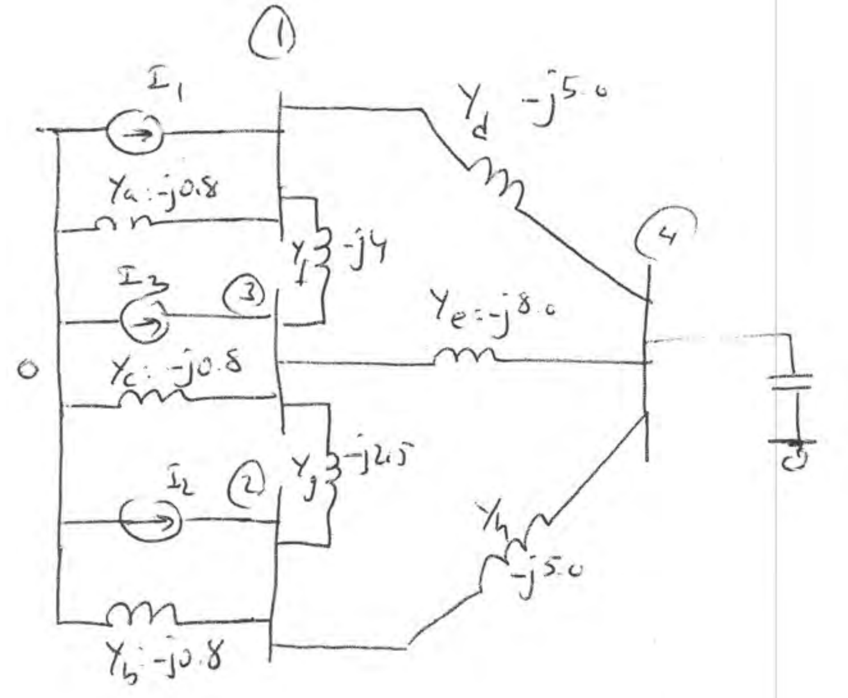
transfer impedances

also $$z_{12} = \frac{V_1}{I_2}\Bigg|_{I_1 = I_3 = 0}$$

$$z_{32} = \frac{V_3}{I_2}\Bigg|_{I_1 = I_3 = 0}$$

Example

$X_C = 5$ pu

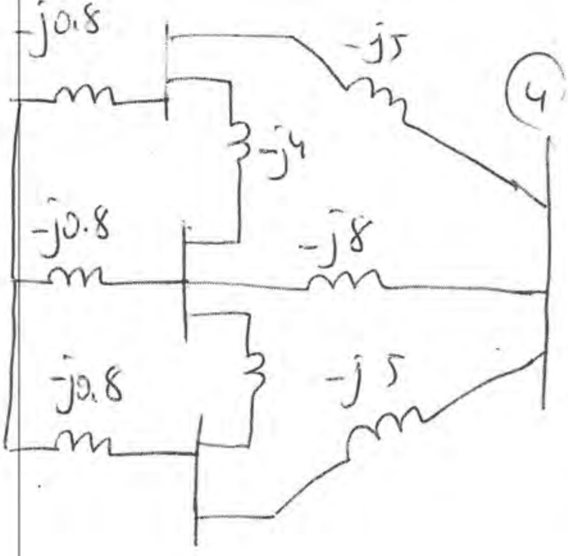


Find the current drawn by the capacitor

Solution: System before the cap was connected:

$E_{th} = 1.432 \angle -11.97$ Thevenin volt.

Thevenin imp.?



$V_4 = Z_{41} I_1 + Z_{42} I_2 + Z_{43} I_3 + Z_{44} I_4$

$I_1 = I_2 = I_3 = 0$

$\Rightarrow \quad Z_{th_4} = Z_{44} = j\,0.4733$ (calculated before)

current drawn by the cap.

$$\Rightarrow I_C = \frac{1.432\angle -11.97^\circ}{j0.4733 - j5} = 0.316\angle 78.03 \text{ pu}$$

Example (Cont'd)

a current of $-0.316\angle 78.03$ p.u. is injected to (4). Find the voltages of (1), (2), (3), (4)

Solution: Node voltages before injection are already known. We need to know the effect of this injected current alone. So, short all emf's, and calculate $Z_{14}, Z_{24}, Z_{34}, Z_{44}$; i.e. the parameters for this node.

$$V_1 = Z_{14} \cdot I_4 = j0.4142 * (-0.316) \angle 78.03$$
$$= 0.1309 \angle -11.97$$

$$V_2 = Z_{24}\, I_4 = 0.1304 \angle -11.97$$
$$V_3 = Z_{34}\, I_4 = 0.1337 \angle -11.97$$
$$V_4 = Z_{44}\, I_4 = 0.1496 \angle -11.97$$

By superposition

$$V_1 = 1.436 \angle -10.71 + 0.1309 \angle -11.97 = 1.567 \angle -10.81$$

$$V_2 = 1.427 \angle -14.24 + 0.1304 \angle -11.97 = 1.557 \angle -14.04$$
$$V_3 = 1.434 \angle -11.36 + 0.1337 \angle -11.97 = 1.568 \angle -11.41$$
$$V_4 = 1.432 \angle -11.97 + 0.1496 \angle -11.97 = 1.582 \angle -11.97$$

if the network is considerably inductive, then ZI may be added to existing voltages without worrying about angles.

* A capacitive addition causes voltage increase in the line.

Modification of an existing Z_{bus}.

$$\left[Z_{orig} \right]_{n \times n}$$

Z_b will be added to Z_{bus}.

h, i, j, k : original
p : new bus

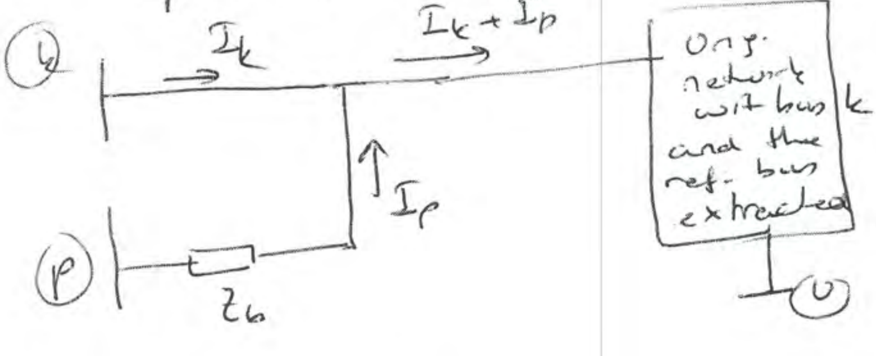
<u>Case 1</u> : Adding Z_b from a new bus p to reference bus through Z_b w/o any connection to other existing buses $\longrightarrow$ original bus voltages don't change. I_p is injected to new bus.

$$\Rightarrow \quad V_p = I_p Z_b$$

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ \hline V_p \end{bmatrix} = \underbrace{\left[\begin{array}{c|c} Z_{orig} & \begin{matrix} 0 \\ 0 \\ \vdots \\ 0 \end{matrix} \\ \hline 0 \; \cdots \; 0 & Z_b \end{array} \right]}_{Z_{bus\,(new)}} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ \hline I_p \end{bmatrix}$$

Case 2 $\quad Z_b$: from p to existing k.

I_p is injected through p



$$V_{k(new)} = V_{k(orig)} + I_p Z_{kk}$$

$$V_p = V_{k(orig)} + I_p Z_{kk} + I_p Z_b$$

$$= \underbrace{I_1 Z_{k1} + I_2 Z_{k2} + \dots + I_n Z_{kn}}_{V_{k(orig)}} + I_p (Z_{kk} + Z_b)$$

$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ \hline V_p \end{bmatrix} = \left[\begin{array}{cccc|c} & & & & Z_{1k} \\ & Z_{orig} & & & Z_{2k} \\ & & & & \vdots \\ & & & & Z_{nk} \\ \hline Z_{k1} & Z_{k2} & \dots & Z_{kn} & Z_{kk}+Z_b \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ \hline I_p \end{bmatrix}$$

Case 3 Add Z_b from bus k to ref. bus

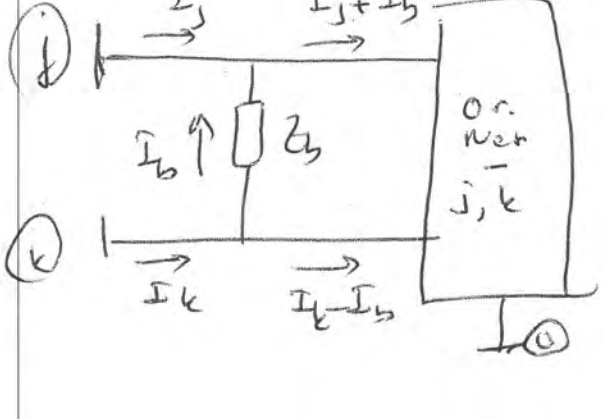
⇒ First ~~create~~ add a new bus p
Then connect p to k through Z_b
Then short p to ground $\Rightarrow V_p = 0$

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ \hline V_p = 0 \end{bmatrix} = \left[\begin{array}{ccc|c} & & & Z_{1k} \\ & Z_{orig} & & \vdots \\ & & & Z_{nk} \\ \hline Z_{k1} & \cdots & Z_{kn} & Z_{kk}+Z_b \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ \hline I_p \end{bmatrix}$$

eliminate.

$$\Rightarrow Z_{hi\,(new)} = Z_{(hi)\,orig} - \frac{Z_{h(n+1)} Z_{(n+1)i}}{Z_{kk} + Z_b}$$

Case 4 Add Z_b between 2 existing buses j and k.



$$V_1 = Z_{11} I_1 + \dots + Z_{1j}(I_j + I_b) + Z_{1k}(I_k - I_b) + \dots$$

$$\Rightarrow V_1 = Z_{11} I_1 + \dots + Z_{1j} I_j + Z_{1k} I_k + \dots + I_b(Z_{1j} - Z_{1k})$$

Similarly:

$$V_j = Z_{j1} I_1 + \dots + Z_{jj} I_j + Z_{jk} I_k + \dots + I_b(Z_{jj} - Z_{jk})$$

$$V_k = Z_{k1} I_1 + \dots + Z_{kj} I_j + Z_{kk} I_k + \dots + I_b(Z_{kj} - Z_{kk})$$

we need one more eqn. (I_b is unknown)

$$V_k - V_j = Z_b I_b$$

or

$$0 = Z_b I_b + V_j - V_k$$

$$\Rightarrow 0 = Z_b I_b + (Z_{j1} - Z_{k1}) I_1 + \dots + (Z_{jj} - Z_{kj}) I_j + (Z_{jk} - Z_{kk}) I_k + \dots + (Z_{jj} + Z_{kk} - 2Z_{jk}) I_b$$

$$\Rightarrow 0 = (Z_{j1} - Z_{k1}) I_1 + \dots + (Z_{jj} - Z_{kj}) I_j + (Z_{jk} - Z_{kk}) I_k + \dots + Z_{bb} I_b$$

where

$$Z_{bb} = Z_b + Z_{jj} + Z_{kk} - 2Z_{jk}$$

$\Rightarrow$

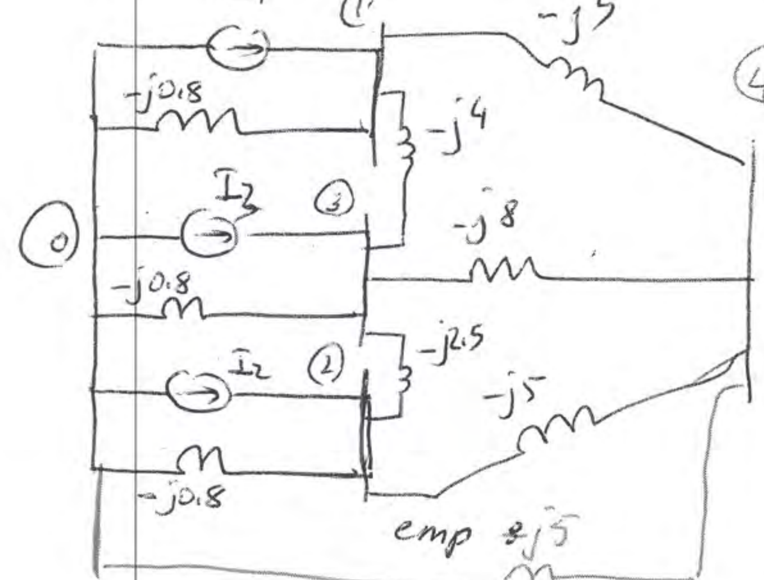
$$\begin{bmatrix} V_1 \\ \vdots \\ V_j \\ V_k \\ \vdots \\ V_n \\ \hline 0 \end{bmatrix} = \left[\begin{array}{ccc|c} & & & Z_{1j}-Z_{1k} \\ & & & \vdots \\ & Z_{orig} & & Z_{jj}-Z_{jk} \\ & & & Z_{kj}-Z_{kk} \\ & & & \vdots \\ & & & Z_{nj}-Z_{nk} \\ \hline (Z_{j1}-Z_{k1}) \ldots & (Z_{jj}-Z_{kj}) & (Z_{jk}-Z_{kk}) \ldots & Z_{bb} \end{array}\right] \begin{bmatrix} I_1 \\ \vdots \\ I_j \\ I_k \\ I_n \\ \hline I_b \end{bmatrix}$$

$Z_{jn}-Z_{kn}$

eliminate $(n+1)^{th}$ row and column.

$$Z_{hi(new)} = Z_{hi(orig)} - \frac{Z_{h(n+1)} Z_{(n+1)i}}{Z_b + Z_{jj} + Z_{kk} - 2Z_{jk}}$$

Example:



Find V_4

$Z_b = -j5$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ 0 \end{bmatrix} = \begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ j0.4142 & j0.4126 & j0.4232 & j0.4733 & \end{bmatrix}$$

$Z_{44} + Z_b = j0.4733 - j5$
$= -j4.5267$

Eliminate 5th row and column:

$$Z_{11} = j0.474 - \frac{j0.4142 * j0.4142}{-j4.5267} = j0.5153$$

~~Z_{24} =~~

~~page 187~~

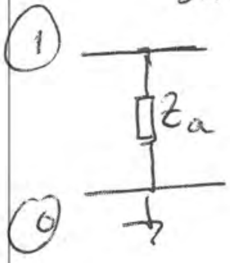
$$\begin{bmatrix} \\ \text{page 186} \\ \\ \end{bmatrix}$$

$\Rightarrow V_4 = 1.5474 - j0.3281 = 1.582 \angle -11.97$ pu

Direct Determination of Z_{bus}

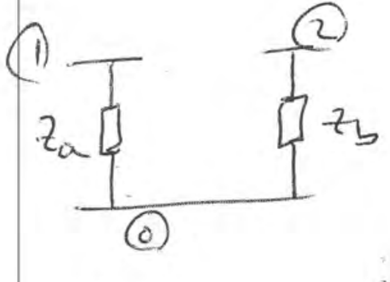
Obtaining Z_{bus} directly is easier than inverting Y_{bus} for large systems.

Start with one impedance



$V_1 = I_1 Z_a$: can be considered as a matrix eqn.

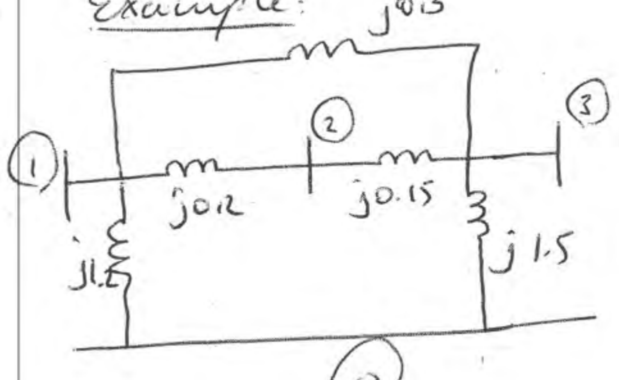
Now connect Z_b between another bus and reference bus.



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_a & 0 \\ 0 & Z_b \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Proceed accordingly.

Example:



Start with ①

$V_1 = j1.2\ I_1 \Rightarrow Z_{bus} = j1.2$

Now, bus ② ($j0.2$ is connected between ① and ②)

$$Z_{bus(new)} = \begin{bmatrix} j1.2 & \overset{Z_{11}}{j1.2} \\ \underset{Z_{11}}{j1.2} & \underset{Z_{11}+Z_b}{j1.4} \end{bmatrix}$$

Between ③ and ① : $j0.3$

$$\Rightarrow Z_{bus(new)} = \begin{bmatrix} j1.2 & j1.2 & j1.2 \; Z_{11} \\ j1.2 & j1.4 & j1.2 \; Z_{21} \\ j1.2 & j1.2 & \underbrace{j1.5}_{\substack{j1.2 + j0.3 \\ Z_{11} + Z_b}} \end{bmatrix}$$

Now: $Z_b = j1.5$ between ③ and ⓪ (Case ③)

$$Z_{bus(new)} = \begin{bmatrix} j1.2 & j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.2 & j1.2 \\ j1.2 & j1.2 & j1.5 & j1.5 \\ \underset{Z_{31}}{j1.2} & \underset{Z_{32}}{j1.2} & \underset{Z_{33}}{j1.5} & j3.0 \end{bmatrix} \longrightarrow Z_{33} + Z_b$$

Node ④ &

Now, eliminate Row 4 and Column 4

~~j0.5 between 2 and 3 case 4~~

$$\cancel{Z_{11} = j1.2}$$

$$Z_{11} = Z_{11(orig)} - \frac{Z_{14} \cdot Z_{41}}{Z_{44}}$$

$$= j1.2 - \frac{j1.2 \times j1.2}{j3} = j0.72$$

$$Z_{12} = Z_{12(orig)} - \frac{Z_{14}\, Z_{42}}{Z_{44}}$$

$$= j1.2 - \frac{j1.2 * j1.2}{j3} = j0.72$$

$$Z_{13} = j\cancel{1.2} - \frac{j1.2 \times j1.5}{j3.0} = j0.6$$

$$Z_{22} = j1.4 - \frac{j1.2 \times j1.2}{j3} = j0.92$$

$$Z_{33} = j1.5 - \frac{j1.5 \times j1.5}{j3} = j0.75$$

$$Z_{bus(new)} = \begin{bmatrix} j0.72 & j0.72 & j0.60 \\ j0.72 & j0.92 & j0.60 \\ j0.60 & j0.60 & j0.75 \end{bmatrix}$$

<u>Now</u>. $Z_b = j0.15$ between (2) and (3).
Case 4!

$j = 2$
$k = 3$

$\Rightarrow Z_{41} = Z_{21} - Z_{31} = j0.12$

$Z_{42} = Z_{22} - Z_{32} = j0.92 - j0.6 = j0.32$

$Z_{43} = -j0.15$

$$\begin{aligned} Z_{44} &= Z_b + Z_{22} + Z_{33} - 2Z_{23} \\ &= j0.15 + j0.92 + j0.75 - 2 * j0.6 \\ &= j0.62 \end{aligned}$$

$$\Rightarrow \left[\begin{array}{ccc|c} j0.72 & j0.72 & j0.6 & j0.12 \\ j0.72 & j0.92 & j0.6 & j0.32 \\ j0.6 & j0.6 & j0.75 & -j0.15 \\ \hline j0.12 & j0.32 & -j0.15 & j0.62 \end{array}\right] \Rightarrow$$

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$Z_{bus(new)} = \cancel{\left[\right.}$

$$Z_{11} = j0.72 - \frac{j0.12 \times j0.12}{j0.62} = j0.6968$$

$Z_{12} = j0.658$

$Z_{13} = j0.629$

$$Z_{bus(new)} = \begin{bmatrix} j0.6968 & j0.6581 & j0.6290 \\ j0.6581 & j0.7548 & j0.6774 \\ j0.6290 & j0.6774 & j0.7137 \end{bmatrix}$$